Computer algebra independent integration tests

3-Logarithms/3.4-u-a+b-log-c-d+e-x^m-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [641]. This is test number [63].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (641)	% 0. (0)
Mathematica	% 96.88 (621)	% 3.12 (20)
Maple	% 52.73 (338)	% 47.27 (303)
Maxima	% 42.59 (273)	% 57.41 (368)
Fricas	% 61.15 (392)	% 38.85 (249)
Sympy	% 18.56 (119)	% 81.44 (522)
Giac	% 52.42 (336)	% 47.58 (305)

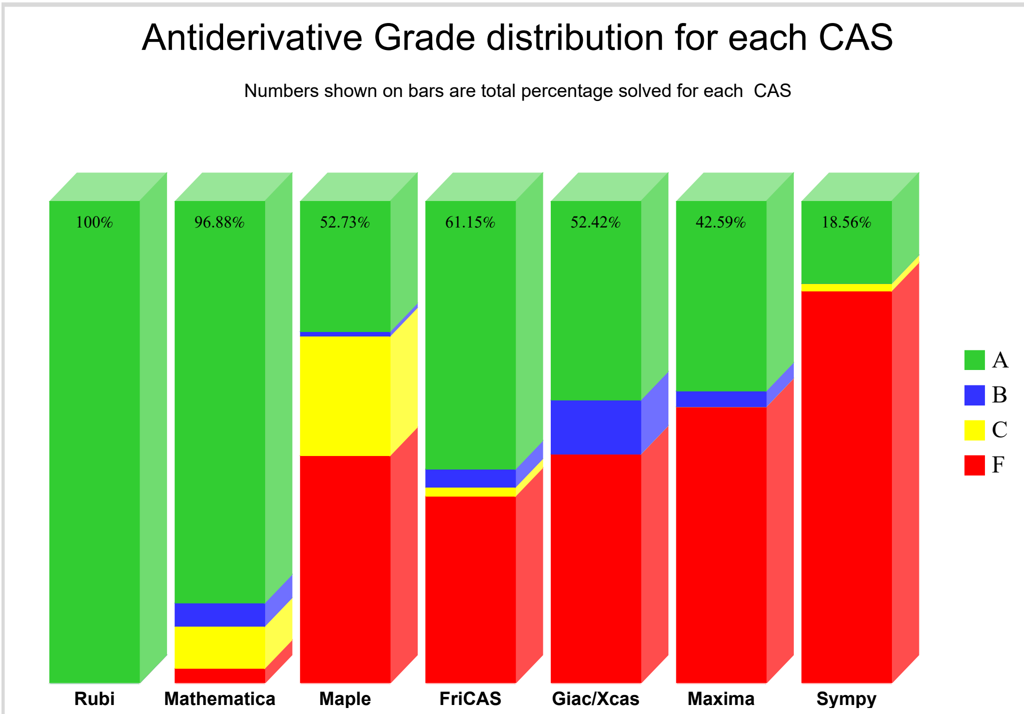
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

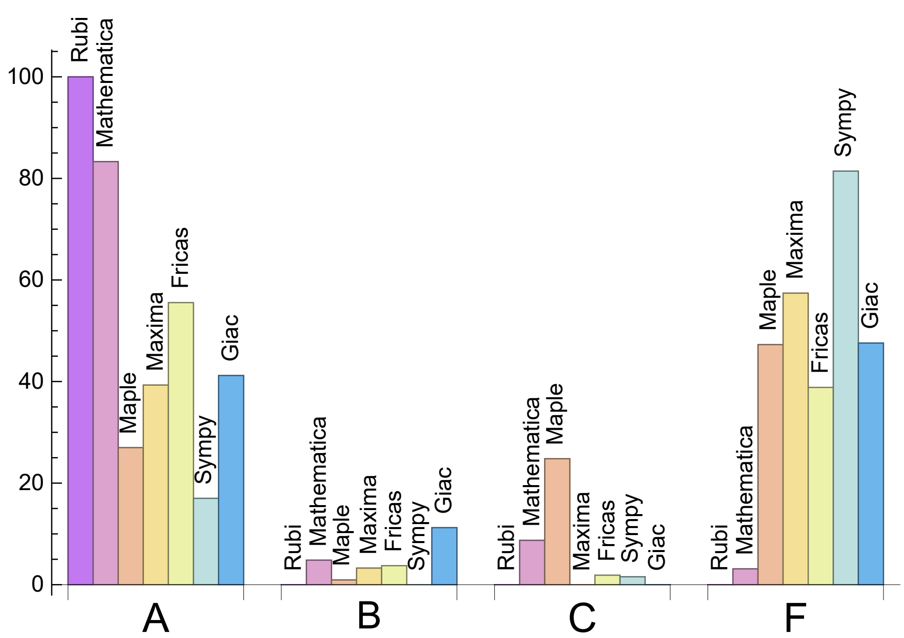
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	83.31	4.84	8.74	3.12
Maple	26.99	0.94	24.8	47.27
Maxima	39.31	3.28	0.	57.41
Fricas	55.54	3.74	1.87	38.85
Sympy	17.	0.	1.56	81.44
Giac	41.19	11.23	0.	47.58

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	220.12	0.79	121.	1.
Mathematica	0.72	237.38	0.94	121.	0.88
Maple	2.16	367.26	2.55	110.5	1.23
Maxima	0.72	152.99	1.06	100.	1.09
Fricas	1.78	583.	2.74	194.	2.3
Sympy	25.04	126.07	1.64	82.	1.37
Giac	0.81	276.83	1.37	96.5	1.2

1.4 list of integrals that has no closed form antiderivative

{98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 381, 382, 383, 384, 385, 386, 387, 388, 398, 485, 486, 487, 488, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 635, 640, 641}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {98, 99, 100, 101, 158, 159, 277, 298, 299, 485, 486, 487, 488, 528, 529, 530, 531}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {99, 158, 159, 168, 262, 271, 277, 292, 298, 299, 343, 344, 345, 346, 347, 376, 487}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

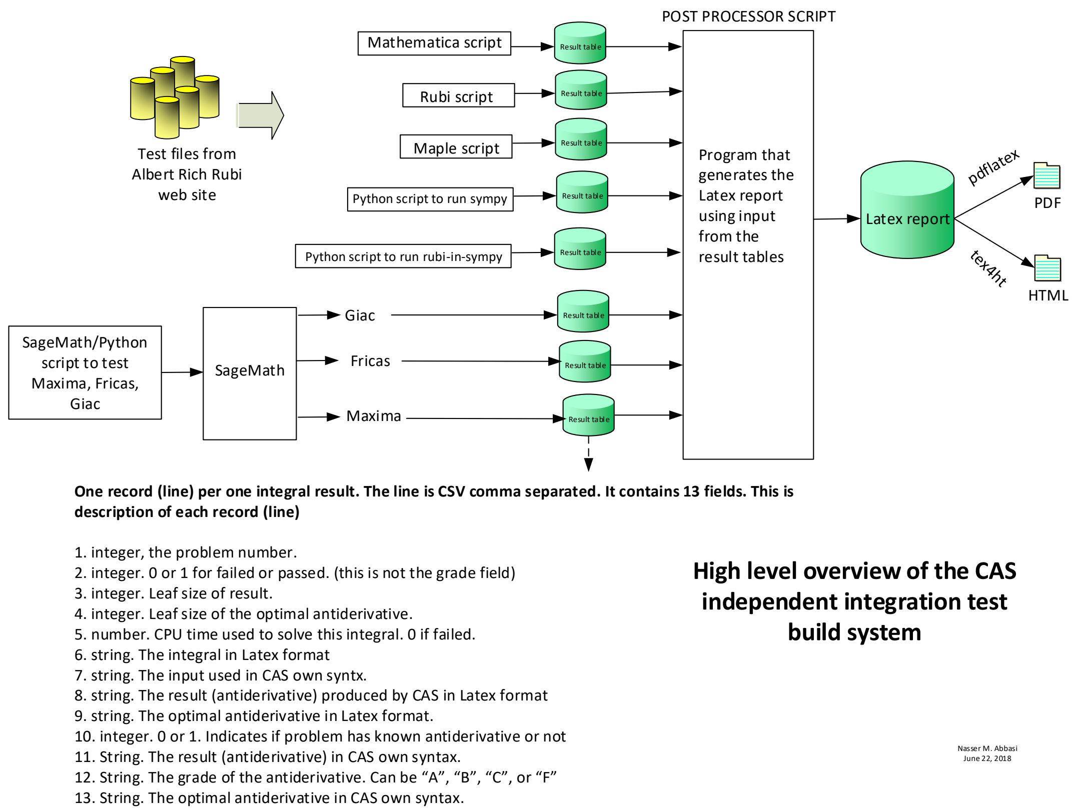
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 15, 16, 18, 19, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 257, 258, 261, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 375, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 469, 471, 472, 474, 475, 476, 477, 478, 481, 482, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 503, 504, 506, 507, 508, 510, 512, 513, 514, 515, 522, 523, 524, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 557, 558, 559, 560, 561, 566, 567, 568, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 612, 613, 616, 617, 618, 619, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 638, 639, 640, 641 }

B grade: { 45, 80, 94, 95, 96, 131, 174, 175, 376, 390, 411, 418, 419, 432, 438, 453, 460, 473, 483, 500, 505, 516, 517, 518, 526, 620, 621, 622, 623, 636, 637 }

C grade: { 9, 11, 14, 17, 20, 23, 24, 36, 38, 89, 90, 133, 137, 191, 192, 193, 196, 197, 233, 234, 238, 239, 247, 254, 255, 256, 259, 260, 266, 267, 321, 322, 335, 336, 337, 347, 359, 433, 434, 435, 468, 470, 479, 480, 501, 502, 509, 511, 519, 520, 521, 609, 610, 614, 615, 634 }

F grade: { 370, 373, 374, 377, 538, 539, 540, 541, 553, 554, 555, 562, 563, 564, 565, 575, 576, 591, 592, 593}

2.1.3 Maple

A grade: { 4, 5, 16, 18, 30, 32, 40, 43, 45, 49, 54, 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 124, 126, 128, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 169, 172, 178, 179, 186, 187, 194, 205, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 298, 299, 302, 303, 304, 305, 306, 307, 308, 309, 358, 359, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 395, 396, 397, 398, 403, 424, 426, 445, 466, 485, 486, 487, 488, 492, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 630, 631, 632, 635, 639, 640, 641 }

B grade: { 170, 171, 391, 394, 511, 633 }

C grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 67, 73, 77, 78, 79, 81, 82, 83, 91, 92, 93, 103, 110, 117, 129, 130, 132, 140, 150, 166, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 195, 196, 197, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 261, 262, 263, 268, 269, 270, 271, 288, 289, 290, 291, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 619, 620, 624, 629, 634 }

F grade: { 26, 27, 28, 29, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 74, 75, 76, 80, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 109, 116, 123, 125, 127, 131, 133, 134, 135, 136, 137, 138, 139, 148, 149, 160, 163, 164, 165, 167, 168, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 215, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 264, 265, 266, 267, 272, 273, 274, 292, 293, 294, 295, 296, 297, 300, 301, 353, 354, 355, 356, 357, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 621, 622, 623, 628, 636, 637, 638 }

2.1.4 Maxima

A grade: { 2, 4, 8, 10, 12, 13, 16, 22, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 37, 39, 43, 46, 47, 48, 49, 51, 52, 53, 54, 77, 78, 79, 81, 82, 83, 91, 92, 93, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 129, 130, 132, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 161, 162, 176, 177, 178, 179, 181, 182, 183, 198, 199, 200, 201, 202, 203, 204, 205, 216, 217, 218, 223, 224, 225, 243, 244, 245, 246, 280, 281, 282, 283, 284, 285, 286, 287, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 315, 316, 317, 323, 324, 325, 329, 330, 331, 341, 342, 350, 351, 352, 358, 371, 372, 375, 376, 393, 396, 398, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 465, 469, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 510, 514, 519, 520, 527, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 631, 632, 635, 639, 640, 641

B grade: { 6, 19, 31, 41, 45, 50, 180, 222, 340, 378, 379, 390, 391, 394, 404, 425, 439, 446, 467, 493, 512 }

C grade: { }

F grade: { 1, 3, 5, 7, 9, 11, 14, 15, 17, 18, 20, 21, 23, 24, 36, 38, 40, 42, 44, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 131, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 313, 314, 318, 319, 320, 321, 322, 326, 327, 328, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 373, 374, 377, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 392, 395, 397, 399, 411, 412, 413, 414, 418, 419, 420, 429, 430, 431, 432, 436, 437, 438, 453, 454, 455, 460, 461, 462, 464, 466, 468, 470, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 485, 486, 487, 488, 497, 498, 499, 500, 503, 504, 505, 509, 511, 513, 515, 516, 517, 518, 521, 522, 523, 524, 525, 526, 528, 529, 530, 531, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 630, 633, 634, 636, 637, 638 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 67, 68, 69, 73, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 167, 168, 169, 172, 173, 177, 178, 179, 181, 184, 185, 186, 187, 189, 194, 198, 199, 200, 202, 211, 216, 217, 218, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 395, 398, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 481, 482, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 501, 502, 507, 508, 509, 510, 513, 514, 515, 519, 520, 527, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 624, 625, 626, 627, 630, 631, 632, 635, 639, 640, 641 }

B grade: { 170, 171, 176, 182, 183, 190, 203, 204, 392, 417, 439, 506, 511, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 633 }

C grade: { 192, 193, 196, 197, 389, 619, 620, 621, 622, 623, 629, 634 }

F grade: { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 166, 174, 175, 180, 188, 191, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 377, 393, 394, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 628, 636, 637, 638 }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 16, 22, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 47, 48, 49, 51, 54, 58, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 120, 121, 122, 124, 126, 128, 129, 130, 144, 145, 154, 155, 176, 177, 178, 179, 184, 185, 186, 187, 194, 198, 199, 200, 216, 270, 275, 278, 280, 281, 284, 285, 290, 303, 307, 312, 319, 320, 321, 386, 401, 402, 403, 405, 424, 444, 445, 466, 492, 511, 631, 632, 639 }

B grade: { }

C grade: { 45, 71, 72, 74, 169, 170, 171, 213, 214, 215 }

F grade: { 1, 6, 11, 14, 15, 17, 18, 19, 20, 21, 23, 24, 31, 36, 41, 46, 50, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 119, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 180, 181, 182, 183, 188, 189, 190, 191, 192, 193, 195, 196, 197, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 276, 277, 279, 282, 283, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 633, 634, 635, 636, 637, 638, 640, 641 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 44, 54, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 178, 179, 181, 184, 185, 186, 187, 189, 192, 193, 194, 196, 200, 202, 216, 217, 218, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 318, 319, 320, 321, 322, 332, 333, 334, 335, 336, 337, 381, 382, 383, 384, 385, 386, 387, 388, 398, 421, 422, 423, 424, 426, 427, 428, 464, 465, 466, 468, 469, 470, 472, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 508, 509, 510, 511, 513, 514, 515, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 625, 626, 627, 630, 631, 632, 635, 639, 640, 641 }

B grade: { 10, 12, 25, 43, 46, 47, 48, 49, 51, 52, 53, 109, 116, 117, 148, 149, 176, 177, 182, 183, 190, 191, 197, 198, 199, 203, 204, 310, 311, 315, 316, 317, 323, 324, 325, 329, 330, 331, 350, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 471, 481, 482, 629, 633 }

C grade: { }

F grade: { 6, 19, 31, 41, 45, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 512, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 634, 636, 637, 638 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	74	229	0	436	0	96
normalized size	1	1.	0.92	2.86	0.	5.45	0.	1.2
time (sec)	N/A	0.047	0.044	0.499	0.	2.103	0.	1.212

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	183	74	127	70	131
normalized size	1	1.	1.	3.1	1.25	2.15	1.19	2.22
time (sec)	N/A	0.049	0.015	0.414	1.104	2.021	8.442	1.22

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	62	217	0	350	121	80
normalized size	1	1.	0.94	3.29	0.	5.3	1.83	1.21
time (sec)	N/A	0.037	0.023	0.421	0.	1.986	56.764	1.192

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	50	59	89	56	58
normalized size	1	1.	0.97	1.43	1.69	2.54	1.6	1.66
time (sec)	N/A	0.024	0.009	0.097	1.097	1.947	3.694	1.182

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	0	250	90	55
normalized size	1	1.	1.	0.84	0.	5.56	2.	1.22
time (sec)	N/A	0.018	0.012	0.066	0.	2.065	15.841	1.193

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	232	108	0	0	0
normalized size	1	1.	0.98	5.27	2.45	0.	0.	0.
time (sec)	N/A	0.046	0.007	0.434	1.228	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	183	0	228	377	54
normalized size	1	1.	1.	4.16	0.	5.18	8.57	1.23
time (sec)	N/A	0.02	0.009	0.333	0.	2.057	44.26	1.168

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	45	45	173	59	103	82	78
normalized size	1	1.18	1.18	4.55	1.55	2.71	2.16	2.05
time (sec)	N/A	0.037	0.003	0.279	1.129	2.029	6.96	1.237

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	49	211	0	311	774	78
normalized size	1	1.	0.82	3.52	0.	5.18	12.9	1.3
time (sec)	N/A	0.03	0.003	0.34	0.	1.859	178.799	1.228

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	198	73	134	102	178
normalized size	1	1.	0.88	3.09	1.14	2.09	1.59	2.78
time (sec)	N/A	0.052	0.037	0.295	1.07	2.09	21.474	1.282

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	49	235	0	390	0	96
normalized size	1	1.	0.66	3.18	0.	5.27	0.	1.3
time (sec)	N/A	0.037	0.003	0.362	0.	2.077	0.	1.284

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	206	93	163	116	258
normalized size	1	1.	0.87	2.64	1.19	2.09	1.49	3.31
time (sec)	N/A	0.06	0.065	0.286	1.045	2.062	54.435	1.176

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	183	74	128	70	131
normalized size	1	1.	1.	3.1	1.25	2.17	1.19	2.22
time (sec)	N/A	0.049	0.013	0.556	1.173	1.872	34.772	1.164

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	69	196	0	406	0	219
normalized size	1	1.	0.43	1.23	0.	2.55	0.	1.38
time (sec)	N/A	0.129	0.003	0.452	0.	2.148	0.	1.273

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	147	194	0	356	0	216
normalized size	1	1.	0.94	1.24	0.	2.27	0.	1.38
time (sec)	N/A	0.114	0.05	0.513	0.	2.044	0.	1.286

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	50	59	89	56	58
normalized size	1	1.	0.97	1.43	1.69	2.54	1.6	1.66
time (sec)	N/A	0.029	0.01	0.062	1.157	1.922	7.53	1.273

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	53	184	0	381	0	203
normalized size	1	1.	0.36	1.25	0.	2.59	0.	1.38
time (sec)	N/A	0.085	0.003	0.518	0.	2.089	0.	1.25

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	129	113	0	288	0	193
normalized size	1	1.	0.97	0.85	0.	2.17	0.	1.45
time (sec)	N/A	0.082	0.037	0.062	0.	2.083	0.	1.199

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	180	108	0	0	0
normalized size	1	1.	0.98	4.09	2.45	0.	0.	0.
time (sec)	N/A	0.048	0.007	0.431	1.071	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	47	184	0	311	0	186
normalized size	1	1.	0.35	1.38	0.	2.34	0.	1.4
time (sec)	N/A	0.078	0.003	0.317	0.	2.017	0.	1.248

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	134	197	0	366	0	186
normalized size	1	1.	0.96	1.42	0.	2.63	0.	1.34
time (sec)	N/A	0.075	0.033	0.322	0.	2.018	0.	1.307

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	173	59	103	82	78
normalized size	1	1.	1.	3.84	1.31	2.29	1.82	1.73
time (sec)	N/A	0.039	0.003	0.27	1.085	1.907	22.911	1.221

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	49	215	0	348	0	207
normalized size	1	1.	0.32	1.42	0.	2.3	0.	1.37
time (sec)	N/A	0.095	0.003	0.323	0.	2.454	0.	1.263

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	49	216	0	412	0	208
normalized size	1	1.	0.32	1.43	0.	2.73	0.	1.38
time (sec)	N/A	0.091	0.003	0.322	0.	2.373	0.	1.176

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	198	73	134	102	178
normalized size	1	1.	0.88	3.09	1.14	2.09	1.59	2.78
time (sec)	N/A	0.051	0.037	0.273	1.158	2.224	80.932	1.321

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	0	100	207	122	117
normalized size	1	1.	0.96	0.	1.12	2.33	1.37	1.31
time (sec)	N/A	0.055	0.046	0.358	1.061	2.334	34.223	1.33

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	86	177	109	101
normalized size	1	1.	0.99	0.	1.15	2.36	1.45	1.35
time (sec)	N/A	0.04	0.032	0.236	1.21	2.283	16.462	1.299

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	62	0	69	149	95	85
normalized size	1	1.	1.02	0.	1.13	2.44	1.56	1.39
time (sec)	N/A	0.032	0.024	0.068	1.205	2.149	6.513	1.29

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	0	54	116	82	69
normalized size	1	1.	0.85	0.	1.15	2.47	1.74	1.47
time (sec)	N/A	0.021	0.017	0.081	1.169	2.126	4.218	1.318

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	37	30	36	81	48	43
normalized size	1	1.	1.37	1.11	1.33	3.	1.78	1.59
time (sec)	N/A	0.009	0.002	0.06	1.144	2.205	1.318	1.277

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	112	0	0	0
normalized size	1	1.	1.02	0.	2.8	0.	0.	0.
time (sec)	N/A	0.037	0.003	0.079	1.224	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	48	68	77	39	69
normalized size	1	1.	1.	1.6	2.27	2.57	1.3	2.3
time (sec)	N/A	0.021	0.004	0.058	1.085	2.156	2.927	1.337

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	85	124	66	95
normalized size	1	1.	1.	0.	1.44	2.1	1.12	1.61
time (sec)	N/A	0.035	0.014	0.273	1.056	2.222	6.976	1.321

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	100	151	80	113
normalized size	1	1.	1.	0.	1.37	2.07	1.1	1.55
time (sec)	N/A	0.05	0.017	0.07	1.096	2.272	11.248	1.283

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	115	181	94	130
normalized size	1	1.	1.	0.	1.32	2.08	1.08	1.49
time (sec)	N/A	0.056	0.021	0.069	1.09	2.303	73.274	1.2

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	49	0	0	401	0	101
normalized size	1	1.	0.68	0.	0.	5.57	0.	1.4
time (sec)	N/A	0.038	0.006	0.333	0.	2.324	0.	1.198

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	56	0	59	127	87	80
normalized size	1	1.	1.1	0.	1.16	2.49	1.71	1.57
time (sec)	N/A	0.034	0.02	0.245	1.08	2.321	29.732	1.211

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	320	146	85
normalized size	1	1.	0.81	0.	0.	5.52	2.52	1.47
time (sec)	N/A	0.027	0.002	0.247	0.	2.216	85.137	1.178

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	0	45	100	71	63
normalized size	1	1.	1.22	0.	1.22	2.7	1.92	1.7
time (sec)	N/A	0.014	0.003	0.245	1.189	2.207	11.847	1.17

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	43	38	0	244	109	57
normalized size	1	1.	1.05	0.93	0.	5.95	2.66	1.39
time (sec)	N/A	0.015	0.008	0.063	0.	2.193	24.001	1.249

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	120	0	0	0
normalized size	1	1.	1.02	0.	2.73	0.	0.	0.
time (sec)	N/A	0.04	0.003	0.276	1.149	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	52	0	0	261	129	73
normalized size	1	1.	1.04	0.	0.	5.22	2.58	1.46
time (sec)	N/A	0.027	0.014	0.235	0.	2.273	59.999	1.26

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	50	73	93	58	88
normalized size	1	1.	0.97	1.43	2.09	2.66	1.66	2.51
time (sec)	N/A	0.026	0.008	0.059	1.123	2.119	11.744	1.278

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	362	177	99
normalized size	1	1.	1.03	0.	0.	5.32	2.6	1.46
time (sec)	N/A	0.036	0.021	0.273	0.	2.31	126.087	1.289

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	34	9	47	31	8	0
normalized size	1	1.	4.25	1.12	5.88	3.88	1.	0.
time (sec)	N/A	0.008	0.003	0.059	1.043	2.245	4.317	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	134	0	162	313	0	601
normalized size	1	1.	0.88	0.	1.06	2.05	0.	3.93
time (sec)	N/A	0.118	0.131	0.299	1.101	2.466	0.	1.304

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	112	0	132	248	119	450
normalized size	1	1.	0.91	0.	1.07	2.02	0.97	3.66
time (sec)	N/A	0.088	0.053	0.062	1.073	2.364	162.442	1.237

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	103	190	92	298
normalized size	1	1.	0.95	0.	1.11	2.04	0.99	3.2
time (sec)	N/A	0.06	0.036	0.063	1.06	2.364	8.125	1.306

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	68	130	61	131
normalized size	1	1.	1.	0.87	1.28	2.45	1.15	2.47
time (sec)	N/A	0.028	0.024	0.065	1.057	2.423	2.1	1.307

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	0	107	0	0	0
normalized size	1	1.	1.02	0.	2.33	0.	0.	0.
time (sec)	N/A	0.04	0.003	0.266	1.068	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	55	0	72	136	406	178
normalized size	1	1.	0.87	0.	1.14	2.16	6.44	2.83
time (sec)	N/A	0.045	0.04	0.065	1.035	2.263	91.81	1.202

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	0	103	208	0	313
normalized size	1	1.	0.9	0.	1.03	2.08	0.	3.13
time (sec)	N/A	0.063	0.044	0.27	1.062	2.206	0.	1.3

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	0	132	269	0	437
normalized size	1	1.	0.88	0.	1.02	2.07	0.	3.36
time (sec)	N/A	0.079	0.063	0.067	1.125	2.271	0.	1.328

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	40	42	73	133	42
normalized size	1	1.	1.03	1.25	1.31	2.28	4.16	1.31
time (sec)	N/A	0.018	0.009	0.06	1.006	2.089	1.288	1.249

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.027	1.043	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.027	1.053	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	56	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.025	0.851	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	201	0
normalized size	1	1.	0.84	0.	0.	0.	3.	0.
time (sec)	N/A	0.041	0.016	2.666	0.	0.	52.695	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.03	3.868	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.031	4.389	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	76	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.034	0.328	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	77	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.037	0.389	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.037	2.053	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	92	0	0	254	0	0
normalized size	1	1.	0.65	0.	0.	1.8	0.	0.
time (sec)	N/A	0.077	0.077	1.82	0.	2.172	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	74	0	0	204	0	0
normalized size	1	1.	0.66	0.	0.	1.82	0.	0.
time (sec)	N/A	0.057	0.041	1.887	0.	2.093	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	48	0	0	127	0	0
normalized size	1	1.	0.7	0.	0.	1.84	0.	0.
time (sec)	N/A	0.043	0.033	1.869	0.	2.013	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	201	0	155	0	0
normalized size	1	1.	0.92	4.02	0.	3.1	0.	0.
time (sec)	N/A	0.047	0.008	2.547	0.	2.071	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	57	0	0	159	0	0
normalized size	1	1.	0.71	0.	0.	1.99	0.	0.
time (sec)	N/A	0.036	0.018	1.819	0.	2.077	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	76	0	0	240	0	0
normalized size	1	1.	0.63	0.	0.	2.	0.	0.
time (sec)	N/A	0.058	0.048	1.854	0.	2.072	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.031	1.69	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	104	0
normalized size	1	1.	0.94	0.	0.	0.	1.6	0.
time (sec)	N/A	0.024	0.032	1.892	0.	0.	38.355	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	48	0
normalized size	1	1.	0.96	0.	0.	0.	0.89	0.
time (sec)	N/A	0.02	0.029	1.6	0.	0.	4.677	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	177	0	150	0	0
normalized size	1	1.	0.98	4.02	0.	3.41	0.	0.
time (sec)	N/A	0.04	0.003	2.52	0.	2.048	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	59	0	0	0	46	0
normalized size	1	1.	0.89	0.	0.	0.	0.7	0.
time (sec)	N/A	0.032	0.032	1.523	0.	0.	14.765	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	62	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.027	1.554	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.026	1.529	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	175	200	1436	196	412	267	439
normalized size	1	0.81	0.93	6.68	0.91	1.92	1.24	2.04
time (sec)	N/A	0.299	0.067	0.519	1.061	2.083	87.557	1.273

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	105	1242	162	316	209	279
normalized size	1	1.	0.72	8.57	1.12	2.18	1.44	1.92
time (sec)	N/A	0.153	0.056	0.492	1.051	1.909	19.35	1.232

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	63	1034	131	216	139	130
normalized size	1	1.	1.03	16.95	2.15	3.54	2.28	2.13
time (sec)	N/A	0.05	0.009	0.497	1.031	1.973	3.583	1.286

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	163	0	0	0	0	0
normalized size	1	1.	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.06	0.849	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	93	841	159	0	0	0
normalized size	1	1.	1.16	10.51	1.99	0.	0.	0.
time (sec)	N/A	0.084	0.024	0.494	1.069	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	147	137	1080	192	0	0	0
normalized size	1	1.14	1.06	8.37	1.49	0.	0.	0.
time (sec)	N/A	0.269	0.079	0.5	1.117	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	211	205	1289	234	0	0	0
normalized size	1	1.09	1.06	6.68	1.21	0.	0.	0.
time (sec)	N/A	0.407	0.054	0.516	1.055	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	248	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.408	0.195	0.576	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	223	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.323	0.135	0.984	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	193	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.267	0.081	0.844	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	173	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.052	0.884	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	207	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	0.091	0.964	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	277	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.32	0.272	0.963	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	334	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.377	0.224	0.945	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	309	5905	323	780	561	803
normalized size	1	1.	0.93	17.68	0.97	2.34	1.68	2.4
time (sec)	N/A	0.358	0.195	0.941	1.12	1.81	50.409	1.256

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	237	4942	274	587	450	486
normalized size	1	1.	1.12	23.42	1.3	2.78	2.13	2.3
time (sec)	N/A	0.205	0.086	0.87	1.062	2.185	24.007	1.309

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	87	3925	221	393	301	228
normalized size	1	1.	0.94	42.2	2.38	4.23	3.24	2.45
time (sec)	N/A	0.066	0.012	0.843	1.094	1.861	7.32	1.285

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	279	0	0	0	0	0
normalized size	1	1.	2.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.097	0.868	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	302	0	0	0	0	0
normalized size	1	1.	2.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.287	1.296	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	236	478	0	0	0	0	0
normalized size	1	1.08	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.434	0.344	1.501	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	331	571	0	0	0	0	0
normalized size	1	0.94	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.745	0.393	1.332	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	379	0	909	0	0	0	0	0
normalized size	1	0.	2.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.834	3.753	13.369	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	289	0	789	0	0	0	0	0
normalized size	1	0.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.434	3.356	0.866	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	50	0	505	0	0	0	0	0
normalized size	1	0.	10.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.781	2.409	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	253	0	851	0	0	0	0	0
normalized size	1	0.	3.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	2.622	6.689	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	96	0	0	162	0	99
normalized size	1	1.	0.9	0.	0.	1.51	0.	0.93
time (sec)	N/A	0.154	0.139	0.484	0.	2.268	0.	1.279

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	317	0	72	0	42
normalized size	1	1.	1.	6.22	0.	1.41	0.	0.82
time (sec)	N/A	0.058	0.041	1.23	0.	2.235	0.	1.284

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.18	0.49	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.33	0.507	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.287	0.464	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.004	0.009	0.428	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.397	0.472	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	157	0	0	338	0	402
normalized size	1	1.	1.14	0.	0.	2.45	0.	2.91
time (sec)	N/A	0.207	0.153	5.273	0.	2.282	0.	1.354

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	97	466	0	194	0	190
normalized size	1	1.	1.17	5.61	0.	2.34	0.	2.29
time (sec)	N/A	0.072	0.044	1.129	0.	2.216	0.	1.265

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.277	1.759	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	1.46	3.458	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.331	3.284	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.004	0.366	3.365	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	1.2	3.452	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	185	0	0	629	0	1134
normalized size	1	1.	0.91	0.	0.	3.08	0.	5.56
time (sec)	N/A	0.289	0.193	5.264	0.	2.205	0.	1.248

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	113	761	0	378	0	548
normalized size	1	1.	0.99	6.68	0.	3.32	0.	4.81
time (sec)	N/A	0.093	0.054	1.2	0.	2.256	0.	1.282

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.551	3.704	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	2.835	3.746	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.537	3.486	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.004	0.462	3.435	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	2.158	3.457	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	0	0	140	0	51
normalized size	1	1.	0.91	0.	0.	3.11	0.	1.13
time (sec)	N/A	0.095	0.085	0.352	0.	1.873	0.	1.187

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	53	27	26
normalized size	1	1.	1.	1.15	0.	2.65	1.35	1.3
time (sec)	N/A	0.027	0.014	0.074	0.	1.912	2.794	1.316

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	0	0	235	0	120
normalized size	1	1.	0.93	0.	0.	3.31	0.	1.69
time (sec)	N/A	0.125	0.107	0.103	0.	1.943	0.	1.218

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	59	0	130	49	61
normalized size	1	1.	0.91	1.26	0.	2.77	1.04	1.3
time (sec)	N/A	0.043	0.019	0.07	0.	1.948	2.807	1.277

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	87	0	0	325	0	190
normalized size	1	1.	0.69	0.	0.	2.56	0.	1.5
time (sec)	N/A	0.17	0.12	0.088	0.	2.058	0.	1.285

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	94	0	185	70	92
normalized size	1	1.	0.75	1.29	0.	2.53	0.96	1.26
time (sec)	N/A	0.059	0.023	0.068	0.	1.906	2.98	1.283

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	105	1242	162	317	206	298
normalized size	1	1.	0.7	8.28	1.08	2.11	1.37	1.99
time (sec)	N/A	0.156	0.064	0.681	1.071	2.041	54.527	1.22

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	1036	131	216	160	140
normalized size	1	1.	0.95	15.7	1.98	3.27	2.42	2.12
time (sec)	N/A	0.056	0.01	0.556	1.064	1.889	12.317	1.25

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	163	0	0	0	0	0
normalized size	1	1.	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.097	0.908	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	771	159	0	0	0
normalized size	1	1.	0.98	8.97	1.85	0.	0.	0.
time (sec)	N/A	0.084	0.037	0.619	1.088	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1294	1300	823	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	1.92	1.302	0.672	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1304	1310	1090	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	1.791	0.653	1.059	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1137	1143	742	0	0	0	0	0
normalized size	1	1.01	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	1.328	0.838	1.021	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1170	1176	745	0	0	0	0	0
normalized size	1	1.01	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	1.344	0.838	1.02	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1328	1334	847	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	1.723	1.6	1.063	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	146	0	0	271	0	146
normalized size	1	1.	0.89	0.	0.	1.65	0.	0.89
time (sec)	N/A	0.237	0.231	0.621	0.	1.905	0.	1.327

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	96	0	0	162	0	97
normalized size	1	1.	0.9	0.	0.	1.51	0.	0.91
time (sec)	N/A	0.147	0.115	0.553	0.	1.84	0.	1.273

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	317	0	72	0	42
normalized size	1	1.	1.	6.22	0.	1.41	0.	0.82
time (sec)	N/A	0.062	0.041	1.246	0.	1.97	0.	1.256

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.18	0.454	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.316	0.514	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.26	0.501	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.226	0.455	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.004	0.009	0.445	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.367	0.484	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.383	0.496	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	290	0	0	494	0	667
normalized size	1	1.	1.49	0.	0.	2.53	0.	3.42
time (sec)	N/A	0.381	0.26	3.678	0.	1.851	0.	1.294

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	157	0	0	338	0	436
normalized size	1	1.	1.11	0.	0.	2.4	0.	3.09
time (sec)	N/A	0.208	0.134	5.191	0.	1.971	0.	1.311

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	97	466	0	194	0	208
normalized size	1	1.	1.17	5.61	0.	2.34	0.	2.51
time (sec)	N/A	0.082	0.045	1.288	0.	1.986	0.	1.286

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.293	1.929	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	1.435	3.464	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.332	3.507	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.48	3.579	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.004	0.359	3.554	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	1.158	3.679	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	1.167	3.719	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	76	0	994	0	0	0	0	0
normalized size	1	0.	13.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	2.271	0.897	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	74	0	466	0	0	0	0	0
normalized size	1	0.	6.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	1.03	0.957	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.024	0.063	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.334	0.957	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.529	5.842	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	278	171	0	0	603	0	0
normalized size	1	0.75	0.46	0.	0.	1.62	0.	0.
time (sec)	N/A	0.321	0.153	2.118	0.	2.126	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	140	0	0	459	0	0
normalized size	1	1.	0.55	0.	0.	1.8	0.	0.
time (sec)	N/A	0.187	0.111	2.066	0.	2.093	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	0	282	0	0
normalized size	1	1.	0.73	0.	0.	2.79	0.	0.
time (sec)	N/A	0.081	0.025	2.097	0.	2.099	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	168	1473	0	0	0	0
normalized size	1	1.	1.91	16.74	0.	0.	0.	0.
time (sec)	N/A	0.114	0.099	4.615	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	148	0	0	446	0	0
normalized size	1	1.	1.19	0.	0.	3.6	0.	0.
time (sec)	N/A	0.111	0.089	2.043	0.	2.444	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	238	288	0	0	645	0	0
normalized size	1	1.19	1.44	0.	0.	3.22	0.	0.
time (sec)	N/A	0.316	0.262	1.978	0.	2.234	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	24	14	0
normalized size	1	1.	1.	1.08	0.	1.85	1.08	0.
time (sec)	N/A	0.009	0.003	0.07	0.	2.018	4.063	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	56	0	107	78	0
normalized size	1	1.	1.	2.67	0.	5.1	3.71	0.
time (sec)	N/A	0.027	0.003	0.072	0.	2.015	4.608	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	57	0	109	78	0
normalized size	1	1.	1.	2.71	0.	5.19	3.71	0.
time (sec)	N/A	0.028	0.004	0.069	0.	2.123	4.75	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	41	0	123	0	0
normalized size	1	1.	0.95	1.	0.	3.	0.	0.
time (sec)	N/A	0.039	0.008	0.114	0.	2.138	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	177	0	150	0	0
normalized size	1	1.	0.98	4.02	0.	3.41	0.	0.
time (sec)	N/A	0.04	0.005	0.079	0.	2.104	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	164	1356	0	0	0	0
normalized size	1	1.	2.08	17.16	0.	0.	0.	0.
time (sec)	N/A	0.1	0.058	4.447	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	270	6131	0	0	0	0
normalized size	1	1.	2.39	54.26	0.	0.	0.	0.
time (sec)	N/A	0.148	0.1	4.84	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	185	766	289	559	369	753
normalized size	1	1.	1.32	5.47	2.06	3.99	2.64	5.38
time (sec)	N/A	0.077	0.203	0.611	1.068	1.996	6.666	1.162

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	121	537	184	369	223	423
normalized size	1	1.	1.08	4.79	1.64	3.29	1.99	3.78
time (sec)	N/A	0.073	0.106	0.51	1.104	2.017	3.182	1.274

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	83	100	205	116	192
normalized size	1	1.	0.98	0.99	1.19	2.44	1.38	2.29
time (sec)	N/A	0.037	0.046	0.091	1.103	2.023	1.427	1.274

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	47	73	37	53
normalized size	1	1.	1.	1.25	1.96	3.04	1.54	2.21
time (sec)	N/A	0.009	0.006	0.069	1.043	1.906	0.54	1.181

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	242	159	0	0	0
normalized size	1	1.	0.98	4.17	2.74	0.	0.	0.
time (sec)	N/A	0.05	0.004	0.633	1.06	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	329	88	176	0	123
normalized size	1	1.	0.76	4.84	1.29	2.59	0.	1.81
time (sec)	N/A	0.027	0.048	0.358	1.018	1.969	0.	1.198

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	582	162	489	0	359
normalized size	1	1.	0.76	5.54	1.54	4.66	0.	3.42
time (sec)	N/A	0.059	0.082	0.375	1.076	1.996	0.	1.19

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	105	873	313	894	0	668
normalized size	1	1.	0.79	6.56	2.35	6.72	0.	5.02
time (sec)	N/A	0.077	0.131	0.393	1.104	2.379	0.	1.226

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	249	1330	0	1044	422	383
normalized size	1	1.	1.4	7.47	0.	5.87	2.37	2.15
time (sec)	N/A	0.163	0.731	0.796	0.	2.096	82.928	1.267

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	211	965	0	709	309	247
normalized size	1	1.	1.5	6.84	0.	5.03	2.19	1.75
time (sec)	N/A	0.132	0.456	0.72	0.	2.054	39.445	1.21

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	83	93	0	455	160	135
normalized size	1	1.	0.84	0.94	0.	4.6	1.62	1.36
time (sec)	N/A	0.079	0.026	0.078	0.	2.212	19.996	1.281

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	0	250	90	55
normalized size	1	1.	1.	0.84	0.	5.56	2.	1.22
time (sec)	N/A	0.019	0.013	0.073	0.	2.391	10.002	1.273

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	201	366	0	0	0	0
normalized size	1	1.	1.	1.82	0.	0.	0.	0.
time (sec)	N/A	0.266	0.081	0.511	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	137	755	0	582	0	213
normalized size	1	1.	1.15	6.34	0.	4.89	0.	1.79
time (sec)	N/A	0.09	0.072	0.548	0.	2.556	0.	1.223

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	217	3183	0	1524	0	567
normalized size	1	1.	1.25	18.29	0.	8.76	0.	3.26
time (sec)	N/A	0.145	0.589	0.504	0.	3.029	0.	1.332

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	264	738	0	0	0	753
normalized size	1	1.	0.82	2.31	0.	0.	0.	2.35
time (sec)	N/A	0.742	0.485	0.783	0.	0.	0.	1.342

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	218	537	0	12299	0	509
normalized size	1	1.	0.87	2.15	0.	49.2	0.	2.04
time (sec)	N/A	0.483	0.304	0.733	0.	44.593	0.	1.225

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	204	335	0	5179	0	406
normalized size	1	1.	0.89	1.46	0.	22.62	0.	1.77
time (sec)	N/A	0.317	0.068	0.717	0.	13.255	0.	1.295

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	129	113	0	288	236	193
normalized size	1	1.	0.97	0.85	0.	2.17	1.77	1.45
time (sec)	N/A	0.087	0.039	0.072	0.	1.976	177.527	1.686

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	313	261	0	0	0	0
normalized size	1	1.	1.02	0.85	0.	0.	0.	0.
time (sec)	N/A	0.517	0.15	0.589	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	202	1068	0	13520	0	537
normalized size	1	1.	0.69	3.66	0.	46.3	0.	1.84
time (sec)	N/A	0.549	0.646	0.539	0.	14.158	0.	1.339

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	C	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	303	4085	0	26734	0	1079
normalized size	1	1.	0.77	10.45	0.	68.37	0.	2.76
time (sec)	N/A	0.713	0.706	0.591	0.	40.571	0.	1.754

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	114	0	224	498	484	437
normalized size	1	1.	0.82	0.	1.61	3.58	3.48	3.14
time (sec)	N/A	0.126	0.14	0.582	1.053	1.682	13.92	1.232

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	86	0	138	329	298	284
normalized size	1	1.	0.84	0.	1.35	3.23	2.92	2.78
time (sec)	N/A	0.094	0.081	0.355	1.038	1.711	8.272	1.158

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	85	0	74	184	156	151
normalized size	1	1.	1.09	0.	0.95	2.36	2.	1.94
time (sec)	N/A	0.056	0.027	0.109	1.026	1.685	3.926	1.266

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	114	0	215	0	0	0
normalized size	1	1.	1.01	0.	1.9	0.	0.	0.
time (sec)	N/A	0.158	0.025	0.503	1.088	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	115	320	0	163
normalized size	1	1.	1.	0.	1.42	3.95	0.	2.01
time (sec)	N/A	0.077	0.066	0.543	1.019	2.349	0.	1.257

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	113	0	216	864	0	579
normalized size	1	1.	0.89	0.	1.7	6.8	0.	4.56
time (sec)	N/A	0.119	0.196	0.544	1.051	6.633	0.	1.297

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	164	0	404	1629	0	1208
normalized size	1	1.	0.94	0.	2.31	9.31	0.	6.9
time (sec)	N/A	0.172	0.283	0.539	1.088	43.007	0.	1.285

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	114	111	0	0	0
normalized size	1	1.	0.76	1.09	1.06	0.	0.	0.
time (sec)	N/A	0.167	0.037	0.421	1.066	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	239	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.756	0.843	1.456	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	176	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	0.276	1.411	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.06	1.257	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	123	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.083	2.635	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	211	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.533	0.507	3.108	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.523	2.648	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	224	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.516	2.714	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	178	0	0	0	284	0
normalized size	1	1.	0.98	0.	0.	0.	1.57	0.
time (sec)	N/A	0.176	0.254	2.775	0.	0.	120.028	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	130	0	0	0	162	0
normalized size	1	1.	0.98	0.	0.	0.	1.23	0.
time (sec)	N/A	0.136	0.123	3.066	0.	0.	15.44	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	48	0
normalized size	1	1.	0.96	0.	0.	0.	0.89	0.
time (sec)	N/A	0.017	0.029	0.066	0.	0.	2.951	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	1.59	2.352	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.181	2.131	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.204	2.204	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	183	919	0	0	0	0
normalized size	1	1.	0.73	3.68	0.	0.	0.	0.
time (sec)	N/A	0.246	0.184	0.66	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	127	666	0	0	0	0
normalized size	1	1.	0.8	4.19	0.	0.	0.	0.
time (sec)	N/A	0.167	0.088	0.649	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	79	427	0	0	0	0
normalized size	1	1.	0.87	4.69	0.	0.	0.	0.
time (sec)	N/A	0.108	0.033	0.622	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	242	159	0	0	0
normalized size	1	1.	0.98	4.17	2.74	0.	0.	0.
time (sec)	N/A	0.043	0.003	0.101	1.046	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	98	420	166	0	0	0
normalized size	1	1.	1.01	4.33	1.71	0.	0.	0.
time (sec)	N/A	0.123	0.02	0.592	1.239	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	139	615	211	0	0	0
normalized size	1	1.	0.95	4.21	1.45	0.	0.	0.
time (sec)	N/A	0.166	0.048	0.595	1.234	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	188	850	292	0	0	0
normalized size	1	1.	0.83	3.74	1.29	0.	0.	0.
time (sec)	N/A	0.223	0.168	0.608	1.233	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	338	1083	0	0	0	0
normalized size	1	1.	0.86	2.75	0.	0.	0.	0.
time (sec)	N/A	0.426	0.319	0.541	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	271	825	0	0	0	0
normalized size	1	1.	0.87	2.64	0.	0.	0.	0.
time (sec)	N/A	0.334	0.161	0.465	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	225	576	0	0	0	0
normalized size	1	1.	0.88	2.25	0.	0.	0.	0.
time (sec)	N/A	0.273	0.117	0.444	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	201	366	0	0	0	0
normalized size	1	1.	1.	1.82	0.	0.	0.	0.
time (sec)	N/A	0.185	0.026	0.092	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	232	624	0	0	0	0
normalized size	1	1.	0.94	2.53	0.	0.	0.	0.
time (sec)	N/A	0.303	0.071	0.408	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	268	831	0	0	0	0
normalized size	1	1.	0.88	2.72	0.	0.	0.	0.
time (sec)	N/A	0.35	0.191	0.413	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	320	1071	0	0	0	0
normalized size	1	1.	0.86	2.89	0.	0.	0.	0.
time (sec)	N/A	0.395	0.212	0.358	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	497	912	0	0	0	0
normalized size	1	1.	0.72	1.32	0.	0.	0.	0.
time (sec)	N/A	0.891	0.601	0.665	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	643	643	504	704	0	0	0	0
normalized size	1	1.	0.78	1.09	0.	0.	0.	0.
time (sec)	N/A	0.774	0.389	0.648	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	430	500	0	0	0	0
normalized size	1	1.	0.94	1.09	0.	0.	0.	0.
time (sec)	N/A	0.632	0.198	0.632	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	313	261	0	0	0	0
normalized size	1	1.	1.02	0.85	0.	0.	0.	0.
time (sec)	N/A	0.387	0.053	0.102	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	358	461	0	0	0	0
normalized size	1	1.	1.02	1.31	0.	0.	0.	0.
time (sec)	N/A	0.565	0.053	0.586	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	510	424	732	0	0	0	0
normalized size	1	1.	0.83	1.44	0.	0.	0.	0.
time (sec)	N/A	0.677	0.061	0.592	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	674	674	542	1025	0	0	0	0
normalized size	1	1.	0.8	1.52	0.	0.	0.	0.
time (sec)	N/A	0.793	0.39	0.595	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	251	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.321	0.218	0.758	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	183	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	0.125	0.723	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	149	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.057	0.742	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	114	0	215	0	0	0
normalized size	1	1.	1.01	0.	1.9	0.	0.	0.
time (sec)	N/A	0.147	0.017	0.069	1.049	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	139	0	242	0	0	0
normalized size	1	1.	0.87	0.	1.52	0.	0.	0.
time (sec)	N/A	0.246	0.061	0.744	1.21	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	166	0	311	0	0	0
normalized size	1	1.	0.84	0.	1.57	0.	0.	0.
time (sec)	N/A	0.277	0.093	0.741	1.234	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	241	0	414	0	0	0
normalized size	1	1.	0.84	0.	1.44	0.	0.	0.
time (sec)	N/A	0.329	0.216	0.733	1.289	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	375	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.586	0.418	0.773	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	319	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.492	0.223	0.741	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	271	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.438	0.16	0.763	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	242	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.334	0.057	0.763	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	264	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.461	0.116	0.785	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	320	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.507	0.202	0.747	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	364	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.547	0.289	0.769	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	505	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.921	0.401	0.919	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	666	443	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.731	0.222	0.718	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	488	403	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.601	0.113	0.72	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	350	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.409	0.081	0.721	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	395	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.538	0.085	0.708	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	429	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.668	0.184	0.709	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	737	737	520	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.795	0.275	0.704	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	749	749	867	327	0	0	0	0
normalized size	1	1.	1.16	0.44	0.	0.	0.	0.
time (sec)	N/A	0.931	0.597	0.716	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0
normalized size	1	1.	1.06	0.95	0.	0.	0.	0.
time (sec)	N/A	0.509	0.344	0.766	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	178	419	0	0	0	0
normalized size	1	1.	0.78	1.83	0.	0.	0.	0.
time (sec)	N/A	0.229	0.111	0.533	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	373	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.437	0.225	0.752	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	597	706	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.839	0.371	0.743	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	541	541	422	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.813	0.277	0.727	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	561	912	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	1.107	0.53	0.747	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	215	995	0	1335	0	417
normalized size	1	1.	0.64	2.94	0.	3.95	0.	1.23
time (sec)	N/A	0.259	0.273	0.546	0.	2.104	0.	1.787

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	151	686	0	922	0	271
normalized size	1	1.	0.68	3.1	0.	4.17	0.	1.23
time (sec)	N/A	0.171	0.12	0.533	0.	2.094	0.	1.328

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	416	0	504	228	147
normalized size	1	1.	1.	3.56	0.	4.31	1.95	1.26
time (sec)	N/A	0.087	0.037	0.519	0.	2.075	40.402	1.325

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0
normalized size	1	1.	1.06	0.95	0.	0.	0.	0.
time (sec)	N/A	0.459	0.253	0.074	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	1236	0	0	0	0	0
normalized size	1	1.	1.65	0.	0.	0.	0.	0.
time (sec)	N/A	1.024	3.668	1.271	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	945	945	435	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	1.251	0.509	0.945	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	281	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.72	0.264	1.481	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	2.728	6.98	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	8.451	7.441	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	682	0	1460	0	0	0	0	0
normalized size	1	0.	2.14	0.	0.	0.	0.	0.
time (sec)	N/A	1.386	4.481	19.954	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	3.981	3.108	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	17.199	7.325	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.472	0.782	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.3	0.512	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.548	1.013	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	2.553	1.075	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.865	3.822	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.626	3.711	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	4.561	4.606	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	7.495	4.234	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	258	1311	0	1611	0	478
normalized size	1	1.	0.7	3.58	0.	4.4	0.	1.31
time (sec)	N/A	0.308	0.249	0.874	0.	2.167	0.	1.301

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	178	869	0	1027	0	304
normalized size	1	1.	0.77	3.76	0.	4.45	0.	1.32
time (sec)	N/A	0.177	0.208	0.723	0.	2.023	0.	1.221

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	402	0	562	175	158
normalized size	1	1.	1.	3.65	0.	5.11	1.59	1.44
time (sec)	N/A	0.097	0.049	0.565	0.	2.06	72.981	1.33

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1165	1165	990	1180	0	0	0	0
normalized size	1	1.	0.85	1.01	0.	0.	0.	0.
time (sec)	N/A	1.604	0.812	0.609	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1861	1863	2168	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	2.887	7.042	1.281	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1221	1139	1020	0	0	0	0	0
normalized size	1	0.93	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	1.642	0.967	1.704	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	835	835	475	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.079	0.562	1.776	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	415	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.507	0.158	1.532	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	13.646	180.	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	22.506	180.	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	1124	0	2539	0	0	0	0	0
normalized size	1	0.	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	2.649	9.213	105.	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	517	0	1066	0	0	0	0	0
normalized size	1	0.	2.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.768	2.381	64.717	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	17.117	180.	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	37.304	180.	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.372	0.887	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.322	0.534	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	2.525	1.066	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	8.748	1.182	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.702	3.793	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.532	3.905	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	7.922	4.077	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	14.592	4.97	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	170	413	178	339	0	416
normalized size	1	1.	1.2	2.91	1.25	2.39	0.	2.93
time (sec)	N/A	0.229	0.041	0.567	1.002	1.984	0.	1.174

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	140	387	146	282	0	317
normalized size	1	1.	1.18	3.25	1.23	2.37	0.	2.66
time (sec)	N/A	0.179	0.026	0.573	1.095	2.032	0.	1.149

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	98	361	134	216	139	200
normalized size	1	1.	1.04	3.84	1.43	2.3	1.48	2.13
time (sec)	N/A	0.093	0.044	0.571	1.03	1.833	67.443	1.214

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	419	0	0	0	0
normalized size	1	1.	0.98	5.11	0.	0.	0.	0.
time (sec)	N/A	0.111	0.019	0.567	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	92	421	0	0	0	0
normalized size	1	1.	0.99	4.53	0.	0.	0.	0.
time (sec)	N/A	0.127	0.028	0.499	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	105	392	104	223	0	435
normalized size	1	1.	1.13	4.22	1.12	2.4	0.	4.68
time (sec)	N/A	0.137	0.038	0.373	1.02	2.099	0.	1.266

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	130	428	140	285	0	695
normalized size	1	1.	1.04	3.42	1.12	2.28	0.	5.56
time (sec)	N/A	0.163	0.063	0.395	1.025	2.047	0.	1.284

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	158	448	178	346	0	910
normalized size	1	1.	1.07	3.03	1.2	2.34	0.	6.15
time (sec)	N/A	0.202	0.111	0.388	1.031	2.232	0.	1.302

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	118	453	0	693	0	186
normalized size	1	1.	0.77	2.94	0.	4.5	0.	1.21
time (sec)	N/A	0.13	0.056	0.586	0.	2.375	0.	1.483

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	416	0	504	228	147
normalized size	1	1.	1.	3.56	0.	4.31	1.95	1.26
time (sec)	N/A	0.086	0.036	0.102	0.	2.261	35.036	1.167

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	93	62	427	0	436	262	105
normalized size	1	1.29	0.86	5.93	0.	6.06	3.64	1.46
time (sec)	N/A	0.084	0.046	0.586	0.	2.501	62.627	1.262

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	96	430	0	441	1469	124
normalized size	1	1.	0.89	3.98	0.	4.08	13.6	1.15
time (sec)	N/A	0.1	0.039	0.566	0.	2.4	131.902	1.242

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	101	483	0	583	0	165
normalized size	1	1.	0.72	3.45	0.	4.16	0.	1.18
time (sec)	N/A	0.121	0.006	0.574	0.	1.934	0.	1.33

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	205	687	301	582	0	721
normalized size	1	1.	0.82	2.74	1.2	2.32	0.	2.87
time (sec)	N/A	0.471	0.171	0.585	1.025	2.088	0.	1.306

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	173	643	250	477	0	564
normalized size	1	1.	0.82	3.06	1.19	2.27	0.	2.69
time (sec)	N/A	0.361	0.133	0.585	1.023	1.716	0.	1.317

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	135	599	205	379	0	389
normalized size	1	1.	1.09	4.83	1.65	3.06	0.	3.14
time (sec)	N/A	0.141	0.107	0.587	1.01	1.762	0.	1.197

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	121	652	0	0	0	0
normalized size	1	1.	0.79	4.26	0.	0.	0.	0.
time (sec)	N/A	0.203	0.082	0.569	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	126	642	0	0	0	0
normalized size	1	1.	0.93	4.76	0.	0.	0.	0.
time (sec)	N/A	0.193	0.078	0.583	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	148	663	0	0	0	0
normalized size	1	1.	0.86	3.85	0.	0.	0.	0.
time (sec)	N/A	0.223	0.112	0.509	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	141	656	185	393	0	1068
normalized size	1	1.	1.08	5.05	1.42	3.02	0.	8.22
time (sec)	N/A	0.208	0.121	0.403	1.024	1.806	0.	1.339

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	184	713	247	489	0	1470
normalized size	1	1.	0.85	3.3	1.14	2.26	0.	6.81
time (sec)	N/A	0.29	0.161	0.408	1.022	1.853	0.	1.355

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	215	748	301	587	0	1809
normalized size	1	1.	0.85	2.96	1.19	2.32	0.	7.15
time (sec)	N/A	0.333	0.219	0.415	1.037	2.075	0.	1.2

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	188	761	0	1138	0	332
normalized size	1	1.	0.68	2.74	0.	4.09	0.	1.19
time (sec)	N/A	0.236	0.166	0.602	0.	1.682	0.	1.27

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	151	686	0	922	0	271
normalized size	1	1.	0.68	3.1	0.	4.17	0.	1.23
time (sec)	N/A	0.163	0.117	0.105	0.	1.731	0.	1.336

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	112	742	0	790	0	227
normalized size	1	1.	0.63	4.17	0.	4.44	0.	1.28
time (sec)	N/A	0.155	0.136	0.606	0.	1.78	0.	1.254

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	113	700	0	744	0	208
normalized size	1	1.	0.67	4.14	0.	4.4	0.	1.23
time (sec)	N/A	0.147	0.134	0.805	0.	1.678	0.	1.356

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	156	753	0	774	0	244
normalized size	1	1.	0.78	3.76	0.	3.87	0.	1.22
time (sec)	N/A	0.171	0.064	0.594	0.	1.886	0.	1.314

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	161	784	0	956	0	300
normalized size	1	1.	0.64	3.11	0.	3.79	0.	1.19
time (sec)	N/A	0.206	0.029	0.424	0.	1.819	0.	1.366

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	143	902	0	0	0	0
normalized size	1	1.	0.76	4.8	0.	0.	0.	0.
time (sec)	N/A	0.276	0.124	0.849	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	91	672	0	0	0	0
normalized size	1	1.	0.81	6.	0.	0.	0.	0.
time (sec)	N/A	0.188	0.041	0.68	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	64	472	186	0	0	0
normalized size	1	1.	0.91	6.74	2.66	0.	0.	0.
time (sec)	N/A	0.095	0.007	0.649	1.015	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	92	732	189	0	0	0
normalized size	1	1.	0.77	6.15	1.59	0.	0.	0.
time (sec)	N/A	0.206	0.036	0.665	1.72	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	147	942	240	0	0	0
normalized size	1	1.	0.84	5.35	1.36	0.	0.	0.
time (sec)	N/A	0.267	0.073	0.686	1.923	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	667	667	691	1011	0	0	0	0
normalized size	1	1.	1.04	1.52	0.	0.	0.	0.
time (sec)	N/A	0.718	0.549	0.631	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	585	585	680	746	0	0	0	0
normalized size	1	1.	1.16	1.28	0.	0.	0.	0.
time (sec)	N/A	0.594	0.292	0.555	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0
normalized size	1	1.	1.06	0.95	0.	0.	0.	0.
time (sec)	N/A	0.425	0.158	0.082	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	581	581	673	755	0	0	0	0
normalized size	1	1.	1.16	1.3	0.	0.	0.	0.
time (sec)	N/A	0.597	0.299	0.652	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	651	651	754	1005	0	0	0	0
normalized size	1	1.	1.16	1.54	0.	0.	0.	0.
time (sec)	N/A	0.65	0.262	0.651	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	166	985	0	0	0	0
normalized size	1	1.	0.83	4.95	0.	0.	0.	0.
time (sec)	N/A	0.283	0.222	0.713	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	131	732	0	0	0	0
normalized size	1	1.	0.85	4.72	0.	0.	0.	0.
time (sec)	N/A	0.221	0.103	0.671	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	63	371	100	194	0	246
normalized size	1	1.	0.76	4.47	1.2	2.34	0.	2.96
time (sec)	N/A	0.074	0.052	0.382	1.014	2.069	0.	1.168

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	170	984	266	0	0	0
normalized size	1	1.	0.85	4.9	1.32	0.	0.	0.
time (sec)	N/A	0.283	0.121	0.708	1.759	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	208	1216	398	0	0	0
normalized size	1	1.	0.83	4.84	1.59	0.	0.	0.
time (sec)	N/A	0.341	0.174	0.724	1.265	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	802	802	1349	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	1.688	4.111	1.505	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	1231	0	0	0	0	0
normalized size	1	1.	1.65	0.	0.	0.	0.	0.
time (sec)	N/A	1.513	3.303	1.414	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	1236	0	0	0	0	0
normalized size	1	1.	1.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.825	3.14	0.079	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	803	803	1438	0	0	0	0	0
normalized size	1	1.	1.79	0.	0.	0.	0.	0.
time (sec)	N/A	1.542	4.509	1.51	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	128	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.042	1.373	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	248	214	281	0	0	0
normalized size	1	1.	1.04	0.9	1.18	0.	0.	0.
time (sec)	N/A	0.29	0.12	0.293	1.584	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	468	282	0	0	0	0
normalized size	1	1.	2.16	1.3	0.	0.	0.	0.
time (sec)	N/A	0.253	0.123	0.096	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	118	428	0	363	0	0
normalized size	1	1.	0.82	2.97	0.	2.52	0.	0.
time (sec)	N/A	0.171	0.169	4.184	0.	2.124	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	410	0	323	0	0
normalized size	1	1.	0.81	3.31	0.	2.6	0.	0.
time (sec)	N/A	0.142	0.112	4.243	0.	2.085	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	68	376	0	244	0	0
normalized size	1	1.	0.82	4.53	0.	2.94	0.	0.
time (sec)	N/A	0.11	0.058	4.306	0.	2.103	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	87	423	0	266	0	0
normalized size	1	1.	0.9	4.36	0.	2.74	0.	0.
time (sec)	N/A	0.141	0.093	4.279	0.	2.133	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	104	448	0	352	0	0
normalized size	1	1.	0.83	3.56	0.	2.79	0.	0.
time (sec)	N/A	0.169	0.198	4.392	0.	2.105	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	209	795	0	659	0	0
normalized size	1	1.	0.64	2.43	0.	2.02	0.	0.
time (sec)	N/A	0.326	0.375	5.224	0.	2.226	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	171	734	0	551	0	0
normalized size	1	1.	0.67	2.89	0.	2.17	0.	0.
time (sec)	N/A	0.267	0.259	5.136	0.	2.088	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	124	665	0	437	0	0
normalized size	1	1.	0.7	3.78	0.	2.48	0.	0.
time (sec)	N/A	0.197	0.19	5.079	0.	2.16	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	150	693	0	467	0	0
normalized size	1	1.	0.78	3.59	0.	2.42	0.	0.
time (sec)	N/A	0.242	0.376	5.204	0.	2.153	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	188	755	0	590	0	0
normalized size	1	1.	0.73	2.94	0.	2.3	0.	0.
time (sec)	N/A	0.317	0.57	5.238	0.	2.186	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	266	0	695	0	0	0	0
normalized size	1	1.	0.	2.61	0.	0.	0.	0.
time (sec)	N/A	0.424	4.936	1.066	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	92	532	208	0	0	0
normalized size	1	1.	0.76	4.4	1.72	0.	0.	0.
time (sec)	N/A	0.195	0.075	1.197	1.44	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	64	298	151	0	0	0
normalized size	1	1.	0.91	4.26	2.16	0.	0.	0.
time (sec)	N/A	0.164	0.023	1.198	1.407	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	461	0	0	0	0
normalized size	1	1.	0.	2.09	0.	0.	0.	0.
time (sec)	N/A	0.412	1.427	1.067	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	419	419	0	1036	0	0	0	0
normalized size	1	1.	0.	2.47	0.	0.	0.	0.
time (sec)	N/A	0.565	7.435	1.095	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	171	805	315	0	0	0
normalized size	1	1.	0.84	3.95	1.54	0.	0.	0.
time (sec)	N/A	0.268	0.184	1.217	1.435	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	433	589	282	0	0	0
normalized size	1	1.	2.78	3.78	1.81	0.	0.	0.
time (sec)	N/A	0.278	1.499	1.22	1.458	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	377	377	0	810	0	0	0	0
normalized size	1	1.	0.	2.15	0.	0.	0.	0.
time (sec)	N/A	0.609	1.245	1.082	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	23	143	49	0	0
normalized size	1	1.	1.04	0.92	5.72	1.96	0.	0.
time (sec)	N/A	0.157	0.069	0.083	1.402	1.696	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	23	147	49	0	0
normalized size	1	1.	1.04	0.92	5.88	1.96	0.	0.
time (sec)	N/A	0.098	0.021	0.388	1.058	1.708	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	34	24	0	55	0	0
normalized size	1	1.	1.31	0.92	0.	2.12	0.	0.
time (sec)	N/A	0.155	0.069	0.088	0.	1.751	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	607	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.417	29.864	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	306	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.297	29.012	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.435	28.329	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.368	27.697	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	2.417	15.357	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	1.759	19.992	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	1.803	21.518	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.238	18.854	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	75	66	0	185	0	0
normalized size	1	1.	1.09	0.96	0.	2.68	0.	0.
time (sec)	N/A	0.124	0.058	2.256	0.	1.724	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	12	34	9	80	31	0	0
normalized size	1	1.5	4.25	1.12	10.	3.88	0.	0.
time (sec)	N/A	0.01	0.003	0.081	1.031	1.439	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	76	93	42	0	0
normalized size	1	1.	1.	6.33	7.75	3.5	0.	0.
time (sec)	N/A	0.018	0.003	0.139	1.06	1.596	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	151	0	0
normalized size	1	1.	1.	1.07	0.	10.79	0.	0.
time (sec)	N/A	0.019	0.004	0.086	0.	1.646	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	36	34	90	0	0	0
normalized size	1	1.	1.03	0.97	2.57	0.	0.	0.
time (sec)	N/A	0.056	0.004	0.084	1.054	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	108	104	0	0	0
normalized size	1	1.	1.03	2.77	2.67	0.	0.	0.
time (sec)	N/A	0.047	0.004	0.093	1.028	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	46	0	159	0	0
normalized size	1	1.	0.94	0.98	0.	3.38	0.	0.
time (sec)	N/A	0.05	0.017	0.085	0.	1.695	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	114	167	0	0	0
normalized size	1	1.	0.76	1.09	1.59	0.	0.	0.
time (sec)	N/A	0.168	0.029	0.411	1.066	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	228	335	0	0	0	0
normalized size	1	1.	1.	1.48	0.	0.	0.	0.
time (sec)	N/A	0.378	0.106	0.112	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.471	1.703	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.055	2.73	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	159	0	173	358	0	779
normalized size	1	1.	0.96	0.	1.04	2.16	0.	4.69
time (sec)	N/A	0.136	0.131	0.415	1.045	1.822	0.	1.376

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	131	0	143	288	128	582
normalized size	1	1.	0.98	0.	1.07	2.15	0.96	4.34
time (sec)	N/A	0.099	0.09	0.098	1.068	1.697	29.445	1.29

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	107	0	113	224	100	385
normalized size	1	1.	1.05	0.	1.11	2.2	0.98	3.77
time (sec)	N/A	0.072	0.033	0.098	1.042	1.746	4.729	1.249

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	53	77	158	66	144
normalized size	1	1.	1.	0.88	1.28	2.63	1.1	2.4
time (sec)	N/A	0.04	0.029	0.084	1.019	1.909	1.457	1.307

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	146	0	0	0
normalized size	1	1.	1.04	0.	2.86	0.	0.	0.
time (sec)	N/A	0.048	0.003	0.097	1.631	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	0	82	161	493	252
normalized size	1	1.	0.96	0.	1.17	2.3	7.04	3.6
time (sec)	N/A	0.058	0.045	0.096	1.025	2.077	96.163	1.29

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	104	0	113	240	0	494
normalized size	1	1.	0.95	0.	1.04	2.2	0.	4.53
time (sec)	N/A	0.074	0.037	0.099	1.038	2.131	0.	1.286

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	132	0	143	308	0	732
normalized size	1	1.	0.94	0.	1.01	2.18	0.	5.19
time (sec)	N/A	0.092	0.131	0.1	1.063	2.087	0.	1.338

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	355	295	0	437	1081	0	1619
normalized size	1	0.74	0.61	0.	0.91	2.25	0.	3.37
time (sec)	N/A	0.476	0.316	0.102	1.074	2.25	0.	1.37

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	263	223	0	347	788	0	1073
normalized size	1	0.77	0.65	0.	1.01	2.3	0.	3.14
time (sec)	N/A	0.361	0.193	0.095	1.064	2.242	0.	1.279

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	150	0	242	509	0	487
normalized size	1	1.	0.77	0.	1.24	2.61	0.	2.5
time (sec)	N/A	0.184	0.082	0.092	1.054	1.845	0.	1.335

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.12	0.099	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	176	188	0	0	0	0	0
normalized size	1	1.14	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	0.163	0.097	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	318	353	0	0	0	0	0
normalized size	1	1.09	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.667	0.325	0.102	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	432	538	0	0	0	0	0
normalized size	1	1.06	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.034	0.28	0.1	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	907	907	577	0	899	2655	0	3444
normalized size	1	1.	0.64	0.	0.99	2.93	0.	3.8
time (sec)	N/A	1.007	0.477	0.098	1.095	2.476	0.	1.414

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	595	595	433	0	724	1875	0	2283
normalized size	1	1.	0.73	0.	1.22	3.15	0.	3.84
time (sec)	N/A	0.619	0.298	0.102	1.122	2.26	0.	1.267

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	241	0	514	1146	0	1030
normalized size	1	1.	0.85	0.	1.81	4.04	0.	3.63
time (sec)	N/A	0.251	0.202	0.095	1.124	1.918	0.	1.175

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	0.155	0.102	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	283	536	0	0	0	0	0
normalized size	1	1.08	2.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.595	0.713	0.1	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	573	550	841	0	0	0	0	0
normalized size	1	0.96	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	1.497	1.056	0.098	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	158	0	159	440	0	163
normalized size	1	1.	0.92	0.	0.93	2.57	0.	0.95
time (sec)	N/A	0.132	0.133	0.42	1.051	1.786	0.	1.314

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	130	0	130	369	0	136
normalized size	1	1.	0.94	0.	0.94	2.65	0.	0.98
time (sec)	N/A	0.096	0.086	0.331	1.029	1.938	0.	1.29

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	102	0	100	300	0	109
normalized size	1	1.	0.95	0.	0.93	2.8	0.	1.02
time (sec)	N/A	0.073	0.028	0.335	1.06	1.796	0.	1.353

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	94	65	217	76	76
normalized size	1	1.	1.17	1.77	1.23	4.09	1.43	1.43
time (sec)	N/A	0.034	0.031	0.093	1.04	1.791	16.978	1.345

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	167	0	0	0
normalized size	1	1.	1.04	0.	3.27	0.	0.	0.
time (sec)	N/A	0.05	0.003	0.346	2.36	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	63	101	165	0	120
normalized size	1	1.	1.05	0.97	1.55	2.54	0.	1.85
time (sec)	N/A	0.051	0.033	0.103	1.062	1.932	0.	1.311

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	109	0	128	228	0	157
normalized size	1	1.	1.05	0.	1.23	2.19	0.	1.51
time (sec)	N/A	0.076	0.067	0.33	1.044	1.913	0.	1.415

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	133	0	158	292	0	189
normalized size	1	1.	0.98	0.	1.16	2.15	0.	1.39
time (sec)	N/A	0.097	0.086	0.328	1.024	1.886	0.	1.211

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	428	540	0	0	0	0	0
normalized size	1	1.06	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	1.007	0.258	0.336	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	311	321	0	0	0	0	0
normalized size	1	1.08	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.637	0.222	0.381	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	174	170	0	0	0	0	0
normalized size	1	1.14	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.354	0.123	0.336	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	386	0	0	0	0	0
normalized size	1	1.	4.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.347	0.342	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	298	0	335	521	0	0
normalized size	1	1.	1.53	0.	1.72	2.67	0.	0.
time (sec)	N/A	0.198	0.298	0.331	1.081	1.731	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	263	473	0	433	799	0	0
normalized size	1	0.77	1.39	0.	1.27	2.34	0.	0.
time (sec)	N/A	0.366	0.36	0.339	1.095	1.847	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	355	692	0	522	1094	0	0
normalized size	1	0.74	1.44	0.	1.09	2.28	0.	0.
time (sec)	N/A	0.471	0.341	0.339	1.077	1.845	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	546	777	0	0	0	0	0
normalized size	1	0.96	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	1.492	0.978	0.427	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	281	476	0	0	0	0	0
normalized size	1	1.08	1.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.616	0.665	0.427	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	532	0	0	0	0	0
normalized size	1	1.	3.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	0.263	0.347	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	558	0	767	1162	0	0
normalized size	1	1.	1.96	0.	2.69	4.08	0.	0.
time (sec)	N/A	0.272	0.615	0.362	1.17	1.868	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	595	595	766	0	988	1894	0	0
normalized size	1	1.	1.29	0.	1.66	3.18	0.	0.
time (sec)	N/A	0.642	1.004	0.344	1.155	1.883	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	907	907	950	0	1166	2684	0	0
normalized size	1	1.	1.05	0.	1.29	2.96	0.	0.
time (sec)	N/A	1.011	1.548	0.391	1.163	1.99	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	219	0	232	513	0	714
normalized size	1	1.	0.94	0.	0.99	2.19	0.	3.05
time (sec)	N/A	0.188	0.229	0.434	1.024	1.818	0.	1.25

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	176	0	189	392	0	540
normalized size	1	1.	0.95	0.	1.02	2.12	0.	2.92
time (sec)	N/A	0.134	0.136	0.096	1.041	1.847	0.	1.344

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	133	0	143	292	131	366
normalized size	1	1.	0.98	0.	1.05	2.15	0.96	2.69
time (sec)	N/A	0.095	0.093	0.096	1.004	1.83	11.86	1.308

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	66	95	193	82	182
normalized size	1	1.	1.	0.86	1.23	2.51	1.06	2.36
time (sec)	N/A	0.053	0.042	0.088	1.005	1.785	1.543	1.324

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	224	0	0	0
normalized size	1	1.	1.04	0.	4.39	0.	0.	0.
time (sec)	N/A	0.051	0.003	0.098	1.599	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	84	0	101	208	0	378
normalized size	1	1.	0.97	0.	1.16	2.39	0.	4.34
time (sec)	N/A	0.068	0.033	0.098	1.027	1.858	0.	1.338

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	143	312	0	732
normalized size	1	1.	0.94	0.	1.	2.18	0.	5.12
time (sec)	N/A	0.093	0.138	0.101	1.03	1.93	0.	1.301

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	177	0	188	409	0	1091
normalized size	1	1.	0.92	0.	0.98	2.13	0.	5.68
time (sec)	N/A	0.125	0.191	0.096	1.037	1.84	0.	1.31

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	491	411	0	572	1574	0	1926
normalized size	1	0.72	0.6	0.	0.84	2.31	0.	2.83
time (sec)	N/A	0.697	0.55	0.099	1.049	2.479	0.	1.344

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	355	301	0	436	1088	0	1291
normalized size	1	0.74	0.63	0.	0.91	2.27	0.	2.69
time (sec)	N/A	0.462	0.308	0.109	1.055	2.183	0.	1.337

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	210	197	0	293	664	0	647
normalized size	1	0.79	0.74	0.	1.1	2.49	0.	2.42
time (sec)	N/A	0.29	0.127	0.093	1.052	1.975	0.	1.373

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.107	0.1	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	253	274	0	0	0	0	0
normalized size	1	1.1	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.501	0.223	0.098	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	430	533	0	0	0	0	0
normalized size	1	1.06	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.013	0.273	0.097	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1835	1835	1009	0	1436	5488	0	5998
normalized size	1	1.	0.55	0.	0.78	2.99	0.	3.27
time (sec)	N/A	2.267	1.251	0.095	1.153	4.336	0.	1.621

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1357	1357	808	0	1170	4018	0	4500
normalized size	1	1.	0.6	0.	0.86	2.96	0.	3.32
time (sec)	N/A	1.573	0.771	0.112	1.13	3.952	0.	1.479

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	907	907	589	0	902	2668	0	3001
normalized size	1	1.	0.65	0.	0.99	2.94	0.	3.31
time (sec)	N/A	0.979	0.467	0.098	1.1	2.582	0.	1.406

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	362	0	614	1544	0	1492
normalized size	1	1.	0.83	0.	1.4	3.53	0.	3.41
time (sec)	N/A	0.444	0.224	0.095	1.084	2.096	0.	1.339

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	0.173	0.151	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	414	733	0	0	0	0	0
normalized size	1	0.94	1.67	0.	0.	0.	0.	0.
time (sec)	N/A	1.006	0.709	0.102	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	765	742	1074	0	0	0	0	0
normalized size	1	0.97	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	3.053	1.676	0.099	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	135	0	146	302	0	359
normalized size	1	1.	0.98	0.	1.06	2.19	0.	2.6
time (sec)	N/A	0.107	0.11	0.416	1.039	1.868	0.	1.303

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	135	0	0	806	0	140
normalized size	1	1.	1.04	0.	0.	6.2	0.	1.08
time (sec)	N/A	0.078	0.098	0.331	0.	1.94	0.	1.339

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	103	203	0	111
normalized size	1	1.	1.06	0.	1.16	2.28	0.	1.25
time (sec)	N/A	0.065	0.027	0.326	1.036	1.827	0.	1.281

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	62	0	547	133	92
normalized size	1	1.	1.	0.86	0.	7.6	1.85	1.28
time (sec)	N/A	0.053	0.027	0.092	0.	1.852	13.401	1.283

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	154	0	0	0
normalized size	1	1.	1.	0.	2.8	0.	0.	0.
time (sec)	N/A	0.051	0.012	0.384	1.64	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	0	0	481	0	82
normalized size	1	1.	0.87	0.	0.	7.07	0.	1.21
time (sec)	N/A	0.04	0.018	0.342	0.	1.858	0.	1.245

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	0	104	216	0	128
normalized size	1	1.	0.97	0.	1.11	2.3	0.	1.36
time (sec)	N/A	0.07	0.032	0.332	1.041	1.916	0.	1.333

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	0	755	0	127
normalized size	1	1.	0.53	0.	0.	6.14	0.	1.03
time (sec)	N/A	0.077	0.014	0.332	0.	1.932	0.	1.366

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	355	328	0	446	1125	0	1287
normalized size	1	0.74	0.68	0.	0.93	2.33	0.	2.67
time (sec)	N/A	0.488	0.369	0.336	1.057	2.418	0.	1.514

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	217	239	0	312	689	0	427
normalized size	1	0.79	0.87	0.	1.13	2.51	0.	1.55
time (sec)	N/A	0.305	0.157	0.341	1.045	2.101	0.	1.707

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	199	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.115	0.344	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	261	264	0	0	0	0	0
normalized size	1	1.1	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.5	0.303	0.338	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	436	539	0	0	0	0	0
normalized size	1	1.06	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	1.016	0.276	0.343	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	547	438	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.769	0.485	0.341	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	319	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.447	0.216	0.343	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	247	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.406	0.187	0.347	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	473	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.62	0.532	0.343	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	640	640	678	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.944	1.034	0.351	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	913	913	598	0	918	2745	0	3002
normalized size	1	1.	0.65	0.	1.01	3.01	0.	3.29
time (sec)	N/A	1.032	1.018	0.338	1.12	3.214	0.	2.155

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	428	0	653	1590	0	1050
normalized size	1	1.	0.95	0.	1.45	3.54	0.	2.34
time (sec)	N/A	0.459	0.407	0.347	1.111	2.305	0.	2.301

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	339	0	0	0	0	0
normalized size	1	1.	2.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	0.193	0.348	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	428	764	0	0	0	0	0
normalized size	1	0.95	1.69	0.	0.	0.	0.	0.
time (sec)	N/A	1.008	0.782	0.399	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	793	0	3146	0	0	0	0	0
normalized size	1	0.	3.97	0.	0.	0.	0.	0.
time (sec)	N/A	3.006	9.011	0.341	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	485	0	598	0	0	0	0	0
normalized size	1	0.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	1.079	1.243	0.334	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	318	0	1028	0	0	0	0	0
normalized size	1	0.	3.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.487	7.168	0.341	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	631	0	803	0	0	0	0	0
normalized size	1	0.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	1.87	2.892	0.364	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	218	0	219	605	0	217
normalized size	1	1.	0.91	0.	0.92	2.53	0.	0.91
time (sec)	N/A	0.171	0.227	0.542	1.047	2.188	0.	1.334

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	175	0	173	473	0	177
normalized size	1	1.	0.92	0.	0.91	2.49	0.	0.93
time (sec)	N/A	0.13	0.133	0.335	1.033	2.196	0.	1.398

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	132	0	130	373	0	136
normalized size	1	1.	0.94	0.	0.92	2.65	0.	0.96
time (sec)	N/A	0.092	0.086	0.34	1.014	2.148	0.	1.445

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	79	115	80	269	92	89
normalized size	1	1.	1.13	1.64	1.14	3.84	1.31	1.27
time (sec)	N/A	0.05	0.047	0.102	1.03	2.156	15.053	1.385

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	250	0	0	0
normalized size	1	1.	1.04	0.	4.9	0.	0.	0.
time (sec)	N/A	0.05	0.003	0.352	2.318	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	85	0	116	250	0	128
normalized size	1	1.	1.04	0.	1.41	3.05	0.	1.56
time (sec)	N/A	0.068	0.033	0.341	1.025	1.762	0.	1.348

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	135	0	158	377	0	166
normalized size	1	1.	0.98	0.	1.14	2.73	0.	1.2
time (sec)	N/A	0.097	0.089	0.329	1.032	1.814	0.	1.334

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	178	0	203	502	0	207
normalized size	1	1.	0.95	0.	1.09	2.68	0.	1.11
time (sec)	N/A	0.133	0.15	0.38	1.043	1.812	0.	1.39

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	572	596	738	0	0	0	0	0
normalized size	1	1.04	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	1.715	0.418	0.346	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	423	546	0	0	0	0	0
normalized size	1	1.06	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	1.024	0.239	0.339	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	248	237	0	0	0	0	0
normalized size	1	1.09	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.533	0.187	0.341	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	389	0	0	0	0	0
normalized size	1	1.	4.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.209	0.351	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	212	374	0	383	798	0	0
normalized size	1	0.79	1.39	0.	1.42	2.97	0.	0.
time (sec)	N/A	0.311	0.36	0.576	1.07	1.904	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	355	698	0	522	1310	0	0
normalized size	1	0.74	1.46	0.	1.09	2.73	0.	0.
time (sec)	N/A	0.48	0.34	0.359	1.086	2.048	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	759	736	1006	0	0	0	0	0
normalized size	1	0.97	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	2.975	1.587	0.338	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	410	675	0	0	0	0	0
normalized size	1	0.94	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	1.042	0.742	0.534	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	527	0	0	0	0	0
normalized size	1	1.	3.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	0.319	0.352	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	666	0	864	1781	0	0
normalized size	1	1.	1.52	0.	1.97	4.07	0.	0.
time (sec)	N/A	0.452	0.794	0.348	1.165	2.22	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	907	907	962	0	1166	3106	0	0
normalized size	1	1.	1.06	0.	1.29	3.42	0.	0.
time (sec)	N/A	1.001	1.633	0.353	1.178	2.421	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	132	385	0	139
normalized size	1	1.	0.94	0.	0.92	2.69	0.	0.97
time (sec)	N/A	0.105	0.109	0.6	1.032	1.959	0.	1.357

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	65	0	0	971	0	131
normalized size	1	1.	0.54	0.	0.	8.02	0.	1.08
time (sec)	N/A	0.072	0.015	0.346	0.	2.071	0.	1.366

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	0	85	277	0	97
normalized size	1	1.	0.97	0.	0.9	2.95	0.	1.03
time (sec)	N/A	0.062	0.023	0.358	1.041	1.88	0.	1.27

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	168	0	664	61	77
normalized size	1	1.	0.82	2.58	0.	10.22	0.94	1.18
time (sec)	N/A	0.042	0.013	0.204	0.	1.993	161.897	1.325

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	171	0	0	0
normalized size	1	1.	1.	0.	3.11	0.	0.	0.
time (sec)	N/A	0.051	0.011	0.417	2.412	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	80	0	0	554	0	99
normalized size	1	1.	1.04	0.	0.	7.19	0.	1.29
time (sec)	N/A	0.051	0.049	0.349	0.	1.872	0.	1.344

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	119	205	0	140
normalized size	1	1.	1.06	0.	1.34	2.3	0.	1.57
time (sec)	N/A	0.068	0.033	0.353	1.036	1.789	0.	1.453

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	137	0	0	811	0	139
normalized size	1	1.	1.04	0.	0.	6.14	0.	1.05
time (sec)	N/A	0.088	0.064	0.349	0.	1.899	0.	1.287

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	436	968	0	0	0	0	0
normalized size	1	1.06	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	1.017	0.435	0.359	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	264	542	0	0	0	0	0
normalized size	1	1.1	2.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.484	0.443	0.35	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	199	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.143	0.352	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	217	691	0	402	695	0	0
normalized size	1	0.79	2.5	0.	1.46	2.52	0.	0.
time (sec)	N/A	0.301	0.528	0.415	1.059	1.861	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	355	1021	0	536	1137	0	0
normalized size	1	0.74	2.12	0.	1.11	2.36	0.	0.
time (sec)	N/A	0.476	0.838	0.352	1.097	1.874	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	735	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.807	2.229	0.347	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	523	0	0	0	0	0
normalized size	1	1.	1.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.445	1.136	0.356	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	598	0	0	0	0	0
normalized size	1	1.	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.594	1.267	0.36	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	773	746	1014	0	0	0	0	0
normalized size	1	0.97	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	3.019	2.232	0.351	0.	0.	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	428	683	0	0	0	0	0
normalized size	1	0.95	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	1.003	1.352	0.352	0.	0.	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	341	0	0	0	0	0
normalized size	1	1.	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.251	0.362	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	692	0	923	1600	0	0
normalized size	1	1.	1.54	0.	2.06	3.56	0.	0.
time (sec)	N/A	0.463	1.434	0.358	1.151	1.962	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	1277	0	764	0	0	0	0	0
normalized size	1	0.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	3.169	4.829	0.355	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	737	0	824	0	0	0	0	0
normalized size	1	0.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	1.297	5.688	0.345	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	482	0	1097	0	0	0	0	0
normalized size	1	0.	2.28	0.	0.	0.	0.	0.
time (sec)	N/A	1.383	2.238	0.341	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	783	0	2858	0	0	0	0	0
normalized size	1	0.	3.65	0.	0.	0.	0.	0.
time (sec)	N/A	3.625	9.008	0.351	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	435	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.336	0.986	0.489	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	325	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.864	0.893	0.083	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	229	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.543	0.422	0.087	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	130	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.124	0.086	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.264	0.086	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.317	0.087	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.404	0.473	0.571	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	677	677	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.982	0.326	0.088	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	445	445	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.627	0.238	0.087	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.261	0.117	0.089	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.116	0.097	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.12	0.09	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.646	0.483	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.175	0.351	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.304	0.342	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	131	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	0.171	0.361	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	325	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.848	0.778	0.326	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	926	926	525	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.557	3.255	0.332	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.225	0.607	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.104	0.327	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.165	0.332	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.288	0.139	0.334	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	676	676	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.995	0.143	0.333	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1141	1141	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.738	0.133	0.333	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1121	1121	670	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.868	2.662	0.484	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	831	831	501	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.352	0.972	0.088	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	325	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.846	0.911	0.089	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	174	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.393	0.195	0.09	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.248	0.088	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.359	0.091	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1363	1363	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.133	0.65	0.576	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1035	1035	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.566	0.44	0.089	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.971	0.303	0.09	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	338	338	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.46	0.127	0.088	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.117	0.086	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.116	0.085	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	325	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.869	0.92	0.475	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	181	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.381	0.237	0.089	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.285	0.094	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.386	0.086	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.719	0.086	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.17	0.09	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.467	0.085	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	675	675	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.971	0.492	0.594	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	347	347	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.453	0.284	0.086	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.154	0.089	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.155	0.084	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.172	0.085	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.09	0.085	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.124	0.089	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.815	0.501	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.183	0.332	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.309	0.335	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	175	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.392	0.234	0.332	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	325	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.833	0.771	0.333	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	832	832	502	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.303	0.83	0.331	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.229	0.574	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.11	0.335	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.165	0.329	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.478	0.14	0.335	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.972	0.134	0.331	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1036	1036	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.519	0.132	0.331	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.851	0.505	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.722	0.372	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.555	0.319	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.23	0.309	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.456	0.309	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.396	0.309	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.291	0.573	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.204	0.306	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.242	0.308	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.114	0.314	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.165	0.311	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.144	0.312	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	344	0	0	2461	0	694
normalized size	1	1.	0.55	0.	0.	3.9	0.	1.1
time (sec)	N/A	0.895	0.458	1.281	0.	1.932	0.	1.416

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	603	603	316	0	0	2245	0	576
normalized size	1	1.	0.52	0.	0.	3.72	0.	0.96
time (sec)	N/A	0.795	0.477	1.149	0.	1.983	0.	1.587

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	588	588	271	0	0	2557	0	599
normalized size	1	1.	0.46	0.	0.	4.35	0.	1.02
time (sec)	N/A	0.741	0.433	1.139	0.	1.953	0.	1.652

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	620	620	309	0	0	2877	0	647
normalized size	1	1.	0.5	0.	0.	4.64	0.	1.04
time (sec)	N/A	0.799	0.183	1.239	0.	2.09	0.	1.621

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	641	641	100	0	0	2996	0	659
normalized size	1	1.	0.16	0.	0.	4.67	0.	1.03
time (sec)	N/A	0.827	0.069	1.201	0.	2.126	0.	1.615

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1002	1002	588	0	0	4782	0	1107
normalized size	1	1.	0.59	0.	0.	4.77	0.	1.1
time (sec)	N/A	1.309	1.334	1.281	0.	2.36	0.	1.384

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	949	949	436	0	0	4340	0	869
normalized size	1	1.	0.46	0.	0.	4.57	0.	0.92
time (sec)	N/A	1.261	0.83	1.332	0.	2.43	0.	1.738

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	932	932	503	0	0	4329	0	863
normalized size	1	1.	0.54	0.	0.	4.64	0.	0.93
time (sec)	N/A	1.215	0.96	1.299	0.	2.464	0.	1.648

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	935	935	340	0	0	4817	0	892
normalized size	1	1.	0.36	0.	0.	5.15	0.	0.95
time (sec)	N/A	1.183	0.952	1.289	0.	2.46	0.	1.675

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	968	968	294	0	0	5276	0	911
normalized size	1	1.	0.3	0.	0.	5.45	0.	0.94
time (sec)	N/A	1.232	0.263	1.354	0.	2.562	0.	1.586

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1680	1680	1471	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	3.085	1.347	1.358	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1361	1361	1297	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	1.831	0.388	1.306	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1659	1659	1336	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	2.529	1.638	1.321	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	191	0	93	0	0
normalized size	1	1.	1.	5.79	0.	2.82	0.	0.
time (sec)	N/A	0.026	0.012	2.384	0.	1.565	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	210	1373	0	257	0	0
normalized size	1	1.	2.8	18.31	0.	3.43	0.	0.
time (sec)	N/A	0.12	0.129	0.345	0.	1.586	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	659	0	0	1072	0	0
normalized size	1	1.	4.09	0.	0.	6.66	0.	0.
time (sec)	N/A	0.224	0.256	0.51	0.	1.774	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	456	0	0	724	0	0
normalized size	1	1.	3.45	0.	0.	5.48	0.	0.
time (sec)	N/A	0.188	0.228	0.458	0.	1.753	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	265	0	0	423	0	0
normalized size	1	1.	2.6	0.	0.	4.15	0.	0.
time (sec)	N/A	0.145	0.18	0.46	0.	1.676	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	189	0	171	0	0
normalized size	1	1.	1.	3.86	0.	3.49	0.	0.
time (sec)	N/A	0.053	0.015	3.542	0.	1.666	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.292	0.543	0.474	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	2.	0.48	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	10.606	0.468	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	65	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.024	2.597	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	147	145	0	3071	0	402
normalized size	1	1.	0.87	0.86	0.	18.17	0.	2.38
time (sec)	N/A	0.206	0.099	0.569	0.	9.785	0.	1.424

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	98	0	458	0	130
normalized size	1	1.	1.	1.56	0.	7.27	0.	2.06
time (sec)	N/A	0.048	0.035	0.171	0.	1.634	0.	1.262

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	47	57	73	108	85	93
normalized size	1	1.	1.34	1.63	2.09	3.09	2.43	2.66
time (sec)	N/A	0.016	0.031	0.122	1.017	1.5	1.517	1.174

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	56	74	88	163	109	116
normalized size	1	1.	1.24	1.64	1.96	3.62	2.42	2.58
time (sec)	N/A	0.027	0.047	0.131	1.021	1.602	2.38	1.3

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	115	0	660	0	185
normalized size	1	1.	1.03	1.95	0.	11.19	0.	3.14
time (sec)	N/A	0.036	0.047	0.133	0.	1.743	0.	4.573

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	66	157	0	3146	0	0
normalized size	1	1.	0.4	0.95	0.	19.07	0.	0.
time (sec)	N/A	0.173	0.333	0.554	0.	9.939	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.407	0.659	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	739	0	0	0	0	0
normalized size	1	1.	3.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	1.365	0.599	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	415	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	0.686	0.095	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	219	0	0	0	0	0
normalized size	1	1.	1.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.27	0.134	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	70	81	95	190	114	123
normalized size	1	1.	1.4	1.62	1.9	3.8	2.28	2.46
time (sec)	N/A	0.038	0.041	0.106	1.025	1.943	2.297	1.32

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.538	0.123	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.859	0.135	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [134] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	16	0.188
2	A	4	3	1.	16	0.188
3	A	4	3	1.	16	0.188
4	A	3	3	1.	14	0.214
5	A	3	3	1.	12	0.25
6	A	3	3	1.	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
7	A	2	2	1.	16	0.125
8	A	5	5	1.18	16	0.312
9	A	3	3	1.	16	0.188
10	A	4	3	1.	16	0.188
11	A	4	3	1.	16	0.188
12	A	4	3	1.	16	0.188
13	A	4	3	1.	16	0.188
14	A	9	8	1.	16	0.5
15	A	9	8	1.	16	0.5
16	A	3	3	1.	16	0.188
17	A	8	8	1.	14	0.571
18	A	8	8	1.	12	0.667
19	A	3	3	1.	16	0.188
20	A	7	7	1.	16	0.438
21	A	7	7	1.	16	0.438
22	A	5	5	1.	16	0.312
23	A	8	8	1.	16	0.5
24	A	8	8	1.	16	0.5
25	A	4	3	1.	16	0.188
26	A	4	3	1.	16	0.188
27	A	4	3	1.	16	0.188
28	A	4	3	1.	16	0.188
29	A	4	3	1.	14	0.214
30	A	3	3	1.	12	0.25
31	A	3	3	1.	16	0.188
32	A	3	3	1.	16	0.188
33	A	4	3	1.	16	0.188
34	A	4	3	1.	16	0.188
35	A	4	3	1.	16	0.188
36	A	5	4	1.	16	0.25
37	A	5	4	1.	16	0.25
38	A	4	4	1.	16	0.25
39	A	3	3	1.	14	0.214
40	A	3	3	1.	12	0.25
41	A	3	3	1.	16	0.188
42	A	4	4	1.	16	0.25
43	A	3	3	1.	16	0.188
44	A	5	4	1.	16	0.25
45	A	1	1	1.	12	0.083
46	A	4	3	1.	18	0.167
47	A	4	3	1.	18	0.167
48	A	4	3	1.	16	0.188
49	A	4	3	1.	14	0.214
50	A	3	3	1.	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
51	A	4	3	1.	18	0.167
52	A	4	3	1.	18	0.167
53	A	4	3	1.	18	0.167
54	A	3	3	1.	16	0.188
55	A	3	3	1.	18	0.167
56	A	3	3	1.	18	0.167
57	A	2	2	1.	16	0.125
58	A	4	4	1.	18	0.222
59	A	4	4	1.	18	0.222
60	A	4	4	1.	18	0.222
61	A	4	4	1.	20	0.2
62	A	5	5	1.	20	0.25
63	A	3	3	1.	18	0.167
64	A	5	4	1.	22	0.182
65	A	5	4	1.	22	0.182
66	A	5	4	1.	20	0.2
67	A	4	4	1.	19	0.21
68	A	6	6	1.	22	0.273
69	A	5	4	1.	22	0.182
70	A	2	2	1.	16	0.125
71	A	2	2	1.	14	0.143
72	A	2	2	1.	12	0.167
73	A	3	3	1.	16	0.188
74	A	2	2	1.	16	0.125
75	A	2	2	1.	16	0.125
76	A	2	2	1.	16	0.125
77	A	8	8	0.81	18	0.444
78	A	9	8	1.	18	0.444
79	A	4	4	1.	16	0.25
80	A	5	5	1.	18	0.278
81	A	4	4	1.	18	0.222
82	A	10	10	1.14	18	0.556
83	A	14	12	1.09	18	0.667
84	A	20	13	1.	18	0.722
85	A	16	13	1.	18	0.722
86	A	12	11	1.	14	0.786
87	A	7	8	1.	18	0.444
88	A	11	10	1.	18	0.556
89	A	14	11	1.	18	0.611
90	A	18	11	1.	18	0.611
91	A	15	8	1.	18	0.444
92	A	11	8	1.	18	0.444
93	A	5	4	1.	16	0.25
94	A	6	6	1.	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	6	1.	18	0.333
96	A	13	12	1.08	18	0.667
97	A	22	16	0.94	18	0.889
98	A	0	0	0.	0	0.
99	A	0	0	0.	0	0.
100	A	0	0	0.	0	0.
101	A	0	0	0.	0	0.
102	A	9	7	1.	18	0.389
103	A	4	4	1.	16	0.25
104	A	0	0	0.	0	0.
105	A	0	0	0.	0	0.
106	A	0	0	0.	0	0.
107	A	0	0	0.	0	0.
108	A	0	0	0.	0	0.
109	A	13	8	1.	18	0.444
110	A	5	5	1.	16	0.312
111	A	0	0	0.	0	0.
112	A	0	0	0.	0	0.
113	A	0	0	0.	0	0.
114	A	0	0	0.	0	0.
115	A	0	0	0.	0	0.
116	A	18	9	1.	18	0.5
117	A	6	5	1.	16	0.312
118	A	0	0	0.	0	0.
119	A	0	0	0.	0	0.
120	A	0	0	0.	0	0.
121	A	0	0	0.	0	0.
122	A	0	0	0.	0	0.
123	A	8	7	1.	16	0.438
124	A	3	3	1.	14	0.214
125	A	11	8	1.	16	0.5
126	A	4	4	1.	14	0.286
127	A	15	9	1.	16	0.562
128	A	5	4	1.	14	0.286
129	A	9	8	1.	18	0.444
130	A	4	4	1.	18	0.222
131	A	5	5	1.	18	0.278
132	A	4	4	1.	18	0.222
133	A	49	19	1.	16	1.187
134	A	49	20	1.	14	1.429
135	A	39	11	1.01	18	0.611
136	A	39	11	1.01	18	0.611
137	A	48	18	1.	18	1.
138	A	12	7	1.	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
139	A	9	7	1.	18	0.389
140	A	4	4	1.	18	0.222
141	A	0	0	0.	0	0.
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	0	0	0.	0	0.
145	A	0	0	0.	0	0.
146	A	0	0	0.	0	0.
147	A	0	0	0.	0	0.
148	A	21	8	1.	18	0.444
149	A	13	8	1.	18	0.444
150	A	5	5	1.	18	0.278
151	A	0	0	0.	0	0.
152	A	0	0	0.	0	0.
153	A	0	0	0.	0	0.
154	A	0	0	0.	0	0.
155	A	0	0	0.	0	0.
156	A	0	0	0.	0	0.
157	A	0	0	0.	0	0.
158	A	0	0	0.	0	0.
159	A	0	0	0.	0	0.
160	A	3	3	1.	18	0.167
161	A	0	0	0.	0	0.
162	A	0	0	0.	0	0.
163	A	9	9	0.75	24	0.375
164	A	10	9	1.	24	0.375
165	A	5	5	1.	22	0.227
166	A	6	6	1.	21	0.286
167	A	5	5	1.	24	0.208
168	A	11	11	1.19	24	0.458
169	A	1	1	1.	12	0.083
170	A	3	3	1.	12	0.25
171	A	3	3	1.	14	0.214
172	A	3	3	1.	14	0.214
173	A	3	3	1.	16	0.188
174	A	5	5	1.	18	0.278
175	A	6	6	1.	18	0.333
176	A	3	2	1.	18	0.111
177	A	3	2	1.	18	0.111
178	A	3	2	1.	16	0.125
179	A	2	2	1.	10	0.2
180	A	3	3	1.	18	0.167
181	A	4	3	1.	18	0.167
182	A	3	2	1.	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
183	A	3	2	1.	18	0.111
184	A	6	5	1.	20	0.25
185	A	6	5	1.	20	0.25
186	A	6	5	1.	18	0.278
187	A	3	3	1.	12	0.25
188	A	9	6	1.	20	0.3
189	A	6	5	1.	20	0.25
190	A	6	5	1.	20	0.25
191	A	13	11	1.	20	0.55
192	A	12	11	1.	20	0.55
193	A	11	10	1.	18	0.556
194	A	8	8	1.	12	0.667
195	A	12	6	1.	20	0.3
196	A	11	10	1.	20	0.5
197	A	11	10	1.	20	0.5
198	A	4	3	1.	20	0.15
199	A	4	3	1.	20	0.15
200	A	4	3	1.	18	0.167
201	A	8	7	1.	20	0.35
202	A	4	3	1.	20	0.15
203	A	4	3	1.	20	0.15
204	A	4	3	1.	20	0.15
205	A	8	7	1.	16	0.438
206	A	6	3	1.	20	0.15
207	A	5	3	1.	20	0.15
208	A	2	2	1.	18	0.111
209	A	5	5	1.	20	0.25
210	A	9	6	1.	20	0.3
211	A	0	0	0.	0	0.
212	A	8	4	1.	20	0.2
213	A	7	4	1.	20	0.2
214	A	6	4	1.	18	0.222
215	A	2	2	1.	12	0.167
216	A	0	0	0.	0	0.
217	A	0	0	0.	0	0.
218	A	0	0	0.	0	0.
219	A	13	8	1.	21	0.381
220	A	10	8	1.	21	0.381
221	A	7	7	1.	19	0.368
222	A	3	3	1.	18	0.167
223	A	7	8	1.	21	0.381
224	A	11	10	1.	21	0.476
225	A	14	10	1.	21	0.476
226	A	21	15	1.	23	0.652

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
227	A	17	13	1.	23	0.565
228	A	14	10	1.	21	0.476
229	A	9	6	1.	20	0.3
230	A	14	9	1.	23	0.391
231	A	16	11	1.	23	0.478
232	A	21	15	1.	23	0.652
233	A	33	20	1.	23	0.87
234	A	30	17	1.	23	0.739
235	A	22	15	1.	21	0.714
236	A	12	6	1.	20	0.3
237	A	17	9	1.	23	0.391
238	A	24	16	1.	23	0.696
239	A	31	17	1.	23	0.739
240	A	21	14	1.	23	0.609
241	A	17	14	1.	23	0.609
242	A	13	11	1.	21	0.524
243	A	8	7	1.	20	0.35
244	A	13	9	1.	23	0.391
245	A	16	11	1.	23	0.478
246	A	20	13	1.	23	0.565
247	A	25	14	1.	23	0.609
248	A	21	12	1.	23	0.522
249	A	18	11	1.	21	0.524
250	A	13	7	1.	20	0.35
251	A	18	9	1.	23	0.391
252	A	22	13	1.	23	0.565
253	A	25	15	1.	23	0.652
254	A	37	18	1.	23	0.783
255	A	34	18	1.	23	0.783
256	A	26	16	1.	21	0.762
257	A	16	7	1.	20	0.35
258	A	21	9	1.	23	0.391
259	A	30	18	1.	23	0.783
260	A	39	19	1.	23	0.826
261	A	16	9	1.	22	0.409
262	A	12	8	1.	22	0.364
263	A	8	4	1.	20	0.2
264	A	12	12	1.	22	0.546
265	A	18	12	1.	22	0.546
266	A	19	8	1.	24	0.333
267	A	20	10	1.	24	0.417
268	A	17	6	1.	22	0.273
269	A	13	6	1.	22	0.273
270	A	9	6	1.	20	0.3

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
271	A	12	8	1.	22	0.364
272	A	26	13	1.	22	0.591
273	A	50	15	1.	24	0.625
274	A	30	15	1.	22	0.682
275	A	0	0	0.	0	0.
276	A	0	0	0.	0	0.
277	A	0	0	0.	0	0.
278	A	0	0	0.	0	0.
279	A	0	0	0.	0	0.
280	A	0	0	0.	0	0.
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	0	0	0.	0	0.
284	A	0	0	0.	0	0.
285	A	0	0	0.	0	0.
286	A	0	0	0.	0	0.
287	A	0	0	0.	0	0.
288	A	17	9	1.	22	0.409
289	A	13	9	1.	22	0.409
290	A	9	7	1.	20	0.35
291	A	29	7	1.	22	0.318
292	A	47	11	1.	22	0.5
293	A	55	29	0.93	24	1.208
294	A	47	23	1.	24	0.958
295	A	23	20	1.	22	0.909
296	A	0	0	0.	0	0.
297	A	0	0	0.	0	0.
298	A	0	0	0.	0	0.
299	A	0	0	0.	0	0.
300	A	0	0	0.	0	0.
301	A	0	0	0.	0	0.
302	A	0	0	0.	0	0.
303	A	0	0	0.	0	0.
304	A	0	0	0.	0	0.
305	A	0	0	0.	0	0.
306	A	0	0	0.	0	0.
307	A	0	0	0.	0	0.
308	A	0	0	0.	0	0.
309	A	0	0	0.	0	0.
310	A	5	5	1.	23	0.217
311	A	5	5	1.	23	0.217
312	A	4	3	1.	21	0.143
313	A	7	7	1.	23	0.304
314	A	9	9	1.	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
315	A	5	5	1.	23	0.217
316	A	5	5	1.	23	0.217
317	A	5	5	1.	23	0.217
318	A	10	4	1.	23	0.174
319	A	9	6	1.	20	0.3
320	A	7	5	1.29	23	0.217
321	A	7	4	1.	23	0.174
322	A	9	4	1.	23	0.174
323	A	5	5	1.	25	0.2
324	A	5	5	1.	25	0.2
325	A	4	3	1.	23	0.13
326	A	10	8	1.	25	0.32
327	A	11	11	1.	25	0.44
328	A	12	10	1.	25	0.4
329	A	5	5	1.	25	0.2
330	A	5	5	1.	25	0.2
331	A	5	5	1.	25	0.2
332	A	14	4	1.	25	0.16
333	A	13	6	1.	22	0.273
334	A	11	6	1.	25	0.24
335	A	10	6	1.	25	0.24
336	A	11	4	1.	25	0.16
337	A	14	4	1.	25	0.16
338	A	11	9	1.	25	0.36
339	A	8	8	1.	25	0.32
340	A	4	4	1.	23	0.174
341	A	8	9	1.	25	0.36
342	A	12	11	1.	25	0.44
343	A	21	13	1.	25	0.52
344	A	17	11	1.	25	0.44
345	A	12	8	1.	22	0.364
346	A	16	10	1.	25	0.4
347	A	19	11	1.	25	0.44
348	A	12	11	1.	25	0.44
349	A	10	9	1.	25	0.36
350	A	5	4	1.	23	0.174
351	A	12	10	1.	25	0.4
352	A	16	11	1.	25	0.44
353	A	43	16	1.	25	0.64
354	A	40	14	1.	25	0.56
355	A	26	13	1.	22	0.591
356	A	42	15	1.	25	0.6
357	A	6	7	1.	22	0.318
358	A	12	8	1.	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
359	A	11	7	1.	18	0.389
360	A	8	7	1.	25	0.28
361	A	8	7	1.	25	0.28
362	A	7	7	1.	23	0.304
363	A	9	9	1.	25	0.36
364	A	8	7	1.	25	0.28
365	A	11	7	1.	27	0.259
366	A	11	7	1.	27	0.259
367	A	10	8	1.	25	0.32
368	A	12	11	1.	27	0.407
369	A	11	9	1.	27	0.333
370	A	13	11	1.	27	0.407
371	A	8	9	1.	25	0.36
372	A	5	5	1.	27	0.185
373	A	9	7	1.	27	0.259
374	A	19	14	1.	27	0.518
375	A	12	10	1.	25	0.4
376	A	10	10	1.	27	0.37
377	A	17	14	1.	27	0.518
378	A	4	4	1.	33	0.121
379	A	3	3	1.	29	0.103
380	A	4	4	1.	33	0.121
381	A	0	0	0.	0	0.
382	A	0	0	0.	0	0.
383	A	0	0	0.	0	0.
384	A	0	0	0.	0	0.
385	A	0	0	0.	0	0.
386	A	0	0	0.	0	0.
387	A	0	0	0.	0	0.
388	A	0	0	0.	0	0.
389	A	4	4	1.	14	0.286
390	A	1	1	1.5	12	0.083
391	A	2	2	1.	14	0.143
392	A	2	2	1.	16	0.125
393	A	4	4	1.	14	0.286
394	A	4	4	1.	16	0.25
395	A	4	4	1.	18	0.222
396	A	9	8	1.	18	0.444
397	A	14	8	1.	20	0.4
398	A	0	0	0.	0	0.
399	A	3	3	1.	22	0.136
400	A	4	3	1.	22	0.136
401	A	4	3	1.	22	0.136
402	A	4	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
403	A	5	3	1.	18	0.167
404	A	3	3	1.	22	0.136
405	A	4	3	1.	22	0.136
406	A	4	3	1.	22	0.136
407	A	4	3	1.	22	0.136
408	A	8	8	0.74	24	0.333
409	A	8	8	0.77	22	0.364
410	A	10	8	1.	20	0.4
411	A	5	5	1.	24	0.208
412	A	10	10	1.14	24	0.417
413	A	18	12	1.09	24	0.5
414	A	26	12	1.06	24	0.5
415	A	28	8	1.	24	0.333
416	A	20	8	1.	22	0.364
417	A	12	8	1.	20	0.4
418	A	6	6	1.	24	0.25
419	A	13	12	1.08	24	0.5
420	A	35	17	0.96	24	0.708
421	A	4	3	1.	22	0.136
422	A	4	3	1.	22	0.136
423	A	4	3	1.	20	0.15
424	A	6	4	1.	18	0.222
425	A	3	3	1.	22	0.136
426	A	4	3	1.	22	0.136
427	A	4	3	1.	22	0.136
428	A	4	3	1.	22	0.136
429	A	26	12	1.06	24	0.5
430	A	18	12	1.08	22	0.546
431	A	11	11	1.14	20	0.55
432	A	5	5	1.	24	0.208
433	A	10	8	1.	24	0.333
434	A	8	8	0.77	24	0.333
435	A	8	8	0.74	24	0.333
436	A	35	17	0.96	22	0.773
437	A	14	13	1.08	20	0.65
438	A	6	6	1.	24	0.25
439	A	12	8	1.	24	0.333
440	A	20	8	1.	24	0.333
441	A	28	8	1.	24	0.333
442	A	4	3	1.	22	0.136
443	A	4	3	1.	22	0.136
444	A	4	3	1.	20	0.15
445	A	5	3	1.	18	0.167
446	A	3	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
447	A	4	3	1.	22	0.136
448	A	4	3	1.	22	0.136
449	A	4	3	1.	22	0.136
450	A	8	8	0.72	24	0.333
451	A	8	8	0.74	22	0.364
452	A	8	8	0.79	20	0.4
453	A	5	5	1.	24	0.208
454	A	14	12	1.1	24	0.5
455	A	26	12	1.06	24	0.5
456	A	52	8	1.	24	0.333
457	A	40	8	1.	24	0.333
458	A	28	8	1.	22	0.364
459	A	16	8	1.	20	0.4
460	A	6	6	1.	24	0.25
461	A	22	16	0.94	24	0.667
462	A	73	17	0.97	24	0.708
463	A	4	3	1.	22	0.136
464	A	5	4	1.	22	0.182
465	A	4	3	1.	20	0.15
466	A	6	4	1.	18	0.222
467	A	3	3	1.	22	0.136
468	A	4	4	1.	22	0.182
469	A	4	3	1.	22	0.136
470	A	7	4	1.	22	0.182
471	A	8	8	0.74	24	0.333
472	A	8	8	0.79	22	0.364
473	A	5	5	1.	24	0.208
474	A	14	12	1.1	24	0.5
475	A	26	12	1.06	24	0.5
476	A	30	14	1.	24	0.583
477	A	18	14	1.	20	0.7
478	A	12	11	1.	24	0.458
479	A	24	12	1.	24	0.5
480	A	45	12	1.	24	0.5
481	A	28	8	1.	24	0.333
482	A	16	8	1.	22	0.364
483	A	6	6	1.	24	0.25
484	A	22	16	0.95	24	0.667
485	A	0	0	0.	0	0.
486	A	0	0	0.	0	0.
487	A	0	0	0.	0	0.
488	A	0	0	0.	0	0.
489	A	4	3	1.	22	0.136
490	A	4	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
491	A	4	3	1.	20	0.15
492	A	6	4	1.	18	0.222
493	A	3	3	1.	22	0.136
494	A	4	3	1.	22	0.136
495	A	4	3	1.	22	0.136
496	A	4	3	1.	22	0.136
497	A	38	12	1.04	24	0.5
498	A	26	12	1.06	22	0.546
499	A	15	13	1.09	20	0.65
500	A	5	5	1.	24	0.208
501	A	8	8	0.79	24	0.333
502	A	8	8	0.74	24	0.333
503	A	73	17	0.97	22	0.773
504	A	23	17	0.94	20	0.85
505	A	6	6	1.	24	0.25
506	A	16	8	1.	24	0.333
507	A	28	8	1.	24	0.333
508	A	4	3	1.	22	0.136
509	A	6	5	1.	22	0.227
510	A	4	3	1.	20	0.15
511	A	6	5	1.	18	0.278
512	A	3	3	1.	22	0.136
513	A	6	5	1.	22	0.227
514	A	4	3	1.	22	0.136
515	A	9	5	1.	22	0.227
516	A	26	12	1.06	24	0.5
517	A	14	12	1.1	22	0.546
518	A	5	5	1.	24	0.208
519	A	8	8	0.79	24	0.333
520	A	8	8	0.74	24	0.333
521	A	28	17	1.	24	0.708
522	A	14	13	1.	20	0.65
523	A	19	14	1.	24	0.583
524	A	73	17	0.97	24	0.708
525	A	22	16	0.95	22	0.727
526	A	6	6	1.	24	0.25
527	A	16	8	1.	24	0.333
528	A	0	0	0.	0	0.
529	A	0	0	0.	0	0.
530	A	0	0	0.	0	0.
531	A	0	0	0.	0	0.
532	A	27	7	1.	22	0.318
533	A	21	7	1.	22	0.318
534	A	15	7	1.	20	0.35

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
535	A	9	7	1.	18	0.389
536	A	0	0	0.	0	0.
537	A	0	0	0.	0	0.
538	A	27	7	1.	24	0.292
539	A	21	7	1.	24	0.292
540	A	15	7	1.	22	0.318
541	A	9	7	1.	20	0.35
542	A	0	0	0.	0	0.
543	A	0	0	0.	0	0.
544	A	0	0	0.	0	0.
545	A	0	0	0.	0	0.
546	A	0	0	0.	0	0.
547	A	9	7	1.	22	0.318
548	A	21	7	1.	22	0.318
549	A	33	7	1.	22	0.318
550	A	0	0	0.	0	0.
551	A	0	0	0.	0	0.
552	A	0	0	0.	0	0.
553	A	9	7	1.	24	0.292
554	A	21	7	1.	24	0.292
555	A	33	7	1.	24	0.292
556	A	39	7	1.	22	0.318
557	A	30	7	1.	22	0.318
558	A	21	7	1.	20	0.35
559	A	12	7	1.	18	0.389
560	A	0	0	0.	0	0.
561	A	0	0	0.	0	0.
562	A	39	7	1.	24	0.292
563	A	30	7	1.	24	0.292
564	A	21	7	1.	22	0.318
565	A	12	7	1.	20	0.35
566	A	0	0	0.	0	0.
567	A	0	0	0.	0	0.
568	A	21	7	1.	22	0.318
569	A	12	7	1.	20	0.35
570	A	0	0	0.	0	0.
571	A	0	0	0.	0	0.
572	A	0	0	0.	0	0.
573	A	0	0	0.	0	0.
574	A	0	0	0.	0	0.
575	A	21	7	1.	24	0.292
576	A	12	7	1.	22	0.318
577	A	0	0	0.	0	0.
578	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
579	A	0	0	0.	0	0.
580	A	0	0	0.	0	0.
581	A	0	0	0.	0	0.
582	A	0	0	0.	0	0.
583	A	0	0	0.	0	0.
584	A	0	0	0.	0	0.
585	A	12	7	1.	22	0.318
586	A	21	7	1.	22	0.318
587	A	30	7	1.	22	0.318
588	A	0	0	0.	0	0.
589	A	0	0	0.	0	0.
590	A	0	0	0.	0	0.
591	A	12	7	1.	24	0.292
592	A	21	7	1.	24	0.292
593	A	30	7	1.	24	0.292
594	A	0	0	0.	0	0.
595	A	0	0	0.	0	0.
596	A	0	0	0.	0	0.
597	A	0	0	0.	0	0.
598	A	0	0	0.	0	0.
599	A	0	0	0.	0	0.
600	A	0	0	0.	0	0.
601	A	0	0	0.	0	0.
602	A	0	0	0.	0	0.
603	A	0	0	0.	0	0.
604	A	0	0	0.	0	0.
605	A	0	0	0.	0	0.
606	A	26	12	1.	29	0.414
607	A	25	12	1.	29	0.414
608	A	23	10	1.	29	0.345
609	A	24	11	1.	29	0.379
610	A	25	11	1.	29	0.379
611	A	38	13	1.	31	0.419
612	A	36	12	1.	31	0.387
613	A	35	12	1.	31	0.387
614	A	34	11	1.	31	0.355
615	A	35	11	1.	31	0.355
616	A	39	20	1.	31	0.645
617	A	25	11	1.	31	0.355
618	A	37	19	1.	31	0.613
619	A	2	2	1.	18	0.111
620	A	3	3	1.	23	0.13
621	A	6	5	1.	28	0.179
622	A	5	5	1.	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
623	A	4	4	1.	26	0.154
624	A	3	3	1.	20	0.15
625	A	0	0	0.	0	0.
626	A	0	0	0.	0	0.
627	A	0	0	0.	0	0.
628	A	3	3	1.	16	0.188
629	A	9	9	1.	16	0.562
630	A	4	4	1.	16	0.25
631	A	3	3	1.	14	0.214
632	A	4	4	1.	16	0.25
633	A	4	4	1.	16	0.25
634	A	9	9	1.	16	0.562
635	A	0	0	0.	0	0.
636	A	8	8	1.	22	0.364
637	A	7	7	1.	22	0.318
638	A	5	5	1.	22	0.227
639	A	5	4	1.	20	0.2
640	A	0	0	0.	0	0.
641	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^4 \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=80

$$-\frac{2a^2px}{5b^2}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx^3}{15b}-\frac{2px^5}{25}$$

```
[Out] (-2*a^2*p*x)/(5*b^2) + (2*a*p*x^3)/(15*b) - (2*p*x^5)/25 + (2*a^(5/2)*p*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p])/5
```

Rubi [A] time = 0.0465048, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 302, 205}

$$-\frac{2a^2px}{5b^2}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx^3}{15b}-\frac{2px^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*a^2*p*x)/(5*b^2) + (2*a*p*x^3)/(15*b) - (2*p*x^5)/25 + (2*a^(5/2)*p*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p])/5
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int x^4 \log\left(c\left(a+bx^2\right)^p\right)dx &= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int\frac{x^6}{a+bx^2}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int\left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx\\
&= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{\left(2a^3p\right)\int\frac{1}{a+bx^2}\,dx}{5b^2}\\
&= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.0444517, size = 74, normalized size = 0.92

$$\frac{1}{75}\left(-\frac{30a^2px}{b^2}+\frac{30a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+15x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{10apx^3}{b}-6px^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((-30*a^2*p*x)/b^2 + (10*a*p*x^3)/b - 6*p*x^5 + (30*a^(5/2)*p*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2) + 15*x^5*Log[c*(a + b*x^2)^p])/75
```

Maple [C] time = 0.499, size = 229, normalized size = 2.9

$$\frac{x^5\ln\left(\left(bx^2+a\right)^p\right)}{5}-\frac{i}{10}\pi x^5\left(\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^3+\frac{i}{10}\pi x^5\left(\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2\mathrm{csgn}\left(ic\right)+\frac{i}{10}\pi x^5\mathrm{csgn}\left(i\left(bx^2+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/5*x^5*ln((b*x^2+a)^p)-1/10*I*Pi*x^5*csgn(I*c*(b*x^2+a)^p)^3+1/10*I*Pi*x^5
*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/10*I*Pi*x^5*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2-1/10*I*Pi*x^5*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*c
sgn(I*c)+1/5*ln(c)*x^5-2/25*p*x^5+2/15*a*p*x^3/b+1/5/b^3*(-a*b)^(1/2)*a^2*p
*ln(-(-a*b)^(1/2)*x+a)-1/5/b^3*(-a*b)^(1/2)*a^2*p*ln((-a*b)^(1/2)*x+a)-2/5*
a^2*p*x/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10261, size = 436, normalized size = 5.45

$$\left[\frac{15\,b^2px^5\log\left(bx^2+a\right)-6\,b^2px^5+15\,b^2x^5\log\left(c\right)+10\,abpx^3+15\,a^2p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\,a^2px}{75\,b^2},\ \frac{15\,b^2px^5\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15*b^2*x^5*log(c) + 10*a
*b*p*x^3 + 15*a^2*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 30*a^2*p*x)/b^2, 1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15
*b^2*x^5*log(c) + 10*a*b*p*x^3 + 30*a^2*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a)
 - 30*a^2*p*x)/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21173, size = 96, normalized size = 1.2

$$\frac{1}{5}\,px^5\log\left(bx^2+a\right)-\frac{1}{25}\left(2\,p-5\,\log\left(c\right)\right)x^5+\frac{2\,apx^3}{15\,b}+\frac{2\,a^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}b^2}-\frac{2\,a^2px}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(b*x^2 + a) - 1/25*(2*p - 5*log(c))*x^5 + 2/15*a*p*x^3/b + 2/5
*a^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 2/5*a^2*p*x/b^2
```

3.2 $\int x^3 \log\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=59

$$-\frac{a^2 p \log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4 \log\left(c\left(a+bx^2\right)^p\right)+\frac{apx^2}{4b}-\frac{px^4}{8}$$

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Rubi [A] time = 0.0492964, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$-\frac{a^2 p \log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4 \log\left(c\left(a+bx^2\right)^p\right)+\frac{apx^2}{4b}-\frac{px^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3 \log\left(c\left(a+bx^2\right)^p\right)dx = \frac{1}{2}\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)dx, x, x^2\right)$$
$$= \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right) - \frac{1}{4}(bp)\,\text{Subst}\left(\int \frac{x^2}{a+bx}dx, x, x^2\right)$$
$$= \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right) - \frac{1}{4}(bp)\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{apx^2}{4b} - \frac{px^4}{8} - \frac{a^2p\log\left(a+bx^2\right)}{4b^2} + \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)$$

Mathematica [A] time = 0.0145955, size = 59, normalized size = 1.

$$-\frac{a^2p\log\left(a+bx^2\right)}{4b^2} + \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right) + \frac{apx^2}{4b} - \frac{px^4}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Maple [C] time = 0.414, size = 183, normalized size = 3.1

$$\frac{x^4\ln\left(\left(bx^2+a\right)^p\right)}{4} - \frac{i}{8}\pi x^4\text{csgn}\left(i\left(bx^2+a\right)^p\right)\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\text{csgn}(ic) + \frac{i}{8}\pi x^4\left(\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2\text{csgn}(ic)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/4*x^4*ln((b*x^2+a)^p)-1/8*I*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)*csgn(I*c)+1/8*I*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/8*I*Pi*x^4*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/8*I*Pi*x^4*csgn(I*c*(b*x^2+a)^
p)^3+1/4*ln(c)*x^4-1/8*p*x^4+1/4*a*p*x^2/b-1/4*a^2*p*ln(b*x^2+a)/b^2
```

Maxima [A] time = 1.10431, size = 74, normalized size = 1.25

$$\frac{1}{4}x^4\log\left(\left(bx^2+a\right)^p c\right) - \frac{1}{8}bp\left(\frac{2a^2\log\left(bx^2+a\right)}{b^3} + \frac{bx^4-2ax^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c) - 1/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4 -
 2*a*x^2)/b^2)
```

Fricas [A] time = 2.0214, size = 127, normalized size = 2.15

$$-\frac{b^2px^4 - 2\,b^2x^4\log{(c)} - 2\,abpx^2 - 2\,\left(b^2px^4 - a^2p\right)\log{\left(bx^2 + a\right)}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*p*x^4 - 2*b^2*x^4*log(c) - 2*a*b*p*x^2 - 2*(b^2*p*x^4 - a^2*p)*lo
g(b*x^2 + a))/b^2
```

Sympy [A] time = 8.44226, size = 70, normalized size = 1.19

$$\begin{cases} -\frac{a^2p\log{\left(a+bx^2\right)}}{4b^2} + \frac{apx^2}{4b} + \frac{px^4\log{\left(a+bx^2\right)}}{4} - \frac{px^4}{8} + \frac{x^4\log{(c)}}{4} & \text{for}\ b \neq 0 \\ \frac{x^4\log{(a^pc)}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-a**2*p*log(a + b*x**2)/(4*b**2) + a*p*x**2/(4*b) + p*x**4*log(a
 + b*x**2)/4 - p*x**4/8 + x**4*log(c)/4, Ne(b, 0)), (x**4*log(a**p*c)/4, Tr
ue))
```

Giac [A] time = 1.22004, size = 131, normalized size = 2.22

$$\frac{\frac{\left(2\left(bx^2+a\right)^2\log\left(bx^2+a\right)-4\left(bx^2+a\right)a\log\left(bx^2+a\right)-\left(bx^2+a\right)^2+4\left(bx^2+a\right)a\right)p}{b} + \frac{2\left(\left(bx^2+a\right)^2-2\left(bx^2+a\right)a\right)\log(c)}{b}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*((2*(b*x^2 + a)^2*log(b*x^2 + a) - 4*(b*x^2 + a)*a*log(b*x^2 + a) - (b*
x^2 + a)^2 + 4*(b*x^2 + a)*a)*p/b + 2*((b*x^2 + a)^2 - 2*(b*x^2 + a)*a)*log
(c)/b)/b
```

3.3 $\int x^2 \log\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=66

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx}{3b}-\frac{2px^3}{9}$$

```
[Out] (2*a*p*x)/(3*b) - (2*p*x^3)/9 - (2*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p])/3
```

Rubi [A] time = 0.036543, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 302, 205}

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx}{3b}-\frac{2px^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (2*a*p*x)/(3*b) - (2*p*x^3)/9 - (2*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p])/3
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int x^2\log\left(c\left(a+bx^2\right)^p\right)dx &= \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(2bp)\int\frac{x^4}{a+bx^2}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(2bp)\int\left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx\\
&= \frac{2apx}{3b}-\frac{2px^3}{9}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{\left(2a^2p\right)\int\frac{1}{a+bx^2}\,dx}{3b}\\
&= \frac{2apx}{3b}-\frac{2px^3}{9}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.0234834, size = 62, normalized size = 0.94

$$\frac{1}{9}\left(-\frac{6a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+3x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{6apx}{b}-2px^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((6*a*p*x)/b - 2*p*x^3 - (6*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
+ 3*x^3*Log[c*(a + b*x^2)^p])/9
```

Maple [C] time = 0.421, size = 217, normalized size = 3.3

$$\frac{x^3\ln\left(\left(bx^2+a\right)^p\right)}{3}-\frac{i}{6}\pi x^3\mathrm{csgn}\left(i\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\right)+\frac{i}{6}\pi x^3\left(\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2\mathrm{csgn}\left(ic\right)+\frac{i}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/3*x^3*ln((b*x^2+a)^p)-1/6*I*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)*csgn(I*c)+1/6*I*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/6*I*Pi*x^3*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/6*I*Pi*x^3*csgn(I*c*(b*x^2+a)^
p)^3+1/3*ln(c)*x^3-2/9*p*x^3+1/3/b^2*(-a*b)^(1/2)*a*p*ln(-(-a*b)^(1/2)*x-a)
-1/3/b^2*(-a*b)^(1/2)*a*p*ln((-a*b)^(1/2)*x-a)+2/3*a*p*x/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98627, size = 350, normalized size = 5.3

$$\left[\frac{3\,bpx^3\log\left(bx^2+a\right)-2\,bpx^3+3\,bx^3\log\left(c\right)+3\,ap\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,apx}{9\,b},\frac{3\,bpx^3\log\left(bx^2+a\right)-2\,bpx^3+3\,bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/9*(3*b*p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) + 3*a*p*sqrt(-a
/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*a*p*x)/b, 1/9*(3*b*
p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) - 6*a*p*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 6*a*p*x)/b]
```

Sympy [A] time = 56.7637, size = 121, normalized size = 1.83

$$\begin{cases} -\frac{ia^{\frac{3}{2}}p\log\left(a+bx^2\right)}{3b^2\sqrt{\frac{1}{b}}} + \frac{2ia^{\frac{3}{2}}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{3b^2\sqrt{\frac{1}{b}}} + \frac{2apx}{3b} + \frac{px^3\log\left(a+bx^2\right)}{3} - \frac{2px^3}{9} + \frac{x^3\log\left(c\right)}{3} & \text{for } b \neq 0 \\ \frac{x^3\log\left(a^pc\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-I*a**(3/2)*p*log(a + b*x**2)/(3*b**2*sqrt(1/b)) + 2*I*a**(3/2)*
p*log(-I*sqrt(a)*sqrt(1/b) + x)/(3*b**2*sqrt(1/b)) + 2*a*p*x/(3*b) + p*x**3
*log(a + b*x**2)/3 - 2*p*x**3/9 + x**3*log(c)/3, Ne(b, 0)), (x**3*log(a**p*
c)/3, True))
```

Giac [A] time = 1.1917, size = 80, normalized size = 1.21

$$\frac{1}{3}px^3\log\left(bx^2+a\right) - \frac{1}{9}\left(2p - 3\log\left(c\right)\right)x^3 - \frac{2a^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\sqrt{ab}b} + \frac{2apx}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(b*x^2 + a) - 1/9*(2*p - 3*log(c))*x^3 - 2/3*a^2*p*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b) + 2/3*a*p*x/b
```

3.4 $\int x \log \left(c \left(a+b x^2\right)^p\right) \, dx$

Optimal. Leaf size=35

$$\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{px^2}{2}$$

```
[Out] -(p*x^2)/2 + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*b)
```

Rubi [A] time = 0.0244695, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2389, 2295}

$$\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{px^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -(p*x^2)/2 + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*b)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \log\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b}\\
&= -\frac{px^2}{2}+\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0088682, size = 34, normalized size = 0.97

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-px^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b)/2
```

Maple [A] time = 0.097, size = 50, normalized size = 1.4

$$\frac{x^2\ln\left(c\left(bx^2+a\right)^p\right)}{2}-\frac{px^2}{2}+\frac{\ln\left(c\left(bx^2+a\right)^p\right)a}{2b}-\frac{ap}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/2*x^2*ln(c*(b*x^2+a)^p)-1/2*p*x^2+1/2/b*ln(c*(b*x^2+a)^p)*a-1/2/b*a*p
```

Maxima [A] time = 1.09696, size = 59, normalized size = 1.69

$$-\frac{1}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2) + 1/2*x^2*log((b*x^2 + a)^p*c)
```

Fricas [A] time = 1.94739, size = 89, normalized size = 2.54

$$-\frac{bpx^2-bx^2\log\left(c\right)-\left(bpx^2+ap\right)\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(b*p*x^2 - b*x^2*log(c) - (b*p*x^2 + a*p)*log(b*x^2 + a))/b
```

Sympy [A] time = 3.69366, size = 56, normalized size = 1.6

$$\begin{cases}\frac{ap\log\left(a+bx^2\right)}{2b}+\frac{px^2\log\left(a+bx^2\right)}{2}-\frac{px^2}{2}+\frac{x^2\log\left(c\right)}{2} & \text{for } b\neq 0\\ \frac{x^2\log\left(a^pc\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((a*p*log(a + b*x**2)/(2*b) + p*x**2*log(a + b*x**2)/2 - p*x**2/2
+ x**2*log(c)/2, Ne(b, 0)), (x**2*log(a**p*c)/2, True))
```

Giac [A] time = 1.18202, size = 58, normalized size = 1.66

$$-\frac{\left(bx^2 - \left(bx^2 + a\right)\log\left(bx^2 + a\right) + a\right)p - \left(bx^2 + a\right)\log\left(c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p - (b*x^2 + a)*log(c))/b
```

3.5 $\int \log \left(c \left(a+b x^2\right)^p\right) \, dx$

Optimal. Leaf size=45

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rubi [A] time = 0.0183606, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2448, 321, 205}

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+b x^2\right)^p\right) \, dx &= x \log \left(c \left(a+b x^2\right)^p\right)-(2 b p) \int \frac{x^2}{a+b x^2} \, dx\\
&= -2 p x+x \log \left(c \left(a+b x^2\right)^p\right)+(2 a p) \int \frac{1}{a+b x^2} \, dx\\
&= -2 p x+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}+x \log \left(c \left(a+b x^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.0119686, size = 45, normalized size = 1.

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Maple [A] time = 0.066, size = 38, normalized size = 0.8

$$x \ln \left(c \left(b x^2+a\right)^p\right)-2 p x+2 \frac{a p}{\sqrt{a b}} \arctan \left(\frac{b x}{\sqrt{a b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p),x)
```

```
[Out] x*ln(c*(b*x^2+a)^p)-2*p*x+2*p*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06516, size = 250, normalized size = 5.56

$$\left[p x \log \left(b x^2+a\right)+p \sqrt{-\frac{a}{b}} \log \left(\frac{b x^2+2 b x \sqrt{-\frac{a}{b}}-a}{b x^2+a}\right)-2 p x+x \log (c), p x \log \left(b x^2+a\right)+2 p \sqrt{\frac{a}{b}} \arctan \left(\frac{b x \sqrt{\frac{a}{b}}}{a}\right)-2 p x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [A] time = 15.8413, size = 90, normalized size = 2.

$$\begin{cases} \frac{i\sqrt{a}p\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}} + px\log\left(a+bx^2\right) - 2px + x\log(c) & \text{for } b \neq 0 \\ x\log(a^p c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((I*sqrt(a)*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*p*log(-I
*sqrt(a)*sqrt(1/b) + x)/(b*sqrt(1/b)) + p*x*log(a + b*x**2) - 2*p*x + x*log
(c), Ne(b, 0)), (x*log(a**p*c), True))
```

Giac [A] time = 1.19302, size = 55, normalized size = 1.22

$$px\log\left(bx^2+a\right) + \frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \left(2\,p - \log(c)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

3.6 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/2 + (p*PolyLog[2, 1 + (b*x^2)/a])/
2
```

Rubi [A] time = 0.045528, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{1}{2}p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/2 + (p*PolyLog[2, 1 + (b*x^2)/a])/
2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}p\text{Li}_2\left(1+\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.0065284, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(p\text{PolyLog}\left(2,\frac{a+bx^2}{a}\right)+\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a + b*x^2)/a])/2
```

Maple [C] time = 0.434, size = 232, normalized size = 5.3

$$\ln(x)\ln\left(\left(bx^2+a\right)^p\right)-p\ln(x)\ln\left(\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\right)-p\ln(x)\ln\left(\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\right)-p\text{dilog}\left(\left(-bx+\sqrt{-ab}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x,x)
```

```
[Out] ln(x)*ln((b*x^2+a)^p)-p*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p*ln(x)*
ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))-p*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2*I*ln(x)*Pi*csgn(I*(b*x^2+a)
^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)*csgn(I*c)-1/2*I*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(x)*Pi*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+ln(c)*ln(x)
```

Maxima [B] time = 1.22755, size = 108, normalized size = 2.45

$$\frac{1}{2}bp\left(\frac{2\log\left(bx^2+a\right)\log(x)}{b}-\frac{2\log\left(\frac{bx^2}{a}+1\right)\log(x)+\text{Li}_2\left(-\frac{bx^2}{a}\right)}{b}\right)-p\log\left(bx^2+a\right)\log(x)+\log\left(\left(bx^2+a\right)^pc\right)\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(b*x^2 + a)*log(x)/b - (2*log(b*x^2/a + 1)*log(x) + dilog(-b*
x^2/a))/b) - p*log(b*x^2 + a)*log(x) + log((b*x^2 + a)^p*c)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^pc\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(c \left(a + bx^2\right)^p\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(bx^2 + a\right)^p c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/x, x)
```

3.7 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2} dx$

Optimal. Leaf size=44

$$\frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Rubi [A] time = 0.0204981, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 205}

$$\frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^2,x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}+(2bp)\int \frac{1}{a+bx^2}\,dx\\ &= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0090083, size = 44, normalized size = 1.

$$\frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^2,x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Maple [C] time = 0.333, size = 183, normalized size = 4.2

$$-\frac{\ln\left(\left(bx^2+a\right)^p\right)}{x}-\frac{i\pi\,\mathrm{csgn}\left(i\left(bx^2+a\right)^p\right)\left(\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2-i\pi\,\mathrm{csgn}\left(i\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^2,x)
```

```
[Out] -1/x*ln((b*x^2+a)^p)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-
I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x
^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*sum(_R*ln((3*_R^2*a+2*b
*p^2)*x-a*p*_R),_R=RootOf(_Z^2*a+b*p^2))*x+2*ln(c))/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05687, size = 228, normalized size = 5.18

$$\left[\frac{px\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-p\log\left(bx^2+a\right)-\log\left(c\right)}{x},\ \frac{2\,px\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-p\log\left(bx^2+a\right)-\log\left(c\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [(p*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - p*log(b*
x^2 + a) - log(c))/x, (2*p*x*sqrt(b/a)*arctan(x*sqrt(b/a)) - p*log(b*x^2 +
a) - log(c))/x]
```

Sympy [A] time = 44.2599, size = 377, normalized size = 8.57

$$\begin{cases} -\frac{\log(0^p c)}{x} \\ -\frac{p\log(b)}{x} - \frac{2p\log(x)}{x} - \frac{2p}{x} - \frac{\log(c)}{x} \\ -\frac{\log(a^p c)}{x} \\ \frac{ia^{\frac{3}{2}}px\sqrt{\frac{1}{b}}\log(a+bx^2)}{\frac{a^2x}{b}+ax^3} - \frac{2ia^{\frac{3}{2}}px\sqrt{\frac{1}{b}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{\frac{a^2x}{b}+ax^3} + \frac{ia^{\frac{3}{2}}x\sqrt{\frac{1}{b}}\log(c)}{\frac{a^2x}{b}+ax^3} + \frac{i\sqrt{a}bpx^3\sqrt{\frac{1}{b}}\log(a+bx^2)}{\frac{a^2x}{b}+ax^3} - \frac{2i\sqrt{a}bpx^3\sqrt{\frac{1}{b}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{\frac{a^2x}{b}+ax^3} + \frac{i\sqrt{a}bx^3\sqrt{}}{\frac{a^2x}{b}+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-p*log(b)/x - 2*p*log(x)/
x - 2*p/x - log(c)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (I*a**(3/2)*p*
x*sqrt(1/b)*log(a + b*x**2)/(a**2*x/b + a*x**3) - 2*I*a**(3/2)*p*x*sqrt(1/b
)*log(-I*sqrt(a)*sqrt(1/b) + x)/(a**2*x/b + a*x**3) + I*a**(3/2)*x*sqrt(1/b
)*log(c)/(a**2*x/b + a*x**3) + I*sqrt(a)*b*p*x**3*sqrt(1/b)*log(a + b*x**2)
/(a**2*x/b + a*x**3) - 2*I*sqrt(a)*b*p*x**3*sqrt(1/b)*log(-I*sqrt(a)*sqrt(1
/b) + x)/(a**2*x/b + a*x**3) + I*sqrt(a)*b*x**3*sqrt(1/b)*log(c)/(a**2*x/b
+ a*x**3) - a**2*p*log(a + b*x**2)/(a**2*x + a*b*x**3) - a**2*log(c)/(a**2*
x + a*b*x**3) - a*p*x**2*log(a + b*x**2)/(a**2*x/b + a*x**3) - a*x**2*log(c
)/(a**2*x/b + a*x**3), True))
```

Giac [A] time = 1.16791, size = 54, normalized size = 1.23

$$\frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{p\log\left(bx^2+a\right)}{x} - \frac{\log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - p*log(b*x^2 + a)/x - log(c)/x
```

3.8 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3} dx$

Optimal. Leaf size=38

$$\frac{bp\log(x)}{a} - \frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2ax^2}$$

[Out] (b*p*Log[x])/a - ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a*x^2)

Rubi [A] time = 0.037124, antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2395, 36, 29, 31}

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2} - \frac{bp\log\left(a+bx^2\right)}{2a} + \frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(a + b*x^2)^p]/x^3,x]

[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2} + \frac{(bp)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} - \frac{\left(b^2p\right)\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2a}$$
$$= \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^2\right)}{2a} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2}$$

Mathematica [A] time = 0.0026732, size = 45, normalized size = 1.18

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2} - \frac{bp\log\left(a+bx^2\right)}{2a} + \frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^3,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

Maple [C] time = 0.279, size = 173, normalized size = 4.6

$$-\frac{\ln\left(\left(bx^2+a\right)^p\right)}{2\,x^2} - \frac{i\pi\,\text{acsgn}\left(i\left(bx^2+a\right)^p\right)\left(\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2 - i\pi\,\text{acsgn}\left(i\left(bx^2+a\right)^p\right)\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\text{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^3,x)
```

```
[Out] -1/2/x^2*ln((b*x^2+a)^p)-1/4*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*b*p*ln(x)*x
^2+2*b*p*ln(b*x^2+a)*x^2+2*ln(c)*a)/a/x^2
```

Maxima [A] time = 1.12891, size = 59, normalized size = 1.55

$$-\frac{1}{2}\,bp\left(\frac{\log\left(bx^2+a\right)}{a} - \frac{\log\left(x^2\right)}{a}\right) - \frac{\log\left(\left(bx^2+a\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(log(b*x^2 + a)/a - log(x^2)/a) - 1/2*log((b*x^2 + a)^p*c)/x^2
```

Fricas [A] time = 2.02935, size = 103, normalized size = 2.71

$$\frac{2\,bpx^2\log\left(x\right) - \left(bpx^2 + ap\right)\log\left(bx^2 + a\right) - a\log\left(c\right)}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*p*x^2*log(x) - (b*p*x^2 + a*p)*log(b*x^2 + a) - a*log(c))/(a*x^2)
```

Sympy [A] time = 6.96032, size = 82, normalized size = 2.16

$$\begin{cases} -\frac{p\log\left(a+bx^2\right)}{2x^2} - \frac{\log\left(c\right)}{2x^2} + \frac{bp\log\left(x\right)}{a} - \frac{bp\log\left(a+bx^2\right)}{2a} & \text{for}\ a \neq 0 \\ -\frac{p\log\left(b\right)}{2x^2} - \frac{p\log\left(x\right)}{x^2} - \frac{p}{2x^2} - \frac{\log\left(c\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**3,x)
```

```
[Out] Piecewise((-p*log(a + b*x**2)/(2*x**2) - log(c)/(2*x**2) + b*p*log(x)/a - b
*p*log(a + b*x**2)/(2*a), Ne(a, 0)), (-p*log(b)/(2*x**2) - p*log(x)/x**2 -
p/(2*x**2) - log(c)/(2*x**2), True))
```

Giac [A] time = 1.23713, size = 78, normalized size = 2.05

$$-\frac{\frac{b^2p\log\left(bx^2+a\right)}{a} - \frac{b^2p\log\left(bx^2\right)}{a} + \frac{bp\log\left(bx^2+a\right)}{x^2} + \frac{b\log\left(c\right)}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*p*log(b*x^2 + a)/a - b^2*p*log(b*x^2)/a + b*p*log(b*x^2 + a)/x^2
+ b*log(c)/x^2)/b
```

3.9 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4} dx$

Optimal. Leaf size=60

$$-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}-\frac{2bp}{3ax}$$

```
[Out] (-2*b*p)/(3*a*x) - (2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) -
Log[c*(a + b*x^2)^p]/(3*x^3)
```

Rubi [A] time = 0.0295252, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 325, 205}

$$-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}-\frac{2bp}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^4,x]
```

```
[Out] (-2*b*p)/(3*a*x) - (2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) -
Log[c*(a + b*x^2)^p]/(3*x^3)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx = -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{1}{3}(2bp)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx$$
$$= -\frac{2bp}{3ax} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3} - \frac{\left(2b^2p\right)\int \frac{1}{a+bx^2}\,dx}{3a}$$
$$= -\frac{2bp}{3ax} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}$$

Mathematica [C] time = 0.0030129, size = 49, normalized size = 0.82

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3} - \frac{2bp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^4,x]
```

```
[Out] (-2*b*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(3*a*x) - Log[c*(a +
 b*x^2)^p]/(3*x^3)
```

Maple [C] time = 0.34, size = 211, normalized size = 3.5

$$-\frac{\ln\left(\left(bx^2+a\right)^p\right)}{3\,x^3} - \frac{i\pi\,a\mathrm{csgn}\left(i\left(bx^2+a\right)^p\right)\left(\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2 - i\pi\,a\mathrm{csgn}\left(i\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^4,x)
```

```
[Out] -1/3/x^3*ln((b*x^2+a)^p)-1/6*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*sum(_R*ln((
3*_R^2*a^3+2*b^3*p^2)*x+a^2*b*p*_R),_R=RootOf(_Z^2*a^3+b^3*p^2))*a*x^3+4*x^
2*p*b+2*ln(c)*a)/a/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85948, size = 311, normalized size = 5.18

$$\left[\frac{bpx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,bpx^2-ap\log\left(bx^2+a\right)-a\log\left(c\right)}{3\,ax^3},-\frac{2\,bpx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,bpx^2+ap\log\left(b\right.}{3\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(b*p*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) -
2*b*p*x^2 - a*p*log(b*x^2 + a) - a*log(c))/(a*x^3), -1/3*(2*b*p*x^3*sqrt(b/
a)*arctan(x*sqrt(b/a)) + 2*b*p*x^2 + a*p*log(b*x^2 + a) + a*log(c))/(a*x^3)
]
```

Sympy [A] time = 178.799, size = 774, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (-p*log(b)/(3*x**3)
 - 2*p*log(x)/(3*x**3) - 2*p/(9*x**3) - log(c)/(3*x**3), Eq(a, 0)), (-log(a
**p*c)/(3*x**3), Eq(b, 0)), (-I*a**(7/2)*p*sqrt(1/b)*log(a + b*x**2)/(3*I*a
**(7/2)*x**3*sqrt(1/b) + 3*I*a**(5/2)*b*x**5*sqrt(1/b)) - I*a**(7/2)*sqrt(1
/b)*log(c)/(3*I*a**(7/2)*x**3*sqrt(1/b) + 3*I*a**(5/2)*b*x**5*sqrt(1/b)) -
I*a**(5/2)*p*x**2*sqrt(1/b)*log(a + b*x**2)/(3*I*a**(7/2)*x**3*sqrt(1/b)/b
+ 3*I*a**(5/2)*x**5*sqrt(1/b)) - 2*I*a**(5/2)*p*x**2*sqrt(1/b)/(3*I*a**(7/2
)*x**3*sqrt(1/b)/b + 3*I*a**(5/2)*x**5*sqrt(1/b)) - I*a**(5/2)*x**2*sqrt(1/
b)*log(c)/(3*I*a**(7/2)*x**3*sqrt(1/b)/b + 3*I*a**(5/2)*x**5*sqrt(1/b)) - 2
*I*a**(3/2)*b*p*x**4*sqrt(1/b)/(3*I*a**(7/2)*x**3*sqrt(1/b)/b + 3*I*a**(5/2
)*x**5*sqrt(1/b)) + a**2*p*x**3*log(a + b*x**2)/(3*I*a**(7/2)*x**3*sqrt(1/b
)/b + 3*I*a**(5/2)*x**5*sqrt(1/b)) - 2*a**2*p*x**3*log(-I*sqrt(a)*sqrt(1/b)
 + x)/(3*I*a**(7/2)*x**3*sqrt(1/b)/b + 3*I*a**(5/2)*x**5*sqrt(1/b)) + a**2*
x**3*log(c)/(3*I*a**(7/2)*x**3*sqrt(1/b)/b + 3*I*a**(5/2)*x**5*sqrt(1/b)) +
 a*b*p*x**5*log(a + b*x**2)/(3*I*a**(7/2)*x**3*sqrt(1/b)/b + 3*I*a**(5/2)*x
**5*sqrt(1/b)) - 2*a*b*p*x**5*log(-I*sqrt(a)*sqrt(1/b) + x)/(3*I*a**(7/2)*x
**3*sqrt(1/b)/b + 3*I*a**(5/2)*x**5*sqrt(1/b)) + a*b*x**5*log(c)/(3*I*a**(7
/2)*x**3*sqrt(1/b)/b + 3*I*a**(5/2)*x**5*sqrt(1/b)), True))
```

Giac [A] time = 1.228, size = 78, normalized size = 1.3

$$-\frac{2\,b^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a}-\frac{p\log\left(bx^2+a\right)}{3\,x^3}-\frac{2\,bpx^2+a\log\left(c\right)}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*b^2*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/3*p*log(b*x^2 + a)/x^3 -
 1/3*(2*b*p*x^2 + a*log(c))/(a*x^3)
```

3.10 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=64

$$\frac{b^2p\log\left(a+bx^2\right)}{4a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}-\frac{bp}{4ax^2}$$

```
[Out] -(b*p)/(4*a*x^2) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^2])/(4*a^2)
- Log[c*(a + b*x^2)^p]/(4*x^4)
```

Rubi [A] time = 0.0520589, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 44}

$$\frac{b^2p\log\left(a+bx^2\right)}{4a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}-\frac{bp}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^5,x]
```

```
[Out] -(b*p)/(4*a*x^2) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^2])/(4*a^2)
- Log[c*(a + b*x^2)^p]/(4*x^4)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\operatorname{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{bp}{4ax^2} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^2\right)}{4a^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

Mathematica [A] time = 0.0368477, size = 56, normalized size = 0.88

$$\frac{1}{4}bp\left(\frac{b\log\left(a+bx^2\right)}{a^2} - \frac{2b\log(x)}{a^2} - \frac{1}{ax^2}\right) - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^5,x]
```

```
[Out] (b*p*(-(1/(a*x^2)) - (2*b*Log[x])/a^2 + (b*Log[a + b*x^2])/a^2))/4 - Log[c*
(a + b*x^2)^p]/(4*x^4)
```

Maple [C] time = 0.295, size = 198, normalized size = 3.1

$$-\frac{\ln\left(\left(bx^2+a\right)^p\right)}{4x^4} - \frac{4b^2p\ln(x)x^4 - 2b^2p\ln\left(-bx^2-a\right)x^4 + i\pi a^2\operatorname{csgn}\left(i\left(bx^2+a\right)^p\right)\left(\operatorname{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2 - i\pi a^2\operatorname{cs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^5,x)
```

```
[Out] -1/4/x^4*ln((b*x^2+a)^p)-1/8*(4*b^2*p*ln(x)*x^4-2*b^2*p*ln(-b*x^2-a)*x^4+I*
Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*
a^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*a*b*p*x^2+2*ln(c)*a^2)/a^2/x^4
```

Maxima [A] time = 1.0698, size = 73, normalized size = 1.14

$$\frac{1}{4}bp\left(\frac{b\log\left(bx^2+a\right)}{a^2} - \frac{b\log\left(x^2\right)}{a^2} - \frac{1}{ax^2}\right) - \frac{\log\left(\left(bx^2+a\right)^pc\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(b*log(b*x^2 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2)) - 1/4*log((b*x^
2 + a)^p*c)/x^4
```

Fricas [A] time = 2.08974, size = 134, normalized size = 2.09

$$-\frac{2\,b^2px^4\log(x) + abpx^2 + a^2\log(c) - \left(b^2px^4 - a^2p\right)\log\left(bx^2 + a\right)}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*p*x^4*log(x) + a*b*p*x^2 + a^2*log(c) - (b^2*p*x^4 - a^2*p)*log
(b*x^2 + a))/(a^2*x^4)
```

Sympy [A] time = 21.4743, size = 102, normalized size = 1.59

$$\begin{cases} -\frac{p\log\left(a+bx^2\right)}{4x^4} - \frac{\log(c)}{4x^4} - \frac{bp}{4ax^2} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^2\right)}{4a^2} & \text{for}\ a \neq 0 \\ -\frac{p\log(b)}{4x^4} - \frac{p\log(x)}{2x^4} - \frac{p}{8x^4} - \frac{\log(c)}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**5,x)
```

```
[Out] Piecewise((-p*log(a + b*x**2)/(4*x**4) - log(c)/(4*x**4) - b*p/(4*a*x**2) -
 b**2*p*log(x)/(2*a**2) + b**2*p*log(a + b*x**2)/(4*a**2), Ne(a, 0)), (-p*l
og(b)/(4*x**4) - p*log(x)/(2*x**4) - p/(8*x**4) - log(c)/(4*x**4), True))
```

Giac [B] time = 1.28153, size = 178, normalized size = 2.78

$$-\frac{\frac{b^3p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^2-2\left(bx^2+a\right)a+a^2} - \frac{b^3p\log\left(bx^2+a\right)}{a^2} + \frac{b^3p\log\left(bx^2\right)}{a^2} + \frac{\left(bx^2+a\right)b^3p-ab^3p+ab^3\log(c)}{\left(bx^2+a\right)^2a-2\left(bx^2+a\right)a^2+a^3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*p*log(b*x^2 + a)/((b*x^2 + a)^2 - 2*(b*x^2 + a)*a + a^2) - b^3*p*
log(b*x^2 + a)/a^2 + b^3*p*log(b*x^2)/a^2 + ((b*x^2 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^2 + a)^2*a - 2*(b*x^2 + a)*a^2 + a^3))/b
```

3.11 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=74

$$\frac{2b^2p}{5a^2x}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{2bp}{15ax^3}$$

```
[Out] (-2*b*p)/(15*a*x^3) + (2*b^2*p)/(5*a^2*x) + (2*b^(5/2)*p*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]/(5*x^5)
```

Rubi [A] time = 0.0365117, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 325, 205}

$$\frac{2b^2p}{5a^2x}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{2bp}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^6,x]
```

```
[Out] (-2*b*p)/(15*a*x^3) + (2*b^2*p)/(5*a^2*x) + (2*b^(5/2)*p*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]/(5*x^5)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx = -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{1}{5}(2bp)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx$$
$$= -\frac{2bp}{15ax^3} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} - \frac{\left(2b^2p\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{5a}$$
$$= -\frac{2bp}{15ax^3} + \frac{2b^2p}{5a^2x} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{\left(2b^3p\right)\int \frac{1}{a+bx^2}\,dx}{5a^2}$$
$$= -\frac{2bp}{15ax^3} + \frac{2b^2p}{5a^2x} + \frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

Mathematica [C] time = 0.0027562, size = 49, normalized size = 0.66

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} - \frac{2bp\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx^2}{a}\right)}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^6,x]
```

```
[Out] (-2*b*p*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/a)])/(15*a*x^3) - Log[c*
(a + b*x^2)^p]/(5*x^5)
```

Maple [C] time = 0.362, size = 235, normalized size = 3.2

$$-\frac{\ln\left(\left(bx^2+a\right)^p\right)}{5x^5} - \frac{1}{30a^3x^5}\left(-6\sqrt{-ab}pb^2\ln\left(-bx-\sqrt{-ab}\right)x^5 + 6\sqrt{-ab}pb^2\ln\left(-bx+\sqrt{-ab}\right)x^5 + 3i\pi a^3\mathrm{csgn}\left(i\left(bx^2+a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^6,x)
```

```
[Out] -1/5/x^5*ln((b*x^2+a)^p)-1/30*(-6*(-a*b)^(1/2)*p*b^2*ln(-b*x-(-a*b)^(1/2))*
x^5+6*(-a*b)^(1/2)*p*b^2*ln(-b*x+(-a*b)^(1/2))*x^5+3*I*Pi*a^3*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3*I*Pi*a^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x
^2+a)^p)*csgn(I*c)-3*I*Pi*a^3*csgn(I*c*(b*x^2+a)^p)^3+3*I*Pi*a^3*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)-12*a*b^2*p*x^4+4*a^2*b*p*x^2+6*ln(c)*a^3)/a^3/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07726, size = 390, normalized size = 5.27

$$\left[\frac{3\,b^2px^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,b^2px^4-2\,abpx^2-3\,a^2p\log\left(bx^2+a\right)-3\,a^2\log\left(c\right)}{15\,a^2x^5},\frac{6\,b^2px^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*(3*b^2*p*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) + 6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^
2*x^5), 1/15*(6*b^2*p*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 6*b^2*p*x^4 - 2*a
*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^2*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28366, size = 96, normalized size = 1.3

$$\frac{2\,b^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}a^2}-\frac{p\log\left(bx^2+a\right)}{5\,x^5}+\frac{6\,b^2px^4-2\,abpx^2-3\,a^2\log\left(c\right)}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*b^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/5*p*log(b*x^2 + a)/x^5
+ 1/15*(6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*log(c))/(a^2*x^5)
```

3.12 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=78

$$\frac{b^2p}{6a^2x^2}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}+\frac{b^3p\log(x)}{3a^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}-\frac{bp}{12ax^4}$$

```
[Out] -(b*p)/(12*a*x^4) + (b^2*p)/(6*a^2*x^2) + (b^3*p*Log[x])/(3*a^3) - (b^3*p*L
og[a + b*x^2])/(6*a^3) - Log[c*(a + b*x^2)^p]/(6*x^6)
```

Rubi [A] time = 0.0601039, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 44}

$$\frac{b^2p}{6a^2x^2}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}+\frac{b^3p\log(x)}{3a^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}-\frac{bp}{12ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] -(b*p)/(12*a*x^4) + (b^2*p)/(6*a^2*x^2) + (b^3*p*Log[x])/(3*a^3) - (b^3*p*L
og[a + b*x^2])/(6*a^3) - Log[c*(a + b*x^2)^p]/(6*x^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x^4}\,dx,x,x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}+\frac{1}{6}(bp)\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}+\frac{1}{6}(bp)\operatorname{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx,x,x^2\right)$$
$$= -\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log(x)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}$$

Mathematica [A] time = 0.0648393, size = 68, normalized size = 0.87

$$-\frac{\frac{bpx^2\left(2b^2x^4\log\left(a+bx^2\right)+a\left(a-2bx^2\right)-4b^2x^4\log(x)\right)}{a^3}+2\log\left(c\left(a+bx^2\right)^p\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] -((b*p*x^2*(a*(a - 2*b*x^2) - 4*b^2*x^4*Log[x] + 2*b^2*x^4*Log[a + b*x^2]))
/a^3 + 2*Log[c*(a + b*x^2)^p])/(12*x^6)
```

Maple [C] time = 0.286, size = 206, normalized size = 2.6

$$-\frac{\ln\left(\left(bx^2+a\right)^p\right)}{6\,x^6}-\frac{-4\,b^3p\ln(x)\,x^6+2\,b^3p\ln\left(bx^2+a\right)x^6+i\pi\,a^3\operatorname{csgn}\left(i\left(bx^2+a\right)^p\right)\left(\operatorname{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2-i\pi\,a^3\mathrm{cs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^7,x)
```

```
[Out] -1/6/x^6*ln((b*x^2+a)^p)-1/12*(-4*b^3*p*ln(x)*x^6+2*b^3*p*ln(b*x^2+a)*x^6+I
*Pi*a^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a^3*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^3*csgn(I*c*(b*x^2+a)^p)^3+I*Pi
*a^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*a*b^2*p*x^4+a^2*b*p*x^2+2*ln(c)*a^
3)/a^3/x^6
```

Maxima [A] time = 1.04477, size = 93, normalized size = 1.19

$$-\frac{1}{12}\,bp\left(\frac{2\,b^2\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^2\log\left(x^2\right)}{a^3}-\frac{2\,bx^2-a}{a^2x^4}\right)-\frac{\log\left(\left(bx^2+a\right)^pc\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/(a
^2*x^4)) - 1/6*log((b*x^2 + a)^p*c)/x^6
```

Fricas [A] time = 2.06221, size = 163, normalized size = 2.09

$$\frac{4\,b^3px^6\log\left(x\right)+2\,ab^2px^4-a^2bpx^2-2\,a^3\log\left(c\right)-2\left(b^3px^6+a^3p\right)\log\left(bx^2+a\right)}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^3*p*x^6*log(x) + 2*a*b^2*p*x^4 - a^2*b*p*x^2 - 2*a^3*log(c) - 2*(
b^3*p*x^6 + a^3*p)*log(b*x^2 + a))/(a^3*x^6)
```

Sympy [A] time = 54.4347, size = 116, normalized size = 1.49

$$\begin{cases} -\frac{p\log\left(a+bx^2\right)}{6x^6}-\frac{\log\left(c\right)}{6x^6}-\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log\left(x\right)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3} & \text{for } a\neq 0 \\ -\frac{p\log\left(b\right)}{6x^6}-\frac{p\log\left(x\right)}{3x^6}-\frac{p}{18x^6}-\frac{\log\left(c\right)}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**7,x)
```

```
[Out] Piecewise((-p*log(a + b*x**2)/(6*x**6) - log(c)/(6*x**6) - b*p/(12*a*x**4)
+ b**2*p/(6*a**2*x**2) + b**3*p*log(x)/(3*a**3) - b**3*p*log(a + b*x**2)/(6
*a**3), Ne(a, 0)), (-p*log(b)/(6*x**6) - p*log(x)/(3*x**6) - p/(18*x**6) -
log(c)/(6*x**6), True))
```

Giac [B] time = 1.17609, size = 258, normalized size = 3.31

$$-\frac{\frac{2\,b^4p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^3-3\left(bx^2+a\right)^2a+3\left(bx^2+a\right)a^2-a^3}+\frac{2\,b^4p\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^4p\log\left(bx^2\right)}{a^3}-\frac{2\left(bx^2+a\right)^2b^4p-5\left(bx^2+a\right)ab^4p+3\,a^2b^4p-2\,a^2b^4\log\left(c\right)}{\left(bx^2+a\right)^3a^2-3\left(bx^2+a\right)^2a^3+3\left(bx^2+a\right)a^4-a^5}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(2*b^4*p*log(b*x^2 + a)/((b*x^2 + a)^3 - 3*(b*x^2 + a)^2*a + 3*(b*x^2
 + a)*a^2 - a^3) + 2*b^4*p*log(b*x^2 + a)/a^3 - 2*b^4*p*log(b*x^2)/a^3 - (2
*(b*x^2 + a)^2*b^4*p - 5*(b*x^2 + a)*a*b^4*p + 3*a^2*b^4*p - 2*a^2*b^4*log(
c))/((b*x^2 + a)^3*a^2 - 3*(b*x^2 + a)^2*a^3 + 3*(b*x^2 + a)*a^4 - a^5))/b
```

3.13 $\int x^5 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=59

$$-\frac{a^2p\log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)+\frac{apx^3}{6b}-\frac{px^6}{12}$$

```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

Rubi [A] time = 0.0494319, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$-\frac{a^2p\log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)+\frac{apx^3}{6b}-\frac{px^6}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^3)^p],x]
```


```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5 \log\left(c\left(a+bx^3\right)^p\right) dx = \frac{1}{3}\operatorname{Subst}\left(\int x\log\left(c(a+bx)^p\right) dx, x, x^3\right)$$
$$= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right) - \frac{1}{6}(bp)\operatorname{Subst}\left(\int \frac{x^2}{a+bx}dx, x, x^3\right)$$
$$= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right) - \frac{1}{6}(bp)\operatorname{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, x^3\right)$$
$$= \frac{apx^3}{6b} - \frac{px^6}{12} - \frac{a^2p\log\left(a+bx^3\right)}{6b^2} + \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)$$

Mathematica [A] time = 0.0126688, size = 59, normalized size = 1.

$$-\frac{a^2p\log\left(a+bx^3\right)}{6b^2} + \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right) + \frac{apx^3}{6b} - \frac{px^6}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

Maple [C] time = 0.556, size = 183, normalized size = 3.1

$$\frac{x^6\ln\left(\left(bx^3+a\right)^p\right)}{6} + \frac{i}{12}\pi x^6 \operatorname{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\operatorname{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2 - \frac{i}{12}\pi x^6 \operatorname{csgn}\left(i\left(bx^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(bx^3+a\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/6*x^6*ln((b*x^3+a)^p)+1/12*I*Pi*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a
)^p)^2-1/12*I*Pi*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/
12*I*Pi*x^6*csgn(I*c*(b*x^3+a)^p)^3+1/12*I*Pi*x^6*csgn(I*c*(b*x^3+a)^p)^2*c
sgn(I*c)+1/6*ln(c)*x^6-1/12*p*x^6+1/6*a*p*x^3/b-1/6*a^2*p*ln(b*x^3+a)/b^2
```

Maxima [A] time = 1.17267, size = 74, normalized size = 1.25

$$\frac{1}{6}x^6\log\left(\left(bx^3+a\right)^p c\right) - \frac{1}{12}bp\left(\frac{2a^2\log\left(bx^3+a\right)}{b^3} + \frac{bx^6-2ax^3}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^3 + a)^p*c) - 1/12*b*p*(2*a^2*log(b*x^3 + a)/b^3 + (b*x^6
- 2*a*x^3)/b^2)
```

Fricas [A] time = 1.8724, size = 128, normalized size = 2.17

$$-\frac{b^2px^6 - 2\,b^2x^6\log\left(c\right) - 2\,abpx^3 - 2\,\left(b^2px^6 - a^2p\right)\log\left(bx^3 + a\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/12*(b^2*p*x^6 - 2*b^2*x^6*log(c) - 2*a*b*p*x^3 - 2*(b^2*p*x^6 - a^2*p)*l
og(b*x^3 + a))/b^2
```

Sympy [A] time = 34.7721, size = 70, normalized size = 1.19

$$\begin{cases} -\frac{a^2p\log\left(a+bx^3\right)}{6b^2} + \frac{apx^3}{6b} + \frac{px^6\log\left(a+bx^3\right)}{6} - \frac{px^6}{12} + \frac{x^6\log\left(c\right)}{6} & \text{for}\ b \neq 0 \\ \frac{x^6\log\left(a^pc\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((-a**2*p*log(a + b*x**3)/(6*b**2) + a*p*x**3/(6*b) + p*x**6*log(a
 + b*x**3)/6 - p*x**6/12 + x**6*log(c)/6, Ne(b, 0)), (x**6*log(a**p*c)/6, T
rue))
```

Giac [A] time = 1.1641, size = 131, normalized size = 2.22

$$\frac{\frac{\left(2\left(bx^3+a\right)^2\log\left(bx^3+a\right)-4\left(bx^3+a\right)a\log\left(bx^3+a\right)-\left(bx^3+a\right)^2+4\left(bx^3+a\right)a\right)p}{b} + \frac{2\left(\left(bx^3+a\right)^2-2\left(bx^3+a\right)a\right)\log(c)}{b}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/12*((2*(b*x^3 + a)^2*log(b*x^3 + a) - 4*(b*x^3 + a)*a*log(b*x^3 + a) - (b
*x^3 + a)^2 + 4*(b*x^3 + a)*a)*p/b + 2*((b*x^3 + a)^2 - 2*(b*x^3 + a)*a)*lo
g(c)/b)/b
```

3.14 $\int x^4 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=159

$$-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}+\frac{\sqrt{3}a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)+\frac{3ap}{10b}$$

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 + (Sqrt[3]*a^(5/3)*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(5*b^(5/3)) + (a^(5/3)*p*Log[a^(1/3) + b^(1/
3)*x])/(5*b^(5/3)) - (a^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(10*b^(5/3)) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Rubi [A] time = 0.129039, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2455, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}+\frac{\sqrt{3}a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)+\frac{3ap}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 + (Sqrt[3]*a^(5/3)*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(5*b^(5/3)) + (a^(5/3)*p*Log[a^(1/3) + b^(1/
3)*x])/(5*b^(5/3)) - (a^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(10*b^(5/3)) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \log\left(c\left(a+bx^3\right)^p\right)dx &= \frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{5}(3bp)\int\frac{x^7}{a+bx^3}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{5}(3bp)\int\left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(3a^2p\right)\int\frac{x}{a+bx^3}\,dx}{5b}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(a^{5/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{5b^{4/3}}-\frac{\left(a^{5/3}p\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(a^{5/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b}}{10b^{5/3}}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}+\frac{1}{5}x^5\log\left(c\right.\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{\sqrt{3}a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{\right.}{10b}
\end{aligned}
$$

Mathematica [C] time = 0.0032979, size = 69, normalized size = 0.43

$$
\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{3apx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{10b}+\frac{3apx^2}{10b}-\frac{3px^5}{25}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 - (3*a*p*x^2*Hypergeometric2F1[2/3, 1, 5/
3, -((b*x^3)/a)])/(10*b) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Maple [C] time = 0.452, size = 196, normalized size = 1.2

$$\frac{x^5 \ln\left(\left(bx^3+a\right)^p\right)}{5} - \frac{i}{10}\pi x^5 \mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\right) + \frac{i}{10}\pi x^5\left(\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2 \mathrm{csgn}\left(ic\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/5*x^5*ln((b*x^3+a)^p)-1/10*I*Pi*x^5*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a
)^p)*csgn(I*c)+1/10*I*Pi*x^5*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+1/10*I*Pi*x^
5*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/10*I*Pi*x^5*csgn(I*c*(b*x^3
+a)^p)^3+1/5*ln(c)*x^5-3/25*p*x^5+3/10*a*p*x^2/b-1/5/b^2*a^2*p*sum(1/_R*ln(
x-_R),_R=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14814, size = 406, normalized size = 2.55

$$\frac{10\,bpx^5 \log\left(bx^3+a\right) - 6\,bpx^5 + 10\,bx^5 \log\left(c\right) + 15\,apx^2 - 10\,\sqrt{3}ap\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}a}{3\,a}\right) - 5\,ap\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \log\left(ax^2\right.}{50\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/50*(10*b*p*x^5*log(b*x^3 + a) - 6*b*p*x^5 + 10*b*x^5*log(c) + 15*a*p*x^2
- 10*sqrt(3)*a*p*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3)
- sqrt(3)*a)/a) - 5*a*p*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 10*a*p*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27326, size = 219, normalized size = 1.38

$$\frac{1}{10}\,a^2b^4p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^5}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^7}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/10*a^2*b^4*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) + 2*sqrt(
3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*
b^7) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^7)) + 1
/5*p*x^5*log(b*x^3 + a) - 1/25*(3*p - 5*log(c))*x^5 + 3/10*a*p*x^2/b
```

3.15 $\int x^3 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=157

$$\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{\sqrt{3}a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)+\frac{3apx}{4b}$$

```
[Out] (3*a*p*x)/(4*b) - (3*p*x^4)/16 + (Sqrt[3]*a^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) - (a^(4/3)*p*Log[a^(1/3) + b^(1/3)*
x])/(4*b^(4/3)) + (a^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(8*b^(4/3)) + (x^4*Log[c*(a + b*x^3)^p])/4
```

Rubi [A] time = 0.114458, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2455, 302, 200, 31, 634, 617, 204, 628}

$$\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{\sqrt{3}a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)+\frac{3apx}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x)/(4*b) - (3*p*x^4)/16 + (Sqrt[3]*a^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) - (a^(4/3)*p*Log[a^(1/3) + b^(1/3)*
x])/(4*b^(4/3)) + (a^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(8*b^(4/3)) + (x^4*Log[c*(a + b*x^3)^p])/4
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \log\left(c\left(a+bx^3\right)^p\right) dx &= \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right) - \frac{1}{4}(3bp)\int \frac{x^6}{a+bx^3}\,dx \\
&= \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right) - \frac{1}{4}(3bp)\int\left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx \\
&= \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right) - \frac{\left(3a^2p\right)\int\frac{1}{a+bx^3}\,dx}{4b} \\
&= \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right) - \frac{\left(a^{4/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{4b} - \frac{\left(a^{4/3}p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{4b} \\
&= \frac{3apx}{4b} - \frac{3px^4}{16} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} + \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right) + \frac{\left(a^{4/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}}}{8b^{4/3}} \\
&= \frac{3apx}{4b} - \frac{3px^4}{16} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} + \frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}} + \frac{1}{4}x^4\log\left(c\right. \\
&= \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{\sqrt{3}a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} + \frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{\right.}}{8b^{4/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0495574, size = 147, normalized size = 0.94

$$
\frac{2a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-4a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+4\sqrt{3}a^{4/3}p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4b^{4/3}x^4\log\left(c\left(a+bx^3\right)^p\right)+}{16b^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (12*a*b^(1/3)*p*x - 3*b^(4/3)*p*x^4 + 4*Sqrt[3]*a^(4/3)*p*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*a^(4/3)*p*Log[a^(1/3) + b^(1/3)*x] + 2*a^(4/
3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*b^(4/3)*x^4*Log[c*(
a + b*x^3)^p])/(16*b^(4/3))
```

Maple [C] time = 0.513, size = 194, normalized size = 1.2

$$\frac{x^4 \ln\left(\left(bx^3+a\right)^p\right)}{4} - \frac{i}{8}\pi x^4 \mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\mathrm{csgn}(ic) + \frac{i}{8}\pi x^4\left(\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2\mathrm{csgn}(ic) + \frac{i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/4*x^4*ln((b*x^3+a)^p)-1/8*I*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)*csgn(I*c)+1/8*I*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+1/8*I*Pi*x^4*c
sgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/8*I*Pi*x^4*csgn(I*c*(b*x^3+a)^
p)^3+1/4*ln(c)*x^4-3/16*p*x^4-1/4/b^2*a^2*p*sum(1/_R^2*ln(x-_R),_R=RootOf(_
Z^3*b+a))+3/4*a*p*x/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04393, size = 356, normalized size = 2.27

$$\frac{4\,bpx^4\log\left(bx^3+a\right) - 3\,bpx^4 + 4\,bx^4\log(c) + 4\,\sqrt{3}ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3\,a}\right) - 2\,ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(\right.\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*p*x^4*log(b*x^3 + a) - 3*b*p*x^4 + 4*b*x^4*log(c) + 4*sqrt(3)*a*p
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) - 2*a*
p*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 4*a*p*(-a/b)^(1/3
)*log(x - (-a/b)^(1/3)) + 12*a*p*x)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28597, size = 216, normalized size = 1.38

$$\frac{1}{8}\,a^2b^3p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^4}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^5}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*a^2*b^3*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 2*sqrt(3
)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^5) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5)) + 1/
4*p*x^4*log(b*x^3 + a) - 1/16*(3*p - 4*log(c))*x^4 + 3/4*a*p*x/b
```

3.16 $\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=35

$$\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{3b}-\frac{px^3}{3}$$

```
[Out] -(p*x^3)/3 + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/(3*b)
```

Rubi [A] time = 0.0291231, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{3b}-\frac{px^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^3)^p],x]

[Out] -(p*x^3)/3 + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/(3*b)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, x^3\right)\\ &= \frac{\operatorname{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx^3\right)}{3b}\\ &= -\frac{px^3}{3}+\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0095017, size = 34, normalized size = 0.97

$$\frac{1}{3}\left(\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{b}-px^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-(p*x^3) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/b)/3
```

Maple [A] time = 0.062, size = 50, normalized size = 1.4

$$\frac{x^3\ln\left(c\left(bx^3+a\right)^p\right)}{3}-\frac{px^3}{3}+\frac{\ln\left(c\left(bx^3+a\right)^p\right)a}{3b}-\frac{ap}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/3*x^3*ln(c*(b*x^3+a)^p)-1/3*p*x^3+1/3/b*ln(c*(b*x^3+a)^p)*a-1/3/b*a*p
```

Maxima [A] time = 1.15659, size = 59, normalized size = 1.69

$$\frac{1}{3}x^3\log\left(\left(bx^3+a\right)^pc\right)-\frac{1}{3}\left(\frac{x^3}{b}-\frac{a\log\left(bx^3+a\right)}{b^2}\right)bp$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*x^3 + a)^p*c) - 1/3*(x^3/b - a*log(b*x^3 + a)/b^2)*b*p
```

Fricas [A] time = 1.92167, size = 89, normalized size = 2.54

$$-\frac{bpx^3-bx^3\log\left(c\right)-\left(bpx^3+ap\right)\log\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/3*(b*p*x^3 - b*x^3*log(c) - (b*p*x^3 + a*p)*log(b*x^3 + a))/b
```

Sympy [A] time = 7.52976, size = 56, normalized size = 1.6

$$\begin{cases}\frac{ap\log\left(a+bx^3\right)}{3b}+\frac{px^3\log\left(a+bx^3\right)}{3}-\frac{px^3}{3}+\frac{x^3\log\left(c\right)}{3} & \text{for } b\neq 0\\ \frac{x^3\log\left(a^pc\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((a*p*log(a + b*x**3)/(3*b) + p*x**3*log(a + b*x**3)/3 - p*x**3/3
+ x**3*log(c)/3, Ne(b, 0)), (x**3*log(a**p*c)/3, True))
```

Giac [A] time = 1.27323, size = 58, normalized size = 1.66

$$-\frac{\left(bx^3 - \left(bx^3 + a\right)\log\left(bx^3 + a\right) + a\right)p - \left(bx^3 + a\right)\log\left(c\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*p - (b*x^3 + a)*log(c))/b
```

3.17 $\int x \log \left(c \left(a + bx^3\right)^p\right) dx$

Optimal. Leaf size=147

$$\frac{a^{2/3} p \log \left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{4b^{2/3}} - \frac{a^{2/3} p \log \left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{\sqrt{3}a^{2/3} p \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} + \frac{1}{2}x^2 \log \left(c \left(a + bx^3\right)^p\right) - \frac{3p\ldots}{4\ldots}$$

```
[Out] (-3*p*x^2)/4 - (Sqrt[3]*a^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(2*b^(2/3)) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) +
(a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)) + (x
^2*Log[c*(a + b*x^3)^p])/2
```

Rubi [A] time = 0.0846344, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2455, 321, 292, 31, 634, 617, 204, 628}

$$\frac{a^{2/3} p \log \left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{4b^{2/3}} - \frac{a^{2/3} p \log \left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{\sqrt{3}a^{2/3} p \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} + \frac{1}{2}x^2 \log \left(c \left(a + bx^3\right)^p\right) - \frac{3p\ldots}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*p*x^2)/4 - (Sqrt[3]*a^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(2*b^(2/3)) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) +
(a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)) + (x
^2*Log[c*(a + b*x^3)^p])/2
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\log\left(c\left(a+bx^3\right)^p\right)dx &= \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{2}(3bp)\int\frac{x^4}{a+bx^3}\,dx\\
&= -\frac{3px^2}{4}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}(3ap)\int\frac{x}{a+bx^3}\,dx\\
&= -\frac{3px^2}{4}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(a^{2/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2\sqrt[3]{b}}+\frac{\left(a^{2/3}p\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&= -\frac{3px^2}{4}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(a^{2/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4b^{2/3}}+\ldots\\
&= -\frac{3px^2}{4}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)\ldots\\
&= -\frac{3px^2}{4}-\frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\ldots\right)}{4b^{2/3}}\ldots
\end{aligned}
$$

Mathematica [C] time = 0.0027077, size = 53, normalized size = 0.36

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{3}{4}px^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)-\frac{3px^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*p*x^2)/4 + (3*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (
x^2*Log[c*(a + b*x^3)^p])/2
```

Maple [C] time = 0.518, size = 184, normalized size = 1.3

$$\frac{x^2\ln\left(\left(bx^3+a\right)^p\right)}{2}+\frac{i}{4}\pi x^2\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2-\frac{i}{4}\pi x^2\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/2*x^2*ln((b*x^3+a)^p)+1/4*I*Pi*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-1/4*I*Pi*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/4*
I*Pi*x^2*csgn(I*c*(b*x^3+a)^p)^3+1/4*I*Pi*x^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(
I*c)+1/2*ln(c)*x^2-3/4*p*x^2+1/2/b*a*p*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08934, size = 381, normalized size = 2.59

$$\frac{1}{2}px^2\log\left(bx^3+a\right)-\frac{3}{4}px^2+\frac{1}{2}x^2\log\left(c\right)+\frac{1}{2}\sqrt{3}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{4}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/2*p*x^2*log(b*x^3 + a) - 3/4*p*x^2 + 1/2*x^2*log(c) + 1/2*sqrt(3)*p*(-a^2
/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) - 1/
4*p*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
 + 1/2*p*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24984, size = 203, normalized size = 1.38

$$-\frac{1}{4}\,ab^{2}p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^{2}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^{4}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^{4}}\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*a*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sqrt(3)
*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^
4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)) + 1/2
*p*x^2*log(b*x^3 + a) - 1/4*(3*p - 2*log(c))*x^2
```

3.18 $\int \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=133

$$-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-3px$$

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rubi [A] time = 0.081811, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2448, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-3px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(c\left(a+bx^3\right)^p\right)dx &= x\log\left(c\left(a+bx^3\right)^p\right)-(3bp)\int\frac{x^3}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+(3ap)\int\frac{1}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+\left(\sqrt[3]{a}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx+\left(\sqrt[3]{a}p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}\left(3a^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{(3\cdots}{\cdots}\\
&= -3px-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0365433, size = 129, normalized size = 0.97

$$
-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-3px
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

Maple [A] time = 0.062, size = 113, normalized size = 0.9

$$x\ln\left(c\left(bx^3+a\right)^p\right)-3px+\frac{ap}{b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{ap}{2b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ap\sqrt{3}}{b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p),x)
```

```
[Out] x*ln(c*(b*x^3+a)^p)-3*p*x+1/b*p*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/2/b*p*a/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/b*p*a/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08337, size = 288, normalized size = 2.17

$$px\log\left(bx^3+a\right)+\sqrt{3}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\frac{1}{2}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19911, size = 193, normalized size = 1.45

$$-\frac{1}{2}\,abp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^{2}}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^{2}}\right)+px$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

3.19 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} dx$

Optimal. Leaf size=44

$$\frac{1}{3}p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)+\frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)$$

```
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/3 + (p*PolyLog[2, 1 + (b*x^3)/a])/
3
```

Rubi [A] time = 0.0477984, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{1}{3}p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)+\frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x,x]
```

```
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/3 + (p*PolyLog[2, 1 + (b*x^3)/a])/
3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x} dx, x, x^3\right)\\
&= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx} dx, x, x^3\right)\\
&= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{3}p\text{Li}_2\left(1+\frac{bx^3}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.0068243, size = 43, normalized size = 0.98

$$\frac{1}{3}\left(p\text{PolyLog}\left(2,\frac{a+bx^3}{a}\right)+\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x,x]
```

```
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] + p*PolyLog[2, (a + b*x^3)/a])/3
```

Maple [C] time = 0.431, size = 180, normalized size = 4.1

$$\ln(x)\ln\left(\left(bx^3+a\right)^p\right)-p\sum_{\_R1=\text{RootOf}\left(b\_Z^3+a\right)}\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)+\frac{i}{2}\ln(x)\pi\,\text{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\text{csg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x,x)
```

```
[Out] ln(x)*ln((b*x^3+a)^p)-p*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=Ro
otOf(_Z^3*b+a))+1/2*I*ln(x)*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-
1/2*I*ln(x)*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln
(x)*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(
I*c)+ln(c)*ln(x)
```

Maxima [B] time = 1.07101, size = 108, normalized size = 2.45

$$\frac{1}{3}bp\left(\frac{3\log\left(bx^3+a\right)\log(x)}{b}-\frac{3\log\left(\frac{bx^3}{a}+1\right)\log(x)+\text{Li}_2\left(-\frac{bx^3}{a}\right)}{b}\right)-p\log\left(bx^3+a\right)\log(x)+\log\left(\left(bx^3+a\right)^pc\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*p*(3*log(b*x^3 + a)*log(x)/b - (3*log(b*x^3/a + 1)*log(x) + dilog(-b*
x^3/a))/b) - p*log(b*x^3 + a)*log(x) + log((b*x^3 + a)^p*c)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^pc\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/x, x)
```

3.20 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2} dx$

Optimal. Leaf size=133

$$\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}-\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(
1/3)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (b^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)) - Log[c*(a + b*x^3)^p]/x
```

Rubi [A] time = 0.0777852, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2455, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}-\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^2,x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(
1/3)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (b^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)) - Log[c*(a + b*x^3)^p]/x
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} + (3bp)\int \frac{x}{a+bx^3}\,dx$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} - \frac{\left(b^{2/3}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{\sqrt[3]{a}} + \frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} + \frac{\left(\sqrt[3]{b}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}} + \frac{1}{2}\left(3b^{2/3}p\right)\int$$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} + \frac{\left(3\sqrt[3]{b}p\right)}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}} - \cdots$$

Mathematica [C] time = 0.002632, size = 47, normalized size = 0.35

$$\frac{3bpx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{2a} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^2,x]
```

```
[Out] (3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a) - Log[c*(a +
 b*x^3)^p]/x
```

Maple [C] time = 0.317, size = 184, normalized size = 1.4

$$-\frac{\ln\left(\left(bx^3+a\right)^p\right)}{x} - \frac{i\pi\,\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2 - i\pi\,\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(i\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^2,x)
```

```
[Out] -1/x*ln((b*x^3+a)^p)-1/2*(I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-
I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x
^3+a)^p)^3+I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-2*sum(_R*ln((-4*_R^3*a-3*
b*p^3)*x+a*p*_R^2),_R=RootOf(_Z^3*a+b*p^3))*x+2*ln(c))/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01721, size = 311, normalized size = 2.34

$$\frac{2\sqrt{3}px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+2px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)\right.}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*p*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqr
t(3)) - p*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 2
*p*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) - 2*p*log(b*x^3 + a) - 2*log(c)
)/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24798, size = 186, normalized size = 1.4

$$-\frac{1}{2}\,bp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^{2}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^{2}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a + 2*sqrt(3)*(-a*b^2)^
(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) - (-a*b
^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) - p*log(b*x^3 +
 a)/x - log(c)/x
```

3.21 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3} dx$

Optimal. Leaf size=139

$$-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

```
[Out] -(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*a
^(2/3)) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)) - (b^(2/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)) - Log[c*(a + b*x^3
)^p]/(2*x^2)
```

Rubi [A] time = 0.0747968, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2455, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^3,x]
```

```
[Out] -(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*a
^(2/3)) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)) - (b^(2/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)) - Log[c*(a + b*x^3
)^p]/(2*x^2)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\,dx &= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}+\frac{1}{2}(3bp)\int \frac{1}{a+bx^3}\,dx \\
&= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}+\frac{(bp)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2a^{2/3}}+\frac{(bp)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{2/3}} \\
&= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}-\frac{\left(b^{2/3}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4a^{2/3}}+\frac{(3bp)\int \frac{\cdots}{a^{2/3}-\cdots}}{4\cdots} \\
&= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}+\frac{\left(3b^{2/3}p\right)\cdots}{\cdots} \\
&= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}-\cdots
\end{aligned}
$$

Mathematica [A] time = 0.0327894, size = 134, normalized size = 0.96

$$
-\frac{b^{2/3}px^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2a^{2/3}\log\left(c\left(a+bx^3\right)^p\right)-2b^{2/3}px^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}b^{2/3}px^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\sqrt{3}}\right)\cdots}{4a^{2/3}x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^3,x]
```

```
[Out] -(2*Sqrt[3]*b^(2/3)*p*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b
^(2/3)*p*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*p*x^2*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] + 2*a^(2/3)*Log[c*(a + b*x^3)^p])/(4*a^(2/3)*x^2)
```

Maple [C] time = 0.322, size = 197, normalized size = 1.4

$$
-\frac{\ln\left(\left(bx^3+a\right)^p\right)}{2\,x^2}-\frac{i\pi\,\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2-i\pi\,\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(i\cdots\right.}{\cdots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^3,x)
```

```
[Out] -1/2/x^2*ln((b*x^3+a)^p)-1/4*(I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p
)^2-I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*
(b*x^3+a)^p)^3+I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-2*sum(_R*ln((-4*_R^3*
a^2+3*b^2*p^3)*x-a*p^2*_R*b),_R=RootOf(_Z^3*a^2-b^2*p^3))*x^2+2*ln(c))/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01815, size = 366, normalized size = 2.63

$$\frac{2\sqrt{3}px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*p*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - p*x^2*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(
1/3) + a^2*(b^2/a^2)^(2/3)) + 2*p*x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*p*log(b*x^3 + a) - 2*log(c))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30659, size = 186, normalized size = 1.34

$$-\frac{1}{4}\,bp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2
)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)) - 1/2*p*log(b*x^3 +
 a)/x^2 - 1/2*log(c)/x^2
```

3.22 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}-\frac{bp\log\left(a+bx^3\right)}{3a}+\frac{bp\log(x)}{a}$$

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Rubi [A] time = 0.0385288, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2395, 36, 29, 31}

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}-\frac{bp\log\left(a+bx^3\right)}{3a}+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^4,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^4}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2}\,dx,x,x^3\right)$$
$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^3\right)$$
$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3} + \frac{(bp)\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a} - \frac{\left(b^2p\right)\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^3\right)}{3a}$$
$$= \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^3\right)}{3a} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}$$

Mathematica [A] time = 0.002599, size = 45, normalized size = 1.

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3} - \frac{bp\log\left(a+bx^3\right)}{3a} + \frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^4,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Maple [C] time = 0.27, size = 173, normalized size = 3.8

$$-\frac{\ln\left(\left(bx^3+a\right)^p\right)}{3x^3} - \frac{i\pi\,\mathrm{acsgn}\left(i\left(bx^3+a\right)^p\right)\left(\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2 - i\pi\,\mathrm{acsgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\mathrm{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^4,x)
```

```
[Out] -1/3/x^3*ln((b*x^3+a)^p)-1/6*(I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a*csg
n(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-6*b*p*ln(x)*x
^3+2*b*p*ln(b*x^3+a)*x^3+2*ln(c)*a)/a/x^3
```

Maxima [A] time = 1.08538, size = 59, normalized size = 1.31

$$-\frac{1}{3}\,bp\left(\frac{\log\left(bx^3+a\right)}{a} - \frac{\log\left(x^3\right)}{a}\right) - \frac{\log\left(\left(bx^3+a\right)^p c\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b*p*(log(b*x^3 + a)/a - log(x^3)/a) - 1/3*log((b*x^3 + a)^p*c)/x^3
```

Fricas [A] time = 1.90716, size = 103, normalized size = 2.29

$$\frac{3\,bpx^3\log\left(x\right) - \left(bpx^3 + ap\right)\log\left(bx^3 + a\right) - a\log\left(c\right)}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*p*x^3*log(x) - (b*p*x^3 + a*p)*log(b*x^3 + a) - a*log(c))/(a*x^3)
```

Sympy [A] time = 22.911, size = 82, normalized size = 1.82

$$\begin{cases} -\frac{p\log\left(a+bx^3\right)}{3x^3} - \frac{\log\left(c\right)}{3x^3} + \frac{bp\log\left(x\right)}{a} - \frac{bp\log\left(a+bx^3\right)}{3a} & \text{for } a \neq 0 \\ -\frac{p\log\left(b\right)}{3x^3} - \frac{p\log\left(x\right)}{x^3} - \frac{p}{3x^3} - \frac{\log\left(c\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**4,x)
```

```
[Out] Piecewise((-p*log(a + b*x**3)/(3*x**3) - log(c)/(3*x**3) + b*p*log(x)/a - b
*p*log(a + b*x**3)/(3*a), Ne(a, 0)), (-p*log(b)/(3*x**3) - p*log(x)/x**3 -
p/(3*x**3) - log(c)/(3*x**3), True))
```

Giac [A] time = 1.22113, size = 78, normalized size = 1.73

$$-\frac{\frac{b^2p\log\left(bx^3+a\right)}{a} - \frac{b^2p\log\left(bx^3\right)}{a} + \frac{bp\log\left(bx^3+a\right)}{x^3} + \frac{b\log(c)}{x^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(b^2*p*log(b*x^3 + a)/a - b^2*p*log(b*x^3)/a + b*p*log(b*x^3 + a)/x^3
+ b*log(c)/x^3)/b
```

3.23 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=151

$$-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}+\frac{\sqrt{3}b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{3bp}{4ax}$$

```
[Out] (-3*b*p)/(4*a*x) + (Sqrt[3]*b^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(4*a^(4/3)) + (b^(4/3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*a^(4/3)
) - (b^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*a^(4/3))
- Log[c*(a + b*x^3)^p]/(4*x^4)
```

Rubi [A] time = 0.0945938, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2455, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}+\frac{\sqrt{3}b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{3bp}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^5,x]
```


```
[Out] (-3*b*p)/(4*a*x) + (Sqrt[3]*b^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(4*a^(4/3)) + (b^(4/3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*a^(4/3)
) - (b^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*a^(4/3))
- Log[c*(a + b*x^3)^p]/(4*x^4)
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}+\frac{1}{4}(3bp)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx$$
$$= -\frac{3bp}{4ax}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{\left(3b^2p\right)\int \frac{x}{a+bx^3}\,dx}{4a}$$
$$= -\frac{3bp}{4ax}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}+\frac{\left(b^{5/3}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{4a^{4/3}}-\frac{\left(b^{5/3}p\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4a^{4/3}}$$
$$= -\frac{3bp}{4ax}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{\left(b^{4/3}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{8a^{4/3}}-\frac{\left(3b^{5/3}p\right)}{}$$
$$= -\frac{3bp}{4ax}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{(3b}{}$$
$$= -\frac{3bp}{4ax}+\frac{\sqrt{3}b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right.}{8a^{4/3}}$$

Mathematica [C] time = 0.0028287, size = 49, normalized size = 0.32

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{3bp\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a}\right)}{4ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^5,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^3)/a)])/(4*a*x) - Log[c*(a +
 b*x^3)^p]/(4*x^4)
```

Maple [C] time = 0.323, size = 215, normalized size = 1.4

$$-\frac{\ln\left(\left(bx^3+a\right)^p\right)}{4x^4}-\frac{i\pi\,a\operatorname{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\operatorname{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2-i\pi\,a\operatorname{csgn}\left(i\left(bx^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(bx^3+a\right)^p\right)\operatorname{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^5,x)
```

```
[Out] -1/4/x^4*ln((b*x^3+a)^p)-1/8*(I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a*csg
n(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-2*sum(_R*ln((
-4*_R^3*a^4+3*b^4*p^3)*x-a^3*p*_R^2*b),_R=RootOf(_Z^3*a^4-b^4*p^3))*a*x^4+6
*b*p*x^3+2*ln(c)*a)/a/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45381, size = 348, normalized size = 2.3

$$-\frac{2\sqrt{3}bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right.\right.}{8\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*b*p*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*
sqrt(3)) + b*p*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3))
 - 2*b*p*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6*b*p*x^3 + 2*a*p*log(b
*x^3 + a) + 2*a*log(c))/(a*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2632, size = 207, normalized size = 1.37

$$\frac{1}{8}\,b^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^2}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b^2}\right)-\frac{p\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) -
(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)) - 1/4*p*
log(b*x^3 + a)/x^4 - 1/4*(3*b*p*x^3 + a*log(c))/(a*x^4)
```

3.24 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=151

$$\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}+\frac{\sqrt{3}b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\frac{3bp}{10ax^2}$$

```
[Out] (-3*b*p)/(10*a*x^2) + (Sqrt[3]*b^(5/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(5*a^(5/3)) - (b^(5/3)*p*Log[a^(1/3) + b^(1/3)*x])/(5*a^(5
/3)) + (b^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(10*a^(5/
3)) - Log[c*(a + b*x^3)^p]/(5*x^5)
```

Rubi [A] time = 0.0911043, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2455, 325, 200, 31, 634, 617, 204, 628}

$$\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}+\frac{\sqrt{3}b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\frac{3bp}{10ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^6,x]
```


```
[Out] (-3*b*p)/(10*a*x^2) + (Sqrt[3]*b^(5/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(5*a^(5/3)) - (b^(5/3)*p*Log[a^(1/3) + b^(1/3)*x])/(5*a^(5
/3)) + (b^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(10*a^(5/
3)) - Log[c*(a + b*x^3)^p]/(5*x^5)
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^6}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5} + \frac{1}{5}(3bp)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx$$

$$= -\frac{3bp}{10ax^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5} - \frac{\left(3b^2p\right)\int \frac{1}{a+bx^3}\,dx}{5a}$$

$$= -\frac{3bp}{10ax^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5} - \frac{\left(b^2p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{5a^{5/3}} - \frac{\left(b^2p\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{5a^{5/3}}$$

$$= -\frac{3bp}{10ax^2} - \frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5} + \frac{\left(b^{5/3}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{10a^{5/3}} - \frac{\left(3b^2p\right.}{}$$

$$= -\frac{3bp}{10ax^2} - \frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}} + \frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5} - \frac{(}{}$$

$$= -\frac{3bp}{10ax^2} + \frac{\sqrt{3}b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}} - \frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}} + \frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.}{10a^{5/3}}$$

Mathematica [C] time = 0.0026382, size = 49, normalized size = 0.32

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5} - \frac{3bp\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a}\right)}{10ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^6,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^3)/a)])/(10*a*x^2) - Log[c*(
a + b*x^3)^p]/(5*x^5)
```

Maple [C] time = 0.322, size = 216, normalized size = 1.4

$$-\frac{\ln\left(\left(bx^3+a\right)^p\right)}{5\,x^5}-\frac{-2\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^3+b^5p^3\right)}\_R\,\ln\left(\left(-4\,a^5\_R^3-3\,b^5p^3\right)x-p^2\_R\,b^3a^2\right)ax^5+i\pi\,a\mathrm{csgn}\left(i\left(bx^3+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^6,x)
```

```
[Out] -1/5/x^5*ln((b*x^3+a)^p)-1/10*(-2*sum(_R*ln((-4*_R^3*a^5-3*b^5*p^3)*x-p^2*_
R*b^3*a^2),_R=RootOf(_Z^3*a^5+b^5*p^3))*a*x^5+I*Pi*a*csgn(I*(b*x^3+a)^p)*cs
gn(I*c*(b*x^3+a)^p)^2-I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn
(I*c)-I*Pi*a*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*
c)+3*b*p*x^3+2*ln(c)*a)/a/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37267, size = 412, normalized size = 2.73

$$\frac{2\sqrt{3}bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(\right.}{10\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*sqrt(3)*b*p*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^
2)^(2/3) - sqrt(3)*b)/b) - b*p*x^5*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*b*p*x^5*(-b^2/a^2)^(1/3)*log(b*x
- a*(-b^2/a^2)^(1/3)) - 3*b*p*x^3 - 2*a*p*log(b*x^3 + a) - 2*a*log(c))/(a*x
^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17594, size = 208, normalized size = 1.38

$$\frac{1}{10}\,b^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b}\right)-\frac{p\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^2*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (
-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b)) - 1/5*p*log
(b*x^3 + a)/x^5 - 1/10*(3*b*p*x^3 + 2*a*log(c))/(a*x^5)
```

3.25 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^7} dx$

Optimal. Leaf size=64

$$\frac{b^2 p \log\left(a+bx^3\right)}{6a^2}-\frac{b^2 p \log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}-\frac{bp}{6ax^3}$$

```
[Out] -(b*p)/(6*a*x^3) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^3])/(6*a^2)
- Log[c*(a + b*x^3)^p]/(6*x^6)
```

Rubi [A] time = 0.051364, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 44}

$$\frac{b^2 p \log\left(a+bx^3\right)}{6a^2}-\frac{b^2 p \log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}-\frac{bp}{6ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^7,x]
```

```
[Out] -(b*p)/(6*a*x^3) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^3])/(6*a^2)
- Log[c*(a + b*x^3)^p]/(6*x^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^7}dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x^3}dx, x, x^3\right)$$
$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6} + \frac{1}{6}(bp)\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)}dx, x, x^3\right)$$
$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6} + \frac{1}{6}(bp)\operatorname{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx, x, x^3\right)$$
$$= -\frac{bp}{6ax^3} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^3\right)}{6a^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}$$

Mathematica [A] time = 0.0373722, size = 56, normalized size = 0.88

$$\frac{1}{6}bp\left(\frac{b\log\left(a+bx^3\right)}{a^2} - \frac{3b\log(x)}{a^2} - \frac{1}{ax^3}\right) - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^7,x]
```

```
[Out] (b*p*(-(1/(a*x^3)) - (3*b*Log[x])/a^2 + (b*Log[a + b*x^3])/a^2))/6 - Log[c*
(a + b*x^3)^p]/(6*x^6)
```

Maple [C] time = 0.273, size = 198, normalized size = 3.1

$$-\frac{\ln\left(\left(bx^3+a\right)^p\right)}{6x^6} - \frac{6b^2p\ln(x)x^6 - 2b^2p\ln\left(-bx^3-a\right)x^6 + i\pi a^2\operatorname{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\operatorname{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2 - i\pi a^2\operatorname{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^7,x)
```

```
[Out] -1/6/x^6*ln((b*x^3+a)^p)-1/12*(6*b^2*p*ln(x)*x^6-2*b^2*p*ln(-b*x^3-a)*x^6+I
*Pi*a^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*a^2*csgn(I*(b*x^3+
a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^3+a)^p)^3+I*Pi
*a^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*a*b*p*x^3+2*ln(c)*a^2)/a^2/x^6
```

Maxima [A] time = 1.1582, size = 73, normalized size = 1.14

$$\frac{1}{6}bp\left(\frac{b\log\left(bx^3+a\right)}{a^2} - \frac{b\log\left(x^3\right)}{a^2} - \frac{1}{ax^3}\right) - \frac{\log\left(\left(bx^3+a\right)^pc\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b*p*(b*log(b*x^3 + a)/a^2 - b*log(x^3)/a^2 - 1/(a*x^3)) - 1/6*log((b*x^
3 + a)^p*c)/x^6
```

Fricas [A] time = 2.22418, size = 134, normalized size = 2.09

$$-\frac{3\,b^2px^6\log\left(x\right)+abpx^3+a^2\log\left(c\right)-\left(b^2px^6-a^2p\right)\log\left(bx^3+a\right)}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*p*x^6*log(x) + a*b*p*x^3 + a^2*log(c) - (b^2*p*x^6 - a^2*p)*log
(b*x^3 + a))/(a^2*x^6)
```

Sympy [A] time = 80.932, size = 102, normalized size = 1.59

$$\begin{cases} -\frac{p\log\left(a+bx^3\right)}{6x^6}-\frac{\log\left(c\right)}{6x^6}-\frac{bp}{6ax^3}-\frac{b^2p\log\left(x\right)}{2a^2}+\frac{b^2p\log\left(a+bx^3\right)}{6a^2} & \text{for } a\neq 0 \\ -\frac{p\log\left(b\right)}{6x^6}-\frac{p\log\left(x\right)}{2x^6}-\frac{p}{12x^6}-\frac{\log\left(c\right)}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**7,x)
```

```
[Out] Piecewise((-p*log(a + b*x**3)/(6*x**6) - log(c)/(6*x**6) - b*p/(6*a*x**3) -
 b**2*p*log(x)/(2*a**2) + b**2*p*log(a + b*x**3)/(6*a**2), Ne(a, 0)), (-p*l
og(b)/(6*x**6) - p*log(x)/(2*x**6) - p/(12*x**6) - log(c)/(6*x**6), True))
```

Giac [B] time = 1.32081, size = 178, normalized size = 2.78

$$-\frac{\frac{b^3p\log\left(bx^3+a\right)}{\left(bx^3+a\right)^2-2\left(bx^3+a\right)a+a^2}-\frac{b^3p\log\left(bx^3+a\right)}{a^2}+\frac{b^3p\log\left(bx^3\right)}{a^2}+\frac{\left(bx^3+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(bx^3+a\right)^2a-2\left(bx^3+a\right)a^2+a^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*p*log(b*x^3 + a)/((b*x^3 + a)^2 - 2*(b*x^3 + a)*a + a^2) - b^3*p*
log(b*x^3 + a)/a^2 + b^3*p*log(b*x^3)/a^2 + ((b*x^3 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^3 + a)^2*a - 2*(b*x^3 + a)*a^2 + a^3))/b
```

3.26 $\int x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=89

$$\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}-\frac{b^4px}{5a^4}+\frac{b^5p\log(ax+b)}{5a^5}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^4}{20a}$$

```
[Out] -(b^4*p*x)/(5*a^4) + (b^3*p*x^2)/(10*a^3) - (b^2*p*x^3)/(15*a^2) + (b*p*x^4
)/(20*a) + (x^5*Log[c*(a + b/x)^p])/5 + (b^5*p*Log[b + a*x])/(5*a^5)
```

Rubi [A] time = 0.0549788, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 263, 43}

$$\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}-\frac{b^4px}{5a^4}+\frac{b^5p\log(ax+b)}{5a^5}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b/x)^p],x]
```



```
[Out] -(b^4*p*x)/(5*a^4) + (b^3*p*x^2)/(10*a^3) - (b^2*p*x^3)/(15*a^2) + (b*p*x^4
)/(20*a) + (x^5*Log[c*(a + b/x)^p])/5 + (b^5*p*Log[b + a*x])/(5*a^5)
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\frac{x^3}{a+\frac{b}{x}}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\frac{x^4}{b+ax}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\left(-\frac{b^3}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{a^2}+\frac{x^3}{a}+\frac{b^4}{a^4(b+ax)}\right)dx\\
&= -\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{bpx^4}{20a}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^5p\log(b+ax)}{5a^5}
\end{aligned}$$

Mathematica [A] time = 0.0457851, size = 85, normalized size = 0.96

$$\frac{abpx\left(-4a^2bx^2+3a^3x^3+6ab^2x-12b^3\right)+12a^5x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+12b^5p\log\left(a+\frac{b}{x}\right)+12b^5p\log(x)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(-12*b^3 + 6*a*b^2*x - 4*a^2*b*x^2 + 3*a^3*x^3) + 12*b^5*p*Log[a +
 b/x] + 12*a^5*x^5*Log[c*(a + b/x)^p] + 12*b^5*p*Log[x])/(60*a^5)
```

Maple [F] time = 0.358, size = 0, normalized size = 0.

$$\int x^4\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^4*ln(c*(a+b/x)^p),x)
```

Maxima [A] time = 1.06121, size = 100, normalized size = 1.12

$$\frac{1}{5}x^5\log\left(\left(a+\frac{b}{x}\right)^pc\right)+\frac{1}{60}bp\left(\frac{12\,b^4\log\left(ax+b\right)}{a^5}+\frac{3\,a^3x^4-4\,a^2bx^3+6\,ab^2x^2-12\,b^3x}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((a + b/x)^p*c) + 1/60*b*p*(12*b^4*log(a*x + b)/a^5 + (3*a^3*x^4
 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 12*b^3*x)/a^4)
```

Fricas [A] time = 2.33376, size = 207, normalized size = 2.33

$$\frac{12\,a^5px^5\log\left(\frac{ax+b}{x}\right)+12\,a^5x^5\log\left(c\right)+3\,a^4bpx^4-4\,a^3b^2px^3+6\,a^2b^3px^2-12\,ab^4px+12\,b^5p\log\left(ax+b\right)}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^5*p*x^5*log((a*x + b)/x) + 12*a^5*x^5*log(c) + 3*a^4*b*p*x^4 - 4
*a^3*b^2*p*x^3 + 6*a^2*b^3*p*x^2 - 12*a*b^4*p*x + 12*b^5*p*log(a*x + b))/a^
5
```

Sympy [A] time = 34.2231, size = 122, normalized size = 1.37

$$\begin{cases} \frac{px^5 \log\left(a+\frac{b}{x}\right)}{5} + \frac{x^5 \log(c)}{5} + \frac{bpx^4}{20a} - \frac{b^2px^3}{15a^2} + \frac{b^3px^2}{10a^3} - \frac{b^4px}{5a^4} + \frac{b^5p\log(ax+b)}{5a^5} & \text{for } a \neq 0 \\ \frac{px^5\log(b)}{5} - \frac{px^5\log(x)}{5} + \frac{px^5}{25} + \frac{x^5\log(c)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**5*log(a + b/x)/5 + x**5*log(c)/5 + b*p*x**4/(20*a) - b**2*p
*x**3/(15*a**2) + b**3*p*x**2/(10*a**3) - b**4*p*x/(5*a**4) + b**5*p*log(a*
x + b)/(5*a**5), Ne(a, 0)), (p*x**5*log(b)/5 - p*x**5*log(x)/5 + p*x**5/25
+ x**5*log(c)/5, True))
```

Giac [A] time = 1.33013, size = 117, normalized size = 1.31

$$\frac{1}{5}px^5\log(ax+b) - \frac{1}{5}px^5\log(x) + \frac{1}{5}x^5\log(c) + \frac{bpx^4}{20a} - \frac{b^2px^3}{15a^2} + \frac{b^3px^2}{10a^3} - \frac{b^4px}{5a^4} + \frac{b^5p\log(ax+b)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(a*x + b) - 1/5*p*x^5*log(x) + 1/5*x^5*log(c) + 1/20*b*p*x^4/a
 - 1/15*b^2*p*x^3/a^2 + 1/10*b^3*p*x^2/a^3 - 1/5*b^4*p*x/a^4 + 1/5*b^5*p*lo
g(a*x + b)/a^5
```

3.27 $\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=75

$$-\frac{b^2px^2}{8a^2}+\frac{b^3px}{4a^3}-\frac{b^4p\log(ax+b)}{4a^4}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^3}{12a}$$

```
[Out] (b^3*p*x)/(4*a^3) - (b^2*p*x^2)/(8*a^2) + (b*p*x^3)/(12*a) + (x^4*Log[c*(a
+ b/x)^p])/4 - (b^4*p*Log[b + a*x])/(4*a^4)
```

Rubi [A] time = 0.040069, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 263, 43}

$$-\frac{b^2px^2}{8a^2}+\frac{b^3px}{4a^3}-\frac{b^4p\log(ax+b)}{4a^4}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^3}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (b^3*p*x)/(4*a^3) - (b^2*p*x^2)/(8*a^2) + (b*p*x^3)/(12*a) + (x^4*Log[c*(a
+ b/x)^p])/4 - (b^4*p*Log[b + a*x])/(4*a^4)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\frac{x^2}{a+\frac{b}{x}}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\frac{x^3}{b+ax}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\left(\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}-\frac{b^3}{a^3(b+ax)}\right)dx\\
&= \frac{b^3px}{4a^3}-\frac{b^2px^2}{8a^2}+\frac{bpx^3}{12a}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^4p\log(b+ax)}{4a^4}
\end{aligned}$$

Mathematica [A] time = 0.0319432, size = 74, normalized size = 0.99

$$\frac{abpx\left(2a^2x^2-3abx+6b^2\right)+6a^4x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)-6b^4p\log\left(a+\frac{b}{x}\right)-6b^4p\log(x)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(6*b^2 - 3*a*b*x + 2*a^2*x^2) - 6*b^4*p*Log[a + b/x] + 6*a^4*x^4*L
og[c*(a + b/x)^p] - 6*b^4*p*Log[x])/(24*a^4)
```

Maple [F] time = 0.236, size = 0, normalized size = 0.

$$\int x^3\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^3*ln(c*(a+b/x)^p),x)
```

Maxima [A] time = 1.20977, size = 86, normalized size = 1.15

$$\frac{1}{4}x^4\log\left(\left(a+\frac{b}{x}\right)^pc\right)-\frac{1}{24}bp\left(\frac{6\,b^3\log\left(ax+b\right)}{a^4}-\frac{2\,a^2x^3-3\,abx^2+6\,b^2x}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((a + b/x)^p*c) - 1/24*b*p*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3
- 3*a*b*x^2 + 6*b^2*x)/a^3)
```

Fricas [A] time = 2.28267, size = 177, normalized size = 2.36

$$\frac{6\,a^4px^4\log\left(\frac{ax+b}{x}\right)+6\,a^4x^4\log\left(c\right)+2\,a^3bpx^3-3\,a^2b^2px^2+6\,ab^3px-6\,b^4p\log\left(ax+b\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^4*p*x^4*log((a*x + b)/x) + 6*a^4*x^4*log(c) + 2*a^3*b*p*x^3 - 3*a
^2*b^2*p*x^2 + 6*a*b^3*p*x - 6*b^4*p*log(a*x + b))/a^4
```

Sympy [A] time = 16.4616, size = 109, normalized size = 1.45

$$\begin{cases} \frac{px^4\log\left(a+\frac{b}{x}\right)}{4} + \frac{x^4\log(c)}{4} + \frac{bpx^3}{12a} - \frac{b^2px^2}{8a^2} + \frac{b^3px}{4a^3} - \frac{b^4p\log(ax+b)}{4a^4} & \text{for } a \neq 0 \\ \frac{px^4\log(b)}{4} - \frac{px^4\log(x)}{4} + \frac{px^4}{16} + \frac{x^4\log(c)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**4*log(a + b/x)/4 + x**4*log(c)/4 + b*p*x**3/(12*a) - b**2*p
*x**2/(8*a**2) + b**3*p*x/(4*a**3) - b**4*p*log(a*x + b)/(4*a**4), Ne(a, 0)
), (p*x**4*log(b)/4 - p*x**4*log(x)/4 + p*x**4/16 + x**4*log(c)/4, True))
```

Giac [A] time = 1.29949, size = 101, normalized size = 1.35

$$\frac{1}{4}px^4\log(ax+b) - \frac{1}{4}px^4\log(x) + \frac{1}{4}x^4\log(c) + \frac{bpx^3}{12a} - \frac{b^2px^2}{8a^2} + \frac{b^3px}{4a^3} - \frac{b^4p\log(ax+b)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/4*p*x^4*log(a*x + b) - 1/4*p*x^4*log(x) + 1/4*x^4*log(c) + 1/12*b*p*x^3/a
 - 1/8*b^2*p*x^2/a^2 + 1/4*b^3*p*x/a^3 - 1/4*b^4*p*log(a*x + b)/a^4
```

3.28 $\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=61

$$-\frac{b^2px}{3a^2}+\frac{b^3p\log(ax+b)}{3a^3}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^2}{6a}$$

```
[Out] -(b^2*p*x)/(3*a^2) + (b*p*x^2)/(6*a) + (x^3*Log[c*(a + b/x)^p])/3 + (b^3*p*
Log[b + a*x])/(3*a^3)
```

Rubi [A] time = 0.0316943, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 263, 43}

$$-\frac{b^2px}{3a^2}+\frac{b^3p\log(ax+b)}{3a^3}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b/x)^p],x]
```

```
[Out] -(b^2*p*x)/(3*a^2) + (b*p*x^2)/(6*a) + (x^3*Log[c*(a + b/x)^p])/3 + (b^3*p*
Log[b + a*x])/(3*a^3)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\frac{x}{a+\frac{b}{x}}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\frac{x^2}{b+ax}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right)dx\\
&= -\frac{b^2px}{3a^2}+\frac{bpx^2}{6a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^3p\log(b+ax)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.0240148, size = 62, normalized size = 1.02

$$\frac{2a^3x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+2b^3p\log\left(a+\frac{b}{x}\right)+abpx(ax-2b)+2b^3p\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(-2*b + a*x) + 2*b^3*p*Log[a + b/x] + 2*a^3*x^3*Log[c*(a + b/x)^p]
 + 2*b^3*p*Log[x])/(6*a^3)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int x^2\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^2*ln(c*(a+b/x)^p),x)
```

Maxima [A] time = 1.20513, size = 69, normalized size = 1.13

$$\frac{1}{3}x^3\log\left(\left(a+\frac{b}{x}\right)^p c\right)+\frac{1}{6}bp\left(\frac{2\,b^2\log\left(ax+b\right)}{a^3}+\frac{ax^2-2\,bx}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((a + b/x)^p*c) + 1/6*b*p*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b
*x)/a^2)
```

Fricas [A] time = 2.14907, size = 149, normalized size = 2.44

$$\frac{2\,a^3px^3\log\left(\frac{ax+b}{x}\right)+2\,a^3x^3\log\left(c\right)+a^2bpx^2-2\,ab^2px+2\,b^3p\log\left(ax+b\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*p*x^3*log((a*x + b)/x) + 2*a^3*x^3*log(c) + a^2*b*p*x^2 - 2*a*b^
2*p*x + 2*b^3*p*log(a*x + b))/a^3
```

Sympy [A] time = 6.51316, size = 95, normalized size = 1.56

$$\begin{cases} \frac{px^3 \log\left(a+\frac{b}{x}\right)}{3} + \frac{x^3 \log(c)}{3} + \frac{bpx^2}{6a} - \frac{b^2px}{3a^2} + \frac{b^3p\log(ax+b)}{3a^3} & \text{for } a \neq 0 \\ \frac{px^3\log(b)}{3} - \frac{px^3\log(x)}{3} + \frac{px^3}{9} + \frac{x^3\log(c)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**3*log(a + b/x)/3 + x**3*log(c)/3 + b*p*x**2/(6*a) - b**2*p*
x/(3*a**2) + b**3*p*log(a*x + b)/(3*a**3), Ne(a, 0)), (p*x**3*log(b)/3 - p*
x**3*log(x)/3 + p*x**3/9 + x**3*log(c)/3, True))
```

Giac [A] time = 1.28979, size = 85, normalized size = 1.39

$$\frac{1}{3}\,px^3\log(ax+b) - \frac{1}{3}\,px^3\log(x) + \frac{1}{3}\,x^3\log(c) + \frac{bpx^2}{6\,a} - \frac{b^2px}{3\,a^2} + \frac{b^3p\log(ax+b)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(a*x + b) - 1/3*p*x^3*log(x) + 1/3*x^3*log(c) + 1/6*b*p*x^2/a
- 1/3*b^2*p*x/a^2 + 1/3*b^3*p*log(a*x + b)/a^3
```

3.29 $\int x \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=47

$$-\frac{b^2 p \log (a x+b)}{2 a^2}+\frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p x}{2 a}$$

```
[Out] (b*p*x)/(2*a) + (x^2*Log[c*(a + b/x)^p])/2 - (b^2*p*Log[b + a*x])/(2*a^2)
```

Rubi [A] time = 0.0210677, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2455, 193, 43}

$$-\frac{b^2 p \log (a x+b)}{2 a^2}+\frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p x}{2 a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*p*x)/(2*a) + (x^2*Log[c*(a + b/x)^p])/2 - (b^2*p*Log[b + a*x])/(2*a^2)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx &= \frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2} (b p) \int \frac{1}{a+\frac{b}{x}} \, dx\\
&= \frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2} (b p) \int \frac{x}{b+a x} \, dx\\
&= \frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2} (b p) \int \left(\frac{1}{a}-\frac{b}{a (b+a x)}\right) \, dx\\
&= \frac{b p x}{2 a}+\frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right)-\frac{b^2 p \log (b+a x)}{2 a^2}
\end{aligned}$$

Mathematica [A] time = 0.0168691, size = 40, normalized size = 0.85

$$\frac{1}{2}\left(\frac{bp(ax-b\log(ax+b))}{a^2}+x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b/x)^p],x]
```

```
[Out] (x^2*Log[c*(a + b/x)^p] + (b*p*(a*x - b*Log[b + a*x]))/a^2)/2
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int x\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x*ln(c*(a+b/x)^p),x)
```

Maxima [A] time = 1.1689, size = 54, normalized size = 1.15

$$\frac{1}{2}\,bp\left(\frac{x}{a}-\frac{b\log(ax+b)}{a^2}\right)+\frac{1}{2}\,x^2\log\left(\left(a+\frac{b}{x}\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(x/a - b*log(a*x + b)/a^2) + 1/2*x^2*log((a + b/x)^p*c)
```

Fricas [A] time = 2.12569, size = 116, normalized size = 2.47

$$\frac{a^2px^2\log\left(\frac{ax+b}{x}\right)+a^2x^2\log(c)+abpx-b^2p\log(ax+b)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*p*x^2*log((a*x + b)/x) + a^2*x^2*log(c) + a*b*p*x - b^2*p*log(a*x
+ b))/a^2
```

Sympy [A] time = 4.21775, size = 82, normalized size = 1.74

$$\begin{cases}\frac{px^2\log\left(a+\frac{b}{x}\right)}{2}+\frac{x^2\log(c)}{2}+\frac{bpx}{2a}-\frac{b^2p\log(ax+b)}{2a^2} & \text{for } a\neq 0\\ \frac{px^2\log(b)}{2}-\frac{px^2\log(x)}{2}+\frac{px^2}{4}+\frac{x^2\log(c)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**2*log(a + b/x)/2 + x**2*log(c)/2 + b*p*x/(2*a) - b**2*p*log
(a*x + b)/(2*a**2), Ne(a, 0)), (p*x**2*log(b)/2 - p*x**2*log(x)/2 + p*x**2/
4 + x**2*log(c)/2, True))
```

Giac [A] time = 1.31848, size = 69, normalized size = 1.47

$$\frac{1}{2}px^2\log(ax+b) - \frac{1}{2}px^2\log(x) + \frac{1}{2}x^2\log(c) + \frac{bpx}{2a} - \frac{b^2p\log(ax+b)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/2*p*x^2*log(a*x + b) - 1/2*p*x^2*log(x) + 1/2*x^2*log(c) + 1/2*b*p*x/a -
1/2*b^2*p*log(a*x + b)/a^2
```

3.30 $\int \log \left(c \left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=27

$$x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (a x+b)}{a}$$

```
[Out] x*Log[c*(a + b/x)^p] + (b*p*Log[b + a*x])/a
```

Rubi [A] time = 0.0087283, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2448, 263, 31}

$$x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (a x+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p],x]

[Out] x*Log[c*(a + b/x)^p] + (b*p*Log[b + a*x])/a
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+\frac{b}{x}\right)^p\right) dx &= x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+(b p) \int \frac{1}{\left(a+\frac{b}{x}\right) x} dx \\
&= x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+(b p) \int \frac{1}{b+a x} dx \\
&= x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (b+a x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.002082, size = 37, normalized size = 1.37

$$x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log \left(a+\frac{b}{x}\right)}{a}+\frac{b p \log (x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p],x]
```

```
[Out] (b*p*Log[a + b/x])/a + x*Log[c*(a + b/x)^p] + (b*p*Log[x])/a
```

Maple [A] time = 0.06, size = 30, normalized size = 1.1

$$x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)+\frac{bp\ln(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p),x)
```

```
[Out] x*ln(c*((a*x+b)/x)^p)+b*p*ln(a*x+b)/a
```

Maxima [A] time = 1.14417, size = 36, normalized size = 1.33

$$x\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)+\frac{bp\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] x*log((a + b/x)^p*c) + b*p*log(a*x + b)/a
```

Fricas [A] time = 2.20548, size = 81, normalized size = 3.

$$\frac{apx\log\left(\frac{ax+b}{x}\right)+bp\log(ax+b)+ax\log(c)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] (a*p*x*log((a*x + b)/x) + b*p*log(a*x + b) + a*x*log(c))/a
```

Sympy [A] time = 1.31805, size = 48, normalized size = 1.78

$$\begin{cases} px\log\left(a+\frac{b}{x}\right)+x\log(c)+\frac{bp\log(ax+b)}{a} & \text{for } a\neq 0 \\ px\log(b)-px\log(x)+px+x\log(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x*log(a + b/x) + x*log(c) + b*p*log(a*x + b)/a, Ne(a, 0)), (p*
x*log(b) - p*x*log(x) + p*x + x*log(c), True))
```

Giac [A] time = 1.27674, size = 43, normalized size = 1.59

$$px\log(ax+b) - px\log(x) + \frac{bp\log(ax+b)}{a} + x\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] p*x*log(a*x + b) - p*x*log(x) + b*p*log(a*x + b)/a + x*log(c)
```

3.31 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=40

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)$$

```
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, 1 + b/(a*x)]
```

Rubi [A] time = 0.0370085, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, 1 + b/(a*x)]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx &= -\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx,x,\frac{1}{x}\right)\\
&= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)+(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\frac{1}{x}\right)\\
&= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)-p\text{Li}_2\left(1+\frac{b}{ax}\right)
\end{aligned}$$

Mathematica [A] time = 0.0028915, size = 41, normalized size = 1.02

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\text{PolyLog}\left(2,\frac{a+\frac{b}{x}}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, (a + b/x)/a]
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{x}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x,x)
```

Maxima [B] time = 1.22406, size = 112, normalized size = 2.8

$$\frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x}\right)\log(x)}{b}+\frac{\log(x)^2}{b}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\text{Li}_2\left(-\frac{ax}{b}\right)\right)}{b}\right)-p\log\left(a+\frac{b}{x}\right)\log(x)+\log\left(\left(a+\frac{b}{x}\right)^p c\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(a + b/x)*log(x)/b + log(x)^2/b - 2*(log(a*x/b + 1)*log(x) +
dilog(-a*x/b))/b) - p*log(a + b/x)*log(x) + log((a + b/x)^p*c)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/x, x)
```

3.32 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=30

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

```
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Rubi [A] time = 0.0211428, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^2,x]

[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx &= -\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,a+\frac{b}{x}\right)}{b}\\
&= \frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0044495, size = 30, normalized size = 1.

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^2,x]
```

```
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Maple [A] time = 0.058, size = 48, normalized size = 1.6

$$-\frac{a}{b}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{1}{x}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{ap}{b}+\frac{p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^2,x)
```

```
[Out] -1/b*ln(c*(a+b/x)^p)*a-ln(c*(a+b/x)^p)/x+1/b*a*p+p/x
```

Maxima [A] time = 1.08494, size = 68, normalized size = 2.27

$$-bp\left(\frac{a\log(ax+b)}{b^2}-\frac{a\log(x)}{b^2}-\frac{1}{bx}\right)-\frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -b*p*(a*log(a*x + b)/b^2 - a*log(x)/b^2 - 1/(b*x)) - log((a + b/x)^p*c)/x
```

Fricas [A] time = 2.15596, size = 77, normalized size = 2.57

$$\frac{bp-b\log(c)-\left(apx+bp\right)\log\left(\frac{ax+b}{x}\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="fricas")
```

```
[Out] (b*p - b*log(c) - (a*p*x + b*p)*log((a*x + b)/x))/(b*x)
```

Sympy [A] time = 2.92736, size = 39, normalized size = 1.3

$$\begin{cases}-\frac{ap\log\left(a+\frac{b}{x}\right)}{b}-\frac{p\log\left(a+\frac{b}{x}\right)}{x}+\frac{p}{x}-\frac{\log(c)}{x} & \text{for}\ b\neq 0\\ -\frac{\log\left(a^p c\right)}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**2,x)
```

```
[Out] Piecewise((-a*p*log(a + b/x)/b - p*log(a + b/x)/x + p/x - log(c)/x, Ne(b, 0
)), (-log(a**p*c)/x, True))
```

Giac [A] time = 1.33727, size = 69, normalized size = 2.3

$$-\frac{ap\log(ax+b)}{b}+\frac{ap\log(x)}{b}-\frac{p\log(ax+b)}{x}+\frac{p\log(x)}{x}+\frac{p-\log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="giac")
```

```
[Out] -a*p*log(a*x + b)/b + a*p*log(x)/b - p*log(a*x + b)/x + p*log(x)/x + (p - l
og(c))/x
```

3.33 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=59

$$\frac{a^2 p \log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}-\frac{ap}{2bx}+\frac{p}{4x^2}$$

```
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Rubi [A] time = 0.0354572, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^2 p \log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}-\frac{ap}{2bx}+\frac{p}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^3,x]
```

```
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = -\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{p}{4x^2} - \frac{ap}{2bx} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

Mathematica [A] time = 0.01413, size = 59, normalized size = 1.

$$\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} - \frac{ap}{2bx} + \frac{p}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^3,x]
```

```
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^3,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^3,x)
```

Maxima [A] time = 1.05556, size = 85, normalized size = 1.44

$$\frac{1}{4}\,bp\left(\frac{2\,a^2\log\left(ax+b\right)}{b^3} - \frac{2\,a^2\log\left(x\right)}{b^3} - \frac{2\,ax-b}{b^2x^2}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(2*a^2*log(a*x + b)/b^3 - 2*a^2*log(x)/b^3 - (2*a*x - b)/(b^2*x^2))
 - 1/2*log((a + b/x)^p*c)/x^2
```

Fricas [A] time = 2.22248, size = 124, normalized size = 2.1

$$-\frac{2\,abpx - b^2p + 2\,b^2\log\left(c\right) - 2\,\left(a^2px^2 - b^2p\right)\log\left(\frac{ax+b}{x}\right)}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*p*x - b^2*p + 2*b^2*log(c) - 2*(a^2*p*x^2 - b^2*p)*log((a*x + b
)/x))/(b^2*x^2)
```

Sympy [A] time = 6.97564, size = 66, normalized size = 1.12

$$\begin{cases} \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{ap}{2bx} - \frac{p\log\left(a+\frac{b}{x}\right)}{2x^2} + \frac{p}{4x^2} - \frac{\log\left(c\right)}{2x^2} & \text{for}\: b \neq 0 \\ -\frac{\log\left(a^pc\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**3,x)
```

```
[Out] Piecewise((a**2*p*log(a + b/x)/(2*b**2) - a*p/(2*b*x) - p*log(a + b/x)/(2*x
**2) + p/(4*x**2) - log(c)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), Tru
e))
```

Giac [A] time = 1.32137, size = 95, normalized size = 1.61

$$\frac{a^2p\log\left(ax+b\right)}{2\,b^2} - \frac{a^2p\log\left(x\right)}{2\,b^2} - \frac{p\log\left(ax+b\right)}{2\,x^2} + \frac{p\log\left(x\right)}{2\,x^2} - \frac{2\,apx - bp + 2\,b\log\left(c\right)}{4\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*a^2*p*log(a*x + b)/b^2 - 1/2*a^2*p*log(x)/b^2 - 1/2*p*log(a*x + b)/x^2
+ 1/2*p*log(x)/x^2 - 1/4*(2*a*p*x - b*p + 2*b*log(c))/(b*x^2)
```

3.34 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4} dx$

Optimal. Leaf size=73

$$\frac{a^2p}{3b^2x} - \frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} - \frac{ap}{6bx^2} + \frac{p}{9x^3}$$

```
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Rubi [A] time = 0.0501546, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^2p}{3b^2x} - \frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} - \frac{ap}{6bx^2} + \frac{p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^4,x]
```

```
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = -\operatorname{Subst}\left(\int x^2\log\left(c(a+bx)^p\right)dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\operatorname{Subst}\left(\int \frac{x^3}{a+bx}dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\operatorname{Subst}\left(\int \left(\frac{a^2}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} - \frac{a^3}{b^3(a+bx)}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{p}{9x^3} - \frac{ap}{6bx^2} + \frac{a^2p}{3b^2x} - \frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

Mathematica [A] time = 0.0173841, size = 73, normalized size = 1.

$$\frac{a^2p}{3b^2x} - \frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} - \frac{ap}{6bx^2} + \frac{p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^4,x]
```

```
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^4,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^4,x)
```

Maxima [A] time = 1.09592, size = 100, normalized size = 1.37

$$-\frac{1}{18}bp\left(\frac{6a^3\log(ax+b)}{b^4} - \frac{6a^3\log(x)}{b^4} - \frac{6a^2x^2 - 3abx + 2b^2}{b^3x^3}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*b*p*(6*a^3*log(a*x + b)/b^4 - 6*a^3*log(x)/b^4 - (6*a^2*x^2 - 3*a*b*x
 + 2*b^2)/(b^3*x^3)) - 1/3*log((a + b/x)^p*c)/x^3
```

Fricas [A] time = 2.2719, size = 151, normalized size = 2.07

$$\frac{6\,a^2bpx^2 - 3\,ab^2px + 2\,b^3p - 6\,b^3\log\left(c\right) - 6\left(a^3px^3 + b^3p\right)\log\left(\frac{ax+b}{x}\right)}{18\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a^2*b*p*x^2 - 3*a*b^2*p*x + 2*b^3*p - 6*b^3*log(c) - 6*(a^3*p*x^3 +
 b^3*p)*log((a*x + b)/x))/(b^3*x^3)
```

Sympy [A] time = 11.2477, size = 80, normalized size = 1.1

$$\begin{cases} -\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3} + \frac{a^2p}{3b^2x} - \frac{ap}{6bx^2} - \frac{p\log\left(a+\frac{b}{x}\right)}{3x^3} + \frac{p}{9x^3} - \frac{\log\left(c\right)}{3x^3} & \text{for}\: b \neq 0 \\ -\frac{\log\left(a^pc\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**4,x)
```

```
[Out] Piecewise((-a**3*p*log(a + b/x)/(3*b**3) + a**2*p/(3*b**2*x) - a*p/(6*b*x**
2) - p*log(a + b/x)/(3*x**3) + p/(9*x**3) - log(c)/(3*x**3), Ne(b, 0)), (-l
og(a**p*c)/(3*x**3), True))
```

Giac [A] time = 1.28302, size = 113, normalized size = 1.55

$$-\frac{a^3p\log\left(ax+b\right)}{3\,b^3} + \frac{a^3p\log\left(x\right)}{3\,b^3} - \frac{p\log\left(ax+b\right)}{3\,x^3} + \frac{p\log\left(x\right)}{3\,x^3} + \frac{6\,a^2px^2 - 3\,abpx + 2\,b^2p - 6\,b^2\log\left(c\right)}{18\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*a^3*p*log(a*x + b)/b^3 + 1/3*a^3*p*log(x)/b^3 - 1/3*p*log(a*x + b)/x^3
 + 1/3*p*log(x)/x^3 + 1/18*(6*a^2*p*x^2 - 3*a*b*p*x + 2*b^2*p - 6*b^2*log(c
))/(b^2*x^3)
```

3.35 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=87

$$\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}-\frac{ap}{12bx^3}+\frac{p}{16x^4}$$

```
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Rubi [A] time = 0.0560602, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}-\frac{ap}{12bx^3}+\frac{p}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^5,x]
```

```
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx = -\operatorname{Subst}\left(\int x^3\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\operatorname{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{p}{16x^4} - \frac{ap}{12bx^3} + \frac{a^2p}{8b^2x^2} - \frac{a^3p}{4b^3x} + \frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

Mathematica [A] time = 0.0207037, size = 87, normalized size = 1.

$$\frac{a^2p}{8b^2x^2} - \frac{a^3p}{4b^3x} + \frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} - \frac{ap}{12bx^3} + \frac{p}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^5,x]
```

```
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^5,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^5,x)
```

Maxima [A] time = 1.09044, size = 115, normalized size = 1.32

$$\frac{1}{48}\,bp\left(\frac{12\,a^4\log(ax+b)}{b^5} - \frac{12\,a^4\log(x)}{b^5} - \frac{12\,a^3x^3 - 6\,a^2bx^2 + 4\,ab^2x - 3\,b^3}{b^4x^4}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*b*p*(12*a^4*log(a*x + b)/b^5 - 12*a^4*log(x)/b^5 - (12*a^3*x^3 - 6*a^2
*b*x^2 + 4*a*b^2*x - 3*b^3)/(b^4*x^4)) - 1/4*log((a + b/x)^p*c)/x^4
```

Fricas [A] time = 2.30261, size = 181, normalized size = 2.08

$$-\frac{12\,a^{3}bpx^{3}-6\,a^{2}b^{2}px^{2}+4\,ab^{3}px-3\,b^{4}p+12\,b^{4}\log\left(c\right)-12\,\left(a^{4}px^{4}-b^{4}p\right)\log\left(\frac{ax+b}{x}\right)}{48\,b^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*b*p*x^3 - 6*a^2*b^2*p*x^2 + 4*a*b^3*p*x - 3*b^4*p + 12*b^4*lo
g(c) - 12*(a^4*p*x^4 - b^4*p)*log((a*x + b)/x))/(b^4*x^4)
```

Sympy [A] time = 73.2739, size = 94, normalized size = 1.08

$$\begin{cases} \frac{a^{4}p\log\left(a+\frac{b}{x}\right)}{4b^{4}}-\frac{a^{3}p}{4b^{3}x}+\frac{a^{2}p}{8b^{2}x^{2}}-\frac{ap}{12bx^{3}}-\frac{p\log\left(a+\frac{b}{x}\right)}{4x^{4}}+\frac{p}{16x^{4}}-\frac{\log\left(c\right)}{4x^{4}} & \text{for}\ b\neq 0\\ -\frac{\log\left(a^{p}c\right)}{4x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**5,x)
```

```
[Out] Piecewise((a**4*p*log(a + b/x)/(4*b**4) - a**3*p/(4*b**3*x) + a**2*p/(8*b**
2*x**2) - a*p/(12*b*x**3) - p*log(a + b/x)/(4*x**4) + p/(16*x**4) - log(c)/
(4*x**4), Ne(b, 0)), (-log(a**p*c)/(4*x**4), True))
```

Giac [A] time = 1.19987, size = 130, normalized size = 1.49

$$\frac{a^{4}p\log\left(ax+b\right)}{4\,b^{4}}-\frac{a^{4}p\log\left(x\right)}{4\,b^{4}}-\frac{p\log\left(ax+b\right)}{4\,x^{4}}+\frac{p\log\left(x\right)}{4\,x^{4}}-\frac{12\,a^{3}px^{3}-6\,a^{2}bpx^{2}+4\,ab^{2}px-3\,b^{3}p+12\,b^{3}\log\left(c\right)}{48\,b^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*a^4*p*log(a*x + b)/b^4 - 1/4*a^4*p*log(x)/b^4 - 1/4*p*log(a*x + b)/x^4
+ 1/4*p*log(x)/x^4 - 1/48*(12*a^3*p*x^3 - 6*a^2*b*p*x^2 + 4*a*b^2*p*x - 3*b
^3*p + 12*b^3*log(c))/(b^3*x^4)
```

3.36 $\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=72

$$-\frac{2b^2px}{5a^2}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx^3}{15a}$$

```
[Out] (-2*b^2*p*x)/(5*a^2) + (2*b*p*x^3)/(15*a) + (2*b^(5/2)*p*ArcTan[(Sqrt[a]*x)
/Sqrt[b]])/(5*a^(5/2)) + (x^5*Log[c*(a + b/x^2)^p])/5
```

Rubi [A] time = 0.0375931, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 263, 302, 205}

$$-\frac{2b^2px}{5a^2}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx^3}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (-2*b^2*p*x)/(5*a^2) + (2*b*p*x^3)/(15*a) + (2*b^(5/2)*p*ArcTan[(Sqrt[a]*x)
/Sqrt[b]])/(5*a^(5/2)) + (x^5*Log[c*(a + b/x^2)^p])/5
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{5}(2bp)\int \frac{x^2}{a+\frac{b}{x^2}} dx$$
$$= \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{5}(2bp)\int \frac{x^4}{b+ax^2} dx$$
$$= \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{5}(2bp)\int \left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2\left(b+ax^2\right)}\right) dx$$
$$= -\frac{2b^2px}{5a^2} + \frac{2bpx^3}{15a} + \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{\left(2b^3p\right)\int \frac{1}{b+ax^2} dx}{5a^2}$$
$$= -\frac{2b^2px}{5a^2} + \frac{2bpx^3}{15a} + \frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}} + \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Mathematica [C] time = 0.0060882, size = 49, normalized size = 0.68

$$\frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{2bpx^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{b}{ax^2}\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(b/(a*x^2))])/(15*a) + (x^5*Lo
g[c*(a + b/x^2)^p])/5
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int x^4 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x^4*ln(c*(a+b/x^2)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32446, size = 401, normalized size = 5.57

$$\left[\frac{3\,a^2px^5\log\left(\frac{ax^2+b}{x^2}\right)+3\,a^2x^5\log\left(c\right)+2\,abpx^3+3\,b^2p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)-6\,b^2px}{15\,a^2},\ \frac{3\,a^2px^5\log\left(\frac{ax^2+b}{x^2}\right)+3\,a^2x^5\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b*p*x^3 +
3*b^2*p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) - 6*b^2*
p*x)/a^2, 1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b
*p*x^3 + 6*b^2*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - 6*b^2*p*x)/a^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19816, size = 101, normalized size = 1.4

$$\frac{1}{5}\,px^5\log\left(ax^2+b\right)-\frac{1}{5}\,px^5\log\left(x^2\right)+\frac{1}{5}\,x^5\log\left(c\right)+\frac{2\,bpx^3}{15\,a}+\frac{2\,b^3p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{5\,\sqrt{ab}a^2}-\frac{2\,b^2px}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(a*x^2 + b) - 1/5*p*x^5*log(x^2) + 1/5*x^5*log(c) + 2/15*b*p*x
^3/a + 2/5*b^3*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/5*b^2*p*x/a^2
```

3.37 $\int x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=51

$$-\frac{b^2 p \log\left(ax^2+b\right)}{4a^2}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bpx^2}{4a}$$

```
[Out] (b*p*x^2)/(4*a) + (x^4*Log[c*(a + b/x^2)^p])/4 - (b^2*p*Log[b + a*x^2])/(4*
a^2)
```

Rubi [A] time = 0.0335074, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 263, 266, 43}

$$-\frac{b^2 p \log\left(ax^2+b\right)}{4a^2}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bpx^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (b*p*x^2)/(4*a) + (x^4*Log[c*(a + b/x^2)^p])/4 - (b^2*p*Log[b + a*x^2])/(4*
a^2)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{2}(bp)\int \frac{x}{a+\frac{b}{x^2}}\, dx$$
$$= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{2}(bp)\int \frac{x^3}{b+ax^2}\, dx$$
$$= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{4}(bp)\,\mathrm{Subst}\left(\int \frac{x}{b+ax}\, dx, x, x^2\right)$$
$$= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{4}(bp)\,\mathrm{Subst}\left(\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right) dx, x, x^2\right)$$
$$= \frac{bpx^2}{4a} + \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - \frac{b^2 p \log\left(b+ax^2\right)}{4a^2}$$

Mathematica [A] time = 0.0202259, size = 56, normalized size = 1.1

$$\frac{1}{4}bp\left(-\frac{b\log\left(a+\frac{b}{x^2}\right)}{a^2} - \frac{2b\log(x)}{a^2} + \frac{x^2}{a}\right) + \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (x^4*Log[c*(a + b/x^2)^p])/4 + (b*p*(x^2/a - (b*Log[a + b/x^2])/a^2 - (2*b*
Log[x])/a^2))/4
```

Maple [F] time = 0.245, size = 0, normalized size = 0.

$$\int x^3 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^2)^p),x)
```

Maxima [A] time = 1.08037, size = 59, normalized size = 1.16

$$\frac{1}{4}x^4 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right) + \frac{1}{4}bp\left(\frac{x^2}{a} - \frac{b\log\left(ax^2+b\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((a + b/x^2)^p*c) + 1/4*b*p*(x^2/a - b*log(a*x^2 + b)/a^2)
```

Fricas [A] time = 2.32064, size = 127, normalized size = 2.49

$$\frac{a^2px^4\log\left(\frac{ax^2+b}{x^2}\right) + a^2x^4\log(c) + abpx^2 - b^2p\log\left(ax^2+b\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*p*x^4*log((a*x^2 + b)/x^2) + a^2*x^4*log(c) + a*b*p*x^2 - b^2*p*lo
g(a*x^2 + b))/a^2
```

Sympy [A] time = 29.7322, size = 87, normalized size = 1.71

$$\begin{cases} \frac{px^4\log\left(a+\frac{b}{x^2}\right)}{4} + \frac{x^4\log(c)}{4} + \frac{bpx^2}{4a} - \frac{b^2p\log\left(ax^2+b\right)}{4a^2} & \text{for}\ a \neq 0 \\ \frac{px^4\log(b)}{4} - \frac{px^4\log(x)}{2} + \frac{px^4}{8} + \frac{x^4\log(c)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x**4*log(a + b/x**2)/4 + x**4*log(c)/4 + b*p*x**2/(4*a) - b**2
*p*log(a*x**2 + b)/(4*a**2), Ne(a, 0)), (p*x**4*log(b)/4 - p*x**4*log(x)/2
+ p*x**4/8 + x**4*log(c)/4, True))
```

Giac [A] time = 1.21072, size = 80, normalized size = 1.57

$$\frac{1}{4}\,px^4\log\left(ax^2+b\right) - \frac{1}{4}\,px^4\log\left(x^2\right) + \frac{1}{4}\,x^4\log(c) + \frac{bpx^2}{4\,a} - \frac{b^2p\log\left(ax^2+b\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/4*p*x^4*log(a*x^2 + b) - 1/4*p*x^4*log(x^2) + 1/4*x^4*log(c) + 1/4*b*p*x^
2/a - 1/4*b^2*p*log(a*x^2 + b)/a^2
```

3.38 $\int x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=58

$$-\frac{2 b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{3 a^{3/2}}+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{2 b p x}{3 a}$$

```
[Out] (2*b*p*x)/(3*a) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)) + (
x^3*Log[c*(a + b/x^2)^p])/3
```

Rubi [A] time = 0.0270836, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 193, 321, 205}

$$-\frac{2 b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{3 a^{3/2}}+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{2 b p x}{3 a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x)/(3*a) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)) + (
x^3*Log[c*(a + b/x^2)^p])/3
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{3}(2bp)\int \frac{1}{a+\frac{b}{x^2}}\, dx$$
$$= \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{3}(2bp)\int \frac{x^2}{b+ax^2}\, dx$$
$$= \frac{2bpx}{3a} + \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - \frac{(2b^2p)\int \frac{1}{b+ax^2}\, dx}{3a}$$
$$= \frac{2bpx}{3a} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}} + \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Mathematica [C] time = 0.0024465, size = 47, normalized size = 0.81

$$\frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{2bpx\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{b}{ax^2}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x*Hypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/(3*a) + (x^3*Log[c*
(a + b/x^2)^p])/3
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x^2*ln(c*(a+b/x^2)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21636, size = 320, normalized size = 5.52

$$\left[\frac{apx^3\log\left(\frac{ax^2+b}{x^2}\right) + ax^3\log(c) + bp\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right) + 2\,bpx}{3\,a}, \frac{apx^3\log\left(\frac{ax^2+b}{x^2}\right) + ax^3\log(c) - 2\,bp\sqrt{\frac{b}{a}}\arctan\right.}{3\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [1/3*(a*p*x^3*log((a*x^2 + b)/x^2) + a*x^3*log(c) + b*p*sqrt(-b/a)*log((a*x
^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) + 2*b*p*x)/a, 1/3*(a*p*x^3*log((a*x
^2 + b)/x^2) + a*x^3*log(c) - 2*b*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) + 2*b
*p*x)/a]
```

Sympy [A] time = 85.1369, size = 146, normalized size = 2.52

$$\begin{cases} \frac{px^3\log\left(a+\frac{b}{x^2}\right)}{3} + \frac{x^3\log\left(c\right)}{3} + \frac{2bpx}{3a} + \frac{ib^{\frac{3}{2}}p\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3a^2\sqrt{\frac{1}{a}}} - \frac{ib^{\frac{3}{2}}p\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3a^2\sqrt{\frac{1}{a}}} & \text{for } a \neq 0 \\ \frac{px^3\log\left(b\right)}{3} - \frac{2px^3\log\left(x\right)}{3} + \frac{2px^3}{9} + \frac{x^3\log\left(c\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x**3*log(a + b/x**2)/3 + x**3*log(c)/3 + 2*b*p*x/(3*a) + I*b**
(3/2)*p*log(-I*sqrt(b)*sqrt(1/a) + x)/(3*a**2*sqrt(1/a)) - I*b**(3/2)*p*log
(I*sqrt(b)*sqrt(1/a) + x)/(3*a**2*sqrt(1/a)), Ne(a, 0)), (p*x**3*log(b)/3 -
 2*p*x**3*log(x)/3 + 2*p*x**3/9 + x**3*log(c)/3, True))
```

Giac [A] time = 1.17803, size = 85, normalized size = 1.47

$$\frac{1}{3}\,px^3\log\left(ax^2+b\right) - \frac{1}{3}\,px^3\log\left(x^2\right) + \frac{1}{3}\,x^3\log\left(c\right) - \frac{2\,b^2p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} + \frac{2\,bpx}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(a*x^2 + b) - 1/3*p*x^3*log(x^2) + 1/3*x^3*log(c) - 2/3*b^2*p*
arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + 2/3*b*p*x/a
```

3.39 $\int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(ax^2+b\right)}{2a}$$

```
[Out] (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[b + a*x^2])/(2*a)
```

Rubi [A] time = 0.0144121, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2455, 263, 260}

$$\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[b + a*x^2])/(2*a)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx &= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx\\ &= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{x}{b+ax^2}\,dx\\ &= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(b+ax^2\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.002538, size = 45, normalized size = 1.22

$$\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(a+\frac{b}{x^2}\right)}{2a}+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (b*p*Log[a + b/x^2])/(2*a) + (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[x])/a
```

Maple [F] time = 0.245, size = 0, normalized size = 0.

$$\int x \ln \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x*ln(c*(a+b/x^2)^p),x)
```

Maxima [A] time = 1.18933, size = 45, normalized size = 1.22

$$\frac{1}{2}x^2\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)+\frac{bp\log\left(ax^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((a + b/x^2)^p*c) + 1/2*b*p*log(a*x^2 + b)/a
```

Fricas [A] time = 2.20729, size = 100, normalized size = 2.7

$$\frac{apx^2\log\left(\frac{ax^2+b}{x^2}\right)+ax^2\log(c)+bp\log\left(ax^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a*p*x^2*log((a*x^2 + b)/x^2) + a*x^2*log(c) + b*p*log(a*x^2 + b))/a
```

Sympy [A] time = 11.8473, size = 71, normalized size = 1.92

$$\begin{cases} \frac{px^2\log\left(a+\frac{b}{x^2}\right)}{2}+\frac{x^2\log(c)}{2}+\frac{bp\log\left(ax^2+b\right)}{2a} & \text{for } a \neq 0 \\ \frac{px^2\log(b)}{2}-px^2\log(x)+\frac{px^2}{2}+\frac{x^2\log(c)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x**2*log(a + b/x**2)/2 + x**2*log(c)/2 + b*p*log(a*x**2 + b)/(
2*a), Ne(a, 0)), (p*x**2*log(b)/2 - p*x**2*log(x) + p*x**2/2 + x**2*log(c)/
2, True))
```

Giac [A] time = 1.16985, size = 63, normalized size = 1.7

$$\frac{1}{2}\,px^2\log\left(ax^2+b\right)-\frac{1}{2}\,px^2\log\left(x^2\right)+\frac{1}{2}\,x^2\log\left(c\right)+\frac{bp\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/2*p*x^2*log(a*x^2 + b) - 1/2*p*x^2*log(x^2) + 1/2*x^2*log(c) + 1/2*b*p*lo
g(a*x^2 + b)/a
```

3.40 $\int \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=41

$$x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a}}$$

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Rubi [A] time = 0.0148343, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2448, 263, 205}

$$x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx &= x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+(2 b p) \int \frac{1}{\left(a+\frac{b}{x^2}\right) x^2} \, dx\\
&= x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+(2 b p) \int \frac{1}{b+a x^2} \, dx\\
&= \frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a}}+x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.0075675, size = 43, normalized size = 1.05

$$x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)-\frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{b}}{\sqrt{a} x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p],x]
```

```
[Out] (-2*Sqrt[b]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Maple [A] time = 0.063, size = 38, normalized size = 0.9

$$x\ln\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)+2\,\frac{bp}{\sqrt{ab}}\arctan\left(\frac{ax}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p),x)
```

```
[Out] x*ln(c*((a*x^2+b)/x^2)^p)+2*b*p/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.19322, size = 244, normalized size = 5.95

$$\left[px\log\left(\frac{ax^2+b}{x^2}\right)+p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+x\log\left(c\right),px\log\left(\frac{ax^2+b}{x^2}\right)+2\,p\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)+x\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log((a*x^2 + b)/x^2) + p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)
/(a*x^2 + b)) + x*log(c), p*x*log((a*x^2 + b)/x^2) + 2*p*sqrt(b/a)*arctan(a
*x*sqrt(b/a)/b) + x*log(c)]
```

Sympy [A] time = 24.0015, size = 109, normalized size = 2.66

$$\begin{cases} px\log\left(a+\frac{b}{x^2}\right)+x\log\left(c\right)-\frac{i\sqrt{b}p\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{a\sqrt{\frac{1}{a}}}+\frac{i\sqrt{b}p\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{a\sqrt{\frac{1}{a}}} & \text{for } a\neq 0 \\ px\log\left(b\right)-2px\log\left(x\right)+2px+x\log\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x*log(a + b/x**2) + x*log(c) - I*sqrt(b)*p*log(-I*sqrt(b)*sqrt
(1/a) + x)/(a*sqrt(1/a)) + I*sqrt(b)*p*log(I*sqrt(b)*sqrt(1/a) + x)/(a*sqrt
(1/a)), Ne(a, 0)), (p*x*log(b) - 2*p*x*log(x) + 2*p*x + x*log(c), True))
```

Giac [A] time = 1.24949, size = 57, normalized size = 1.39

$$px\log\left(ax^2+b\right)-px\log\left(x^2\right)+\frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] p*x*log(a*x^2 + b) - p*x*log(x^2) + 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) +
 x*log(c)
```

3.41 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)-\frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/2 - (p*PolyLog[2, 1 + b/(a*x^2)])
/2
```

Rubi [A] time = 0.040415, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$-\frac{1}{2}p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)-\frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/2 - (p*PolyLog[2, 1 + b/(a*x^2)])
/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right) + \frac{1}{2}(bp)\operatorname{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right) - \frac{1}{2}p\operatorname{Li}_2\left(1+\frac{b}{ax^2}\right)$$

Mathematica [A] time = 0.002826, size = 45, normalized size = 1.02

$$-\frac{1}{2}p\operatorname{PolyLog}\left(2, \frac{a+\frac{b}{x^2}}{a}\right) - \frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/2 - (p*PolyLog[2, (a + b/x^2)/a])
/2
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int \frac{1}{x}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x,x)
```

Maxima [B] time = 1.14888, size = 120, normalized size = 2.73

$$\frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x^2}\right)\log(x)}{b} + \frac{2\log(x)^2}{b} - \frac{2\log\left(\frac{ax^2}{b}+1\right)\log(x) + \operatorname{Li}_2\left(-\frac{ax^2}{b}\right)}{b}\right) - p\log\left(a+\frac{b}{x^2}\right)\log(x) + \log\left(\left(a+\frac{b}{x^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(a + b/x^2)*log(x)/b + 2*log(x)^2/b - (2*log(a*x^2/b + 1)*log
(x) + dilog(-a*x^2/b))/b) - p*log(a + b/x^2)*log(x) + log((a + b/x^2)^p*c)*
log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b/x**2)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/x, x)
```

3.42 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=50

$$-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{2p}{x}$$

```
[Out] (2*p)/x + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

Rubi [A] time = 0.0271548, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 263, 325, 205}

$$-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{2p}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^2,x]
```

```
[Out] (2*p)/x + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{x^2} \, dx &= -\frac{\log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{x}-(2 b p) \int \frac{1}{\left(a+\frac{b}{x^2}\right) x^4} \, dx\\ &= -\frac{\log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{x}-(2 b p) \int \frac{1}{x^2 \left(b+a x^2\right)} \, dx\\ &= \frac{2 p}{x}-\frac{\log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{x}+(2 a p) \int \frac{1}{b+a x^2} \, dx\\ &= \frac{2 p}{x}+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{\log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0139083, size = 52, normalized size = 1.04

$$-\frac{\log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{x}-\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b}}{\sqrt{a} x}\right)}{\sqrt{b}}+\frac{2 p}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^2,x]
```

```
[Out] (2*p)/x - (2*Sqrt[a]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

Maple [F] time = 0.235, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \ln \left(c \left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^2,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27257, size = 261, normalized size = 5.22

$$\left[\frac{px\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)+2\,p-\log\left(c\right)}{x},\ \frac{2\,px\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)+2\,p-\log\left(c\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [(p*x*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - p*log((a
*x^2 + b)/x^2) + 2*p - log(c))/x, (2*p*x*sqrt(a/b)*arctan(x*sqrt(a/b)) - p*
log((a*x^2 + b)/x^2) + 2*p - log(c))/x]
```

Sympy [A] time = 59.9988, size = 129, normalized size = 2.58

$$\begin{cases} -\frac{\log\left(0^{p}c\right)}{x} & \text{for}\ a=0 \wedge b=0 \\ -\frac{p\log\left(b\right)}{x}+\frac{2p\log\left(x\right)}{x}+\frac{2p}{x}-\frac{\log\left(c\right)}{x} & \text{for}\ a=0 \\ -\frac{\log\left(a^{p}c\right)}{x} & \text{for}\ b=0 \\ -\frac{p\log\left(a+\frac{b}{x^2}\right)}{x}+\frac{2p}{x}-\frac{\log\left(c\right)}{x}-\frac{ip\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{\sqrt{b}\sqrt{\frac{1}{a}}}+\frac{ip\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{\sqrt{b}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-p*log(b)/x + 2*p*log(x)/
x + 2*p/x - log(c)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (-p*log(a + b/
x**2)/x + 2*p/x - log(c)/x - I*p*log(-I*sqrt(b)*sqrt(1/a) + x)/(sqrt(b)*sqr
t(1/a)) + I*p*log(I*sqrt(b)*sqrt(1/a) + x)/(sqrt(b)*sqrt(1/a)), True))
```

Giac [A] time = 1.26026, size = 73, normalized size = 1.46

$$\frac{2\,ap\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{p\log\left(ax^2+b\right)}{x}+\frac{p\log\left(x^2\right)}{x}+\frac{2\,p-\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) - p*log(a*x^2 + b)/x + p*log(x^2)/x +
 (2*p - log(c))/x
```

3.43 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=35

$$\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

```
[Out] p/(2*x^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b)
```

Rubi [A] time = 0.0259068, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^3,x]
```

```
[Out] p/(2*x^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{\operatorname{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+\frac{b}{x^2}\right)}{2b}\\ &= \frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0080653, size = 34, normalized size = 0.97

$$\frac{1}{2}\left(\frac{p}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^3,x]
```

```
[Out] (p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/b)/2
```

Maple [A] time = 0.059, size = 50, normalized size = 1.4

$$-\frac{a}{2b}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{1}{2x^2}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{ap}{2b}+\frac{p}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^3,x)
```

```
[Out] -1/2/b*ln(c*(a+b/x^2)^p)*a-1/2*ln(c*(a+b/x^2)^p)/x^2+1/2/b*a*p+1/2*p/x^2
```

Maxima [A] time = 1.12336, size = 73, normalized size = 2.09

$$-\frac{1}{2}bp\left(\frac{a\log\left(ax^2+b\right)}{b^2}-\frac{a\log\left(x^2\right)}{b^2}-\frac{1}{bx^2}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(a*log(a*x^2 + b)/b^2 - a*log(x^2)/b^2 - 1/(b*x^2)) - 1/2*log((a +
 b/x^2)^p*c)/x^2
```

Fricas [A] time = 2.11883, size = 93, normalized size = 2.66

$$\frac{bp-b\log\left(c\right)-\left(apx^2+bp\right)\log\left(\frac{ax^2+b}{x^2}\right)}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*p - b*log(c) - (a*p*x^2 + b*p)*log((a*x^2 + b)/x^2))/(b*x^2)
```

Sympy [A] time = 11.7439, size = 58, normalized size = 1.66

$$\begin{cases} -\frac{ap\log\left(a+\frac{b}{x^2}\right)}{2b} - \frac{p\log\left(a+\frac{b}{x^2}\right)}{2x^2} + \frac{p}{2x^2} - \frac{\log(c)}{2x^2} & \text{for } b \neq 0 \\ -\frac{\log(a^p c)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3,x)
```

```
[Out] Piecewise((-a*p*log(a + b/x**2)/(2*b) - p*log(a + b/x**2)/(2*x**2) + p/(2*x
**2) - log(c)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

Giac [B] time = 1.27776, size = 88, normalized size = 2.51

$$-\frac{ap\log\left(ax^2+b\right)}{2\,b} + \frac{ap\log(x)}{b} - \frac{p\log\left(ax^2+b\right)}{2\,x^2} + \frac{p\log\left(x^2\right)}{2\,x^2} + \frac{bp - b\log(c)}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*a*p*log(a*x^2 + b)/b + a*p*log(x)/b - 1/2*p*log(a*x^2 + b)/x^2 + 1/2*p
*log(x^2)/x^2 + 1/2*(b*p - b*log(c))/(b*x^2)
```

3.44 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=68

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{2ap}{3bx}+\frac{2p}{9x^3}$$

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) - (2*a^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Rubi [A] time = 0.0355895, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 263, 325, 205}

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{2ap}{3bx}+\frac{2p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^4,x]
```

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) - (2*a^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx &= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{1}{3}(2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx \\ &= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{1}{3}(2bp)\int \frac{1}{x^4\left(b+ax^2\right)}\,dx \\ &= \frac{2p}{9x^3} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} + \frac{1}{3}(2ap)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx \\ &= \frac{2p}{9x^3} - \frac{2ap}{3bx} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{\left(2a^2p\right)\int \frac{1}{b+ax^2}\,dx}{3b} \\ &= \frac{2p}{9x^3} - \frac{2ap}{3bx} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0212296, size = 70, normalized size = 1.03

$$\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{2ap}{3bx} + \frac{2p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^4,x]
```

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) + (2*a^(3/2)*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^4,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.31024, size = 362, normalized size = 5.32

$$\left[\frac{3\,apx^3\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-6\,apx^2-3\,bp\log\left(\frac{ax^2+b}{x^2}\right)+2\,bp-3\,b\log\left(c\right)}{9\,bx^3}, -\frac{6\,apx^3\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+6\,apx^2+3\,b}{9\,bx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*p*x^3*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b))
- 6*a*p*x^2 - 3*b*p*log((a*x^2 + b)/x^2) + 2*b*p - 3*b*log(c))/(b*x^3), -1/
9*(6*a*p*x^3*sqrt(a/b)*arctan(x*sqrt(a/b)) + 6*a*p*x^2 + 3*b*p*log((a*x^2 +
 b)/x^2) - 2*b*p + 3*b*log(c))/(b*x^3)]
```

Sympy [A] time = 126.087, size = 177, normalized size = 2.6

$$\begin{cases} -\frac{\log(0^p c)}{3x^3} & \text{for } a = 0 \wedge b = 0 \\ -\frac{p\log(b)}{3x^3} + \frac{2p\log(x)}{3x^3} + \frac{2p}{9x^3} - \frac{\log(c)}{3x^3} & \text{for } a = 0 \\ -\frac{\log(a^p c)}{3x^3} & \text{for } b = 0 \\ -\frac{2ap}{3bx} + \frac{iap\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{iap\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{p\log\left(a+\frac{b}{x^2}\right)}{3x^3} + \frac{2p}{9x^3} - \frac{\log(c)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (-p*log(b)/(3*x**3)
 + 2*p*log(x)/(3*x**3) + 2*p/(9*x**3) - log(c)/(3*x**3), Eq(a, 0)), (-log(a
**p*c)/(3*x**3), Eq(b, 0)), (-2*a*p/(3*b*x) + I*a*p*log(-I*sqrt(b)*sqrt(1/a
) + x)/(3*b**(3/2)*sqrt(1/a)) - I*a*p*log(I*sqrt(b)*sqrt(1/a) + x)/(3*b**(3
/2)*sqrt(1/a)) - p*log(a + b/x**2)/(3*x**3) + 2*p/(9*x**3) - log(c)/(3*x**3
), True))
```

Giac [A] time = 1.28948, size = 99, normalized size = 1.46

$$-\frac{2\,a^2p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}-\frac{p\log\left(ax^2+b\right)}{3\,x^3}+\frac{p\log\left(x^2\right)}{3\,x^3}-\frac{6\,apx^2-2\,bp+3\,b\log\left(c\right)}{9\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*a^2*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/3*p*log(a*x^2 + b)/x^3 +
 1/3*p*log(x^2)/x^3 - 1/9*(6*a*p*x^2 - 2*b*p + 3*b*log(c))/(b*x^3)
```

3.45 $\int \frac{\log\left(1+\frac{b}{x}\right)}{x} dx$

Optimal. Leaf size=8

$$\text{PolyLog}\left(2,-\frac{b}{x}\right)$$

```
[Out] PolyLog[2, -(b/x)]
```

Rubi [A] time = 0.0077864, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2391}

$$\text{PolyLog}\left(2,-\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + b/x]/x,x]

[Out] PolyLog[2, -(b/x)]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x} dx = \text{Li}_2\left(-\frac{b}{x}\right)$$

Mathematica [B] time = 0.0033783, size = 34, normalized size = 4.25

$$-\text{PolyLog}\left(2,-\frac{-b-x}{x}\right)-\log\left(-\frac{b}{x}\right)\log\left(\frac{b+x}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + b/x]/x,x]

[Out] -(Log[-(b/x)]*Log[(b + x)/x]) - PolyLog[2, -((-b - x)/x)]
```

Maple [A] time = 0.059, size = 9, normalized size = 1.1

$$\text{dilog}\left(1+\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+b/x)/x,x)
```

```
[Out] dilog(1+b/x)
```

Maxima [B] time = 1.04336, size = 47, normalized size = 5.88

$$\log(b+x)\log(x) - \frac{1}{2}\log(x)^2 - \log(x)\log\left(\frac{x}{b}+1\right) - \mathrm{Li}_2\left(-\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="maxima")
```

```
[Out] log(b + x)*log(x) - 1/2*log(x)^2 - log(x)*log(x/b + 1) - dilog(-x/b)
```

Fricas [A] time = 2.24473, size = 31, normalized size = 3.88

$$\mathrm{Li}_2\left(-\frac{b+x}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="fricas")
```

```
[Out] dilog(-(b + x)/x + 1)
```

Sympy [C] time = 4.31709, size = 8, normalized size = 1.

$$\mathrm{Li}_2\left(\frac{be^{i\pi}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+b/x)/x,x)
```

```
[Out] polylog(2, b*exp_polar(I*pi)/x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{b}{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="giac")
```

```
[Out] integrate(log(b/x + 1)/x, x)
```

3.46 $\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right) dx$

Optimal. Leaf size=153

$$\frac{a^5px^{3/2}}{12b^5}-\frac{a^4px^2}{16b^4}+\frac{a^3px^{5/2}}{20b^3}-\frac{a^2px^3}{24b^2}+\frac{a^7p\sqrt{x}}{4b^7}-\frac{a^6px}{8b^6}-\frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}+\frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{7/2}}{28b}-\frac{px^4}{32}$$

```
[Out] (a^7*p*Sqrt[x])/(4*b^7) - (a^6*p*x)/(8*b^6) + (a^5*p*x^(3/2))/(12*b^5) - (a
^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20*b^3) - (a^2*p*x^3)/(24*b^2) + (a*p
*x^(7/2))/(28*b) - (p*x^4)/32 - (a^8*p*Log[a + b*Sqrt[x]])/(4*b^8) + (x^4*L
og[c*(a + b*Sqrt[x])^p])/4
```

Rubi [A] time = 0.117554, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 43}

$$\frac{a^5px^{3/2}}{12b^5}-\frac{a^4px^2}{16b^4}+\frac{a^3px^{5/2}}{20b^3}-\frac{a^2px^3}{24b^2}+\frac{a^7p\sqrt{x}}{4b^7}-\frac{a^6px}{8b^6}-\frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}+\frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{7/2}}{28b}-\frac{px^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^7*p*Sqrt[x])/(4*b^7) - (a^6*p*x)/(8*b^6) + (a^5*p*x^(3/2))/(12*b^5) - (a
^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20*b^3) - (a^2*p*x^3)/(24*b^2) + (a*p
*x^(7/2))/(28*b) - (p*x^4)/32 - (a^8*p*Log[a + b*Sqrt[x]])/(4*b^8) + (x^4*L
og[c*(a + b*Sqrt[x])^p])/4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right) dx = 2\,\text{Subst}\left(\int x^7 \log\left(c(a+bx)^p\right) dx, x, \sqrt{x}\right)$$
$$= \frac{1}{4}x^4 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{4}(bp)\,\text{Subst}\left(\int \frac{x^8}{a+bx} dx, x, \sqrt{x}\right)$$
$$= \frac{1}{4}x^4 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{4}(bp)\,\text{Subst}\left(\int \left(-\frac{a^7}{b^8} + \frac{a^6x}{b^7} - \frac{a^5x^2}{b^6} + \frac{a^4x^3}{b^5} - \frac{a^3x^4}{b^4} + \frac{a^2x^5}{b^3} - \frac{ax}{b}\right.\right.$$
$$= \frac{a^7p\sqrt{x}}{4b^7} - \frac{a^6px}{8b^6} + \frac{a^5px^{3/2}}{12b^5} - \frac{a^4px^2}{16b^4} + \frac{a^3px^{5/2}}{20b^3} - \frac{a^2px^3}{24b^2} + \frac{apx^{7/2}}{28b} - \frac{px^4}{32} - \frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}$$

Mathematica [A] time = 0.131131, size = 134, normalized size = 0.88

$$\frac{1}{4}\left(x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{p\left(-280a^5b^3x^{3/2} + 210a^4b^4x^2 - 168a^3b^5x^{5/2} + 140a^2b^6x^3 + 420a^6b^2x - 840a^7b\sqrt{x} + 840a^8\log\right.}{840b^8}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (-(p*(-840*a^7*b*Sqrt[x] + 420*a^6*b^2*x - 280*a^5*b^3*x^(3/2) + 210*a^4*b^
4*x^2 - 168*a^3*b^5*x^(5/2) + 140*a^2*b^6*x^3 - 120*a*b^7*x^(7/2) + 105*b^8
*x^4 + 840*a^8*Log[a + b*Sqrt[x]]))/(840*b^8) + x^4*Log[c*(a + b*Sqrt[x])^p
])/4
```

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int x^3 \ln\left(c\left(a+b\sqrt{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b*x^(1/2))^p),x)
```

```
[Out] int(x^3*ln(c*(a+b*x^(1/2))^p),x)
```

Maxima [A] time = 1.10123, size = 162, normalized size = 1.06

$$\frac{1}{4}x^4\log\left(\left(b\sqrt{x}+a\right)^p c\right) - \frac{1}{3360}bp\left(\frac{840\,a^8\log\left(b\sqrt{x}+a\right)}{b^9} + \frac{105\,b^7x^4 - 120\,ab^6x^{\frac{7}{2}} + 140\,a^2b^5x^3 - 168\,a^3b^4x^{\frac{5}{2}} + 210\,a^4b^3x^2}{b^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*sqrt(x) + a)^p*c) - 1/3360*b*p*(840*a^8*log(b*sqrt(x) + a)/b
^9 + (105*b^7*x^4 - 120*a*b^6*x^(7/2) + 140*a^2*b^5*x^3 - 168*a^3*b^4*x^(5/
2) + 210*a^4*b^3*x^2 - 280*a^5*b^2*x^(3/2) + 420*a^6*b*x - 840*a^7*sqrt(x))
/b^8)
```

Fricas [A] time = 2.46633, size = 313, normalized size = 2.05

$$-\frac{105\,b^8px^4-840\,b^8x^4\log\left(c\right)+140\,a^2b^6px^3+210\,a^4b^4px^2+420\,a^6b^2px-840\left(b^8px^4-a^8p\right)\log\left(b\sqrt{x}+a\right)-8\left(15\,ab\right.}{3360\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/3360*(105*b^8*p*x^4 - 840*b^8*x^4*log(c) + 140*a^2*b^6*p*x^3 + 210*a^4*b
^4*p*x^2 + 420*a^6*b^2*p*x - 840*(b^8*p*x^4 - a^8*p)*log(b*sqrt(x) + a) - 8
*(15*a*b^7*p*x^3 + 21*a^3*b^5*p*x^2 + 35*a^5*b^3*p*x + 105*a^7*b*p)*sqrt(x)
)/b^8
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30382, size = 601, normalized size = 3.93

$$\frac{\left(\frac{840\left(b\sqrt{x}+a\right)^8\log\left(b\sqrt{x}+a\right)}{b^6}-\frac{6720\left(b\sqrt{x}+a\right)^7a\log\left(b\sqrt{x}+a\right)}{b^6}+\frac{23520\left(b\sqrt{x}+a\right)^6a^2\log\left(b\sqrt{x}+a\right)}{b^6}-\frac{47040\left(b\sqrt{x}+a\right)^5a^3\log\left(b\sqrt{x}+a\right)}{b^6}+\frac{58800\left(b\sqrt{x}+a\right)^4a^4\log\left(b\sqrt{x}+a\right)}{b^6}-\frac{47040\left(b\sqrt{x}+a\right)^3a^5\,l}{b^6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/3360*((840*(b*sqrt(x) + a)^8*log(b*sqrt(x) + a)/b^6 - 6720*(b*sqrt(x) + a
)^7*a*log(b*sqrt(x) + a)/b^6 + 23520*(b*sqrt(x) + a)^6*a^2*log(b*sqrt(x) +
a)/b^6 - 47040*(b*sqrt(x) + a)^5*a^3*log(b*sqrt(x) + a)/b^6 + 58800*(b*sqrt
(x) + a)^4*a^4*log(b*sqrt(x) + a)/b^6 - 47040*(b*sqrt(x) + a)^3*a^5*log(b*s
qrt(x) + a)/b^6 + 23520*(b*sqrt(x) + a)^2*a^6*log(b*sqrt(x) + a)/b^6 - 6720
*(b*sqrt(x) + a)*a^7*log(b*sqrt(x) + a)/b^6 - 105*(b*sqrt(x) + a)^8/b^6 + 9
60*(b*sqrt(x) + a)^7*a/b^6 - 3920*(b*sqrt(x) + a)^6*a^2/b^6 + 9408*(b*sqrt(
x) + a)^5*a^3/b^6 - 14700*(b*sqrt(x) + a)^4*a^4/b^6 + 15680*(b*sqrt(x) + a)
^3*a^5/b^6 - 11760*(b*sqrt(x) + a)^2*a^6/b^6 + 6720*(b*sqrt(x) + a)*a^7/b^6
)*p/b + 840*((b*sqrt(x) + a)^8 - 8*(b*sqrt(x) + a)^7*a + 28*(b*sqrt(x) + a)
^6*a^2 - 56*(b*sqrt(x) + a)^5*a^3 + 70*(b*sqrt(x) + a)^4*a^4 - 56*(b*sqrt(x
) + a)^3*a^5 + 28*(b*sqrt(x) + a)^2*a^6 - 8*(b*sqrt(x) + a)*a^7)*log(c)/b^7
)/b
```

3.47 $\int x^2 \log \left(c \left(a+b\sqrt{x}\right)^p\right) dx$

Optimal. Leaf size=123

$$\frac{a^3 p x^{3/2}}{9b^3}-\frac{a^2 p x^2}{12b^2}+\frac{a^5 p\sqrt{x}}{3b^5}-\frac{a^4 p x}{6b^4}-\frac{a^6 p\log\left(a+b\sqrt{x}\right)}{3b^6}+\frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{a p x^{5/2}}{15b}-\frac{p x^3}{18}$$

```
[Out] (a^5*p*Sqrt[x])/(3*b^5) - (a^4*p*x)/(6*b^4) + (a^3*p*x^(3/2))/(9*b^3) - (a^
2*p*x^2)/(12*b^2) + (a*p*x^(5/2))/(15*b) - (p*x^3)/18 - (a^6*p*Log[a + b*Sq
rt[x]])/(3*b^6) + (x^3*Log[c*(a + b*Sqrt[x])^p])/3
```

Rubi [A] time = 0.0880257, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 43}

$$\frac{a^3 p x^{3/2}}{9b^3}-\frac{a^2 p x^2}{12b^2}+\frac{a^5 p\sqrt{x}}{3b^5}-\frac{a^4 p x}{6b^4}-\frac{a^6 p\log\left(a+b\sqrt{x}\right)}{3b^6}+\frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{a p x^{5/2}}{15b}-\frac{p x^3}{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^5*p*Sqrt[x])/(3*b^5) - (a^4*p*x)/(6*b^4) + (a^3*p*x^(3/2))/(9*b^3) - (a^
2*p*x^2)/(12*b^2) + (a*p*x^(5/2))/(15*b) - (p*x^3)/18 - (a^6*p*Log[a + b*Sq
rt[x]])/(3*b^6) + (x^3*Log[c*(a + b*Sqrt[x])^p])/3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right) dx = 2\,\text{Subst}\left(\int x^5 \log\left(c(a+bx)^p\right) dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{x^6}{a+bx} dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{3}(bp)\,\text{Subst}\left(\int \left(-\frac{a^5}{b^6} + \frac{a^4x}{b^5} - \frac{a^3x^2}{b^4} + \frac{a^2x^3}{b^3} - \frac{ax^4}{b^2} + \frac{x^5}{b} + \frac{}{b^6}\right.\right.$$
$$= \frac{a^5p\sqrt{x}}{3b^5} - \frac{a^4px}{6b^4} + \frac{a^3px^{3/2}}{9b^3} - \frac{a^2px^2}{12b^2} + \frac{apx^{5/2}}{15b} - \frac{px^3}{18} - \frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6} + \frac{1}{3}x^3\log\left(c\left(a\right.\right.$$

Mathematica [A] time = 0.052671, size = 112, normalized size = 0.91

$$\frac{bp\sqrt{x}\left(-15a^2b^3x^{3/2} + 20a^3b^2x - 30a^4b\sqrt{x} + 60a^5 + 12ab^4x^2 - 10b^5x^{5/2}\right) - 60a^6p\log\left(a+b\sqrt{x}\right) + 60b^6x^3\log\left(c\left(a+b\right.\right.}{180b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (b*p*Sqrt[x]*(60*a^5 - 30*a^4*b*Sqrt[x] + 20*a^3*b^2*x - 15*a^2*b^3*x^(3/2)
 + 12*a*b^4*x^2 - 10*b^5*x^(5/2)) - 60*a^6*p*Log[a + b*Sqrt[x]] + 60*b^6*x^
3*Log[c*(a + b*Sqrt[x])^p])/(180*b^6)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int x^2 \ln\left(c\left(a+b\sqrt{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b*x^(1/2))^p),x)
```

```
[Out] int(x^2*ln(c*(a+b*x^(1/2))^p),x)
```

Maxima [A] time = 1.07278, size = 132, normalized size = 1.07

$$\frac{1}{3}x^3\log\left(\left(b\sqrt{x}+a\right)^p c\right) - \frac{1}{180}bp\left(\frac{60\,a^6\log\left(b\sqrt{x}+a\right)}{b^7} + \frac{10\,b^5x^3 - 12\,ab^4x^{\frac{5}{2}} + 15\,a^2b^3x^2 - 20\,a^3b^2x^{\frac{3}{2}} + 30\,a^4bx - 60\,a^5}{b^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*sqrt(x) + a)^p*c) - 1/180*b*p*(60*a^6*log(b*sqrt(x) + a)/b^7
 + (10*b^5*x^3 - 12*a*b^4*x^(5/2) + 15*a^2*b^3*x^2 - 20*a^3*b^2*x^(3/2) + 3
0*a^4*b*x - 60*a^5*sqrt(x))/b^6)
```

Fricas [A] time = 2.36353, size = 248, normalized size = 2.02

$$-\frac{10\,b^6px^3 - 60\,b^6x^3\log(c) + 15\,a^2b^4px^2 + 30\,a^4b^2px - 60\left(b^6px^3 - a^6p\right)\log\left(b\sqrt{x}+a\right) - 4\left(3\,ab^5px^2 + 5\,a^3b^3px + 15\right.}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/180*(10*b^6*p*x^3 - 60*b^6*x^3*log(c) + 15*a^2*b^4*p*x^2 + 30*a^4*b^2*p*
x - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) - 4*(3*a*b^5*p*x^2 + 5*a^3*b^
3*p*x + 15*a^5*b*p)*sqrt(x))/b^6
```

Sympy [A] time = 162.442, size = 119, normalized size = 0.97

$$-\frac{bp\left(\frac{2a^6\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^6}-\frac{2a^5\sqrt{x}}{b^6}+\frac{a^4x}{b^5}-\frac{2a^3x^{\frac{3}{2}}}{3b^4}+\frac{a^2x^2}{2b^3}-\frac{2ax^{\frac{5}{2}}}{5b^2}+\frac{x^3}{3b}\right)}{6}+\frac{x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**6*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**6 - 2*a**5*sqrt(x)/b**6 + a**4*x/b**5 - 2*a**3*x**(3/2)/(3*b**4) + a**2
*x**2/(2*b**3) - 2*a*x**(5/2)/(5*b**2) + x**3/(3*b))/6 + x**3*log(c*(a + b*
sqrt(x))**p)/3
```

Giac [B] time = 1.23736, size = 450, normalized size = 3.66

$$\frac{\left(\frac{60\left(b\sqrt{x}+a\right)^6\log\left(b\sqrt{x}+a\right)}{b^4}-\frac{360\left(b\sqrt{x}+a\right)^5a\log\left(b\sqrt{x}+a\right)}{b^4}+\frac{900\left(b\sqrt{x}+a\right)^4a^2\log\left(b\sqrt{x}+a\right)}{b^4}-\frac{1200\left(b\sqrt{x}+a\right)^3a^3\log\left(b\sqrt{x}+a\right)}{b^4}+\frac{900\left(b\sqrt{x}+a\right)^2a^4\log\left(b\sqrt{x}+a\right)}{b^4}-\frac{360\left(b\sqrt{x}+a\right)a^5\log\left(b\sqrt{x}+a\right)}{b^4}-10\left(\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/180*((60*(b*sqrt(x) + a)^6*log(b*sqrt(x) + a)/b^4 - 360*(b*sqrt(x) + a)^5
*a*log(b*sqrt(x) + a)/b^4 + 900*(b*sqrt(x) + a)^4*a^2*log(b*sqrt(x) + a)/b^
4 - 1200*(b*sqrt(x) + a)^3*a^3*log(b*sqrt(x) + a)/b^4 + 900*(b*sqrt(x) + a)
^2*a^4*log(b*sqrt(x) + a)/b^4 - 360*(b*sqrt(x) + a)*a^5*log(b*sqrt(x) + a)/
b^4 - 10*(b*sqrt(x) + a)^6/b^4 + 72*(b*sqrt(x) + a)^5*a/b^4 - 225*(b*sqrt(x
) + a)^4*a^2/b^4 + 400*(b*sqrt(x) + a)^3*a^3/b^4 - 450*(b*sqrt(x) + a)^2*a^
4/b^4 + 360*(b*sqrt(x) + a)*a^5/b^4)*p/b + 60*((b*sqrt(x) + a)^6 - 6*(b*sqr
t(x) + a)^5*a + 15*(b*sqrt(x) + a)^4*a^2 - 20*(b*sqrt(x) + a)^3*a^3 + 15*(b
*sqrt(x) + a)^2*a^4 - 6*(b*sqrt(x) + a)*a^5)*log(c)/b^5)/b
```

3.48 $\int x \log \left(c \left(a+b \sqrt{x}\right)^p\right) \, dx$

Optimal. Leaf size=93

$$\frac{a^3 p \sqrt{x}}{2 b^3}-\frac{a^2 p x}{4 b^2}-\frac{a^4 p \log \left(a+b \sqrt{x}\right)}{2 b^4}+\frac{1}{2} x^2 \log \left(c \left(a+b \sqrt{x}\right)^p\right)+\frac{a p x^{3/2}}{6 b}-\frac{p x^2}{8}$$

```
[Out] (a^3*p*Sqrt[x])/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b) - (p*x^2)
/8 - (a^4*p*Log[a + b*Sqrt[x]])/(2*b^4) + (x^2*Log[c*(a + b*Sqrt[x])^p])/2
```

Rubi [A] time = 0.0602953, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^3 p \sqrt{x}}{2 b^3}-\frac{a^2 p x}{4 b^2}-\frac{a^4 p \log \left(a+b \sqrt{x}\right)}{2 b^4}+\frac{1}{2} x^2 \log \left(c \left(a+b \sqrt{x}\right)^p\right)+\frac{a p x^{3/2}}{6 b}-\frac{p x^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^3*p*Sqrt[x])/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b) - (p*x^2)
/8 - (a^4*p*Log[a + b*Sqrt[x]])/(2*b^4) + (x^2*Log[c*(a + b*Sqrt[x])^p])/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)dx = 2\,\text{Subst}\left(\int x^3\log\left(c(a+bx)^p\right)dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{x^4}{a+bx}dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{2}(bp)\,\text{Subst}\left(\int\left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx, x, \sqrt{x}\right)$$
$$= \frac{a^3p\sqrt{x}}{2b^3} - \frac{a^2px}{4b^2} + \frac{apx^{3/2}}{6b} - \frac{px^2}{8} - \frac{a^4p\log\left(a+b\sqrt{x}\right)}{2b^4} + \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

Mathematica [A] time = 0.0361773, size = 88, normalized size = 0.95

$$\frac{bp\sqrt{x}\left(-6a^2b\sqrt{x}+12a^3+4ab^2x-3b^3x^{3/2}\right)-12a^4p\log\left(a+b\sqrt{x}\right)+12b^4x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (b*p*Sqrt[x]*(12*a^3 - 6*a^2*b*Sqrt[x] + 4*a*b^2*x - 3*b^3*x^(3/2)) - 12*a^
4*p*Log[a + b*Sqrt[x]] + 12*b^4*x^2*Log[c*(a + b*Sqrt[x])^p])/(24*b^4)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x\ln\left(c\left(a+b\sqrt{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b*x^(1/2))^p),x)
```

```
[Out] int(x*ln(c*(a+b*x^(1/2))^p),x)
```

Maxima [A] time = 1.06029, size = 103, normalized size = 1.11

$$-\frac{1}{24}bp\left(\frac{12\,a^4\log\left(b\sqrt{x}+a\right)}{b^5}+\frac{3\,b^3x^2-4\,ab^2x^{\frac{3}{2}}+6\,a^2bx-12\,a^3\sqrt{x}}{b^4}\right)+\frac{1}{2}\,x^2\log\left(\left(b\sqrt{x}+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/24*b*p*(12*a^4*log(b*sqrt(x) + a)/b^5 + (3*b^3*x^2 - 4*a*b^2*x^(3/2) + 6
*a^2*b*x - 12*a^3*sqrt(x))/b^4) + 1/2*x^2*log((b*sqrt(x) + a)^p*c)
```

Fricas [A] time = 2.36408, size = 190, normalized size = 2.04

$$-\frac{3\,b^4px^2-12\,b^4x^2\log\left(c\right)+6\,a^2b^2px-12\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)-4\left(ab^3px+3\,a^3bp\right)\sqrt{x}}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/24*(3*b^4*p*x^2 - 12*b^4*x^2*log(c) + 6*a^2*b^2*p*x - 12*(b^4*p*x^2 - a^
4*p)*log(b*sqrt(x) + a) - 4*(a*b^3*p*x + 3*a^3*b*p)*sqrt(x))/b^4
```

Sympy [A] time = 8.12539, size = 92, normalized size = 0.99

$$-\frac{bp\left(\frac{2a^4\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^4} - \frac{2a^3\sqrt{x}}{b^4} + \frac{a^2x}{b^3} - \frac{2ax^{\frac{3}{2}}}{3b^2} + \frac{x^2}{2b}\right)}{4} + \frac{x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**4*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**4 - 2*a**3*sqrt(x)/b**4 + a**2*x/b**3 - 2*a*x**(3/2)/(3*b**2) + x**2/(2
*b))/4 + x**2*log(c*(a + b*sqrt(x))**p)/2
```

Giac [B] time = 1.30632, size = 298, normalized size = 3.2

$$\frac{\frac{\left(\frac{12\left(b\sqrt{x}+a\right)^4\log\left(b\sqrt{x}+a\right)}{b^2} - \frac{48\left(b\sqrt{x}+a\right)^3a\log\left(b\sqrt{x}+a\right)}{b^2} + \frac{72\left(b\sqrt{x}+a\right)^2a^2\log\left(b\sqrt{x}+a\right)}{b^2} - \frac{48\left(b\sqrt{x}+a\right)a^3\log\left(b\sqrt{x}+a\right)}{b^2} - \frac{3\left(b\sqrt{x}+a\right)^4}{b^2} + \frac{16\left(b\sqrt{x}+a\right)^3a}{b^2} - \frac{36\left(b\sqrt{x}+a\right)^2a^2}{b^2} + \frac{48\left(b\sqrt{x}+a\right)a^3}{b^2}\right)p}{b}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/24*((12*(b*sqrt(x) + a)^4*log(b*sqrt(x) + a)/b^2 - 48*(b*sqrt(x) + a)^3*a
*log(b*sqrt(x) + a)/b^2 + 72*(b*sqrt(x) + a)^2*a^2*log(b*sqrt(x) + a)/b^2 -
 48*(b*sqrt(x) + a)*a^3*log(b*sqrt(x) + a)/b^2 - 3*(b*sqrt(x) + a)^4/b^2 +
16*(b*sqrt(x) + a)^3*a/b^2 - 36*(b*sqrt(x) + a)^2*a^2/b^2 + 48*(b*sqrt(x) +
 a)*a^3/b^2)*p/b + 12*((b*sqrt(x) + a)^4 - 4*(b*sqrt(x) + a)^3*a + 6*(b*sqr
t(x) + a)^2*a^2 - 4*(b*sqrt(x) + a)*a^3)*log(c)/b^3)/b
```

3.49 $\int \log \left(c \left(a+b \sqrt{x}\right)^p\right) \, dx$

Optimal. Leaf size=53

$$-\frac{a^2 p \log \left(a+b \sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b \sqrt{x}\right)^p\right)+\frac{a p \sqrt{x}}{b}-\frac{p x}{2}$$

```
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Rubi [A] time = 0.0283768, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2448, 266, 43}

$$-\frac{a^2 p \log \left(a+b \sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b \sqrt{x}\right)^p\right)+\frac{a p \sqrt{x}}{b}-\frac{p x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+b \sqrt{x}\right)^p\right) \, dx &= x \log \left(c \left(a+b \sqrt{x}\right)^p\right)-\frac{1}{2} (b p) \int \frac{\sqrt{x}}{a+b \sqrt{x}} \, dx\\
&= x \log \left(c \left(a+b \sqrt{x}\right)^p\right)-(b p) \text{Subst}\left(\int \frac{x^2}{a+b x} \, dx,x,\sqrt{x}\right)\\
&= x \log \left(c \left(a+b \sqrt{x}\right)^p\right)-(b p) \text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2 (a+b x)}\right) \, dx,x,\sqrt{x}\right)\\
&= \frac{a p \sqrt{x}}{b}-\frac{p x}{2}-\frac{a^2 p \log \left(a+b \sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b \sqrt{x}\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.0235122, size = 53, normalized size = 1.

$$-\frac{a^2 p \log \left(a+b\sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b\sqrt{x}\right)^p\right)+\frac{a p\sqrt{x}}{b}-\frac{p x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Maple [A] time = 0.065, size = 46, normalized size = 0.9

$$-\frac{px}{2}-\frac{a^2 p}{b^2}\ln\left(a+b\sqrt{x}\right)+x\ln\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{ap}{b}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p),x)
```

```
[Out] -1/2*p*x-a^2*p*ln(a+b*x^(1/2))/b^2+x*ln(c*(a+b*x^(1/2))^p)+a*p*x^(1/2)/b
```

Maxima [A] time = 1.05733, size = 68, normalized size = 1.28

$$-\frac{1}{2} bp\left(\frac{2 a^2 \log \left(b\sqrt{x}+a\right)}{b^3}+\frac{bx-2 a\sqrt{x}}{b^2}\right)+x\log\left(\left(b\sqrt{x}+a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(2*a^2*log(b*sqrt(x) + a)/b^3 + (b*x - 2*a*sqrt(x))/b^2) + x*log((
b*sqrt(x) + a)^p*c)
```

Fricas [A] time = 2.42336, size = 130, normalized size = 2.45

$$-\frac{b^2 px-2 b^2 x\log(c)-2 abp\sqrt{x}-2\left(b^2 px-a^2 p\right)\log\left(b\sqrt{x}+a\right)}{2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*p*x - 2*b^2*x*log(c) - 2*a*b*p*sqrt(x) - 2*(b^2*p*x - a^2*p)*log(
b*sqrt(x) + a))/b^2
```

Sympy [A] time = 2.10022, size = 61, normalized size = 1.15

$$-\frac{bp\left(\frac{2a^2\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^2}-\frac{2a\sqrt{x}}{b^2}+\frac{x}{b}\right)}{2}+x\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**2*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**2 - 2*a*sqrt(x)/b**2 + x/b)/2 + x*log(c*(a + b*sqrt(x))**p)
```

Giac [B] time = 1.30654, size = 131, normalized size = 2.47

$$\frac{\frac{\left(2\left(b\sqrt{x}+a\right)^2\log\left(b\sqrt{x}+a\right)-4\left(b\sqrt{x}+a\right)a\log\left(b\sqrt{x}+a\right)-\left(b\sqrt{x}+a\right)^2+4\left(b\sqrt{x}+a\right)a\right)p}{b}+\frac{2\left(\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a\right)\log(c)}{b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/2*((2*(b*sqrt(x) + a)^2*log(b*sqrt(x) + a) - 4*(b*sqrt(x) + a)*a*log(b*sq
rt(x) + a) - (b*sqrt(x) + a)^2 + 4*(b*sqrt(x) + a)*a)*p/b + 2*((b*sqrt(x) +
 a)^2 - 2*(b*sqrt(x) + a)*a)*log(c)/b)/b
```

3.50 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} dx$

Optimal. Leaf size=46

$$2p\text{PolyLog}\left(2,\frac{b\sqrt{x}}{a}+1\right)+2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, 1 + (b*Sq
rt[x])/a]
```

Rubi [A] time = 0.0404115, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2394, 2315}

$$2p\text{PolyLog}\left(2,\frac{b\sqrt{x}}{a}+1\right)+2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, 1 + (b*Sq
rt[x])/a]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right) - (2bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right) + 2p\text{Li}_2\left(1+\frac{b\sqrt{x}}{a}\right)$$

Mathematica [A] time = 0.0028508, size = 47, normalized size = 1.02

$$2p\text{PolyLog}\left(2, \frac{a+b\sqrt{x}}{a}\right) + 2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, (a + b*Sq
rt[x])/a]
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int \frac{1}{x}\ln\left(c\left(a+b\sqrt{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x,x)
```

Maxima [B] time = 1.06812, size = 107, normalized size = 2.33

$$bp\left(\frac{\log\left(b\sqrt{x}+a\right)\log(x)}{b} - \frac{\log(x)\log\left(\frac{b\sqrt{x}}{a}+1\right)+2\,\text{Li}_2\left(-\frac{b\sqrt{x}}{a}\right)}{b}\right) - p\log\left(b\sqrt{x}+a\right)\log(x) + \log\left(\left(b\sqrt{x}+a\right)^p c\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="maxima")
```

```
[Out] b*p*(log(b*sqrt(x) + a)*log(x)/b - (log(x)*log(b*sqrt(x)/a + 1) + 2*dilog(-
b*sqrt(x)/a))/b) - p*log(b*sqrt(x) + a)*log(x) + log((b*sqrt(x) + a)^p*c)*l
og(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(b\sqrt{x}+a\right)^{p}c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*sqrt(x) + a)^p*c)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*sqrt(x))**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(b\sqrt{x}+a\right)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*sqrt(x) + a)^p*c)/x, x)
```

3.51 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=63

$$\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp}{a\sqrt{x}}$$

```
[Out] -((b*p)/(a*Sqrt[x])) + (b^2*p*Log[a + b*Sqrt[x]])/a^2 - Log[c*(a + b*Sqrt[x
])^p]/x - (b^2*p*Log[x])/(2*a^2)
```

Rubi [A] time = 0.0452999, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 44}

$$\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

```
[Out] -((b*p)/(a*Sqrt[x])) + (b^2*p*Log[a + b*Sqrt[x]])/a^2 - Log[c*(a + b*Sqrt[x
])^p]/x - (b^2*p*Log[x])/(2*a^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} + (bp)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} + (bp)\,\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{bp}{a\sqrt{x}} + \frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} - \frac{b^2p\log(x)}{2a^2}$$

Mathematica [A] time = 0.039548, size = 55, normalized size = 0.87

$$-\frac{bp\left(-2b\log\left(a+b\sqrt{x}\right) + \frac{2a}{\sqrt{x}} + b\log(x)\right)}{2a^2} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

```
[Out] -(Log[c*(a + b*Sqrt[x])^p]/x) - (b*p*((2*a)/Sqrt[x] - 2*b*Log[a + b*Sqrt[x]
] + b*Log[x]))/(2*a^2)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\ln\left(c\left(a+b\sqrt{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^2,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x^2,x)
```

Maxima [A] time = 1.03496, size = 72, normalized size = 1.14

$$\frac{1}{2}bp\left(\frac{2\,b\log\left(b\sqrt{x}+a\right)}{a^2} - \frac{b\log(x)}{a^2} - \frac{2}{a\sqrt{x}}\right) - \frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*b*log(b*sqrt(x) + a)/a^2 - b*log(x)/a^2 - 2/(a*sqrt(x))) - log((
b*sqrt(x) + a)^p*c)/x
```

Fricas [A] time = 2.2627, size = 136, normalized size = 2.16

$$-\frac{b^2px\log\left(\sqrt{x}\right) + abp\sqrt{x} + a^2\log\left(c\right) - \left(b^2px - a^2p\right)\log\left(b\sqrt{x} + a\right)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*p*x*log(sqrt(x)) + a*b*p*sqrt(x) + a^2*log(c) - (b^2*p*x - a^2*p)*log
(b*sqrt(x) + a))/(a^2*x)
```

Sympy [A] time = 91.8096, size = 406, normalized size = 6.44

$$\begin{cases} -\frac{a^3p\sqrt{x}\log\left(a+b\sqrt{x}\right)}{a^3x^{\frac{3}{2}}+a^2bx^2} - \frac{a^3\sqrt{x}\log\left(c\right)}{a^3x^{\frac{3}{2}}+a^2bx^2} - \frac{a^2bpx\log\left(a+b\sqrt{x}\right)}{a^3x^{\frac{3}{2}}+a^2bx^2} - \frac{a^2bpx}{a^3x^{\frac{3}{2}}+a^2bx^2} - \frac{a^2bx\log\left(c\right)}{a^3x^{\frac{3}{2}}+a^2bx^2} - \frac{ab^2px^{\frac{3}{2}}\log\left(x\right)}{2\left(a^3x^{\frac{3}{2}}+a^2bx^2\right)} + \frac{ab^2px^{\frac{3}{2}}\log\left(a+b\sqrt{x}\right)}{a^3x^{\frac{3}{2}}+a^2bx^2} + \frac{ab^2x^{\frac{3}{2}}\log\left(c\right)}{a^3x^{\frac{3}{2}}+a^2bx^2} \\ -\frac{p\log\left(b\right)}{x} - \frac{p\log\left(x\right)}{2x} - \frac{p}{2x} - \frac{\log\left(c\right)}{x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**2,x)
```

```
[Out] Piecewise((-a**3*p*sqrt(x)*log(a + b*sqrt(x))/(a**3*x**(3/2) + a**2*b*x**2)
 - a**3*sqrt(x)*log(c)/(a**3*x**(3/2) + a**2*b*x**2) - a**2*b*p*x*log(a + b
*sqrt(x))/(a**3*x**(3/2) + a**2*b*x**2) - a**2*b*p*x/(a**3*x**(3/2) + a**2*
b*x**2) - a**2*b*x*log(c)/(a**3*x**(3/2) + a**2*b*x**2) - a*b**2*p*x**(3/2)
*log(x)/(2*(a**3*x**(3/2) + a**2*b*x**2)) + a*b**2*p*x**(3/2)*log(a + b*sqr
t(x))/(a**3*x**(3/2) + a**2*b*x**2) + a*b**2*x**(3/2)*log(c)/(a**3*x**(3/2)
 + a**2*b*x**2) - b**3*p*x**2*log(x)/(2*(a**3*x**(3/2) + a**2*b*x**2)) + b*
*3*p*x**2*log(a + b*sqrt(x))/(a**3*x**(3/2) + a**2*b*x**2) + b**3*p*x**2/(a
**3*x**(3/2) + a**2*b*x**2) + b**3*x**2*log(c)/(a**3*x**(3/2) + a**2*b*x**2
), Ne(a, 0)), (-p*log(b)/x - p*log(x)/(2*x) - p/(2*x) - log(c)/x, True))
```

Giac [B] time = 1.20244, size = 178, normalized size = 2.83

$$-\frac{\frac{b^3p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a+a^2} - \frac{b^3p\log\left(b\sqrt{x}+a\right)}{a^2} + \frac{b^3p\log\left(b\sqrt{x}\right)}{a^2} + \frac{\left(b\sqrt{x}+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(b\sqrt{x}+a\right)^2a-2\left(b\sqrt{x}+a\right)a^2+a^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="giac")
```

```
[Out] -(b^3*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^2 - 2*(b*sqrt(x) + a)*a + a^2)
- b^3*p*log(b*sqrt(x) + a)/a^2 + b^3*p*log(b*sqrt(x))/a^2 + ((b*sqrt(x) + a
)*b^3*p - a*b^3*p + a*b^3*log(c))/((b*sqrt(x) + a)^2*a - 2*(b*sqrt(x) + a)*
a^2 + a^3))/b
```

3.52 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3} dx$

Optimal. Leaf size=100

$$-\frac{b^3p}{2a^3\sqrt{x}}+\frac{b^2p}{4a^2x}+\frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{b^4p\log(x)}{4a^4}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{bp}{6ax^{3/2}}$$

```
[Out] -(b*p)/(6*a*x^(3/2)) + (b^2*p)/(4*a^2*x) - (b^3*p)/(2*a^3*Sqrt[x]) + (b^4*p
*Log[a + b*Sqrt[x]])/(2*a^4) - Log[c*(a + b*Sqrt[x])^p]/(2*x^2) - (b^4*p*Lo
g[x])/(4*a^4)
```

Rubi [A] time = 0.0632133, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 44}

$$-\frac{b^3p}{2a^3\sqrt{x}}+\frac{b^2p}{4a^2x}+\frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{b^4p\log(x)}{4a^4}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{bp}{6ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```



```
[Out] -(b*p)/(6*a*x^(3/2)) + (b^2*p)/(4*a^2*x) - (b^3*p)/(2*a^3*Sqrt[x]) + (b^4*p
*Log[a + b*Sqrt[x]])/(2*a^4) - Log[c*(a + b*Sqrt[x])^p]/(2*x^2) - (b^4*p*Lo
g[x])/(4*a^4)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log \left(c \left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{\log (c(a+bx)^p)}{x^5}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\log \left(c \left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\log \left(c \left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx,x,\sqrt{x}\right)$$
$$= -\frac{bp}{6ax^{3/2}}+\frac{b^2p}{4a^2x}-\frac{b^3p}{2a^3\sqrt{x}}+\frac{b^4p\log \left(a+b\sqrt{x}\right)}{2a^4}-\frac{\log \left(c \left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{b^4p\log(x)}{4a^4}$$

Mathematica [A] time = 0.0436651, size = 90, normalized size = 0.9

$$\frac{abp\sqrt{x}\left(-2a^2+3ab\sqrt{x}-6b^2x\right)-6a^4\log \left(c\left(a+b\sqrt{x}\right)^p\right)+6b^4px^2\log \left(a+b\sqrt{x}\right)-3b^4px^2\log(x)}{12a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

```
[Out] (a*b*p*Sqrt[x]*(-2*a^2 + 3*a*b*Sqrt[x] - 6*b^2*x) + 6*b^4*p*x^2*Log[a + b*S
qrt[x]] - 6*a^4*Log[c*(a + b*Sqrt[x])^p] - 3*b^4*p*x^2*Log[x])/(12*a^4*x^2)
```

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\ln \left(c\left(a+b\sqrt{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^3,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x^3,x)
```

Maxima [A] time = 1.06173, size = 103, normalized size = 1.03

$$\frac{1}{12}\,bp\left(\frac{6\,b^3\log \left(b\sqrt{x}+a\right)}{a^4}-\frac{3\,b^3\log (x)}{a^4}-\frac{6\,b^2x-3\,ab\sqrt{x}+2\,a^2}{a^3x^{\frac{3}{2}}}\right)-\frac{\log \left(\left(b\sqrt{x}+a\right)^pc\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b*p*(6*b^3*log(b*sqrt(x) + a)/a^4 - 3*b^3*log(x)/a^4 - (6*b^2*x - 3*a*
b*sqrt(x) + 2*a^2)/(a^3*x^(3/2))) - 1/2*log((b*sqrt(x) + a)^p*c)/x^2
```

Fricas [A] time = 2.20647, size = 208, normalized size = 2.08

$$-\frac{6\,b^4px^2\log\left(\sqrt{x}\right)-3\,a^2b^2px+6\,a^4\log\left(c\right)-6\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)+2\left(3\,ab^3px+a^3bp\right)\sqrt{x}}{12\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^4*p*x^2*log(sqrt(x)) - 3*a^2*b^2*p*x + 6*a^4*log(c) - 6*(b^4*p*x
^2 - a^4*p)*log(b*sqrt(x) + a) + 2*(3*a*b^3*p*x + a^3*b*p)*sqrt(x))/(a^4*x^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30011, size = 313, normalized size = 3.13

$$-\frac{\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^4-4\left(b\sqrt{x}+a\right)^3a+6\left(b\sqrt{x}+a\right)^2a^2-4\left(b\sqrt{x}+a\right)a^3+a^4}-\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{a^4}+\frac{6\,b^5p\log\left(b\sqrt{x}\right)}{a^4}+\frac{6\left(b\sqrt{x}+a\right)^3b^5p-21\left(b\sqrt{x}+a\right)^2ab^5p+26\left(b\sqrt{x}+a\right)a^2b\cdots}{\left(b\sqrt{x}+a\right)^4a^3-4\left(b\sqrt{x}+a\right)^3a^4+6\left(b\sqrt{x}+a\right)^2a^5\cdots}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^4 - 4*(b*sqrt(x) + a)^3*
a + 6*(b*sqrt(x) + a)^2*a^2 - 4*(b*sqrt(x) + a)*a^3 + a^4) - 6*b^5*p*log(b*
sqrt(x) + a)/a^4 + 6*b^5*p*log(b*sqrt(x))/a^4 + (6*(b*sqrt(x) + a)^3*b^5*p
- 21*(b*sqrt(x) + a)^2*a*b^5*p + 26*(b*sqrt(x) + a)*a^2*b^5*p - 11*a^3*b^5*
p + 6*a^3*b^5*log(c))/((b*sqrt(x) + a)^4*a^3 - 4*(b*sqrt(x) + a)^3*a^4 + 6*
(b*sqrt(x) + a)^2*a^5 - 4*(b*sqrt(x) + a)*a^6 + a^7))/b
```

3.53 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4} dx$

Optimal. Leaf size=130

$$-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^2p}{12a^2x^2}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^4p}{6a^4x}+\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{b^6p\log(x)}{6a^6}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{bp}{15ax^{5/2}}$$

```
[Out] -(b*p)/(15*a*x^(5/2)) + (b^2*p)/(12*a^2*x^2) - (b^3*p)/(9*a^3*x^(3/2)) + (b
^4*p)/(6*a^4*x) - (b^5*p)/(3*a^5*Sqrt[x]) + (b^6*p*Log[a + b*Sqrt[x]])/(3*a
^6) - Log[c*(a + b*Sqrt[x])^p]/(3*x^3) - (b^6*p*Log[x])/(6*a^6)
```

Rubi [A] time = 0.0794123, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 44}

$$-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^2p}{12a^2x^2}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^4p}{6a^4x}+\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{b^6p\log(x)}{6a^6}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{bp}{15ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] -(b*p)/(15*a*x^(5/2)) + (b^2*p)/(12*a^2*x^2) - (b^3*p)/(9*a^3*x^(3/2)) + (b
^4*p)/(6*a^4*x) - (b^5*p)/(3*a^5*Sqrt[x]) + (b^6*p*Log[a + b*Sqrt[x]])/(3*a
^6) - Log[c*(a + b*Sqrt[x])^p]/(3*x^3) - (b^6*p*Log[x])/(6*a^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^7}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{1}{x^6(a+bx)}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \left(\frac{1}{ax^6} - \frac{b}{a^2x^5} + \frac{b^2}{a^3x^4} - \frac{b^3}{a^4x^3} + \frac{b^4}{a^5x^2} - \frac{b^5}{a^6x} + \frac{\quad}{a^6(a}\right.\right.$$
$$= -\frac{bp}{15ax^{5/2}} + \frac{b^2p}{12a^2x^2} - \frac{b^3p}{9a^3x^{3/2}} + \frac{b^4p}{6a^4x} - \frac{b^5p}{3a^5\sqrt{x}} + \frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}$$

Mathematica [A] time = 0.0627276, size = 114, normalized size = 0.88

$$\frac{abp\sqrt{x}\left(-20a^2b^2x+15a^3b\sqrt{x}-12a^4+30ab^3x^{3/2}-60b^4x^2\right)-60a^6\log\left(c\left(a+b\sqrt{x}\right)^p\right)+60b^6px^3\log\left(a+b\sqrt{x}\right)-30b}{180a^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] (a*b*p*Sqrt[x]*(-12*a^4 + 15*a^3*b*Sqrt[x] - 20*a^2*b^2*x + 30*a*b^3*x^(3/2
) - 60*b^4*x^2) + 60*b^6*p*x^3*Log[a + b*Sqrt[x]] - 60*a^6*Log[c*(a + b*Sqr
t[x])^p] - 30*b^6*p*x^3*Log[x])/(180*a^6*x^3)
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\ln\left(c\left(a+b\sqrt{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^4,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x^4,x)
```

Maxima [A] time = 1.12465, size = 132, normalized size = 1.02

$$\frac{1}{180}bp\left(\frac{60\,b^5\log\left(b\sqrt{x}+a\right)}{a^6} - \frac{30\,b^5\log\left(x\right)}{a^6} - \frac{60\,b^4x^2-30\,ab^3x^{\frac{3}{2}}+20\,a^2b^2x-15\,a^3b\sqrt{x}+12\,a^4}{a^5x^{\frac{5}{2}}}\right) - \frac{\log\left(\left(b\sqrt{x}+a\right)^pc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*p*(60*b^5*log(b*sqrt(x) + a)/a^6 - 30*b^5*log(x)/a^6 - (60*b^4*x^2
- 30*a*b^3*x^(3/2) + 20*a^2*b^2*x - 15*a^3*b*sqrt(x) + 12*a^4)/(a^5*x^(5/2)
)) - 1/3*log((b*sqrt(x) + a)^p*c)/x^3
```

Fricas [A] time = 2.27093, size = 269, normalized size = 2.07

$$-\frac{60\,b^6px^3\log\left(\sqrt{x}\right)-30\,a^2b^4px^2-15\,a^4b^2px+60\,a^6\log\left(c\right)-60\left(b^6px^3-a^6p\right)\log\left(b\sqrt{x}+a\right)+4\left(15\,ab^5px^2+5\,a^3b^3px\cdots\right.}{180\,a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="fricas")
```

```
[Out] -1/180*(60*b^6*p*x^3*log(sqrt(x)) - 30*a^2*b^4*p*x^2 - 15*a^4*b^2*p*x + 60*
a^6*log(c) - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) + 4*(15*a*b^5*p*x^2
+ 5*a^3*b^3*p*x + 3*a^5*b*p)*sqrt(x))/(a^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32791, size = 437, normalized size = 3.36

$$-\frac{\frac{60\,b^7p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^6-6\left(b\sqrt{x}+a\right)^5a+15\left(b\sqrt{x}+a\right)^4a^2-20\left(b\sqrt{x}+a\right)^3a^3+15\left(b\sqrt{x}+a\right)^2a^4-6\left(b\sqrt{x}+a\right)a^5+a^6}-\frac{60\,b^7p\log\left(b\sqrt{x}+a\right)}{a^6}+\frac{60\,b^7p\log\left(b\sqrt{x}\right)}{a^6}+\frac{60\left(b\sqrt{x}+a\right)^5b^7p-\cdots}{\left(b\sqrt{\cdots}\right.}}{180\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/180*(60*b^7*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^6 - 6*(b*sqrt(x) + a)^
5*a + 15*(b*sqrt(x) + a)^4*a^2 - 20*(b*sqrt(x) + a)^3*a^3 + 15*(b*sqrt(x) +
 a)^2*a^4 - 6*(b*sqrt(x) + a)*a^5 + a^6) - 60*b^7*p*log(b*sqrt(x) + a)/a^6
+ 60*b^7*p*log(b*sqrt(x))/a^6 + (60*(b*sqrt(x) + a)^5*b^7*p - 330*(b*sqrt(x
) + a)^4*a*b^7*p + 740*(b*sqrt(x) + a)^3*a^2*b^7*p - 855*(b*sqrt(x) + a)^2*
a^3*b^7*p + 522*(b*sqrt(x) + a)*a^4*b^7*p - 137*a^5*b^7*p + 60*a^5*b^7*log(
c))/((b*sqrt(x) + a)^6*a^5 - 6*(b*sqrt(x) + a)^5*a^6 + 15*(b*sqrt(x) + a)^4
*a^7 - 20*(b*sqrt(x) + a)^3*a^8 + 15*(b*sqrt(x) + a)^2*a^9 - 6*(b*sqrt(x) +
 a)*a^10 + a^11))/b
```

3.54 $\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-2\sqrt{x}$$

```
[Out] -2*Sqrt[x] + (2*(a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b
```

Rubi [A] time = 0.0180764, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] + (2*(a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \log(a+bx)\,dx,x,\sqrt{x}\right)\\ &= \frac{2\,\text{Subst}\left(\int \log(x)\,dx,x,a+b\sqrt{x}\right)}{b}\\ &= -2\sqrt{x}+\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0093172, size = 33, normalized size = 1.03

$$2\left(\frac{\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*(-Sqrt[x] + ((a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b)
```

Maple [A] time = 0.06, size = 40, normalized size = 1.3

$$2\ln\left(a+b\sqrt{x}\right)\sqrt{x}+2\,\frac{\ln\left(a+b\sqrt{x}\right)a}{b}-2\sqrt{x}-2\,\frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b*x^(1/2))/x^(1/2),x)
```

```
[Out] 2*ln(a+b*x^(1/2))*x^(1/2)+2/b*ln(a+b*x^(1/2))*a-2*x^(1/2)-2*a/b
```

Maxima [A] time = 1.00635, size = 42, normalized size = 1.31

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

Fricas [A] time = 2.08938, size = 73, normalized size = 2.28

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x))/b
```

Sympy [A] time = 1.28765, size = 133, normalized size = 4.16

$$\begin{cases}\frac{2a^2\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}+\frac{2a^2}{ab+b^2\sqrt{x}}+\frac{4ab\sqrt{x}\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}+\frac{2b^2x\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}-\frac{2b^2x}{ab+b^2\sqrt{x}} & \text{for } b\neq 0\\ 2\sqrt{x}\log\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*x**(1/2))/x**(1/2),x)
```

```
[Out] Piecewise((2*a**2*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) + 2*a**2/(a*b + b
**2*sqrt(x)) + 4*a*b*sqrt(x)*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) + 2*b*
*2*x*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) - 2*b**2*x/(a*b + b**2*sqrt(x)
), Ne(b, 0)), (2*sqrt(x)*log(a), True))
```

Giac [A] time = 1.24872, size = 42, normalized size = 1.31

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

3.55 $\int (fx)^m \log \left(c \left(d+ex^3\right)^p\right) \, dx$

Optimal. Leaf size=81

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^3\right)^p\right)}{f(m+1)}-\frac{3ep(fx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};-\frac{ex^3}{d}\right)}{df^4(m+1)(m+4)}$$

```
[Out] (-3*e*p*(f*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)/
d)])/(d*f^4*(1 + m)*(4 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^3)^p])/(f*(1 +
 m))
```

Rubi [A] time = 0.0437643, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 16, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^3\right)^p\right)}{f(m+1)}-\frac{3ep(fx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};-\frac{ex^3}{d}\right)}{df^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (-3*e*p*(f*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)/
d)])/(d*f^4*(1 + m)*(4 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^3)^p])/(f*(1 +
 m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+ex^3\right)^p\right) dx = \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)} - \frac{(3ep)\int \frac{x^2(fx)^{1+m}}{d+ex^3}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)} - \frac{(3ep)\int \frac{(fx)^{3+m}}{d+ex^3}dx}{f^3(1+m)}$$
$$= -\frac{3ep(fx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};-\frac{ex^3}{d}\right)}{df^4(1+m)(4+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0273693, size = 70, normalized size = 0.86

$$\frac{x(fx)^m\left(d(m+4)\log\left(c\left(d+ex^3\right)^p\right)-3epx^3\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};-\frac{ex^3}{d}\right)\right)}{d(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (x*(f*x)^m*(-3*e*p*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)
/d)] + d*(4 + m)*Log[c*(d + e*x^3)^p]))/(d*(1 + m)*(4 + m))
```

Maple [F] time = 1.043, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(ex^3+d\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^m \log\left(\left(ex^3+d\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^3 + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log\left((ex^3 + d)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^3 + d)^p*c), x)
```

3.56 $\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=81

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rubi [A] time = 0.0418747, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 16, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{x(fx)^{1+m}}{d+ex^2}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{(fx)^{2+m}}{d+ex^2}dx}{f^2(1+m)}$$
$$= -\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0270598, size = 70, normalized size = 0.86

$$\frac{x(fx)^m\left(d(m+3)\log\left(c\left(d+ex^2\right)^p\right)-2epx^2\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)\right)}{d(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Maple [F] time = 1.053, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(ex^2+d\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^m \log\left(\left(ex^2+d\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log\left(\left(ex^2+d\right)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

3.57 $\int (fx)^m \log (c(d+ex)^p)\, dx$

Optimal. Leaf size=69

$$\frac{(fx)^{m+1}\log (c(d+ex)^p)}{f(m+1)}-\frac{ep(fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{ex}{d}\right)}{df^2(m+1)(m+2)}$$

```
[Out] -((e*p*(f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)])/(d*f^2
*(1 + m)*(2 + m))) + ((f*x)^(1 + m)*Log[c*(d + e*x)^p])/(f*(1 + m))
```

Rubi [A] time = 0.0293683, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2395, 64}

$$\frac{(fx)^{m+1}\log (c(d+ex)^p)}{f(m+1)}-\frac{ep(fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{ex}{d}\right)}{df^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x)^p],x]
```

```
[Out] -((e*p*(f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)])/(d*f^2
*(1 + m)*(2 + m))) + ((f*x)^(1 + m)*Log[c*(d + e*x)^p])/(f*(1 + m))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int (fx)^m \log (c(d+ex)^p)\, dx &= \frac{(fx)^{1+m}\log (c(d+ex)^p)}{f(1+m)}-\frac{(ep)\int \frac{(fx)^{1+m}}{d+ex}\, dx}{f(1+m)}\\ &= -\frac{ep(fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{ex}{d}\right)}{df^2(1+m)(2+m)}+\frac{(fx)^{1+m}\log (c(d+ex)^p)}{f(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0246867, size = 56, normalized size = 0.81

$$\frac{x(fx)^m\left(d(m+2)\log (c(d+ex)^p)-epx\,{}_2F_1\left(1,m+2;m+3;-\frac{ex}{d}\right)\right)}{d(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x)^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*p*x*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)]) + d*(2
+ m)*Log[c*(d + e*x)^p]))/(d*(1 + m)*(2 + m))
```

Maple [F] time = 0.851, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \ln\left(c\left(ex+d\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x+d)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx\right)^{m} \log\left(\left(ex+d\right)^{p}c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x + d)^p*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \log\left(c\left(d+ex\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x+d)**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*x)**p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log \left((ex+d)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x + d)^p*c), x)
```

3.58 $\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx$

Optimal. Leaf size=67

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}+\frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(m+1)}$$

```
[Out] (e*p*(f*x)^m*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))])/(d*m*(1 + m)) + (
(f*x)^(1 + m)*Log[c*(d + e/x)^p])/(f*(1 + m))
```

Rubi [A] time = 0.040617, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2455, 16, 339, 64}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}+\frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x)^p],x]
```

```
[Out] (e*p*(f*x)^m*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))])/(d*m*(1 + m)) + (
(f*x)^(1 + m)*Log[c*(d + e/x)^p])/(f*(1 + m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)} + \frac{(ep)\int\frac{(fx)^{1+m}}{\left(d+\frac{e}{x}\right)x^2}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)} + \frac{(efp)\int\frac{(fx)^{-1+m}}{d+\frac{e}{x}}dx}{1+m}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)} - \frac{\left(ep\left(\frac{1}{x}\right)^m(fx)^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{d+ex}dx, x, \frac{1}{x}\right)}{1+m}$$
$$= \frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(1+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0159608, size = 56, normalized size = 0.84

$$\frac{(fx)^m\left(dmx\log\left(c\left(d+\frac{e}{x}\right)^p\right)+ep\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)\right)}{dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x)^p],x]
```

```
[Out] ((f*x)^m*(e*p*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))] + d*m*x*Log[c*(d
+ e/x)^p]))/(d*m*(1 + m))
```

Maple [F] time = 2.666, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((fx)^m\log\left(c\left(\frac{dx+e}{x}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x + e)/x)^p), x)
```

Sympy [A] time = 52.6949, size = 201, normalized size = 3.

$$ep\left(\begin{cases}\frac{0^m\log(dx+e)}{d}\\ \frac{f^m m x^m \Phi\left(\frac{ee^{i\pi}}{dx},1,me^{i\pi}\right)\Gamma(-m)}{dm\Gamma(1-m)+d\Gamma(1-m)}\\ \frac{\begin{cases}-\frac{1}{dx} & \text{for } e=0\\ \begin{cases}\log(d)\log(x)+\operatorname{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)+\operatorname{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)+\operatorname{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{otherwise}\end{cases}}{e} & \text{otherwise}\end{cases}}{f}\end{cases}\right. - \frac{\begin{cases}\frac{1}{dx}\\ \log\left(d+\frac{e}{x}\right)\end{cases}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x)**p),x)
```

```
[Out] e*p*Piecewise((0**m*log(d*x + e)/d, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1))), (f*
*m*m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x), 1, m*exp_polar(I*pi))*gamma(-m)
/(d*m*gamma(1 - m) + d*gamma(1 - m)), (m > -oo) & (m < oo) & Ne(m, -1)), (P
iecewise((-1/(d*x), Eq(e, 0)), (Piecewise((log(d)*log(x) + polylog(2, e*exp
_polar(I*pi)/(d*x)), Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_pola
r(I*pi)/(d*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log
(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp_pola
r(I*pi)/(d*x)), True))/e, True))/f - Piecewise((1/(d*x), Eq(e, 0)), (log(d
+ e/x)/e, True))*log(f*x)/f, True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecew
ise(((f*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*
(d + e/x)**p)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/x)^p), x)
```

3.59 $\int (fx)^m \log \left(c \left(d+\frac{e}{x^2}\right)^p\right) dx$

Optimal. Leaf size=82

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}-\frac{2efp(fx)^{m-1}{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}$$

```
[Out] (-2*e*f*p*(f*x)^(-1 + m)*Hypergeometric2F1[1, (1 - m)/2, (3 - m)/2, -(e/(d*
x^2))])/(d*(1 - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^2)^p])/(f*(1 + m))
```

Rubi [A] time = 0.0544225, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2455, 16, 339, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}-\frac{2efp(fx)^{m-1}{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

```
[Out] (-2*e*f*p*(f*x)^(-1 + m)*Hypergeometric2F1[1, (1 - m)/2, (3 - m)/2, -(e/(d*
x^2))])/(d*(1 - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^2)^p])/(f*(1 + m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)} + \frac{(2ep)\int\frac{(fx)^{1+m}}{\left(d+\frac{e}{x^2}\right)x^3}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)} + \frac{\left(2ef^2p\right)\int\frac{(fx)^{-2+m}}{d+\frac{e}{x^2}}dx}{1+m}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)} - \frac{\left(2efp\left(\frac{1}{x}\right)^{-1+m}(fx)^{-1+m}\right)\text{Subst}\left(\int\frac{x^{-m}}{d+ex^2}dx, x, \frac{1}{x}\right)}{1+m}$$
$$= -\frac{2efp(fx)^{-1+m}{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0297495, size = 76, normalized size = 0.93

$$\frac{(fx)^m\left(d(m-1)x^2\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)+2ep\,{}_2F_1\left(1,\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};-\frac{e}{dx^2}\right)\right)}{d(m-1)(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

```
[Out] ((f*x)^m*(2*e*p*Hypergeometric2F1[1, 1/2 - m/2, 3/2 - m/2, -(e/(d*x^2))] +
d*(-1 + m)*x^2*Log[c*(d + e/x^2)^p]))/(d*(-1 + m)*(1 + m)*x)
```

Maple [F] time = 3.868, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^{m}\log\left(c\left(\frac{dx^{2}+e}{x^{2}}\right)^{p}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x^2 + e)/x^2)^p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**2)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{m}\log\left(c\left(d+\frac{e}{x^{2}}\right)^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/x^2)^p), x)
```

3.60 $\int (fx)^m \log \left(c \left(d+\frac{e}{x^3}\right)^p\right) \, dx$

Optimal. Leaf size=85

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}-\frac{3ef^2p(fx)^{m-2}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\left(-m^2+m+2\right)}$$

```
[Out] (-3*e*f^2*p*(f*x)^(-2 + m)*Hypergeometric2F1[1, (2 - m)/3, (5 - m)/3, -(e/(
d*x^3))])/(d*(2 + m - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^3)^p])/(f*(1 +
m))
```

Rubi [A] time = 0.0605074, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2455, 16, 339, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}-\frac{3ef^2p(fx)^{m-2}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\left(-m^2+m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x^3)^p],x]
```

```
[Out] (-3*e*f^2*p*(f*x)^(-2 + m)*Hypergeometric2F1[1, (2 - m)/3, (5 - m)/3, -(e/(
d*x^3))])/(d*(2 + m - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^3)^p])/(f*(1 +
m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)} + \frac{(3ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{x^3}\right)x^4}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)} + \frac{\left(3ef^3p\right)\int \frac{(fx)^{-3+m}}{d+\frac{e}{x^3}}dx}{1+m}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)} - \frac{\left(3ef^2p\left(\frac{1}{x}\right)^{-2+m}(fx)^{-2+m}\right)\text{Subst}\left(\int \frac{x^{1-m}}{d+ex^3}dx, x, \frac{1}{x}\right)}{1+m}$$
$$= -\frac{3ef^2p(fx)^{-2+m}{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\left(2+m-m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0310082, size = 76, normalized size = 0.89

$$\frac{(fx)^m\left(d(m-2)x^3\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)+3ep\,{}_2F_1\left(1,\frac{2}{3}-\frac{m}{3};\frac{5}{3}-\frac{m}{3};-\frac{e}{dx^3}\right)\right)}{d(m-2)(m+1)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^3)^p],x]
```

```
[Out] ((f*x)^m*(3*e*p*Hypergeometric2F1[1, 2/3 - m/3, 5/3 - m/3, -(e/(d*x^3))] +
d*(-2 + m)*x^3*Log[c*(d + e/x^3)^p]))/(d*(-2 + m)*(1 + m)*x^2)
```

Maple [F] time = 4.389, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^3}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^{m}\log\left(c\left(\frac{dx^{3}+e}{x^{3}}\right)^{p}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x^3 + e)/x^3)^p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**3)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{m}\log\left(c\left(d+\frac{e}{x^{3}}\right)^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/x^3)^p), x)
```

3.61 $\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx$

Optimal. Leaf size=83

$$\frac{(fx)^{m+1}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,2m+3;2(m+2);-\frac{e\sqrt{x}}{d}\right)}{d\left(2m^2+5m+3\right)}$$

```
[Out] -((e*p*x^(3/2)*(f*x)^m*Hypergeometric2F1[1, 3 + 2*m, 2*(2 + m), -((e*Sqrt[x
])/d)])/(d*(3 + 5*m + 2*m^2))) + ((f*x)^(1 + m)*Log[c*(d + e*Sqrt[x])^p])/(
f*(1 + m))
```

Rubi [A] time = 0.0505886, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2455, 20, 341, 64}

$$\frac{(fx)^{m+1}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,2m+3;2(m+2);-\frac{e\sqrt{x}}{d}\right)}{d\left(2m^2+5m+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

```
[Out] -((e*p*x^(3/2)*(f*x)^m*Hypergeometric2F1[1, 3 + 2*m, 2*(2 + m), -((e*Sqrt[x
])/d)])/(d*(3 + 5*m + 2*m^2))) + ((f*x)^(1 + m)*Log[c*(d + e*Sqrt[x])^p])/(
f*(1 + m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx = \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+e\sqrt{x}\right)\sqrt{x}}\,dx}{2f(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^{\frac{1}{2}+m}}{d+e\sqrt{x}}\,dx}{2(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\text{Subst}\left(\int \frac{x^{-1+2\left(\frac{3}{2}+m\right)}}{d+ex}\,dx, x, \sqrt{x}\right)}{1+m}$$

$$= -\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,3+2m;2(2+m);-\frac{e\sqrt{x}}{d}\right)}{d\left(3+5m+2m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0339509, size = 76, normalized size = 0.92

$$\frac{x(fx)^m\left(d(2m+3)\log\left(c\left(d+e\sqrt{x}\right)^p\right) - ep\sqrt{x}\,{}_2F_1\left(1,2m+3;2m+4;-\frac{e\sqrt{x}}{d}\right)\right)}{d(m+1)(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*p*Sqrt[x]*Hypergeometric2F1[1, 3 + 2*m, 4 + 2*m, -((e*Sqrt[
x])/d)]) + d*(3 + 2*m)*Log[c*(d + e*Sqrt[x])^p]))/(d*(1 + m)*(3 + 2*m))
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^m\log\left(\left(e\sqrt{x}+d\right)^p c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e*x**(1/2))**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(fx\right)^m\log\left(\left(e\sqrt{x}+d\right)^p c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

3.62 $\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\,dx$

Optimal. Leaf size=70

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}+\frac{px(fx)^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{d\sqrt{x}}{e}\right)}{2(m+1)^2}$$

```
[Out] (p*x*(f*x)^m*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((d*Sqrt[x])/e)])/(2
*(1 + m)^2) + ((f*x)^(1 + m)*Log[c*(d + e/Sqrt[x])^p])/(f*(1 + m))
```

Rubi [A] time = 0.0422632, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 20, 263, 341, 64}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}+\frac{px(fx)^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{d\sqrt{x}}{e}\right)}{2(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

```
[Out] (p*x*(f*x)^m*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((d*Sqrt[x])/e)])/(2
*(1 + m)^2) + ((f*x)^(1 + m)*Log[c*(d + e/Sqrt[x])^p])/(f*(1 + m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
```

```
&& !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{\sqrt{x}}\right)x^{3/2}}\,dx}{2f(1+m)} \\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^{-\frac{1}{2}+m}}{d+\frac{e}{\sqrt{x}}}\,dx}{2(1+m)} \\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^m}{e+d\sqrt{x}}\,dx}{2(1+m)} \\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{\left(epx^{-m}(fx)^m\right)\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{e+dx}\,dx, x, \sqrt{x}\right)}{1+m} \\
&= \frac{px(fx)^m\,{}_2F_1\left(1, 2(1+m); 3+2m; -\frac{d\sqrt{x}}{e}\right)}{2(1+m)^2} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}
\end{aligned}
$$

Mathematica [A] time = 0.0372645, size = 77, normalized size = 1.1

$$
\frac{\sqrt{x}(fx)^m\left(d(2m+1)\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)+ep\,{}_2F_1\left(1,-2m-1;-2m;-\frac{e}{d\sqrt{x}}\right)\right)}{d(m+1)(2m+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

```
[Out] (Sqrt[x]*(f*x)^m*(e*p*Hypergeometric2F1[1, -1 - 2*m, -2*m, -(e/(d*Sqrt[x]))
] + d*(1 + 2*m)*Sqrt[x]*Log[c*(d + e/Sqrt[x])^p]))/(d*(1 + m)*(1 + 2*m))
```

Maple [F] time = 0.389, size = 0, normalized size = 0.

$$
\int (fx)^m \ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^p\right) dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^{m}\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{p}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x + e*sqrt(x))/x)^p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**(1/2))**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/sqrt(x))^p), x)
```

3.63 $\int (fx)^m \log \left(c \left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=87

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^n\right)^p\right)}{f(m+1)}-\frac{enpx^{n+1}(fx)^m\,{}_2F_1\left(1,\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{ex^n}{d}\right)}{d(m+1)(m+n+1)}$$

```
[Out] -((e*n*p*x^(1 + n)*(f*x)^m*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n
)/n, -((e*x^n)/d)])/(d*(1 + m)*(1 + m + n))) + ((f*x)^(1 + m)*Log[c*(d + e*
x^n)^p])/(f*(1 + m))
```

Rubi [A] time = 0.0434718, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 20, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^n\right)^p\right)}{f(m+1)}-\frac{enpx^{n+1}(fx)^m\,{}_2F_1\left(1,\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{ex^n}{d}\right)}{d(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*(f*x)^m*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n
)/n, -((e*x^n)/d)])/(d*(1 + m)*(1 + m + n))) + ((f*x)^(1 + m)*Log[c*(d + e*
x^n)^p])/(f*(1 + m))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+ex^n\right)^p\right) dx = \frac{(fx)^{1+m}\log\left(c\left(d+ex^n\right)^p\right)}{f(1+m)} - \frac{(enp)\int \frac{x^{-1+n}(fx)^{1+m}}{d+ex^n} dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+ex^n\right)^p\right)}{f(1+m)} - \frac{\left(enpx^{-m}(fx)^m\right)\int \frac{x^{m+n}}{d+ex^n} dx}{1+m}$$
$$= -\frac{enpx^{1+n}(fx)^m\,{}_2F_1\left(1,\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{ex^n}{d}\right)}{d(1+m)(1+m+n)} + \frac{(fx)^{1+m}\log\left(c\left(d+ex^n\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0368807, size = 77, normalized size = 0.89

$$\frac{x(fx)^m\left(d(m+n+1)\log\left(c\left(d+ex^n\right)^p\right) - enpx^n\,{}_2F_1\left(1,\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{ex^n}{d}\right)\right)}{d(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*n*p*x^n*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n)/n
, -((e*x^n)/d)]) + d*(1 + m + n)*Log[c*(d + e*x^n)^p]))/(d*(1 + m)*(1 + m +
 n))
```

Maple [F] time = 2.053, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(d+ex^n\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((fx)^m \log\left((ex^n+d)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^n + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log\left((ex^n + d)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^n + d)^p*c), x)
```

3.64 $\int (fx)^{-1+3n} \log \left(c \left(d+ex^n\right)^p\right) \, dx$

Optimal. Leaf size=141

$$\frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn}-\frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn}+\frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn}+\frac{dpx^{-n}(fx)^{3n}}{6efn}-\frac{p(fx)^{3n}}{9fn}$$

```
[Out] -(p*(f*x)^(3*n))/(9*f*n) - (d^2*p*(f*x)^(3*n))/(3*e^2*f*n*x^(2*n)) + (d*p*(
f*x)^(3*n))/(6*e*f*n*x^n) + (d^3*p*(f*x)^(3*n)*Log[d + e*x^n])/(3*e^3*f*n*x
^(3*n)) + ((f*x)^(3*n)*Log[c*(d + e*x^n)^p])/(3*f*n)
```

Rubi [A] time = 0.0772847, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 20, 266, 43}

$$\frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn}-\frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn}+\frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn}+\frac{dpx^{-n}(fx)^{3n}}{6efn}-\frac{p(fx)^{3n}}{9fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```


```
[Out] -(p*(f*x)^(3*n))/(9*f*n) - (d^2*p*(f*x)^(3*n))/(3*e^2*f*n*x^(2*n)) + (d*p*(
f*x)^(3*n))/(6*e*f*n*x^n) + (d^3*p*(f*x)^(3*n)*Log[d + e*x^n])/(3*e^3*f*n*x
^(3*n)) + ((f*x)^(3*n)*Log[c*(d + e*x^n)^p])/(3*f*n)
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (fx)^{-1+3n}\log\left(c\left(d+ex^n\right)^p\right)\,dx = \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{(ep)\int \frac{x^{-1+n}(fx)^{3n}}{d+ex^n}\,dx}{3f}$$
$$= \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\int \frac{x^{-1+4n}}{d+ex^n}\,dx}{3f}$$
$$= \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx,x,x^n\right)}{3fn}$$
$$= \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx,x,x}{3fn}$$
$$= -\frac{p(fx)^{3n}}{9fn} - \frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn} + \frac{dpx^{-n}(fx)^{3n}}{6efn} + \frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn} + \frac{(fx)^{3n}\text{lo}}{}$$

Mathematica [A] time = 0.0766561, size = 92, normalized size = 0.65

$$\frac{x^{-3n}(fx)^{3n}\left(6e^3x^{3n}\log\left(c\left(d+ex^n\right)^p\right)-epx^n\left(6d^2-3dex^n+2e^2x^{2n}\right)+6d^3p\log\left(d+ex^n\right)\right)}{18e^3fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] ((f*x)^(3*n)*(-(e*p*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))) + 6*d^3*p*Log[
d + e*x^n] + 6*e^3*x^(3*n)*Log[c*(d + e*x^n)^p]))/(18*e^3*f*n*x^(3*n))
```

Maple [F] time = 1.82, size = 0, normalized size = 0.

$$\int \left(fx\right)^{-1+3n}\ln\left(c\left(d+ex^n\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17161, size = 254, normalized size = 1.8

$$\frac{3\,de^2 f^{3\,n-1}px^{2\,n} - 6\,d^2 e f^{3\,n-1}px^n - 2\left(e^3 p - 3\,e^3 \log\left(c\right)\right)f^{3\,n-1}x^{3\,n} + 6\left(e^3 f^{3\,n-1}px^{3\,n} + d^3 f^{3\,n-1}p\right)\log\left(ex^n + d\right)}{18\,e^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/18*(3*d*e^2*f^(3*n - 1)*p*x^(2*n) - 6*d^2*e*f^(3*n - 1)*p*x^n - 2*(e^3*p
- 3*e^3*log(c))*f^(3*n - 1)*x^(3*n) + 6*(e^3*f^(3*n - 1)*p*x^(3*n) + d^3*f^
(3*n - 1)*p)*log(e*x^n + d))/(e^3*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{3\,n-1}\log\left(\left(ex^n + d\right)^p c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c), x)
```

3.65 $\int (fx)^{-1+2n} \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=112

$$\frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn}-\frac{p(fx)^{2n}}{4fn}$$

```
[Out] -(p*(f*x)^(2*n))/(4*f*n) + (d*p*(f*x)^(2*n))/(2*e*f*n*x^n) - (d^2*p*(f*x)^(
2*n)*Log[d + e*x^n])/(2*e^2*f*n*x^(2*n)) + ((f*x)^(2*n)*Log[c*(d + e*x^n)^p
])/(2*f*n)
```

Rubi [A] time = 0.0571477, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 20, 266, 43}

$$\frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn}-\frac{p(fx)^{2n}}{4fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```


```
[Out] -(p*(f*x)^(2*n))/(4*f*n) + (d*p*(f*x)^(2*n))/(2*e*f*n*x^n) - (d^2*p*(f*x)^(
2*n)*Log[d + e*x^n])/(2*e^2*f*n*x^(2*n)) + ((f*x)^(2*n)*Log[c*(d + e*x^n)^p
])/(2*f*n)
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (fx)^{-1+2n}\log\left(c\left(d+ex^n\right)^p\right)dx &= \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{(ep)\int \frac{x^{-1+n}(fx)^{2n}}{d+ex^n}dx}{2f} \\ &= \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\int \frac{x^{-1+3n}}{d+ex^n}dx}{2f} \\ &= \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\text{Subst}\left(\int \frac{x^2}{d+ex}dx,x,x^n\right)}{2fn} \\ &= \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx,x,x^n\right.}{2fn} \\ &= -\frac{p(fx)^{2n}}{4fn} + \frac{dpx^{-n}(fx)^{2n}}{2efn} - \frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn} + \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right.}{2fn}\end{aligned}$$

Mathematica [A] time = 0.0407131, size = 74, normalized size = 0.66

$$-\frac{x^{-2n}(fx)^{2n}\left(ex^n\left(-2ex^n\log\left(c\left(d+ex^n\right)^p\right)-2dp+epx^n\right)+2d^2p\log\left(d+ex^n\right)\right)}{4e^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((f*x)^(2*n)*(2*d^2*p*Log[d + e*x^n] + e*x^n*(-2*d*p + e*p*x^n - 2*e*x^n*L
og[c*(d + e*x^n)^p])))/(4*e^2*f*n*x^(2*n))
```

Maple [F] time = 1.887, size = 0, normalized size = 0.

$$\int (fx)^{-1+2n}\ln\left(c\left(d+ex^n\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09319, size = 204, normalized size = 1.82

$$\frac{2\,def^{2\,n-1}px^{n} - \left(e^{2}p - 2\,e^{2}\log\left(c\right)\right)f^{2\,n-1}x^{2\,n} + 2\,\left(e^{2}f^{2\,n-1}px^{2\,n} - d^{2}f^{2\,n-1}p\right)\log\left(ex^{n} + d\right)}{4\,e^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*f^(2*n - 1)*p*x^n - (e^2*p - 2*e^2*log(c))*f^(2*n - 1)*x^(2*n) +
 2*(e^2*f^(2*n - 1)*p*x^(2*n) - d^2*f^(2*n - 1)*p)*log(e*x^n + d))/(e^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{2\,n-1}\log\left(\left(ex^{n} + d\right)^{p}c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c), x)
```

3.66 $\int (fx)^{-1+n} \log\left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=69

$$\frac{(fx)^n \log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{dpx^{-n}(fx)^n \log\left(d+ex^n\right)}{efn}-\frac{p(fx)^n}{fn}$$

```
[Out] -((p*(f*x)^n)/(f*n)) + (d*p*(f*x)^n*Log[d + e*x^n])/(e*f*n*x^n) + ((f*x)^n*
Log[c*(d + e*x^n)^p])/(f*n)
```

Rubi [A] time = 0.0428553, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2455, 20, 266, 43}

$$\frac{(fx)^n \log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{dpx^{-n}(fx)^n \log\left(d+ex^n\right)}{efn}-\frac{p(fx)^n}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```



```
[Out] -((p*(f*x)^n)/(f*n)) + (d*p*(f*x)^n*Log[d + e*x^n])/(e*f*n*x^n) + ((f*x)^n*
Log[c*(d + e*x^n)^p])/(f*n)
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (fx)^{-1+n}\log\left(c\left(d+ex^n\right)^p\right)dx = \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(ep)\int\frac{x^{-1+n}(fx)^n}{d+ex^n}dx}{f}$$
$$= \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\int\frac{x^{-1+2n}}{d+ex^n}dx}{f}$$
$$= \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\text{Subst}\left(\int\frac{x}{d+ex}dx,x,x^n\right)}{fn}$$
$$= \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\text{Subst}\left(\int\left(\frac{1}{e}-\frac{d}{e(d+ex)}\right)dx,x,x^n\right)}{fn}$$
$$= -\frac{p(fx)^n}{fn} + \frac{dpx^{-n}(fx)^n\log\left(d+ex^n\right)}{efn} + \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Mathematica [A] time = 0.0326439, size = 48, normalized size = 0.7

$$\frac{x^{1-n}(fx)^{n-1}\left(\frac{\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{e}-px^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x^(1 - n)*(f*x)^(-1 + n)*(-(p*x^n) + ((d + e*x^n)*Log[c*(d + e*x^n)^p])/e)
)/n
```

Maple [F] time = 1.869, size = 0, normalized size = 0.

$$\int \left(fx\right)^{-1+n}\ln\left(c\left(d+ex^n\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01271, size = 127, normalized size = 1.84

$$-\frac{\left(ep-e\log\left(c\right)\right)f^{n-1}x^{n}-\left(ef^{n-1}px^{n}+df^{n-1}p\right)\log\left(ex^{n}+d\right)}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -((e*p - e*log(c))*f^(n - 1)*x^n - (e*f^(n - 1)*p*x^n + d*f^(n - 1)*p)*log(
e*x^n + d))/(e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{n-1}\log\left(\left(ex^{n}+d\right)^{p}c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c), x)
```

3.67 $\int \frac{\log\left(c(d+ex^n)^p\right)}{fx}\,dx$

Optimal. Leaf size=50

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/
d])/(f*n)
```

Rubi [A] time = 0.0467335, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {12, 2454, 2394, 2315}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/
d])/(f*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log \left(c \left(d+e x^n\right)^p\right)}{f x} \, dx = \frac{\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x} dx}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x} dx, x, x^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex} dx, x, x^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}$$

Mathematica [A] time = 0.0080784, size = 46, normalized size = 0.92

$$\frac{p\text{PolyLog}\left(2, \frac{d+ex^n}{d}\right) + \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/(f*n
)
```

Maple [C] time = 2.547, size = 201, normalized size = 4.

$$\frac{\ln(x)\ln\left(\left(d+ex^n\right)^p\right)}{f} + \frac{\frac{i}{2}\ln(x)\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\left(\text{csgn}\left(ic\left(d+ex^n\right)^p\right)\right)^2}{f} - \frac{\frac{i}{2}\ln(x)\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\text{csgn}\left(ic\,(d\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/f/x,x)
```

```
[Out] 1/f*ln(x)*ln((d+e*x^n)^p)+1/2*I/f*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+
e*x^n)^p)^2-1/2*I/f*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn
(I*c)-1/2*I/f*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I/f*ln(x)*Pi*csgn(I*c*(d
+e*x^n)^p)^2*csgn(I*c)+1/f*ln(c)*ln(x)-1/f*p/n*dilog((d+e*x^n)/d)-1/f*p*ln(
x)*ln((d+e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,dnp\int \frac{\log(x)}{exx^n+dx}\,dx - np\log(x)^2 + 2\log\left(\left(ex^n+d\right)^p\right)\log(x) + 2\log(c)\log(x)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - n*p*log(x)^2 + 2*log((e
*x^n + d)^p)*log(x) + 2*log(c)*log(x))/f
```

Fricas [A] time = 2.07143, size = 155, normalized size = 3.1

$$\frac{np\log\left(ex^{n}+d\right)\log\left(x\right)-np\log\left(x\right)\log\left(\frac{ex^{n}+d}{d}\right)+n\log\left(c\right)\log\left(x\right)-p\mathrm{Li}_{2}\left(-\frac{ex^{n}+d}{d}+1\right)}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/(f*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\log\left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/f/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)/f
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(ex^{n}+d\right)^{p}c\right)}{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(f*x), x)
```

3.68 $\int (fx)^{-1-n} \log \left(c \left(d+e x^n\right)^p\right) \, dx$

Optimal. Leaf size=80

$$-\frac{(fx)^{-n} \log \left(c \left(d+e x^n\right)^p\right)}{f n}+\frac{e p x^n \log (x) (fx)^{-n}}{d f}-\frac{e p x^n (fx)^{-n} \log \left(d+e x^n\right)}{d f n}$$

```
[Out] (e*p*x^n*Log[x])/(d*f*(f*x)^n) - (e*p*x^n*Log[d + e*x^n])/(d*f*n*(f*x)^n) -
 Log[c*(d + e*x^n)^p]/(f*n*(f*x)^n)
```

Rubi [A] time = 0.0356827, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2455, 19, 266, 36, 29, 31}

$$-\frac{(fx)^{-n} \log \left(c \left(d+e x^n\right)^p\right)}{f n}+\frac{e p x^n \log (x) (fx)^{-n}}{d f}-\frac{e p x^n (fx)^{-n} \log \left(d+e x^n\right)}{d f n}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (e*p*x^n*Log[x])/(d*f*(f*x)^n) - (e*p*x^n*Log[d + e*x^n])/(d*f*n*(f*x)^n) -
 Log[c*(d + e*x^n)^p]/(f*n*(f*x)^n)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int (fx)^{-1-n}\log\left(c\left(d+ex^n\right)^p\right)\,dx &= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{(ep)\int\frac{x^{-1+n}(fx)^{-n}}{d+ex^n}\,dx}{f}\\
&= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\int\frac{1}{x(d+ex^n)}\,dx}{f}\\
&= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\operatorname{Subst}\left(\int\frac{1}{x(d+ex)}\,dx,x,x^n\right)}{fn}\\
&= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\operatorname{Subst}\left(\int\frac{1}{x}\,dx,x,x^n\right)}{dfn}-\frac{\left(e^2px^n(fx)^{-n}\right)\operatorname{Sub}}{d}\\
&= \frac{epx^n(fx)^{-n}\log(x)}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}-\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}
\end{aligned}$$

Mathematica [A] time = 0.0175413, size = 57, normalized size = 0.71

$$-\frac{(fx)^{-n}\left(d\log\left(c\left(d+ex^n\right)^p\right)+epx^n\log\left(d+ex^n\right)-enpx^n\log(x)\right)}{dfn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((-(e*n*p*x^n*Log[x]) + e*p*x^n*Log[d + e*x^n] + d*Log[c*(d + e*x^n)^p])/(
d*f*n*(f*x)^n))
```

Maple [F] time = 1.819, size = 0, normalized size = 0.

$$\int (fx)^{-1-n}\ln\left(c\left(d+ex^n\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0769, size = 159, normalized size = 1.99

$$\frac{ef^{-n-1}npx^n\log(x) - df^{-n-1}\log(c) - \left(ef^{-n-1}px^n + df^{-n-1}p\right)\log(ex^n + d)}{dnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] (e*f^(-n - 1)*n*p*x^n*log(x) - d*f^(-n - 1)*log(c) - (e*f^(-n - 1)*p*x^n +
d*f^(-n - 1)*p)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^{-n-1}\log\left((ex^n + d)^p c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c), x)
```

3.69 $\int (fx)^{-1-2n} \log \left(c \left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=120

$$-\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{e^2px^{2n}\log(x)(fx)^{-2n}}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{epx^n(fx)^{-2n}}{2dfn}$$

```
[Out] -(e*p*x^n)/(2*d*f*n*(f*x)^(2*n)) - (e^2*p*x^(2*n)*Log[x])/(2*d^2*f*(f*x)^(2
*n)) + (e^2*p*x^(2*n)*Log[d + e*x^n])/(2*d^2*f*n*(f*x)^(2*n)) - Log[c*(d +
e*x^n)^p]/(2*f*n*(f*x)^(2*n))
```

Rubi [A] time = 0.0578244, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 20, 266, 44}

$$-\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{e^2px^{2n}\log(x)(fx)^{-2n}}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{epx^n(fx)^{-2n}}{2dfn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(e*p*x^n)/(2*d*f*n*(f*x)^(2*n)) - (e^2*p*x^(2*n)*Log[x])/(2*d^2*f*(f*x)^(2
*n)) + (e^2*p*x^(2*n)*Log[d + e*x^n])/(2*d^2*f*n*(f*x)^(2*n)) - Log[c*(d +
e*x^n)^p]/(2*f*n*(f*x)^(2*n))
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int (fx)^{-1-2n}\log\left(c\left(d+ex^n\right)^p\right)dx = -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{(ep)\int\frac{x^{-1+n}(fx)^{-2n}}{d+ex^n}dx}{2f}$$
$$= -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\int\frac{x^{-1-n}}{d+ex^n}dx}{2f}$$
$$= -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\text{Subst}\left(\int\frac{1}{x^2(d+ex)}dx,x,x^n\right)}{2fn}$$
$$= -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\text{Subst}\left(\int\left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)dx,x\right.}{2fn}$$
$$= -\frac{epx^n(fx)^{-2n}}{2dfn}-\frac{e^2px^{2n}(fx)^{-2n}\log(x)}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{(fx)^{-2n}\log}{}$$

Mathematica [A] time = 0.0481357, size = 76, normalized size = 0.63

$$-\frac{(fx)^{-2n}\left(d\left(d\log\left(c\left(d+ex^n\right)^p\right)+epx^n\right)-e^2px^{2n}\log\left(d+ex^n\right)+e^2npx^{2n}\log(x)\right)}{2d^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(e^2*n*p*x^(2*n)*Log[x] - e^2*p*x^(2*n)*Log[d + e*x^n] + d*(e*p*x^n + d*Lo
g[c*(d + e*x^n)^p]))/(2*d^2*f*n*(f*x)^(2*n))
```

Maple [F] time = 1.854, size = 0, normalized size = 0.

$$\int (fx)^{-1-2n}\ln\left(c\left(d+ex^n\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07245, size = 240, normalized size = 2.

$$-\frac{e^{2}f^{-2\,n-1}npx^{2\,n}\log\left(x\right)+def^{-2\,n-1}px^{n}+d^{2}f^{-2\,n-1}\log\left(c\right)-\left(e^{2}f^{-2\,n-1}px^{2\,n}-d^{2}f^{-2\,n-1}p\right)\log\left(ex^{n}+d\right)}{2\,d^{2}nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(e^2*f^(-2*n - 1)*n*p*x^(2*n)*log(x) + d*e*f^(-2*n - 1)*p*x^n + d^2*f^
(-2*n - 1)*log(c) - (e^2*f^(-2*n - 1)*p*x^(2*n) - d^2*f^(-2*n - 1)*p)*log(e
*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{-2\,n-1}\log\left(\left(ex^{n}+d\right)^{p}c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c), x)
```

3.70 $\int x^2 \log\left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=65

$$\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+3}\,_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(n+3)}$$

```
[Out] -(e*n*p*x^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(
3*d*(3 + n)) + (x^3*Log[c*(d + e*x^n)^p])/3
```

Rubi [A] time = 0.0280675, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+3}\,_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(e*n*p*x^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(
3*d*(3 + n)) + (x^3*Log[c*(d + e*x^n)^p])/3
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(c\left(d+ex^n\right)^p\right)dx&=\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)-\frac{1}{3}(enp)\int\frac{x^{2+n}}{d+ex^n}\,dx\\&=-\frac{enpx^{3+n}\,_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3+n)}+\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.0314956, size = 61, normalized size = 0.94

$$\frac{1}{3}x^3\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{d(n+3)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x^3*(-((e*n*p*x^n*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/
(d*(3 + n))) + Log[c*(d + e*x^n)^p]))/3
```

Maple [F] time = 1.69, size = 0, normalized size = 0.

$$\int x^2 \ln\left(c\,(d+ex^n)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int(x^2*ln(c*(d+e*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{9}\left(np - 3\,\log\left(c\right)\right)x^3 + dnp\int \frac{x^2}{3\,(ex^n + d)}\,dx + \frac{1}{3}\,x^3\log\left((ex^n + d)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/9*(n*p - 3*log(c))*x^3 + d*n*p*integrate(1/3*x^2/(e*x^n + d), x) + 1/3*x
^3*log((e*x^n + d)^p)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\log\left((ex^n + d)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((e*x^n + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \log\left((ex^n + d)^p c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((e*x^n + d)^p*c), x)
```

3.71 $\int x \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}$$

```
[Out] -(e*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((e*x^n)
/d)])/(2*d*(2 + n)) + (x^2*Log[c*(d + e*x^n)^p])/2
```

Rubi [A] time = 0.0241193, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2455, 364}

$$\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(e*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((e*x^n)
/d)])/(2*d*(2 + n)) + (x^2*Log[c*(d + e*x^n)^p])/2
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\log\left(c\left(d+ex^n\right)^p\right)dx&=\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{1}{2}(enp)\int\frac{x^{1+n}}{d+ex^n}\,dx\\&=-\frac{enpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}+\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.0320371, size = 61, normalized size = 0.94

$$\frac{1}{2}x^2\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,{}_2F_1\left(1,\frac{n+2}{n};2+\frac{2}{n};-\frac{ex^n}{d}\right)}{d(n+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x^2*(-((e*n*p*x^n*Hypergeometric2F1[1, (2 + n)/n, 2 + 2/n, -((e*x^n)/d)])/
(d*(2 + n))) + Log[c*(d + e*x^n)^p]))/2
```

Maple [F] time = 1.892, size = 0, normalized size = 0.

$$\int x \ln \left(c \left(d + e x^{n}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int(x*ln(c*(d+e*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$dnp \int \frac{x}{2\,(ex^{n} + d)}\,dx - \frac{1}{4}\left(np - 2\,\log\left(c\right)\right)x^{2} + \frac{1}{2}\,x^{2}\log\left(\left(ex^{n} + d\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(1/2*x/(e*x^n + d), x) - 1/4*(n*p - 2*log(c))*x^2 + 1/2*x^2*
log((e*x^n + d)^p)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x \log\left(\left(ex^{n} + d\right)^{p} c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(x*log((e*x^n + d)^p*c), x)
```

Sympy [C] time = 38.3548, size = 104, normalized size = 1.6

$$\frac{x^{2}\log\left(c\left(d + ex^{n}\right)^{p}\right)}{2} - \frac{epx^{2}x^{n}\Phi\left(\frac{ex^{n}e^{i\pi}}{d}, 1, 1 + \frac{2}{n}\right)\Gamma\left(1 + \frac{2}{n}\right)}{2d\Gamma\left(2 + \frac{2}{n}\right)} - \frac{epx^{2}x^{n}\Phi\left(\frac{ex^{n}e^{i\pi}}{d}, 1, 1 + \frac{2}{n}\right)\Gamma\left(1 + \frac{2}{n}\right)}{dn\Gamma\left(2 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(d+e*x**n)**p),x)
```

```
[Out] x**2*log(c*(d + e*x**n)**p)/2 - e*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*p
i)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - e*p*x**2*x**n*lerch
phi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamma(2 + 2/n
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \log \left((ex^n + d)^p c \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(x*log((e*x^n + d)^p*c), x)
```

3.72 $\int \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=54

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rubi [A] time = 0.0202858, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 364}

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \log\left(c\left(d+ex^n\right)^p\right)\,dx &= x\log\left(c\left(d+ex^n\right)^p\right)-(enp)\int \frac{x^n}{d+ex^n}\,dx\\ &= -\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.029246, size = 52, normalized size = 0.96

$$x\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
```

```
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Maple [F] time = 1.6, size = 0, normalized size = 0.

$$\int \ln \left(c \left(d+e x^{n}\right)^{p}\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$dnp \int \frac{1}{ex^{n}+d}\, dx - \left(np - \log\left(c\right)\right)x + x \log\left(\left(ex^{n}+d\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\left(ex^{n}+d\right)^{p} c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c), x)
```

Sympy [C] time = 4.67734, size = 48, normalized size = 0.89

$$x \log\left(c\left(d+ex^{n}\right)^{p}\right) + \frac{px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p),x)
```

```
[Out] x*log(c*(d + e*x**n)**p) + p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1, exp
_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left((ex^n + d)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c), x)
```

3.73 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rubi [A] time = 0.0398748, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A] time = 0.0029561, size = 43, normalized size = 0.98

$$\frac{p\text{PolyLog}\left(2, \frac{d+ex^n}{d}\right) + \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Maple [C] time = 2.52, size = 177, normalized size = 4.

$$\ln(x)\ln\left(\left(d+ex^n\right)^p\right) + \frac{i}{2}\ln(x)\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\left(\text{csgn}\left(ic\left(d+ex^n\right)^p\right)\right)^2 - \frac{i}{2}\ln(x)\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\text{csgn}\left(ic\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] ln(x)*ln((d+e*x^n)^p)+1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)^2-1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/
2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^2
*csgn(I*c)+ln(c)*ln(x)-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$dnp\int \frac{\log(x)}{exx^n + dx}dx - \frac{1}{2}np\log(x)^2 + \log\left(\left(ex^n+d\right)^p\right)\log(x) + \log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
+ d)^p)*log(x) + log(c)*log(x)
```

Fricas [A] time = 2.04783, size = 150, normalized size = 3.41

$$\frac{np\log\left(ex^n+d\right)\log(x) - np\log(x)\log\left(\frac{ex^n+d}{d}\right) + n\log(c)\log(x) - p\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(ex^{n}+d\right)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x, x)
```

3.74 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}-\frac{enpx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}$$

```
[Out] -((e*n*p*x^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((e*x^n
)/d)])/(d*(1 - n))) - Log[c*(d + e*x^n)^p]/x
```

Rubi [A] time = 0.0315251, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}-\frac{enpx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^2,x]
```

```
[Out] -((e*n*p*x^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((e*x^n
)/d)])/(d*(1 - n))) - Log[c*(d + e*x^n)^p]/x
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x^2}\,dx &= -\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}+(enp)\int \frac{x^{-2+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0321867, size = 59, normalized size = 0.89

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{n-1}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n-1)}-\log\left(c\left(d+ex^n\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^2,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-1 + n)/n, 2 - n^(-1), -((e*x^n)/d)])/(d*
(-1 + n)) - Log[c*(d + e*x^n)^p])/x
```

Maple [F] time = 1.523, size = 0, normalized size = 0.

$$\int \frac{\ln \left(c \left(d+e x^{n}\right)^{p}\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x^2,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-dnp \int \frac{1}{ex^{2}x^{n}+dx^{2}}\, dx - \frac{np+\log \left(\left(ex^{n}+d\right)^{p}\right)+\log \left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/(e*x^2*x^n + d*x^2), x) - (n*p + log((e*x^n + d)^p) + lo
g(c))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log \left(\left(ex^{n}+d\right)^{p}c\right)}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/x^2, x)
```

Sympy [C] time = 14.7649, size = 46, normalized size = 0.7

$$-\frac{\log \left(c \left(d+ex^{n}\right)^{p}\right)}{x} + \frac{p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{1}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**2,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/x + p*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1, 1/n)
*gamma(-1/n)/(n*x*gamma(1 - 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x^2, x)
```

3.75 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x^3}\,dx$

Optimal. Leaf size=72

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}-\frac{enpx^{n-2}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}$$

```
[Out] -(e*n*p*x^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)), -((e*
x^n)/d)])/(2*d*(2 - n)) - Log[c*(d + e*x^n)^p]/(2*x^2)
```

Rubi [A] time = 0.0307312, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}-\frac{enpx^{n-2}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^3,x]
```

```
[Out] -(e*n*p*x^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)), -((e*
x^n)/d)])/(2*d*(2 - n)) - Log[c*(d + e*x^n)^p]/(2*x^2)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x^3}\,dx &= -\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}+\frac{1}{2}(enp)\int \frac{x^{-3+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-2+n}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0266078, size = 62, normalized size = 0.86

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{n-2}{n};2-\frac{2}{n};-\frac{ex^n}{d}\right)}{d(n-2)}-\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^3,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-2 + n)/n, 2 - 2/n, -((e*x^n)/d)])/(d*(-2
 + n)) - Log[c*(d + e*x^n)^p])/(2*x^2)
```

Maple [F] time = 1.554, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\,(d+ex^n)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x^3,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-dnp\int \frac{1}{2\left(ex^3x^n+dx^3\right)}\,dx-\frac{np+2\,\log\left((ex^n+d)^p\right)+2\,\log\left(c\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/2/(e*x^3*x^n + d*x^3), x) - 1/4*(n*p + 2*log((e*x^n + d)
^p) + 2*log(c))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left((ex^n+d)^pc\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x^3, x)
```

3.76 $\int \frac{\log(c(d+ex^n)^p)}{x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}-\frac{enpx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}$$

```
[Out] -(e*n*p*x^(-3 + n)*Hypergeometric2F1[1, -((3 - n)/n), 2 - 3/n, -((e*x^n)/d)
])/(3*d*(3 - n)) - Log[c*(d + e*x^n)^p]/(3*x^3)
```

Rubi [A] time = 0.0293658, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}-\frac{enpx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^4,x]
```

```
[Out] -(e*n*p*x^(-3 + n)*Hypergeometric2F1[1, -((3 - n)/n), 2 - 3/n, -((e*x^n)/d)
])/(3*d*(3 - n)) - Log[c*(d + e*x^n)^p]/(3*x^3)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x^4}\,dx &= -\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}+\frac{1}{3}(enp)\int \frac{x^{-4+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0263766, size = 62, normalized size = 0.89

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{n-3}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{d(n-3)}-\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^4,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-3 + n)/n, 2 - 3/n, -((e*x^n)/d)])/(d*(-3
 + n)) - Log[c*(d + e*x^n)^p])/(3*x^3)
```

Maple [F] time = 1.529, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(d+e x^{n}\right)^{p}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x^4,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-dnp\int \frac{1}{3\left(e x^{4} x^{n}+d x^{4}\right)}\,dx-\frac{np+3\,\log\left(\left(e x^{n}+d\right)^{p}\right)+3\,\log\left(c\right)}{9\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/3/(e*x^4*x^n + d*x^4), x) - 1/9*(n*p + 3*log((e*x^n + d)
^p) + 3*log(c))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(e x^{n}+d\right)^{p} c\right)}{x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x^4, x)
```

3.77 $\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=215

$$-\frac{a^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}+\frac{a^3p\log\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^3}+\frac{a^2p^2x^2}{b^2}-\frac{a^3p^2\log^2\left(a+bx^2\right)}{6b^3}-\frac{p\left(a+bx^2\right)^3\log}{9}$$

```
[Out] (a^2*p^2*x^2)/b^2 - (a*p^2*(a + b*x^2)^2)/(4*b^3) + (p^2*(a + b*x^2)^3)/(27
*b^3) - (a^3*p^2*Log[a + b*x^2]^2)/(6*b^3) - (a^2*p*(a + b*x^2)*Log[c*(a +
b*x^2)^p])/b^3 + (a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(2*b^3) - (p*(a +
 b*x^2)^3*Log[c*(a + b*x^2)^p])/(9*b^3) + (a^3*p*Log[a + b*x^2]*Log[c*(a +
b*x^2)^p])/(3*b^3) + (x^6*Log[c*(a + b*x^2)^p]^2)/6
```

Rubi [A] time = 0.298564, antiderivative size = 175, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3}-\frac{6a^3\log\left(a+bx^2\right)}{b^3}-\frac{9a\left(a+bx^2\right)^2}{b^3}+\frac{2\left(a+bx^2\right)^3}{b^3}\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{a^2p^2x^2}{b^2}-\frac{a^3p^2\log^2\left(a+\right.}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (a^2*p^2*x^2)/b^2 - (a*p^2*(a + b*x^2)^2)/(4*b^3) + (p^2*(a + b*x^2)^3)/(27
*b^3) - (a^3*p^2*Log[a + b*x^2]^2)/(6*b^3) - (p*((18*a^2*(a + b*x^2))/b^3 -
 (9*a*(a + b*x^2)^2)/b^3 + (2*(a + b*x^2)^3)/b^3 - (6*a^3*Log[a + b*x^2])/b
^3)*Log[c*(a + b*x^2)^p])/18 + (x^6*Log[c*(a + b*x^2)^p]^2)/6
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{2}\operatorname{Subst}\left(\int x^2 \log^2\left(c(a+bx)^p\right) dx, x, x^2\right)$$

$$= \frac{1}{6}x^6 \log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(bp)\operatorname{Subst}\left(\int \frac{x^3 \log\left(c(a+bx)^p\right)}{a+bx} dx, x, x^2\right)$$

$$= \frac{1}{6}x^6 \log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}p\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \log\left(cx^p\right)}{x} dx, x, a+bx^2\right)$$

$$= -\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)^2}{b^3} + \frac{2\left(a+bx^2\right)^3}{b^3} - \frac{6a^3\log\left(a+bx^2\right)}{b^3}\right)\log\left(c\left(a+b\right.\right.$$

$$= -\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)^2}{b^3} + \frac{2\left(a+bx^2\right)^3}{b^3} - \frac{6a^3\log\left(a+bx^2\right)}{b^3}\right)\log\left(c\left(a+b\right.\right.$$

$$= -\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)^2}{b^3} + \frac{2\left(a+bx^2\right)^3}{b^3} - \frac{6a^3\log\left(a+bx^2\right)}{b^3}\right)\log\left(c\left(a+b\right.\right.$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2\left(a+bx^2\right)^2}{4b^3} + \frac{p^2\left(a+bx^2\right)^3}{27b^3} - \frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)^2}{b^3} + \frac{2\left(a\right.}{}\right.$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2\left(a+bx^2\right)^2}{4b^3} + \frac{p^2\left(a+bx^2\right)^3}{27b^3} - \frac{a^3p^2\log^2\left(a+bx^2\right)}{6b^3} - \frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3}\right.$$

Mathematica [A] time = 0.0665617, size = 200, normalized size = 0.93

$$\frac{a^3\log^2\left(c\left(a+bx^2\right)^p\right)}{6b^3}-\frac{a^2px^2\log\left(c\left(a+bx^2\right)^p\right)}{3b^2}-\frac{a^3p\log\left(c\left(a+bx^2\right)^p\right)}{3b^3}+\frac{11a^2p^2x^2}{18b^2}-\frac{5a^3p^2\log\left(a+bx^2\right)}{18b^3}+\frac{1}{6}x^6\log^2\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (11*a^2*p^2*x^2)/(18*b^2) - (5*a*p^2*x^4)/(36*b) + (p^2*x^6)/27 - (5*a^3*p^
2*Log[a + b*x^2])/(18*b^3) - (a^3*p*Log[c*(a + b*x^2)^p])/(3*b^3) - (a^2*p*
x^2*Log[c*(a + b*x^2)^p])/(3*b^2) + (a*p*x^4*Log[c*(a + b*x^2)^p])/(6*b) -
(p*x^6*Log[c*(a + b*x^2)^p])/9 + (a^3*Log[c*(a + b*x^2)^p]^2)/(6*b^3) + (x^
6*Log[c*(a + b*x^2)^p]^2)/6
```

Maple [C] time = 0.519, size = 1436, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] 1/6*x^6*ln((b*x^2+a)^p)^2-1/9*ln(c)*p*x^6-1/24*Pi^2*x^6*csgn(I*c*(b*x^2+a)^
p)^6+11/18*a^2*p^2*x^2/b^2+1/18*(3*I*Pi*b^3*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*
c*(b*x^2+a)^p)^2-3*I*Pi*b^3*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*c
sgn(I*c)-3*I*Pi*b^3*x^6*csgn(I*c*(b*x^2+a)^p)^3+3*I*Pi*b^3*x^6*csgn(I*c*(b*
x^2+a)^p)^2*csgn(I*c)+6*ln(c)*b^3*x^6-2*b^3*p*x^6+3*a*b^2*p*x^4-6*a^2*b*p*x
^2+6*a^3*p*ln(b*x^2+a))/b^3*ln((b*x^2+a)^p)+1/6*ln(c)^2*x^6-1/6*a^3*p^2*ln(
b*x^2+a)^2/b^3+1/27*p^2*x^6-11/18*a^3*p^2/b^3*ln(b*x^2+a)-5/36/b*a*p^2*x^4-
1/24*Pi^2*x^6*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+1/12*Pi^2*x^6*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+1/12*Pi^2*x^6*csgn(I*c*(b*x^2+a)
^p)^5*csgn(I*c)-1/24*Pi^2*x^6*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+1/12*I/b*
Pi*a*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*c*(b*x^2
+a)^p)^2*csgn(I*c)+1/6*I/b^3*Pi*ln(b*x^2+a)*a^3*p*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2+1/18*I*Pi*p*x^6*csgn(I*c*(b*x^2+a)^p)^3+1/6/b*ln(c)*a*p*
x^4-1/3/b^2*ln(c)*a^2*p*x^2+1/3/b^3*ln(c)*ln(b*x^2+a)*a^3*p+1/12*Pi^2*x^6*c
sgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-1/24*Pi^2*x^6*csgn(I
*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-1/6*Pi^2*x^6*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+1/12*Pi^2*x^6*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-1/6*I*ln(c)*Pi*x^6*csgn(I*c*(b*x^2+a
)^p)^3+1/6*I/b^3*Pi*ln(b*x^2+a)*a^3*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/1
2*I/b*Pi*a*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/12*I/b*Pi*a*
p*x^4*csgn(I*c*(b*x^2+a)^p)^3+1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^
3-1/6*I/b^3*Pi*ln(b*x^2+a)*a^3*p*csgn(I*c*(b*x^2+a)^p)^3+1/6*I*ln(c)*Pi*x^6
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/6*I*ln(c)*Pi*x^6*csgn(I*c*(b
*x^2+a)^p)^2*csgn(I*c)-1/18*I*Pi*p*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)^2-1/18*I*Pi*p*x^6*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/12*I/b*Pi*a*p*x
^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/6*I/b^2*Pi*a^2*p*x
^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*I/b^3*Pi*ln(b*x^
2+a)*a^3*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*I*ln(c)*
Pi*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/18*I*Pi*p*x^6*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)
```

Maxima [A] time = 1.06111, size = 196, normalized size = 0.91

$$\frac{1}{6}x^6\log\left(\left(bx^2+a\right)^pc\right)^2+\frac{1}{18}bp\left(\frac{6a^3\log\left(bx^2+a\right)}{b^4}-\frac{2b^2x^6-3abx^4+6a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^pc\right)+\frac{\left(4b^3x^6-15ab^2x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^2 + 1/18*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c) + 1/108*(4*b^3*x^6
 - 15*a*b^2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2
 + a))*p^2/b^3
```

Fricas [A] time = 2.08301, size = 412, normalized size = 1.92

$$\frac{4b^3p^2x^6+18b^3x^6\log(c)^2-15ab^2p^2x^4+66a^2bp^2x^2+18\left(b^3p^2x^6+a^3p^2\right)\log\left(bx^2+a\right)^2-6\left(2b^3p^2x^6-3ab^2p^2x^4+\right.}{108b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/108*(4*b^3*p^2*x^6 + 18*b^3*x^6*log(c)^2 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^
2*x^2 + 18*(b^3*p^2*x^6 + a^3*p^2)*log(b*x^2 + a)^2 - 6*(2*b^3*p^2*x^6 - 3*
a*b^2*p^2*x^4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2 - 6*(b^3*p*x^6 + a^3*p)*log(c)
)*log(b*x^2 + a) - 6*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2)*log(c))/
b^3
```

Sympy [A] time = 87.5571, size = 267, normalized size = 1.24

$$\begin{cases}\frac{a^3p^2\log\left(a+bx^2\right)^2}{6b^3}-\frac{11a^3p^2\log\left(a+bx^2\right)}{18b^3}+\frac{a^3p\log(c)\log\left(a+bx^2\right)}{3b^3}-\frac{a^2p^2x^2\log\left(a+bx^2\right)}{3b^2}+\frac{11a^2p^2x^2}{18b^2}-\frac{a^2px^2\log(c)}{3b^2}+\frac{ap^2x^4\log\left(a+bx^2\right)}{6b}-\frac{5ap^2}{36b} \\ \frac{x^6\log\left(a^pc\right)^2}{6}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((a**3*p**2*log(a + b*x**2)**2/(6*b**3) - 11*a**3*p**2*log(a + b*x
**2)/(18*b**3) + a**3*p*log(c)*log(a + b*x**2)/(3*b**3) - a**2*p**2*x**2*lo
g(a + b*x**2)/(3*b**2) + 11*a**2*p**2*x**2/(18*b**2) - a**2*p*x**2*log(c)/(
3*b**2) + a*p**2*x**4*log(a + b*x**2)/(6*b) - 5*a*p**2*x**4/(36*b) + a*p*x*
*4*log(c)/(6*b) + p**2*x**6*log(a + b*x**2)**2/6 - p**2*x**6*log(a + b*x**2
)/9 + p**2*x**6/27 + p*x**6*log(c)*log(a + b*x**2)/3 - p*x**6*log(c)/9 + x*
*6*log(c)**2/6, Ne(b, 0)), (x**6*log(a**p*c)**2/6, True))
```

Giac [A] time = 1.27299, size = 439, normalized size = 2.04

$$18bx^6\log(c)^2+\left(\frac{18\left(bx^2+a\right)^3\log\left(bx^2+a\right)^2}{b^2}-\frac{54\left(bx^2+a\right)^2a\log\left(bx^2+a\right)^2}{b^2}+\frac{54\left(bx^2+a\right)a^2\log\left(bx^2+a\right)^2}{b^2}-\frac{12\left(bx^2+a\right)^3\log\left(bx^2+a\right)}{b^2}+\frac{54\left(bx^2+a\right)^2a}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/108*(18*b*x^6*log(c)^2 + (18*(b*x^2 + a)^3*log(b*x^2 + a)^2/b^2 - 54*(b*x
^2 + a)^2*a*log(b*x^2 + a)^2/b^2 + 54*(b*x^2 + a)*a^2*log(b*x^2 + a)^2/b^2
- 12*(b*x^2 + a)^3*log(b*x^2 + a)/b^2 + 54*(b*x^2 + a)^2*a*log(b*x^2 + a)/b
^2 - 108*(b*x^2 + a)*a^2*log(b*x^2 + a)/b^2 + 4*(b*x^2 + a)^3/b^2 - 27*(b*x
^2 + a)^2*a/b^2 + 108*(b*x^2 + a)*a^2/b^2)*p^2 + 6*(6*(b*x^2 + a)^3*log(b*x
^2 + a)/b^2 - 18*(b*x^2 + a)^2*a*log(b*x^2 + a)/b^2 + 18*(b*x^2 + a)*a^2*lo
g(b*x^2 + a)/b^2 - 2*(b*x^2 + a)^3/b^2 + 9*(b*x^2 + a)^2*a/b^2 - 18*(b*x^2
+ a)*a^2/b^2)*p*log(c))/b
```

3.78 $\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=145

$$\frac{\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{a\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}-\frac{p\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2}+\frac{ap\left(a+bx^2\right)\log\left(c\right.}{b^2}$$

```
[Out] -((a*p^2*x^2)/b) + (p^2*(a + b*x^2)^2)/(8*b^2) + (a*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/b^2 - (p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (a*(a +
 b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^
p]^2)/(4*b^2)
```

Rubi [A] time = 0.153345, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{a\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}-\frac{p\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2}+\frac{ap\left(a+bx^2\right)\log\left(c\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -((a*p^2*x^2)/b) + (p^2*(a + b*x^2)^2)/(8*b^2) + (a*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/b^2 - (p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (a*(a +
 b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^
p]^2)/(4*b^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\log^2\left(c\left(a+bx^2\right)^p\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\log^2\left(c(a+bx)^p\right)dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\log^2\left(c(a+bx)^p\right)}{b}+\frac{(a+bx)\log^2\left(c(a+bx)^p\right)}{b}\right)dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int(a+bx)\log^2\left(c(a+bx)^p\right)dx, x, x^2\right)}{2b}-\frac{a\operatorname{Subst}\left(\int\log^2\left(c(a+bx)^p\right)dx, x, x^2\right)}{2b}\\
&= \frac{\operatorname{Subst}\left(\int x\log^2\left(cx^p\right)dx, x, a+bx^2\right)}{2b^2}-\frac{a\operatorname{Subst}\left(\int\log^2\left(cx^p\right)dx, x, a+bx^2\right)}{2b^2}\\
&= -\frac{a\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}+\frac{\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{p\operatorname{Subst}\left(\int x\log\left(cx^p\right)\right.}{2b^2}\\
&= -\frac{ap^2x^2}{b}+\frac{p^2\left(a+bx^2\right)^2}{8b^2}+\frac{ap\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}-\frac{p\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right.}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.056058, size = 105, normalized size = 0.72

$$\frac{-2\left(a^2-b^2x^4\right)\log^2\left(c\left(a+bx^2\right)^p\right)+2p\left(2a^2+2abx^2-b^2x^4\right)\log\left(c\left(a+bx^2\right)^p\right)+2a^2p^2\log\left(a+bx^2\right)+bp^2x^2\left(bx^2-6a\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (b*p^2*x^2*(-6*a + b*x^2) + 2*a^2*p^2*Log[a + b*x^2] + 2*p*(2*a^2 + 2*a*b*x
^2 - b^2*x^4)*Log[c*(a + b*x^2)^p] - 2*(a^2 - b^2*x^4)*Log[c*(a + b*x^2)^p]
^2)/(8*b^2)
```

Maple [C] time = 0.492, size = 1242, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] -3/4*a*p^2*x^2/b-1/4*ln(c)*p*x^4-1/16*Pi^2*x^4*csgn(I*c*(b*x^2+a)^p)^6+1/4*
I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^3-1/4*I*ln(c)*Pi*x^4*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/8*I*Pi*p*x^4*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/4*I/b*Pi*a*p*x^2*csgn(I*c*(b*x^2+a
)^p)^3+1/4*(I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b
^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b^2*x^4*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)
*b^2*x^4-b^2*p*x^4+2*a*b*p*x^2-2*a^2*p*ln(b*x^2+a))/b^2*ln((b*x^2+a)^p)+1/4
/b^2*a^2*p^2*ln(b*x^2+a)^2-1/16*Pi^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*
x^2+a)^p)^4+1/8*Pi^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+1/8*Pi
^2*x^4*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-1/16*Pi^2*x^4*csgn(I*c*(b*x^2+a)^p
)^4*csgn(I*c)^2+1/4*ln(c)^2*x^4+3/4*a^2*p^2/b^2*ln(b*x^2+a)+1/4*x^4*ln((b*x
^2+a)^p)^2+1/8*x^4*p^2+1/2/b*ln(c)*a*p*x^2-1/2/b^2*ln(c)*ln(b*x^2+a)*a^2*p+
1/8*Pi^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-1/16*P
i^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-1/4*Pi^2*
x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+1/8*Pi^2*x^4*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-1/4*I*ln(c)*Pi*x^4*csgn
(I*c*(b*x^2+a)^p)^3+1/8*I*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^3+1/4*I*ln(c)*Pi*x
^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/4*I*ln(c)*Pi*x^4*csgn(I*c*
(b*x^2+a)^p)^2*csgn(I*c)-1/8*I*Pi*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)^2-1/8*I*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/4*I/b*Pi*a*p*x^
2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/4*I/b*Pi*a*p*x^2*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)-1/4*I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)^2-1/4*I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p
)^2*csgn(I*c)-1/4*I/b*Pi*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*
csgn(I*c)+1/4*I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)*csgn(I*c)
```

Maxima [A] time = 1.05056, size = 162, normalized size = 1.12

$$\frac{1}{4}\,x^4\log\left(\left(bx^2+a\right)^p c\right)^2-\frac{1}{4}\,bp\left(\frac{2\,a^2\log\left(bx^2+a\right)}{b^3}+\frac{bx^4-2\,ax^2}{b^2}\right)\log\left(\left(bx^2+a\right)^p c\right)+\frac{\left(b^2x^4-6\,abx^2+2\,a^2\log\left(bx^2+\right.\right.}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^2 - 1/4*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c) + 1/8*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*lo
g(b*x^2 + a)^2 + 6*a^2*log(b*x^2 + a))*p^2/b^2
```

Fricas [A] time = 1.90911, size = 316, normalized size = 2.18

$$\frac{b^2p^2x^4+2\,b^2x^4\log\left(c\right)^2-6\,abp^2x^2+2\left(b^2p^2x^4-a^2p^2\right)\log\left(bx^2+a\right)^2-2\left(b^2p^2x^4-2\,abp^2x^2-3\,a^2p^2-2\left(b^2px^4-a^2\right.\right.}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*p^2*x^4 + 2*b^2*x^4*log(c)^2 - 6*a*b*p^2*x^2 + 2*(b^2*p^2*x^4 - a^
2*p^2)*log(b*x^2 + a)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2 - 2*(b
^2*p*x^4 - a^2*p)*log(c))*log(b*x^2 + a) - 2*(b^2*p*x^4 - 2*a*b*p*x^2)*log(
c))/b^2
```

Sympy [A] time = 19.3503, size = 209, normalized size = 1.44

$$\begin{cases} -\frac{a^2p^2\log\left(a+bx^2\right)^2}{4b^2} + \frac{3a^2p^2\log\left(a+bx^2\right)}{4b^2} - \frac{a^2p\log(c)\log\left(a+bx^2\right)}{2b^2} + \frac{ap^2x^2\log\left(a+bx^2\right)}{2b} - \frac{3ap^2x^2}{4b} + \frac{apx^2\log(c)}{2b} + \frac{p^2x^4\log\left(a+bx^2\right)^2}{4} - \frac{p^2x^4\log\left(a+\right.}{4} \\ \frac{x^4\log\left(a^pc\right)^2}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((-a**2*p**2*log(a + b*x**2)**2/(4*b**2) + 3*a**2*p**2*log(a + b*x
**2)/(4*b**2) - a**2*p*log(c)*log(a + b*x**2)/(2*b**2) + a*p**2*x**2*log(a
+ b*x**2)/(2*b) - 3*a*p**2*x**2/(4*b) + a*p*x**2*log(c)/(2*b) + p**2*x**4*l
og(a + b*x**2)**2/4 - p**2*x**4*log(a + b*x**2)/4 + p**2*x**4/8 + p*x**4*lo
g(c)*log(a + b*x**2)/2 - p*x**4*log(c)/4 + x**4*log(c)**2/4, Ne(b, 0)), (x*
*4*log(a**p*c)**2/4, True))
```

Giac [A] time = 1.23159, size = 279, normalized size = 1.92

$$\frac{\frac{\left(2\left(bx^2+a\right)^2\log\left(bx^2+a\right)^2-4\left(bx^2+a\right)a\log\left(bx^2+a\right)^2-2\left(bx^2+a\right)^2\log\left(bx^2+a\right)+8\left(bx^2+a\right)a\log\left(bx^2+a\right)+\left(bx^2+a\right)^2-8\left(bx^2+a\right)a\right)p^2}{b} + \frac{2\left(2\left(bx^2+a\right)^2\log\left(bx^2+a\right)-4\left(\right.\right.}{}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/8*((2*(b*x^2 + a)^2*log(b*x^2 + a)^2 - 4*(b*x^2 + a)*a*log(b*x^2 + a)^2 -
 2*(b*x^2 + a)^2*log(b*x^2 + a) + 8*(b*x^2 + a)*a*log(b*x^2 + a) + (b*x^2 +
 a)^2 - 8*(b*x^2 + a)*a)*p^2/b + 2*(2*(b*x^2 + a)^2*log(b*x^2 + a) - 4*(b*x
^2 + a)*a*log(b*x^2 + a) - (b*x^2 + a)^2 + 4*(b*x^2 + a)*a)*p*log(c)/b + 2*
((b*x^2 + a)^2 - 2*(b*x^2 + a)*a)*log(c)^2/b)/b
```

3.79 $\int x \log^2\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=61

$$\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+p^2x^2$$

```
[Out] p^2*x^2 - (p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b + ((a + b*x^2)*Log[c*(a +
b*x^2)^p]^2)/(2*b)
```

Rubi [A] time = 0.0503193, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2389, 2296, 2295}

$$\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+p^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] p^2*x^2 - (p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b + ((a + b*x^2)*Log[c*(a +
b*x^2)^p]^2)/(2*b)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)dx=\frac{1}{2}\operatorname{Subst}\left(\int\log^2\left(c(a+bx)^p\right)dx,x,x^2\right)$$
$$=\frac{\operatorname{Subst}\left(\int\log^2\left(cx^p\right)dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{p\operatorname{Subst}\left(\int\log\left(cx^p\right)dx,x,a+bx^2\right)}{b}$$
$$=p^2x^2-\frac{p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}$$

Mathematica [A] time = 0.008556, size = 63, normalized size = 1.03

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{b}-2p\left(\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-px^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (((a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b - 2*p*(-(p*x^2) + ((a + b*x^2)*Log[
c*(a + b*x^2)^p])/b))/2
```

Maple [C] time = 0.497, size = 1034, normalized size = 17.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] -1/8*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^6-ln(c)*p*x^2+1/2*(I*Pi*b*x^2*csgn(I*(b
*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*b*x^2*csgn(I
*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)*b*x^2-2*x^2*p*b+2*a*p*ln(b*x^2+a))/b*ln
((b*x^2+a)^p)+1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)^2+1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-a*p^
2/b*ln(b*x^2+a)+1/2*ln(c)^2*x^2+1/2*x^2*ln((b*x^2+a)^p)^2-1/2/b*a*p^2*ln(b*
x^2+a)^2+1/4*Pi^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+1/4*Pi^2*
x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-1/8*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^4*
csgn(I*c)^2-1/8*Pi^2*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+p^2*
x^2-1/2*I*ln(c)*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+
1/2*I*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I/b*
Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^3+1/b*ln(c)*ln(b*x^2+a)*a*p+1/4*Pi
^2*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-1/8*Pi^2*x^2
*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-1/2*Pi^2*x^2*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+1/4*Pi^2*x^2*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-1/2*I*ln(c)*Pi*x^2*csgn(I*c*(b
*x^2+a)^p)^3+1/2*I*Pi*p*x^2*csgn(I*c*(b*x^2+a)^p)^3-1/2*I/b*Pi*ln(b*x^2+a)*
a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/2*I*ln(c)*Pi*x^2*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/2*I*ln(c)*Pi*x^2*csgn(I*c*(b*
x^2+a)^p)^2*csgn(I*c)-1/2*I*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
```

```
^p)^2-1/2*I*Pi*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)
```

Maxima [A] time = 1.03056, size = 131, normalized size = 2.15

$$-bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right)+\frac{1}{2}x^2\log\left(\left(bx^2+a\right)^pc\right)^2+\frac{\left(2\,bx^2-a\log\left(bx^2+a\right)^2-2\,a\log\left(bx^2+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c) + 1/2*x^2*log((b*x
^2 + a)^p*c)^2 + 1/2*(2*b*x^2 - a*log(b*x^2 + a)^2 - 2*a*log(b*x^2 + a))*p^
2/b
```

Fricas [A] time = 1.97339, size = 216, normalized size = 3.54

$$\frac{2\,bp^2x^2-2\,bpx^2\log\left(c\right)+bx^2\log\left(c\right)^2+\left(bp^2x^2+ap^2\right)\log\left(bx^2+a\right)^2-2\left(bp^2x^2+ap^2-\left(bpx^2+ap\right)\log\left(c\right)\right)\log\left(bx^2\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*p^2*x^2 - 2*b*p*x^2*log(c) + b*x^2*log(c)^2 + (b*p^2*x^2 + a*p^2)*
log(b*x^2 + a)^2 - 2*(b*p^2*x^2 + a*p^2 - (b*p*x^2 + a*p)*log(c))*log(b*x^2
 + a))/b
```

Sympy [A] time = 3.58321, size = 139, normalized size = 2.28

$$\begin{cases}\frac{ap^2\log\left(a+bx^2\right)^2}{2b}-\frac{ap^2\log\left(a+bx^2\right)}{b}+\frac{ap\log\left(c\right)\log\left(a+bx^2\right)}{b}+\frac{p^2x^2\log\left(a+bx^2\right)^2}{2}-p^2x^2\log\left(a+bx^2\right)+p^2x^2+px^2\log\left(c\right)\log\left(a+b\right. \\ \frac{x^2\log\left(a^pc\right)^2}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((a*p**2*log(a + b*x**2)**2/(2*b) - a*p**2*log(a + b*x**2)/b + a*p
*log(c)*log(a + b*x**2)/b + p**2*x**2*log(a + b*x**2)**2/2 - p**2*x**2*log(
a + b*x**2) + p**2*x**2 + p*x**2*log(c)*log(a + b*x**2) - p*x**2*log(c) + x
**2*log(c)**2/2, Ne(b, 0)), (x**2*log(a**p*c)**2/2, True))
```

Giac [A] time = 1.28606, size = 130, normalized size = 2.13

$$\frac{\left(2\,bx^2+\left(bx^2+a\right)\log\left(bx^2+a\right)^2-2\left(bx^2+a\right)\log\left(bx^2+a\right)+2\,a\right)p^2-2\left(bx^2-\left(bx^2+a\right)\log\left(bx^2+a\right)+a\right)p\log\left(c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a)
 + 2*a)*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c) + (b*x^2
+ a)*log(c)^2)/b
```

3.80 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=72

$$p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+p^2\left(-\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/2 + p*Log[c*(a + b*x^2)^p]*PolyL
og[2, 1 + (b*x^2)/a] - p^2*PolyLog[3, 1 + (b*x^2)/a]
```

Rubi [A] time = 0.112751, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2396, 2433, 2374, 6589}

$$p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+p^2\left(-\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/2 + p*Log[c*(a + b*x^2)^p]*PolyL
og[2, 1 + (b*x^2)/a] - p^2*PolyLog[3, 1 + (b*x^2)/a]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-(bp)\operatorname{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-p\operatorname{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx, x, a+bx^2\right)\\ &= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p\log\left(c\left(a+bx^2\right)^p\right)\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right)-p^2\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{x}{a}\right)}{x}\right.\\ &= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p\log\left(c\left(a+bx^2\right)^p\right)\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right)-p^2\operatorname{Li}_3\left(1+\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [B] time = 0.060482, size = 163, normalized size = 2.26

$$2p\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(\frac{bx^2}{a}+1\right)\right)-\frac{1}{2}\text{PolyLog}\left(2,-\frac{bx^2}{a}\right)\right)\left(\log\left(c\left(a+bx^2\right)^p\right)-p\log\left(a+bx^2\right)\right)+\frac{1}{2}p^2\left(-2\text{Poly}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 2*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a])
 - PolyLog[2, -((b*x^2)/a)]/2) + (p^2*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 +
 2*Log[a + b*x^2]*PolyLog[2, 1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))
/2
```

Maple [F] time = 0.849, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x, x)
```

3.81 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}$$

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - ((a + b*x^2)*Log[c*(a + b*
x^2)^p]^2)/(2*a*x^2) + (b*p^2*PolyLog[2, 1 + (b*x^2)/a])/a
```

Rubi [A] time = 0.0838589, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2397, 2394, 2315}

$$\frac{bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - ((a + b*x^2)*Log[c*(a + b*
x^2)^p]^2)/(2*a*x^2) + (b*p^2*PolyLog[2, 1 + (b*x^2)/a])/a
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{(bp)\,\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,x,x^2\right)}{a}$$

$$= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a} - \frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2} - \frac{\left(b^2p^2\right)\mathrm{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\right.}{a}$$

$$= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a} - \frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{bp^2\mathbf{Li}_2\left(1+\frac{bx^2}{a}\right)}{a}$$

Mathematica [A] time = 0.0239508, size = 93, normalized size = 1.16

$$\frac{bp^2\mathrm{PolyLog}\left(2,\frac{a+bx^2}{a}\right)}{a} - \frac{b\log^2\left(c\left(a+bx^2\right)^p\right)}{2a} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2x^2} + \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - (b*Log[c*(a + b*x^2)^p]^2)
/(2*a) - Log[c*(a + b*x^2)^p]^2/(2*x^2) + (b*p^2*PolyLog[2, (a + b*x^2)/a])
/a
```

Maple [C] time = 0.494, size = 841, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^3,x)
```

```
[Out] -1/2/x^2*ln((b*x^2+a)^p)^2-b*p*ln((b*x^2+a)^p)/a*ln(b*x^2+a)+2*b*p*ln((b*x^
2+a)^p)/a*ln(x)-2*b*p^2/a*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*b*p^
2/a*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*b*p^2/a*dilog((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))-2*b*p^2/a*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2*b
*p^2/a*ln(b*x^2+a)^2+1/2*I*b*p/a*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/2
*I*b*p/a*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*b*p/a*ln(x)*Pi*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/2*I*b*p/a*ln(b*x^2+a)
*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/x^2*ln((b*x^2+a)^
p)*ln(c)+1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)*csgn(I*c)+I*b*p/a*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*b*p/a*ln(
x)*Pi*csgn(I*c*(b*x^2+a)^p)^3-b*p/a*ln(b*x^2+a)*ln(c)-1/2*I*b*p/a*ln(b*x^2+
a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I/x^2*ln((b*x^2+a)^p)
*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+I*b*p/a*ln(x)*Pi*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)+1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^
3+2*b*p/a*ln(x)*ln(c)-1/8*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*
x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^2
```

Maxima [A] time = 1.06904, size = 159, normalized size = 1.99

$$\frac{1}{2}b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{ab}-\frac{2\left(2\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{ab}\right)-bp\left(\frac{\log\left(bx^2+a\right)}{a}-\frac{\log\left(x^2\right)}{a}\right)\log\left(\left(bx^2+a\right)^pc\right)-\frac{\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*p^2*(log(b*x^2 + a)^2/(a*b) - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(
-b*x^2/a))/(a*b)) - b*p*(log(b*x^2 + a)/a - log(x^2)/a)*log((b*x^2 + a)^p*c
) - 1/2*log((b*x^2 + a)^p*c)^2/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^3, x)
```

3.82 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=129

$$\frac{b^2p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^2}-\frac{b^2p\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{b^2p^2\log(x)}{a^2}-\frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2}-\text{log}$$

```
[Out] (b^2*p^2*Log[x])/a^2 - (b*p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^2*x^2) -
 Log[c*(a + b*x^2)^p]^2/(4*x^4) - (b^2*p*Log[c*(a + b*x^2)^p]*Log[1 - a/(a
+ b*x^2)])/(2*a^2) + (b^2*p^2*PolyLog[2, a/(a + b*x^2)])/(2*a^2)
```

Rubi [A] time = 0.268723, antiderivative size = 147, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$-\frac{b^2p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{b^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2}-\frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{b^2p^2\log(x)}{a^2}-\frac{bp\left(a+bx^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (b^2*p^2*Log[x])/a^2 - (b*p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^2*x^2) -
 (b^2*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^2) + (b^2*Log[c*(a + b
*x^2)^p]^2)/(4*a^2) - Log[c*(a + b*x^2)^p]^2/(4*x^4) - (b^2*p^2*PolyLog[2,
1 + (b*x^2)/a])/(2*a^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3}dx, x, x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2(a+bx)}dx, x, x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{2}p\,\text{Subst}\left(\int \frac{\log\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}dx, x, a+bx^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{p\,\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}dx, x, a+bx^2\right)}{2a} - \frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}dx, x,\right.}{2a}$$
$$= -\frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} - \frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{-\frac{a}{b}+\frac{x}{b}}dx, x, a+b\right.}{2a^2}$$
$$= \frac{b^2p^2\log(x)}{a^2} - \frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2} - \frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} + \frac{b^2\log}{}$$
$$= \frac{b^2p^2\log(x)}{a^2} - \frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2} - \frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} + \frac{b^2\log}{}$$

Mathematica [A] time = 0.0790633, size = 137, normalized size = 1.06

$$\frac{\frac{bx^2\left(-2bpx^2\left(p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)\right)+bx^2\log^2\left(c\left(a+bx^2\right)^p\right)-2ap\log\left(c\left(a+bx^2\right)^p\right)+2bp^2x^2\left(2\log(x)-\log\left(a+bx^2\right)\right)\right)}{a^2}-\log^2\left(c\left(a\right.\right.}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(a + b*x^2)^p]^2 + (b*x^2*(2*b*p^2*x^2*(2*Log[x] - Log[a + b*x^2])
- 2*a*p*Log[c*(a + b*x^2)^p] + b*x^2*Log[c*(a + b*x^2)^p]^2 - 2*b*p*x^2*(Lo
g[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a])))/a^2)/
(4*x^4)
```

Maple [C] time = 0.5, size = 1080, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^5,x)
```

```
[Out] b^2*p^2*ln(x)/a^2+1/4*I*b^2*p/a^2*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^2*cs
gn(I*c)+b^2*p^2/a^2*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+b^2*p^2/a^2*
ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn
(I*c*(b*x^2+a)^p)^3+1/2*I*b^2*p/a^2*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)*csgn(I*c)-1/4*I*b^2*p/a^2*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/4*I*b*p/a/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)*csgn(I*c)-1/4*b^2*p^2/a^2*ln(b*x^2+a)^2-1/2*b^2*p^2/a^2*ln(
```

```
b*x^2+a)+b^2*p^2/a^2*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+b^2*p^2/a^2*di
log((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/2/x^4*ln((b*x^2+a)^p)*ln(c)-1/4*I/x^
4*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*b^2*p/a^2*ln(x
)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*b^2*p/a^2*ln(x)*Pi*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/4*I*b^2*p/a^2*ln(b*x^2+a)*Pi*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/4*I*b*p/a/x^2*Pi*csgn(I*c*(b*x^2+a)^p)^2
*csgn(I*c)-1/4*I*b*p/a/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1
/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/4
/x^4*ln((b*x^2+a)^p)^2+1/2*b^2*p/a^2*ln(b*x^2+a)*ln(c)-1/2*b*p/a/x^2*ln(c)-
b^2*p/a^2*ln(x)*ln(c)-1/16*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^
2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b
*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^4+1/2*I*b^
2*p/a^2*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/4*I*b*p/a/x^2*Pi*csgn(I*c*
(b*x^2+a)^p)^3-1/4*I*b^2*p/a^2*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3-b^2*p
*ln((b*x^2+a)^p)/a^2*ln(x)+1/2*b^2*p*ln((b*x^2+a)^p)/a^2*ln(b*x^2+a)-1/2*b*
p*ln((b*x^2+a)^p)/a/x^2
```

Maxima [A] time = 1.11725, size = 192, normalized size = 1.49

$$-\frac{1}{4}b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{a^2}-\frac{2\left(2\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\operatorname{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{a^2}+\frac{2\log\left(bx^2+a\right)}{a^2}-\frac{4\log\left(x\right)}{a^2}\right)+\frac{1}{2}bp\left(\frac{b\log\left(bx^2+a\right)}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*p^2*(log(b*x^2 + a)^2/a^2 - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(-
b*x^2/a))/a^2 + 2*log(b*x^2 + a)/a^2 - 4*log(x)/a^2) + 1/2*b*p*(b*log(b*x^2
 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2))*log((b*x^2 + a)^p*c) - 1/4*log((b*x
^2 + a)^p*c)^2/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^5, x)
```

3.83 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=193

$$-\frac{b^3p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{3a^3}+\frac{b^3p\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3}+\frac{b^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3x^2}-\frac{b^2p^2}{6a^2x^2}+\frac{b^3p^2\log}{6\ldots}$$

```
[Out] -(b^2*p^2)/(6*a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/(6
*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/(3*a^3*x^2) - Log[c*(a + b*x^2)^p]^2/(6*x^6) + (b^3*p*Log[c*(a
 + b*x^2)^p]*Log[1 - a/(a + b*x^2)])/(3*a^3) - (b^3*p^2*PolyLog[2, a/(a + b
*x^2)])/(3*a^3)
```

Rubi [A] time = 0.407236, antiderivative size = 211, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^3p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{3a^3}-\frac{b^3\log^2\left(c\left(a+bx^2\right)^p\right)}{6a^3}+\frac{b^3p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3}+\frac{b^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3x^2}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -(b^2*p^2)/(6*a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/(6
*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/(3*a^3*x^2) + (b^3*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(
3*a^3) - (b^3*Log[c*(a + b*x^2)^p]^2)/(6*a^3) - Log[c*(a + b*x^2)^p]^2/(6*x
^6) + (b^3*p^2*PolyLog[2, 1 + (b*x^2)/a])/(3*a^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log^2(c(a+bx)^p)}{x^4}\,dx,x,x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x^3(a+bx)}\,dx,x,x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{3}p\,\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx,x,a+bx^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{p\,\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx,x,a+bx^2\right)}{3a} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a\right.}{3a}$$
$$= -\frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a+bx^2\right)}{3a^2} + \frac{(b^2p)}{}$$
$$= -\frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} + \frac{b^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3x^2} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{(b^2p)\,\text{Subs}}{}$$
$$= -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} + \frac{b^2p\left(a+bx^2\right)\log\left(c\left(a\right.\right.}{3a^3x^2}$$
$$= -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} + \frac{b^2p\left(a+bx^2\right)\log\left(c\left(a\right.\right.}{3a^3x^2}$$

Mathematica [A] time = 0.0544469, size = 205, normalized size = 1.06

$$\frac{b^3p^2\text{PolyLog}\left(2,\frac{a+bx^2}{a}\right)}{3a^3} - \frac{b^3\log^2\left(c\left(a+bx^2\right)^p\right)}{6a^3} + \frac{b^3p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3} + \frac{b^2p\log\left(c\left(a+bx^2\right)^p\right)}{3a^2x^2} - \frac{b^2p^2}{6a^2x^2} +$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -(b^2*p^2)/(6*a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/(2
*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*Log[c*(a + b*x^2)^p])
/(3*a^2*x^2) + (b^3*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(3*a^3) - (b^
3*Log[c*(a + b*x^2)^p]^2)/(6*a^3) - Log[c*(a + b*x^2)^p]^2/(6*x^6) + (b^3*p
^2*PolyLog[2, (a + b*x^2)/a])/(3*a^3)
```

Maple [C] time = 0.516, size = 1289, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^7,x)
```

```
[Out] -b^3*p^2*ln(x)/a^3+1/2*b^3*p^2*ln(b*x^2+a)/a^3-1/3*I*b^3*p/a^3*ln(x)*Pi*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*I*b^2*p/a^2/x^2*Pi*csg
```

```
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6/x^6*ln((b*x^2+a)^p)^2+
1/6*I*b^3*p/a^3*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-2/3*b^3*p^2/a^3*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*b^3*p^2
/a^3*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/6*b^3*p^2/a^3*ln(b*x^2+a)^2+1
/3*I*b^3*p/a^3*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/24*(I
*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b
*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^6-1/12*I*b*p/a/x^4*Pi*csgn(I*c*(b*x^2+a
)^p)^2*csgn(I*c)-1/12*I*b*p/a/x^4*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2+1/12*I*b*p/a/x^4*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*
c)-1/3/x^6*ln((b*x^2+a)^p)*ln(c)-1/6*I/x^6*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x
^2+a)^p)^2*csgn(I*c)+1/6*I*b^2*p/a^2/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^2+1/6*I*b^2*p/a^2/x^2*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/3*I
*b^3*p/a^3*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/6*I*b^3*p/a^3*ln(b*
x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/6*I*b^3*p/a^3*ln(b*
x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/6*b^2*p^2/a^2/x^2-1/6*I/x^6*l
n((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/3*b^3*p/a^3
*ln(b*x^2+a)*ln(c)+2/3*b^3*p/a^3*ln(x)*ln(c)-1/6*b*p/a/x^4*ln(c)+1/3*b^2*p/
a^2/x^2*ln(c)-2/3*b^3*p^2/a^3*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/
3*b^3*p^2/a^3*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/6*I*b^3*p/a^3*ln(
b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/12*I*b*p/a/x^4*Pi*csgn(I*c*(b*x^2+a)^
p)^3+1/6*I/x^6*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/3*b^3*p*ln((b*x
^2+a)^p)/a^3*ln(b*x^2+a)-1/6*b*p*ln((b*x^2+a)^p)/a/x^4+2/3*b^3*p*ln((b*x^2+
a)^p)/a^3*ln(x)+1/3*b^2*p*ln((b*x^2+a)^p)/a^2/x^2-1/6*I*b^2*p/a^2/x^2*Pi*cs
gn(I*c*(b*x^2+a)^p)^3-1/3*I*b^3*p/a^3*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/6*
I/x^6*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c
)
```

Maxima [A] time = 1.05452, size = 234, normalized size = 1.21

$$-\frac{1}{6}\,b^2p^2\left(\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)b}{a^3}-\frac{3\,b\log\left(bx^2+a\right)}{a^3}-\frac{bx^2\log\left(bx^2+a\right)^2-6\,bx^2\log\left(x\right)-a}{a^3x^2}\right)-\frac{1}{6}\,bp$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*b^2*p^2*(2*(2*log(b*x^2/a + 1)*log(x) + dilog(-b*x^2/a))*b/a^3 - 3*b*l
og(b*x^2 + a)/a^3 - (b*x^2*log(b*x^2 + a)^2 - 6*b*x^2*log(x) - a)/(a^3*x^2)
) - 1/6*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/
(a^2*x^4))*log((b*x^2 + a)^p*c) - 1/6*log((b*x^2 + a)^p*c)^2/x^6
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^7, x)
```

3.84 $\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=336

$$\frac{4ia^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}}-\frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2}+\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5b^{5/2}}+\frac{184a^2p^2x}{75b^2}+\cdots$$

```
[Out] (184*a^2*p^2*x)/(75*b^2) - (64*a*p^2*x^3)/(225*b) + (8*p^2*x^5)/125 - (184*
a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(75*b^(5/2)) + (((4*I)/5)*a^(5/2)*
p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(5/2) + (8*a^(5/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(5*b^(5/2)) - (4*a^2
*p*x*Log[c*(a + b*x^2)^p])/(5*b^2) + (4*a*p*x^3*Log[c*(a + b*x^2)^p])/(15*b
) - (4*p*x^5*Log[c*(a + b*x^2)^p])/25 + (4*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqr
t[a]]*Log[c*(a + b*x^2)^p])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p]^2)/5 +
(((4*I)/5)*a^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])
/b^(5/2)
```

Rubi [A] time = 0.407981, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ia^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}}-\frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2}+\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5b^{5/2}}+\frac{184a^2p^2x}{75b^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^2)^p]^2,x]
```


```
[Out] (184*a^2*p^2*x)/(75*b^2) - (64*a*p^2*x^3)/(225*b) + (8*p^2*x^5)/125 - (184*
a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(75*b^(5/2)) + (((4*I)/5)*a^(5/2)*
p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(5/2) + (8*a^(5/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(5*b^(5/2)) - (4*a^2
*p*x*Log[c*(a + b*x^2)^p])/(5*b^2) + (4*a*p*x^3*Log[c*(a + b*x^2)^p])/(15*b
) - (4*p*x^5*Log[c*(a + b*x^2)^p])/25 + (4*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqr
t[a]]*Log[c*(a + b*x^2)^p])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p]^2)/5 +
(((4*I)/5)*a^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])
/b^(5/2)
```


Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4bp)\int \frac{x^6\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx$$

$$= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4bp)\int\left(\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{b^3} - \frac{ax^2\log\left(c\left(a+bx^2\right)^p\right)}{b^2} + \cdots\right.$$

$$= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4p)\int x^4\log\left(c\left(a+bx^2\right)^p\right) dx - \frac{\left(4a^2p\right)\int\log\left(c\left(a+bx^2\right)^{p}\right)\cdots}{5b^2}$$

$$= -\frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2} + \frac{4apx^3\log\left(c\left(a+bx^2\right)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c\left(a+bx^2\right)^p\right) + \frac{4a^5\cdots}{\cdots}$$

$$= \frac{8a^2p^2x}{5b^2} - \frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2} + \frac{4apx^3\log\left(c\left(a+bx^2\right)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c\left(a+bx^2\right)\cdots\right.$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} - \frac{4a^2px\cdots}{\cdots}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} + \frac{8a^{5/}\cdots}{\cdots}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} + \frac{8a^{5/}\cdots}{\cdots}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} + \frac{8a^{5/}\cdots}{\cdots}$$

Mathematica [A] time = 0.195426, size = 248, normalized size = 0.74

$$\frac{900ia^{5/2}p^2\text{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)+\sqrt{b}x\left(-60p\left(15a^2-5abx^2+3b^2x^4\right)\log\left(c\left(a+bx^2\right)^p\right)+8p^2\left(345a^2-40abx^2+9b^2\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((900*I)*a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 60*a^(5/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-46*p + 30*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] +
```

```
15*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(345*a^2 - 40*a*b*x^2 + 9*b^2*
x^4) - 60*p*(15*a^2 - 5*a*b*x^2 + 3*b^2*x^4)*Log[c*(a + b*x^2)^p] + 225*b^2
*x^4*Log[c*(a + b*x^2)^p]^2) + (900*I)*a^(5/2)*p^2*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(1125*b^(5/2))
```

Maple [F] time = 0.576, size = 0, normalized size = 0.

$$\int x^{4} \left(\ln \left(c \left(b x^{2}+a\right)^{p}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^4*ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{4} \log\left(\left(b x^{2}+a\right)^{p} c\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \log\left(\left(bx^2+a\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*log((b*x^2 + a)^p*c)^2, x)
```

3.85 $\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=294

$$-\frac{4ia^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}}-\frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{9b^{3/2}}$$

```
[Out] (-32*a*p^2*x)/(9*b) + (8*p^2*x^3)/27 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(9*b^(3/2)) - (((4*I)/3)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2
)/b^(3/2) - (8*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqr
t[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) + (4*a*p*x*Log[c*(a + b*x^2)^p])/(3*b) -
(4*p*x^3*Log[c*(a + b*x^2)^p])/9 - (4*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p]^2)/3 - (((4*
I)/3)*a^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(3
/2)
```

Rubi [A] time = 0.322765, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ia^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}}-\frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{9b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^2,x]
```


```
[Out] (-32*a*p^2*x)/(9*b) + (8*p^2*x^3)/27 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(9*b^(3/2)) - (((4*I)/3)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2
)/b^(3/2) - (8*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqr
t[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) + (4*a*p*x*Log[c*(a + b*x^2)^p])/(3*b) -
(4*p*x^3*Log[c*(a + b*x^2)^p])/9 - (4*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p]^2)/3 - (((4*
I)/3)*a^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(3
/2)
```


Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(4bp)\int \frac{x^4\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(4bp)\int\left(-\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b^2} + \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{b} + \frac{a^2\log\ldots}{b\ldots}\right.$$

$$= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(4p)\int x^2\log\left(c\left(a+bx^2\right)^p\right)dx + \frac{(4ap)\int\log\left(c\left(a+bx^2\right)^p\right)dx\ldots}{3b}$$

$$= \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}\ldots$$

$$= -\frac{8ap^2x}{3b} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(\ldots\right.\right.}{3b^{3/2}}$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{4apx\log\left(c\left(a+bx^2\right)^{\ldots}\right.}{3b}$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\ldots}{3b^{3/2}}$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\ldots}{3b^{3/2}}$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\ldots}{3b^{3/2}}$$

Mathematica [A] time = 0.134917, size = 223, normalized size = 0.76

$$\frac{-36ia^{3/2}p^2\text{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right) - 12a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3\log\left(c\left(a+bx^2\right)^p\right) + 6p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right) - 8p\right) - 36ia^{3/2}p^2\tan^{-1}\left(\ldots\right.}{27b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 12*a^(3/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-8*p + 6*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + 3
```

```
*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(-12*a + b*x^2) + 12*p*(3*a - b*x
^2)*Log[c*(a + b*x^2)^p] + 9*b*x^2*Log[c*(a + b*x^2)^p]^2) - (36*I)*a^(3/2)
*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(27*b^
(3/2))
```

Maple [F] time = 0.984, size = 0, normalized size = 0.

$$\int x^2 \left(\ln \left(c \left(b x^2+a\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^2*ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \log\left(\left(b x^2+a\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \log\left(c \left(a+b x^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \log\left(\left(bx^2+a\right)^p c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^2, x)
```

3.86 $\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=237

$$\frac{4i\sqrt{a}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}+x\log^2\left(c\left(a+bx^2\right)^p\right)-4px\log\left(c\left(a+bx^2\right)^p\right)+\frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}$$

```
[Out] 8*p^2*x - (8*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + ((4*I)*Sqrt
[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] + (8*Sqrt[a]*p^2*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] - 4*p*x*
Log[c*(a + b*x^2)^p] + (4*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a +
b*x^2)^p])/Sqrt[b] + x*Log[c*(a + b*x^2)^p]^2 + ((4*I)*Sqrt[a]*p^2*PolyLog[
2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b]
```

Rubi [A] time = 0.267497, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4i\sqrt{a}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}+x\log^2\left(c\left(a+bx^2\right)^p\right)-4px\log\left(c\left(a+bx^2\right)^p\right)+\frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2,x]
```


```
[Out] 8*p^2*x - (8*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + ((4*I)*Sqrt
[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] + (8*Sqrt[a]*p^2*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] - 4*p*x*
Log[c*(a + b*x^2)^p] + (4*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a +
b*x^2)^p])/Sqrt[b] + x*Log[c*(a + b*x^2)^p]^2 + ((4*I)*Sqrt[a]*p^2*PolyLog[
2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b]
```


Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)dx = x\log^2\left(c\left(a+bx^2\right)^p\right) - (4bp)\int \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= x\log^2\left(c\left(a+bx^2\right)^p\right) - (4bp)\int\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right)dx$$

$$= x\log^2\left(c\left(a+bx^2\right)^p\right) - (4p)\int\log\left(c\left(a+bx^2\right)^p\right)dx + (4ap)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= -4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} + x\log^2\left(c\left(a+bx^2\right)^p\right) + ($$

$$= 8p^2x - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} + x\log^2\left(c\left(a+bx^2\right)\right.$$

$$= 8p^2x - \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}p\tan^{-}}{}$$

$$= 8p^2x - \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} -$$

$$= 8p^2x - \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} -$$

$$= 8p^2x - \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} -$$

Mathematica [A] time = 0.0812616, size = 193, normalized size = 0.81

$$\frac{4i\sqrt{a}p^2\text{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right) + \sqrt{b}x\left(\log^2\left(c\left(a+bx^2\right)^p\right) - 4p\log\left(c\left(a+bx^2\right)^p\right) + 8p^2\right) + 4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\log\left(c\left(a\right.\right.\right.}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((4*I)*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + Log[
c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2 - 4*p*Log[c*(a + b*x^2)^p] + Log[c*(a
+ b*x^2)^p]^2) + (4*I)*Sqrt[a]*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)
*Sqrt[a] + Sqrt[b]*x)])/Sqrt[b]
```

Maple [F] time = 0.844, size = 0, normalized size = 0.

$$\int\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\left(bx^2+a\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(c\left(a+bx^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\left(bx^2+a\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2, x)
```

3.87 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=190

$$\frac{4i\sqrt{b}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}+\frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}}+\frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] ((4*I)*Sqrt[b]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[a] + (8*Sqrt[b]*p^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[
a] + (4*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/Sqrt[a]
 - Log[c*(a + b*x^2)^p]^2/x + ((4*I)*Sqrt[b]*p^2*PolyLog[2, 1 - (2*Sqrt[a])
/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[a]
```

Rubi [A] time = 0.168799, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2457, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4i\sqrt{b}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}+\frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}}+\frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[a] + (8*Sqrt[b]*p^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[
a] + (4*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/Sqrt[a]
 - Log[c*(a + b*x^2)^p]^2/x + ((4*I)*Sqrt[b]*p^2*PolyLog[2, 1 - (2*Sqrt[a])
/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[a]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} + (4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx$$

$$= \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} - \left(8b^2p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx$$

$$= \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} - \frac{\left(8b^{3/2}p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a+bx^2}\,dx}{\sqrt{a}}$$

$$= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} + \frac{\left(8bp^2\right)}{}$$

$$= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{bx}}\right)}{\sqrt{a}} + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(c\left(a+bx\right.\right.}{\sqrt{a}}$$

$$= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{bx}}\right)}{\sqrt{a}} + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(c\left(a+bx\right.\right.}{\sqrt{a}}$$

$$= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{bx}}\right)}{\sqrt{a}} + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\log\left(c\left(a+bx\right.\right.}{\sqrt{a}}$$

Mathematica [A] time = 0.051766, size = 173, normalized size = 0.91

$$\frac{4i\sqrt{b}p^2x\text{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)-\sqrt{a}\log^2\left(c\left(a+bx^2\right)^p\right)+4\sqrt{b}px\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\log\left(c\left(a+bx^2\right)^p\right)+2p\log\left(\frac{2i}{-\frac{\sqrt{b}x}{\sqrt{a}}+i}\right)\right)+}{\sqrt{a}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - Sqrt[a]*Log[c*(a + b*x
^2)^p]^2 + 4*Sqrt[b]*p*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(2*p*Log[(2*I)/(I - (S
qrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^p]) + (4*I)*Sqrt[b]*p^2*x*PolyLog[2
, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*x)
```

Maple [F] time = 0.884, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^2, x)
```

3.88 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=254

$$-\frac{4ib^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}}-\frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}}+\frac{8b^{3/2}p^2\tan^{-1}(}{3a^{3/2}}$$

```
[Out] (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) - (((4*I)/3)*b^(3/2
)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) - (8*b^(3/2)*p^2*ArcTan[(Sqrt[
b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(3*a^(3/2)) - (4*b
*p*Log[c*(a + b*x^2)^p])/(3*a*x) - (4*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*a^(3/2)) - Log[c*(a + b*x^2)^p]^2/(3*x^3) - (((4*
I)/3)*b^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2)
```

Rubi [A] time = 0.28507, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2457, 2476, 2455, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}}-\frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}}+\frac{8b^{3/2}p^2\tan^{-1}(}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) - (((4*I)/3)*b^(3/2
)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) - (8*b^(3/2)*p^2*ArcTan[(Sqrt[
b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(3*a^(3/2)) - (4*b
*p*Log[c*(a + b*x^2)^p])/(3*a*x) - (4*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*a^(3/2)) - Log[c*(a + b*x^2)^p]^2/(3*x^3) - (((4*
I)/3)*b^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2)
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{1}{3}(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{1}{3}(4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^2} - \frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a\left(a+bx^2\right)}\right)dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{3a} - \frac{\left(4b^2p\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{3a}$$

$$= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \ldots$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}} - \ldots$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \ldots$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} - \ldots$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} - \ldots$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} - \ldots$$

Mathematica [A] time = 0.0907811, size = 207, normalized size = 0.81

$$\frac{-4ib^{3/2}p^2x^3\text{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right) - 4b^{3/2}px^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\log\left(c\left(a+bx^2\right)^p\right)+2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)-2p\right) - 4ib^{3/2}p^2x^3\tan\ldots}{3a^{3/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] ((-4*I)*b^(3/2)*p^2*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 4*b^(3/2)*p*x^3*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x
)] + Log[c*(a + b*x^2)^p]) - Sqrt[a]*Log[c*(a + b*x^2)^p]*(4*b*p*x^2 + a*Lo
g[c*(a + b*x^2)^p]) - (4*I)*b^(3/2)*p^2*x^3*PolyLog[2, (I*Sqrt[a] + Sqrt[b]
*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(3*a^(3/2)*x^3)
```

Maple [F] time = 0.964, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^4,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**4,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^4, x)
```

3.89 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=296

$$\frac{4ib^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}+\frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}-\frac{8b^2p^2}{15a^2x}+\frac{4ib^{5/2}}{}$$

```
[Out] (-8*b^2*p^2)/(15*a^2*x) - (32*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(15*
a^(5/2)) + (((4*I)/5)*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(5/2) +
(8*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)])/(5*a^(5/2)) - (4*b*p*Log[c*(a + b*x^2)^p])/(15*a*x^3) + (4*b^2*p*
Log[c*(a + b*x^2)^p])/(5*a^2*x) + (4*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]^2/(5*x^5) + (((4*I
)/5)*b^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(5/
2)
```

Rubi [A] time = 0.32035, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ib^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}+\frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}-\frac{8b^2p^2}{15a^2x}+\frac{4ib^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^6,x]
```



```
[Out] (-8*b^2*p^2)/(15*a^2*x) - (32*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(15*
a^(5/2)) + (((4*I)/5)*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(5/2) +
(8*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)])/(5*a^(5/2)) - (4*b*p*Log[c*(a + b*x^2)^p])/(15*a*x^3) + (4*b^2*p*
Log[c*(a + b*x^2)^p])/(5*a^2*x) + (4*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]^2/(5*x^5) + (((4*I
)/5)*b^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(5/
2)
```


Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{1}{5}(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{1}{5}(4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^4} - \frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a^2x^2} + \frac{b^2\log\left(c\left(a\right.\right.}{a^2\left(a+\right.}\right.$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx}{5a} - \frac{\left(4b^2p\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{5a^2} + \frac{\left(4b^3p\right)\int \frac{\log}{}}{5}$$

$$= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x} + \frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x} + \frac{4b^{5/2}}{}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3} + \frac{4b^2p}{}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{}}\right.}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{}}\right.}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{}}\right.}{5a^{5/2}}$$

<u>Mathematica</u> [C] time = 0.27216, size = 277, normalized size = 0.94

$$-\frac{3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{4bpx^2\left(-3ib^{3/2}px^3\left(\text{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)+\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-2i\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)\right)\right)+a^{3/2}\log\left(c\left(a+bx^2\right)^p\right)-3b^{3/2}x^3\tan^{-}\right.}{a^{5/2}}}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

```
[Out] -(3*Log[c*(a + b*x^2)^p]^2 + (4*b*p*x^2*(6*b^(3/2)*p*x^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]] + 2*Sqrt[a]*b*p*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)]
 + a^(3/2)*Log[c*(a + b*x^2)^p] - 3*Sqrt[a]*b*x^2*Log[c*(a + b*x^2)^p] - 3*
b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p] - (3*I)*b^(3/2
)*p*x^3*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]] - (2*I)*L
og[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)]) + PolyLog[2, (I*Sqrt[a] + Sqrt[b]*
x)/((-I)*Sqrt[a] + Sqrt[b]*x)])))/a^(5/2))/(15*x^5)
```

Maple [F] time = 0.963, size = 0, normalized size = 0.

$$\int \frac{\left(\ln \left(c \left(b x^{2}+a \right)^{p}\right)\right)^{2}}{x^{6}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^6,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^{2}+a\right)^{p}c\right)^{2}}{x^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^{2}+a\right)^{p}c\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^6, x)
```

3.90 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx$

<u>**Optimal**</u>. Leaf size=338

$$-\frac{4ib^{7/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}-\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+b\right.\right.}{7a^{7/2}}$$

```
[Out] (-8*b^2*p^2)/(105*a^2*x^3) + (64*b^3*p^2)/(105*a^3*x) + (184*b^(7/2)*p^2*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(105*a^(7/2)) - (((4*I)/7)*b^(7/2)*p^2*ArcTan[(S
qrt[b]*x)/Sqrt[a]]^2)/a^(7/2) - (8*b^(7/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(7*a^(7/2)) - (4*b*p*Log[c*(a + b
*x^2)^p])/(35*a*x^5) + (4*b^2*p*Log[c*(a + b*x^2)^p])/(21*a^2*x^3) - (4*b^3
*p*Log[c*(a + b*x^2)^p])/(7*a^3*x) - (4*b^(7/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a
]]*Log[c*(a + b*x^2)^p])/(7*a^(7/2)) - Log[c*(a + b*x^2)^p]^2/(7*x^7) - (((
4*I)/7)*b^(7/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^
(7/2)
```

<u>**Rubi**</u> **[A]** time = 0.376727, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^{7/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}-\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+b\right.\right.}{7a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^8,x]
```


```
[Out] (-8*b^2*p^2)/(105*a^2*x^3) + (64*b^3*p^2)/(105*a^3*x) + (184*b^(7/2)*p^2*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(105*a^(7/2)) - (((4*I)/7)*b^(7/2)*p^2*ArcTan[(S
qrt[b]*x)/Sqrt[a]]^2)/a^(7/2) - (8*b^(7/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(7*a^(7/2)) - (4*b*p*Log[c*(a + b
*x^2)^p])/(35*a*x^5) + (4*b^2*p*Log[c*(a + b*x^2)^p])/(21*a^2*x^3) - (4*b^3
*p*Log[c*(a + b*x^2)^p])/(7*a^3*x) - (4*b^(7/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a
]]*Log[c*(a + b*x^2)^p])/(7*a^(7/2)) - Log[c*(a + b*x^2)^p]^2/(7*x^7) - (((
4*I)/7)*b^(7/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^
(7/2)
```


<u>Rule 2457</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

<u>Rule 2476</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{1}{7}(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{1}{7}(4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^6} - \frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a^2x^4} + \frac{b^2\log\left(c\left(a+\right.\right.}{a^3x^2}\right.$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}dx}{7a} - \frac{\left(4b^2p\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4}dx}{7a^2} + \frac{\left(4b^3p\right)\int \frac{\log(c(a}{}}{7a^3}$$

$$= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5} + \frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3} - \frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x} - \frac{4b^{7/2}p\tan^{-1}\left(\right.}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{8b^3p^2}{21a^3x} + \frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{7a^{7/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5} + \frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{32b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{21a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right.}{35ax^5}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{7a^{7/2}}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{7a^{7/2}}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{7a^{7/2}}$$

<u>Mathematica</u> [C] time = 0.223705, size = 334, normalized size = 0.99

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{4bp\left(-15ib^{5/2}px^5\left(\text{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)+\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-2i\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)\right)\right)+5a^{3/2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

```
[Out] -Log[c*(a + b*x^2)^p]^2/(7*x^7) + (4*b*p*(30*b^(5/2)*p*x^5*ArcTan[(Sqrt[b]*
x)/Sqrt[a]] - 2*a^(3/2)*b*p*x^2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/
a)] + 10*Sqrt[a]*b^2*p*x^4*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)] -
3*a^(5/2)*Log[c*(a + b*x^2)^p] + 5*a^(3/2)*b*x^2*Log[c*(a + b*x^2)^p] - 15*
Sqrt[a]*b^2*x^4*Log[c*(a + b*x^2)^p] - 15*b^(5/2)*x^5*ArcTan[(Sqrt[b]*x)/Sq
rt[a]]*Log[c*(a + b*x^2)^p] - (15*I)*b^(5/2)*p*x^5*(ArcTan[(Sqrt[b]*x)/Sqrt
[a]]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]] - (2*I)*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt
[b]*x)]) + PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))
)/(105*a^(7/2)*x^5)
```

Maple [F] time = 0.945, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^8,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^8,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^8, x)
```

3.91 $\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)dx$

Optimal. Leaf size=334

$$\frac{3a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}-\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^3}+\frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3}-\frac{3a^2p^3x^2}{b^2}$$

```
[Out] (-3*a^2*p^3*x^2)/b^2 + (3*a*p^3*(a + b*x^2)^2)/(8*b^3) - (p^3*(a + b*x^2)^3
)/(27*b^3) + (3*a^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b^3 - (3*a*p^2*(a
 + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^3) + (p^2*(a + b*x^2)^3*Log[c*(a + b
*x^2)^p])/(9*b^3) - (3*a^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^3) +
(3*a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^2)/(4*b^3) - (p*(a + b*x^2)^3*Log
[c*(a + b*x^2)^p]^2)/(6*b^3) + (a^2*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*
b^3) - (a*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(2*b^3) + ((a + b*x^2)^3*Lo
g[c*(a + b*x^2)^p]^3)/(6*b^3)
```

Rubi [A] time = 0.357969, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{3a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}-\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^3}+\frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3}-\frac{3a^2p^3x^2}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^3,x]
```


```
[Out] (-3*a^2*p^3*x^2)/b^2 + (3*a*p^3*(a + b*x^2)^2)/(8*b^3) - (p^3*(a + b*x^2)^3
)/(27*b^3) + (3*a^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b^3 - (3*a*p^2*(a
 + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^3) + (p^2*(a + b*x^2)^3*Log[c*(a + b
*x^2)^p])/(9*b^3) - (3*a^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^3) +
(3*a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^2)/(4*b^3) - (p*(a + b*x^2)^3*Log
[c*(a + b*x^2)^p]^2)/(6*b^3) + (a^2*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*
b^3) - (a*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(2*b^3) + ((a + b*x^2)^3*Lo
g[c*(a + b*x^2)^p]^3)/(6*b^3)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right) dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2 \log^3\left(c(a+bx)^p\right) dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2 \log^3\left(c(a+bx)^p\right)}{b^2} - \frac{2a(a+bx)\log^3\left(c(a+bx)^p\right)}{b^2} + \frac{(a+bx)^2\log^3\left(c(a+bx\right.}{b^2}\right.\right. \\
&= \frac{\operatorname{Subst}\left(\int (a+bx)^2\log^3\left(c(a+bx)^p\right) dx, x, x^2\right)}{2b^2} - \frac{a\operatorname{Subst}\left(\int (a+bx)\log^3\left(c(a+bx)^p\right) dx, x\right.}{b^2} \\
&= \frac{\operatorname{Subst}\left(\int x^2\log^3\left(cx^p\right) dx, x, a+bx^2\right)}{2b^3} - \frac{a\operatorname{Subst}\left(\int x\log^3\left(cx^p\right) dx, x, a+bx^2\right)}{b^3} + \frac{a^2\operatorname{Subs}}{} \\
&= \frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3} - \frac{a\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3} + \frac{\left(a+bx^2\right)^3\log^3\left(c\left(a\right.\right.}{6b^3} \\
&= -\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^3} + \frac{3ap\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^3} - \frac{p\left(a+bx^2\right)^3\log}{6} \\
&= -\frac{3a^2p^3x^2}{b^2} + \frac{3ap^3\left(a+bx^2\right)^2}{8b^3} - \frac{p^3\left(a+bx^2\right)^3}{27b^3} + \frac{3a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3} - \frac{3ap^2}{}
\end{aligned}$$

Mathematica [A] time = 0.194772, size = 309, normalized size = 0.93

$$\frac{11a^2p^2x^2\log\left(c\left(a+bx^2\right)^p\right)}{6b^2} + \frac{11a^3p^2\log\left(c\left(a+bx^2\right)^p\right)}{6b^3} - \frac{a^2px^2\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2} + \frac{a^3\log^3\left(c\left(a+bx^2\right)^p\right)}{6b^3} - \frac{11a^3p\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-85*a^2*p^3*x^2)/(36*b^2) + (19*a*p^3*x^4)/(72*b) - (p^3*x^6)/27 + (19*a^3
*p^3*Log[a + b*x^2])/(36*b^3) + (11*a^3*p^2*Log[c*(a + b*x^2)^p])/(6*b^3) +
 (11*a^2*p^2*x^2*Log[c*(a + b*x^2)^p])/(6*b^2) - (5*a*p^2*x^4*Log[c*(a + b*
x^2)^p])/(12*b) + (p^2*x^6*Log[c*(a + b*x^2)^p])/9 - (11*a^3*p*Log[c*(a + b
*x^2)^p]^2)/(12*b^3) - (a^2*p*x^2*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + (a*p*x^
4*Log[c*(a + b*x^2)^p]^2)/(4*b) - (p*x^6*Log[c*(a + b*x^2)^p]^2)/6 + (a^3*L
og[c*(a + b*x^2)^p]^3)/(6*b^3) + (x^6*Log[c*(a + b*x^2)^p]^3)/6
```

Maple [C] time = 0.941, size = 5905, normalized size = 17.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.11961, size = 323, normalized size = 0.97

$$\frac{1}{6}x^6\log\left(\left(bx^2+a\right)^p c\right)^3 + \frac{1}{12}bp\left(\frac{6a^3\log\left(bx^2+a\right)}{b^4} - \frac{2b^2x^6 - 3abx^4 + 6a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^p c\right)^2 - \frac{1}{216}bp\left(\frac{\left(8b^3x^6 - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^3 + 1/12*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c)^2 - 1/216*b*p*((8*
b^3*x^6 - 57*a*b^2*x^4 - 36*a^3*log(b*x^2 + a)^3 + 510*a^2*b*x^2 - 198*a^3*
log(b*x^2 + a)^2 - 510*a^3*log(b*x^2 + a))*p^2/b^4 - 6*(4*b^3*x^6 - 15*a*b^
2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2 + a))*p*l
og((b*x^2 + a)^p*c)/b^4)
```

Fricas [A] time = 1.80952, size = 780, normalized size = 2.34

$$-\frac{8b^3p^3x^6 - 36b^3x^6\log\left(c\right)^3 - 57ab^2p^3x^4 + 510a^2bp^3x^2 - 36\left(b^3p^3x^6 + a^3p^3\right)\log\left(bx^2+a\right)^3 + 18\left(2b^3p^3x^6 - 3ab^2p^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(8*b^3*p^3*x^6 - 36*b^3*x^6*log(c)^3 - 57*a*b^2*p^3*x^4 + 510*a^2*b*
p^3*x^2 - 36*(b^3*p^3*x^6 + a^3*p^3)*log(b*x^2 + a)^3 + 18*(2*b^3*p^3*x^6 -
 3*a*b^2*p^3*x^4 + 6*a^2*b*p^3*x^2 + 11*a^3*p^3 - 6*(b^3*p^2*x^6 + a^3*p^2)
```

```
*log(c))*log(b*x^2 + a)^2 + 18*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2
)*log(c)^2 - 6*(4*b^3*p^3*x^6 - 15*a*b^2*p^3*x^4 + 66*a^2*b*p^3*x^2 + 85*a^
3*p^3 + 18*(b^3*p*x^6 + a^3*p)*log(c)^2 - 6*(2*b^3*p^2*x^6 - 3*a*b^2*p^2*x^
4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2)*log(c))*log(b*x^2 + a) - 6*(4*b^3*p^2*x^6
 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^2*x^2)*log(c))/b^3
```

Sympy [A] time = 50.4087, size = 561, normalized size = 1.68

$$\begin{cases} \frac{a^3p^3\log\left(a+bx^2\right)^3}{6b^3} - \frac{11a^3p^3\log\left(a+bx^2\right)^2}{12b^3} + \frac{85a^3p^3\log\left(a+bx^2\right)}{36b^3} + \frac{a^3p^2\log\left(c\right)\log\left(a+bx^2\right)^2}{2b^3} - \frac{11a^3p^2\log\left(c\right)\log\left(a+bx^2\right)}{6b^3} + \frac{a^3p\log\left(c\right)^2\log\left(a+bx^2\right)}{2b^3} - \\ \frac{x^6\log\left(a^pc\right)^3}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((a**3*p**3*log(a + b*x**2)**3/(6*b**3) - 11*a**3*p**3*log(a + b*x
**2)**2/(12*b**3) + 85*a**3*p**3*log(a + b*x**2)/(36*b**3) + a**3*p**2*log(
c)*log(a + b*x**2)**2/(2*b**3) - 11*a**3*p**2*log(c)*log(a + b*x**2)/(6*b**
3) + a**3*p*log(c)**2*log(a + b*x**2)/(2*b**3) - a**2*p**3*x**2*log(a + b*x
**2)**2/(2*b**2) + 11*a**2*p**3*x**2*log(a + b*x**2)/(6*b**2) - 85*a**2*p**
3*x**2/(36*b**2) - a**2*p**2*x**2*log(c)*log(a + b*x**2)/b**2 + 11*a**2*p**
2*x**2*log(c)/(6*b**2) - a**2*p*x**2*log(c)**2/(2*b**2) + a*p**3*x**4*log(a
 + b*x**2)**2/(4*b) - 5*a*p**3*x**4*log(a + b*x**2)/(12*b) + 19*a*p**3*x**4
/(72*b) + a*p**2*x**4*log(c)*log(a + b*x**2)/(2*b) - 5*a*p**2*x**4*log(c)/(
12*b) + a*p*x**4*log(c)**2/(4*b) + p**3*x**6*log(a + b*x**2)**3/6 - p**3*x*
*6*log(a + b*x**2)**2/6 + p**3*x**6*log(a + b*x**2)/9 - p**3*x**6/27 + p**2
*x**6*log(c)*log(a + b*x**2)**2/2 - p**2*x**6*log(c)*log(a + b*x**2)/3 + p*
*2*x**6*log(c)/9 + p*x**6*log(c)**2*log(a + b*x**2)/2 - p*x**6*log(c)**2/6
+ x**6*log(c)**3/6, Ne(b, 0)), (x**6*log(a**p*c)**3/6, True))
```

Giac [A] time = 1.25596, size = 803, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/216*(36*b*x^6*log(c)^3 + (36*(b*x^2 + a)^3*log(b*x^2 + a)^3/b^2 - 108*(b*
x^2 + a)^2*a*log(b*x^2 + a)^3/b^2 + 108*(b*x^2 + a)*a^2*log(b*x^2 + a)^3/b^
2 - 36*(b*x^2 + a)^3*log(b*x^2 + a)^2/b^2 + 162*(b*x^2 + a)^2*a*log(b*x^2 +
 a)^2/b^2 - 324*(b*x^2 + a)*a^2*log(b*x^2 + a)^2/b^2 + 24*(b*x^2 + a)^3*log
(b*x^2 + a)/b^2 - 162*(b*x^2 + a)^2*a*log(b*x^2 + a)/b^2 + 648*(b*x^2 + a)*
a^2*log(b*x^2 + a)/b^2 - 8*(b*x^2 + a)^3/b^2 + 81*(b*x^2 + a)^2*a/b^2 - 648
*(b*x^2 + a)*a^2/b^2)*p^3 + 6*(18*(b*x^2 + a)^3*log(b*x^2 + a)^2/b^2 - 54*(
b*x^2 + a)^2*a*log(b*x^2 + a)^2/b^2 + 54*(b*x^2 + a)*a^2*log(b*x^2 + a)^2/b
^2 - 12*(b*x^2 + a)^3*log(b*x^2 + a)/b^2 + 54*(b*x^2 + a)^2*a*log(b*x^2 + a
)/b^2 - 108*(b*x^2 + a)*a^2*log(b*x^2 + a)/b^2 + 4*(b*x^2 + a)^3/b^2 - 27*(
b*x^2 + a)^2*a/b^2 + 108*(b*x^2 + a)*a^2/b^2)*p^2*log(c) + 18*(6*(b*x^2 + a
)^3*log(b*x^2 + a)/b^2 - 18*(b*x^2 + a)^2*a*log(b*x^2 + a)/b^2 + 18*(b*x^2
+ a)*a^2*log(b*x^2 + a)/b^2 - 2*(b*x^2 + a)^3/b^2 + 9*(b*x^2 + a)^2*a/b^2 -
 18*(b*x^2 + a)*a^2/b^2)*p*log(c)^2)/b
```

3.92 $\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=211

$$\frac{3p^2\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}-\frac{3p\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2}+\frac{3ap\left(a+b\right.}{}$$

```
[Out] (3*a*p^3*x^2)/b - (3*p^3*(a + b*x^2)^2)/(16*b^2) - (3*a*p^2*(a + b*x^2)*Log
[c*(a + b*x^2)^p])/b^2 + (3*p^2*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(8*b^2)
 + (3*a*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) - (3*p*(a + b*x^2)^2*
Log[c*(a + b*x^2)^p]^2)/(8*b^2) - (a*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2
*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2)
```

Rubi [A] time = 0.205337, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{3p^2\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}-\frac{3p\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2}+\frac{3ap\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^3,x]
```


```
[Out] (3*a*p^3*x^2)/b - (3*p^3*(a + b*x^2)^2)/(16*b^2) - (3*a*p^2*(a + b*x^2)*Log
[c*(a + b*x^2)^p])/b^2 + (3*p^2*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(8*b^2)
 + (3*a*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) - (3*p*(a + b*x^2)^2*
Log[c*(a + b*x^2)^p]^2)/(8*b^2) - (a*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2
*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\log^3\left(c(a+bx)^p\right)}{b}+\frac{(a+bx)\log^3\left(c(a+bx)^p\right)}{b}\right)dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int(a+bx)\log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)}{2b}-\frac{a\operatorname{Subst}\left(\int\log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)}{2b}\\
&= \frac{\operatorname{Subst}\left(\int x\log^3\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b^2}-\frac{a\operatorname{Subst}\left(\int\log^3\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b^2}\\
&= -\frac{a\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^2}+\frac{\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{(3p)\operatorname{Subst}\left(\int x\log^2(c\cdots\right.}{4b^2}\\
&= \frac{3ap\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}-\frac{3p\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{a\left(a+bx^2\right)\log^3\left(c\cdots\right.}{2b^2}\\
&= \frac{3ap^3x^2}{b}-\frac{3p^3\left(a+bx^2\right)^2}{16b^2}-\frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}+\frac{3p^2\left(a+bx^2\right)^2\log\left(c\left(a+\cdots\right.\right.}{8b^2}
\end{aligned}$$

Mathematica [A] time = 0.0862747, size = 237, normalized size = 1.12

$$-\frac{9a^2p^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{a^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2}+\frac{9a^2p\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{3a^2p^3\log\left(a+bx^2\right)}{8b^2}+\frac{3}{8}p^2x^4\log\left(c\left(a+\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (21*a*p^3*x^2)/(8*b) - (3*p^3*x^4)/16 - (3*a^2*p^3*Log[a + b*x^2])/(8*b^2)
- (9*a^2*p^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (9*a*p^2*x^2*Log[c*(a + b*x^2)
^p])/(4*b) + (3*p^2*x^4*Log[c*(a + b*x^2)^p])/8 + (9*a^2*p*Log[c*(a + b*x^2
)^p]^2)/(8*b^2) + (3*a*p*x^2*Log[c*(a + b*x^2)^p]^2)/(4*b) - (3*p*x^4*Log[c
*(a + b*x^2)^p]^2)/8 - (a^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2) + (x^4*Log[c*(a
 + b*x^2)^p]^3)/4
```

Maple [C] time = 0.87, size = 4942, normalized size = 23.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] -3/16*x^4*p^3+3/8*(I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b^2*
x^4*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+
2*ln(c)*b^2*x^4-b^2*p*x^4+2*a*b*p*x^2-2*a^2*p*ln(b*x^2+a))/b^2*ln((b*x^2+a)
^p)^2+3/8*ln(c)*Pi^2*x^4*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/16*ln(c)*Pi^2*
x^4*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+3/32*Pi^2*p*x^4*csgn(I*(b*x^2+a)^p)
^2*csgn(I*c*(b*x^2+a)^p)^4-3/16*Pi^2*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)^5-3/16*Pi^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)+3/32*Pi^2*p*x
^4*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-1/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^
3*csgn(I*c*(b*x^2+a)^p)^6+3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b
*x^2+a)^p)^7-3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^8-3/
32*I*Pi^3*x^4*csgn(I*c*(b*x^2+a)^p)^8*csgn(I*c)+3/32*I*Pi^3*x^4*csgn(I*c*(b
*x^2+a)^p)^7*csgn(I*c)^2-1/32*I*Pi^3*x^4*csgn(I*c*(b*x^2+a)^p)^6*csgn(I*c)^
3-3/8*I*ln(c)^2*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^3-3/16*I*Pi*p^2*x^4*csgn(I*c*(
b*x^2+a)^p)^3+3/8*ln(c)*p^2*x^4-3/8*ln(c)^2*p*x^4-1/4/b^2*a^2*p^3*ln(b*x^2+
a)^3-9/8/b^2*a^2*p^3*ln(b*x^2+a)^2-3/16*ln(c)*Pi^2*x^4*csgn(I*c*(b*x^2+a)^p
)^6+3/32*Pi^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^6+1/32*I*Pi^3*x^4*csgn(I*c*(b*x^2
+a)^p)^9+21/8*a*p^3*x^2/b-21/8*a^2*p^3/b^2*ln(b*x^2+a)-3/16*ln(c)*Pi^2*x^4*
csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/4/b*ln(c)^2*a*p*x^2+3/4/b^2
*ln(c)*a^2*p^2*ln(b*x^2+a)^2-9/4/b*ln(c)*a*p^2*x^2-3/4/b^2*ln(c)^2*ln(b*x^2
+a)*a^2*p+9/4/b^2*ln(c)*ln(b*x^2+a)*a^2*p^2+3/8*ln(c)*Pi^2*x^4*csgn(I*(b*x^
2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+1/4*x^4*ln((b*x^2+a)^p)^3+3/16*(-12*x^2*b*a
*p^2-4*I*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*I*Pi*a*b*
p*x^2*csgn(I*c*(b*x^2+a)^p)^3+4*I*ln(c)*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)^2+12*ln(b*x^2+a)*a^2*p^2+4*ln(c)^2*b^2*x^4+4*a^2*p^2*ln(b
*x^2+a)^2-4*ln(c)*b^2*p*x^4-8*ln(c)*ln(b*x^2+a)*a^2*p+2*x^4*b^2*p^2+8*ln(c)
*a*b*p*x^2-Pi^2*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^6-4*I*Pi*ln(b*x^2+a)*a^2*p*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+4*I*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c
*(b*x^2+a)^p)^3-4*I*Pi*a*b*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*
csgn(I*c)-Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+2*Pi^2
*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+2*Pi^2*b^2*x^4*csgn(I*
c*(b*x^2+a)^p)^5*csgn(I*c)-Pi^2*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2
+2*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-Pi^
2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-4*Pi^2*
b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+2*Pi^2*b^2*x^
4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-4*I*ln(c)*Pi*b^2*
x^4*csgn(I*c*(b*x^2+a)^p)^3+2*I*Pi*b^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^3+2*I*Pi
*b^2*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+4*I*Pi*ln(b*
x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-2*I*Pi*b^2
*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-2*I*Pi*b^2*p*x^4*csgn(I*
c*(b*x^2+a)^p)^2*csgn(I*c)+4*I*Pi*a*b*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^2+4*I*Pi*a*b*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*I*ln(c)*P
```

i*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+4*I*ln(c)*Pi*
b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c))/b^2*ln((b*x^2+a)^p)+9/8*I/b*Pi*a
*p^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/4*I/b^2*ln(c
)*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3/4*I/b^
2*ln(c)*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-9/8*I/b^2*Pi
*ln(b*x^2+a)*a^2*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+3/
4*I/b*ln(c)*Pi*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/4*I/b*
ln(c)*Pi*a*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-3/8*I/b^2*Pi*a^2*p^2*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*ln(b*x^2+a)^2-3/4*I/b*ln(c
)*Pi*a*p*x^2*csgn(I*c*(b*x^2+a)^p)^3+3/8*I/b^2*Pi*a^2*p^2*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^2*ln(b*x^2+a)^2-9/8*I/b*Pi*a*p^2*x^2*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/8*I/b^2*Pi*a^2*p^2*csgn(I*c*(b*x^2+a)^p)^2
*csgn(I*c)*ln(b*x^2+a)^2-9/8*I/b*Pi*a*p^2*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)+3/4*I/b^2*ln(c)*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^3+9/8*I/b^2
*Pi*ln(b*x^2+a)*a^2*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+9/8*I/b
^2*Pi*ln(b*x^2+a)*a^2*p^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/8/b*Pi^2*a*p*
x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/16/b*Pi^2*a*p
*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2+3/8*I*ln(c)*
Pi*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/4/b*Pi^2*a*p
*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/8/b*Pi^2*a*p*x
^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-3/8/b^2*Pi^2*ln(
b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)+3/16
/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)^2+3/4/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x
^2+a)^p)^4*csgn(I*c)-3/8/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+3/8*ln(c)*Pi^2*x^4*csgn(I*(b*x^2+a)^p)^2*c
sgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/16*ln(c)*Pi^2*x^4*csgn(I*(b*x^2+a)^p)^2*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-3/16/b*Pi^2*a*p*x^2*csgn(I*c*(b*x^2+a)^
p)^6+3/16/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^6-3/4*ln(c)*Pi^2
*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/8*ln(c)*Pi^2*x
^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-3/16*Pi^2*p*x^4*
csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)+3/32*Pi^2*p*x^4*csg
n(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2+3/8*Pi^2*p*x^4*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)-3/16*Pi^2*p*x^4*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+3/32*I*Pi^3*x^4*csgn(I*(b*x^2
+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)
^p)^3*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+1/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^
p)^3*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^3-9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p
)^2*csgn(I*c*(b*x^2+a)^p)^6*csgn(I*c)+9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^2
*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)^2-3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^2*
csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^3+9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)^7*csgn(I*c)-9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)^6*csgn(I*c)^2+3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^5*csgn(I*c)^3+3/8*I*ln(c)^2*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2+3/8*I*ln(c)^2*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/8*I
*ln(c)*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^3+3/16*I*Pi*p^2*x^4*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^2+3/16*I*Pi*p^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I
*c)+1/4*ln(c)^3*x^4-3/4*I/b*ln(c)*Pi*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)*csgn(I*c)+3/4*I/b^2*ln(c)*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)
^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+3/16/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*
c*(b*x^2+a)^p)^4*csgn(I*c)^2-3/16*I*Pi*p^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)*csgn(I*c)-3/8*I*ln(c)^2*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)*csgn(I*c)-3/8*I*ln(c)*Pi*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)^2-3/8*I*ln(c)*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-3/8*I/b^
2*Pi*a^2*p^2*csgn(I*c*(b*x^2+a)^p)^3*ln(b*x^2+a)^2+9/8*I/b*Pi*a*p^2*x^2*csg
n(I*c*(b*x^2+a)^p)^3-9/8*I/b^2*Pi*ln(b*x^2+a)*a^2*p^2*csgn(I*c*(b*x^2+a)^p)
^3-3/16/b*Pi^2*a*p*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/8/b*
Pi^2*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+3/8/b*Pi^2*a*p*x^2
*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/16/b*Pi^2*a*p*x^2*csgn(I*c*(b*x^2+a)^p

```
)^4*csgn(I*c)^2+3/16/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)^2*csgn(
I*c*(b*x^2+a)^p)^4-3/8/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^5-3/8/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^5*c
sgn(I*c)
```

Maxima [A] time = 1.06158, size = 274, normalized size = 1.3

$$\frac{1}{4}x^4\log\left(\left(bx^2+a\right)^p c\right)^3 - \frac{3}{8}bp\left(\frac{2a^2\log\left(bx^2+a\right)}{b^3} + \frac{bx^4-2ax^2}{b^2}\right)\log\left(\left(bx^2+a\right)^p c\right)^2 - \frac{1}{16}bp\left(\frac{\left(3b^2x^4+4a^2\log\left(bx^2+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^3 - 3/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c)^2 - 1/16*b*p*((3*b^2*x^4 + 4*a^2*log(
b*x^2 + a)^3 - 42*a*b*x^2 + 18*a^2*log(b*x^2 + a)^2 + 42*a^2*log(b*x^2 + a)
)*p^2/b^3 - 6*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*log(b*x^2 + a)^2 + 6*a^2*log(b*x
^2 + a))*p*log((b*x^2 + a)^p*c)/b^3)
```

Fricas [A] time = 2.18504, size = 587, normalized size = 2.78

$$-\frac{3b^2p^3x^4 - 4b^2x^4\log(c)^3 - 42abp^3x^2 - 4\left(b^2p^3x^4 - a^2p^3\right)\log\left(bx^2+a\right)^3 + 6\left(b^2p^3x^4 - 2abp^3x^2 - 3a^2p^3 - 2\left(b^2p^2x^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*b^2*p^3*x^4 - 4*b^2*x^4*log(c)^3 - 42*a*b*p^3*x^2 - 4*(b^2*p^3*x^4
 - a^2*p^3)*log(b*x^2 + a)^3 + 6*(b^2*p^3*x^4 - 2*a*b*p^3*x^2 - 3*a^2*p^3 -
 2*(b^2*p^2*x^4 - a^2*p^2)*log(c))*log(b*x^2 + a)^2 + 6*(b^2*p*x^4 - 2*a*b*
p*x^2)*log(c)^2 - 6*(b^2*p^3*x^4 - 6*a*b*p^3*x^2 - 7*a^2*p^3 + 2*(b^2*p*x^4
 - a^2*p)*log(c)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2)*log(c))*lo
g(b*x^2 + a) - 6*(b^2*p^2*x^4 - 6*a*b*p^2*x^2)*log(c))/b^2
```

Sympy [A] time = 24.0075, size = 450, normalized size = 2.13

$$\begin{cases} -\frac{a^2p^3\log\left(a+bx^2\right)^3}{4b^2} + \frac{9a^2p^3\log\left(a+bx^2\right)^2}{8b^2} - \frac{21a^2p^3\log\left(a+bx^2\right)}{8b^2} - \frac{3a^2p^2\log(c)\log\left(a+bx^2\right)^2}{4b^2} + \frac{9a^2p^2\log(c)\log\left(a+bx^2\right)}{4b^2} - \frac{3a^2p\log(c)^2\log\left(a+bx^2\right.}{4b^2} \\ \frac{x^4\log\left(a^pc\right)^3}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((-a**2*p**3*log(a + b*x**2)**3/(4*b**2) + 9*a**2*p**3*log(a + b*x
**2)**2/(8*b**2) - 21*a**2*p**3*log(a + b*x**2)/(8*b**2) - 3*a**2*p**2*log(
c)*log(a + b*x**2)**2/(4*b**2) + 9*a**2*p**2*log(c)*log(a + b*x**2)/(4*b**2
) - 3*a**2*p*log(c)**2*log(a + b*x**2)/(4*b**2) + 3*a*p**3*x**2*log(a + b*x
```

```
**2)**2/(4*b) - 9*a*p**3*x**2*log(a + b*x**2)/(4*b) + 21*a*p**3*x**2/(8*b)
+ 3*a*p**2*x**2*log(c)*log(a + b*x**2)/(2*b) - 9*a*p**2*x**2*log(c)/(4*b) +
 3*a*p*x**2*log(c)**2/(4*b) + p**3*x**4*log(a + b*x**2)**3/4 - 3*p**3*x**4*
log(a + b*x**2)**2/8 + 3*p**3*x**4*log(a + b*x**2)/8 - 3*p**3*x**4/16 + 3*p
**2*x**4*log(c)*log(a + b*x**2)**2/4 - 3*p**2*x**4*log(c)*log(a + b*x**2)/4
 + 3*p**2*x**4*log(c)/8 + 3*p*x**4*log(c)**2*log(a + b*x**2)/4 - 3*p*x**4*l
og(c)**2/8 + x**4*log(c)**3/4, Ne(b, 0)), (x**4*log(a**p*c)**3/4, True))
```

Giac [A] time = 1.309, size = 486, normalized size = 2.3

$$\frac{\left(4\left(bx^2+a\right)^2\log\left(bx^2+a\right)^3-8\left(bx^2+a\right)a\log\left(bx^2+a\right)^3-6\left(bx^2+a\right)^2\log\left(bx^2+a\right)^2+24\left(bx^2+a\right)a\log\left(bx^2+a\right)^2+6\left(bx^2+a\right)^2\log\left(bx^2+a\right)-48\left(bx^2+a\right)a\log\left(bx^2+a\right)-3\left(bx^2\right.\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/16*((4*(b*x^2 + a)^2*log(b*x^2 + a)^3 - 8*(b*x^2 + a)*a*log(b*x^2 + a)^3
- 6*(b*x^2 + a)^2*log(b*x^2 + a)^2 + 24*(b*x^2 + a)*a*log(b*x^2 + a)^2 + 6*
(b*x^2 + a)^2*log(b*x^2 + a) - 48*(b*x^2 + a)*a*log(b*x^2 + a) - 3*(b*x^2 +
 a)^2 + 48*(b*x^2 + a)*a)*p^3/b + 6*(2*(b*x^2 + a)^2*log(b*x^2 + a)^2 - 4*(
b*x^2 + a)*a*log(b*x^2 + a)^2 - 2*(b*x^2 + a)^2*log(b*x^2 + a) + 8*(b*x^2 +
 a)*a*log(b*x^2 + a) + (b*x^2 + a)^2 - 8*(b*x^2 + a)*a)*p^2*log(c)/b + 6*(2
*(b*x^2 + a)^2*log(b*x^2 + a) - 4*(b*x^2 + a)*a*log(b*x^2 + a) - (b*x^2 + a
)^2 + 4*(b*x^2 + a)*a)*p*log(c)^2/b + 4*((b*x^2 + a)^2 - 2*(b*x^2 + a)*a)*l
og(c)^3/b)/b
```

3.93 $\int x \log^3\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=93

$$\frac{3p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}+\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b}-3p^3x^2$$

```
[Out] -3*p^3*x^2 + (3*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b - (3*p*(a + b*x^2)*
Log[c*(a + b*x^2)^p]^2)/(2*b) + ((a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Rubi [A] time = 0.0656985, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2389, 2296, 2295}

$$\frac{3p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}+\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b}-3p^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -3*p^3*x^2 + (3*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b - (3*p*(a + b*x^2)*
Log[c*(a + b*x^2)^p]^2)/(2*b) + ((a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int x \log^3\left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{2}\text{Subst}\left(\int \log^3\left(c(a+bx)^p\right) dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \log^3\left(cx^p\right) dx, x, a+bx^2\right)}{2b}$$
$$= \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b} - \frac{(3p)\,\text{Subst}\left(\int \log^2\left(cx^p\right) dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b} + \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b} + \frac{\left(3p^2\right)\text{Subst}\left(\int \log(cx\right.}{b}$$
$$= -3p^3x^2 + \frac{3p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b} + \frac{\left(a+bx^2\right)\text{lo}}{}$$

Mathematica [A] time = 0.0121931, size = 87, normalized size = 0.94

$$\frac{6p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right) - 3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right) + \left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right) - 6bp^3x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-6*b*p^3*x^2 + 6*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p] - 3*p*(a + b*x^2)*Lo
g[c*(a + b*x^2)^p]^2 + (a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Maple [C] time = 0.843, size = 3925, normalized size = 42.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] -3/2/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csg
n(I*c)+3/4/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
^3*csgn(I*c)^2+3/4/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b
*x^2+a)^p)^3*csgn(I*c)+1/2/b*a*p^3*ln(b*x^2+a)^3+3/2/b*a*p^3*ln(b*x^2+a)^2-
3/8*ln(c)*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^6+3/8*Pi^2*p*x^2*csgn(I*c*(b*x^2+a
)^p)^6+1/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^9-3*p^3*x^2+3/4*(I*Pi*b*x^2*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*b*x^2
*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)*b*x^2-2*x^2*p*b+2*a*p*ln(b*x^2+a
))/b*ln((b*x^2+a)^p)^2+3*a*p^3/b*ln(b*x^2+a)+3/8*(8*x^2*b*p^2+4*ln(c)^2*b*x
^2-4*a*p^2*ln(b*x^2+a)^2-8*ln(c)*b*p*x^2+8*ln(c)*ln(b*x^2+a)*a*p-8*ln(b*x^2
+a)*a*p^2-Pi^2*b*x^2*csgn(I*c*(b*x^2+a)^p)^6+2*Pi^2*b*x^2*csgn(I*c*(b*x^2+a
)^p)^5*csgn(I*c)-Pi^2*b*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+4*I*Pi*ln(b
*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+4*I*Pi*ln(b*x^2+a)*
a*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*I*ln(c)*Pi*b*x^2*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c
*(b*x^2+a)^p)^4+2*Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+4*
I*ln(c)*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*I*Pi*b*p*x^2*csgn(I*(b
*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-4*I*Pi*b*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)-4*I*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*c
```

sgn(I*c)+4*I*Pi*b*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)
+2*Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-Pi^2*
b*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-4*Pi^2*b*x^
2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+2*Pi^2*b*x^2*csgn(I
*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-4*I*ln(c)*Pi*b*x^2*csgn(I
c(b*x^2+a)^p)^3+4*I*Pi*b*p*x^2*csgn(I*c*(b*x^2+a)^p)^3-4*I*Pi*ln(b*x^2+a)
*a*p*csgn(I*c*(b*x^2+a)^p)^3+4*I*ln(c)*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*
c*(b*x^2+a)^p)^2)/b*ln((b*x^2+a)^p)-3/2*ln(c)^2*p*x^2+3*ln(c)*p^2*x^2-3/8*l
n(c)*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-3/8*ln(c)*Pi^2*x^2*csgn(I
*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4-3/4*Pi^2*p*x^2*csgn(I*c*(b*x^2+a)^p
)^5*csgn(I*c)-3/4*Pi^2*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+3/
8*Pi^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+3/8*Pi^2*p*x^2*csgn(I*(b*x
^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4-3/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^8*
csgn(I*c)-3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^8+3/16*
I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^7*csgn(I*c)^2+3/16*I*Pi^3*x^2*csgn(I*(b*x^
2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^7-1/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^6*c
sgn(I*c)^3-1/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^6-3/
4*I*ln(c)^2*Pi*x^2*csgn(I*c*(b*x^2+a)^p)^3-3/2*I*Pi*p^2*x^2*csgn(I*c*(b*x^2
+a)^p)^3-3/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)^2+3/2*I*ln(c)*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(
I*c)+3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/2
*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3
/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(
I*c)+3/4*I/b*Pi*a*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*l
n(b*x^2+a)^2+1/2*x^2*ln((b*x^2+a)^p)^3-3/8/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b
*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-3/4*I/b*Pi*a*p^2*csgn(I*c*
(b*x^2+a)^p)^2*csgn(I*c)*ln(b*x^2+a)^2-3/4*I/b*Pi*a*p^2*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)^2*ln(b*x^2+a)^2-3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csg
n(I*c*(b*x^2+a)^p)^3-3/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)+3/4*ln(c)*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)+3/4*ln(c)*Pi^
2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5-3/2/b*ln(c)*a*p^2*ln(b*x^
2+a)^2+3/2/b*ln(c)^2*ln(b*x^2+a)*a*p-3/b*ln(c)*ln(b*x^2+a)*a*p^2+1/2*ln(c)^
3*x^2+9/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*
c)^2+3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c
)-3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^3
-3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+
1/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^3+3
/4*I*ln(c)^2*Pi*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/4*I*ln(c)^2*Pi*x^2*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/2*I*ln(c)*Pi*p*x^2*csgn(I*c*(
b*x^2+a)^p)^3+3/2*I*Pi*p^2*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/2*I*Pi*p
^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3/2*I/b*ln(c)*Pi*ln(b*x^
2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/2*ln(c)*Pi^2
*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/4*ln(c)*Pi^2*x
^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+3/4*ln(c)*Pi^2*x
^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/8*ln(c)*Pi^2*x
^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2+3/2*Pi^2*p*x^2
csgn(I(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)-3/4*Pi^2*p*x^2*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-3/4*Pi^2*p*x^2*csgn(I*(b
*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)+3/8*Pi^2*p*x^2*csgn(I*(b*x^2
+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-3/8/b*Pi^2*ln(b*x^2+a)*a*p*csg
n(I*c*(b*x^2+a)^p)^6+9/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^7*csgn(I*c)-9/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^6
*csgn(I*c)^2-9/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^6*
csgn(I*c)+3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5*csgn(
I*c)^3+3/4/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)+3/4/b*P
i^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5-3/8/b*Pi^2*
ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-3/8/b*Pi^2*ln(b*x^2+a)*
a*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4-3/4*I*ln(c)^2*Pi*x^2*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/2*I*ln(c)*Pi*p*x^2*csgn(I

```
*c*(b*x^2+a)^p)^2*csgn(I*c)-3/2*I*ln(c)*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2-3/2*I*Pi*p^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)*csgn(I*c)+3/4*I/b*Pi*a*p^2*csgn(I*c*(b*x^2+a)^p)^3*ln(b*x^2+a)^2+3/2*I/b*
Pi*ln(b*x^2+a)*a*p^2*csgn(I*c*(b*x^2+a)^p)^3
```

Maxima [A] time = 1.09372, size = 221, normalized size = 2.38

$$-\frac{3}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^p c\right)^2+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^p c\right)^3+\frac{1}{2}\,bp\left(\frac{\left(a\log\left(bx^2+a\right)^3-6\,bx^2+3\,a\log\left(bx^2\right.\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c)^2 + 1/2*x^2*lo
g((b*x^2 + a)^p*c)^3 + 1/2*b*p*((a*log(b*x^2 + a)^3 - 6*b*x^2 + 3*a*log(b*x
^2 + a)^2 + 6*a*log(b*x^2 + a))*p^2/b^2 + 3*(2*b*x^2 - a*log(b*x^2 + a)^2 -
 2*a*log(b*x^2 + a))*p*log((b*x^2 + a)^p*c)/b^2)
```

Fricas [A] time = 1.86092, size = 393, normalized size = 4.23

$$-\frac{6\,bp^3x^2-6\,bp^2x^2\log\left(c\right)+3\,bpx^2\log\left(c\right)^2-bx^2\log\left(c\right)^3-\left(bp^3x^2+ap^3\right)\log\left(bx^2+a\right)^3+3\left(bp^3x^2+ap^3-\left(bp^2x^2+ap^2\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*b*p^3*x^2 - 6*b*p^2*x^2*log(c) + 3*b*p*x^2*log(c)^2 - b*x^2*log(c)^
3 - (b*p^3*x^2 + a*p^3)*log(b*x^2 + a)^3 + 3*(b*p^3*x^2 + a*p^3 - (b*p^2*x^
2 + a*p^2)*log(c))*log(b*x^2 + a)^2 - 3*(2*b*p^3*x^2 + 2*a*p^3 + (b*p*x^2 +
 a*p)*log(c)^2 - 2*(b*p^2*x^2 + a*p^2)*log(c))*log(b*x^2 + a))/b
```

Sympy [A] time = 7.31956, size = 301, normalized size = 3.24

$$\begin{cases}\frac{ap^3\log\left(a+bx^2\right)^3}{2b}-\frac{3ap^3\log\left(a+bx^2\right)^2}{2b}+\frac{3ap^3\log\left(a+bx^2\right)}{b}+\frac{3ap^2\log\left(c\right)\log\left(a+bx^2\right)^2}{2b}-\frac{3ap^2\log\left(c\right)\log\left(a+bx^2\right)}{b}+\frac{3ap\log\left(c\right)^2\log\left(a+bx^2\right)}{2b}+\frac{p^3x^2\log\ldots}{} \\ \frac{x^2\log\left(a^pc\right)^3}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((a*p**3*log(a + b*x**2)**3/(2*b) - 3*a*p**3*log(a + b*x**2)**2/(2
*b) + 3*a*p**3*log(a + b*x**2)/b + 3*a*p**2*log(c)*log(a + b*x**2)**2/(2*b)
 - 3*a*p**2*log(c)*log(a + b*x**2)/b + 3*a*p*log(c)**2*log(a + b*x**2)/(2*b
) + p**3*x**2*log(a + b*x**2)**3/2 - 3*p**3*x**2*log(a + b*x**2)**2/2 + 3*p
**3*x**2*log(a + b*x**2) - 3*p**3*x**2 + 3*p**2*x**2*log(c)*log(a + b*x**2)
**2/2 - 3*p**2*x**2*log(c)*log(a + b*x**2) + 3*p**2*x**2*log(c) + 3*p*x**2*
log(c)**2*log(a + b*x**2)/2 - 3*p*x**2*log(c)**2/2 + x**2*log(c)**3/2, Ne(b
```

```
, 0)), (x**2*log(a**p*c)**3/2, True))
```

Giac [A] time = 1.28473, size = 228, normalized size = 2.45

$$\frac{\left(\left(bx^2+a\right)\log\left(bx^2+a\right)^3-6\,bx^2-3\left(bx^2+a\right)\log\left(bx^2+a\right)^2+6\left(bx^2+a\right)\log\left(bx^2+a\right)-6\,a\right)p^3+3\left(2\,bx^2+\left(bx^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a)
^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*log
(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*p^2*log(c) - 3*(b*x^2 -
 (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c)^2 + (b*x^2 + a)*log(c)^3)/b
```

3.94 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x} dx$

Optimal. Leaf size=106

$$-3p^2\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{3}{2}p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log^2\left(c\left(a+bx^2\right)^p\right)+3p^3\text{PolyLog}\left(4,\frac{bx^2}{a}+1\right)$$

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^3)/2 + (3*p*Log[c*(a + b*x^2)^p]^2*
PolyLog[2, 1 + (b*x^2)/a])/2 - 3*p^2*Log[c*(a + b*x^2)^p]*PolyLog[3, 1 + (b
*x^2)/a] + 3*p^3*PolyLog[4, 1 + (b*x^2)/a]
```

Rubi [A] time = 0.161411, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-3p^2\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{3}{2}p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log^2\left(c\left(a+bx^2\right)^p\right)+3p^3\text{PolyLog}\left(4,\frac{bx^2}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^3)/2 + (3*p*Log[c*(a + b*x^2)^p]^2*
PolyLog[2, 1 + (b*x^2)/a])/2 - 3*p^2*Log[c*(a + b*x^2)^p]*PolyLog[3, 1 + (b
*x^2)/a] + 3*p^3*PolyLog[4, 1 + (b*x^2)/a]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
```

```
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(3bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^p\right)}{a+bx}\,dx,x,x^2\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(3p)\,\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx,x,a+b\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right) - \left(3p^2\right)\text{Subst}\left(\int\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right) - 3p^2\log\left(c\left(a+\right.\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right) - 3p^2\log\left(c\left(a+\right.\right.$$

Mathematica [B] time = 0.0966514, size = 279, normalized size = 2.63

$$-\frac{3}{2}p^2\left(-2\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)+2\log\left(a+bx^2\right)\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)\right)\left(p\log\left(a+bx^2\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3 + 3*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])^2*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a
]) - PolyLog[2, -((b*x^2)/a)]/2) - (3*p^2*(p*Log[a + b*x^2] - Log[c*(a + b*
x^2)^p])*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*Log[a + b*x^2]*PolyLog[2,
1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))/2 + (p^3*(Log[-((b*x^2)/a)]*
Log[a + b*x^2]^3 + 3*Log[a + b*x^2]^2*PolyLog[2, 1 + (b*x^2)/a] - 6*Log[a +
 b*x^2]*PolyLog[3, 1 + (b*x^2)/a] + 6*PolyLog[4, 1 + (b*x^2)/a]))/2
```

Maple [F] time = 0.868, size = 0, normalized size = 0.

$$\int \frac{\left(\ln \left(c \left(b x^{2}+a \right)^{p}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(b x^{2}+a\right)^{p} c\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log \left(\left(b x^{2}+a\right)^{p} c\right)^{3}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(c \left(a + b x^{2}\right)^{p}\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x, x)
```

3.95 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^3} dx$

Optimal. Leaf size=119

$$\frac{3bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}-\frac{3bp^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{a}+\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-\frac{\left(a+bx^2\right)\ldots}{\ldots}$$

```
[Out] (3*b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/(2*a) - ((a + b*x^2)*Log[c
*(a + b*x^2)^p]^3)/(2*a*x^2) + (3*b*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 +
 (b*x^2)/a])/a - (3*b*p^3*PolyLog[3, 1 + (b*x^2)/a])/a
```

Rubi [A] time = 0.145445, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2397, 2396, 2433, 2374, 6589}

$$\frac{3bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}-\frac{3bp^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{a}+\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-\frac{\left(a+bx^2\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^3,x]
```


```
[Out] (3*b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/(2*a) - ((a + b*x^2)*Log[c
*(a + b*x^2)^p]^3)/(2*a*x^2) + (3*b*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 +
 (b*x^2)/a])/a - (3*b*p^3*PolyLog[3, 1 + (b*x^2)/a])/a
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{(3bp)\,\text{Subst}\left(\int \frac{\log^2(c(a+bx)^p)}{x}\,dx, x, x^2\right)}{2a}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a} - \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2} - \frac{\left(3b^2p^2\right)\text{Subst}\left(\int \frac{\log}{}\right.}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a} - \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2} - \frac{\left(3bp^2\right)\text{Subst}\left(\int \frac{\log(}{}\right.}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a} - \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{3bp^2\log\left(c\left(a+bx^2\right)\right.}{a}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a} - \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{3bp^2\log\left(c\left(a+bx^2\right)\right.}{a}$$

Mathematica [B] time = 0.287264, size = 302, normalized size = 2.54

$$-\frac{-6bp^2x^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+6bp^3x^2\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)-3bp^2x^2\log^2\left(a+bx^2\right)\log\left(c\left(a+b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

```
[Out] -(-6*b*p^3*x^2*Log[x]*Log[a + b*x^2]^2 + 3*b*p^3*x^2*Log[-((b*x^2)/a)]*Log[
a + b*x^2]^2 + b*p^3*x^2*Log[a + b*x^2]^3 + 12*b*p^2*x^2*Log[x]*Log[a + b*x
^2]*Log[c*(a + b*x^2)^p] - 6*b*p^2*x^2*Log[-((b*x^2)/a)]*Log[a + b*x^2]*Log
[c*(a + b*x^2)^p] - 3*b*p^2*x^2*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] - 6*b
*p*x^2*Log[x]*Log[c*(a + b*x^2)^p]^2 + 3*b*p*x^2*Log[a + b*x^2]*Log[c*(a +
```

```
b*x^2)^p]^2 + a*Log[c*(a + b*x^2)^p]^3 - 6*b*p^2*x^2*Log[c*(a + b*x^2)^p]*P
olyLog[2, 1 + (b*x^2)/a] + 6*b*p^3*x^2*PolyLog[3, 1 + (b*x^2)/a])/(2*a*x^2)
```

Maple [F] time = 1.296, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(bx^2+a\right)^p\right)^3}{2\,x^2}+\int \frac{bx^2\log\left(c\right)^3+a\log\left(c\right)^3+3\left(b\left(p+\log\left(c\right)\right)x^2+a\log\left(c\right)\right)\log\left(\left(bx^2+a\right)^p\right)^2+3\left(bx^2\log\left(c\right)^2+a\,\mathrm{l}\right.}{bx^5+ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log((b*x^2 + a)^p)^3/x^2 + integrate((b*x^2*log(c)^3 + a*log(c)^3 + 3*
(b*(p + log(c))*x^2 + a*log(c))*log((b*x^2 + a)^p)^2 + 3*(b*x^2*log(c)^2 +
a*log(c)^2)*log((b*x^2 + a)^p))/(b*x^5 + a*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^3, x)
```

3.96 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=219

$$\frac{3b^2p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(3,\frac{a}{a+bx^2}\right)}{2a^2}+\frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\cdots}{\cdots}$$

```
[Out] (3*b^2*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^2) - (3*b*p*(a + b*
x^2)*Log[c*(a + b*x^2)^p]^2)/(4*a^2*x^2) - Log[c*(a + b*x^2)^p]^3/(4*x^4) -
 (3*b^2*p*Log[c*(a + b*x^2)^p]^2*Log[1 - a/(a + b*x^2)])/(4*a^2) + (3*b^2*p
^2*Log[c*(a + b*x^2)^p]*PolyLog[2, a/(a + b*x^2)])/(2*a^2) + (3*b^2*p^3*Pol
yLog[2, 1 + (b*x^2)/a])/(2*a^2) + (3*b^2*p^3*PolyLog[3, a/(a + b*x^2)])/(2*
a^2)
```

Rubi [A] time = 0.433736, antiderivative size = 236, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391}

$$-\frac{3b^2p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{3b^2p^2\log\left(\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (3*b^2*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^2) - (3*b*p*(a + b*
x^2)*Log[c*(a + b*x^2)^p]^2)/(4*a^2*x^2) - (3*b^2*p*Log[-((b*x^2)/a)]*Log[c
*(a + b*x^2)^p]^2)/(4*a^2) + (b^2*Log[c*(a + b*x^2)^p]^3)/(4*a^2) - Log[c*(
a + b*x^2)^p]^3/(4*x^4) + (3*b^2*p^3*PolyLog[2, 1 + (b*x^2)/a])/(2*a^2) - (
3*b^2*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 + (b*x^2)/a])/(2*a^2) + (3*b^2*
p^3*PolyLog[3, 1 + (b*x^2)/a])/(2*a^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(3bp)\operatorname{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(3p)\operatorname{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{(3p)\operatorname{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{4a} - \frac{(3bp)\operatorname{Subst}\left(\int \frac{\log^2(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x\right.}{4a}$$

$$= -\frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} - \frac{(3bp)\operatorname{Subst}\left(\int \frac{\log^2(cx^p)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+\right.}{4a^2}$$

$$= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{3b^2p\log\left(-\frac{bx^2}{a}\right)\log}{4a^2}$$

$$= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{3b^2p\log\left(-\frac{bx^2}{a}\right)\log}{4a^2}$$

$$= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{3b^2p\log\left(-\frac{bx^2}{a}\right)\log}{4a^2}$$

Mathematica [B] time = 0.343504, size = 478, normalized size = 2.18

$$\frac{6b^2p^2x^4\operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\left(p-\log\left(c\left(a+bx^2\right)^p\right)\right)+6b^2p^3x^4\operatorname{PolyLog}\left(3,\frac{bx^2}{a}+1\right)-a^2\log^3\left(c\left(a+bx^2\right)^p\right)-3b^2p^2x^4\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (-12*b^2*p^3*x^4*Log[x]*Log[a + b*x^2] + 6*b^2*p^3*x^4*Log[-((b*x^2)/a)]*Lo
g[a + b*x^2] + 3*b^2*p^3*x^4*Log[a + b*x^2]^2 - 6*b^2*p^3*x^4*Log[x]*Log[a
+ b*x^2]^2 + 3*b^2*p^3*x^4*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + b^2*p^3*x^4
*Log[a + b*x^2]^3 + 12*b^2*p^2*x^4*Log[x]*Log[c*(a + b*x^2)^p] - 6*b^2*p^2*
x^4*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] + 12*b^2*p^2*x^4*Log[x]*Log[a + b*x
^2]*Log[c*(a + b*x^2)^p] - 6*b^2*p^2*x^4*Log[-((b*x^2)/a)]*Log[a + b*x^2]*L
og[c*(a + b*x^2)^p] - 3*b^2*p^2*x^4*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] -
 3*a*b*p*x^2*Log[c*(a + b*x^2)^p]^2 - 6*b^2*p*x^4*Log[x]*Log[c*(a + b*x^2)^
p]^2 + 3*b^2*p*x^4*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2 - a^2*Log[c*(a + b
*x^2)^p]^3 + 6*b^2*p^2*x^4*(p - Log[c*(a + b*x^2)^p])*PolyLog[2, 1 + (b*x^2
)/a] + 6*b^2*p^3*x^4*PolyLog[3, 1 + (b*x^2)/a])/(4*a^2*x^4)
```

Maple [F] time = 1.501, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(bx^2+a\right)^p\right)^3}{4\,x^4}+\int \frac{2\,bx^2\log\left(c\right)^3+2\,a\log\left(c\right)^3+3\left(b\left(p+2\,\log\left(c\right)\right)x^2+2\,a\log\left(c\right)\right)\log\left(\left(bx^2+a\right)^p\right)^2+6\left(bx^2\,\mathrm{lo}\right.}{2\left(bx^7+ax^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log((b*x^2 + a)^p)^3/x^4 + integrate(1/2*(2*b*x^2*log(c)^3 + 2*a*log(c
)^3 + 3*(b*(p + 2*log(c))*x^2 + 2*a*log(c))*log((b*x^2 + a)^p)^2 + 6*(b*x^2
*log(c)^2 + a*log(c)^2)*log((b*x^2 + a)^p))/(b*x^7 + a*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^5, x)
```

3.97 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=352

$$-\frac{b^3p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3}+\frac{b^3p^3\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^3}-\frac{b^3p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a^3}-\frac{b^3p^3\text{PolyLog}(}{a^3}$$

```
[Out] (b^3*p^3*Log[x])/a^3 - (b^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^3*x^
2) - (b^3*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a^3 - (b*p*Log[c*(a +
 b*x^2)^p]^2)/(4*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*a^3
*x^2) - Log[c*(a + b*x^2)^p]^3/(6*x^6) - (b^3*p^2*Log[c*(a + b*x^2)^p]*Log[
1 - a/(a + b*x^2)])/(2*a^3) + (b^3*p*Log[c*(a + b*x^2)^p]^2*Log[1 - a/(a +
b*x^2)])/(2*a^3) + (b^3*p^3*PolyLog[2, a/(a + b*x^2)])/(2*a^3) - (b^3*p^2*L
og[c*(a + b*x^2)^p]*PolyLog[2, a/(a + b*x^2)])/a^3 - (b^3*p^3*PolyLog[2, 1
+ (b*x^2)/a])/a^3 - (b^3*p^3*PolyLog[3, a/(a + b*x^2)])/a^3
```

Rubi [A] time = 0.744987, antiderivative size = 331, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$\frac{b^3p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3}-\frac{3b^3p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^3}-\frac{b^3p^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{a^3}-\frac{3b^3p^2\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] (b^3*p^3*Log[x])/a^3 - (b^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^3*x^
2) - (3*b^3*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^3) + (b^3*p*Lo
g[c*(a + b*x^2)^p]^2)/(4*a^3) - (b*p*Log[c*(a + b*x^2)^p]^2)/(4*a*x^4) + (b
^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*a^3*x^2) + (b^3*p*Log[-((b*x^2)
/a)]*Log[c*(a + b*x^2)^p]^2)/(2*a^3) - (b^3*Log[c*(a + b*x^2)^p]^3)/(6*a^3)
 - Log[c*(a + b*x^2)^p]^3/(6*x^6) - (3*b^3*p^3*PolyLog[2, 1 + (b*x^2)/a])/(
2*a^3) + (b^3*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 + (b*x^2)/a])/a^3 - (b^
3*p^3*PolyLog[3, 1 + (b*x^2)/a])/a^3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7}dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^4}dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3(a+bx)}dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{2}p\,\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}dx, x, a+bx^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{p\,\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}dx, x, a+bx^2\right)}{2a} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log^2(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}dx, x, a -\right.}{2a}$$

$$= -\frac{bp\log^2\left(c\left(a+bx^2\right)^p\right)}{4ax^4} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}dx, x, a+bx^2\right)}{2a^2} + \frac{\left(b^2p\right.}{}$$

$$= -\frac{bp\log^2\left(c\left(a+bx^2\right)^p\right)}{4ax^4} + \frac{b^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{\left(b^2p\right)\text{Su}}{}$$

$$= -\frac{b^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3} - \frac{bp\log^2\left(c\left(a+bx^2\right.\right.}{4ax^4}$$

$$= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{3b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3} + \frac{b^3p\,\mathrm{l}}{}$$

$$= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{3b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3} + \frac{b^3p\,\mathrm{l}}{}$$

Mathematica [A] time = 0.393036, size = 571, normalized size = 1.62

$$-\frac{6b^3p^2x^6\text{PolyLog}\left(2, \frac{bx^2}{a}+1\right)\left(3p-2\log\left(c\left(a+bx^2\right)^p\right)\right) + 12b^3p^3x^6\text{PolyLog}\left(3, \frac{bx^2}{a}+1\right) + 3a^2bpx^2\log^2\left(c\left(a+bx^2\right)^p\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] -(-6*b^3*p^3*x^6*Log[-((b*x^2)/a)] + 6*b^3*p^3*x^6*Log[a + b*x^2] - 36*b^3*
p^3*x^6*Log[x]*Log[a + b*x^2] + 18*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a + b*
x^2] + 9*b^3*p^3*x^6*Log[a + b*x^2]^2 - 12*b^3*p^3*x^6*Log[x]*Log[a + b*x^2
]^2 + 6*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*b^3*p^3*x^6*Log[
a + b*x^2]^3 + 6*a*b^2*p^2*x^4*Log[c*(a + b*x^2)^p] + 36*b^3*p^2*x^6*Log[x]
*Log[c*(a + b*x^2)^p] - 18*b^3*p^2*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]
+ 24*b^3*p^2*x^6*Log[x]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 12*b^3*p^2*x^
6*Log[-((b*x^2)/a)]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 6*b^3*p^2*x^6*Log
[a + b*x^2]^2*Log[c*(a + b*x^2)^p] + 3*a^2*b*p*x^2*Log[c*(a + b*x^2)^p]^2 -
 6*a*b^2*p*x^4*Log[c*(a + b*x^2)^p]^2 - 12*b^3*p*x^6*Log[x]*Log[c*(a + b*x^
2)^p]^2 + 6*b^3*p*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2 + 2*a^3*Log[c*(
a + b*x^2)^p]^3 + 6*b^3*p^2*x^6*(3*p - 2*Log[c*(a + b*x^2)^p])*PolyLog[2, 1
 + (b*x^2)/a] + 12*b^3*p^3*x^6*PolyLog[3, 1 + (b*x^2)/a])/(12*a^3*x^6)
```

Maple [F] time = 1.332, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(bx^2+a\right)^p\right)^3}{6\,x^6}+\int \frac{bx^2\log\left(c\right)^3+a\log\left(c\right)^3+\left(b\left(p+3\,\log\left(c\right)\right)x^2+3\,a\log\left(c\right)\right)\log\left(\left(bx^2+a\right)^p\right)^2+3\left(bx^2\log\left(c\right)^2\right.}{bx^9+ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*log((b*x^2 + a)^p)^3/x^6 + integrate((b*x^2*log(c)^3 + a*log(c)^3 + (b
*(p + 3*log(c))*x^2 + 3*a*log(c))*log((b*x^2 + a)^p)^2 + 3*(b*x^2*log(c)^2
+ a*log(c)^2)*log((b*x^2 + a)^p))/(b*x^9 + a*x^7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2 + a\right)^p c\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^7, x)
```

3.98 $\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=379

$$\frac{32ia^{3/2}p^3\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}}-\frac{2a^2p\text{Unintegrable}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)}{b}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}$$

```
[Out] (208*a*p^3*x)/(9*b) - (16*p^3*x^3)/27 - (208*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(9*b^(3/2)) + (((32*I)/3)*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]
]^2)/b^(3/2) + (64*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/
(Sqrt[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) - (32*a*p^2*x*Log[c*(a + b*x^2)^p])/(
3*b) + (8*p^2*x^3*Log[c*(a + b*x^2)^p])/9 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (2*a*p*x*Log[c*(a + b*x^2)
^p]^2)/b - (2*p*x^3*Log[c*(a + b*x^2)^p]^2)/3 + (x^3*Log[c*(a + b*x^2)^p]^3
)/3 + (((32*I)/3)*a^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[
b]*x)])/b^(3/2) - (2*a^2*p*Unintegrable[Log[c*(a + b*x^2)^p]^2/(a + b*x^2),
 x])/b
```

Rubi [A] time = 0.833947, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (208*a*p^3*x)/(9*b) - (16*p^3*x^3)/27 - (208*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(9*b^(3/2)) + (((32*I)/3)*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]
]^2)/b^(3/2) + (64*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/
(Sqrt[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) - (32*a*p^2*x*Log[c*(a + b*x^2)^p])/(
3*b) + (8*p^2*x^3*Log[c*(a + b*x^2)^p])/9 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (2*a*p*x*Log[c*(a + b*x^2)
^p]^2)/b - (2*p*x^3*Log[c*(a + b*x^2)^p]^2)/3 + (x^3*Log[c*(a + b*x^2)^p]^3
)/3 + (((32*I)/3)*a^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[
b]*x)])/b^(3/2) - (2*a^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x
])/b
```

Rubi steps

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{3}x^3 \log^3\left(c\left(a+bx^2\right)^p\right) - (2bp)\int \frac{x^4 \log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} dx$$

$$= \frac{1}{3}x^3 \log^3\left(c\left(a+bx^2\right)^p\right) - (2bp)\int \left(-\frac{a\log^2\left(c\left(a+bx^2\right)^p\right)}{b^2} + \frac{x^2\log^2\left(c\left(a+bx^2\right)^p\right)}{b} + \frac{a^2 \log\ldots}{\ldots}\right.$$

$$= \frac{1}{3}x^3 \log^3\left(c\left(a+bx^2\right)^p\right) - (2p)\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right) dx + \frac{(2ap)\int \log^2\left(c\left(a+bx^2\right)^p\right) dx}{b}\ldots$$

$$= \frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{(2a^2p)\int \ldots}{\ldots}$$

$$= \frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{(2a^2p)\int \ldots}{\ldots}$$

$$= \frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{(2a^2p)\int \ldots}{\ldots}$$

$$= -\frac{32ap^2x\log\left(c\left(a+bx^2\right)^p\right)}{3b} + \frac{8}{9}p^2x^3\log\left(c\left(a+bx^2\right)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+\ldots\right.\right.}{3b^{3/2}}$$

$$= \frac{64ap^3x}{3b} - \frac{32ap^2x\log\left(c\left(a+bx^2\right)^p\right)}{3b} + \frac{8}{9}p^2x^3\log\left(c\left(a+bx^2\right)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\ldots}{3b^{3/2}}$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{32ap^2x\log\left(c\left(a+\ldots\right.\right.}{3b}$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{64a^{3/2}p^3\tan^{-1}\left(\ldots\right)}{3b\ldots}$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{64a^{3/2}p^3\tan^{-1}\left(\ldots\right)}{3b\ldots}$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{64a^{3/2}p^3\tan^{-1}\left(\ldots\right)}{3b\ldots}$$

Mathematica [A] time = 3.753, size = 909, normalized size = 2.4

$$\frac{\left(-48\left(4\sqrt{bx^2}\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a}}\right)\left(\log\left(bx^2+a\right)-\log\left(\frac{bx^2}{a}+1\right)\right)-\sqrt{-a}\sqrt{-\frac{bx^2}{a}}\left(\log^2\left(\frac{bx^2}{a}+1\right)-4\log\left(\frac{1}{2}\left(\sqrt{-\frac{bx^2}{a}}+1\right)\right)\log\left(\frac{bx^2}{a}\ldots\right.\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (2*a*p*x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/b - (2*a^(3/2)*p*A
rcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/
```

```
b^(3/2) + p*x^3*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])
^2 + (x^3*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-2*p - p*Log[a +
b*x^2] + Log[c*(a + b*x^2)^p]))/3 + 3*p^2*(-(p*Log[a + b*x^2]) + Log[c*(a +
 b*x^2)^p])*((x^3*Log[a + b*x^2]^2)/3 - (4*((9*I)*a^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]]^2 + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-8 + 6*Log[(2*Sqrt[a]
)/(Sqrt[a] + I*Sqrt[b]*x)] + 3*Log[a + b*x^2]) + Sqrt[b]*x*(24*a - 2*b*x^2
+ (-9*a + 3*b*x^2)*Log[a + b*x^2]) + (9*I)*a^(3/2)*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/(27*b^(3/2))) + (p^3*(416*Sqrt[-a]
*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a +
b*x^2]] + (2*Sqrt[-a]*b*x^2*(624*a - 16*b*x^2 + (-288*a + 24*b*x^2)*Log[a +
 b*x^2] + 18*(3*a - b*x^2)*Log[a + b*x^2]^2 + 9*b*x^2*Log[a + b*x^2]^3))/3
+ 36*Sqrt[-a]*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*(8*Sqrt[a]*HypergeometricPF
Q[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + Log[a + b*x^2]*(4
*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sq
rt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]]*Log[a + b*x^2])) - 48*a^2*(4*
Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(Log[a + b*x^2] - Log[1 + (b*x^2)
/a]) - Sqrt[-a]*Sqrt[-((b*x^2)/a)]*(Log[1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2
)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2
- 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/2]))))/(18*Sqrt[-a]*b^2*x)
```

Maple [A] time = 13.369, size = 0, normalized size = 0.

$$\int x^2 \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \log\left(\left(bx^2+a\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \log\left(c\left(a + bx^2\right)^p\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \log\left(\left(bx^2 + a\right)^p c\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^3, x)
```

3.99 $\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=289

$$6ap\text{Unintegrable}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)-\frac{24i\sqrt{a}p^3\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}+24p^2x\log\left(c\left(a+bx^2\right)^p\right)-\frac{24\sqrt{a}p^2}{}$$

```
[Out] -48*p^3*x + (48*Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] - ((24*I)*
Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] - (48*Sqrt[a]*p^3*ArcTan
[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] + 2
4*p^2*x*Log[c*(a + b*x^2)^p] - (24*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/Sqrt[b] - 6*p*x*Log[c*(a + b*x^2)^p]^2 + x*Log[c*(a +
 b*x^2)^p]^3 - ((24*I)*Sqrt[a]*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*
Sqrt[b]*x)])/Sqrt[b] + 6*a*p*Unintegrable[Log[c*(a + b*x^2)^p]^2/(a + b*x^2
), x]
```

Rubi [A] time = 0.43435, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -48*p^3*x + (48*Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] - ((24*I)*
Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] - (48*Sqrt[a]*p^3*ArcTan
[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] + 2
4*p^2*x*Log[c*(a + b*x^2)^p] - (24*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/Sqrt[b] - 6*p*x*Log[c*(a + b*x^2)^p]^2 + x*Log[c*(a +
 b*x^2)^p]^3 - ((24*I)*Sqrt[a]*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*
Sqrt[b]*x)])/Sqrt[b] + 6*a*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2),
 x]
```

Rubi steps

$$\int \log^3\left(c\left(a+bx^2\right)^p\right) dx = x\log^3\left(c\left(a+bx^2\right)^p\right) - (6bp)\int \frac{x^2\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} dx$$

$$= x\log^3\left(c\left(a+bx^2\right)^p\right) - (6bp)\int\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{a\log^2\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right) dx$$

$$= x\log^3\left(c\left(a+bx^2\right)^p\right) - (6p)\int \log^2\left(c\left(a+bx^2\right)^p\right) dx + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} dx$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} dx + \left(24bp^2\right)$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} dx + \left(24bp^2\right)$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} dx + \left(24p^2\right)\int$$

$$= 24p^2x\log\left(c\left(a+bx^2\right)^p\right) - \frac{24\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} - 6px\log^2\left(c\left(a+bx^2\right)^p\right) +$$

$$= -48p^3x + 24p^2x\log\left(c\left(a+bx^2\right)^p\right) - \frac{24\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} - 6px\log^2\left(c\left(a\right.\right.$$

$$= -48p^3x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + 24p^2x\log\left(c\left(a+bx^2\right)^p\right) - \frac{24\sqrt{a}p}{}$$

$$= -48p^3x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} - \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}$$

$$= -48p^3x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} - \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}$$

$$= -48p^3x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} - \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}$$

Mathematica [A] time = 3.35596, size = 789, normalized size = 2.73

$$p^3\left(-6\sqrt{-a^2}\sqrt{\frac{bx^2}{a+bx^2}}\left(8\sqrt{a}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right) + \log\left(a+bx^2\right)\left(4\sqrt{a}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right) + \sqrt{a+bx^2}\log\left(a+\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (6*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a +
b*x^2)^p])^2)/Sqrt[b] + 3*p*x*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(
a + b*x^2)^p])^2 + x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-6*p -
 p*Log[a + b*x^2] + Log[c*(a + b*x^2)^p]) - (3*p^2*(p*Log[a + b*x^2] - Log[
```

```
c*(a + b*x^2)^p])*((4*I)*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)
] + Log[a + b*x^2]) + Sqrt[b]*x*(8 - 4*Log[a + b*x^2] + Log[a + b*x^2]^2) +
 (4*I)*Sqrt[a]*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x
)]))/Sqrt[b] + (p^3*(-48*Sqrt[-a^2]*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^
2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]] + Sqrt[-a]*b*x^2*(-48 + 24*Log[a + b*x^2
] - 6*Log[a + b*x^2]^2 + Log[a + b*x^2]^3) - 6*Sqrt[-a^2]*Sqrt[(b*x^2)/(a +
 b*x^2)]*(8*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}
, a/(a + b*x^2)] + Log[a + b*x^2]*(4*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1
/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b
*x^2]]*Log[a + b*x^2])) + 24*a*Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(L
og[a + b*x^2] - Log[1 + (b*x^2)/a]) + 6*(-a)^(3/2)*Sqrt[-((b*x^2)/a)]*(Log[
1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2
*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/
2])))/(Sqrt[-a]*b*x)
```

Maple [A] time = 0.866, size = 0, normalized size = 0.

$$\int \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\left(bx^2+a\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \log\left(c\left(a+bx^2\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\left(bx^2+a\right)^p c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3, x)
```

3.100 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2} \, dx$

Optimal. Leaf size=50

$$6bp\text{Unintegrable}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}$$

```
[Out] -(Log[c*(a + b*x^2)^p]^3/x) + 6*b*p*Unintegrable[Log[c*(a + b*x^2)^p]^2/(a
+ b*x^2), x]
```

Rubi [A] time = 0.0450119, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] -(Log[c*(a + b*x^2)^p]^3/x) + 6*b*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a +
b*x^2), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2} \, dx = -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}+(6bp)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} \, dx$$

Mathematica [A] time = 0.781377, size = 505, normalized size = 10.1

$$\frac{p^3\left(-96\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)-48\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\log\left(a+bx^2\right)\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)-2\log^2\right.}{2\sqrt{a}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] (p^3*(-96*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2,
 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] - 48*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]
*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)]*Log[a + b*x^
2] - 2*Log[a + b*x^2]^2*(6*Sqrt[a + b*x^2]*Sqrt[1 - a/(a + b*x^2)]*ArcSin[S
qrt[a]/Sqrt[a + b*x^2]] + Sqrt[a]*Log[a + b*x^2])))/(2*Sqrt[a]*x) + (6*Sqrt
[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^
p])^2)/Sqrt[a] - (3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^
2)^p])^2)/x - (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/x + 3*p^2*(-(p
```

```
*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])*(-(Log[a + b*x^2]^2/x) + (4*Sqrt[b
]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*Log[(2*I)
/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[a + b*x^2]) + I*PolyLog[2, (I*Sqrt[a] + S
qrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/Sqrt[a])
```

Maple [A] time = 2.409, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(b x^2 + a \right)^p c \right)^3}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^2, x)
```

3.101 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=253

$$\frac{8ib^{3/2}p^3\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}}-\frac{2b^2p\text{Unintegrable}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)}{a}+\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}}+\frac{8i}{}$$

```
[Out] ((8*I)*b^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) + (16*b^(3/2)*p^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2) + (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(3
/2) - (2*b*p*Log[c*(a + b*x^2)^p]^2)/(a*x) - Log[c*(a + b*x^2)^p]^3/(3*x^3)
 + ((8*I)*b^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/
a^(3/2) - (2*b^2*p*Unintegrable[Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x])/a
```

Rubi [A] time = 0.350779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] ((8*I)*b^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) + (16*b^(3/2)*p^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2) + (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(3
/2) - (2*b*p*Log[c*(a + b*x^2)^p]^2)/(a*x) - Log[c*(a + b*x^2)^p]^3/(3*x^3)
 + ((8*I)*b^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/
a^(3/2) - (2*b^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x])/a
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx = -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} + (2bp)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} + (2bp)\int \left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{ax^2} - \frac{b\log^2\left(c\left(a+bx^2\right)^p\right)}{a\left(a+bx^2\right)}\right)dx$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{(2bp)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{a} - \frac{\left(2b^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{a}$$

$$= -\frac{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}{ax} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} - \frac{\left(2b^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{a} + \frac{\left(8b^2p^2\right)\int -$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}{ax} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} - $$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}{ax} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} - $$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}{ax}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log(c}{a^{3/2}}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log(c}{a^{3/2}}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log(c}{a^{3/2}}$$

Mathematica [A] time = 2.62218, size = 851, normalized size = 3.36

$$\frac{\left(-a^2\log^3\left(bx^2+a\right) - 6abx^2\log^2\left(bx^2+a\right) + 6\sqrt{a}\left(\frac{bx^2}{bx^2+a}\right)^{3/2}\left(bx^2+a\right)^{3/2}\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{bx^2+a}}\right)\log^2\left(bx^2+a\right) + 24\sqrt{-a}\left(bx^2\right)^{3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] (a^2*(p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])^3 - 6*a*b*p*x^2*(-(p*Log[a +
 b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 6*Sqrt[a]*b^(3/2)*p*x^3*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 3*a^2*p*Log[
a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 3*Sqrt[a]*p^2*(
p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])*(a^(3/2)*Log[a + b*x^2]^2 + 4*b*x^
2*(I*Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + Sqrt[a]*Log[a + b*x^2] + Sqr
t[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)] + Log[a + b*x^2]) + I*Sqrt[b]*x*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)
/((-I)*Sqrt[a] + Sqrt[b]*x)])) + p^3*(48*a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*
```

```
HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + 2
4*Sqrt[-a]*(b*x^2)^(3/2)*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[a + b*x^2] + 24*
a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2,
3/2}, a/(a + b*x^2)]*Log[a + b*x^2] - 6*a*b*x^2*Log[a + b*x^2]^2 + 6*Sqrt[a
]*((b*x^2)/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*ArcSin[Sqrt[a]/Sqrt[a + b*x
^2]]*Log[a + b*x^2]^2 - a^2*Log[a + b*x^2]^3 - 24*Sqrt[-a]*(b*x^2)^(3/2)*Ar
cTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[1 + (b*x^2)/a] - 6*a^2*(-((b*x^2)/a))^(3/2)
*Log[1 + (b*x^2)/a]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*Log[1 + (b*x^2)/a]*Log[
(1 + Sqrt[-((b*x^2)/a)])/2] - 12*a^2*(-((b*x^2)/a))^(3/2)*Log[(1 + Sqrt[-((
b*x^2)/a)])/2]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*PolyLog[2, 1/2 - Sqrt[-((b*x
^2)/a)]/2]))/(3*a^2*x^3)
```

Maple [A] time = 6.689, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**4,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^4, x)
```

3.102 $\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=107

$$\frac{\left(a+bx^2\right)^2 \left(c\left(a+bx^2\right)^p\right)^{-2/p} \text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p} - \frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p} \text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}$$

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b^2*p*(c*(a + b*x
^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p])/
(2*b^2*p*(c*(a + b*x^2)^p)^(2/p))
```

Rubi [A] time = 0.154434, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2454, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{\left(a+bx^2\right)^2 \left(c\left(a+bx^2\right)^p\right)^{-2/p} \text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p} - \frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p} \text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b^2*p*(c*(a + b*x
^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p])/
(2*b^2*p*(c*(a + b*x^2)^p)^(2/p))
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log\left(c(a+bx)^p\right)}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a}{b\log\left(c(a+bx)^p\right)}+\frac{a+bx}{b\log\left(c(a+bx)^p\right)}\right)dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx,x,x^2\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx,x,x^2\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx,x,a+bx^2\right)}{2b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx,x,a+bx^2\right)}{2b^2}$$
$$= \frac{\left(\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx,x,\log\left(c\left(a+bx^2\right)^p\right)\right)}{2b^2p} - \frac{\left(a\left(a+bx^2\right)\left(c\left(a\right.\right.\right.}{}$$
$$= -\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p} + \frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(\right.\right.}{}\right.}{2b^2p}$$

Mathematica [A] time = 0.139206, size = 96, normalized size = 0.9

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-2/p}\left(a\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)-\left(a+bx^2\right)\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)\right)}{2b^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^2)^p
]/p] - (a + b*x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]))/(2*b^2*p*(c*
(a + b*x^2)^p)^(2/p))
```

Maple [F] time = 0.484, size = 0, normalized size = 0.

$$\int \frac{x^3}{\ln\left(c\left(bx^2+a\right)^p\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p),x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(\left(bx^2+a\right)^p c\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^3/log((b*x^2 + a)^p*c), x)
```

Fricas [A] time = 2.26807, size = 162, normalized size = 1.51

$$-\frac{ac^{\left(\frac{1}{p}\right)}\,\text{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)-\text{log\_integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)c^{\frac{2}{p}}\right)}{2\,b^2c^{\frac{2}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) - log_integral((b^2*x^4 +
 2*a*b*x^2 + a^2)*c^(2/p)))/(b^2*c^(2/p)*p)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p), x)
```

Giac [A] time = 1.27946, size = 99, normalized size = 0.93

$$-\frac{\frac{a\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{bc^{\left(\frac{1}{p}\right)}p}-\frac{\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log\left(bx^2+a\right)\right)}{bc^{\frac{2}{p}}p}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*(a*Ei(log(c)/p + log(b*x^2 + a))/(b*c^(1/p)*p) - Ei(2*log(c)/p + 2*log
(b*x^2 + a))/(b*c^(2/p)*p))/b
```

3.103 $\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=51

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

Rubi [A] time = 0.0576011, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2389, 2300, 2178}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p],x]
```


```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\log\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b}$$

$$= \frac{\left(\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\left(a+bx^2\right)^p\right)\right)}{2bp}$$

$$= \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp}$$

Mathematica [A] time = 0.0414916, size = 51, normalized size = 1.

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

Maple [C] time = 1.23, size = 317, normalized size = 6.2

$$-\frac{1}{2\,bp}\mathrm{e}^{\frac{i\pi\,\text{csgn}\left(i\left(bx^2+a\right)^p\right)\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\text{csgn}(ic)-i\pi\left(\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2\text{csgn}(ic)-i\pi\,\text{csgn}\left(i\left(bx^2+a\right)^p\right)\left(\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^2+i\pi\left(\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)^3+2p\ln\left(bx^2+a\right)-2\ln(c}{2p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p),x)
```

```
[Out] -1/2/b/p*exp(1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^2+I*Pi*csgn(I*c*(b*x^2+a)^p)^3+2*p*ln(b*x^2+a)-2*ln(c)-2*ln((b*x
^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b
*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(x/log((b*x^2 + a)^p*c), x)
```

Fricas [A] time = 2.23548, size = 72, normalized size = 1.41

$$\frac{\mathrm{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] 1/2*log_integral((b*x^2 + a)*c^(1/p))/(b*c^(1/p)*p)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p), x)
```

Giac [A] time = 1.28448, size = 42, normalized size = 0.82

$$\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/2*Ei(log(c)/p + log(b*x^2 + a))/(b*c^(1/p)*p)
```

3.104 $\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x*Log[c*(a + b*x^2)^p]), x]
```

Rubi [A] time = 0.0171699, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]),x]

[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.179574, size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]),x]

[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]), x]
```

Maple [A] time = 0.49, size = 0, normalized size = 0.

$$\int \frac{1}{x\ln\left(c\left(bx^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p),x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \log \left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \log \left(\left(bx^2+a\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((b*x^2 + a)^p*c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \log \left(c \left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \log \left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

3.105 $\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable[1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Rubi [A] time = 0.0173301, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.330132, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Maple [A] time = 0.507, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \ln\left(c\left(bx^2+a\right)^p\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p),x)
```

```
[Out] int(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log \left(\left(bx^2 + a \right)^p c \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^3 \log \left(\left(bx^2 + a \right)^p c \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log \left(c \left(a + bx^2 \right)^p \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log \left(\left(bx^2 + a \right)^p c \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

3.106 $\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

[Out] Unintegrable[x^2/Log[c*(a + b*x^2)^p], x]

Rubi [A] time = 0.0177013, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[x^2/Log[c*(a + b*x^2)^p],x]

[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p], x]

Rubi steps

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.286646, size = 0, normalized size = 0.

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2/Log[c*(a + b*x^2)^p],x]

[Out] Integrate[x^2/Log[c*(a + b*x^2)^p], x]

Maple [A] time = 0.464, size = 0, normalized size = 0.

$$\int \frac{x^2}{\ln\left(c\left(bx^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p),x)
```

```
[Out] int(x^2/ln(c*(b*x^2+a)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral(x^2/log((b*x^2 + a)^p*c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(x**2/log(c*(a + b*x**2)**p), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

3.107 $\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(a + b*x^2)^p]^(-1), x]
```

Rubi [A] time = 0.0035811, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-1),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.0090355, size = 0, normalized size = 0.

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-1),x]
```

```
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-1), x]
```

Maple [A] time = 0.428, size = 0, normalized size = 0.

$$\int \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^{-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p),x)
```

```
[Out] int(1/ln(c*(b*x^2+a)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log \left(\left(bx^2+a\right)^p c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\log \left(\left(bx^2+a\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral(1/log((b*x^2 + a)^p*c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log \left(c \left(a+bx^2\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(1/log(c*(a + b*x**2)**p), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log \left(\left(bx^2+a\right)^p c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

3.108 $\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

[Out] Unintegrable[1/(x^2*Log[c*(a + b*x^2)^p]), x]

Rubi [A] time = 0.0177068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]),x]

[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]), x]

Rubi steps

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 0.396934, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]),x]

[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]), x]

Maple [A] time = 0.472, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \ln\left(c\left(bx^2+a\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(c*(b*x^2+a)^p),x)

```
[Out] int(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log \left(\left(bx^2 + a \right)^p c \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2 \log \left(\left(bx^2 + a \right)^p c \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log \left(c \left(a + bx^2 \right)^p \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log \left(\left(bx^2 + a \right)^p c \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

3.109 $\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=138

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\,\mathrm{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^2}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\,\mathrm{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p^2}-\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b^2*p^2*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(b^2*p^2*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(2*b*p*Log[c*(a + b
*x^2)^p])
```

Rubi [A] time = 0.206615, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\,\mathrm{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^2}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\,\mathrm{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p^2}-\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b^2*p^2*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(b^2*p^2*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(2*b*p*Log[c*(a + b
*x^2)^p])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
```

```
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log^2\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{p} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x,\right.}{2bp}$$

$$= -\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx)^p)} + \frac{a+bx}{b\log(c(a+bx)^p)}\right)dx, x, x^2\right)}{p} + \frac{a\,\text{Subst}\left(\right.}{}$$

$$= -\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{bp} - \frac{a\,\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x,\right.}{bp}$$

$$= \frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p^2} - \frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c}\right.}{b}$$

$$= \frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p^2} - \frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)^2\left(c\right.\right.}{}$$

$$= -\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p^2} + \frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\right.}{}\right.}{b^2p^2}$$

Mathematica [A] time = 0.152928, size = 157, normalized size = 1.14

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-2/p}\left(a\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\log\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right) - 2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)\text{Ei}\right.}{2b^2p^2\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -((a + b*x^2)*(b*p*x^2*(c*(a + b*x^2)^p)^(2/p) + a*(c*(a + b*x^2)^p)^p^(-1)
*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2)^p] - 2*(a + b*x^2)
*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]))/(2*b^2*p^
2*(c*(a + b*x^2)^p)^(2/p)*Log[c*(a + b*x^2)^p])
```

Maple [F] time = 5.273, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^4+ax^2}{2\left(bp\log\left(\left(bx^2+a\right)^p\right)+bp\log\left(c\right)\right)}+\int\frac{2\,bx^3+ax}{bp\log\left(\left(bx^2+a\right)^p\right)+bp\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^4 + a*x^2)/(b*p*log((b*x^2 + a)^p) + b*p*log(c)) + integrate((2*b
*x^3 + a*x)/(b*p*log((b*x^2 + a)^p) + b*p*log(c)), x)
```

Fricas [A] time = 2.28195, size = 338, normalized size = 2.45

$$-\frac{\left(ap\log\left(bx^2+a\right)+a\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2px^4+abpx^2\right)c^{\frac{2}{p}}-2\left(p\log\left(bx^2+a\right)+\log\left(c\right)\right)\operatorname{log\_}}{2\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*p*log(b*x^2 + a) + a*log(c))*c^(1/p)*log_integral((b*x^2 + a)*c^(1
/p)) + (b^2*p*x^4 + a*b*p*x^2)*c^(2/p) - 2*(p*log(b*x^2 + a) + log(c))*log_
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/((b^2*p^3*log(b*x^2 + a) + b
^2*p^2*log(c))*c^(2/p))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**2, x)
```

Giac [B] time = 1.35384, size = 402, normalized size = 2.91

$$-\frac{\frac{\left(bx^2+a\right)^2p}{bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)}-\frac{\left(bx^2+a\right)ap}{bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)}+\frac{ap\operatorname{Ei}\left(\frac{\log\left(c\right)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}-\frac{2\,p\operatorname{Ei}\left(\frac{2\log\left(c\right)}{p}+2\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\frac{2}{p}}}+\frac{a\operatorname{Ei}\left(\frac{l}{}\right.}{\left(bp^3\,\mathrm{lo}\right.}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x^2 + a)^2*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) - (b*x^2 + a)*a
*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) + a*p*Ei(log(c)/p + log(b*x^2 + a)
)*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(1/p)) - 2*p*Ei(2
*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2
*log(c))*c^(2/p)) + a*Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b*p^3*log(b*x^
2 + a) + b*p^2*log(c))*c^(1/p)) - 2*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(c
)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)))/b
```

3.110 $\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=83

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp^2}-\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p^2*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(2*b*p*Log[c*(a + b*x^2)^p])
```

Rubi [A] time = 0.0720991, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2389, 2297, 2300, 2178}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp^2}-\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p^2*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(2*b*p*Log[c*(a + b*x^2)^p])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\log^2\left(c(a+bx)^p\right)}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx,x,a+bx^2\right)}{2b}\\ &= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}+\frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx,x,a+bx^2\right)}{2bp}\\ &= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}+\frac{\left(\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx,x,\log\left(c\left(a+bx^2\right)^p\right)\right)}{2bp^2}\\ &= \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp^2}-\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}\end{aligned}$$

Mathematica [A] time = 0.0436608, size = 97, normalized size = 1.17

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\left(p\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}-\log\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^2,x]

[Out] -((a + b*x^2)*(p*(c*(a + b*x^2)^p)^p^(-1) - ExpIntegralEi[Log[c*(a + b*x^2)
^p]/p]*Log[c*(a + b*x^2)^p]))/(2*b*p^2*(c*(a + b*x^2)^p)^p^(-1)*Log[c*(a +
b*x^2)^p])
```

Maple [C] time = 1.129, size = 466, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p)^2,x)

[Out] -1/(2*ln(c)+2*ln((b*x^2+a)^p)+I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*
(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c))/p/b*(b*x^2+a)-1/2/p^
```

```
2/b*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*
x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p
)-2*p*ln(b*x^2+a))/p)*exp(-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*
c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2
+a)^p)-2*p*ln(b*x^2+a))/p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^2+a}{2\left(bp\log\left(\left(bx^2+a\right)^p\right)+bp\log\left(c\right)\right)}+\int\frac{x}{p\log\left(\left(bx^2+a\right)^p\right)+p\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p*log((b*x^2 + a)^p) + b*p*log(c)) + integrate(x/(p*log
((b*x^2 + a)^p) + p*log(c)), x)
```

Fricas [A] time = 2.2156, size = 194, normalized size = 2.34

$$-\frac{\left(bpx^2+ap\right)c^{\left(\frac{1}{p}\right)}-\left(p\log\left(bx^2+a\right)+\log\left(c\right)\right)\mathrm{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*p*x^2 + a*p)*c^(1/p) - (p*log(b*x^2 + a) + log(c))*log_integral((b
*x^2 + a)*c^(1/p)))/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(1/p))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p)**2, x)
```

Giac [A] time = 1.26515, size = 190, normalized size = 2.29

$$-\frac{\left(bx^2+a\right)p}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)}+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x^2 + a)*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) + 1/2*p*Ei(log(c)/
p + log(b*x^2 + a))*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c
^(1/p)) + 1/2*Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b*p^3*log(b*x^2 + a) +
 b*p^2*log(c))*c^(1/p))
```

3.111 $\int \frac{1}{x\log^2\left(c(a+bx^2)^p\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi [A] time = 0.0157959, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.276951, size = 0, normalized size = 0.

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [A] time = 1.759, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-a\int \frac{1}{bpx^3\log\left(\left(bx^2+a\right)^p\right)+bpx^3\log\left(c\right)}\,dx-\frac{bx^2+a}{2\left(bpx^2\log\left(\left(bx^2+a\right)^p\right)+bpx^2\log\left(c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -a*integrate(1/(b*p*x^3*log((b*x^2 + a)^p) + b*p*x^3*log(c)), x) - 1/2*(b*x
^2 + a)/(b*p*x^2*log((b*x^2 + a)^p) + b*p*x^2*log(c))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\log\left(\left(bx^2+a\right)^pc\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

3.112 $\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi [A] time = 0.0172644, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 1.45958, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [A] time = 3.458, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^2+a}{2\left(bpx^4\log\left(\left(bx^2+a\right)^p\right)+bpx^4\log\left(c\right)\right)}-\int\frac{bx^2+2\,a}{bpx^5\log\left(\left(bx^2+a\right)^p\right)+bpx^5\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p*x^4*log((b*x^2 + a)^p) + b*p*x^4*log(c)) - integrate(
(b*x^2 + 2*a)/(b*p*x^5*log((b*x^2 + a)^p) + b*p*x^5*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^3\log\left(\left(bx^2+a\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

3.113 $\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Rubi [A] time = 0.0168387, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.331371, size = 0, normalized size = 0.

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Maple [A] time = 3.284, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^3+ax}{2\left(bp\log\left(\left(bx^2+a\right)^p\right)+bp\log\left(c\right)\right)}+\int\frac{3\,bx^2+a}{2\left(bp\log\left(\left(bx^2+a\right)^p\right)+bp\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^3 + a*x)/(b*p*log((b*x^2 + a)^p) + b*p*log(c)) + integrate(1/2*(3
*b*x^2 + a)/(b*p*log((b*x^2 + a)^p) + b*p*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/log((b*x^2 + a)^p*c)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^2, x)
```

3.114 $\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(a + b*x^2)^p]^(-2), x]
```

Rubi [A] time = 0.0035547, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-2),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.366038, size = 0, normalized size = 0.

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-2),x]
```

```
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-2), x]
```

Maple [A] time = 3.365, size = 0, normalized size = 0.

$$\int \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^2+a}{2\left(bpx\log\left(\left(bx^2+a\right)^p\right)+bpx\log\left(c\right)\right)}+\int\frac{bx^2-a}{2\left(bpx^2\log\left(\left(bx^2+a\right)^p\right)+bpx^2\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p*x*log((b*x^2 + a)^p) + b*p*x*log(c)) + integrate(1/2*
(b*x^2 - a)/(b*p*x^2*log((b*x^2 + a)^p) + b*p*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^(-2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**(-2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^(-2), x)
```

3.115 $\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi [A] time = 0.0165008, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [A] time = 1.19994, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [A] time = 3.452, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^2+a}{2\left(bpx^3\log\left(\left(bx^2+a\right)^p\right)+bpx^3\log\left(c\right)\right)}-\int\frac{bx^2+3\,a}{2\left(bpx^4\log\left(\left(bx^2+a\right)^p\right)+bpx^4\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p*x^3*log((b*x^2 + a)^p) + b*p*x^3*log(c)) - integrate(
1/2*(b*x^2 + 3*a)/(b*p*x^4*log((b*x^2 + a)^p) + b*p*x^4*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2\log\left(\left(bx^2+a\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

3.116 $\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=204

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^3}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4b^2p^3}-\frac{a\left(a+bx^2\right)}{4b^2p^2\log(c(a+bx^2}$$

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b^2*p^3*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(4*b*p*Log[c*(a + b
*x^2)^p]^2) - (a*(a + b*x^2))/(4*b^2*p^2*Log[c*(a + b*x^2)^p]) - (x^2*(a +
b*x^2))/(2*b*p^2*Log[c*(a + b*x^2)^p])
```

Rubi [A] time = 0.288515, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310, 2297}

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^3}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4b^2p^3}-\frac{a\left(a+bx^2\right)}{4b^2p^2\log(c(a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b^2*p^3*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(4*b*p*Log[c*(a + b
*x^2)^p]^2) - (a*(a + b*x^2))/(4*b^2*p^2*Log[c*(a + b*x^2)^p]) - (x^2*(a +
b*x^2))/(2*b*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rubi steps

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log^3\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx)^p)}\,dx, x, x^2\right)}{2p} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log^2(c(a+bx)^p)}\,dx, x, x\right.}{4bp}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2\left(a+bx^2\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{p^2} + \frac{a\,\text{Sul}}{}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a\left(a+bx^2\right)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2\left(a+bx^2\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int\left(-\right.\right.}{}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a\left(a+bx^2\right)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2\left(a+bx^2\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{}{\text{lo}}\right.}{}$$

$$= \frac{3a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} - \frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a\left(a+bx^2\right)}{4b^2p^2\log\left(c\left(a\right.\right.}$$

$$= \frac{3a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} - \frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a\left(a+bx^2\right)}{4b^2p^2\log\left(c\left(a\right.\right.}$$

$$= -\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} + \frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(a+\right.\right.}{p}\right.}{b^2p^3}$$

Mathematica [A] time = 0.193008, size = 185, normalized size = 0.91

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-2/p}\left(a\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\log^2\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right) - 4\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\right.\right.}{4b^2p^3\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^2)^p
]/p]*Log[c*(a + b*x^2)^p]^2 - 4*(a + b*x^2)*ExpIntegralEi[(2*Log[c*(a + b*x
^2)^p])/p]*Log[c*(a + b*x^2)^p]^2 + p*(c*(a + b*x^2)^p)^(2/p)*(b*p*x^2 + (a
 + 2*b*x^2)*Log[c*(a + b*x^2)^p])))/(4*b^2*p^3*(c*(a + b*x^2)^p)^(2/p)*Log[
c*(a + b*x^2)^p]^2)
```

Maple [F] time = 5.264, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\left(p+2\log\left(c\right)\right)x^4+ab\left(p+3\log\left(c\right)\right)x^2+a^2\log\left(c\right)+\left(2b^2x^4+3abx^2+a^2\right)\log\left(\left(bx^2+a\right)^p\right)}{4\left(b^2p^2\log\left(\left(bx^2+a\right)^p\right)^2+2b^2p^2\log\left(\left(bx^2+a\right)^p\right)\log\left(c\right)+b^2p^2\log\left(c\right)^2\right)}+\int\frac{4b}{2\left(bp^2\log\left(\left(bx^2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p + 2*log(c))*x^4 + a*b*(p + 3*log(c))*x^2 + a^2*log(c) + (2*b^2
*x^4 + 3*a*b*x^2 + a^2)*log((b*x^2 + a)^p))/(b^2*p^2*log((b*x^2 + a)^p)^2 +
 2*b^2*p^2*log((b*x^2 + a)^p)*log(c) + b^2*p^2*log(c)^2) + integrate(1/2*(4
*b*x^3 + 3*a*x)/(b*p^2*log((b*x^2 + a)^p) + b*p^2*log(c)), x)
```

Fricas [A] time = 2.20534, size = 629, normalized size = 3.08

$$-\frac{\left(ap^2\log\left(bx^2+a\right)^2+2ap\log\left(bx^2+a\right)\log\left(c\right)+a\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2p^2x^4+abp^2x^2+\right.}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a*p^2*log(b*x^2 + a)^2 + 2*a*p*log(b*x^2 + a)*log(c) + a*log(c)^2)*c
^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) + (b^2*p^2*x^4 + a*b*p^2*x^2 + (2*
b^2*p^2*x^4 + 3*a*b*p^2*x^2 + a^2*p^2)*log(b*x^2 + a) + (2*b^2*p*x^4 + 3*a*
b*p*x^2 + a^2*p)*log(c))*c^(2/p) - 4*(p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2
+ a)*log(c) + log(c)^2)*log_integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/
((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(
c)^2)*c^(2/p))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**3, x)
```

Giac [B] time = 1.24803, size = 1134, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(b*x^2 + a)^2*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*
log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - (b*x^2 + a)*a*p^2*log(b*x^2 + a)/
(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) +
 a*p^2*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a
)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) + (b*x^2 + a
)^2*p^2/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log
(c)^2) - (b*x^2 + a)*a*p^2/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)
*log(c) + b*p^3*log(c)^2) - 4*p^2*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x
^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*
log(c)^2)*c^(2/p)) + 2*(b*x^2 + a)^2*p*log(c)/(b*p^5*log(b*x^2 + a)^2 + 2*b
*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - (b*x^2 + a)*a*p*log(c)/(b*p^
5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) + 2*a*
p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2 + a
)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) - 8*p*Ei(2*l
og(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2 + a)^2
+ 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)) + a*Ei(log(c)/p
+ log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a
)*log(c) + b*p^3*log(c)^2)*c^(1/p)) - 4*Ei(2*log(c)/p + 2*log(b*x^2 + a))*l
og(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*lo
g(c)^2)*c^(2/p)))/b
```

3.117 $\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=114

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b*p^3*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(4*b*p*Log[c*(a + b*x^2)^p]^2) - (a + b*x^2)/(4*b
*p^2*Log[c*(a + b*x^2)^p])
```

Rubi [A] time = 0.0931416, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2389, 2297, 2300, 2178}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b*p^3*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(4*b*p*Log[c*(a + b*x^2)^p]^2) - (a + b*x^2)/(4*b
*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\log^3\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx^p)}\,dx, x, a+bx^2\right)}{2b}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{4bp}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{4bp^2}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int\right.}{4bp^3}$$

$$= \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4bp^3} - \frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx\right.\right.}$$

Mathematica [A] time = 0.0543933, size = 113, normalized size = 0.99

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\left(p\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\left(\log\left(c\left(a+bx^2\right)^p\right)+p\right)-\log^2\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)\right)}{4bp^3\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -((a + b*x^2)*(-(ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2)^p]
^2) + p*(c*(a + b*x^2)^p)^p^(-1)*(p + Log[c*(a + b*x^2)^p])))/(4*b*p^3*(c*(
a + b*x^2)^p)^p^(-1)*Log[c*(a + b*x^2)^p]^2)
```

Maple [C] time = 1.2, size = 761, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] -1/2*(I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2
+a)^p)^3+I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+I*Pi*a*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)*csgn(I*c)-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2
*csgn(I*c)+2*ln(c)*b*x^2+2*b*x^2*ln((b*x^2+a)^p)+2*ln(c)*a+2*a*ln((b*x^2+a)
^p)+2*x^2*p*b+2*a*p)/p^2/(2*ln(c)+2*ln((b*x^2+a)^p)+I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)
)^2/b-1/4/p^3/b*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*
csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln
((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*exp(-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)
+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\left(p+\log\left(c\right)\right)x^{2}+a\left(p+\log\left(c\right)\right)+\left(bx^{2}+a\right)\log\left(\left(bx^{2}+a\right)^{p}\right)}{4\left(bp^{2}\log\left(\left(bx^{2}+a\right)^{p}\right)^{2}+2\,bp^{2}\log\left(\left(bx^{2}+a\right)^{p}\right)\log\left(c\right)+bp^{2}\log\left(c\right)^{2}\right)}+\int\frac{x}{2\left(p^{2}\log\left(\left(bx^{2}+a\right)^{p}\right)+p^{2}\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(p + log(c))*x^2 + a*(p + log(c)) + (b*x^2 + a)*log((b*x^2 + a)^p))
/(b*p^2*log((b*x^2 + a)^p)^2 + 2*b*p^2*log((b*x^2 + a)^p)*log(c) + b*p^2*lo
g(c)^2) + integrate(1/2*x/(p^2*log((b*x^2 + a)^p) + p^2*log(c)), x)
```

Fricas [A] time = 2.25631, size = 378, normalized size = 3.32

$$-\frac{\left(bp^{2}x^{2}+ap^{2}+\left(bp^{2}x^{2}+ap^{2}\right)\log\left(bx^{2}+a\right)+\left(bpx^{2}+ap\right)\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}-\left(p^{2}\log\left(bx^{2}+a\right)^{2}+2\,p\log\left(bx^{2}+a\right)\log\left(c\right.}{4\left(bp^{5}\log\left(bx^{2}+a\right)^{2}+2\,bp^{4}\log\left(bx^{2}+a\right)\log\left(c\right)+bp^{3}\log\left(c\right)^{2}\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((b*p^2*x^2 + a*p^2 + (b*p^2*x^2 + a*p^2)*log(b*x^2 + a) + (b*p*x^2 +
a*p)*log(c))*c^(1/p) - (p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2 + a)*log(c) +
log(c)^2)*log_integral((b*x^2 + a)*c^(1/p)))/((b*p^5*log(b*x^2 + a)^2 + 2*b
*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p)**3, x)
```

Giac [B] time = 1.28209, size = 548, normalized size = 4.81

$$-\frac{\left(bx^2+a\right)p^2\log\left(bx^2+a\right)}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)}+\frac{p^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+\right.}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^2 + a)*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b
*x^2 + a)*log(c) + b*p^3*log(c)^2) + 1/4*p^2*Ei(log(c)/p + log(b*x^2 + a))*
log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) +
 b*p^3*log(c)^2)*c^(1/p)) - 1/4*(b*x^2 + a)*p^2/(b*p^5*log(b*x^2 + a)^2 + 2
*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 1/4*(b*x^2 + a)*p*log(c)/(
b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) +
1/2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2
 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) + 1/4*Ei
(log(c)/p + log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log
(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

3.118 $\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

[Out] Unintegrable[1/(x*Log[c*(a + b*x^2)^p]^3), x]

Rubi [A] time = 0.0163287, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*Log[c*(a + b*x^2)^p]^3),x]

[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^3), x]

Rubi steps

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.550545, size = 0, normalized size = 0.

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3),x]

[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3), x]

Maple [A] time = 3.704, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2px^4 + ab\left(p - \log\left(c\right)\right)x^2 - a^2\log\left(c\right) - \left(abx^2 + a^2\right)\log\left(\left(bx^2 + a\right)^p\right)}{4\left(b^2p^2x^4\log\left(\left(bx^2 + a\right)^p\right)^2 + 2\,b^2p^2x^4\log\left(\left(bx^2 + a\right)^p\right)\log\left(c\right) + b^2p^2x^4\log\left(c\right)^2\right)} + \int \frac{abx^2 + 2\,a^2}{2\left(b^2p^2x^5\log\left(\left(bx^2 + a\right)^p\right) + b^2p^2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*p*x^4 + a*b*(p - log(c))*x^2 - a^2*log(c) - (a*b*x^2 + a^2)*log((
b*x^2 + a)^p))/(b^2*p^2*x^4*log((b*x^2 + a)^p)^2 + 2*b^2*p^2*x^4*log((b*x^2
 + a)^p)*log(c) + b^2*p^2*x^4*log(c)^2) + integrate(1/2*(a*b*x^2 + 2*a^2)/(
b^2*p^2*x^5*log((b*x^2 + a)^p) + b^2*p^2*x^5*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{x\log\left(\left(bx^2 + a\right)^p c\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(c\left(a + bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(\left(bx^2 + a\right)^p c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

3.119 $\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable[1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi [A] time = 0.0168112, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 2.83524, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [A] time = 3.746, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\left(p-\log\left(c\right)\right)x^4+ab\left(p-3\,\log\left(c\right)\right)x^2-2\,a^2\log\left(c\right)-\left(b^2x^4+3\,abx^2+2\,a^2\right)\log\left(\left(bx^2+a\right)^p\right)}{4\left(b^2p^2x^6\log\left(\left(bx^2+a\right)^p\right)^2+2\,b^2p^2x^6\log\left(\left(bx^2+a\right)^p\right)\log\left(c\right)+b^2p^2x^6\log\left(c\right)^2\right)}+\int\frac{b^2x^4}{2\left(b^2p^2x^7\log\left(\left(b\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p - log(c))*x^4 + a*b*(p - 3*log(c))*x^2 - 2*a^2*log(c) - (b^2*x
^4 + 3*a*b*x^2 + 2*a^2)*log((b*x^2 + a)^p))/(b^2*p^2*x^6*log((b*x^2 + a)^p)
^2 + 2*b^2*p^2*x^6*log((b*x^2 + a)^p)*log(c) + b^2*p^2*x^6*log(c)^2) + inte
grate(1/2*(b^2*x^4 + 6*a*b*x^2 + 6*a^2)/(b^2*p^2*x^7*log((b*x^2 + a)^p) + b
^2*p^2*x^7*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^3\log\left(\left(bx^2+a\right)^pc\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

3.120 $\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[x^2/Log[c*(a + b*x^2)^p]^3, x]
```

Rubi [A] time = 0.0169869, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^3, x]
```

Rubi steps

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 0.53692, size = 0, normalized size = 0.

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^3, x]
```

Maple [A] time = 3.486, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\left(2\,p+3\,\log\left(c\right)\right)x^4+2\,ab\left(p+2\,\log\left(c\right)\right)x^2+a^2\log\left(c\right)+\left(3\,b^2x^4+4\,abx^2+a^2\right)\log\left(\left(bx^2+a\right)^p\right)}{8\left(b^2p^2x\log\left(\left(bx^2+a\right)^p\right)^2+2\,b^2p^2x\log\left(\left(bx^2+a\right)^p\right)\log\left(c\right)+b^2p^2x\log\left(c\right)^2\right)}+\int\frac{9\,b^2}{8\left(b^2p^2x^2\log\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p + 3*log(c))*x^4 + 2*a*b*(p + 2*log(c))*x^2 + a^2*log(c) + (3
*b^2*x^4 + 4*a*b*x^2 + a^2)*log((b*x^2 + a)^p))/(b^2*p^2*x*log((b*x^2 + a)^
p)^2 + 2*b^2*p^2*x*log((b*x^2 + a)^p)*log(c) + b^2*p^2*x*log(c)^2) + integr
ate(1/8*(9*b^2*x^4 + 4*a*b*x^2 - a^2)/(b^2*p^2*x^2*log((b*x^2 + a)^p) + b^2
*p^2*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/log((b*x^2 + a)^p*c)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^3, x)
```

3.121 $\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(a + b*x^2)^p]^(-3), x]
```

Rubi [A] time = 0.0039762, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-3),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-3), x]
```

Rubi steps

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 0.461689, size = 0, normalized size = 0.

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-3),x]
```

```
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-3), x]
```

Maple [A] time = 3.435, size = 0, normalized size = 0.

$$\int \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^{-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\left(2p+\log(c)\right)x^4+2\,abpx^2-a^2\log(c)+\left(b^2x^4-a^2\right)\log\left(\left(bx^2+a\right)^p\right)}{8\left(b^2p^2x^3\log\left(\left(bx^2+a\right)^p\right)^2+2\,b^2p^2x^3\log\left(\left(bx^2+a\right)^p\right)\log(c)+b^2p^2x^3\log(c)^2\right)}+\int\frac{b^2x^4+3\,a^2}{8\left(b^2p^2x^4\log\left(\left(bx^2+a\right)^p\right)+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p + log(c))*x^4 + 2*a*b*p*x^2 - a^2*log(c) + (b^2*x^4 - a^2)*l
og((b*x^2 + a)^p))/(b^2*p^2*x^3*log((b*x^2 + a)^p)^2 + 2*b^2*p^2*x^3*log((b
*x^2 + a)^p)*log(c) + b^2*p^2*x^3*log(c)^2) + integrate(1/8*(b^2*x^4 + 3*a^
2)/(b^2*p^2*x^4*log((b*x^2 + a)^p) + b^2*p^2*x^4*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\log\left(\left(bx^2+a\right)^pc\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^(-3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**(-3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")

[Out] integrate(log((b*x^2 + a)^p*c)^(-3), x)
```

3.122 $\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi [A] time = 0.0167578, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 2.15763, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [A] time = 3.457, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\ln\left(c\left(bx^2+a\right)^p\right)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\left(2p-\log(c)\right)x^4+2ab\left(p-2\log(c)\right)x^2-3a^2\log(c)-\left(b^2x^4+4abx^2+3a^2\right)\log\left(\left(bx^2+a\right)^p\right)}{8\left(b^2p^2x^5\log\left(\left(bx^2+a\right)^p\right)^2+2b^2p^2x^5\log\left(\left(bx^2+a\right)^p\right)\log(c)+b^2p^2x^5\log(c)^2\right)}+\int\frac{b^2x^4 \cdots}{8\left(b^2p^2x^6\log\left(\left(b \cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p - log(c))*x^4 + 2*a*b*(p - 2*log(c))*x^2 - 3*a^2*log(c) - (b
^2*x^4 + 4*a*b*x^2 + 3*a^2)*log((b*x^2 + a)^p))/(b^2*p^2*x^5*log((b*x^2 + a
)^p)^2 + 2*b^2*p^2*x^5*log((b*x^2 + a)^p)*log(c) + b^2*p^2*x^5*log(c)^2) +
integrate(1/8*(b^2*x^4 + 12*a*b*x^2 + 15*a^2)/(b^2*p^2*x^6*log((b*x^2 + a)^
p) + b^2*p^2*x^6*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^2\log\left(\left(bx^2+a\right)^pc\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

3.123 $\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)} dx$

Optimal. Leaf size=45

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{2b^2c^2}-\frac{a\text{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}$$

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(2*b^2*c^2) - (a*LogIntegral[c*(a + b*x
^2)])/(2*b^2*c)
```

Rubi [A] time = 0.0954564, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2454, 2399, 2389, 2298, 2390, 2309, 2178}

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{2b^2c^2}-\frac{a\text{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)],x]
```


```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(2*b^2*c^2) - (a*LogIntegral[c*(a + b*x
^2)])/(2*b^2*c)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\log(c(a+bx))}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx))}+\frac{a+bx}{b\log(c(a+bx))}\right)dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2}\\
&= -\frac{a\operatorname{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\operatorname{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c\left(a+bx^2\right)\right)\right)}{2b^2c^2}\\
&= \frac{\operatorname{Ei}\left(2\log\left(c\left(a+bx^2\right)\right)\right)}{2b^2c^2} - \frac{a\operatorname{li}\left(c\left(a+bx^2\right)\right)}{2b^2c}
\end{aligned}
$$

Mathematica [A] time = 0.0851304, size = 41, normalized size = 0.91

$$
\frac{\operatorname{Ei}\left(2\log\left(bcx^2+ac\right)\right) - ac\operatorname{Ei}\left(\log\left(bcx^2+ac\right)\right)}{2b^2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)],x]
```

```
[Out] (-(a*c*ExpIntegralEi[Log[a*c + b*c*x^2]]) + ExpIntegralEi[2*Log[a*c + b*c*x
^2]])/(2*b^2*c^2)
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$
\int \frac{x^3}{\ln\left(c\left(bx^2+a\right)\right)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)),x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(\left(bx^2+a\right)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/log((b*x^2 + a)*c), x)
```

Fricas [A] time = 1.87321, size = 140, normalized size = 3.11

$$-\frac{ac\,\mathrm{log\_integral}\left(bcx^2+ac\right)-\mathrm{log\_integral}\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*c*log_integral(b*c*x^2 + a*c) - log_integral(b^2*c^2*x^4 + 2*a*b*c^
2*x^2 + a^2*c^2))/(b^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(ac+bcx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)),x)
```

```
[Out] Integral(x**3/log(a*c + b*c*x**2), x)
```

Giac [A] time = 1.18719, size = 51, normalized size = 1.13

$$-\frac{ac\mathrm{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right)-\mathrm{Ei}\left(2\,\log\left(\left(bx^2+a\right)c\right)\right)}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="giac")
```

```
[Out] -1/2*(a*c*Ei(log((b*x^2 + a)*c)) - Ei(2*log((b*x^2 + a)*c)))/(b^2*c^2)
```

3.124 $\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)} dx$

Optimal. Leaf size=20

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}$$

```
[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rubi [A] time = 0.0271766, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2389, 2298}

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)],x]
```

```
[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log(c(a+bx))} dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\log(cx)} dx, x, a+bx^2\right)}{2b}\\ &= \frac{\text{li}\left(c\left(a+bx^2\right)\right)}{2bc}\end{aligned}$$

Mathematica [A] time = 0.0135455, size = 20, normalized size = 1.

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)],x]
```

```
[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Maple [A] time = 0.074, size = 23, normalized size = 1.2

$$-\frac{\mathrm{Ei}\left(1,-\ln\left(c\left(bx^2+a\right)\right)\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)),x)
```

```
[Out] -1/2/b/c*Ei(1,-ln(c*(b*x^2+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(\left(bx^2+a\right)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="maxima")
```

```
[Out] integrate(x/log((b*x^2 + a)*c), x)
```

Fricas [A] time = 1.91236, size = 53, normalized size = 2.65

$$\frac{\mathrm{log\_integral}\left(bcx^2+ac\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="fricas")
```

```
[Out] 1/2*log_integral(b*c*x^2 + a*c)/(b*c)
```

Sympy [A] time = 2.79422, size = 27, normalized size = 1.35

$$\begin{cases} \frac{x^2}{2\log(ac)} & \text{for}\ b=0 \\ 0 & \text{for}\ c=0 \\ \frac{\mathrm{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)),x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))
```

Giac [A] time = 1.31621, size = 26, normalized size = 1.3

$$\frac{\operatorname{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="giac")
```

```
[Out] 1/2*Ei(log((b*x^2 + a)*c))/(b*c)
```

3.125 $\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)} dx$

Optimal. Leaf size=71

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\,\text{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(2*b*Log[
c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(2*b^2*c)
```

Rubi [A] time = 0.1252, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2400, 2399, 2389, 2298, 2390, 2309, 2178}

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\,\text{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(2*b*Log[
c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(2*b^2*c)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right)\\
&= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b} + \text{Subst}\left(\int \frac{x}{\log(c(a+bx))}\,dx, x, \right.\\
&= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} + \text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx))}\right.\right.\\
&= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{b} - \frac{a\,\text{Subst}\left(\int \right.}{}\\
&= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\text{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \right.}{}\\
&= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\,\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c\left(a+bx^2\right)\right)\right)}{b^2c^2}\\
&= \frac{\text{Ei}\left(2\log\left(c\left(a+bx^2\right)\right)\right)}{b^2c^2} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\,\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c}
\end{aligned}
$$

Mathematica [A] time = 0.106691, size = 66, normalized size = 0.93

$$
-\frac{-\frac{2\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{c^2} + \frac{a\text{Ei}\left(\log\left(c\left(bx^2+a\right)\right)\right)}{c} + \frac{bx^2\left(a+bx^2\right)}{\log\left(c\left(a+bx^2\right)\right)}}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] -((a*ExpIntegralEi[Log[c*(a + b*x^2)]])/c - (2*ExpIntegralEi[2*Log[c*(a + b
*x^2)]])/c^2 + (b*x^2*(a + b*x^2))/Log[c*(a + b*x^2)])/(2*b^2)
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\ln\left(c\left(bx^2+a\right)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a))^2,x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^4+ax^2}{2\left(b\log\left(bx^2+a\right)+b\log\left(c\right)\right)}+\int \frac{2\,bx^3+ax}{b\log\left(bx^2+a\right)+b\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^4 + a*x^2)/(b*log(b*x^2 + a) + b*log(c)) + integrate((2*b*x^3 + a
*x)/(b*log(b*x^2 + a) + b*log(c)), x)
```

Fricas [A] time = 1.94275, size = 235, normalized size = 3.31

$$-\frac{b^2c^2x^4+abc^2x^2+\left(ac\,\mathrm{log\_integral}\left(bcx^2+ac\right)-2\,\mathrm{log\_integral}\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)\right)\log\left(bcx^2+ac\right)}{2\,b^2c^2\log\left(bcx^2+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 2*log_
integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c))/(b^2*c
^2*log(b*c*x^2 + a*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{-ax^2-bx^4}{2b\log\left(c\left(a+bx^2\right)\right)}+\frac{\int \frac{ax}{\log\left(ac+bcx^2\right)}\,dx+\int \frac{2bx^3}{\log\left(ac+bcx^2\right)}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a))**2,x)
```

```
[Out] (-a*x**2 - b*x**4)/(2*b*log(c*(a + b*x**2))) + (Integral(a*x/log(a*c + b*c*
x**2), x) + Integral(2*b*x**3/log(a*c + b*c*x**2), x))/b
```

Giac [A] time = 1.2184, size = 120, normalized size = 1.69

$$-\frac{ac\text{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right)-\frac{\left(bcx^2+ac\right)ac}{\log\left(\left(bx^2+a\right)c\right)}+\frac{\left(bcx^2+ac\right)^2}{\log\left(\left(bx^2+a\right)c\right)}-2\,\text{Ei}\left(2\,\log\left(\left(bx^2+a\right)c\right)\right)}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*c*Ei(log((b*x^2 + a)*c)) - (b*c*x^2 + a*c)*a*c/log((b*x^2 + a)*c) +
 (b*c*x^2 + a*c)^2/log((b*x^2 + a)*c) - 2*Ei(2*log((b*x^2 + a)*c)))/(b^2*c^
2)
```

3.126 $\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)} dx$

Optimal. Leaf size=47

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}-\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] -(a + b*x^2)/(2*b*Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rubi [A] time = 0.0430946, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2454, 2389, 2297, 2298}

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}-\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] -(a + b*x^2)/(2*b*Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^2(c(a+bx))}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{li}\left(c\left(a+bx^2\right)\right)}{2bc}$$

Mathematica [A] time = 0.0192256, size = 43, normalized size = 0.91

$$\frac{\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{c} - \frac{a+bx^2}{\log\left(c\left(a+bx^2\right)\right)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] (-((a + b*x^2)/Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/c)/(2*b)
```

Maple [A] time = 0.07, size = 59, normalized size = 1.3

$$-\frac{x^2}{2\ln\left(c\left(bx^2+a\right)\right)} - \frac{a}{2b\ln\left(c\left(bx^2+a\right)\right)} - \frac{\text{Ei}\left(1,-\ln\left(c\left(bx^2+a\right)\right)\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a))^2,x)
```

```
[Out] -1/2/ln(c*(b*x^2+a))*x^2-1/2/b/ln(c*(b*x^2+a))*a-1/2/b/c*Ei(1,-ln(c*(b*x^2+
a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^2+a}{2\left(b\log\left(bx^2+a\right)+b\log(c)\right)} + \int \frac{x}{\log\left(bx^2+a\right)+\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*log(b*x^2 + a) + b*log(c)) + integrate(x/(log(b*x^2 + a
) + log(c)), x)
```

Fricas [A] time = 1.94823, size = 130, normalized size = 2.77

$$-\frac{bcx^2 + ac - \log\left(bcx^2 + ac\right)\mathrm{log\_integral}\left(bcx^2 + ac\right)}{2\,bc\log\left(bcx^2 + ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x^2 + a*c - log(b*c*x^2 + a*c)*log_integral(b*c*x^2 + a*c))/(b*c*
log(b*c*x^2 + a*c))
```

Sympy [A] time = 2.80688, size = 49, normalized size = 1.04

$$\begin{cases} \frac{x^2}{2\log(ac)} & \text{for}\; b = 0 \\ 0 & \text{for}\; c = 0 \\ \frac{\mathrm{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases} + \frac{-a - bx^2}{2b\log\left(c\left(a + bx^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a))**2,x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True)) + (-a - b*x**2)/(2*b*log(c*(a + b*x**2)))
```

Giac [A] time = 1.27656, size = 61, normalized size = 1.3

$$-\frac{\frac{bcx^2+ac}{\log\left(\left(bx^2+a\right)c\right)} - \mathrm{Ei}\left(\log\left(\left(bx^2 + a\right)c\right)\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*c*x^2 + a*c)/log((b*x^2 + a)*c) - Ei(log((b*x^2 + a)*c)))/(b*c)
```

3.127 $\int \frac{x^3}{\log^3(c(a+bx^2))} dx$

Optimal. Leaf size=127

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\,\text{li}\left(c\left(bx^2+a\right)\right)}{4b^2c}-\frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(4*b*Log[
c*(a + b*x^2)]^2) - (a*(a + b*x^2))/(4*b^2*Log[c*(a + b*x^2)]) - (x^2*(a +
b*x^2))/(2*b*Log[c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(4*b^2*c)
```

Rubi [A] time = 0.170153, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2454, 2400, 2399, 2389, 2298, 2390, 2309, 2178, 2297}

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\,\text{li}\left(c\left(bx^2+a\right)\right)}{4b^2c}-\frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(4*b*Log[
c*(a + b*x^2)]^2) - (a*(a + b*x^2))/(4*b^2*Log[c*(a + b*x^2)]) - (x^2*(a +
b*x^2))/(2*b*Log[c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(4*b^2*c)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rubi steps

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log^3(c(a+bx))}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right) + \frac{a\,\text{Subst}\left(\int \frac{1}{\log^2(c(a+bx))}\,d\right.}{4b}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b^2} + \frac{a\,\text{Subs}}{}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log(cx)}\,d\right.}{4b^2}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\,\text{li}\left(c\left(a+bx^2\right)\right)}{4b^2c} +$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\,\text{li}\left(c\left(a+bx^2\right)\right)}{4b^2c} +$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\,\text{li}\left(c\left(a+bx^2\right)\right)}{4b^2c} +$$

$$= \frac{\text{Ei}\left(2\log\left(c\left(a+bx^2\right)\right)\right)}{b^2c^2} - \frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right.\right.}$$

Mathematica [A] time = 0.12011, size = 87, normalized size = 0.69

$$-\frac{-\frac{4\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{c^2} + \frac{a\text{Ei}\left(\log\left(c\left(bx^2+a\right)\right)\right)}{c} + \frac{\left(a+bx^2\right)\left(\left(a+2bx^2\right)\log\left(c\left(a+bx^2\right)\right)+bx^2\right)}{\log^2\left(c\left(a+bx^2\right)\right)}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] -((a*ExpIntegralEi[Log[c*(a + b*x^2)]])/c - (4*ExpIntegralEi[2*Log[c*(a + b
*x^2)]])/c^2 + ((a + b*x^2)*(b*x^2 + (a + 2*b*x^2)*Log[c*(a + b*x^2)]))/Log
[c*(a + b*x^2)]^2)/(4*b^2)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\ln\left(c\left(bx^2+a\right)\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a))^3,x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2x^4(2\log(c)+1)+abx^2(3\log(c)+1)+a^2\log(c)+\left(2b^2x^4+3abx^2+a^2\right)\log\left(bx^2+a\right)}{4\left(b^2\log\left(bx^2+a\right)^2+2b^2\log\left(bx^2+a\right)\log(c)+b^2\log(c)^2\right)}+\int\frac{4bx^3+3ax}{2\left(b\log\left(bx^2+a\right)+b\,\mathrm{l}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*x^4*(2*log(c) + 1) + a*b*x^2*(3*log(c) + 1) + a^2*log(c) + (2*b^2
*x^4 + 3*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^2*log(b*x^2 + a)^2 + 2*b^2*log(b
*x^2 + a)*log(c) + b^2*log(c)^2) + integrate(1/2*(4*b*x^3 + 3*a*x)/(b*log(b
*x^2 + a) + b*log(c)), x)
```

Fricas [A] time = 2.05819, size = 325, normalized size = 2.56

$$-\frac{b^2c^2x^4+abc^2x^2+\left(ac\,\mathrm{log\_integral}\left(bcx^2+ac\right)-4\,\mathrm{log\_integral}\left(b^2c^2x^4+2abc^2x^2+a^2c^2\right)\right)\log\left(bcx^2+ac\right)^2+\left(2b^2c^2\right.}{4b^2c^2\log\left(bcx^2+ac\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 4*log_
integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c)^2 + (2*
b^2*c^2*x^4 + 3*a*b*c^2*x^2 + a^2*c^2)*log(b*c*x^2 + a*c))/(b^2*c^2*log(b*c
*x^2 + a*c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{3ax}{\log\left(ac+bcx^2\right)}\,dx+\int\frac{4bx^3}{\log\left(ac+bcx^2\right)}\,dx}{2b}+\frac{-abx^2-b^2x^4+\left(-a^2-3abx^2-2b^2x^4\right)\log\left(c\left(a+bx^2\right)\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a))**3,x)
```

```
[Out] (Integral(3*a*x/log(a*c + b*c*x**2), x) + Integral(4*b*x**3/log(a*c + b*c*x
**2), x))/(2*b) + (-a*b*x**2 - b**2*x**4 + (-a**2 - 3*a*b*x**2 - 2*b**2*x**
4)*log(c*(a + b*x**2)))/(4*b**2*log(c*(a + b*x**2))**2)
```

Giac [A] time = 1.28478, size = 190, normalized size = 1.5

$$-\frac{ac\,\mathrm{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right)-\frac{\left(bcx^2+ac\right)ac}{\log\left(\left(bx^2+a\right)c\right)}-\frac{\left(bcx^2+ac\right)ac}{\log\left(\left(bx^2+a\right)c\right)^2}+\frac{2\left(bcx^2+ac\right)^2}{\log\left(\left(bx^2+a\right)c\right)}+\frac{\left(bcx^2+ac\right)^2}{\log\left(\left(bx^2+a\right)c\right)^2}-4\,\mathrm{Ei}\left(2\log\left(\left(bx^2+a\right)c\right)\right)}{4b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*c*Ei(log((b*x^2 + a)*c)) - (b*c*x^2 + a*c)*a*c/log((b*x^2 + a)*c) -
 (b*c*x^2 + a*c)*a*c/log((b*x^2 + a)*c)^2 + 2*(b*c*x^2 + a*c)^2/log((b*x^2
+ a)*c) + (b*c*x^2 + a*c)^2/log((b*x^2 + a)*c)^2 - 4*Ei(2*log((b*x^2 + a)*c
)))/(b^2*c^2)
```

3.128 $\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx$

Optimal. Leaf size=73

$$\frac{\operatorname{li}\left(c\left(bx^2+a\right)\right)}{4bc}-\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] -(a + b*x^2)/(4*b*Log[c*(a + b*x^2)]^2) - (a + b*x^2)/(4*b*Log[c*(a + b*x^2
)]) + LogIntegral[c*(a + b*x^2)]/(4*b*c)
```

Rubi [A] time = 0.0592836, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2454, 2389, 2297, 2298}

$$\frac{\operatorname{li}\left(c\left(bx^2+a\right)\right)}{4bc}-\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] -(a + b*x^2)/(4*b*Log[c*(a + b*x^2)]^2) - (a + b*x^2)/(4*b*Log[c*(a + b*x^2
)]) + LogIntegral[c*(a + b*x^2)]/(4*b*c)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\log^3(c(a+bx))}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b}$$
$$= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{4b}$$
$$= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{li}\left(c\left(a+bx^2\right)\right)}{4bc}$$

Mathematica [A] time = 0.0231036, size = 55, normalized size = 0.75

$$\frac{\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{c} - \frac{\left(a+bx^2\right)\left(\log\left(c\left(a+bx^2\right)\right)+1\right)}{\log^2\left(c\left(a+bx^2\right)\right)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] (-(((a + b*x^2)*(1 + Log[c*(a + b*x^2)]))/Log[c*(a + b*x^2)]^2) + LogIntegr
al[c*(a + b*x^2)]/c)/(4*b)
```

Maple [A] time = 0.068, size = 94, normalized size = 1.3

$$-\frac{x^2}{4\left(\ln\left(c\left(bx^2+a\right)\right)\right)^2} - \frac{a}{4\,b\left(\ln\left(c\left(bx^2+a\right)\right)\right)^2} - \frac{x^2}{4\,\ln\left(c\left(bx^2+a\right)\right)} - \frac{a}{4\,b\ln\left(c\left(bx^2+a\right)\right)} - \frac{\text{Ei}\left(1, -\ln\left(c\left(bx^2+a\right)\right)\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a))^3,x)
```

```
[Out] -1/4/ln(c*(b*x^2+a))^2*x^2-1/4/b/ln(c*(b*x^2+a))^2*a-1/4/ln(c*(b*x^2+a))*x^
2-1/4/b/ln(c*(b*x^2+a))*a-1/4/b/c*Ei(1,-ln(c*(b*x^2+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bx^2(\log(c)+1) + a(\log(c)+1) + \left(bx^2+a\right)\log\left(bx^2+a\right)}{4\left(b\log\left(bx^2+a\right)^2 + 2\,b\log\left(bx^2+a\right)\log(c) + b\log(c)^2\right)} + \int \frac{x}{2\left(\log\left(bx^2+a\right)+\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^2*(log(c) + 1) + a*(log(c) + 1) + (b*x^2 + a)*log(b*x^2 + a))/(b*
log(b*x^2 + a)^2 + 2*b*log(b*x^2 + a)*log(c) + b*log(c)^2) + integrate(1/2*
x/(log(b*x^2 + a) + log(c)), x)
```

Fricas [A] time = 1.90631, size = 185, normalized size = 2.53

$$-\frac{bcx^2 - \log\left(bcx^2 + ac\right)^2 \operatorname{log\_integral}\left(bcx^2 + ac\right) + ac + \left(bcx^2 + ac\right)\log\left(bcx^2 + ac\right)}{4\,bc\log\left(bcx^2 + ac\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2 - log(b*c*x^2 + a*c)^2*log_integral(b*c*x^2 + a*c) + a*c + (b
*c*x^2 + a*c)*log(b*c*x^2 + a*c))/(b*c*log(b*c*x^2 + a*c)^2)
```

Sympy [A] time = 2.9801, size = 70, normalized size = 0.96

$$\frac{\begin{cases} \frac{x^2}{2\log(ac)} & \text{for}\ b = 0 \\ 0 & \text{for}\ c = 0 \\ \frac{\operatorname{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases}}{2} + \frac{-a - bx^2 + \left(-a - bx^2\right)\log\left(c\left(a + bx^2\right)\right)}{4b\log\left(c\left(a + bx^2\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a))**3,x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))/2 + (-a - b*x**2 + (-a - b*x**2)*log(c*(a + b*x**2)))
/(4*b*log(c*(a + b*x**2))**2)
```

Giac [A] time = 1.28315, size = 92, normalized size = 1.26

$$-\frac{\frac{bcx^2+ac}{\log\left(\left(bx^2+a\right)c\right)} + \frac{bcx^2+ac}{\log\left(\left(bx^2+a\right)c\right)^2} - \operatorname{Ei}\left(\log\left(\left(bx^2 + a\right)c\right)\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] -1/4*((b*c*x^2 + a*c)/log((b*x^2 + a)*c) + (b*c*x^2 + a*c)/log((b*x^2 + a)*
c)^2 - Ei(log((b*x^2 + a)*c)))/(b*c)
```

3.129 $\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right) dx$

Optimal. Leaf size=150

$$\frac{\left(d+ex^3\right)^2 \log^2\left(c\left(d+ex^3\right)^p\right)}{6e^2}-\frac{d\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e^2}-\frac{p\left(d+ex^3\right)^2\log\left(c\left(d+ex^3\right)^p\right)}{6e^2}+\frac{2dp\left(d+ex^3\right)\log\left(\right.}{3e^2}$$

```
[Out] (-2*d*p^2*x^3)/(3*e) + (p^2*(d + e*x^3)^2)/(12*e^2) + (2*d*p*(d + e*x^3)*Lo
g[c*(d + e*x^3)^p])/(3*e^2) - (p*(d + e*x^3)^2*Log[c*(d + e*x^3)^p])/(6*e^2
) - (d*(d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/(3*e^2) + ((d + e*x^3)^2*Log[c*(
d + e*x^3)^p]^2)/(6*e^2)
```

Rubi [A] time = 0.15612, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{\left(d+ex^3\right)^2 \log^2\left(c\left(d+ex^3\right)^p\right)}{6e^2}-\frac{d\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e^2}-\frac{p\left(d+ex^3\right)^2\log\left(c\left(d+ex^3\right)^p\right)}{6e^2}+\frac{2dp\left(d+ex^3\right)\log\left(\right.}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (-2*d*p^2*x^3)/(3*e) + (p^2*(d + e*x^3)^2)/(12*e^2) + (2*d*p*(d + e*x^3)*Lo
g[c*(d + e*x^3)^p])/(3*e^2) - (p*(d + e*x^3)^2*Log[c*(d + e*x^3)^p])/(6*e^2
) - (d*(d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/(3*e^2) + ((d + e*x^3)^2*Log[c*(
d + e*x^3)^p]^2)/(6*e^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right) dx &= \frac{1}{3}\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right) dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\log^2\left(c(d+ex)^p\right)}{e}\right) dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int (d+ex)\log^2\left(c(d+ex)^p\right) dx, x, x^3\right)}{3e}-\frac{d\,\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right) dx, x, x^3\right)}{3e}\\
&= \frac{\text{Subst}\left(\int x\log^2\left(cx^p\right) dx, x, d+ex^3\right)}{3e^2}-\frac{d\,\text{Subst}\left(\int \log^2\left(cx^p\right) dx, x, d+ex^3\right)}{3e^2}\\
&= -\frac{d\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e^2}+\frac{\left(d+ex^3\right)^2\log^2\left(c\left(d+ex^3\right)^p\right)}{6e^2}-\frac{p\,\text{Subst}\left(\int x\log\left(cx^p\right)\right.}{3e^2}\\
&= -\frac{2dp^2x^3}{3e}+\frac{p^2\left(d+ex^3\right)^2}{12e^2}+\frac{2dp\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e^2}-\frac{p\left(d+ex^3\right)^2\log\left(c\left(d+ex^3\right)\right.}{6e^2}
\end{aligned}$$

Mathematica [A] time = 0.0637101, size = 105, normalized size = 0.7

$$\frac{-2\left(d^2-e^2x^6\right)\log^2\left(c\left(d+ex^3\right)^p\right)+2p\left(2d^2+2dex^3-e^2x^6\right)\log\left(c\left(d+ex^3\right)^p\right)+2d^2p^2\log\left(d+ex^3\right)+ep^2x^3\left(ex^3-6d\right)}{12e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (e*p^2*x^3*(-6*d + e*x^3) + 2*d^2*p^2*Log[d + e*x^3] + 2*p*(2*d^2 + 2*d*e*x
^3 - e^2*x^6)*Log[c*(d + e*x^3)^p] - 2*(d^2 - e^2*x^6)*Log[c*(d + e*x^3)^p]
^2)/(12*e^2)
```

Maple [C] time = 0.681, size = 1242, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] 1/6*(I*Pi*e^2*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*e^2*x^6*
csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*e^2*x^6*csgn(I*c*(
e*x^3+d)^p)^3+I*Pi*e^2*x^6*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)*e^2*x^
6-e^2*p*x^6+2*d*e*p*x^3-2*d^2*p*ln(e*x^3+d))/e^2*ln((e*x^3+d)^p)-1/6*ln(c)*
p*x^6+1/6*x^6*ln((e*x^3+d)^p)^2+1/6*ln(c)^2*x^6+1/12*p^2*x^6+1/6/e^2*d^2*p^
2*ln(e*x^3+d)^2-1/24*Pi^2*x^6*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)^2+1/12*Pi^2
*x^6*csgn(I*c*(e*x^3+d)^p)^5*csgn(I*c)-1/24*Pi^2*x^6*csgn(I*(e*x^3+d)^p)^2*
csgn(I*c*(e*x^3+d)^p)^4+1/12*Pi^2*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d
)^p)^5-1/2*d*p^2*x^3/e+1/2*d^2*p^2/e^2*ln(e*x^3+d)-1/6*I/e*Pi*d*p*x^3*csgn(
I*c*(e*x^3+d)^p)^3+1/6*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*c*(e*x^3+d)^p)^3-1
/6*I*ln(c)*Pi*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+1/12*
I*Pi*p*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-1/24*Pi^2*x^
6*csgn(I*c*(e*x^3+d)^p)^6+1/6*I*ln(c)*Pi*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(
e*x^3+d)^p)^2+1/3/e*ln(c)*d*p*x^3-1/3/e^2*ln(c)*ln(e*x^3+d)*d^2*p-1/24*Pi^2
*x^6*csgn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)^2+1/12*Pi^2*x^
6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)^2+1/12*Pi^2*x^6*csg
n(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)-1/6*Pi^2*x^6*csgn(I*(e
*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)-1/6*I*ln(c)*Pi*x^6*csgn(I*c*(e
*x^3+d)^p)^3+1/12*I*Pi*p*x^6*csgn(I*c*(e*x^3+d)^p)^3+1/6*I/e*Pi*d*p*x^3*csg
n(I*c*(e*x^3+d)^p)^2*csgn(I*c)+1/6*I/e*Pi*d*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(
I*c*(e*x^3+d)^p)^2-1/6*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*c*(e*x^3+d)^p)^2*c
sgn(I*c)-1/6*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3
+d)^p)^2-1/12*I*Pi*p*x^6*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/12*I*Pi*p*x^6*
csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/6*I*ln(c)*Pi*x^6*csgn(I*c*(e*
x^3+d)^p)^2*csgn(I*c)-1/6*I/e*Pi*d*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^
3+d)^p)*csgn(I*c)+1/6*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*(e*x^3+d)^p)*csgn(I
*c*(e*x^3+d)^p)*csgn(I*c)
```

Maxima [A] time = 1.07146, size = 162, normalized size = 1.08

$$\frac{1}{6}x^6\log\left(\left(ex^3+d\right)^pc\right)^2-\frac{1}{6}ep\left(\frac{2d^2\log\left(ex^3+d\right)}{e^3}+\frac{ex^6-2dx^3}{e^2}\right)\log\left(\left(ex^3+d\right)^pc\right)+\frac{\left(e^2x^6-6dex^3+2d^2\log\left(ex^3+d\right)\right.}{12e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((e*x^3 + d)^p*c)^2 - 1/6*e*p*(2*d^2*log(e*x^3 + d)/e^3 + (e*x^6
 - 2*d*x^3)/e^2)*log((e*x^3 + d)^p*c) + 1/12*(e^2*x^6 - 6*d*e*x^3 + 2*d^2*l
og(e*x^3 + d)^2 + 6*d^2*log(e*x^3 + d))*p^2/e^2
```

Fricas [A] time = 2.04078, size = 317, normalized size = 2.11

$$\frac{e^2p^2x^6+2e^2x^6\log(c)^2-6dep^2x^3+2\left(e^2p^2x^6-d^2p^2\right)\log\left(ex^3+d\right)^2-2\left(e^2p^2x^6-2dep^2x^3-3d^2p^2-2\left(e^2px^6-d^2p\right.\right.}{12e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(e^2*p^2*x^6 + 2*e^2*x^6*log(c)^2 - 6*d*e*p^2*x^3 + 2*(e^2*p^2*x^6 - d
^2*p^2)*log(e*x^3 + d)^2 - 2*(e^2*p^2*x^6 - 2*d*e*p^2*x^3 - 3*d^2*p^2 - 2*(
e^2*p*x^6 - d^2*p)*log(c))*log(e*x^3 + d) - 2*(e^2*p*x^6 - 2*d*e*p*x^3)*log
(c))/e^2
```

Sympy [A] time = 54.5267, size = 206, normalized size = 1.37

$$\begin{cases} -\frac{d^2p^2\log\left(d+ex^3\right)^2}{6e^2} + \frac{d^2p^2\log\left(d+ex^3\right)}{2e^2} - \frac{d^2p\log(c)\log\left(d+ex^3\right)}{3e^2} + \frac{dp^2x^3\log\left(d+ex^3\right)}{3e} - \frac{dp^2x^3}{2e} + \frac{dpx^3\log(c)}{3e} + \frac{p^2x^6\log\left(d+ex^3\right)^2}{6} - \frac{p^2x^6\log\left(d+ex^3\right.}{6} \\ \frac{x^6\log\left(cd^p\right)^2}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((-d**2*p**2*log(d + e*x**3)**2/(6*e**2) + d**2*p**2*log(d + e*x**
3)/(2*e**2) - d**2*p*log(c)*log(d + e*x**3)/(3*e**2) + d*p**2*x**3*log(d +
e*x**3)/(3*e) - d*p**2*x**3/(2*e) + d*p*x**3*log(c)/(3*e) + p**2*x**6*log(d
 + e*x**3)**2/6 - p**2*x**6*log(d + e*x**3)/6 + p**2*x**6/12 + p*x**6*log(c
)*log(d + e*x**3)/3 - p*x**6*log(c)/6 + x**6*log(c)**2/6, Ne(e, 0)), (x**6*
log(c*d**p)**2/6, True))
```

Giac [A] time = 1.22022, size = 298, normalized size = 1.99

$$\frac{1}{12}\left(\left(2\left(x^3e+d\right)^2\log\left(x^3e+d\right)^2 - 4\left(x^3e+d\right)d\log\left(x^3e+d\right)^2 - 2\left(x^3e+d\right)^2\log\left(x^3e+d\right) + 8\left(x^3e+d\right)d\log\left(x^3e+d\right) + \left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/12*((2*(x^3*e + d)^2*log(x^3*e + d)^2 - 4*(x^3*e + d)*d*log(x^3*e + d)^2
- 2*(x^3*e + d)^2*log(x^3*e + d) + 8*(x^3*e + d)*d*log(x^3*e + d) + (x^3*e
+ d)^2 - 8*(x^3*e + d)*d)*p^2*e^(-1) + 2*(2*(x^3*e + d)^2*log(x^3*e + d) -
4*(x^3*e + d)*d*log(x^3*e + d) - (x^3*e + d)^2 + 4*(x^3*e + d)*d)*p*e^(-1)*
log(c) + 2*((x^3*e + d)^2 - 2*(x^3*e + d)*d)*e^(-1)*log(c)^2)*e^(-1)
```

3.130 $\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right) dx$

Optimal. Leaf size=66

$$\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}-\frac{2p\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e}+\frac{2p^2x^3}{3}$$

```
[Out] (2*p^2*x^3)/3 - (2*p*(d + e*x^3)*Log[c*(d + e*x^3)^p])/(3*e) + ((d + e*x^3)
*Log[c*(d + e*x^3)^p]^2)/(3*e)
```

Rubi [A] time = 0.0556588, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2389, 2296, 2295}

$$\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}-\frac{2p\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e}+\frac{2p^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (2*p^2*x^3)/3 - (2*p*(d + e*x^3)*Log[c*(d + e*x^3)^p])/(3*e) + ((d + e*x^3)
*Log[c*(d + e*x^3)^p]^2)/(3*e)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)dx = \frac{1}{3}\operatorname{Subst}\left(\int \log^2\left(c(d+ex)^p\right)dx, x, x^3\right)$$
$$= \frac{\operatorname{Subst}\left(\int \log^2\left(cx^p\right)dx, x, d+ex^3\right)}{3e}$$
$$= \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e} - \frac{(2p)\operatorname{Subst}\left(\int \log\left(cx^p\right)dx, x, d+ex^3\right)}{3e}$$
$$= \frac{2p^2x^3}{3} - \frac{2p\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e} + \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}$$

Mathematica [A] time = 0.0095782, size = 63, normalized size = 0.95

$$\frac{1}{3}\left(\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{e} - 2p\left(\frac{\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{e} - px^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (((d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/e - 2*p*(-(p*x^3) + ((d + e*x^3)*Log[
c*(d + e*x^3)^p])/e))/3
```

Maple [C] time = 0.556, size = 1036, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] -2/3*ln(c)*p*x^3-1/12*Pi^2*x^3*csgn(I*c*(e*x^3+d)^p)^6+1/3*(I*Pi*e*x^3*csgn
(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*e*x^3*csgn(I*(e*x^3+d)^p)*csgn
(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*e*x^3*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*e*x^3*c
sgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)*e*x^3-2*x^3*p*e+2*d*p*ln(e*x^3+d))
/e*ln((e*x^3+d)^p)+2/3*p^2*x^3+1/3*ln(c)^2*x^3-1/3/e*d*p^2*ln(e*x^3+d)^2-1/
12*Pi^2*x^3*csgn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^4+1/6*Pi^2*x^3*csgn
(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^5+1/6*Pi^2*x^3*csgn(I*c*(e*x^3+d)^p)^
5*csgn(I*c)-1/12*Pi^2*x^3*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)^2+1/3*x^3*ln((e
*x^3+d)^p)^2-2/3*d*p^2/e*ln(e*x^3+d)+2/3/e*ln(c)*ln(e*x^3+d)*d*p+1/6*Pi^2*x
^3*csgn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)-1/12*Pi^2*x^3*cs
gn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)^2-1/3*Pi^2*x^3*csgn(I
*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)+1/6*Pi^2*x^3*csgn(I*(e*x^3+
d)^p)*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)^2-1/3*I*ln(c)*Pi*x^3*csgn(I*c*(e*x^
3+d)^p)^3+1/3*I*Pi*p*x^3*csgn(I*c*(e*x^3+d)^p)^3-1/3*I/e*Pi*ln(e*x^3+d)*d*p
*csgn(I*c*(e*x^3+d)^p)^3-1/3*I*ln(c)*Pi*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e
*x^3+d)^p)*csgn(I*c)+1/3*I*Pi*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^
p)*csgn(I*c)+1/3*I/e*Pi*ln(e*x^3+d)*d*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3
+d)^p)^2+1/3*I/e*Pi*ln(e*x^3+d)*d*p*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/3*I
/e*Pi*ln(e*x^3+d)*d*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+1
/3*I*ln(c)*Pi*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I*ln(c)*P
i*x^3*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/3*I*Pi*p*x^3*csgn(I*(e*x^3+d)^p)*
```

```
csgn(I*c*(e*x^3+d)^p)^2-1/3*I*Pi*p*x^3*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)
```

Maxima [A] time = 1.06436, size = 131, normalized size = 1.98

$$\frac{1}{3}x^3\log\left(\left(ex^3+d\right)^pc\right)^2-\frac{2}{3}\left(\frac{x^3}{e}-\frac{d\log\left(ex^3+d\right)}{e^2}\right)ep\log\left(\left(ex^3+d\right)^pc\right)+\frac{\left(2\,ex^3-d\log\left(ex^3+d\right)^2-2\,d\log\left(ex^3+d\right)\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((e*x^3 + d)^p*c)^2 - 2/3*(x^3/e - d*log(e*x^3 + d)/e^2)*e*p*log
((e*x^3 + d)^p*c) + 1/3*(2*e*x^3 - d*log(e*x^3 + d)^2 - 2*d*log(e*x^3 + d))
*p^2/e
```

Fricas [A] time = 1.88889, size = 216, normalized size = 3.27

$$\frac{2\,ep^2x^3-2\,epx^3\log\left(c\right)+ex^3\log\left(c\right)^2+\left(ep^2x^3+dp^2\right)\log\left(ex^3+d\right)^2-2\left(ep^2x^3+dp^2-\left(epx^3+dp\right)\log\left(c\right)\right)\log\left(ex^3+\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e*p^2*x^3 - 2*e*p*x^3*log(c) + e*x^3*log(c)^2 + (e*p^2*x^3 + d*p^2)*
log(e*x^3 + d)^2 - 2*(e*p^2*x^3 + d*p^2 - (e*p*x^3 + d*p)*log(c))*log(e*x^3
 + d))/e
```

Sympy [A] time = 12.3171, size = 160, normalized size = 2.42

$$\begin{cases}\frac{dp^2\log\left(d+ex^3\right)^2}{3e}-\frac{2dp^2\log\left(d+ex^3\right)}{3e}+\frac{2dp\log\left(c\right)\log\left(d+ex^3\right)}{3e}+\frac{p^2x^3\log\left(d+ex^3\right)^2}{3}-\frac{2p^2x^3\log\left(d+ex^3\right)}{3}+\frac{2p^2x^3}{3}+\frac{2px^3\log\left(c\right)\log\left(d+ex^3\right)}{3}-\frac{2\ldots}{} \\ \frac{x^3\log\left(cd^p\right)^2}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((d*p**2*log(d + e*x**3)**2/(3*e) - 2*d*p**2*log(d + e*x**3)/(3*e)
 + 2*d*p*log(c)*log(d + e*x**3)/(3*e) + p**2*x**3*log(d + e*x**3)**2/3 - 2*
p**2*x**3*log(d + e*x**3)/3 + 2*p**2*x**3/3 + 2*p*x**3*log(c)*log(d + e*x**
3)/3 - 2*p*x**3*log(c)/3 + x**3*log(c)**2/3, Ne(e, 0)), (x**3*log(c*d**p)**
2/3, True))
```

Giac [A] time = 1.24961, size = 140, normalized size = 2.12

$$\frac{1}{3}\left(\left(2\,x^3e+\left(x^3e+d\right)\log\left(x^3e+d\right)^2-2\left(x^3e+d\right)\log\left(x^3e+d\right)+2\,d\right)p^2-2\left(x^3e-\left(x^3e+d\right)\log\left(x^3e+d\right)+d\right)p\log\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/3*((2*x^3*e + (x^3*e + d)*log(x^3*e + d)^2 - 2*(x^3*e + d)*log(x^3*e + d)
 + 2*d)*p^2 - 2*(x^3*e - (x^3*e + d)*log(x^3*e + d) + d)*p*log(c) + (x^3*e
+ d)*log(c)^2)*e^(-1)
```

3.131 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} dx$

Optimal. Leaf size=77

$$\frac{2}{3}p\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)\log\left(c\left(d+ex^3\right)^p\right)-\frac{2}{3}p^2\text{PolyLog}\left(3,\frac{ex^3}{d}+1\right)+\frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)$$

```
[Out] (Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p]^2)/3 + (2*p*Log[c*(d + e*x^3)^p]*Po
lyLog[2, 1 + (e*x^3)/d])/3 - (2*p^2*PolyLog[3, 1 + (e*x^3)/d])/3
```

Rubi [A] time = 0.108394, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2396, 2433, 2374, 6589}

$$\frac{2}{3}p\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)\log\left(c\left(d+ex^3\right)^p\right)-\frac{2}{3}p^2\text{PolyLog}\left(3,\frac{ex^3}{d}+1\right)+\frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x,x]
```


```
[Out] (Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p]^2)/3 + (2*p*Log[c*(d + e*x^3)^p]*Po
lyLog[2, 1 + (e*x^3)/d])/3 - (2*p^2*PolyLog[3, 1 + (e*x^3)/d])/3
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) - \frac{1}{3}(2ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^p\right)}{d+ex}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) - \frac{1}{3}(2p)\,\text{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^3\right)$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) + \frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right) - \frac{1}{3}\left(2p^2\right)\text{Subst}\left(\int \text{I}\right.$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) + \frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right) - \frac{2}{3}p^2\text{Li}_3\left(1+\frac{ex^3}{d}\right)$$

Mathematica [B] time = 0.0967049, size = 163, normalized size = 2.12

$$2p\left(\log(x)\left(\log\left(d+ex^3\right)-\log\left(\frac{ex^3}{d}+1\right)\right)-\frac{1}{3}\text{PolyLog}\left(2,-\frac{ex^3}{d}\right)\right)\left(\log\left(c\left(d+ex^3\right)^p\right)-p\log\left(d+ex^3\right)\right)+\frac{1}{3}p^2\left(-2\text{PolyI}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^3]) + Log[c*(d + e*x^3)^p])^2 + 2*p*(-(p*Log[d + e*
x^3]) + Log[c*(d + e*x^3)^p])*(Log[x]*(Log[d + e*x^3] - Log[1 + (e*x^3)/d])
 - PolyLog[2, -((e*x^3)/d)]/3) + (p^2*(Log[-((e*x^3)/d)]*Log[d + e*x^3]^2 +
 2*Log[d + e*x^3]*PolyLog[2, 1 + (e*x^3)/d] - 2*PolyLog[3, 1 + (e*x^3)/d]))
/3
```

Maple [F] time = 0.908, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

3.132 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=86

$$\frac{2ep^2\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)}{3d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3}+\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}$$

```
[Out] (2*e*p*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p])/(3*d) - ((d + e*x^3)*Log[c*(
d + e*x^3)^p]^2)/(3*d*x^3) + (2*e*p^2*PolyLog[2, 1 + (e*x^3)/d])/(3*d)
```

Rubi [A] time = 0.0841468, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2397, 2394, 2315}

$$\frac{2ep^2\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)}{3d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3}+\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

```
[Out] (2*e*p*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p])/(3*d) - ((d + e*x^3)*Log[c*(
d + e*x^3)^p]^2)/(3*d*x^3) + (2*e*p^2*PolyLog[2, 1 + (e*x^3)/d])/(3*d)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3} + \frac{(2ep)\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^3\right)}{3d}$$
$$= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d} - \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3} - \frac{\left(2e^2p^2\right)\operatorname{Subst}\left(\int \frac{\log(-}{d+e}\right.}{3d}$$
$$= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d} - \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3} + \frac{2ep^2\operatorname{Li}_2\left(1+\frac{ex^3}{d}\right)}{3d}$$

Mathematica [A] time = 0.0373396, size = 84, normalized size = 0.98

$$\frac{2ep^2x^3\operatorname{PolyLog}\left(2,\frac{ex^3}{d}+1\right) - \left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right) + 2epx^3\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

```
[Out] (2*e*p*x^3*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p] - (d + e*x^3)*Log[c*(d +
e*x^3)^p]^2 + 2*e*p^2*x^3*PolyLog[2, 1 + (e*x^3)/d])/(3*d*x^3)
```

Maple [C] time = 0.619, size = 771, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^4,x)
```

```
[Out] -1/3/x^3*ln((e*x^3+d)^p)^2+2*p*e*ln((e*x^3+d)^p)/d*ln(x)-2/3*p*e*ln((e*x^3+
d)^p)/d*ln(e*x^3+d)-2*p^2*e/d*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),
_R1=RootOf(_Z^3*e+d))+1/3*p^2*e/d*ln(e*x^3+d)^2+I*p*e/d*ln(x)*Pi*csgn(I*c*(
e*x^3+d)^p)^2*csgn(I*c)-I*p*e/d*ln(x)*Pi*csgn(I*c*(e*x^3+d)^p)^3-1/3*I*p*e/
d*ln(e*x^3+d)*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+1/3*I/x^3*ln((e*x^3+d)^p
)*Pi*csgn(I*c*(e*x^3+d)^p)^3-2/3/x^3*ln((e*x^3+d)^p)*ln(c)+I*p*e/d*ln(x)*Pi
*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I*p*e/d*ln(e*x^3+d)*Pi*csg
n(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-1/3*I/x^3*ln((e*x^3+d)^p)*
Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I/x^3*ln((e*x^3+d)^p)*Pi
*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+2*p*e/d*ln(x)*ln(c)-1/
3*I/x^3*ln((e*x^3+d)^p)*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/3*I*p*e/d*ln
(e*x^3+d)*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I*p*e/d*ln(e*x
^3+d)*Pi*csgn(I*c*(e*x^3+d)^p)^3-I*p*e/d*ln(x)*Pi*csgn(I*(e*x^3+d)^p)*csgn(
I*c*(e*x^3+d)^p)*csgn(I*c)-2/3*p*e/d*ln(e*x^3+d)*ln(c)-1/12*(I*Pi*csgn(I*(e
*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3
+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*
csgn(I*c)+2*ln(c))^2/x^3
```

Maxima [A] time = 1.08756, size = 159, normalized size = 1.85

$$\frac{1}{3}e^2p^2\left(\frac{\log\left(ex^3+d\right)^2}{de}-\frac{2\left(3\log\left(\frac{ex^3}{d}+1\right)\log\left(x\right)+\operatorname{Li}_2\left(-\frac{ex^3}{d}\right)\right)}{de}\right)-\frac{2}{3}ep\left(\frac{\log\left(ex^3+d\right)}{d}-\frac{\log\left(x^3\right)}{d}\right)\log\left(\left(ex^3+d\right)^pc\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*e^2*p^2*(log(e*x^3 + d)^2/(d*e) - 2*(3*log(e*x^3/d + 1)*log(x) + dilog(
-e*x^3/d))/(d*e)) - 2/3*e*p*(log(e*x^3 + d)/d - log(x^3)/d)*log((e*x^3 + d)
^p*c) - 1/3*log((e*x^3 + d)^p*c)^2/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^3+d\right)^pc\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(ex^3+d\right)^pc\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^4, x)
```

3.133 $\int x \log^2\left(c\left(d+ex^3\right)^p\right) dx$

Optimal. Leaf size=1294

result too large to display

```
[Out] (9*p^2*x^2)/4 + (3*Sqrt[3]*d^(2/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt
[3]*d^(1/3))])/(2*e^(2/3)) + (3*d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*e^
(2/3)) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(2*e^(2/3)) + (d^(2/3)*p^
2*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(
1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(
1/3)*x])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)
*x]^2)/(2*e^(2/3)) + ((-1)^(2/3)*d^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e
^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]
)/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])
/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/(
2*e^(2/3)) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3)
 + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) - ((-1)^(2/3
)*d^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^
(1/3)))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/
e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[-
(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]
)/e^(2/3) - (3*d^(2/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/
(4*e^(2/3)) - (3*p*x^2*Log[c*(d + e*x^3)^p])/2 - (d^(2/3)*p*Log[d^(1/3) + e
^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) + ((-1)^(1/3)*d^(2/3)*p*Log[d^(1/3)
 - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) - ((-1)^(2/3)*d^(2/3
)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) + (x^
2*Log[c*(d + e*x^3)^p]^2)/2 + (d^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)
/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) - ((-1)^(2/3)*d^(2/3)*p^2*PolyLog[2,
-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/e^(2/3)
+ (d^(2/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3
))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)
*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*
d^(2/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d
^(1/3))])/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3
)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3)
```

Rubi [A] time = 1.91987, antiderivative size = 1300, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {2457, 2476, 2455, 321, 292, 31, 634, 617, 204, 628, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (9*p^2*x^2)/4 + (3*Sqrt[3]*d^(2/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt
[3]*d^(1/3))])/(2*e^(2/3)) + (3*d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*e^
(2/3)) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(2*e^(2/3)) + (d^(2/3)*p^
2*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(
1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(
```

```
1/3)*x])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)
*x]^2)/(2*e^(2/3)) + ((-1)^(2/3)*d^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e
^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]
)/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])
/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/(
2*e^(2/3)) - ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*
e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*
x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^
(1/3)))])/e^(2/3) - (3*d^(2/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3
)*x^2])/(4*e^(2/3)) - (3*p*x^2*Log[c*(d + e*x^3)^p])/2 - (d^(2/3)*p*Log[d^(
1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) + ((-1)^(1/3)*d^(2/3)*p*Log
[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) - ((-1)^(2/3
)*d^(2/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/
3) + (x^2*Log[c*(d + e*x^3)^p]^2)/2 + (d^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*PolyLog[2, (2*
(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(2/3) - ((-1)^(1/3)*d^
(2/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 -
(-1)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*PolyLog[2, (d^(1/
3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) - ((-1)^(2/
3)*d^(2/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1
 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*PolyLog[2, (d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(2/3)
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)dx=\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int\frac{x^4\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}dx$$

$$=\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int\left(\frac{x\log\left(c\left(d+ex^3\right)^p\right)}{e}-\frac{dx\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\right)dx$$

$$=\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3p)\int x\log\left(c\left(d+ex^3\right)^p\right)dx+(3dp)\int\frac{x\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}d$$

$$=-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)+(3dp)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}-\right.$$

$$=\frac{9p^2x^2}{4}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{\left(d^{2/3}p\right)\int\frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{e}}+$$

$$=\frac{9p^2x^2}{4}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)-\frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{e^{2/3}}+\frac{\sqrt[3]{-1}d^{2/3}p\,\mathrm{l}}{}$$

$$=\frac{9p^2x^2}{4}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)-\frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(\right.}{e^{2/3}}$$

$$=\frac{9p^2x^2}{4}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}-\frac{3d^{2/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4e^{2/3}}-\frac{3}{2}px^2\log\left(c\left(a\right.\right.$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right.}{e^{2}}$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{2e^{2/3}}$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{2e^{2/3}}$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{2e^{2/3}}$$

Mathematica [C] time = 1.30223, size = 823, normalized size = 0.64

$$\frac{1}{4}\left(2x^2\log^2\left(c\left(ex^3+d\right)^p\right)+\frac{p\left(-9e^{2/3}p\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{ex^3}{d}\right)-1\right)x^2-6e^{2/3}\log\left(c\left(ex^3+d\right)^p\right)x^2-4d^{2/3}\log\left(-\sqrt[3]{e}x-\sqrt[3]{a}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (2*x^2*Log[c*(d + e*x^3)^p]^2 + (p*(-9*e^(2/3)*p*x^2*(-1 + Hypergeometric2F
1[2/3, 1, 5/3, -((e*x^3)/d)]) - 6*e^(2/3)*x^2*Log[c*(d + e*x^3)^p] - 4*d^(2
/3)*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 4*(-1)^(1/3)*d^(2/3)*L
og[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 4*(-1)^(2/3)*d^(
2/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/
3)*d^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-1)^(1/3)*(d^(1
/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) + (-1)^(1/3)*e
^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((-1 + (-1)
^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-
1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(1/3)*x)/((-1 +
 (-1)^(2/3))*d^(1/3))]) + 2*(-1)^(2/3)*d^(2/3)*p*(Log[-d^(1/3) - (-1)^(2/3)
*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3))*d^
(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1
/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2, (d^(1
/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) + 2*d^(2/3)*p*(Lo
g[-d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(
1/3) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e
^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/
((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3))
)/(3*I + Sqrt[3])])))/e^(2/3))/4
```

Maple [F] time = 0.672, size = 0, normalized size = 0.

$$\int x \left(\ln \left(c \left(e x^3+d\right)^p\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x*ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x \log\left(\left(e x^3+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x*log((e*x^3 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \log\left(\left(ex^3 + d\right)^p c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x*log((e*x^3 + d)^p*c)^2, x)
```

3.134 $\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal. Leaf size=1304

result too large to display

```
[Out] 18*p^2*x + (6*Sqrt[3]*d^(1/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d
^(1/3))])/e^(1/3) - (d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/e^(1/3) - (6*
d^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3)
 - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e
^(1/3) - ((-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2)/e^
(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3)
+ (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x
))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3
) + ((-1)^(1/3)*d^(1/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/e^(1/3)
 - (2*d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)
^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*(-1)^(1/3)*d^(
1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)
))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(1/
3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[-((
(-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/
e^(1/3) + (3*d^(1/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/e^
(1/3) - 6*p*x*Log[c*(d + e*x^3)^p] + (2*d^(1/3)*p*Log[-d^(1/3) - e^(1/3)*x]
*Log[c*(d + e*x^3)^p])/e^(1/3) + (2*(-1)^(2/3)*d^(1/3)*p*Log[-d^(1/3) + (-1
)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) - (2*(-1)^(1/3)*d^(1/3)*p*
Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) + x*Log[
c*(d + e*x^3)^p]^2 - (2*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*(-1)^(1/3)*d^(1/3)*p^2*PolyLog[2, -(((-
1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/e^(1/3) - (2*
d^(1/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))]
)/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*
d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/e^(1/3) - (2*(-1)^(2/3)
*d^(1/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*
d^(1/3))])/e^(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(
2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3)
```

Rubi [A] time = 1.79105, antiderivative size = 1310, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {2450, 2476, 2448, 321, 200, 31, 634, 617, 204, 628, 2471, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2,x]
```


```
[Out] 18*p^2*x + (6*Sqrt[3]*d^(1/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d
^(1/3))])/e^(1/3) - (d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/e^(1/3) - (6*
d^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3)
 - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e
```

```
^(1/3) - ((-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2)/e^
(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3)
+ (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x
))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3
) + ((-1)^(1/3)*d^(1/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/e^(1/3)
 - (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*
x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]*L
og[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))
])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]
*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) + (3*d^(1/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*
x^2])/e^(1/3) - 6*p*x*Log[c*(d + e*x^3)^p] + (2*d^(1/3)*p*Log[-d^(1/3) - e^
(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) + (2*(-1)^(2/3)*d^(1/3)*p*Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) - (2*(-1)^(1/3)*d^
(1/3)*p*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3)
+ x*Log[c*(d + e*x^3)^p]^2 - (2*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*PolyLog[2, (2*(d^(1
/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/
3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*PolyLog[2, (d^(1/3
) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*(-1)^(1
/3)*d^(1/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((
1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*PolyLog[2, (
d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(1/3)
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
```

```
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)dx = x\log^2\left(c\left(d+ex^3\right)^p\right)-(6ep)\int \frac{x^3\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}dx$$
$$= x\log^2\left(c\left(d+ex^3\right)^p\right)-(6ep)\int\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\right)dx$$
$$= x\log^2\left(c\left(d+ex^3\right)^p\right)-(6p)\int\log\left(c\left(d+ex^3\right)^p\right)dx+(6dp)\int\frac{\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}dx$$
$$= -6px\log\left(c\left(d+ex^3\right)^p\right)+x\log^2\left(c\left(d+ex^3\right)^p\right)+(6dp)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}-\frac{\log\left(c\left(\right.\right.}{3d^{2/3}\left(-\sqrt[3]{d}\right.}\right.$$
$$= 18p^2x-6px\log\left(c\left(d+ex^3\right)^p\right)+x\log^2\left(c\left(d+ex^3\right)^p\right)-\left(2\sqrt[3]{d}p\right)\int\frac{\log\left(c\left(d+ex^3\right)^p\right)}{-\sqrt[3]{d}-\sqrt[3]{e}x}dx-\left(2\right.$$
$$= 18p^2x-6px\log\left(c\left(d+ex^3\right)^p\right)+\frac{2\sqrt[3]{d}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{e}}+\frac{2(-1)^{2/3}\sqrt[3]{d}p\log}{}$$
$$= 18p^2x-\frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-6px\log\left(c\left(d+ex^3\right)^p\right)+\frac{2\sqrt[3]{d}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d\right.\right.}{\sqrt[3]{e}}$$
$$= 18p^2x-\frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}}+\frac{3\sqrt[3]{d}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{\sqrt[3]{e}}-6px\log\left(c\left(d+ex^3\right)^p\right.$$
$$= 18p^2x+\frac{6\sqrt{3}\sqrt[3]{d}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}}-\frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-\frac{2\sqrt[3]{d}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log}{\sqrt[3]{e}}$$
$$= 18p^2x+\frac{6\sqrt{3}\sqrt[3]{d}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}}-\frac{\sqrt[3]{d}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-\frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-$$
$$= 18p^2x+\frac{6\sqrt{3}\sqrt[3]{d}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}}-\frac{\sqrt[3]{d}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-\frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-$$
$$= 18p^2x+\frac{6\sqrt{3}\sqrt[3]{d}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}}-\frac{\sqrt[3]{d}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-\frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}}-$$

Mathematica [A] time = 0.653003, size = 1090, normalized size = 0.84

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] x*Log[c*(d + e*x^3)^p]^2 - 6*e*p*(-(p*((6*x)/e - ((2*d^(1/3)*Log[d^(1/3) +
e^(1/3)*x])/e^(1/3) - d^(1/3)*((2*Sqrt[3]*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(S
qrt[3]*d^(1/3))])/e^(1/3) + Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]/
e^(1/3)))/e))/2 + (x*Log[c*(d + e*x^3)^p])/e - (d^(1/3)*Log[-d^(1/3) - e^(1
```

```
/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/3)) - ((-1)^(2/3)*d^(1/3)*Log[-d^(1/3)
 + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/3)) + ((-1)^(1/3)*d^
(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/3)
) + (d^(1/3)*p*(Log[-d^(1/3) - e^(1/3)*x]^2/e^(1/3) + (2*Log[-d^(1/3) - e^(
1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))]
)/e^(1/3) + (2*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2
/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*PolyLog[2, (d^(1/
3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*PolyLog[2, (d^(1/
3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(1/3)))/(6*e) + ((-1)^(2/3)*
d^(1/3)*p*((2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1
/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e^(1/3) + Log[-d^(1/3) + (-1)^(
1/3)*e^(1/3)*x]^2/e^(1/3) + (2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[-((
(-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/
e^(1/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*
d^(1/3))])/e^(1/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 - (
-1)^(2/3))*d^(1/3))])/e^(1/3)))/(6*e) - ((-1)^(1/3)*d^(1/3)*p*((2*Log[-(((-
1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) -
 (-1)^(2/3)*e^(1/3)*x])/e^(1/3) + (2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*
e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x
])/e^(1/3) + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2/e^(1/3) + (2*PolyLog[2,
 (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2
*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e
^(1/3)))/(6*e))
```

Maple [F] time = 1.059, size = 0, normalized size = 0.

$$\int \left(\ln \left(c \left(e x^{3}+d\right)^{p}\right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\log\left(\left(e x^{3}+d\right)^{p} c\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\left(ex^3 + d\right)^p c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2, x)
```

3.135 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2} dx$

Optimal. Leaf size=1137

result too large to display

```
[Out] (e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/
3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^
(1/3)))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/
d^(1/3) - ((-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/d^
(1/3) + (2*(-1)^(2/3)*e^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3) +
 (2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)
)/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3)
+ ((-1)^(2/3)*e^(1/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/d^(1/3) +
(2*e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/
3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)
*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*
Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/d^(1/3) -
 (2*(-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[-(((-1)^
(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(1/
3) - (2*e^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(1/3) +
(2*(-1)^(1/3)*e^(1/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^
3)^p])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x
]*Log[c*(d + e*x^3)^p])/d^(1/3) - Log[c*(d + e*x^3)^p]^2/x + (2*e^(1/3)*p^2
*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(1/3) - (2
*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, -(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1
 - (-1)^(2/3))*d^(1/3)))])/d^(1/3) + (2*e^(1/3)*p^2*PolyLog[2, (2*(d^(1/3)
+ e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p
^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2
/3))*d^(1/3)))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) -
(-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(1/3) + (2*(-1)^(2/3)*
e^(1/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d
^(1/3))])/d^(1/3)
```

Rubi [A] time = 1.32774, antiderivative size = 1143, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2457, 2476, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^2,x]
```


```
[Out] (e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/
3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^
(1/3)))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/
d^(1/3) - ((-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/d^
(1/3) + (2*(-1)^(2/3)*e^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3) +
 (2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)
)/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3)
```

```
+ ((-1)^(2/3)*e^(1/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/d^(1/3) -
(2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))
/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]*Log[(
(-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d
^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[
-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))
])/d^(1/3) - (2*e^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^
(1/3) + (2*(-1)^(1/3)*e^(1/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(
d + e*x^3)^p])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p*Log[d^(1/3) + (-1)^(2/3)*e
^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(1/3) - Log[c*(d + e*x^3)^p]^2/x + (2*e^(
1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(1
/3) + (2*e^(1/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*
d^(1/3))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(1/3) - (2*(
-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, ((-1)^(1
/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(1/3)
+ (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
 - (-1)^(2/3))*d^(1/3))])/d^(1/3)
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
```

```
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2}\,dx = -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} + (6ep)\int \frac{x\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} + (6ep)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} - \frac{(-1)^{2/3}\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)} + \frac{\sqrt[3]{-1}}{3\sqrt[3]{d}\sqrt[3]{e}}\right.$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} - \frac{\left(2e^{2/3}p\right)\int\frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} + \frac{\left(2\sqrt[3]{-1}e^{2/3}p\right)\int\frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}dx}{\sqrt[3]{d}} - \frac{\left(2(-1)\right.}{}$$

$$= -\frac{2\sqrt[3]{e}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\sqrt[3]{e}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\sqrt[3]{e}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\sqrt[3]{e}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}}$$

$$= \frac{2\sqrt[3]{e}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\sqrt[3]{e}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\right.}{\sqrt[3]{d}}$$

$$= \frac{\sqrt[3]{e}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\sqrt[3]{e}p^2\log\left(\frac{\sqrt[3]{-1}}{(1+}\right.}{}$$

$$= \frac{\sqrt[3]{e}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\sqrt[3]{e}p^2\log\left(\frac{\sqrt[3]{-1}}{(1+}\right.}{}$$

$$= \frac{\sqrt[3]{e}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\sqrt[3]{e}p^2\log\left(\frac{\sqrt[3]{-1}}{(1+}\right.}{}$$

Mathematica [A] time = 0.838462, size = 742, normalized size = 0.65

$$-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} - \frac{\sqrt[3]{e}p\left(\sqrt[3]{-1}p\left(2\text{PolyLog}\left(2,\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}}{\left((-1)^{2/3}-1\right)\sqrt[3]{d}}\right)+\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)\left(2\log\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

```
[Out] -(Log[c*(d + e*x^3)^p]^2/x) - (e^(1/3)*p*(2*Log[-d^(1/3) - e^(1/3)*x]*Log[c
*(d + e*x^3)^p] - 2*(-1)^(1/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(
d + e*x^3)^p] + 2*(-1)^(2/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d
+ e*x^3)^p] + (-1)^(1/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-
1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3)
+ (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
```

```
))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/
3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(
1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*p*(Log[-d^(1/3) - (-1)^(
2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3)
)*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1
)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2, (
d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2,
(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) - p*(Log[-d^(
1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3) -
 e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e^(1/3)
*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I
 + Sqrt[3])])))/d^(1/3)
```

Maple [F] time = 1.021, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^{3}+d\right)^{p}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^2,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(ex^{3}+d\right)^{p}c\right)^{2}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^2, x)
```

3.136 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=1170

result too large to display

```
[Out] -(e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/(2*d^(2/3)) - (e^(2/3)*p^2*Log[-
d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3
))*d^(1/3)))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*
x])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^
2)/(2*d^(2/3)) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/
3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/(2
*d^(2/3)) - (e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3)
 + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3) - ((-1)^(1/3
)*e^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^
(1/3)))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/
d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[
-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))
])/d^(2/3) + (e^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(
2/3) + ((-1)^(2/3)*e^(2/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d
+ e*x^3)^p])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p*Log[-d^(1/3) - (-1)^(2/3)*e^(1
/3)*x]*Log[c*(d + e*x^3)^p])/d^(2/3) - Log[c*(d + e*x^3)^p]^2/(2*x^2) - (e^
(2/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(
2/3) - ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, -(((-1)^(2/3)*(d^(1/3) + e^(1/3)*
x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(2/3) - (e^(2/3)*p^2*PolyLog[2, (2*(d^(
1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3
)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)
^(2/3))*d^(1/3)))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) -
 (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3) + ((-1)^(1/3)*e
^(2/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^
(1/3))])/d^(2/3)
```

Rubi [A] time = 1.34448, antiderivative size = 1176, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2457, 2471, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^3,x]
```


```
[Out] -(e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/(2*d^(2/3)) - (e^(2/3)*p^2*Log[-
d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3
))*d^(1/3)))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*
x])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^
2)/(2*d^(2/3)) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/
3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
```

```
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/(2
*d^(2/3)) - ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)
/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3) - (e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)
*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d
^(1/3)))])/d^(2/3) + (e^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)
^p])/d^(2/3) + ((-1)^(2/3)*e^(2/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*L
og[c*(d + e*x^3)^p])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p*Log[-d^(1/3) - (-1)^(2
/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(2/3) - Log[c*(d + e*x^3)^p]^2/(2*x^
2) - (e^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3
))])/d^(2/3) - (e^(2/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sq
rt[3])*d^(1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, -(((-1)^(1/3
)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(2/3) -
 ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, ((-1)^(
1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3)
 + ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
- (-1)^(2/3))*d^(1/3))])/d^(2/3)
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}\,dx = -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2} + (3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2} + (3ep)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)} - \frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)} - \frac{\log\left(c\right.}{3d^{2/3}\left(-\sqrt[3]{}\right.}\right.$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2} - \frac{(ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{-\sqrt[3]{d}-\sqrt[3]{e}x}dx}{d^{2/3}} - \frac{(ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}dx}{d^{2/3}} - \frac{(ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x}}{d^{2/3}}$$

$$= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right.\right.}{d^{2/3}}$$

$$= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right.\right.}{d^{2/3}}$$

$$= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right.\right.}{d^{2/3}}$$

$$= -\frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-1)^{2/3}e^{2/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}+\right.}{d^{2/3}}$$

$$= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-1)^{2/3}e^{2/3}p^2\log}{}$$

$$= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-1)^{2/3}e^{2/3}p^2\log}{}$$

$$= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-1)^{2/3}e^{2/3}p^2\log}{}$$

Mathematica [A] time = 0.837746, size = 745, normalized size = 0.64

$$\frac{1}{2}\left(-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2} + \frac{e^{2/3}p\left(-(-1)^{2/3}p\left(2\text{PolyLog}\left(2,\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}}{\left((-1)^{2/3}-1\right)\sqrt[3]{d}}\right)+\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{}\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

```
[Out] (-(Log[c*(d + e*x^3)^p]^2/x^2) + (e^(2/3)*p*(2*Log[-d^(1/3) - e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[
c*(d + e*x^3)^p] - 2*(-1)^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*
(d + e*x^3)^p] - (-1)^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[
((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3
```

```
)*x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*
e^(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) + (-1)^(1/3)*p*(Log[-d^(1/3) - (-1
)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2
/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 +
(-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2
, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[
2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) - p*(Log[-
d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3
) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e^(1
/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1
 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(
3*I + Sqrt[3])])))/d^(2/3))/2
```

Maple [F] time = 1.02, size = 0, normalized size = 0.

$$\int \frac{\left(\ln \left(c \left(e x^{3}+d\right)^{p}\right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^3,x)

[Out] int(ln(c*(e*x^3+d)^p)^2/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(e x^{3}+d\right)^{p} c\right)^{2}}{x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="fricas")

[Out] integral(log((e*x^3 + d)^p*c)^2/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^3, x)
```

3.137 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=1328

result too large to display

```
[Out] (-3*Sqrt[3]*e^(4/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/
(2*d^(4/3)) - (3*e^(4/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*d^(4/3)) - (e^(4/
3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(4*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) +
 e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3
)))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/(2
*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*
x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2*d^
(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2
*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) + e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3
) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) + ((-1)
^(2/3)*e^(4/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3
))*d^(1/3)))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3
))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)
*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d
^(1/3)))])/(2*d^(4/3)) + (3*e^(4/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e
^(2/3)*x^2])/(4*d^(4/3)) - (3*e*p*Log[c*(d + e*x^3)^p])/(2*d*x) + (e^(4/3)*
p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(2*d^(4/3)) - ((-1)^(1/3)*
e^(4/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(2*d^(4
/3)) + ((-1)^(2/3)*e^(4/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Log[c*(d +
 e*x^3)^p])/(2*d^(4/3)) - Log[c*(d + e*x^3)^p]^2/(4*x^4) - (e^(4/3)*p^2*Pol
yLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) + ((
-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, -(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 -
 (-1)^(2/3))*d^(1/3)))])/(2*d^(4/3)) - (e^(4/3)*p^2*PolyLog[2, (2*(d^(1/3)
+ e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)
*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^
(2/3))*d^(1/3)))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3
) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) - ((-1)^
(2/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1
/3))*d^(1/3))])/(2*d^(4/3))
```

Rubi [A] time = 1.72346, antiderivative size = 1334, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 18, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2457, 2476, 2455, 292, 31, 634, 617, 204, 628, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^5,x]
```


```
[Out] (-3*Sqrt[3]*e^(4/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/
(2*d^(4/3)) - (3*e^(4/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*d^(4/3)) - (e^(4/
3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(4*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) +
 e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3
```

```
)))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/(2
*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*
x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2*d^
(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2
*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)
*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) + e^
(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3)
)*d^(1/3))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)
*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^
(2/3))*d^(1/3)))])/(2*d^(4/3)) + (3*e^(4/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2])/(4*d^(4/3)) - (3*e*p*Log[c*(d + e*x^3)^p])/(2*d*x) + (
e^(4/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(2*d^(4/3)) - ((-1
)^(1/3)*e^(4/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])
/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p])/(2*d^(4/3)) - Log[c*(d + e*x^3)^p]^2/(4*x^4) - (e^(4/3)
*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/
3)) - (e^(4/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^
(1/3))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(2*d^(4/3)) +
((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (
-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, ((-1
)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(2*d
^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 - (-1)^(2/3))*d^(1/3))])/(2*d^(4/3))
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx = -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{1}{2}(3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{x^2\left(d+ex^3\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{1}{2}(3ep)\int \left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{dx^2} - \frac{ex\log\left(c\left(d+ex^3\right)^p\right)}{d\left(d+ex^3\right)}\right)dx$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{(3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{x^2}\,dx}{2d} - \frac{\left(3e^2p\right)\int \frac{x\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx}{2d}$$

$$= -\frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx} - \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} - \frac{\left(3e^2p\right)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} - \frac{(-1)^{2/3}\log\left(c\left(d+\right.\right.}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}-\sqrt[3]{\cdot}\right.}\right.}{2d}$$

$$= -\frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx} - \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{\left(e^{5/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} - \frac{\left(\sqrt[3]{-1}e^{5/3}p\right)\int}{2\ldots}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx} + \frac{e^{4/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)\right.}{2d^{4/3}}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} + \frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4d^{4/3}} - \frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx} -$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} + \frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/}\right.}{4d^{4/3}}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-}{(1}\right.}{2d^{4/3}}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{4d^{4/3}} - \frac{e^{4/3}p}{}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{4d^{4/3}} - \frac{e^{4/3}p}{}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{4d^{4/3}} - \frac{e^{4/3}p}{}$$

Mathematica [C] time = 1.60044, size = 847, normalized size = 0.64

$$\frac{epx^3\left(9ep\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{ex^3}{d}\right)x^3+2d^{2/3}\sqrt[3]{e}\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(c\left(ex^3+d\right)^p\right)x-2\sqrt[3]{-1}d^{2/3}\sqrt[3]{e}\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(c\left(ex^3+d\right)^p\right)x+2(-1)^{2/3}d^{2/3}\sqrt[3]{e}\log\left(-(-1)^{2/3}\sqrt[3]{\cdot}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(d + e*x^3)^p]^2 + (e*p*x^3*(9*e*p*x^3*Hypergeometric2F1[2/3, 1, 5/
3, -((e*x^3)/d)] - 6*d*Log[c*(d + e*x^3)^p] + 2*d^(2/3)*e^(1/3)*x*Log[-d^(1
/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/3)*d^(2/3)*e^(1/3)*x*Log[
-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*d^(2/3
)*e^(1/3)*x*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + (-1
)^(1/3)*d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((
-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3)
 + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*
x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1
/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^
(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*d^(2/3)*e^(1/3)*p*x*(Lo
g[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x)
)/((-1 + (-1)^(2/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x
]) + 2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3
))] + 2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/
3))]) - d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1
/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))
] + Log[(I + Sqrt[3] - ((2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*Po
lyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (
(2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I + Sqrt[3])])))/d^2)/(4*x^4)
```

Maple [F] time = 1.063, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(e x^{3}+d\right)^{p}\right)\right)^{2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^5,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^5,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(e x^{3}+d\right)^{p} c\right)^{2}}{x^{5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^5, x)
```

3.138 $\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=164

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)^2\left(c\left(d+\right.\right.}{}$$

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p*(c*(d + e*
x^3)^p)^p^(-1)) - (2*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])
/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(3*Lo
g[c*(d + e*x^3)^p])/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Rubi [A] time = 0.237425, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2454, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)^2\left(c\left(d+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Log[c*(d + e*x^3)^p],x]
```

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p*(c*(d + e*
x^3)^p)^p^(-1)) - (2*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])
/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(3*Lo
g[c*(d + e*x^3)^p])/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\log\left(c(d+ex)^p\right)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int\left(\frac{d^2}{e^2\log\left(c(d+ex)^p\right)}-\frac{2d(d+ex)}{e^2\log\left(c(d+ex)^p\right)}+\frac{(d+ex)^2}{e^2\log\left(c(d+ex)^p\right)}\right)dx,x,x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{3e^2}-\frac{(2d)\,\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{3e^2}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{\log(c(d}\right.}{3e^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\log(cx^p)}\,dx,x,d+ex^3\right)}{3e^3}-\frac{(2d)\,\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx,x,d+ex^3\right)}{3e^3}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{\log(}\right.}{3}$$

$$= \frac{\left(\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{p}}}{x}\,dx,x,\log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^3p}-\frac{\left(2d\left(d+ex^3\right)^2\left(c\left(\right.\right.\right.}{}$$

$$= \frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(\right.}{}\right.}{3e^3p}$$

Mathematica [A] time = 0.231072, size = 146, normalized size = 0.89

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-3/p}\left(d^2\left(c\left(d+ex^3\right)^p\right)^{2/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)-\left(d+ex^3\right)\left(2d\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)-\right.\right.}{3e^3p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*(d^2*(c*(d + e*x^3)^p)^(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p
]/p] - (d + e*x^3)*(2*d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d
+ e*x^3)^p])/p] - (d + e*x^3)*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p])))/
```

```
(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Maple [F] time = 0.621, size = 0, normalized size = 0.

$$\int \frac{x^8}{\ln\left(c\left(ex^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(x^8/ln(c*(e*x^3+d)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^8/log((e*x^3 + d)^p*c), x)
```

Fricas [A] time = 1.90462, size = 271, normalized size = 1.65

$$\frac{c^{\frac{2}{p}} d^2 \operatorname{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right) - 2\,c^{\left(\frac{1}{p}\right)} d \operatorname{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right) + \operatorname{log\_integral}\left(\left(e^3x^9+3\,de^2x^6+3\,c\right.\right.}{3\,c^{\frac{3}{p}} e^3 p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] 1/3*(c^(2/p)*d^2*log_integral((e*x^3 + d)*c^(1/p)) - 2*c^(1/p)*d*log_integr
al((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)) + log_integral((e^3*x^9 + 3*d*e^2*x
^6 + 3*d^2*e*x^3 + d^3)*c^(3/p)))/(c^(3/p)*e^3*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32653, size = 146, normalized size = 0.89

$$\frac{d^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\left(\frac{1}{p}\right)}p}-\frac{2\,d\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\frac{2}{p}}p}+\frac{\mathrm{Ei}\left(\frac{3\,\log(c)}{p}+3\,\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\frac{3}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] 1/3*d^2*Ei(log(c)/p + log(x^3*e + d))*e^(-3)/(c^(1/p)*p) - 2/3*d*Ei(2*log(c
)/p + 2*log(x^3*e + d))*e^(-3)/(c^(2/p)*p) + 1/3*Ei(3*log(c)/p + 3*log(x^3*
e + d))*e^(-3)/(c^(3/p)*p)
```

3.139 $\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=107

$$\frac{\left(d+ex^3\right)^2 \left(c\left(d+ex^3\right)^p\right)^{-2/p} \text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p} - \frac{d\left(d+ex^3\right) \left(c\left(d+ex^3\right)^p\right)^{-1/p} \text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}$$

```
[Out] -(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^2*p*(c*(d + e*x
^3)^p)^p^(-1)) + ((d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p])/
(3*e^2*p*(c*(d + e*x^3)^p)^(2/p))
```

Rubi [A] time = 0.147369, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2454, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{\left(d+ex^3\right)^2 \left(c\left(d+ex^3\right)^p\right)^{-2/p} \text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p} - \frac{d\left(d+ex^3\right) \left(c\left(d+ex^3\right)^p\right)^{-1/p} \text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^2*p*(c*(d + e*x
^3)^p)^p^(-1)) + ((d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p])/
(3*e^2*p*(c*(d + e*x^3)^p)^(2/p))
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{d}{e\log\left(c(d+ex)^p\right)}+\frac{d+ex}{e\log\left(c(d+ex)^p\right)}\right)dx, x, x^3\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e}-\frac{d\operatorname{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2}-\frac{d\operatorname{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2}\\
&= \frac{\left(\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^2p}-\frac{\left(d\left(d+ex^3\right)\left(c\left(d\right.\right.\right.}{}\\
&= -\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p}+\frac{\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(d\right.\right.}{p}\right.}{3e^2p}
\end{aligned}$$

Mathematica [A] time = 0.115391, size = 96, normalized size = 0.9

$$-\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-2/p}\left(d\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)-\left(d+ex^3\right)\operatorname{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)\right)}{3e^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -((d + e*x^3)*(d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[Log[c*(d + e*x^3)^p
]/p] - (d + e*x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]))/(3*e^2*p*(c*
(d + e*x^3)^p)^(2/p))
```

Maple [F] time = 0.553, size = 0, normalized size = 0.

$$\int \frac{x^5}{\ln \left(c \left(e x^3+d\right)^p\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(x^5/ln(c*(e*x^3+d)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\log \left(\left(e x^3+d\right)^p c\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^5/log((e*x^3 + d)^p*c), x)
```

Fricas [A] time = 1.8404, size = 162, normalized size = 1.51

$$-\frac{c^{\left(\frac{1}{p}\right)} d \operatorname{log\_integral}\left(\left(e x^3+d\right) c^{\left(\frac{1}{p}\right)}\right)-\operatorname{log\_integral}\left(\left(e^2 x^6+2 d e x^3+d^2\right) c^{\frac{2}{p}}\right)}{3 c^{\frac{2}{p}} e^2 p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3*(c^(1/p)*d*log_integral((e*x^3 + d)*c^(1/p)) - log_integral((e^2*x^6 +
 2*d*e*x^3 + d^2)*c^(2/p)))/(c^(2/p)*e^2*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27341, size = 97, normalized size = 0.91

$$-\frac{1}{3}\left(\frac{d\text{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-1)}}{c^{\left(\frac{1}{p}\right)}p}-\frac{\text{Ei}\left(\frac{2\log(c)}{p}+2\log\left(x^3e+d\right)\right)e^{(-1)}}{c^{\frac{2}{p}}p}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] -1/3*(d*Ei(log(c)/p + log(x^3*e + d))*e^(-1)/(c^(1/p)*p) - Ei(2*log(c)/p +
2*log(x^3*e + d))*e^(-1)/(c^(2/p)*p))*e^(-1)
```

3.140 $\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=51

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Rubi [A] time = 0.0623186, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2389, 2300, 2178}

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e}$$
$$= \frac{\left(\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3ep}$$
$$= \frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep}$$

Mathematica [A] time = 0.0405966, size = 51, normalized size = 1.

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Maple [C] time = 1.246, size = 317, normalized size = 6.2

$$-\frac{1}{3pe}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}\left(i\left(ex^3+d\right)^p\right)\left(\mathrm{csgn}\left(ic\left(ex^3+d\right)^p\right)\right)^2-i\pi\,\mathrm{csgn}\left(i\left(ex^3+d\right)^p\right)\mathrm{csgn}\left(ic\left(ex^3+d\right)^p\right)\mathrm{csgn}(ic)-i\pi\left(\mathrm{csgn}\left(ic\left(ex^3+d\right)^p\right)\right)^3+i\pi\left(\mathrm{csgn}\left(ic\left(ex^3+d\right)^p\right)\right)^2\mathrm{csgn}(ic)+2\ln(c)+2\ln\left(\left(ex^3+d\right.\right.}{2p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(e*x^3+d)^p),x)
```

```
[Out] -1/3/e/p*exp(-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*cs
gn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p
)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln
(e*x^3+d))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x
^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*cs
gn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((
e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^2/log((e*x^3 + d)^p*c), x)
```

Fricas [A] time = 1.97017, size = 72, normalized size = 1.41

$$\frac{\text{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)}{3\,c^{\left(\frac{1}{p}\right)}ep}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] 1/3*log_integral((e*x^3 + d)*c^(1/p))/(c^(1/p)*e*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2565, size = 42, normalized size = 0.82

$$\frac{\text{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-1)}}{3\,c^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] 1/3*Ei(log(c)/p + log(x^3*e + d))*e^(-1)/(c^(1/p)*p)
```

3.141 $\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x*Log[c*(d + e*x^3)^p]), x]
```

Rubi [A] time = 0.0168213, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.180138, size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]), x]
```

Maple [A] time = 0.454, size = 0, normalized size = 0.

$$\int \frac{1}{x\ln\left(c\left(ex^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/x/ln(c*(e*x^3+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\log\left(\left(ex^3+d\right)^pc\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((e*x^3 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

3.142 $\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Rubi [A] time = 0.0177668, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\, dx = \int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\, dx$$

Mathematica [A] time = 0.316377, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Maple [A] time = 0.514, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \ln\left(c\left(ex^3+d\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \log \left(\left(ex^3 + d \right)^p c \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^4 \log \left(\left(ex^3 + d \right)^p c \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \log \left(\left(ex^3 + d \right)^p c \right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

3.143 $\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[x^3/Log[c*(d + e*x^3)^p], x]
```

Rubi [A] time = 0.0181132, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/Log[c*(d + e*x^3)^p],x]
```

```
[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p], x]
```

Rubi steps

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.259907, size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/Log[c*(d + e*x^3)^p],x]
```

```
[Out] Integrate[x^3/Log[c*(d + e*x^3)^p], x]
```

Maple [A] time = 0.501, size = 0, normalized size = 0.

$$\int \frac{x^3}{\ln\left(c\left(ex^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(x^3/ln(c*(e*x^3+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^3/log((e*x^3 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(x^3/log((e*x^3 + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(x^3/log((e*x^3 + d)^p*c), x)
```

3.144 $\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{x}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[x/Log[c*(d + e*x^3)^p], x]
```

Rubi [A] time = 0.0097062, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x/Log[c*(d + e*x^3)^p],x]
```

```
[Out] Defer[Int][x/Log[c*(d + e*x^3)^p], x]
```

Rubi steps

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 0.225743, size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Log[c*(d + e*x^3)^p],x]
```

```
[Out] Integrate[x/Log[c*(d + e*x^3)^p], x]
```

Maple [A] time = 0.455, size = 0, normalized size = 0.

$$\int \frac{x}{\ln\left(c\left(ex^3+d\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(x/ln(c*(e*x^3+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x/log((e*x^3 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x}{\log\left(\left(ex^3+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(x/log((e*x^3 + d)^p*c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x/log(c*(d + e*x**3)**p), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(x/log((e*x^3 + d)^p*c), x)
```

3.145 $\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^3)^p]^(-1), x]
```

Rubi [A] time = 0.0035738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^3)^p]^(-1),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.0093644, size = 0, normalized size = 0.

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^(-1),x]
```

```
[Out] Integrate[Log[c*(d + e*x^3)^p]^(-1), x]
```

Maple [A] time = 0.445, size = 0, normalized size = 0.

$$\int \left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/ln(c*(e*x^3+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/log((e*x^3 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\log\left(\left(ex^3+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/log((e*x^3 + d)^p*c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(1/log(c*(d + e*x**3)**p), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/log((e*x^3 + d)^p*c), x)
```

3.146 $\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

```
[Out] Unintegrable[1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Rubi [A] time = 0.0172972, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 0.366619, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Maple [A] time = 0.484, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \ln\left(c\left(ex^3+d\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

3.147 $\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Rubi [A] time = 0.0184043, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 0.382654, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Maple [A] time = 0.496, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \ln\left(c\left(ex^3+d\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

3.148 $\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=195

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p^2}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{e^3p^2}-\frac{4d\left(d+ex^3\right)^2\left(c\left(\right.\right.}{}$$

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p^2*(c*(d +
e*x^3)^p)^p^(-1)) - (4*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p
])/p])/(3*e^3*p^2*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(
3*Log[c*(d + e*x^3)^p])/p])/(e^3*p^2*(c*(d + e*x^3)^p)^(3/p)) - (x^6*(d + e
*x^3))/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rubi [A] time = 0.380937, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p^2}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{e^3p^2}-\frac{4d\left(d+ex^3\right)^2\left(c\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Log[c*(d + e*x^3)^p]^2,x]
```


```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p^2*(c*(d +
e*x^3)^p)^p^(-1)) - (4*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p
])/p])/(3*e^3*p^2*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(
3*Log[c*(d + e*x^3)^p])/p])/(e^3*p^2*(c*(d + e*x^3)^p)^(3/p)) - (x^6*(d + e
*x^3))/(3*e*p*Log[c*(d + e*x^3)^p])
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\operatorname{Subst}\left(\int \frac{x^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{p} + \frac{(2d)\operatorname{Subst}\left(\int \frac{x}{\log(c(d+ex)^p)}\,dx,\right.}{3ep}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\operatorname{Subst}\left(\int \left(\frac{d^2}{e^2\log(c(d+ex)^p)} - \frac{2d(d+ex)}{e^2\log(c(d+ex)^p)} + \frac{(d+ex)^2}{e^2\log(c(d+ex)^p)}\right) dx, x, x\right.}{p}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\operatorname{Subst}\left(\int \frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{e^2p} + \frac{(2d)\operatorname{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx,\right.}{3e^2p}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\operatorname{Subst}\left(\int \frac{x^2}{\log(cx^p)}\,dx, x, d+ex^3\right)}{e^3p} + \frac{(2d)\operatorname{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d\right.}{3e^3p}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\left(\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{3x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex\right.\right.\right.}{e^3p^2}$$

$$= \frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p^2} - \frac{4d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\right.}{}\right.}{3e^3p^2}$$

Mathematica [A] time = 0.259922, size = 290, normalized size = 1.49

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-3/p}\left(d^2\left(c\left(d+ex^3\right)^p\right)^{2/p}\log\left(c\left(d+ex^3\right)^p\right)\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right) + 3d^2\log\left(c\left(d+ex^3\right)^p\right)\operatorname{Ei}\left(\frac{3\log(c\right.}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] ((d + e*x^3)*(-(e^2*p*x^6*(c*(d + e*x^3)^p)^(3/p)) + d^2*(c*(d + e*x^3)^p)^
(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 4*d*(d +
 e*x^3)*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*
Log[c*(d + e*x^3)^p] + 3*d^2*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p] + 6*d*e*x^3*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p] + 3*e^2*x^6*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p]))/(3*e^3*p^2*(c*(d + e*x^3)^p)^(3/p)*Log[c*(d + e*x^3)^p])
```

Maple [F] time = 3.678, size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x^8/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^9 + dx^6}{3\left(ep\log\left(\left(ex^3+d\right)^p\right) + ep\log\left(c\right)\right)} + \int \frac{3\,ex^8 + 2\,dx^5}{ep\log\left(\left(ex^3+d\right)^p\right) + ep\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^9 + d*x^6)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((3*e
*x^8 + 2*d*x^5)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Fricas [A] time = 1.85092, size = 494, normalized size = 2.53

$$-\frac{4\left(dp\log\left(ex^3+d\right) + d\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(e^2x^6 + 2\,dex^3 + d^2\right)c^{\frac{2}{p}}\right) - \left(d^2p\log\left(ex^3+d\right) + d^2\log\left(c\right)\right)c^{\frac{2}{p}}\operatorname{log\_integ}}{3\left(e^3p^3\log\left(ex^3 +\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e^2*x^6 + 2*d
*e*x^3 + d^2)*c^(2/p)) - (d^2*p*log(e*x^3 + d) + d^2*log(c))*c^(2/p)*log_in
tegral((e*x^3 + d)*c^(1/p)) + (e^3*p*x^9 + d*e^2*p*x^6)*c^(3/p) - 3*(p*log(
e*x^3 + d) + log(c))*log_integral((e^3*x^9 + 3*d*e^2*x^6 + 3*d^2*e*x^3 + d^
3)*c^(3/p)))/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(3/p))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29354, size = 667, normalized size = 3.42

$$-\frac{\left(x^3e+d\right)^3p}{3\left(p^3e^3\log\left(x^3e+d\right) + p^2e^3\log\left(c\right)\right)} + \frac{2\left(x^3e+d\right)^2dp}{3\left(p^3e^3\log\left(x^3e+d\right) + p^2e^3\log\left(c\right)\right)} - \frac{\left(x^3e+d\right)d^2p}{3\left(p^3e^3\log\left(x^3e+d\right) + p^2e^3\log\left(c\right)\right)} + \frac{d^2p}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x^3*e + d)^3*p/(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c)) + 2/3*(x^3*e
 + d)^2*d*p/(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c)) - 1/3*(x^3*e + d)*d^2
*p/(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c)) + 1/3*d^2*p*Ei(log(c)/p + log(
x^3*e + d))*log(x^3*e + d)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(1/
p)) - 4/3*d*p*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(x^3*e + d)/((p^3*e^3*lo
g(x^3*e + d) + p^2*e^3*log(c))*c^(2/p)) + 1/3*d^2*Ei(log(c)/p + log(x^3*e +
 d))*log(c)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(1/p)) + p*Ei(3*lo
g(c)/p + 3*log(x^3*e + d))*log(x^3*e + d)/((p^3*e^3*log(x^3*e + d) + p^2*e^
3*log(c))*c^(3/p)) - 4/3*d*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(c)/((p^3*e
^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(2/p)) + Ei(3*log(c)/p + 3*log(x^3*e
+ d))*log(c)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(3/p))
```

3.149 $\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=141

$$\frac{2\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[Out] -(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^2*p^2*(c*(d + e
*x^3)^p)^p^(-1)) + (2*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/
p])/(3*e^2*p^2*(c*(d + e*x^3)^p)^(2/p)) - (x^3*(d + e*x^3))/(3*e*p*Log[c*(d
 + e*x^3)^p])
```

Rubi [A] time = 0.207805, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{2\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^2*p^2*(c*(d + e
*x^3)^p)^p^(-1)) + (2*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/
p])/(3*e^2*p^2*(c*(d + e*x^3)^p)^(2/p)) - (x^3*(d + e*x^3))/(3*e*p*Log[c*(d
 + e*x^3)^p])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
```

```
& IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= -\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\operatorname{Subst}\left(\int \frac{x}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3p} + \frac{d\operatorname{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right.}{3ep}$$

$$= -\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\operatorname{Subst}\left(\int\left(-\frac{d}{e\log(c(d+ex)^p)} + \frac{d+ex}{e\log(c(d+ex)^p)}\right)dx, x, x^3\right)}{3p} + \frac{d\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\operatorname{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3ep} - \frac{(2d)\operatorname{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x,\right.}{3ep}$$

$$= \frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p^2} - \frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\operatorname{Subst}\left(\int \frac{x}{\log(cx^p)}\right.}{3e^2p}$$

$$= \frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p^2} - \frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\left(2\left(d+ex^3\right)^2\left(c\left(d\right.\right.\right.}{}$$

$$= -\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p^2} + \frac{2\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(d\right.\right.}{p}\right.}{3e^2p^2}$$

Mathematica [A] time = 0.134023, size = 157, normalized size = 1.11

$$-\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-2/p}\left(d\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}}\log\left(c\left(d+ex^3\right)^p\right)\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right) - 2\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)\operatorname{Ei}\left(\frac{2\log}{}\right.\right.}{3e^2p^2\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -((d + e*x^3)*(e*p*x^3*(c*(d + e*x^3)^p)^(2/p) + d*(c*(d + e*x^3)^p)^p^(-1)
*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 2*(d + e*x^3)
*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p]))/(3*e^2*p^
2*(c*(d + e*x^3)^p)^(2/p)*Log[c*(d + e*x^3)^p])
```

Maple [F] time = 5.191, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/ln(c*(e*x^3+d)^p)^2,x)
```

[Out] int(x^5/ln(c*(e*x^3+d)^p)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^6+dx^3}{3\left(ep\log\left(\left(ex^3+d\right)^p\right)+ep\log\left(c\right)\right)}+\int\frac{2\,ex^5+dx^2}{ep\log\left(\left(ex^3+d\right)^p\right)+ep\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")

[Out] -1/3*(e*x^6 + d*x^3)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((2*e *x^5 + d*x^2)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)

Fricas [A] time = 1.97112, size = 338, normalized size = 2.4

$$-\frac{\left(dp\log\left(ex^3+d\right)+d\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)+\left(e^2px^6+depx^3\right)c^{\frac{2}{p}}-2\left(p\log\left(ex^3+d\right)+\log\left(c\right)\right)\mathrm{lo}}{3\left(e^2p^3\log\left(ex^3+d\right)+e^2p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")

[Out] -1/3*((d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e*x^3 + d)*c^(1 /p)) + (e^2*p*x^6 + d*e*p*x^3)*c^(2/p) - 2*(p*log(e*x^3 + d) + log(c))*log_ integral((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)))/((e^2*p^3*log(e*x^3 + d) + e ^2*p^2*log(c))*c^(2/p))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/ln(c*(e*x**3+d)**p)**2,x)

[Out] Timed out

Giac [B] time = 1.31127, size = 436, normalized size = 3.09

$$-\frac{1}{3}\left(\frac{\left(x^3e+d\right)^2p}{p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)}-\frac{\left(x^3e+d\right)dp}{p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)}+\frac{dp\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log\left(x^3e+d\right)}{\left(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}-\frac{2\,p}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")

```
[Out] -1/3*((x^3*e + d)^2*p/(p^3*e*log(x^3*e + d) + p^2*e*log(c)) - (x^3*e + d)*d
*p/(p^3*e*log(x^3*e + d) + p^2*e*log(c)) + d*p*Ei(log(c)/p + log(x^3*e + d)
)*log(x^3*e + d)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c^(1/p)) - 2*p*Ei(2
*log(c)/p + 2*log(x^3*e + d))*log(x^3*e + d)/((p^3*e*log(x^3*e + d) + p^2*e
*log(c))*c^(2/p)) + d*Ei(log(c)/p + log(x^3*e + d))*log(c)/((p^3*e*log(x^3*
e + d) + p^2*e*log(c))*c^(1/p)) - 2*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(c
)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c^(2/p)))*e^(-1)
```

3.150 $\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=83

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p^2*(c*(d + e*x^3)
^p)^p^(-1)) - (d + e*x^3)/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rubi [A] time = 0.0820962, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2389, 2297, 2300, 2178}

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p^2*(c*(d + e*x^3)
^p)^p^(-1)) - (d + e*x^3)/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, d+ex^3\right)}{3e}$$

$$= -\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3ep}$$

$$= -\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\left(\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3ep^2}$$

$$= \frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep^2} - \frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Mathematica [A] time = 0.0454005, size = 97, normalized size = 1.17

$$-\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\left(p\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}} - \log\left(c\left(d+ex^3\right)^p\right)\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)\right)}{3ep^2\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -((d + e*x^3)*(p*(c*(d + e*x^3)^p)^p^(-1) - ExpIntegralEi[Log[c*(d + e*x^3)
^p]/p]*Log[c*(d + e*x^3)^p]))/(3*e*p^2*(c*(d + e*x^3)^p)^p^(-1)*Log[c*(d +
e*x^3)^p])
```

Maple [C] time = 1.288, size = 466, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] -2/3/(2*ln(c)+2*ln((e*x^3+d)^p)+I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)
^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*
c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c))/p/e*(e*x^3+d)-1/3/
```

```
p^2/e*exp(-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(
I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3
+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*
x^3+d))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+
d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(
I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x
^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^3+d}{3\left(ep\log\left(\left(ex^3+d\right)^p\right)+ep\log\left(c\right)\right)}+\int\frac{x^2}{p\log\left(\left(ex^3+d\right)^p\right)+p\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(x^2/(p*l
og((e*x^3 + d)^p) + p*log(c)), x)
```

Fricas [A] time = 1.98641, size = 194, normalized size = 2.34

$$-\frac{\left(epx^3+dp\right)c^{\left(\frac{1}{p}\right)}-\left(p\log\left(ex^3+d\right)+\log\left(c\right)\right)\operatorname{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((e*p*x^3 + d*p)*c^(1/p) - (p*log(e*x^3 + d) + log(c))*log_integral((e
*x^3 + d)*c^(1/p)))/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(1/p))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**2/log(c*(d + e*x**3)**p)**2, x)
```

Giac [A] time = 1.28646, size = 208, normalized size = 2.51

$$-\frac{\left(x^3e+d\right)p}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)\right)}+\frac{p\operatorname{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log\left(x^3e+d\right)}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}+\frac{\operatorname{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log\left(c\right)}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x^3*e + d)*p/(p^3*e*log(x^3*e + d) + p^2*e*log(c)) + 1/3*p*Ei(log(c)/
p + log(x^3*e + d))*log(x^3*e + d)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c
^(1/p)) + 1/3*Ei(log(c)/p + log(x^3*e + d))*log(c)/((p^3*e*log(x^3*e + d) +
 p^2*e*log(c))*c^(1/p))
```

3.151 $\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [A] time = 0.0162048, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 0.292616, size = 0, normalized size = 0.

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A] time = 1.929, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-d\int \frac{1}{epx^4 \log\left(\left(ex^3+d\right)^p\right) + epx^4 \log(c)}\,dx - \frac{ex^3+d}{3\left(epx^3 \log\left(\left(ex^3+d\right)^p\right) + epx^3 \log(c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -d*integrate(1/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)), x) - 1/3*(e*x
^3 + d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

3.152 $\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [A] time = 0.0168459, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]

[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 1.4345, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]

[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A] time = 3.464, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^3+d}{3\left(epx^6\log\left(\left(ex^3+d\right)^p\right)+epx^6\log\left(c\right)\right)}-\int\frac{ex^3+2\,d}{epx^7\log\left(\left(ex^3+d\right)^p\right)+epx^7\log\left(c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)) - integrate(
(e*x^3 + 2*d)/(e*p*x^7*log((e*x^3 + d)^p) + e*p*x^7*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^4\log\left(\left(ex^3+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

3.153 $\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi [A] time = 0.0173569, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/Log[c*(d + e*x^3)^p]^2,x]

[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.332326, size = 0, normalized size = 0.

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/Log[c*(d + e*x^3)^p]^2,x]

[Out] Integrate[x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Maple [A] time = 3.507, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^4 + dx}{3\left(ep\log\left(\left(ex^3 + d\right)^p\right) + ep\log\left(c\right)\right)} + \int \frac{4\,ex^3 + d}{3\left(ep\log\left(\left(ex^3 + d\right)^p\right) + ep\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^4 + d*x)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(1/3*(4
*e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\log\left(\left(ex^3 + d\right)^p c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/log((e*x^3 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log\left(\left(ex^3 + d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/log((e*x^3 + d)^p*c)^2, x)
```

3.154 $\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[x/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi [A] time = 0.0095794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] Defer[Int][x/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi steps

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.479795, size = 0, normalized size = 0.

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] Integrate[x/Log[c*(d + e*x^3)^p]^2, x]
```

Maple [A] time = 3.579, size = 0, normalized size = 0.

$$\int \frac{x}{\left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^3+d}{3\left(epx\log\left(\left(ex^3+d\right)^p\right)+epx\log\left(c\right)\right)}+\int\frac{2\,ex^3-d}{3\left(epx^2\log\left(\left(ex^3+d\right)^p\right)+epx^2\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x*log((e*x^3 + d)^p) + e*p*x*log(c)) + integrate(1/3*
(2*e*x^3 - d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\log\left(\left(ex^3+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x/log((e*x^3 + d)^p*c)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x/log(c*(d + e*x**3)**p)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x/log((e*x^3 + d)^p*c)^2, x)
```

3.155 $\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^3)^p]^(-2), x]
```

Rubi [A] time = 0.0035729, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^3)^p]^(-2),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.359393, size = 0, normalized size = 0.

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^(-2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^3)^p]^(-2), x]
```

Maple [A] time = 3.554, size = 0, normalized size = 0.

$$\int \left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^3+d}{3\left(epx^2\log\left(\left(ex^3+d\right)^p\right)+epx^2\log\left(c\right)\right)}+\int\frac{ex^3-2\,d}{3\left(epx^3\log\left(\left(ex^3+d\right)^p\right)+epx^3\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)) + integrate(
1/3*(e*x^3 - 2*d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^(-2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**(-2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^(-2), x)
```

3.156 $\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [A] time = 0.0171222, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A] time = 1.15777, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A] time = 3.679, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^3+d}{3\left(epx^4\log\left(\left(ex^3+d\right)^p\right)+epx^4\log\left(c\right)\right)}-\int\frac{ex^3+4\,d}{3\left(epx^5\log\left(\left(ex^3+d\right)^p\right)+epx^5\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)) - integrate(
1/3*(e*x^3 + 4*d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2\log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

3.157 $\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [A] time = 0.0166394, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A] time = 1.16721, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A] time = 3.719, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\ln\left(c\left(ex^3+d\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^3+d}{3\left(epx^5\log\left(\left(ex^3+d\right)^p\right)+epx^5\log\left(c\right)\right)}-\int\frac{2\,ex^3+5\,d}{3\left(epx^6\log\left(\left(ex^3+d\right)^p\right)+epx^6\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)) - integrate(
1/3*(2*e*x^3 + 5*d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^3\log\left(\left(ex^3+d\right)^p c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

3.158 $\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=76

$$\frac{(fx)^{m+1}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{6ep\text{Unintegrable}\left(\frac{(fx)^{m+2}\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(m+1)}$$

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^3)/(f*(1 + m)) - (6*e*p*Unintegrable[((
f*x)^(2 + m)*Log[c*(d + e*x^2)^p]^2)/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi [A] time = 0.118039, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^3)/(f*(1 + m)) - (6*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p]^2)/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{(fx)^{1+m}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(6ep)\int \frac{(fx)^{2+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{f^2(1+m)}$$

Mathematica [A] time = 2.27135, size = 994, normalized size = 13.08

$$(fx)^m\left(\frac{6p^3\left(d\left(\left(-\frac{ex^2}{d}\right)^{\frac{m+1}{2}}-1\right)\log^2\left(ex^2+d\right)+(m+1)\left(ex^2+d\right){}_3F_2\left(1,1,\frac{1}{2}-\frac{m}{2};2,2;\frac{ex^2}{d}+1\right)\log\left(ex^2+d\right)-(m+1)\left(ex^2+d\right){}_4F_3\left(1,1,1,\frac{1}{2}-\frac{m}{2};2,2,2;\frac{ex^2}{d}+1\right)\right)\left(-\frac{ex^2}{d}\right)^{\frac{1}{2}-\frac{m}{2}}}{e}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^m*((1 + m)*p^3*x^2*Log[d + e*x^2]^3 + (6*p^3*(-((e*x^2)/d))^(1/2 - m
/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2
}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2
}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/
2))*Log[d + e*x^2]^2))/e + (6*d*(1 + m)*p^3*((e*x^2)/(d + e*x^2))^(1/2 - m/
2)*(8*HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2
- m/2, 3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] + (-1 + m)*Log[d + e*x^2]*(-4*
HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}
```

```
, d/(d + e*x^2)] + (-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m
/2, d/(d + e*x^2)]*Log[d + e*x^2])))/(e*(-1 + m)^3) - (3*p^2*(-((e*x^2)/d))
^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2},
 {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1,
1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^
((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])
)/e - (3*m*p^2*(-((e*x^2)/d))^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*Hypergeome
tricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x
^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x
^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p]))/e + (3*p*x^2*(-2*e*x^2*Hypergeometric2F1[1,
(3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e*x^2])*(-(p*Log[d
+ e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + (3*m*p*x^2*(-2*e*x^2*Hyp
ergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e
*x^2])*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + x^2*(-
(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^3 + m*x^2*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^3))/((1 + m)^2*x)
```

Maple [A] time = 0.897, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(\ln \left(c \left(ex^{2}+d\right)^{p}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^{m} \log\left(\left(ex^{2}+d\right)^{p} c\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c)^3, x)
```

3.159 $\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=74

$$\frac{(fx)^{m+1}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{4ep\text{Unintegrable}\left(\frac{(fx)^{m+2}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(m+1)}$$

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^2)/(f*(1 + m)) - (4*e*p*Unintegrable[((
f*x)^(2 + m)*Log[c*(d + e*x^2)^p])/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi [A] time = 0.0889901, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^2)/(f*(1 + m)) - (4*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p])/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) dx = \frac{(fx)^{1+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(4ep)\int \frac{(fx)^{2+m}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{f^2(1+m)}$$

Mathematica [A] time = 1.02958, size = 466, normalized size = 6.3

$$(fx)^m\left(\frac{4d(m+1)p^2\left(\frac{ex^2}{d+ex^2}\right)^{\frac{1}{2}-\frac{m}{2}}\left((m-1)\log\left(d+ex^2\right){}_2F_1\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};\frac{d}{ex^2+d}\right)-2\,{}_3F_2\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2};\frac{d}{ex^2+d}\right)\right)}{e(m-1)^2x}+\frac{2p\left(p\log\left(d+ex^2\right)-\log\left(c\left(d+\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^m*(4*p^2*x*((2*e*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e
*x^2)/d)])/(d*(3 + m)) - Log[d + e*x^2]) + (1 + m)*p^2*x*Log[d + e*x^2]^2 +
 (4*d*(1 + m)*p^2*((e*x^2)/(d + e*x^2))^(1/2 - m/2)*(-2*HypergeometricPFQ[{
1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] +
(-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m/2, d/(d + e*x^2)]*
Log[d + e*x^2]))/(e*(-1 + m)^2*x) + (2*p*(2*e*x^3*Hypergeometric2F1[1, (3 +
 m)/2, (5 + m)/2, -((e*x^2)/d)] - d*(3 + m)*x*Log[d + e*x^2])*(p*Log[d + e*
x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) - (2*m*p*(-2*e*x^3*Hypergeometric
2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*x*Log[d + e*x^2])*(p
```

```
*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) + x*(-(p*Log[d + e*x^2
]) + Log[c*(d + e*x^2)^p])^2 + m*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
^p])^2))/(1 + m)^2
```

Maple [A] time = 0.957, size = 0, normalized size = 0.

$$\int (fx)^m \left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log \left(\left(ex^2 + d \right)^p c \right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

3.160 $\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=81

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rubi [A] time = 0.0429515, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 16, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```


```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right) dx = \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{x(fx)^{1+m}}{d+ex^2}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{(fx)^{2+m}}{d+ex^2}dx}{f^2(1+m)}$$
$$= -\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.0238856, size = 70, normalized size = 0.86

$$\frac{x(fx)^m\left(d(m+3)\log\left(c\left(d+ex^2\right)^p\right)-2epx^2\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)\right)}{d(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (fx)^m \ln\left(c\left(ex^2+d\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^m \log\left(\left(ex^2+d\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \log \left((ex^2 + d)^p c \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

3.161 $\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Rubi [A] time = 0.0181375, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p],x]

[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.3338, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p],x]

[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Maple [A] time = 0.957, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\ln\left(c\left(ex^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

3.162 $\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [A] time = 0.017193, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.529331, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A] time = 5.842, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(ef^{m}x^{2}+df^{m}\right)x^{m}}{2\left(epx\log\left(\left(ex^{2}+d\right)^{p}\right)+epx\log\left(c\right)\right)}+\int\frac{\left(ef^{m}(m+1)x^{2}+df^{m}(m-1)\right)x^{m}}{2\left(epx^{2}\log\left(\left(ex^{2}+d\right)^{p}\right)+epx^{2}\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*f^m*x^2 + d*f^m)*x^m/(e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + in
tegrate(1/2*(e*f^m*(m + 1)*x^2 + d*f^m*(m - 1))*x^m/(e*p*x^2*log((e*x^2 + d
)^p) + e*p*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx\right)^{m}}{\log\left(\left(ex^{2}+d\right)^{p}c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx\right)^{m}}{\log\left(\left(ex^{2}+d\right)^{p}c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

3.163 $\int (fx)^{-1+3n} \log^2\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=372

$$-\frac{2d^2px^{1-3n}(fx)^{3n-1}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{e^3n}+\frac{2d^3px^{1-3n}(fx)^{3n-1}\log\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{3e^3n}-\frac{2px^{1-3n}(fx)^{3n-}}{}$$

```
[Out] (2*d^2*p^2*x^(1 - 2*n)*(f*x)^(-1 + 3*n))/(e^2*n) - (d*p^2*x^(1 - 3*n)*(f*x)
^(-1 + 3*n)*(d + e*x^n)^2)/(2*e^3*n) + (2*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*
(d + e*x^n)^3)/(27*e^3*n) - (d^3*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*Log[d + e
*x^n]^2)/(3*e^3*n) - (2*d^2*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e^3*n) + (d*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)^2
*Log[c*(d + e*x^n)^p])/(e^3*n) - (2*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x
^n)^3*Log[c*(d + e*x^n)^p])/(9*e^3*n) + (2*d^3*p*x^(1 - 3*n)*(f*x)^(-1 + 3*
n)*Log[d + e*x^n]*Log[c*(d + e*x^n)^p])/(3*e^3*n) + (x*(f*x)^(-1 + 3*n)*Log
[c*(d + e*x^n)^p]^2)/(3*n)
```

Rubi [A] time = 0.320828, antiderivative size = 278, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2456, 2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{px^{1-3n}(fx)^{3n-1}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}\right)\log\left(c\left(d+ex^n\right)^p\right)}{9n}+\frac{x(fx)^{3n-1}\log^2\left(c\left(d+ex^n\right)^p\right)}{3n}+$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*d^2*p^2*x^(1 - 2*n)*(f*x)^(-1 + 3*n))/(e^2*n) - (d*p^2*x^(1 - 3*n)*(f*x)
^(-1 + 3*n)*(d + e*x^n)^2)/(2*e^3*n) + (2*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*
(d + e*x^n)^3)/(27*e^3*n) - (d^3*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*Log[d + e
*x^n]^2)/(3*e^3*n) - (p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*((18*d^2*(d + e*x^n))/
e^3 - (9*d*(d + e*x^n)^2)/e^3 + (2*(d + e*x^n)^3)/e^3 - (6*d^3*Log[d + e*x^
n])/e^3)*Log[c*(d + e*x^n)^p])/(9*n) + (x*(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n
)^p]^2)/(3*n)
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int (fx)^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)dx = \left(x^{1-3n}(fx)^{-1+3n}\right)\int x^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)dx$$

$$= \frac{\left(x^{1-3n}(fx)^{-1+3n}\right)\operatorname{Subst}\left(\int x^2\log^2\left(c(d+ex)^p\right)dx, x, x^n\right)}{n}$$

$$= \frac{x(fx)^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)}{3n} - \frac{\left(2epx^{1-3n}(fx)^{-1+3n}\right)\operatorname{Subst}\left(\int \frac{x^3\log(c(d+ex)^p)}{d+ex}dx,\right.}{3n}$$

$$= \frac{x(fx)^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)}{3n} - \frac{\left(2px^{1-3n}(fx)^{-1+3n}\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\log(cx^p)}{x}dx,\right.}{3n}$$

$$= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3} - \frac{9d(d+ex^n)^2}{e^3} + \frac{2(d+ex^n)^3}{e^3} - \frac{6d^3\log(d+ex^n)}{e^3}\right)\log\left(c\left(d+ex\right.\right.}{9n}$$

$$= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3} - \frac{9d(d+ex^n)^2}{e^3} + \frac{2(d+ex^n)^3}{e^3} - \frac{6d^3\log(d+ex^n)}{e^3}\right)\log\left(c\left(d+ex\right.\right.}{9n}$$

$$= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3} - \frac{9d(d+ex^n)^2}{e^3} + \frac{2(d+ex^n)^3}{e^3} - \frac{6d^3\log(d+ex^n)}{e^3}\right)\log\left(c\left(d+ex\right.\right.}{9n}$$

$$= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n} - \frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n} + \frac{2p^2x^{1-3n}(fx)^{-1+3n}\left(d+e\right.}{27e^3n}$$

$$= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n} - \frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n} + \frac{2p^2x^{1-3n}(fx)^{-1+3n}\left(d+e\right.}{27e^3n}$$

Mathematica [A] time = 0.153335, size = 171, normalized size = 0.46

$$\frac{x^{-3n}(fx)^{3n}\left(ex^n\left(-6p\left(6d^2-3dex^n+2e^2x^{2n}\right)\log\left(c\left(d+ex^n\right)^p\right)+18e^2x^{2n}\log^2\left(c\left(d+ex^n\right)^p\right)+p^2\left(66d^2-15dex^n+4e^2\right.\right.\right.}{54e^3fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(3*n)*(-18*d^3*p^2*Log[d + e*x^n]^2 + 6*d^3*p*Log[d + e*x^n]*(-11*p
+ 6*Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(66*d^2 - 15*d*e*x^n + 4*e^2*x^(2*n)
) - 6*p*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))*Log[c*(d + e*x^n)^p] + 18*e^2*x
^(2*n)*Log[c*(d + e*x^n)^p]^2)))/(54*e^3*f*n*x^(3*n))
```

Maple [F] time = 2.118, size = 0, normalized size = 0.

$$\int (fx)^{-1+3n}\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12607, size = 603, normalized size = 1.62

$$\frac{2\left(2\,e^3p^2-6\,e^3p\log\left(c\right)+9\,e^3\log\left(c\right)^2\right)f^{3\,n-1}x^{3\,n}-3\left(5\,de^2p^2-6\,de^2p\log\left(c\right)\right)f^{3\,n-1}x^{2\,n}+6\left(11\,d^2ep^2-6\,d^2ep\log\left(c\right)\right)f^{3\,n-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(2*e^3*p^2 - 6*e^3*p*log(c) + 9*e^3*log(c)^2)*f^(3*n - 1)*x^(3*n) -
 3*(5*d*e^2*p^2 - 6*d*e^2*p*log(c))*f^(3*n - 1)*x^(2*n) + 6*(11*d^2*e*p^2 -
 6*d^2*e*p*log(c))*f^(3*n - 1)*x^n + 18*(e^3*f^(3*n - 1)*p^2*x^(3*n) + d^3*
f^(3*n - 1)*p^2)*log(e*x^n + d)^2 + 6*(3*d*e^2*f^(3*n - 1)*p^2*x^(2*n) - 6*
d^2*e*f^(3*n - 1)*p^2*x^n - 2*(e^3*p^2 - 3*e^3*p*log(c))*f^(3*n - 1)*x^(3*n
) - (11*d^3*p^2 - 6*d^3*p*log(c))*f^(3*n - 1))*log(e*x^n + d))/(e^3*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^{3\,n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

3.164 $\int (fx)^{-1+2n} \log^2\left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=255

$$\frac{x^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2\log^2\left(c\left(d+ex^n\right)^p\right)}{2e^2n}-\frac{dx^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n}-\frac{px^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2}{2e^2n}$$

```
[Out] (-2*d*p^2*x^(1 - n)*(f*x)^(-1 + 2*n))/(e*n) + (p^2*x^(1 - 2*n)*(f*x)^(-1 +
2*n)*(d + e*x^n)^2)/(4*e^2*n) + (2*d*p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e*
x^n)*Log[c*(d + e*x^n)^p])/(e^2*n) - (p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e
*x^n)^2*Log[c*(d + e*x^n)^p])/(2*e^2*n) - (d*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(
d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(e^2*n) + (x^(1 - 2*n)*(f*x)^(-1 + 2*n)*
(d + e*x^n)^2*Log[c*(d + e*x^n)^p]^2)/(2*e^2*n)
```

Rubi [A] time = 0.187219, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2456, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{x^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2\log^2\left(c\left(d+ex^n\right)^p\right)}{2e^2n}-\frac{dx^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n}-\frac{px^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2}{2e^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```


```
[Out] (-2*d*p^2*x^(1 - n)*(f*x)^(-1 + 2*n))/(e*n) + (p^2*x^(1 - 2*n)*(f*x)^(-1 +
2*n)*(d + e*x^n)^2)/(4*e^2*n) + (2*d*p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e*
x^n)*Log[c*(d + e*x^n)^p])/(e^2*n) - (p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e
*x^n)^2*Log[c*(d + e*x^n)^p])/(2*e^2*n) - (d*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(
d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(e^2*n) + (x^(1 - 2*n)*(f*x)^(-1 + 2*n)*
(d + e*x^n)^2*Log[c*(d + e*x^n)^p]^2)/(2*e^2*n)
```


Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (fx)^{-1+2n}\log^2\left(c\left(d+ex^n\right)^p\right)dx &= \left(x^{1-2n}(fx)^{-1+2n}\right)\int x^{-1+2n}\log^2\left(c\left(d+ex^n\right)^p\right)dx\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)dx,x,x^n\right)}{n}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int\left(-\frac{d\log^2(c(d+ex)^p)}{e}+\frac{(d+ex)\log^2(c(d+ex)^p)}{e}\right)dx,x,x^n\right)}{n}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)dx,x,x^n\right)}{en}-\frac{\left(dx^{1-2n}(fx)^{-1+2n}\right)\ldots}{\ldots}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int x\log^2\left(cx^p\right)dx,x,d+ex^n\right)}{e^2n}-\frac{\left(dx^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\ldots\right)}{e^2\ldots}\\
&= -\frac{dx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n}+\frac{x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2\log^2\left(c\left(\ldots\right.\right.}{2e^2n}\\
&= -\frac{2dp^2x^{1-n}(fx)^{-1+2n}}{en}+\frac{p^2x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2}{4e^2n}+\frac{2dpx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\ldots}{e^2n}
\end{aligned}
$$

Mathematica [A] time = 0.110963, size = 140, normalized size = 0.55

$$
\frac{x^{-2n}(fx)^{2n}\left(2d^2p\log\left(d+ex^n\right)\left(3p-2\log\left(c\left(d+ex^n\right)^p\right)\right)+ex^n\left(2ex^n\log^2\left(c\left(d+ex^n\right)^p\right)+2p\left(2d-ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)\ldots\right.\right.}{4e^2fn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(2*n)*(2*d^2*p^2*Log[d + e*x^n]^2 + 2*d^2*p*Log[d + e*x^n]*(3*p - 2*
Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(-6*d + e*x^n) + 2*p*(2*d - e*x^n)*Log[c
*(d + e*x^n)^p] + 2*e*x^n*Log[c*(d + e*x^n)^p]^2)))/(4*e^2*f*n*x^(2*n))
```

Maple [F] time = 2.066, size = 0, normalized size = 0.

$$\int (fx)^{-1+2n} \left(\ln \left(c \left(d+e x^n\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09321, size = 459, normalized size = 1.8

$$\frac{\left(e^2p^2 - 2e^2p\log(c) + 2e^2\log(c)^2\right)f^{2n-1}x^{2n} - 2\left(3dep^2 - 2dep\log(c)\right)f^{2n-1}x^n + 2\left(e^2f^{2n-1}p^2x^{2n} - d^2f^{2n-1}p^2\right)\log(}{4e^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((e^2*p^2 - 2*e^2*p*log(c) + 2*e^2*log(c)^2)*f^(2*n - 1)*x^(2*n) - 2*(3
*d*e*p^2 - 2*d*e*p*log(c))*f^(2*n - 1)*x^n + 2*(e^2*f^(2*n - 1)*p^2*x^(2*n)
 - d^2*f^(2*n - 1)*p^2)*log(e*x^n + d)^2 + 2*(2*d*e*f^(2*n - 1)*p^2*x^n - (
e^2*p^2 - 2*e^2*p*log(c))*f^(2*n - 1)*x^(2*n) + (3*d^2*p^2 - 2*d^2*p*log(c)
)*f^(2*n - 1))*log(e*x^n + d))/(e^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^{2n-1} \log \left((ex^n + d)^p c \right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

3.165 $\int (fx)^{-1+n} \log^2\left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=101

$$\frac{x^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{en}-\frac{2px^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{2p^2x(fx)^{n-1}}{n}$$

```
[Out] (2*p^2*x*(f*x)^(-1 + n))/n - (2*p*x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e*n) + (x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[c*(d +
e*x^n)^p]^2)/(e*n)
```

Rubi [A] time = 0.0810466, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2456, 2454, 2389, 2296, 2295}

$$\frac{x^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{en}-\frac{2px^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{2p^2x(fx)^{n-1}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*p^2*x*(f*x)^(-1 + n))/n - (2*p*x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e*n) + (x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[c*(d +
e*x^n)^p]^2)/(e*n)
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int (fx)^{-1+n}\log^2\left(c\left(d+ex^n\right)^p\right)dx = \left(x^{1-n}(fx)^{-1+n}\right)\int x^{-1+n}\log^2\left(c\left(d+ex^n\right)^p\right)dx$$
$$= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right)dx, x, x^n\right)}{n}$$
$$= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log^2\left(cx^p\right)dx, x, d+ex^n\right)}{en}$$
$$= \frac{x^{1-n}(fx)^{-1+n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{en} - \frac{\left(2px^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log\left(cx^p\right)dx\right.}{en}$$
$$= \frac{2p^2x(fx)^{-1+n}}{n} - \frac{2px^{1-n}(fx)^{-1+n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en} + \frac{x^{1-n}(fx)^{-1+n}\left(d+ex^n\right.}{en}$$

Mathematica [A] time = 0.0245263, size = 74, normalized size = 0.73

$$\frac{x^{-n}(fx)^n\left(\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)-2p\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)+2ep^2x^n\right)}{efn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^n*(2*e*p^2*x^n - 2*p*(d + e*x^n)*Log[c*(d + e*x^n)^p] + (d + e*x^n)*
Log[c*(d + e*x^n)^p]^2))/(e*f*n*x^n)
```

Maple [F] time = 2.097, size = 0, normalized size = 0.

$$\int \left(fx\right)^{-1+n}\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09927, size = 282, normalized size = 2.79

$$\frac{\left(2\,ep^2 - 2\,ep\log\left(c\right) + e\log\left(c\right)^2\right)f^{n-1}x^n + \left(ef^{n-1}p^2x^n + df^{n-1}p^2\right)\log\left(ex^n + d\right)^2 - 2\left(\left(ep^2 - ep\log\left(c\right)\right)f^{n-1}x^n + \left(dp^2 - \right.\right.}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] ((2*e*p^2 - 2*e*p*log(c) + e*log(c)^2)*f^(n - 1)*x^n + (e*f^(n - 1)*p^2*x^n
 + d*f^(n - 1)*p^2)*log(e*x^n + d)^2 - 2*((e*p^2 - e*p*log(c))*f^(n - 1)*x^
n + (d*p^2 - d*p*log(c))*f^(n - 1))*log(e*x^n + d))/(e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{n-1}\log\left(\left(ex^n + d\right)^p c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c)^2, x)
```

3.166 $\int \frac{\log^2(c(d+ex^n)^p)}{fx} dx$

Optimal. Leaf size=88

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/(f*n) + (2*p*Log[c*(d + e*x^n)^p
]*PolyLog[2, 1 + (e*x^n)/d])/(f*n) - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/(f*n
)
```

Rubi [A] time = 0.113938, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 2454, 2396, 2433, 2374, 6589}

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/(f*n) + (2*p*Log[c*(d + e*x^n)^p
]*PolyLog[2, 1 + (e*x^n)/d])/(f*n) - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/(f*n
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^n\right)^p\right)}{fx}\,dx = \frac{\int \frac{\log^2\left(c\left(d+ex^n\right)^p\right)}{x}\,dx}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x}\,dx, x, x^n\right)}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(2ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^p\right)}{d+ex}\,dx, x, x^n\right)}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(2p)\,\text{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right)}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} - \frac{\left(2p^2\right)\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right.}{x}\right.}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} - \frac{2p^2\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{fn}$$

Mathematica [A] time = 0.0993098, size = 168, normalized size = 1.91

$$\frac{2p\left(\log(x)\left(\log\left(d+ex^n\right)-\log\left(\frac{ex^n}{d}+1\right)\right)-\frac{\text{PolyLog}\left(2,-\frac{ex^n}{d}\right)}{n}\right)\left(\log\left(c\left(d+ex^n\right)^p\right)-p\log\left(d+ex^n\right)\right)+\frac{p^2\left(-2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)\right.}{}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e
*x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d]
) - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2
+ 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d])
)/n)/f
```

Maple [C] time = 4.615, size = 1473, normalized size = 16.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^2/f/x,x)
```

```
[Out] I/f/n*ln((d+e*x^n)/d)*ln(x^n)*Pi*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p
)*csgn(I*c)+I/f/n*dilog((d+e*x^n)/d)*Pi*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e
*x^n)^p)*csgn(I*c)-I/f/n*ln((d+e*x^n)/d)*ln(x^n)*Pi*p*csgn(I*(d+e*x^n)^p)*c
sgn(I*c*(d+e*x^n)^p)^2-1/4/f*ln(x)*Pi^2*csgn(I*c*(d+e*x^n)^p)^6+1/f/n*ln((d
+e*x^n)^p)^2*ln(e*x^n)-2/f/n*polylog(3,(d+e*x^n)/d)*p^2-I/f/n*ln(x^n)*ln((d
+e*x^n)^p)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I/f/n*ln(
(d+e*x^n)/d)*ln(x^n)*Pi*p*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+1/f*ln(c)^2*ln(
x)+I/f/n*ln(x^n)*ln((d+e*x^n)^p)*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+I/f/n
*ln(x^n)*ln((d+e*x^n)^p)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I/f
/n*dilog((d+e*x^n)/d)*Pi*p*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+I/f/n*ln((d+e*
x^n)/d)*ln(x^n)*Pi*p*csgn(I*c*(d+e*x^n)^p)^3-I/f/n*dilog((d+e*x^n)/d)*Pi*p*
csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I/f*ln(c)*ln(x)*Pi*csgn(I*(d+e*
x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-2/f/n*ln(d+e*x^n)*dilog(-e*x^n/d)*p
^2+2/f/n*ln((d+e*x^n)^p)*dilog(-e*x^n/d)*p+1/2/f*ln(x)*Pi^2*csgn(I*(d+e*x^n
)^p)^2*csgn(I*c*(d+e*x^n)^p)^3*csgn(I*c)-1/4/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^
p)^2*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)^2-1/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)^4*csgn(I*c)+1/2/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)*csg
n(I*c*(d+e*x^n)^p)^3*csgn(I*c)^2-I/f*ln(c)*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^3
+2/f/n*ln((d+e*x^n)^p)*ln(d+e*x^n)*ln(-e*x^n/d)*p-2/f/n*ln((d+e*x^n)^p)*ln(
d+e*x^n)*ln(e*x^n)*p-2/f/n*ln(c)*ln((d+e*x^n)/d)*ln(x^n)*p-1/4/f*ln(x)*Pi^2
*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^4+1/2/f*ln(x)*Pi^2*csgn(I*(d+e
*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^5+1/2/f*ln(x)*Pi^2*csgn(I*c*(d+e*x^n)^p)^5*c
sgn(I*c)+2/f/n*ln(c)*ln(x^n)*ln((d+e*x^n)^p)-1/4/f*ln(x)*Pi^2*csgn(I*c*(d+e
*x^n)^p)^4*csgn(I*c)^2-2/f/n*ln(c)*dilog((d+e*x^n)/d)*p+1/f/n*ln(d+e*x^n)^2
*ln(e*x^n)*p^2+1/f/n*ln(d+e*x^n)^2*ln(1-(d+e*x^n)/d)*p^2-2/f/n*ln(d+e*x^n)^
2*ln(-e*x^n/d)*p^2+2/f/n*ln(d+e*x^n)*polylog(2,(d+e*x^n)/d)*p^2+I/f*ln(c)*l
n(x)*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+I/f*ln(c)*ln(x)*Pi*csgn(I*(d+e*x^
n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I/f/n*ln(x^n)*ln((d+e*x^n)^p)*Pi*csgn(I*c*(d+
e*x^n)^p)^3+I/f/n*dilog((d+e*x^n)/d)*Pi*p*csgn(I*c*(d+e*x^n)^p)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\left(\frac{\log(x)}{d}-\frac{\log\left(\frac{ex^n+d}{e}\right)}{dn}\right)\log(c)^2+\log\left((ex^n+d)^p\right)^2\log(x)+\frac{\log(c)^2\log\left(\frac{ex^n+d}{e}\right)}{n}-\int\frac{2\left((enp\log(x)-e\log(c))x^n-d\log(c)\right)\log\left((ex^n+d)^p\right)}{exx^n+dx}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="maxima")
```

```
[Out] (log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2*
((e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n + d
*x), x))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)^2}{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^2/(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\log\left(c(d+ex^n)^p\right)^2}{x}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**2/f/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)/f
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)^2}{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^2/(f*x), x)
```

3.167 $\int (fx)^{-1-n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=124

$$\frac{2ep^2x^{n+1}(fx)^{-n-1}\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{dn}-\frac{x(fx)^{-n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn}+\frac{2epx^{n+1}(fx)^{-n-1}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{dn}$$

```
[Out] (2*e*p*x^(1 + n)*(f*x)^(-1 - n)*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(d*
n) - (x*(f*x)^(-1 - n)*(d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(d*n) + (2*e*p^2
*x^(1 + n)*(f*x)^(-1 - n)*PolyLog[2, 1 + (e*x^n)/d])/(d*n)
```

Rubi [A] time = 0.111278, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2456, 2454, 2397, 2394, 2315}

$$\frac{2ep^2x^{n+1}(fx)^{-n-1}\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{dn}-\frac{x(fx)^{-n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn}+\frac{2epx^{n+1}(fx)^{-n-1}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*e*p*x^(1 + n)*(f*x)^(-1 - n)*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(d*
n) - (x*(f*x)^(-1 - n)*(d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(d*n) + (2*e*p^2
*x^(1 + n)*(f*x)^(-1 - n)*PolyLog[2, 1 + (e*x^n)/d])/(d*n)
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (fx)^{-1-n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx = \left(x^{1+n}(fx)^{-1-n}\right)\int x^{-1-n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx$$
$$= \frac{\left(x^{1+n}(fx)^{-1-n}\right)\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x^2}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn} + \frac{\left(2epx^{1+n}(fx)^{-1-n}\right)\text{Subst}\left(\int \frac{\log(c(d+ex)^p}{x}\right.}{dn}$$
$$= \frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{dn} - \frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c\left(d+e\right.\right.}{dn}$$
$$= \frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{dn} - \frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c\left(d+e\right.\right.}{dn}$$

Mathematica [A] time = 0.0891685, size = 148, normalized size = 1.19

$$-\frac{(fx)^{-n}\left(2ep^2x^n\text{PolyLog}\left(2,\frac{dx^{-n}}{e}+1\right)+d\log^2\left(c\left(d+ex^n\right)^p\right)+2epx^n\log\left(-dx^{-n}-e\right)\log\left(c\left(d+ex^n\right)^p\right)-ep^2x^n\log^2\left(\right.\right.}{dfn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] -((2*e*p^2*x^n*Log[-(d/(e*x^n))]*Log[-e - d/x^n] - e*p^2*x^n*Log[-e - d/x^n
]^2 + 2*e*p*x^n*Log[-e - d/x^n]*Log[c*(d + e*x^n)^p] + d*Log[c*(d + e*x^n)^
p]^2 + 2*e*p^2*x^n*PolyLog[2, 1 + d/(e*x^n)])/(d*f*n*(f*x)^n))
```

Maple [F] time = 2.043, size = 0, normalized size = 0.

$$\int \left(fx\right)^{-1-n}\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 2.44405, size = 446, normalized size = 3.6

$$-\frac{2\,ef^{-n-1}np^2x^n\log(x)\log\left(\frac{ex^n+d}{d}\right)-2\,ef^{-n-1}npx^n\log(c)\log(x)+2\,ef^{-n-1}p^2x^n\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+df^{-n-1}\log(c)^2+\left(ef^{\cdot}\right.}{dnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] -(2*e*f^(-n - 1)*n*p^2*x^n*log(x)*log((e*x^n + d)/d) - 2*e*f^(-n - 1)*n*p*x
^n*log(c)*log(x) + 2*e*f^(-n - 1)*p^2*x^n*dilog(-(e*x^n + d)/d + 1) + d*f^(
-n - 1)*log(c)^2 + (e*f^(-n - 1)*p^2*x^n + d*f^(-n - 1)*p^2)*log(e*x^n + d)
^2 + 2*(d*f^(-n - 1)*p*log(c) - (e*n*p^2*log(x) - e*p*log(c))*f^(-n - 1)*x^
n)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^{-n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c)^2, x)
```

3.168 $\int (fx)^{-1-2n} \log^2 \left(c (d+ex^n)^p\right) dx$

Optimal. Leaf size=200

$$\frac{e^2p^2x^{2n+1}(fx)^{-2n-1}\text{PolyLog}\left(2,\frac{d}{d+ex^n}\right)}{d^2n}-\frac{e^2px^{2n+1}(fx)^{-2n-1}\log\left(1-\frac{d}{d+ex^n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n}-\frac{epx^{n+1}(fx)^{-2n-1}\left(d+\right.}{d^2}$$

```
[Out] (e^2*p^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[x])/d^2 - (e*p*x^(1 + n)*(f*x)^(-
1 - 2*n)*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(d^2*n) - (x*(f*x)^(-1 - 2*n)*Lo
g[c*(d + e*x^n)^p]^2)/(2*n) - (e^2*p*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[c*(d
+ e*x^n)^p]*Log[1 - d/(d + e*x^n)])/(d^2*n) + (e^2*p^2*x^(1 + 2*n)*(f*x)^(-
1 - 2*n)*PolyLog[2, d/(d + e*x^n)])/(d^2*n)
```

Rubi [A] time = 0.316349, antiderivative size = 238, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2456, 2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$-\frac{e^2p^2x^{2n+1}(fx)^{-2n-1}\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{d^2n}+\frac{e^2x^{2n+1}(fx)^{-2n-1}\log^2\left(c\left(d+ex^n\right)^p\right)}{2d^2n}-\frac{e^2px^{2n+1}(fx)^{-2n-1}\log\left(-\frac{ex^n}{d}\right)\log\left(\right.}{d^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*p^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[x])/d^2 - (e*p*x^(1 + n)*(f*x)^(-
1 - 2*n)*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(d^2*n) - (e^2*p*x^(1 + 2*n)*(f*
x)^(-1 - 2*n)*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(d^2*n) - (x*(f*x)^(-
1 - 2*n)*Log[c*(d + e*x^n)^p]^2)/(2*n) + (e^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*
Log[c*(d + e*x^n)^p]^2)/(2*d^2*n) - (e^2*p^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*P
olyLog[2, 1 + (e*x^n)/d])/(d^2*n)
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)dx = \left(x^{1+2n}(fx)^{-1-2n}\right)\int x^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)dx$$

$$= \frac{\left(x^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x^3}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x(fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{\left(epx^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2(d+ex)}\,dx, x,\right.}{n}$$

$$= -\frac{x(fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{\left(px^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\right.}{n}$$

$$= -\frac{x(fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{\left(px^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\right.}{dn}$$

$$= -\frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n} - \frac{x(fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)}{2n} -$$

$$= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2} - \frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n} - \frac{e^2px^1}{}$$

$$= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2} - \frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n} - \frac{e^2px^1}{}$$

Mathematica [A] time = 0.262026, size = 288, normalized size = 1.44

$$\frac{(fx)^{-2n}\left(2e^2p^2x^{2n}\text{PolyLog}\left(2,-\frac{ex^n}{d}\right) - d^2\log^2\left(c\left(d+ex^n\right)^p\right) + 2e^2px^{2n}\log\left(e-ex^{-n}\right)\log\left(c\left(d+ex^n\right)^p\right) + 2e^2npx^{2n}\log\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*n^2*p^2*x^(2*n)*Log[x]^2 + e^2*p^2*x^(2*n)*Log[e + d/x^n]^2 - 2*e^2*p^
2*x^(2*n)*Log[e - e/x^n] - 2*e^2*p^2*x^(2*n)*Log[e + d/x^n]*Log[e - e/x^n]
- 2*d*e*p*x^n*Log[c*(d + e*x^n)^p] + 2*e^2*p*x^(2*n)*Log[e - e/x^n]*Log[c*(
d + e*x^n)^p] - d^2*Log[c*(d + e*x^n)^p]^2 + 2*e^2*n*p*x^(2*n)*Log[x]*(p +
p*Log[e + d/x^n] - p*Log[e - e/x^n] - Log[c*(d + e*x^n)^p] + p*Log[1 + (e*x
^n)/d]) + 2*e^2*p^2*x^(2*n)*PolyLog[2, -((e*x^n)/d)])/(2*d^2*f*n*(f*x)^(2*n
))
```

Maple [F] time = 1.978, size = 0, normalized size = 0.

$$\int (fx)^{-1-2n}\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23388, size = 645, normalized size = 3.22

$$\frac{2\,e^2f^{-2\,n-1}np^2x^{2\,n}\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)+2\,e^2f^{-2\,n-1}p^2x^{2\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)-2\,def^{-2\,n-1}px^n\log\left(c\right)-d^2f^{-2\,n-1}\log\left(c\right)^2+2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^2*f^(-2*n - 1)*n*p^2*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*e^2*f^(
-2*n - 1)*p^2*x^(2*n)*dilog(-(e*x^n + d)/d + 1) - 2*d*e*f^(-2*n - 1)*p*x^n*
log(c) - d^2*f^(-2*n - 1)*log(c)^2 + 2*(e^2*n*p^2 - e^2*n*p*log(c))*f^(-2*n
 - 1)*x^(2*n)*log(x) + (e^2*f^(-2*n - 1)*p^2*x^(2*n) - d^2*f^(-2*n - 1)*p^2
)*log(e*x^n + d)^2 - 2*(d*e*f^(-2*n - 1)*p^2*x^n + d^2*f^(-2*n - 1)*p*log(c
) + (e^2*n*p^2*log(x) + e^2*p^2 - e^2*p*log(c))*f^(-2*n - 1)*x^(2*n))*log(e
*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{-2\,n-1}\log\left(\left(ex^n+d\right)^pc\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

3.169 $\int \frac{\log(1+ex^n)}{x} \, dx$

Optimal. Leaf size=13

$$-\frac{\text{PolyLog}(2,-ex^n)}{n}$$

[Out] -(PolyLog[2, -(e*x^n)]/n)

Rubi [A] time = 0.0086505, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2391}

$$-\frac{\text{PolyLog}(2,-ex^n)}{n}$$

Antiderivative was successfully verified.

[In] Int[Log[1 + e*x^n]/x,x]

[Out] -(PolyLog[2, -(e*x^n)]/n)

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log(1+ex^n)}{x} \, dx = -\frac{\text{Li}_2(-ex^n)}{n}$$

Mathematica [A] time = 0.0026189, size = 13, normalized size = 1.

$$-\frac{\text{PolyLog}(2,-ex^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[Log[1 + e*x^n]/x,x]

[Out] -(PolyLog[2, -(e*x^n)]/n)

Maple [A] time = 0.07, size = 14, normalized size = 1.1

$$-\frac{\text{dilog}(1+ex^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(1+e*x^n)/x,x)

```
[Out] -1/n*dilog(1+e*x^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}n\log(x)^2 + n\int\frac{\log(x)}{exx^n + x}\,dx + \log(ex^n + 1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + n*integrate(log(x)/(e*x*x^n + x), x) + log(e*x^n + 1)*log
(x)
```

Fricas [A] time = 2.01841, size = 24, normalized size = 1.85

$$-\frac{\mathrm{Li}_2(-ex^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="fricas")
```

```
[Out] -dilog(-e*x^n)/n
```

Sympy [C] time = 4.06251, size = 14, normalized size = 1.08

$$-\frac{\mathrm{Li}_2\left(ex^n e^{i\pi}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*x**n)/x,x)
```

```
[Out] -polylog(2, e*x**n*exp_polar(I*pi))/n
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log(ex^n + 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^n + 1)/x, x)
```

3.170 $\int \frac{\log(2+ex^n)}{x} \, dx$

Optimal. Leaf size=21

$$\log(2)\log(x)-\frac{\text{PolyLog}\left(2,-\frac{ex^n}{2}\right)}{n}$$

```
[Out] Log[2]*Log[x] - PolyLog[2, -(e*x^n)/2]/n
```

Rubi [A] time = 0.027229, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2392, 2391}

$$\log(2)\log(x)-\frac{\text{PolyLog}\left(2,-\frac{ex^n}{2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + e*x^n]/x,x]

[Out] Log[2]*Log[x] - PolyLog[2, -(e*x^n)/2]/n
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(2+ex^n)}{x} \, dx &= \frac{\text{Subst}\left(\int \frac{\log(2+ex)}{x} \, dx, x, x^n\right)}{n}\\ &= \log(2)\log(x)+\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{2}\right)}{x} \, dx, x, x^n\right)}{n}\\ &= \log(2)\log(x)-\frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0026883, size = 21, normalized size = 1.

$$\log(2)\log(x) - \frac{\operatorname{PolyLog}\left(2, -\frac{ex^n}{2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + e*x^n]/x,x]
```

```
[Out] Log[2]*Log[x] - PolyLog[2, -(e*x^n)/2]/n
```

Maple [B] time = 0.072, size = 56, normalized size = 2.7

$$-\frac{1}{n}\ln\left(-\frac{ex^n}{2}\right)\ln\left(\frac{ex^n}{2}+1\right)+\frac{\ln\left(2+ex^n\right)}{n}\ln\left(-\frac{ex^n}{2}\right)-\frac{1}{n}\mathrm{dilog}\left(\frac{ex^n}{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2+e*x^n)/x,x)
```

```
[Out] -1/n*ln(-1/2*e*x^n)*ln(1/2*e*x^n+1)+1/n*ln(-1/2*e*x^n)*ln(2+e*x^n)-1/n*dilo
g(1/2*e*x^n+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,n\log\left(x\right)^2+2\,n\int\frac{\log\left(x\right)}{exx^n+2\,x}\,dx+\log\left(ex^n+2\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + 2*n*integrate(log(x)/(e*x*x^n + 2*x), x) + log(e*x^n + 2)
*log(x)
```

Fricas [B] time = 2.01465, size = 107, normalized size = 5.1

$$\frac{n\log\left(ex^n+2\right)\log\left(x\right)-n\log\left(\frac{1}{2}\,ex^n+1\right)\log\left(x\right)-\mathrm{Li}_2\left(-\frac{1}{2}\,ex^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="fricas")
```

```
[Out] (n*log(e*x^n + 2)*log(x) - n*log(1/2*e*x^n + 1)*log(x) - dilog(-1/2*e*x^n))
/n
```

Sympy [C] time = 4.60811, size = 78, normalized size = 3.71

$$\begin{cases} \log(2)\log(x) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{2}\right)}{n} & \text{for } |x| < 1 \\ -\log(2)\log\left(\frac{1}{x}\right) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{2}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(2) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(2) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{2}\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2+e*x**n)/x,x)
```

```
[Out] Piecewise((log(2)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, Abs(x) <
 1), (-log(2)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, 1/Abs(x) <
 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ())
, ((), (0, 0)), x)*log(2) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ex^n + 2)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^n + 2)/x, x)
```

3.171 $\int \frac{\log(2(3+ex^n))}{x}\,dx$

Optimal. Leaf size=21

$$\log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

```
[Out] Log[6]*Log[x] - PolyLog[2, -(e*x^n)/3]/n
```

Rubi [A] time = 0.0284914, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2392, 2391}

$$\log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[2*(3 + e*x^n)]/x,x]
```

```
[Out] Log[6]*Log[x] - PolyLog[2, -(e*x^n)/3]/n
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(2(3+ex^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(2(3+ex))}{x}\,dx, x, x^n\right)}{n}\\
&= \log(6)\log(x) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{3}\right)}{x}\,dx, x, x^n\right)}{n}\\
&= \log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.0040055, size = 21, normalized size = 1.

$$\log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2*(3 + e*x^n)]/x,x]
```

```
[Out] Log[6]*Log[x] - PolyLog[2, -(e*x^n)/3]/n
```

Maple [B] time = 0.069, size = 57, normalized size = 2.7

$$-\frac{1}{n}\ln\left(-\frac{ex^n}{3}\right)\ln\left(\frac{ex^n}{3}+1\right)+\frac{\ln(6+2\,ex^n)}{n}\ln\left(-\frac{ex^n}{3}\right)-\frac{1}{n}\text{dilog}\left(\frac{ex^n}{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(6+2*e*x^n)/x,x)
```

```
[Out] -1/n*ln(-1/3*e*x^n)*ln(1/3*e*x^n+1)+1/n*ln(-1/3*e*x^n)*ln(6+2*e*x^n)-1/n*di
log(1/3*e*x^n+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,n\log(x)^2 + 3\,n\int \frac{\log(x)}{exx^n + 3\,x}\,dx + \log(2)\log(x) + \log(ex^n + 3)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + 3*n*integrate(log(x)/(e*x*x^n + 3*x), x) + log(2)*log(x)
+ log(e*x^n + 3)*log(x)
```

Fricas [B] time = 2.12304, size = 109, normalized size = 5.19

$$\frac{n\log(2\,ex^n + 6)\log(x) - n\log\left(\frac{1}{3}\,ex^n + 1\right)\log(x) - \text{Li}_2\left(-\frac{1}{3}\,ex^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="fricas")
```

```
[Out] (n*log(2*e*x^n + 6)*log(x) - n*log(1/3*e*x^n + 1)*log(x) - dilog(-1/3*e*x^n
))/n
```

Sympy [C] time = 4.75034, size = 78, normalized size = 3.71

$$\begin{cases} \log(6)\log(x) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for } |x| < 1 \\ -\log(6)\log\left(\frac{1}{x}\right) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(6) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(6) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(6+2*e*x**n)/x,x)
```

```
[Out] Piecewise((log(6)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, Abs(x) <
 1), (-log(6)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, 1/Abs(x) <
 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(6) + meijerg(((1, 1), ())
, ((), (0, 0)), x)*log(6) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(2\,ex^n + 6)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(2*e*x^n + 6)/x, x)
```

3.172 $\int \frac{\log(c(d+ex^n))}{x}\,dx$

Optimal. Leaf size=41

$$\frac{\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)])/n + PolyLog[2, 1 + (e*x^n)/d]/n
```

Rubi [A] time = 0.0392548, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2394, 2315}

$$\frac{\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)])/n + PolyLog[2, 1 + (e*x^n)/d]/n
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(d+ex^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}-\frac{e\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}+\frac{\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.0075831, size = 39, normalized size = 0.95

$$\frac{\text{PolyLog}\left(2,\frac{d+ex^n}{d}\right)+\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)] + PolyLog[2, (d + e*x^n)/d])/n
```

Maple [A] time = 0.114, size = 41, normalized size = 1.

$$\frac{\ln\left(cex^n+cd\right)}{n}\ln\left(-\frac{ex^n}{d}\right)+\frac{1}{n}\text{dilog}\left(-\frac{ex^n}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n))/x,x)
```

```
[Out] 1/n*ln(c*e*x^n+c*d)*ln(-e*x^n/d)+1/n*dilog(-e*x^n/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$dn\int\frac{\log\left(x\right)}{exx^n+dx}\,dx-\frac{1}{2}\,n\log\left(x\right)^2+\log\left(ex^n+d\right)\log\left(x\right)+\log\left(c\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="maxima")
```

```
[Out] d*n*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*log(x)^2 + log(e*x^n + d)*
log(x) + log(c)*log(x)
```

Fricas [A] time = 2.13791, size = 123, normalized size = 3.

$$\frac{n\log\left(cex^n+cd\right)\log\left(x\right)-n\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)-\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="fricas")
```

```
[Out] (n*log(c*e*x^n + c*d)*log(x) - n*log(x)*log((e*x^n + d)/d) - dilog(-(e*x^n
+ d)/d + 1))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(cd+cex^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n))/x,x)
```

```
[Out] Integral(log(c*d + c*e*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)*c)/x, x)
```

3.173 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{p\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rubi [A] time = 0.0397916, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{p\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A] time = 0.0052101, size = 43, normalized size = 0.98

$$\frac{p\text{PolyLog}\left(2, \frac{d+ex^n}{d}\right) + \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Maple [C] time = 0.079, size = 177, normalized size = 4.

$$\ln(x)\ln\left(\left(d+ex^n\right)^p\right) + \frac{i}{2}\ln(x)\,\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\left(\text{csgn}\left(ic\left(d+ex^n\right)^p\right)\right)^2 - \frac{i}{2}\ln(x)\,\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\text{csgn}\left(ic\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] ln(x)*ln((d+e*x^n)^p)+1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)^2-1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/
2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^2
*csgn(I*c)+ln(c)*ln(x)-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$dnp\int \frac{\log(x)}{exx^n + dx}\,dx - \frac{1}{2}np\log(x)^2 + \log\left(\left(ex^n + d\right)^p\right)\log(x) + \log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
+ d)^p)*log(x) + log(c)*log(x)
```

Fricas [A] time = 2.10443, size = 150, normalized size = 3.41

$$\frac{np\log\left(ex^n+d\right)\log(x) - np\log(x)\log\left(\frac{ex^n+d}{d}\right) + n\log(c)\log(x) - p\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\,(d+ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n+d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x, x)
```

3.174 $\int \frac{\log^2(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=79

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/n + (2*p*Log[c*(d + e*x^n)^p]*Po
lyLog[2, 1 + (e*x^n)/d])/n - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.100176, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2396, 2433, 2374, 6589}

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/n + (2*p*Log[c*(d + e*x^n)^p]*Po
lyLog[2, 1 + (e*x^n)/d])/n - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/n
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(2ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(2p)\,\text{Subst}\left(\int \frac{\log(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{\left(2p^2\right)\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx \ldots\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{2p^2\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [B] time = 0.0575899, size = 164, normalized size = 2.08

$$2p\left(\log(x)\left(\log\left(d+ex^n\right)-\log\left(\frac{ex^n}{d}+1\right)\right)-\frac{\text{PolyLog}\left(2,-\frac{ex^n}{d}\right)}{n}\right)\left(\log\left(c\left(d+ex^n\right)^p\right)-p\log\left(d+ex^n\right)\right)+\frac{p^2\left(-2\text{PolyLog}\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e*
x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d])
 - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2 +
 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d]))
/n
```

Maple [C] time = 4.447, size = 1356, normalized size = 17.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^2/x,x)
```

```
[Out] ln(x)*ln(c)^2-2/n*ln(c)*ln((d+e*x^n)/d)*ln(x^n)*p-1/4*ln(x)*Pi^2*csgn(I*c*(
d+e*x^n)^p)^6+1/n*ln((d+e*x^n)^p)^2*ln(e*x^n)-2/n*polylog(3,(d+e*x^n)/d)*p^
```

```
2-I/n*Pi*ln((d+e*x^n)^p)*ln(x^n)*csgn(I*c*(d+e*x^n)^p)^3+I*ln(x)*ln(c)*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2+I/n*Pi*dilog((d+e*x^n)/d)*p*csgn
(I*c*(d+e*x^n)^p)^3-1/4*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)
^p)^2*csgn(I*c)^2-ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^4*cs
gn(I*c)+I/n*Pi*ln((d+e*x^n)/d)*ln(x^n)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*
x^n)^p)*csgn(I*c)+1/2*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^
5+1/n*ln(d+e*x^n)^2*ln(e*x^n)*p^2+1/n*ln(d+e*x^n)^2*ln(1-(d+e*x^n)/d)*p^2-2
/n*ln(d+e*x^n)^2*ln(-e*x^n/d)*p^2+2/n*ln(d+e*x^n)*polylog(2,(d+e*x^n)/d)*p^
2-2/n*ln(d+e*x^n)*dilog(-e*x^n/d)*p^2+2/n*ln((d+e*x^n)^p)*dilog(-e*x^n/d)*p
+2/n*ln(c)*ln((d+e*x^n)^p)*ln(x^n)-2/n*ln(c)*dilog((d+e*x^n)/d)*p+1/2*ln(x)
*Pi^2*csgn(I*c*(d+e*x^n)^p)^5*csgn(I*c)-1/4*ln(x)*Pi^2*csgn(I*c*(d+e*x^n)^p
)^4*csgn(I*c)^2-1/4*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^
4+I/n*Pi*dilog((d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csg
n(I*c)-I/n*Pi*ln((d+e*x^n)^p)*ln(x^n)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n
)^p)*csgn(I*c)-I/n*Pi*ln((d+e*x^n)/d)*ln(x^n)*p*csgn(I*c*(d+e*x^n)^p)^2*csg
n(I*c)-I/n*Pi*ln((d+e*x^n)/d)*ln(x^n)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x
^n)^p)^2+1/2*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^3*csgn(I*
c)^2-I*ln(x)*ln(c)*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*ln(x)*ln(c)*Pi*csgn(I*c*(d+
e*x^n)^p)^2*csgn(I*c)+1/2*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^
n)^p)^3*csgn(I*c)+2/n*ln((d+e*x^n)^p)*ln(d+e*x^n)*ln(-e*x^n/d)*p-2/n*ln((d+
e*x^n)^p)*ln(d+e*x^n)*ln(e*x^n)*p-I/n*Pi*dilog((d+e*x^n)/d)*p*csgn(I*c*(d+e
*x^n)^p)^2*csgn(I*c)-I/n*Pi*dilog((d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I
*c*(d+e*x^n)^p)^2+I/n*Pi*ln((d+e*x^n)/d)*ln(x^n)*p*csgn(I*c*(d+e*x^n)^p)^3+
I/n*Pi*ln((d+e*x^n)^p)*ln(x^n)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2+
I/n*Pi*ln((d+e*x^n)^p)*ln(x^n)*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)-I*ln(x)*ln
(c)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\log\left(\left(ex^n + d\right)^p\right)^2 \log(x) - \int -\frac{ex^n \log(c)^2 + d\log(c)^2 - 2\left(\left(enp\log(x) - e\log(c)\right)x^n - d\log(c)\right)\log\left(\left(ex^n + d\right)^p\right)}{exx^n + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="maxima")
```

```
[Out] log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2*(
(e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n + d*
x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^n + d\right)^p c\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^{n}\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**2/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^{n}+d\right)^{p}c\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^2/x, x)
```

3.175 $\int \frac{\log^3(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=113

$$-\frac{6p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{3p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{6p^3\text{PolyLog}\left(4,\frac{ex^n}{d}+1\right)}{n}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^3)/n + (3*p*Log[c*(d + e*x^n)^p]^2*
PolyLog[2, 1 + (e*x^n)/d])/n - (6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (
e*x^n)/d])/n + (6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.148454, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-\frac{6p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{3p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{6p^3\text{PolyLog}\left(4,\frac{ex^n}{d}+1\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^3)/n + (3*p*Log[c*(d + e*x^n)^p]^2*
PolyLog[2, 1 + (e*x^n)/d])/n - (6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (
e*x^n)/d])/n + (6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
```

```
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log^3(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(3ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log^2(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(3p)\,\text{Subst}\left(\int \frac{\log^2(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{3p\log^2\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{\left(6p^2\right)\text{Subst}\left(\int \frac{\log(cx^p)\text{I}}{x}\right.}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{3p\log^2\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log\left(c\left(d+ex^n\right)^p\right)\text{L}}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{3p\log^2\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log\left(c\left(d+ex^n\right)^p\right)\text{L}}{n}$$

Mathematica [B] time = 0.0996806, size = 270, normalized size = 2.39

$$\frac{-6p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)+3p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log^2\left(c\left(d+ex^n\right)^p\right)+6p^3\text{PolyLog}\left(4,\frac{ex^n}{d}+1\right)+3\text{r}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (-(n*p^3*Log[x]*Log[d + e*x^n]^3) + p^3*Log[-((e*x^n)/d)]*Log[d + e*x^n]^3
+ 3*n*p^2*Log[x]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*p^2*Log[-((e*x^n
)/d)]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*n*p*Log[x]*Log[d + e*x^n]*L
og[c*(d + e*x^n)^p]^2 + 3*p*Log[-((e*x^n)/d)]*Log[d + e*x^n]*Log[c*(d + e*x
^n)^p]^2 + n*Log[x]*Log[c*(d + e*x^n)^p]^3 + 3*p*Log[c*(d + e*x^n)^p]^2*Pol
yLog[2, 1 + (e*x^n)/d] - 6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (e*x^n)/
d] + 6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Maple [C] time = 4.84, size = 6131, normalized size = 54.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^3/x,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\log\left((ex^n+d)^p\right)^3\log(x)-\int -\frac{ex^n\log(c)^3+d\log(c)^3-3\left((enp\log(x)-e\log(c))x^n-d\log(c)\right)\log\left((ex^n+d)^p\right)^2+3}{exx^n+dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="maxima")
```

```
[Out] log((e*x^n + d)^p)^3*log(x) - integrate(-(e*x^n*log(c)^3 + d*log(c)^3 - 3*(
(e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p)^2 + 3*(e*x^n*l
og(c)^2 + d*log(c)^2)*log((e*x^n + d)^p))/(e*x*x^n + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left((ex^n+d)^p c\right)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\,(d+ex^n)^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**3/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n+d)^p c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^3/x, x)
```

3.176 $\int (d+ex)^3 \log\left(c(a+bx)^p\right)\,dx$

Optimal. Leaf size=140

$$-\frac{px(bd-ae)^3}{4b^3}-\frac{p(d+ex)^2(bd-ae)^2}{8b^2e}-\frac{p(bd-ae)^4\log(a+bx)}{4b^4e}+\frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e}-\frac{p(d+ex)^3(bd-ae)}{12be}-\frac{p}{}$$

```
[Out] -((b*d - a*e)^3*p*x)/(4*b^3) - ((b*d - a*e)^2*p*(d + e*x)^2)/(8*b^2*e) - ((
b*d - a*e)*p*(d + e*x)^3)/(12*b*e) - (p*(d + e*x)^4)/(16*e) - ((b*d - a*e)^
4*p*Log[a + b*x])/(4*b^4*e) + ((d + e*x)^4*Log[c*(a + b*x)^p])/(4*e)
```

Rubi [A] time = 0.0772138, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 43}

$$-\frac{px(bd-ae)^3}{4b^3}-\frac{p(d+ex)^2(bd-ae)^2}{8b^2e}-\frac{p(bd-ae)^4\log(a+bx)}{4b^4e}+\frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e}-\frac{p(d+ex)^3(bd-ae)}{12be}-\frac{p}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -((b*d - a*e)^3*p*x)/(4*b^3) - ((b*d - a*e)^2*p*(d + e*x)^2)/(8*b^2*e) - ((
b*d - a*e)*p*(d + e*x)^3)/(12*b*e) - (p*(d + e*x)^4)/(16*e) - ((b*d - a*e)^
4*p*Log[a + b*x])/(4*b^4*e) + ((d + e*x)^4*Log[c*(a + b*x)^p])/(4*e)
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3\log\left(c(a+bx)^p\right)\,dx &= \frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e}-\frac{(bp)\int\frac{(d+ex)^4}{a+bx}\,dx}{4e}\\
&= \frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e}-\frac{(bp)\int\left(\frac{e(bd-ae)^3}{b^4}+\frac{(bd-ae)^4}{b^4(a+bx)}+\frac{e(bd-ae)^2(d+ex)}{b^3}+\frac{e(bd-ae)(d+ex)^2}{b^2}\right.}{4e}\\
&= -\frac{(bd-ae)^3px}{4b^3}-\frac{(bd-ae)^2p(d+ex)^2}{8b^2e}-\frac{(bd-ae)p(d+ex)^3}{12be}-\frac{p(d+ex)^4}{16e}-\frac{(bd-ae}{}
\end{aligned}$$

Mathematica [A] time = 0.202553, size = 185, normalized size = 1.32

$$-\frac{bpx\left(6a^2be^2(8d+ex)-12a^3e^3-4ab^2e\left(18d^2+6dex+e^2x^2\right)+b^3\left(36d^2ex+48d^3+16de^2x^2+3e^3x^3\right)\right)+12a^2ep\left(a^2e^{}\right.}{48b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -(b*p*x*(-12*a^3*e^3 + 6*a^2*b*e^2*(8*d + e*x) - 4*a*b^2*e*(18*d^2 + 6*d*e*
x + e^2*x^2) + b^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3)) + 12*a
^2*e*(6*b^2*d^2 - 4*a*b*d*e + a^2*e^2)*p*Log[a + b*x] - 12*b^3*(4*a*d^3 + b
*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*Log[c*(a + b*x)^p])/(48*b^4
)
```

Maple [C] time = 0.611, size = 766, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x+a)^p),x)
```

```
[Out] 1/4/b^3*e^3*a^3*p*x-d^3*p*x-1/4/b^4*e^3*ln(b*x+a)*a^4*p+1/b*ln(b*x+a)*a*d^3
*p-1/8*I*e^3*Pi*x^4*csgn(I*c*(b*x+a)^p)^3-1/2*I*Pi*d^3*x*csgn(I*c*(b*x+a)^p
)^3-1/8/b^2*e^3*a^2*p*x^2+ln(c)*d^3*x+1/4*e^3*ln(c)*x^4+3/2*e*ln(c)*d^2*x^2
+e^2*ln(c)*d*x^3-1/16*e^3*p*x^4+1/12/b*e^3*a*p*x^3-1/4/e*ln(b*x+a)*d^4*p-1/
3*d*e^2*p*x^3-3/4*d^2*e*p*x^2+1/4*(e*x+d)^4/e*ln((b*x+a)^p)-1/2*I*e^2*Pi*d*
x^3*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)-3/4*I*e*Pi*d^2*x^2*csgn
(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)+1/2/b*e^2*a*d*p*x^2-1/b^2*e^2*a
^2*d*p*x+3/2/b*e*a*d^2*p*x+1/b^3*e^2*ln(b*x+a)*a^3*d*p-3/2/b^2*e*ln(b*x+a)*
a^2*d^2*p+1/2*I*Pi*d^3*x*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/2*I*Pi*d
^3*x*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2+1/8*I*e^3*Pi*x^4*csgn(I*(b*x+a)^p)*csg
n(I*c*(b*x+a)^p)^2+1/8*I*e^3*Pi*x^4*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2-1/2*I*e
^2*Pi*d*x^3*csgn(I*c*(b*x+a)^p)^3-3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x+a)^p)^3-
1/8*I*e^3*Pi*x^4*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)+1/2*I*e^2*
Pi*d*x^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/2*I*e^2*Pi*d*x^3*csgn(I*
c)*csgn(I*c*(b*x+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x
+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2-1/2*I*Pi*d^3*x*
csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)
```

Maxima [A] time = 1.06763, size = 289, normalized size = 2.06

$$-\frac{1}{48}\,bp\left(\frac{3\,b^3e^3x^4 + 4\left(4\,b^3de^2 - ab^2e^3\right)x^3 + 6\left(6\,b^3d^2e - 4\,ab^2de^2 + a^2be^3\right)x^2 + 12\left(4\,b^3d^3 - 6\,ab^2d^2e + 4\,a^2bde^2 - a^3e^3\right)x}{b^4} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/48*b*p*((3*b^3*e^3*x^4 + 4*(4*b^3*d*e^2 - a*b^2*e^3)*x^3 + 6*(6*b^3*d^2*
e - 4*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + 12*(4*b^3*d^3 - 6*a*b^2*d^2*e + 4*a^2*
b*d*e^2 - a^3*e^3)*x)/b^4 - 12*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e
^2 - a^4*e^3)*log(b*x + a)/b^5) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2
+ 4*d^3*x)*log((b*x + a)^p*c)
```

Fricas [B] time = 1.99572, size = 559, normalized size = 3.99

$$-\frac{3\,b^4e^3px^4 + 4\left(4\,b^4de^2 - ab^3e^3\right)px^3 + 6\left(6\,b^4d^2e - 4\,ab^3de^2 + a^2b^2e^3\right)px^2 + 12\left(4\,b^4d^3 - 6\,ab^3d^2e + 4\,a^2b^2de^2 - a^3be^3\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/48*(3*b^4*e^3*p*x^4 + 4*(4*b^4*d*e^2 - a*b^3*e^3)*p*x^3 + 6*(6*b^4*d^2*e
 - 4*a*b^3*d*e^2 + a^2*b^2*e^3)*p*x^2 + 12*(4*b^4*d^3 - 6*a*b^3*d^2*e + 4*a
^2*b^2*d*e^2 - a^3*b*e^3)*p*x - 12*(b^4*e^3*p*x^4 + 4*b^4*d*e^2*p*x^3 + 6*b
^4*d^2*e*p*x^2 + 4*b^4*d^3*p*x + (4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d
*e^2 - a^4*e^3)*p)*log(b*x + a) - 12*(b^4*e^3*x^4 + 4*b^4*d*e^2*x^3 + 6*b^4
*d^2*e*x^2 + 4*b^4*d^3*x)*log(c))/b^4
```

Sympy [A] time = 6.66625, size = 369, normalized size = 2.64

$$\begin{cases} -\frac{a^4e^3p\log(a+bx)}{4b^4} + \frac{a^3de^2p\log(a+bx)}{b^3} + \frac{a^3e^3px}{4b^3} - \frac{3a^2d^2ep\log(a+bx)}{2b^2} - \frac{a^2de^2px}{b^2} - \frac{a^2e^3px^2}{8b^2} + \frac{ad^3p\log(a+bx)}{b} + \frac{3ad^2epx}{2b} + \frac{ade^2px^2}{2b} + \frac{ae^3px^3}{12b} \cdots \\ \left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right)\log(a^pc) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**4*e**3*p*log(a + b*x)/(4*b**4) + a**3*d*e**2*p*log(a + b*x)/
b**3 + a**3*e**3*p*x/(4*b**3) - 3*a**2*d**2*e*p*log(a + b*x)/(2*b**2) - a**
2*d*e**2*p*x/b**2 - a**2*e**3*p*x**2/(8*b**2) + a*d**3*p*log(a + b*x)/b + 3
*a*d**2*e*p*x/(2*b) + a*d*e**2*p*x**2/(2*b) + a*e**3*p*x**3/(12*b) + d**3*p
*x*log(a + b*x) - d**3*p*x + d**3*x*log(c) + 3*d**2*e*p*x**2*log(a + b*x)/2
 - 3*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x**3*log(a + b*x)
- d*e**2*p*x**3/3 + d*e**2*x**3*log(c) + e**3*p*x**4*log(a + b*x)/4 - e**3*
p*x**4/16 + e**3*x**4*log(c)/4, Ne(b, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e
**2*x**3 + e**3*x**4/4)*log(a**p*c), True))
```

Giac [B] time = 1.1616, size = 753, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d^3*p*log(b*x + a)/b + 3/2*(b*x + a)^2*d^2*p*e*log(b*x + a)/b^2 -
 3*(b*x + a)*a*d^2*p*e*log(b*x + a)/b^2 - (b*x + a)*d^3*p/b - 3/4*(b*x + a)
^2*d^2*p*e/b^2 + 3*(b*x + a)*a*d^2*p*e/b^2 + (b*x + a)^3*d*p*e^2*log(b*x +
a)/b^3 - 3*(b*x + a)^2*a*d*p*e^2*log(b*x + a)/b^3 + 3*(b*x + a)*a^2*d*p*e^2
*log(b*x + a)/b^3 + (b*x + a)*d^3*log(c)/b + 3/2*(b*x + a)^2*d^2*e*log(c)/b
^2 - 3*(b*x + a)*a*d^2*e*log(c)/b^2 - 1/3*(b*x + a)^3*d*p*e^2/b^3 + 3/2*(b*
x + a)^2*a*d*p*e^2/b^3 - 3*(b*x + a)*a^2*d*p*e^2/b^3 + 1/4*(b*x + a)^4*p*e^
3*log(b*x + a)/b^4 - (b*x + a)^3*a*p*e^3*log(b*x + a)/b^4 + 3/2*(b*x + a)^2
*a^2*p*e^3*log(b*x + a)/b^4 - (b*x + a)*a^3*p*e^3*log(b*x + a)/b^4 + (b*x +
 a)^3*d*e^2*log(c)/b^3 - 3*(b*x + a)^2*a*d*e^2*log(c)/b^3 + 3*(b*x + a)*a^2
```

```
*d*e^2*log(c)/b^3 - 1/16*(b*x + a)^4*p*e^3/b^4 + 1/3*(b*x + a)^3*a*p*e^3/b^
4 - 3/4*(b*x + a)^2*a^2*p*e^3/b^4 + (b*x + a)*a^3*p*e^3/b^4 + 1/4*(b*x + a)
^4*e^3*log(c)/b^4 - (b*x + a)^3*a*e^3*log(c)/b^4 + 3/2*(b*x + a)^2*a^2*e^3*
log(c)/b^4 - (b*x + a)*a^3*e^3*log(c)/b^4
```

3.177 $\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx$

Optimal. Leaf size=112

$$-\frac{px(bd-ae)^2}{3b^2}-\frac{p(bd-ae)^3\log(a+bx)}{3b^3e}+\frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e}-\frac{p(d+ex)^2(bd-ae)}{6be}-\frac{p(d+ex)^3}{9e}$$

```
[Out] -((b*d - a*e)^2*p*x)/(3*b^2) - ((b*d - a*e)*p*(d + e*x)^2)/(6*b*e) - (p*(d
+ e*x)^3)/(9*e) - ((b*d - a*e)^3*p*Log[a + b*x])/(3*b^3*e) + ((d + e*x)^3*L
og[c*(a + b*x)^p])/(3*e)
```

Rubi [A] time = 0.073292, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 43}

$$-\frac{px(bd-ae)^2}{3b^2}-\frac{p(bd-ae)^3\log(a+bx)}{3b^3e}+\frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e}-\frac{p(d+ex)^2(bd-ae)}{6be}-\frac{p(d+ex)^3}{9e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] -((b*d - a*e)^2*p*x)/(3*b^2) - ((b*d - a*e)*p*(d + e*x)^2)/(6*b*e) - (p*(d
+ e*x)^3)/(9*e) - ((b*d - a*e)^3*p*Log[a + b*x])/(3*b^3*e) + ((d + e*x)^3*L
og[c*(a + b*x)^p])/(3*e)
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\log\left(c(a+bx)^p\right)dx &= \frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e}-\frac{(bp)\int\frac{(d+ex)^3}{a+bx}dx}{3e}\\ &= \frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e}-\frac{(bp)\int\left(\frac{e(bd-ae)^2}{b^3}+\frac{(bd-ae)^3}{b^3(a+bx)}+\frac{e(bd-ae)(d+ex)}{b^2}+\frac{e(d+ex)^2}{b}\right)dx}{3e}\\ &= -\frac{(bd-ae)^2px}{3b^2}-\frac{(bd-ae)p(d+ex)^2}{6be}-\frac{p(d+ex)^3}{9e}-\frac{(bd-ae)^3p\log(a+bx)}{3b^3e}+\frac{(d+e\end{aligned}$$

Mathematica [A] time = 0.106499, size = 121, normalized size = 1.08

$$\frac{b\left(6b\left(3ad^2+bx\left(3d^2+3dex+e^2x^2\right)\right)\log\left(c(a+bx)^p\right)-px\left(6a^2e^2-3abe(6d+ex)+b^2\left(18d^2+9dex+2e^2x^2\right)\right)\right)+6a^2}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] (6*a^2*e*(-3*b*d + a*e)*p*Log[a + b*x] + b*(-(p*x*(6*a^2*e^2 - 3*a*b*e*(6*d
 + e*x) + b^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*b*(3*a*d^2 + b*x*(3*d^2
+ 3*d*e*x + e^2*x^2))*Log[c*(a + b*x)^p]))/(18*b^3)
```

Maple [C] time = 0.51, size = 537, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x+a)^p),x)
```

```
[Out] -1/2*I*e*Pi*d*x^2*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)-1/3/b^2*e
^2*a^2*p*x+1/3/b^3*e^2*ln(b*x+a)*a^3*p+1/b*ln(b*x+a)*a*d^2*p-1/6*I*e^2*Pi*x
^3*csgn(I*c*(b*x+a)^p)^3-1/2*I*Pi*d^2*x*csgn(I*c*(b*x+a)^p)^3-1/3/e*ln(b*x+
a)*d^3*p+ln(c)*d^2*x+1/3*e^2*ln(c)*x^3+1/6/b*e^2*a*p*x^2-d^2*p*x-1/9*e^2*p*
x^3+e*ln(c)*d*x^2-1/2*d*e*p*x^2+1/6*I*e^2*Pi*x^3*csgn(I*(b*x+a)^p)*csgn(I*c
*(b*x+a)^p)^2+1/6*I*e^2*Pi*x^3*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2-1/2*I*e*Pi*d
*x^2*csgn(I*c*(b*x+a)^p)^3+1/3*(e*x+d)^3/e*ln((b*x+a)^p)+1/b*e*a*d*p*x-1/b^
2*e*ln(b*x+a)*a^2*d*p+1/2*I*Pi*d^2*x*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2+1/2*I*
Pi*d^2*x*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/2*I*e*Pi*d*x^2*csgn(I*(b
*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/2*I*e*Pi*d*x^2*csgn(I*c)*csgn(I*c*(b*x+a)^
p)^2-1/2*I*Pi*d^2*x*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)-1/6*I*e
^2*Pi*x^3*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)
```

Maxima [A] time = 1.10382, size = 184, normalized size = 1.64

$$-\frac{1}{18}\,bp\left(\frac{2\,b^2e^2x^3+3\left(3\,b^2de-abe^2\right)x^2+6\left(3\,b^2d^2-3\,abde+a^2e^2\right)x}{b^3}-\frac{6\left(3\,ab^2d^2-3\,a^2bde+a^3e^2\right)\log\left(bx+a\right)}{b^4}\right)+\frac{1}{3}\left(e^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/18*b*p*((2*b^2*e^2*x^3 + 3*(3*b^2*d*e - a*b*e^2)*x^2 + 6*(3*b^2*d^2 - 3*
a*b*d*e + a^2*e^2)*x)/b^3 - 6*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*log(b*x
 + a)/b^4) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x + a)^p*c)
```

Fricas [A] time = 2.01714, size = 369, normalized size = 3.29

$$-\frac{2\,b^3e^2px^3+3\left(3\,b^3de-ab^2e^2\right)px^2+6\left(3\,b^3d^2-3\,ab^2de+a^2be^2\right)px-6\left(b^3e^2px^3+3\,b^3depx^2+3\,b^3d^2px+\left(3\,ab^2d^2-3\,a\right.\right.}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/18*(2*b^3*e^2*p*x^3 + 3*(3*b^3*d*e - a*b^2*e^2)*p*x^2 + 6*(3*b^3*d^2 - 3
*a*b^2*d*e + a^2*b*e^2)*p*x - 6*(b^3*e^2*p*x^3 + 3*b^3*d*e*p*x^2 + 3*b^3*d^
2*p*x + (3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*p)*log(b*x + a) - 6*(b^3*e^2*
x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*log(c))/b^3
```

Sympy [A] time = 3.18163, size = 223, normalized size = 1.99

$$\begin{cases} \frac{a^3e^2p\log(a+bx)}{3b^3} - \frac{a^2dep\log(a+bx)}{b^2} - \frac{a^2e^2px}{3b^2} + \frac{ad^2p\log(a+bx)}{b} + \frac{adepx}{b} + \frac{ae^2px^2}{6b} + d^2px\log(a+bx) - d^2px + d^2x\log(c) + depx^2 \\ \left(d^2x + dex^2 + \frac{e^2x^3}{3}\right)\log(a^pc) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((a**3*e**2*p*log(a + b*x)/(3*b**3) - a**2*d*e*p*log(a + b*x)/b**2
 - a**2*e**2*p*x/(3*b**2) + a*d**2*p*log(a + b*x)/b + a*d*e*p*x/b + a*e**2*
p*x**2/(6*b) + d**2*p*x*log(a + b*x) - d**2*p*x + d**2*x*log(c) + d*e*p*x**
2*log(a + b*x) - d*e*p*x**2/2 + d*e*x**2*log(c) + e**2*p*x**3*log(a + b*x)/
3 - e**2*p*x**3/9 + e**2*x**3*log(c)/3, Ne(b, 0)), ((d**2*x + d*e*x**2 + e*
*2*x**3/3)*log(a**p*c), True))
```

Giac [B] time = 1.27441, size = 423, normalized size = 3.78

$$\frac{(bx+a)d^2p\log(bx+a)}{b} + \frac{(bx+a)^2dpe\log(bx+a)}{b^2} - \frac{2(bx+a)adpe\log(bx+a)}{b^2} - \frac{(bx+a)d^2p}{b} - \frac{(bx+a)^2dpe}{2b^2} + 2($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d^2*p*log(b*x + a)/b + (b*x + a)^2*d*p*e*log(b*x + a)/b^2 - 2*(b*
x + a)*a*d*p*e*log(b*x + a)/b^2 - (b*x + a)*d^2*p/b - 1/2*(b*x + a)^2*d*p*e
/b^2 + 2*(b*x + a)*a*d*p*e/b^2 + 1/3*(b*x + a)^3*p*e^2*log(b*x + a)/b^3 - (
b*x + a)^2*a*p*e^2*log(b*x + a)/b^3 + (b*x + a)*a^2*p*e^2*log(b*x + a)/b^3
+ (b*x + a)*d^2*log(c)/b + (b*x + a)^2*d*e*log(c)/b^2 - 2*(b*x + a)*a*d*e*l
og(c)/b^2 - 1/9*(b*x + a)^3*p*e^2/b^3 + 1/2*(b*x + a)^2*a*p*e^2/b^3 - (b*x
+ a)*a^2*p*e^2/b^3 + 1/3*(b*x + a)^3*e^2*log(c)/b^3 - (b*x + a)^2*a*e^2*log
(c)/b^3 + (b*x + a)*a^2*e^2*log(c)/b^3
```

3.178 $\int (d+ex)\log\left(c(a+bx)^p\right)\,dx$

Optimal. Leaf size=84

$$-\frac{p(bd-ae)^2\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{px(bd-ae)}{2b}-\frac{p(d+ex)^2}{4e}$$

```
[Out] -((b*d - a*e)*p*x)/(2*b) - (p*(d + e*x)^2)/(4*e) - ((b*d - a*e)^2*p*Log[a +
 b*x])/(2*b^2*e) + ((d + e*x)^2*Log[c*(a + b*x)^p])/(2*e)
```

Rubi [A] time = 0.0371142, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2395, 43}

$$-\frac{p(bd-ae)^2\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{px(bd-ae)}{2b}-\frac{p(d+ex)^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x)^p],x]
```

```
[Out] -((b*d - a*e)*p*x)/(2*b) - (p*(d + e*x)^2)/(4*e) - ((b*d - a*e)^2*p*Log[a +
 b*x])/(2*b^2*e) + ((d + e*x)^2*Log[c*(a + b*x)^p])/(2*e)
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\log\left(c(a+bx)^p\right)\,dx&=\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{(bp)\int\frac{(d+ex)^2}{a+bx}\,dx}{2e}\\&=\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{(bp)\int\left(\frac{e(bd-ae)}{b^2}+\frac{(bd-ae)^2}{b^2(a+bx)}+\frac{e(d+ex)}{b}\right)dx}{2e}\\&=-\frac{(bd-ae)px}{2b}-\frac{p(d+ex)^2}{4e}-\frac{(bd-ae)^2p\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}\end{aligned}$$

Mathematica [A] time = 0.0455267, size = 82, normalized size = 0.98

$$-\frac{a^2ep\log(a+bx)}{2b^2}+\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{b}+\frac{1}{2}ex^2\log\left(c(a+bx)^p\right)+\frac{aepx}{2b}-dpx-\frac{1}{4}epx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x)^p],x]
```

```
[Out] -(d*p*x) + (a*e*p*x)/(2*b) - (e*p*x^2)/4 - (a^2*e*p*Log[a + b*x])/(2*b^2) +
 (e*x^2*Log[c*(a + b*x)^p])/2 + (d*(a + b*x)*Log[c*(a + b*x)^p])/b
```

Maple [A] time = 0.091, size = 83, normalized size = 1.

$$d\ln\left(c\left(bx+a\right)^{p}\right)x-dpx+\frac{dpa\ln\left(bx+a\right)}{b}+\frac{x^{2}e\ln\left(c\mathrm{e}^{p\ln(bx+a)}\right)}{2}-\frac{epx^{2}}{4}-\frac{a^{2}pe\ln\left(bx+a\right)}{2b^{2}}+\frac{apex}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x+a)^p),x)
```

```
[Out] d*ln(c*(b*x+a)^p)*x-d*p*x+d/b*p*a*ln(b*x+a)+1/2*x^2*e*ln(c*exp(p*ln(b*x+a))
)-1/4*e*p*x^2-1/2*p*a^2*e/b^2*ln(b*x+a)+1/2*a*p*e/b*x
```

Maxima [A] time = 1.10274, size = 100, normalized size = 1.19

$$-\frac{1}{4}bp\left(\frac{bex^{2}+2\left(2bd-ae\right)x}{b^{2}}-\frac{2\left(2abd-a^{2}e\right)\log\left(bx+a\right)}{b^{3}}\right)+\frac{1}{2}\left(ex^{2}+2dx\right)\log\left(\left(bx+a\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*((b*e*x^2 + 2*(2*b*d - a*e)*x)/b^2 - 2*(2*a*b*d - a^2*e)*log(b*x +
 a)/b^3) + 1/2*(e*x^2 + 2*d*x)*log((b*x + a)^p*c)
```

Fricas [A] time = 2.02251, size = 205, normalized size = 2.44

$$-\frac{b^{2}epx^{2}+2\left(2b^{2}d-abe\right)px-2\left(b^{2}epx^{2}+2b^{2}dpx+\left(2abd-a^{2}e\right)p\right)\log\left(bx+a\right)-2\left(b^{2}ex^{2}+2b^{2}dx\right)\log\left(c\right)}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*e*p*x^2 + 2*(2*b^2*d - a*b*e)*p*x - 2*(b^2*e*p*x^2 + 2*b^2*d*p*x
+ (2*a*b*d - a^2*e)*p)*log(b*x + a) - 2*(b^2*e*x^2 + 2*b^2*d*x)*log(c))/b^2
```

Sympy [A] time = 1.42708, size = 116, normalized size = 1.38

$$\begin{cases}-\frac{a^{2}ep\log\left(a+bx\right)}{2b^{2}}+\frac{adp\log\left(a+bx\right)}{b}+\frac{aepx}{2b}+dpx\log\left(a+bx\right)-dpx+dx\log\left(c\right)+\frac{epx^{2}\log\left(a+bx\right)}{2}-\frac{epx^{2}}{4}+\frac{ex^{2}\log\left(c\right)}{2} & \text{for}\: b\neq 0\\\left(dx+\frac{ex^{2}}{2}\right)\log\left(a^{p}c\right) & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**2*e*p*log(a + b*x)/(2*b**2) + a*d*p*log(a + b*x)/b + a*e*p*x
/(2*b) + d*p*x*log(a + b*x) - d*p*x + d*x*log(c) + e*p*x**2*log(a + b*x)/2
- e*p*x**2/4 + e*x**2*log(c)/2, Ne(b, 0)), ((d*x + e*x**2/2)*log(a**p*c), T
rue))
```

Giac [A] time = 1.27378, size = 192, normalized size = 2.29

$$\frac{(bx+a)dp\log(bx+a)}{b}+\frac{(bx+a)^2pe\log(bx+a)}{2\,b^2}-\frac{(bx+a)ape\log(bx+a)}{b^2}-\frac{(bx+a)dp}{b}-\frac{(bx+a)^2pe}{4\,b^2}+\frac{(bx+a)ape}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d*p*log(b*x + a)/b + 1/2*(b*x + a)^2*p*e*log(b*x + a)/b^2 - (b*x
+ a)*a*p*e*log(b*x + a)/b^2 - (b*x + a)*d*p/b - 1/4*(b*x + a)^2*p*e/b^2 + (
b*x + a)*a*p*e/b^2 + (b*x + a)*d*log(c)/b + 1/2*(b*x + a)^2*e*log(c)/b^2 -
(b*x + a)*a*e*log(c)/b^2
```

3.179 $\int \log \left(c(a+bx)^p\right) dx$

Optimal. Leaf size=24

$$\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}-px$$

```
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Rubi [A] time = 0.0090786, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2389, 2295}

$$\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}-px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p],x]
```

```
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(c(a+bx)^p\right) dx &= \frac{\text{Subst}\left(\int \log\left(cx^p\right) dx, x, a+bx\right)}{b}\\ &= -px+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0056086, size = 24, normalized size = 1.

$$\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}-px$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p],x]
```

```
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Maple [A] time = 0.069, size = 30, normalized size = 1.3

$$\ln \left(c \left(bx+a\right)^{p}\right) x - px + \frac{ap \ln \left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p),x)
```

```
[Out] ln(c*(b*x+a)^p)*x-p*x+1/b*p*a*ln(b*x+a)
```

Maxima [A] time = 1.0432, size = 47, normalized size = 1.96

$$-bp\left(\frac{x}{b} - \frac{a \log \left(bx+a\right)}{b^{2}}\right) + x \log \left(\left(bx+a\right)^{p} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -b*p*(x/b - a*log(b*x + a)/b^2) + x*log((b*x + a)^p*c)
```

Fricas [A] time = 1.90589, size = 73, normalized size = 3.04

$$-\frac{bpx - bx \log \left(c\right) - \left(bpx + ap\right) \log \left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -(b*p*x - b*x*log(c) - (b*p*x + a*p)*log(b*x + a))/b
```

Sympy [A] time = 0.539807, size = 37, normalized size = 1.54

$$\begin{cases} \frac{ap \log \left(a+bx\right)}{b} + px \log \left(a+bx\right) - px + x \log \left(c\right) & \text{for}\: b \neq 0 \\ x \log \left(a^{p} c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((a*p*log(a + b*x)/b + p*x*log(a + b*x) - p*x + x*log(c), Ne(b, 0)
), (x*log(a**p*c), True))
```

Giac [A] time = 1.18071, size = 53, normalized size = 2.21

$$\frac{\left(bx+a\right)p \log \left(bx+a\right)}{b} - \frac{\left(bx+a\right)p}{b} + \frac{\left(bx+a\right) \log \left(c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*p*log(b*x + a)/b - (b*x + a)*p/b + (b*x + a)*log(c)/b
```

3.180 $\int \frac{\log(c(a+bx)^p)}{d+ex} dx$

Optimal. Leaf size=58

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rubi [A] time = 0.0504011, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{d+ex} dx &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx} dx}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x} dx,x,a+bx\right)}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}
\end{aligned}$$

Mathematica [A] time = 0.0035091, size = 57, normalized size = 0.98

$$\frac{p\text{PolyLog}\left(2,\frac{e(a+bx)}{ae-bd}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

Maple [C] time = 0.633, size = 242, normalized size = 4.2

$$\frac{\ln(ex+d)\ln\left((bx+a)^p\right)}{e}-\frac{p}{e}\text{dilog}\left(\frac{b(ex+d)+ae-bd}{ae-bd}\right)-\frac{p\ln(ex+d)}{e}\ln\left(\frac{b(ex+d)+ae-bd}{ae-bd}\right)-\frac{\frac{i}{2}\ln(ex+d)\pi\,\text{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x+a)^p)-1/e*p*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-1/e*p*
ln(e*x+d)*ln((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c)*
csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c)*csgn(I
*c*(b*x+a)^p)^2+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^
2-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^3+ln(e*x+d)/e*ln(c)
```

Maxima [B] time = 1.06048, size = 159, normalized size = 2.74

$$\frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\text{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e}-\frac{p\log(bx+a)\log(ex+d)}{e}+\frac{\log\left((bx+a)^pc\right)\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e*x
 + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((bx+a)^pc\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx\right)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((bx+a)^{p}c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

3.181 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^2} dx$

Optimal. Leaf size=68

$$-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}+\frac{bp\log(a+bx)}{e(bd-ae)}-\frac{bp\log(d+ex)}{e(bd-ae)}$$

```
[Out] (b*p*Log[a + b*x])/(e*(b*d - a*e)) - Log[c*(a + b*x)^p]/(e*(d + e*x)) - (b*
p*Log[d + e*x])/(e*(b*d - a*e))
```

Rubi [A] time = 0.0274055, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2395, 36, 31}

$$-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}+\frac{bp\log(a+bx)}{e(bd-ae)}-\frac{bp\log(d+ex)}{e(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

```
[Out] (b*p*Log[a + b*x])/(e*(b*d - a*e)) - Log[c*(a + b*x)^p]/(e*(d + e*x)) - (b*
p*Log[d + e*x])/(e*(b*d - a*e))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx &= -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}+\frac{(bp)\int \frac{1}{(a+bx)(d+ex)}\,dx}{e}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}-\frac{(bp)\int \frac{1}{d+ex}\,dx}{bd-ae}+\frac{\left(b^2p\right)\int \frac{1}{a+bx}\,dx}{e(bd-ae)}\\
&= \frac{bp\log(a+bx)}{e(bd-ae)}-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}-\frac{bp\log(d+ex)}{e(bd-ae)}
\end{aligned}$$

Mathematica [A] time = 0.0482561, size = 52, normalized size = 0.76

$$\frac{\frac{bp(\log(a+bx)-\log(d+ex))}{bd-ae}-\frac{\log(c(a+bx)^p)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

```
[Out] (-(Log[c*(a + b*x)^p]/(d + e*x)) + (b*p*(Log[a + b*x] - Log[d + e*x]))/(b*d
 - a*e))/e
```

Maple [C] time = 0.358, size = 329, normalized size = 4.8

$$-\frac{\ln\left((bx+a)^p\right)}{(ex+d)\,e}-\frac{-i\pi\,ae\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(i\,(bx+a)^p\right)\mathrm{csgn}\left(ic\,(bx+a)^p\right)+i\pi\,ae\mathrm{csgn}\,(ic)\left(\mathrm{csgn}\left(ic\,(bx+a)^p\right)\right)^2+i\pi\,ae\mathrm{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^2,x)
```

```
[Out] -1/e/(e*x+d)*ln((b*x+a)^p)-1/2*(-I*Pi*a*e*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(
I*c*(b*x+a)^p)+I*Pi*a*e*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2+I*Pi*a*e*csgn(I*(b*
x+a)^p)*csgn(I*c*(b*x+a)^p)^2-I*Pi*a*e*csgn(I*c*(b*x+a)^p)^3+I*Pi*b*d*csgn(
I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)-I*Pi*b*d*csgn(I*c)*csgn(I*c*(b*x
+a)^p)^2-I*Pi*b*d*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+I*Pi*b*d*csgn(I*c
*(b*x+a)^p)^3-2*ln(-e*x-d)*b*e*p*x+2*ln(b*x+a)*b*e*p*x-2*ln(-e*x-d)*b*d*p+2
*ln(b*x+a)*b*d*p+2*ln(c)*a*e-2*ln(c)*b*d)/(e*x+d)/e/(a*e-b*d)
```

Maxima [A] time = 1.01787, size = 88, normalized size = 1.29

$$\frac{bp\left(\frac{\log(bx+a)}{bd-ae}-\frac{\log(ex+d)}{bd-ae}\right)}{e}-\frac{\log\left((bx+a)^pc\right)}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)/(b*d - a*e) - log(e*x + d)/(b*d - a*e))/e - log((b*x + a)
^p*c)/((e*x + d)*e)
```

Fricas [A] time = 1.96902, size = 176, normalized size = 2.59

$$\frac{\left(bepx+aep\right)\log\left(bx+a\right)-\left(bepx+bdp\right)\log\left(ex+d\right)-\left(bd-ae\right)\log\left(c\right)}{bd^2e-ade^2+\left(bde^2-ae^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] ((b*e*p*x + a*e*p)*log(b*x + a) - (b*e*p*x + b*d*p)*log(e*x + d) - (b*d - a
*e)*log(c))/(b*d^2*e - a*d*e^2 + (b*d*e^2 - a*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19767, size = 123, normalized size = 1.81

$$\frac{bpxe\log(bx+a) - bpxe\log(xe+d) + ape\log(bx+a) - bdp\log(xe+d) - bd\log(c) + ae\log(c)}{bdxe^2 + bd^2e - axe^3 - ade^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (b*p*x*e*log(b*x + a) - b*p*x*e*log(x*e + d) + a*p*e*log(b*x + a) - b*d*p*l
og(x*e + d) - b*d*log(c) + a*e*log(c))/(b*d*x*e^2 + b*d^2*e - a*x*e^3 - a*d
*e^2)
```

3.182 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx$

Optimal. Leaf size=105

$$\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}+\frac{bp}{2e(d+ex)(bd-ae)}$$

```
[Out] (b*p)/(2*e*(b*d - a*e)*(d + e*x)) + (b^2*p*Log[a + b*x])/(2*e*(b*d - a*e)^2
) - Log[c*(a + b*x)^p]/(2*e*(d + e*x)^2) - (b^2*p*Log[d + e*x])/(2*e*(b*d -
 a*e)^2)
```

Rubi [A] time = 0.0588471, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 44}

$$\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}+\frac{bp}{2e(d+ex)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```


```
[Out] (b*p)/(2*e*(b*d - a*e)*(d + e*x)) + (b^2*p*Log[a + b*x])/(2*e*(b*d - a*e)^2
) - Log[c*(a + b*x)^p]/(2*e*(d + e*x)^2) - (b^2*p*Log[d + e*x])/(2*e*(b*d -
 a*e)^2)
```


Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^3}\,dx &= -\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int \frac{1}{(a+bx)(d+ex)^2}\,dx}{2e}\\ &= -\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int \left(\frac{b^2}{(bd-ae)^2(a+bx)}-\frac{e}{(bd-ae)(d+ex)^2}-\frac{be}{(bd-ae)^2(d+ex)}\right)dx}{2e}\\ &= \frac{bp}{2e(bd-ae)(d+ex)}+\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}\end{aligned}$$

Mathematica [A] time = 0.0818907, size = 80, normalized size = 0.76

$$\frac{\frac{bp(d+ex)(b(d+ex)\log(a+bx)-ae-b(d+ex)\log(d+ex)+bd)}{(bd-ae)^2}-\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

```
[Out] (-Log[c*(a + b*x)^p] + (b*p*(d + e*x)*(b*d - a*e + b*(d + e*x)*Log[a + b*x]
 - b*(d + e*x)*Log[d + e*x]))/(b*d - a*e)^2)/(2*e*(d + e*x)^2)
```

Maple [C] time = 0.375, size = 582, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x+a)^p)-1/4*(2*I*Pi*a*b*d*e*csgn(I*c)*csgn(I*(b*x+a)
^p)*csgn(I*c*(b*x+a)^p)-I*Pi*b^2*d^2*csgn(I*c*(b*x+a)^p)^3-I*Pi*a^2*e^2*csg
n(I*c*(b*x+a)^p)^3-2*ln(-b*x-a)*b^2*e^2*p*x^2+2*ln(e*x+d)*b^2*e^2*p*x^2-4*l
n(c)*a*b*d*e-2*I*Pi*a*b*d*e*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2+2*ln(c)*a^2*e^2
-2*ln(-b*x-a)*b^2*d^2*p+2*ln(e*x+d)*b^2*d^2*p-2*b^2*d^2*p+2*a*b*d*p*e-2*I*P
i*a*b*d*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+2*a*b*e^2*p*x-2*b^2*d*e*p
*x+2*I*Pi*a*b*d*e*csgn(I*c*(b*x+a)^p)^3-I*Pi*b^2*d^2*csgn(I*c)*csgn(I*(b*x+
a)^p)*csgn(I*c*(b*x+a)^p)+I*Pi*a^2*e^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p
)^2+2*ln(c)*b^2*d^2+I*Pi*b^2*d^2*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2-I*Pi*a^2*e
^2*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)-4*ln(-b*x-a)*b^2*d*e*p*x
+4*ln(e*x+d)*b^2*d*e*p*x+I*Pi*b^2*d^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)
^2+I*Pi*a^2*e^2*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2)/(e*x+d)^2/(a*e-b*d)^2/e
```

Maxima [A] time = 1.07574, size = 162, normalized size = 1.54

$$\frac{bp\left(\frac{b\log(bx+a)}{b^2d^2-2\,abde+a^2e^2}-\frac{b\log(ex+d)}{b^2d^2-2\,abde+a^2e^2}+\frac{1}{bd^2-ade+\left(bde-ae^2\right)x}\right)}{2\,e}-\frac{\log\left((bx+a)^pc\right)}{2\,(ex+d)^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(b*log(b*x + a)/(b^2*d^2 - 2*a*b*d*e + a^2*e^2) - b*log(e*x + d)/(b
^2*d^2 - 2*a*b*d*e + a^2*e^2) + 1/(b*d^2 - a*d*e + (b*d*e - a*e^2)*x))/e -
1/2*log((b*x + a)^p*c)/((e*x + d)^2*e)
```

Fricas [B] time = 1.99638, size = 489, normalized size = 4.66

$$\frac{\left(b^2de-abe^2\right)px+\left(b^2d^2-abde\right)p+\left(b^2e^2px^2+2\,b^2depx+\left(2\,abde-a^2e^2\right)p\right)\log(bx+a)-\left(b^2e^2px^2+2\,b^2depx+b^2d\cdots\right.}{2\left(b^2d^4e-2\,abd^3e^2+a^2d^2e^3+\left(b^2d^2e^3-2\,abde^4+a^2e^5\right)x^2+2\left(b^2d^3e^2-2\,abd^2e^3\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*d*e - a*b*e^2)*p*x + (b^2*d^2 - a*b*d*e)*p + (b^2*e^2*p*x^2 + 2*b
^2*d*e*p*x + (2*a*b*d*e - a^2*e^2)*p)*log(b*x + a) - (b^2*e^2*p*x^2 + 2*b^2
*d*e*p*x + b^2*d^2*p)*log(e*x + d) - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)*log(c)
)/(b^2*d^4*e - 2*a*b*d^3*e^2 + a^2*d^2*e^3 + (b^2*d^2*e^3 - 2*a*b*d*e^4 + a
^2*e^5)*x^2 + 2*(b^2*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d*e^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1903, size = 359, normalized size = 3.42

$$\frac{b^2px^2e^2\log(bx+a)+2\,b^2dpxe\log(bx+a)-b^2px^2e^2\log(xe+d)-2\,b^2dpxe\log(xe+d)+b^2dpxe+2\,abdpe\log(bx+a}{2\left(b^2d^2x^2e^3+2\,b^2d^3xe^2+b^2d^4e-2\,abdx^2e^4-4\,abd^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*p*x^2*e^2*log(b*x + a) + 2*b^2*d*p*x*e*log(b*x + a) - b^2*p*x^2*e^
2*log(x*e + d) - 2*b^2*d*p*x*e*log(x*e + d) + b^2*d*p*x*e + 2*a*b*d*p*e*log
(b*x + a) - b^2*d^2*p*log(x*e + d) + b^2*d^2*p - a*b*p*x*e^2 - a*b*d*p*e -
a^2*p*e^2*log(b*x + a) - b^2*d^2*log(c) + 2*a*b*d*e*log(c) - a^2*e^2*log(c)
)/(b^2*d^2*x^2*e^3 + 2*b^2*d^3*x*e^2 + b^2*d^4*e - 2*a*b*d*x^2*e^4 - 4*a*b*
d^2*x*e^3 - 2*a*b*d^3*e^2 + a^2*x^2*e^5 + 2*a^2*d*x*e^4 + a^2*d^2*e^3)
```

3.183 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} \, dx$

Optimal. Leaf size=133

$$\frac{b^2p}{3e(d+ex)(bd-ae)^2}+\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{bp}{6e(d+ex)^2(bd-ae)}$$

```
[Out] (b*p)/(6*e*(b*d - a*e)*(d + e*x)^2) + (b^2*p)/(3*e*(b*d - a*e)^2*(d + e*x))
 + (b^3*p*Log[a + b*x])/(3*e*(b*d - a*e)^3) - Log[c*(a + b*x)^p]/(3*e*(d +
e*x)^3) - (b^3*p*Log[d + e*x])/(3*e*(b*d - a*e)^3)
```

Rubi [A] time = 0.0773629, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 44}

$$\frac{b^2p}{3e(d+ex)(bd-ae)^2}+\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{bp}{6e(d+ex)^2(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (b*p)/(6*e*(b*d - a*e)*(d + e*x)^2) + (b^2*p)/(3*e*(b*d - a*e)^2*(d + e*x))
 + (b^3*p*Log[a + b*x])/(3*e*(b*d - a*e)^3) - Log[c*(a + b*x)^p]/(3*e*(d +
e*x)^3) - (b^3*p*Log[d + e*x])/(3*e*(b*d - a*e)^3)
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^4}\,dx &= -\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{(bp)\int\frac{1}{(a+bx)(d+ex)^3}\,dx}{3e}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{(bp)\int\left(\frac{b^3}{(bd-ae)^3(a+bx)}-\frac{e}{(bd-ae)(d+ex)^3}-\frac{be}{(bd-ae)^2(d+ex)^2}-\frac{b^2e}{(bd-ae)^3(d+ex)}\right)dx}{3e}\\
&= \frac{bp}{6e(bd-ae)(d+ex)^2}+\frac{b^2p}{3e(bd-ae)^2(d+ex)}+\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}
\end{aligned}$$

Mathematica [A] time = 0.131363, size = 105, normalized size = 0.79

$$\frac{\frac{bp(d+ex)\left(2b^2(d+ex)^2\log(a+bx)+(bd-ae)(-ae+3bd+2bex)-2b^2(d+ex)^2\log(d+ex)\right)}{(bd-ae)^3}-2\log\left(c(a+bx)^p\right)}{6e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (-2*Log[c*(a + b*x)^p] + (b*p*(d + e*x)*((b*d - a*e)*(3*b*d - a*e + 2*b*e*x
) + 2*b^2*(d + e*x)^2*Log[a + b*x] - 2*b^2*(d + e*x)^2*Log[d + e*x]))/(b*d
- a*e)^3)/(6*e*(d + e*x)^3)
```

Maple [C] time = 0.393, size = 873, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^4,x)
```

```
[Out] -1/3/e/(e*x+d)^3*ln((b*x+a)^p)+1/6*(-3*I*Pi*a^2*b*d*e^2*csgn(I*c)*csgn(I*(b
*x+a)^p)*csgn(I*c*(b*x+a)^p)+3*I*Pi*a*b^2*d^2*e*csgn(I*c)*csgn(I*(b*x+a)^p)
*csgn(I*c*(b*x+a)^p)-2*ln(b*x+a)*b^3*d^3*p+6*a*b^2*d*e^2*p*x+3*I*Pi*a^2*b*d
*e^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+2*ln(-e*x-d)*b^3*d^3*p+2*ln(c)
*b^3*d^3-2*ln(c)*a^3*e^3-a^2*b*d*e^2*p-3*b^3*d^3*p+4*a*b^2*d^2*p*e+2*a*b^2*
e^3*p*x^2-2*b^3*d*e^2*p*x^2-a^2*b*e^3*p*x-2*ln(b*x+a)*b^3*e^3*p*x^3+2*ln(-e
*x-d)*b^3*e^3*p*x^3+6*ln(c)*a^2*b*d*e^2-6*ln(c)*a*b^2*d^2*e-5*b^3*d^2*e*p*x
-3*I*Pi*a*b^2*d^2*e*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2-3*I*Pi*a*b^2*d^2*e*csgn
(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+3*I*Pi*a^2*b*d*e^2*csgn(I*c)*csgn(I*c*(
b*x+a)^p)^2+3*I*Pi*a*b^2*d^2*e*csgn(I*c*(b*x+a)^p)^3-I*Pi*b^3*d^3*csgn(I*c)
*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)+I*Pi*a^3*e^3*csgn(I*c)*csgn(I*(b*x+a
)^p)*csgn(I*c*(b*x+a)^p)-I*Pi*a^3*e^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)
^2+I*Pi*a^3*e^3*csgn(I*c*(b*x+a)^p)^3-I*Pi*b^3*d^3*csgn(I*c*(b*x+a)^p)^3-3*
I*Pi*a^2*b*d*e^2*csgn(I*c*(b*x+a)^p)^3-I*Pi*a^3*e^3*csgn(I*c)*csgn(I*c*(b*x
+a)^p)^2-6*ln(b*x+a)*b^3*d*e^2*p*x^2+6*ln(-e*x-d)*b^3*d*e^2*p*x^2-6*ln(b*x+
a)*b^3*d^2*e*p*x+6*ln(-e*x-d)*b^3*d^2*e*p*x+I*Pi*b^3*d^3*csgn(I*c)*csgn(I*c
*(b*x+a)^p)^2+I*Pi*b^3*d^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2)/(e*x+d)
^3/(a^2*e^2-2*a*b*d*e+b^2*d^2)/(a*e-b*d)/e
```

Maxima [A] time = 1.10358, size = 313, normalized size = 2.35

$$\frac{\left(\frac{2\,b^{2}\log(bx+a)}{b^{3}d^{3}-3\,ab^{2}d^{2}e+3\,a^{2}bde^{2}-a^{3}e^{3}}-\frac{2\,b^{2}\log(ex+d)}{b^{3}d^{3}-3\,ab^{2}d^{2}e+3\,a^{2}bde^{2}-a^{3}e^{3}}+\frac{2\,bex+3\,bd-ae}{b^{2}d^{4}-2\,abd^{3}e+a^{2}d^{2}e^{2}+\left(b^{2}d^{2}e^{2}-2\,abde^{3}+a^{2}e^{4}\right)x^{2}+2\left(b^{2}d^{3}e-2\,abd^{2}e^{2}+a^{2}de^{3}\right)x}\right)bp}{6\,e}-\frac{\log}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*log(b*x + a)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)
 - 2*b^2*log(e*x + d)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) +
 (2*b*e*x + 3*b*d - a*e)/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^
2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*
x))*b*p/e - 1/3*log((b*x + a)^p*c)/((e*x + d)^3*e)
```

Fricas [B] time = 2.37855, size = 894, normalized size = 6.72

$$\frac{2\left(b^3de^2 - ab^2e^3\right)px^2 + \left(5\,b^3d^2e - 6\,ab^2de^2 + a^2be^3\right)px + \left(3\,b^3d^3 - 4\,ab^2d^2e + a^2bde^2\right)p + 2\left(b^3e^3px^3 + 3\,b^3de^2px^2 + 3\,b\right.}{6\left(b^3d^6e - 3\,ab^2d^5e^2 + 3\,a^2bd^4e^3 - a^3d^3e^4 + \left(b^3d^3e^4 - 3\,ab^2d^2e^5 + 3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(b^3*d*e^2 - a*b^2*e^3)*p*x^2 + (5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*b
*e^3)*p*x + (3*b^3*d^3 - 4*a*b^2*d^2*e + a^2*b*d*e^2)*p + 2*(b^3*e^3*p*x^3
+ 3*b^3*d*e^2*p*x^2 + 3*b^3*d^2*e*p*x + (3*a*b^2*d^2*e - 3*a^2*b*d*e^2 + a^
3*e^3)*p)*log(b*x + a) - 2*(b^3*e^3*p*x^3 + 3*b^3*d*e^2*p*x^2 + 3*b^3*d^2*e
*p*x + b^3*d^3*p)*log(e*x + d) - 2*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2
 - a^3*e^3)*log(c))/(b^3*d^6*e - 3*a*b^2*d^5*e^2 + 3*a^2*b*d^4*e^3 - a^3*d^
3*e^4 + (b^3*d^3*e^4 - 3*a*b^2*d^2*e^5 + 3*a^2*b*d*e^6 - a^3*e^7)*x^3 + 3*(
b^3*d^4*e^3 - 3*a*b^2*d^3*e^4 + 3*a^2*b*d^2*e^5 - a^3*d*e^6)*x^2 + 3*(b^3*d
^5*e^2 - 3*a*b^2*d^4*e^3 + 3*a^2*b*d^3*e^4 - a^3*d^2*e^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22556, size = 668, normalized size = 5.02

$$\frac{2\,b^3px^3e^3\log\left(bx+a\right) + 6\,b^3dpx^2e^2\log\left(bx+a\right) + 6\,b^3d^2pxe\log\left(bx+a\right) - 2\,b^3px^3e^3\log\left(xe+d\right) - 6\,b^3dpx^2e^2\log\left(xe+\right.}{6\left(b^3d^3x^3e^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*p*x^3*e^3*log(b*x + a) + 6*b^3*d*p*x^2*e^2*log(b*x + a) + 6*b^3*
d^2*p*x*e*log(b*x + a) - 2*b^3*p*x^3*e^3*log(x*e + d) - 6*b^3*d*p*x^2*e^2*l
og(x*e + d) - 6*b^3*d^2*p*x*e*log(x*e + d) + 2*b^3*d*p*x^2*e^2 + 5*b^3*d^2*
p*x*e + 6*a*b^2*d^2*p*e*log(b*x + a) - 2*b^3*d^3*p*log(x*e + d) + 3*b^3*d^3
*p - 2*a*b^2*p*x^2*e^3 - 6*a*b^2*d*p*x*e^2 - 4*a*b^2*d^2*p*e - 6*a^2*b*d*p*
e^2*log(b*x + a) - 2*b^3*d^3*log(c) + 6*a*b^2*d^2*e*log(c) + a^2*b*p*x*e^3
+ a^2*b*d*p*e^2 + 2*a^3*p*e^3*log(b*x + a) - 6*a^2*b*d*e^2*log(c) + 2*a^3*e
^3*log(c))/(b^3*d^3*x^3*e^4 + 3*b^3*d^4*x^2*e^3 + 3*b^3*d^5*x*e^2 + b^3*d^6
*e - 3*a*b^2*d^2*x^3*e^5 - 9*a*b^2*d^3*x^2*e^4 - 9*a*b^2*d^4*x*e^3 - 3*a*b^
2*d^5*e^2 + 3*a^2*b*d*x^3*e^6 + 9*a^2*b*d^2*x^2*e^5 + 9*a^2*b*d^3*x*e^4 + 3
*a^2*b*d^4*e^3 - a^3*x^3*e^7 - 3*a^3*d*x^2*e^6 - 3*a^3*d^2*x*e^5 - a^3*d^3*
e^4)
```

3.184 $\int (d+ex)^3 \log \left(c \left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=178

$$-\frac{p\left(a^2e^4-6abd^2e^2+b^2d^4\right)\log\left(a+bx^2\right)}{4b^2e}+\frac{2\sqrt{a}dp\left(bd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{(d+ex)^4\log\left(c\left(a+bx^2\right)^p\right)}{4e}-\frac{epx^2\left(6bd^2-\right.}{4b}$$

```
[Out] (-2*d*(b*d^2 - a*e^2)*p*x)/b - (e*(6*b*d^2 - a*e^2)*p*x^2)/(4*b) - (2*d*e^2
*p*x^3)/3 - (e^3*p*x^4)/8 + (2*Sqrt[a]*d*(b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(3/2) - ((b^2*d^4 - 6*a*b*d^2*e^2 + a^2*e^4)*p*Log[a + b*x^2
])/(4*b^2*e) + ((d + e*x)^4*Log[c*(a + b*x^2)^p])/(4*e)
```

Rubi [A] time = 0.163456, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2463, 801, 635, 205, 260}

$$-\frac{p\left(a^2e^4-6abd^2e^2+b^2d^4\right)\log\left(a+bx^2\right)}{4b^2e}+\frac{2\sqrt{a}dp\left(bd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{(d+ex)^4\log\left(c\left(a+bx^2\right)^p\right)}{4e}-\frac{epx^2\left(6bd^2-\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*d*(b*d^2 - a*e^2)*p*x)/b - (e*(6*b*d^2 - a*e^2)*p*x^2)/(4*b) - (2*d*e^2
*p*x^3)/3 - (e^3*p*x^4)/8 + (2*Sqrt[a]*d*(b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(3/2) - ((b^2*d^4 - 6*a*b*d^2*e^2 + a^2*e^4)*p*Log[a + b*x^2
])/(4*b^2*e) + ((d + e*x)^4*Log[c*(a + b*x^2)^p])/(4*e)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right) dx = \frac{(d+ex)^4 \log\left(c\left(a+bx^2\right)^p\right)}{4e} - \frac{(bp)\int \frac{x(d+ex)^4}{a+bx^2}dx}{2e}$$

$$= \frac{(d+ex)^4 \log\left(c\left(a+bx^2\right)^p\right)}{4e} - \frac{(bp)\int \left(\frac{4de\left(bd^2-ae^2\right)}{b^2} + \frac{e^2\left(6bd^2-ae^2\right)x}{b^2} + \frac{4de^3x^2}{b} + \frac{e^4x^3}{b} - \ldots\right.}{2e}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{(d+ex)^4\log\left(c\left(a\ldots\right.\right.}{4e}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{(d+ex)^4\log\left(c\left(a\ldots\right.\right.}{4e}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{2\sqrt{a}d\left(bd^2-ae^2\right)\ldots}{b^{3/2}}$$

Mathematica [A] time = 0.731074, size = 249, normalized size = 1.4

$$\frac{-6p\left(a^2e^4+4\sqrt{-a}b^{3/2}d^3e-6abd^2e^2+4(-a)^{3/2}\sqrt{b}de^3+b^2d^4\right)\log\left(\sqrt{-a}-\sqrt{b}x\right)-6p\left(a^2e^4-4\sqrt{-a}b^{3/2}d^3e-6abd^2e^2+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-6*(b^2*d^4 + 4*Sqrt[-a]*b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*(-a)^(3/2)*Sqrt
[b]*d*e^3 + a^2*e^4)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 6*(b^2*d^4 - 4*Sqrt[-a]*
b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*Sqrt[-a]*a*Sqrt[b]*d*e^3 + a^2*e^4)*p*Log
[Sqrt[-a] + Sqrt[b]*x] + b*(6*a*e^3*p*x*(8*d + e*x) - b*e*p*x*(48*d^3 + 36*
d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3) + 6*b*(d + e*x)^4*Log[c*(a + b*x^2)^p])
)/(24*b^2*e)
```

Maple [C] time = 0.796, size = 1330, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x^2+a)^p),x)
```

```
[Out] -2*d^3*p*x-1/8*I*e^3*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(
I*c)+1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/2*I*e
^2*Pi*d*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/4*I*e*Pi*d^2*x^2*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^2+a)^p)^
2*csgn(I*c)-1/2*I*Pi*d^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c
)*x+ln(c)*d^3*x+1/4*e^3*ln(c)*x^4-1/4/e*p*ln(-a^2*d*e^3+a*b*d^3*e-(-a^3*b*d
^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*d^4-1/4/e*p*ln(-a^2*d*e^3+
a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*d^4+3/2
*e*ln(c)*d^2*x^2+e^2*ln(c)*d*x^3-1/8*e^3*p*x^4-2/3*d*e^2*p*x^3-3/2*d^2*e*p*
```

```
x^2+1/4*(e*x+d)^4/e*ln((b*x^2+a)^p)+2/b*a*d*p*e^2*x-1/2*I*e^2*Pi*d*x^3*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*(
b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/4/b^2*e^3*p*ln(-a^2*d*e^3+a*b
*d^3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a^2-1/4/b^
2*e^3*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6
*e^2)^(1/2)*x)*a^2+1/b^2/e*p*ln(-a^2*d*e^3+a*b*d^3*e-(-a^3*b*d^2*e^6+2*a^2*
b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3
*d^6*e^2)^(1/2)-1/b^2/e*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^2
*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^
6*e^2)^(1/2)-1/2*I*Pi*d^3*csgn(I*c*(b*x^2+a)^p)^3*x-1/8*I*e^3*Pi*x^4*csgn(I
*c*(b*x^2+a)^p)^3+1/4/b*a*e^3*p*x^2+1/8*I*e^3*Pi*x^4*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)^2+1/8*I*e^3*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/
2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^2+a)^p)^3-3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^2
+a)^p)^3+1/2*I*Pi*d^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*x+1/2*I*P
i*d^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*x+3/2/b*e*p*ln(-a^2*d*e^3+a*b*d^3*e
-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a*d^2+3/2/b*e*p*
ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1
/2)*x)*a*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09646, size = 1044, normalized size = 5.87

$$\left[-\frac{3\,b^2e^3px^4+16\,b^2de^2px^3+6\left(6\,b^2d^2e-abe^3\right)px^2-24\left(b^2d^3-abde^2\right)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+48\left(b^2d^3-abde^2\right)px-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 + 6*(6*b^2*d^2*e - a*b*e^3)*p*
x^2 - 24*(b^2*d^3 - a*b*d*e^2)*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4*b^2*
d*e^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*e^3)*p
)*log(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b
^2*d^3*x)*log(c))/b^2, -1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 + 6*(6*b
^2*d^2*e - a*b*e^3)*p*x^2 - 48*(b^2*d^3 - a*b*d*e^2)*p*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4*b^2*d*e
^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*e^3)*p)*l
og(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b^2*
d^3*x)*log(c))/b^2]
```

Sympy [A] time = 82.9278, size = 422, normalized size = 2.37

$$\begin{cases} -\frac{ia^{\frac{3}{2}}de^2p\log\left(a+bx^2\right)}{b^2\sqrt{\frac{1}{b}}}+\frac{2ia^{\frac{3}{2}}de^2p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b^2\sqrt{\frac{1}{b}}}+\frac{i\sqrt{a}d^3p\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}}-\frac{2i\sqrt{a}d^3p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}}-\frac{a^2e^3p\log\left(a+bx^2\right)}{4b^2}+\frac{3ad^2ep\log\left(a+bx^2\right)}{2b} \\ \left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)\log\left(a^pc\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-I*a**(3/2)*d*e**2*p*log(a + b*x**2)/(b**2*sqrt(1/b)) + 2*I*a**(
3/2)*d*e**2*p*log(-I*sqrt(a)*sqrt(1/b) + x)/(b**2*sqrt(1/b)) + I*sqrt(a)*d*
*3*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*d**3*p*log(-I*sqrt(a)*sqrt
(1/b) + x)/(b*sqrt(1/b)) - a**2*e**3*p*log(a + b*x**2)/(4*b**2) + 3*a*d**2*
e*p*log(a + b*x**2)/(2*b) + 2*a*d*e**2*p*x/b + a*e**3*p*x**2/(4*b) + d**3*p
*x*log(a + b*x**2) - 2*d**3*p*x + d**3*x*log(c) + 3*d**2*e*p*x**2*log(a + b
*x**2)/2 - 3*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x**3*log(a
 + b*x**2) - 2*d*e**2*p*x**3/3 + d*e**2*x**3*log(c) + e**3*p*x**4*log(a + b
*x**2)/4 - e**3*p*x**4/8 + e**3*x**4*log(c)/4, Ne(b, 0)), ((d**3*x + 3*d**2
*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(a**p*c), True))
```

Giac [A] time = 1.26655, size = 383, normalized size = 2.15

$$\frac{2\,ad^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{2\,a^2dp\arctan\left(\frac{bx}{\sqrt{ab}}\right)e^2}{\sqrt{ab}b}+\frac{6\,b^2px^4e^3\log\left(bx^2+a\right)+24\,b^2dpx^3e^2\log\left(bx^2+a\right)+36\,b^2d^2px^2e\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*a*d^3*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - 2*a^2*d*p*arctan(b*x/sqrt(a*b))
*e^2/(sqrt(a*b)*b) + 1/24*(6*b^2*p*x^4*e^3*log(b*x^2 + a) + 24*b^2*d*p*x^3*
e^2*log(b*x^2 + a) + 36*b^2*d^2*p*x^2*e*log(b*x^2 + a) - 3*b^2*p*x^4*e^3 -
16*b^2*d*p*x^3*e^2 - 36*b^2*d^2*p*x^2*e + 24*b^2*d^3*p*x*log(b*x^2 + a) + 6
*b^2*x^4*e^3*log(c) + 24*b^2*d*x^3*e^2*log(c) + 36*b^2*d^2*x^2*e*log(c) - 4
8*b^2*d^3*p*x + 36*a*b*d^2*p*e*log(b*x^2 + a) + 24*b^2*d^3*x*log(c) + 6*a*b
*p*x^2*e^3 + 48*a*b*d*p*x*e^2 - 6*a^2*p*e^3*log(b*x^2 + a))/b^2
```

3.185 $\int (d+ex)^2 \log\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{a}p\left(3bd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{dp\left(bd^2-3ae^2\right)\log\left(a+bx^2\right)}{3be}-\frac{2px\left(3bd^2-ae^2\right)}{3b}-depx$$

```
[Out] (-2*(3*b*d^2 - a*e^2)*p*x)/(3*b) - d*e*p*x^2 - (2*e^2*p*x^3)/9 + (2*Sqrt[a]
*(3*b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)) - (d*(b*d^2 -
 3*a*e^2)*p*Log[a + b*x^2])/(3*b*e) + ((d + e*x)^3*Log[c*(a + b*x^2)^p])/(3
*e)
```

Rubi [A] time = 0.132159, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2463, 801, 635, 205, 260}

$$\frac{2\sqrt{a}p\left(3bd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{dp\left(bd^2-3ae^2\right)\log\left(a+bx^2\right)}{3be}-\frac{2px\left(3bd^2-ae^2\right)}{3b}-depx$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*(3*b*d^2 - a*e^2)*p*x)/(3*b) - d*e*p*x^2 - (2*e^2*p*x^3)/9 + (2*Sqrt[a]
*(3*b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)) - (d*(b*d^2 -
 3*a*e^2)*p*Log[a + b*x^2])/(3*b*e) + ((d + e*x)^3*Log[c*(a + b*x^2)^p])/(3
*e)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)dx = \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{(2bp)\int\frac{x(d+ex)^3}{a+bx^2}dx}{3e}$$

$$= \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{(2bp)\int\left(\frac{e\left(3bd^2-ae^2\right)}{b^2}+\frac{3de^2x}{b}+\frac{e^3x^2}{b}-\frac{ae\left(3bd^2-ae^2\right)-bd\left(bd^2\right.}{b^2\left(a+bx^2\right)}\right.}{3e}$$

$$= -\frac{2\left(3bd^2-ae^2\right)px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e} + \frac{(2p)\int\frac{ae(3b}{}}{}$$

$$= -\frac{2\left(3bd^2-ae^2\right)px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{\left(2d\left(bd^2-\right.\right.}{}$$

$$= -\frac{2\left(3bd^2-ae^2\right)px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{2\sqrt{a}\left(3bd^2-ae^2\right)p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{d\left(bd^2\right.}{}$$

Mathematica [A] time = 0.455879, size = 211, normalized size = 1.5

$$\frac{3p\left(-3\sqrt{-a}bd^2e+3a\sqrt{b}de^2+\sqrt{-a}ae^3-b^{3/2}d^3\right)\log\left(\sqrt{-a}-\sqrt{b}x\right)-3p\left(-3\sqrt{-a}bd^2e-3a\sqrt{b}de^2+\sqrt{-a}ae^3+b^{3/2}d^3\right)\log}{9b^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (3*(-(b^(3/2)*d^3) - 3*Sqrt[-a]*b*d^2*e + 3*a*Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^
3)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 3*(b^(3/2)*d^3 - 3*Sqrt[-a]*b*d^2*e - 3*a*
Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^3)*p*Log[Sqrt[-a] + Sqrt[b]*x] + Sqrt[b]*(6*a*
e^3*p*x - b*e*p*x*(18*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*b*(d + e*x)^3*Log[c*(a
 + b*x^2)^p]))/(9*b^(3/2)*e)
```

Maple [C] time = 0.72, size = 965, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x^2+a)^p),x)
```

```
[Out] -1/2*I*e*Pi*d*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+2/3/b
*a*p*e^2*x+ln(c)*d^2*x+1/3*e^2*ln(c)*x^3-1/3/e*p*ln(-a^2*e^3+3*a*b*d^2*e-(-
a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*d^3-1/3/e*p*ln(-a^2*e
^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*d^3-
2*d^2*p*x-2/9*e^2*p*x^3+e*ln(c)*d*x^2-d*e*p*x^2+1/3*(e*x+d)^3/e*ln((b*x^2+a
)^p)+1/3/b^2/e*p*ln(-a^2*e^3+3*a*b*d^2*e-(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*
b^3*d^4*e^2)^(1/2)*x)*(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)-
1/3/b^2/e*p*ln(-a^2*e^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d
^4*e^2)^(1/2)*x)*(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)-1/6*I
*e^2*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^3-1/2*I*Pi*d^2*csgn(I*c*(b*x^2+a)^p)^3*x+
```

```
1/b*e*p*ln(-a^2*e^3+3*a*b*d^2*e-(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e
^2)^(1/2)*x)*a*d+1/b*e*p*ln(-a^2*e^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b^2*d^2*
e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*a*d+1/2*I*Pi*d^2*csgn(I*c*(b*x^2+a)^p)^2*csgn
(I*c)*x+1/2*I*Pi*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*x+1/6*I*e^
2*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/6*I*e^2*Pi*x^3*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x^2+a)^p)^3-1/6*I
*e^2*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/2*I*e*Pi*
d*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/2*I*e*Pi*d*x^2*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*Pi*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05403, size = 709, normalized size = 5.03

$$\left[-\frac{2\,be^{2}px^{3}+9\,bdepx^{2}-3\left(3\,bd^{2}-ae^{2}\right)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)+6\left(3\,bd^{2}-ae^{2}\right)px-3\left(be^{2}px^{3}+3\,bdepx^{2}+3\,bd^{2}px+\right.}{9\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*b*e^2*p*x^3 + 9*b*d*e*p*x^2 - 3*(3*b*d^2 - a*e^2)*p*sqrt(-a/b)*log
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*(3*b*d^2 - a*e^2)*p*x - 3*
(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3*a*d*e*p)*log(b*x^2 + a) - 3*
(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b, -1/9*(2*b*e^2*p*x^3 + 9*b*
d*e*p*x^2 - 6*(3*b*d^2 - a*e^2)*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 6*(3*
b*d^2 - a*e^2)*p*x - 3*(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3*a*d*e
*p)*log(b*x^2 + a) - 3*(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b]
```

Sympy [A] time = 39.4453, size = 309, normalized size = 2.19

$$\begin{cases} -\frac{ia^{\frac{3}{2}}e^{2}p\log\left(a+bx^{2}\right)}{3b^{2}\sqrt{\frac{1}{b}}}+\frac{2ia^{\frac{3}{2}}e^{2}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{3b^{2}\sqrt{\frac{1}{b}}}+\frac{i\sqrt{a}d^{2}p\log\left(a+bx^{2}\right)}{b\sqrt{\frac{1}{b}}}-\frac{2i\sqrt{a}d^{2}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}}+\frac{adep\log\left(a+bx^{2}\right)}{b}+\frac{2ae^{2}px}{3b}+d^{2}px\log\left(a\right. \\ \left(d^{2}x+dex^{2}+\frac{e^{2}x^{3}}{3}\right)\log\left(a^{p}c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-I*a**(3/2)*e**2*p*log(a + b*x**2)/(3*b**2*sqrt(1/b)) + 2*I*a**(
3/2)*e**2*p*log(-I*sqrt(a)*sqrt(1/b) + x)/(3*b**2*sqrt(1/b)) + I*sqrt(a)*d*
*2*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*d**2*p*log(-I*sqrt(a)*sqrt
(1/b) + x)/(b*sqrt(1/b)) + a*d*e*p*log(a + b*x**2)/b + 2*a*e**2*p*x/(3*b) +
 d**2*p*x*log(a + b*x**2) - 2*d**2*p*x + d**2*x*log(c) + d*e*p*x**2*log(a +
 b*x**2) - d*e*p*x**2 + d*e*x**2*log(c) + e**2*p*x**3*log(a + b*x**2)/3 - 2
*e**2*p*x**3/9 + e**2*x**3*log(c)/3, Ne(b, 0)), ((d**2*x + d*e*x**2 + e**2*
x**3/3)*log(a**p*c), True))
```

Giac [A] time = 1.20983, size = 247, normalized size = 1.75

$$\frac{2\,ad^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{2\,a^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)e^2}{3\,\sqrt{ab}b}+\frac{3\,bpx^3e^2\log\left(bx^2+a\right)+9\,bdpx^2e\log\left(bx^2+a\right)-2\,bpx^3e^2-9\,bdpx^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*a*d^2*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - 2/3*a^2*p*arctan(b*x/sqrt(a*b))
*e^2/(sqrt(a*b)*b) + 1/9*(3*b*p*x^3*e^2*log(b*x^2 + a) + 9*b*d*p*x^2*e*log(
b*x^2 + a) - 2*b*p*x^3*e^2 - 9*b*d*p*x^2*e + 9*b*d^2*p*x*log(b*x^2 + a) + 3
*b*x^3*e^2*log(c) + 9*b*d*x^2*e*log(c) - 18*b*d^2*p*x + 9*a*d*p*e*log(b*x^2
 + a) + 9*b*d^2*x*log(c) + 6*a*p*x*e^2)/b
```

3.186 $\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=99

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{p\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2be}+\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-2dpx-\frac{1}{2}epx^2$$

```
[Out] -2*d*p*x - (e*p*x^2)/2 + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b
] - ((b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*b*e) + ((d + e*x)^2*Log[c*(a + b*
x^2)^p])/(2*e)
```

Rubi [A] time = 0.078563, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2463, 801, 635, 205, 260}

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{p\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2be}+\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-2dpx-\frac{1}{2}epx^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*d*p*x - (e*p*x^2)/2 + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b
] - ((b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*b*e) + ((d + e*x)^2*Log[c*(a + b*
x^2)^p])/(2*e)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)dx = \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} - \frac{(bp)\int \frac{x(d+ex)^2}{a+bx^2}dx}{e}$$

$$= \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} - \frac{(bp)\int\left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade-\left(bd^2-ae^2\right)x}{b\left(a+bx^2\right)}\right)dx}{e}$$

$$= -2dpx - \frac{1}{2}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} + \frac{p\int\frac{2ade-\left(bd^2-ae^2\right)x}{a+bx^2}dx}{e}$$

$$= -2dpx - \frac{1}{2}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} + (2adp)\int\frac{1}{a+bx^2}dx + \frac{\left(\left(-bd^2+a\right.\right.}{}$$

$$= -2dpx - \frac{1}{2}epx^2 + \frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{\left(bd^2-ae^2\right)p\log\left(a+bx^2\right)}{2be} + \frac{(d+ex)^2\log}{}$$

Mathematica [A] time = 0.02559, size = 83, normalized size = 0.84

$$dx\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}e\left(\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-px^2\right)+\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-2dpx$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*d*p*x + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + d*x*Log[c*
(a + b*x^2)^p] + (e*(-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b))/2
```

Maple [A] time = 0.078, size = 93, normalized size = 0.9

$$d\ln\left(c\left(bx^2+a\right)^p\right)x-2\,dpx+2\,\frac{dpa}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{e\ln\left(c\left(bx^2+a\right)^p\right)x^2}{2}-\frac{epx^2}{2}+\frac{\ln\left(c\left(bx^2+a\right)^p\right)ae}{2\,b}-\frac{ape}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x^2+a)^p),x)
```

```
[Out] d*ln(c*(b*x^2+a)^p)*x-2*d*p*x+2*d*p*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1
/2*e*ln(c*(b*x^2+a)^p)*x^2-1/2*e*p*x^2+1/2*e/b*ln(c*(b*x^2+a)^p)*a-1/2*a*p*
e/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21228, size = 455, normalized size = 4.6

$$\left[-\frac{bepx^2 - 2\,bdp\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 4\,bdpx - \left(bepx^2 + 2\,bdpx + aep\right)\log\left(bx^2 + a\right) - \left(bex^2 + 2\,bdx\right)\log\left(c\right)}{2\,b}, -\frac{bepx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*e*p*x^2 - 2*b*d*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b
*x^2 + a)) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2 + a) - (
b*e*x^2 + 2*b*d*x)*log(c))/b, -1/2*(b*e*p*x^2 - 4*b*d*p*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2 + a)
 - (b*e*x^2 + 2*b*d*x)*log(c))/b]
```

Sympy [A] time = 19.996, size = 160, normalized size = 1.62

$$\begin{cases} \frac{i\sqrt{a}dp\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}dp\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}} + \frac{aep\log\left(a+bx^2\right)}{2b} + dpx\log\left(a + bx^2\right) - 2dpx + dx\log\left(c\right) + \frac{epx^2\log\left(a+bx^2\right)}{2} - \frac{epx^2}{2} + \frac{ex^2}{} \\ \left(dx + \frac{ex^2}{2}\right)\log\left(a^pc\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((I*sqrt(a)*d*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*d*p*lo
g(-I*sqrt(a)*sqrt(1/b) + x)/(b*sqrt(1/b)) + a*e*p*log(a + b*x**2)/(2*b) + d
*p*x*log(a + b*x**2) - 2*d*p*x + d*x*log(c) + e*p*x**2*log(a + b*x**2)/2 -
e*p*x**2/2 + e*x**2*log(c)/2, Ne(b, 0)), ((d*x + e*x**2/2)*log(a**p*c), Tru
e))
```

Giac [A] time = 1.28123, size = 135, normalized size = 1.36

$$\frac{2\,adp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{bpx^2e\log\left(bx^2 + a\right) - bpx^2e + 2\,bdpx\log\left(bx^2 + a\right) + bx^2e\log\left(c\right) - 4\,bdpx + ape\log\left(bx^2 + a\right) + 2\,b}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*a*d*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*(b*p*x^2*e*log(b*x^2 + a) - b
*p*x^2*e + 2*b*d*p*x*log(b*x^2 + a) + b*x^2*e*log(c) - 4*b*d*p*x + a*p*e*lo
g(b*x^2 + a) + 2*b*d*x*log(c))/b
```

3.187 $\int \log \left(c \left(a+b x^2\right)^p\right) \, dx$

Optimal. Leaf size=45

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rubi [A] time = 0.0190329, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2448, 321, 205}

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+b x^2\right)^p\right) \, dx &= x \log \left(c \left(a+b x^2\right)^p\right)-(2 b p) \int \frac{x^2}{a+b x^2} \, dx\\
&= -2 p x+x \log \left(c \left(a+b x^2\right)^p\right)+(2 a p) \int \frac{1}{a+b x^2} \, dx\\
&= -2 p x+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}+x \log \left(c \left(a+b x^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.0126797, size = 45, normalized size = 1.

$$x\log\left(c\left(a+bx^2\right)^p\right)+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-2px$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Maple [A] time = 0.073, size = 38, normalized size = 0.8

$$x\ln\left(c\left(bx^2+a\right)^p\right)-2\,px+2\,\frac{ap}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p),x)
```

```
[Out] x*ln(c*(b*x^2+a)^p)-2*p*x+2*p*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.3912, size = 250, normalized size = 5.56

$$\left[px\log\left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,px+x\log\left(c\right),px\log\left(bx^2+a\right)+2\,p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [A] time = 10.002, size = 90, normalized size = 2.

$$\begin{cases} \frac{i\sqrt{a}p\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}} + px\log\left(a+bx^2\right) - 2px + x\log(c) & \text{for } b \neq 0 \\ x\log\left(a^pc\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((I*sqrt(a)*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*p*log(-I
*sqrt(a)*sqrt(1/b) + x)/(b*sqrt(1/b)) + p*x*log(a + b*x**2) - 2*p*x + x*log
(c), Ne(b, 0)), (x*log(a**p*c), True))
```

Giac [A] time = 1.27252, size = 55, normalized size = 1.22

$$px\log\left(bx^2+a\right) + \frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \left(2\,p - \log(c)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

3.188 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=201

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}$$

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rubi [A] time = 0.266316, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int\left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} + \frac{\left(\sqrt{b}p\right)\int\frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e} - \frac{\left(\sqrt{b}p\right)\int\frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx\right.\right.}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx\right.\right.}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx\right.\right.}{e}$$

Mathematica [A] time = 0.0806285, size = 201, normalized size = 1.

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Maple [C] time = 0.511, size = 366, normalized size = 1.8

$$\frac{\ln\left(ex+d\right)\ln\left(\left(bx^{2}+a\right)^{p}\right)}{e}-\frac{p\ln\left(ex+d\right)}{e}\ln\left(\left(e\sqrt{-ab}-b\left(ex+d\right)+bd\right)\left(e\sqrt{-ab}+bd\right)^{-1}\right)-\frac{p\ln\left(ex+d\right)}{e}\ln\left(\left(e\sqrt{-ab}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x^2+a)^p)-p/e*ln(e*x+d)*ln((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)
/(e*(-a*b)^(1/2)+b*d))-p/e*ln(e*x+d)*ln((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(
-a*b)^(1/2)-b*d))-p/e*dilog((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+
b*d))-p/e*dilog((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+1/2*I*
ln(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(e*x+d)/
e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*
Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^p)^2*csg
n(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^{2}+a\right)^{p}c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^{2}+a\right)^{p}c\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^{2}\right)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

3.189 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^2} \, dx$

Optimal. Leaf size=119

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(ae^2+bd^2\right)}-\frac{2bdp\log(d+ex)}{e\left(ae^2+bd^2\right)}+\frac{2\sqrt{a}\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{ae^2+bd^2}$$

```
[Out] (2*Sqrt[a]*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2) - (2*b*d*
p*Log[d + e*x])/(e*(b*d^2 + a*e^2)) + (b*d*p*Log[a + b*x^2])/(e*(b*d^2 + a*
e^2)) - Log[c*(a + b*x^2)^p]/(e*(d + e*x))
```

Rubi [A] time = 0.0901617, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2463, 801, 635, 205, 260}

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(ae^2+bd^2\right)}-\frac{2bdp\log(d+ex)}{e\left(ae^2+bd^2\right)}+\frac{2\sqrt{a}\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{ae^2+bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2) - (2*b*d*
p*Log[d + e*x])/(e*(b*d^2 + a*e^2)) + (b*d*p*Log[a + b*x^2])/(e*(b*d^2 + a*
e^2)) - Log[c*(a + b*x^2)^p]/(e*(d + e*x))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^2}\,dx &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{(2bp)\int \frac{x}{(d+ex)\left(a+bx^2\right)}\,dx}{e}\\
&= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{(2bp)\int \left(-\frac{de}{\left(bd^2+ae^2\right)(d+ex)}+\frac{ae+bdx}{\left(bd^2+ae^2\right)\left(a+bx^2\right)}\right)dx}{e}\\
&= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{(2bp)\int \frac{ae+bdx}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}\\
&= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{(2abp)\int \frac{1}{a+bx^2}\,dx}{bd^2+ae^2}+\frac{\left(2b^2dp\right)\int \frac{x}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}\\
&= \frac{2\sqrt{a}\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bd^2+ae^2}-\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}
\end{aligned}$$

Mathematica [A] time = 0.0721267, size = 137, normalized size = 1.15

$$\frac{-bd^2\log\left(c\left(a+bx^2\right)^p\right)-ae^2\log\left(c\left(a+bx^2\right)^p\right)+bd^2p\log\left(a+bx^2\right)+bdepx\log\left(a+bx^2\right)+2\sqrt{a}\sqrt{b}ep(d+ex)\tan^{-1}\left(\frac{\sqrt{b}\ldots}{\sqrt{a}\ldots}\right.}{e(d+ex)\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*e*p*(d + e*x)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 2*b*d*p*(d +
 e*x)*Log[d + e*x] + b*d^2*p*Log[a + b*x^2] + b*d*e*p*x*Log[a + b*x^2] - b*
d^2*Log[c*(a + b*x^2)^p] - a*e^2*Log[c*(a + b*x^2)^p])/(e*(b*d^2 + a*e^2)*(
d + e*x))
```

Maple [C] time = 0.548, size = 755, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^2,x)
```

```
[Out] -1/e/(e*x+d)*ln((b*x^2+a)^p)+1/2*(-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)
*e^2+I*Pi*b*d^2*csgn(I*c*(b*x^2+a)^p)^3-I*Pi*b*d^2*csgn(I*c*(b*x^2+a)^p)^2*
csgn(I*c)+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3*e^2+I*Pi*b*d^2*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e^2+I*Pi*a*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e^2+2*sum(_R*ln(((3*a*e
^4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*_R^2-a*e^2*p*_R),_R=Root
Of((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*a*e^4*x+2*sum(_R*ln(((3*a*e^
4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*_R^2-a*e^2*p*_R),_R=RootO
```

```
f((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*b*d^2*e^2*x-4*ln(e*x+d)*b*d*e
*p*x+2*sum(_R*ln(((3*a*e^4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*
_R^2-a*e^2*p*_R),_R=RootOf((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*a*d*
e^3+2*sum(_R*ln(((3*a*e^4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*_
R^2-a*e^2*p*_R),_R=RootOf((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*b*d^3
*e-4*ln(e*x+d)*b*d^2*p-2*ln(c)*a*e^2-2*ln(c)*b*d^2)/(e*x+d)/e/(a*e^2+b*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55649, size = 582, normalized size = 4.89

$$\left[\frac{\left(e^2px + dep\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right) + \left(bdepx - ae^2p\right)\log\left(bx^2+a\right) - 2\left(bdepx + bd^2p\right)\log\left(ex+d\right) - \left(bd^2+ae^2\right)\log}{bd^3e + ade^3 + \left(bd^2e^2 + ae^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [((e^2*p*x + d*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + (b*d*e*p*x - a*e^2*p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*x
+ d) - (b*d^2 + a*e^2)*log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*x),
 (2*(e^2*p*x + d*e*p)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b*d*e*p*x - a*e^2*
p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*x + d) - (b*d^2 + a*e^2)*
log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22342, size = 213, normalized size = 1.79

$$\frac{bdp\log\left(bx^2+a\right)}{bd^2e+ae^3} + \frac{2\,abp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bd^2+ae^2\right)\sqrt{ab}} - \frac{2\,bdpxe\log\left(xe+d\right) + bd^2p\log\left(bx^2+a\right) + 2\,bd^2p\log\left(xe+d\right) + ape^2\log\left(b\right.}{bd^2xe^2 + bd^3e + axe^4 + ade^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*d*p*log(b*x^2 + a)/(b*d^2*e + a*e^3) + 2*a*b*p*arctan(b*x/sqrt(a*b))/((b*
d^2 + a*e^2)*sqrt(a*b)) - (2*b*d*p*x*e*log(x*e + d) + b*d^2*p*log(b*x^2 + a
) + 2*b*d^2*p*log(x*e + d) + a*p*e^2*log(b*x^2 + a) + b*d^2*log(c) + a*e^2*
log(c))/(b*d^2*x*e^2 + b*d^3*e + a*x*e^4 + a*d*e^3)
```

3.190 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=174

$$\frac{2\sqrt{a}b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\left(ae^2+bd^2\right)^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{bp\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2e\left(ae^2+bd^2\right)^2}+\frac{bdp}{e(d+ex)\left(ae^2+bd^2\right)}-\frac{bp\left(bd^2-ae^2\right)\mathrm{lo}}{e\left(ae^2+b\right.}$$

```
[Out] (b*d*p)/(e*(b*d^2 + a*e^2)*(d + e*x)) + (2*Sqrt[a]*b^(3/2)*d*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2)^2 - (b*(b*d^2 - a*e^2)*p*Log[d + e*x])/(e*
(b*d^2 + a*e^2)^2) + (b*(b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*e*(b*d^2 + a*e
^2)^2) - Log[c*(a + b*x^2)^p]/(2*e*(d + e*x)^2)
```

Rubi [A] time = 0.144985, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2463, 801, 635, 205, 260}

$$\frac{2\sqrt{a}b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\left(ae^2+bd^2\right)^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{bp\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2e\left(ae^2+bd^2\right)^2}+\frac{bdp}{e(d+ex)\left(ae^2+bd^2\right)}-\frac{bp\left(bd^2-ae^2\right)\mathrm{lo}}{e\left(ae^2+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] (b*d*p)/(e*(b*d^2 + a*e^2)*(d + e*x)) + (2*Sqrt[a]*b^(3/2)*d*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2)^2 - (b*(b*d^2 - a*e^2)*p*Log[d + e*x])/(e*
(b*d^2 + a*e^2)^2) + (b*(b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*e*(b*d^2 + a*e
^2)^2) - Log[c*(a + b*x^2)^p]/(2*e*(d + e*x)^2)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2} + \frac{(bp)\int \frac{x}{(d+ex)^2\left(a+bx^2\right)}\,dx}{e}$$

$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2} + \frac{(bp)\int \left(-\frac{de}{\left(bd^2+ae^2\right)(d+ex)^2} + \frac{e\left(-bd^2+ae^2\right)}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{b\left(2ade+\left(bd^2-ae^2\right)x\right)}{\left(bd^2+ae^2\right)^2\left(a+bx^2\right)}\right)dx}{e}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{b\left(bd^2-ae^2\right)p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2} + \frac{\left(b^2p\right)\int \frac{2ade+\left(bd^2-ae^2\right.}{a+bx^2}}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{b\left(bd^2-ae^2\right)p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2} + \frac{\left(2ab^2dp\right)\int \frac{1}{a+bx^2}\,d}{\left(bd^2+ae^2\right)^2}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)} + \frac{2\sqrt{a}b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\left(bd^2+ae^2\right)^2} - \frac{b\left(bd^2-ae^2\right)p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} + \frac{b\left(bd^2-ae^2\right)}{2e\left(bd^2\right.}$$

Mathematica [A] time = 0.588592, size = 217, normalized size = 1.25

$$\frac{\frac{bp(d+ex)\left((d+ex)\left(\sqrt{-a}bd^2+2a\sqrt{b}de+(-a)^{3/2}e^2\right)\log\left(\sqrt{-a}-\sqrt{b}x\right)+(d+ex)\left(\sqrt{-a}bd^2-2a\sqrt{b}de+(-a)^{3/2}e^2\right)\log\left(\sqrt{-a}+\sqrt{b}x\right)+2\sqrt{-a}\left(-(d+ex)\left(bd^2-ae^2\right)\log(d+ex)+ade^2+\right.\right.}{\sqrt{-a}\left(ae^2+bd^2\right)^2}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] ((b*p*(d + e*x)*((Sqrt[-a]*b*d^2 + 2*a*Sqrt[b]*d*e + (-a)^(3/2)*e^2)*(d + e
*x)*Log[Sqrt[-a] - Sqrt[b]*x] + (Sqrt[-a]*b*d^2 - 2*a*Sqrt[b]*d*e + (-a)^(3
/2)*e^2)*(d + e*x)*Log[Sqrt[-a] + Sqrt[b]*x] + 2*Sqrt[-a]*(b*d^3 + a*d*e^2
- (b*d^2 - a*e^2)*(d + e*x)*Log[d + e*x])))/(Sqrt[-a]*(b*d^2 + a*e^2)^2) -
Log[c*(a + b*x^2)^p])/(2*e*(d + e*x)^2)
```

Maple [C] time = 0.504, size = 3183, normalized size = 18.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x^2+a)^p)-1/4*b^2*(-2*I*Pi*a*b*d^2*e^2*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-2*b^2*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3
*e-3*(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1/2)*a*b*d^2*e^2-3*(-a*b)^(1/2)*b^2*d^
4)*x-10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a^2*d*e^3+8*(-a*b)^(1/2)*a*b*d^3*e+3*a
^3*e^4+3*a*b^2*d^4)*d^2*e^2*p*x^2-4*b^2*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e-3*
```

(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1/2)*a*b*d^2*e^2-3*(-a*b)^(1/2)*b^2*d^4)*x-
10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a^2*d*e^3+8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^
4+3*a*b^2*d^4)*d^3*e*p*x+2*b*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2
)*a^2*e^4-10*(-a*b)^(1/2)*a*b*d^2*e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^
2*e^2+8*(-a*b)^(1/2)*a^2*d*e^3-8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d
^4)*a*e^4*p*x^2+2*b*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4
-10*(-a*b)^(1/2)*a*b*d^2*e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(
-a*b)^(1/2)*a^2*d*e^3-8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*a*d^2
*e^2*p-2*b^2*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4-10*(-a
*b)^(1/2)*a*b*d^2*e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(-a*b)^(
1/2)*a^2*d*e^3-8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*d^2*e^2*p*x^
2-4*b^2*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4-10*(-a*b)^(
1/2)*a*b*d^2*e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(-a*b)^(1/2)*
a^2*d*e^3-8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*d^3*e*p*x+2*b*ln(
(-8*a^2*b*d*e^3+8*a*b^2*d^3*e-3*(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1/2)*a*b*d^
2*e^2-3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a^2*d*e^3+8
*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*a*e^4*p*x^2+2*b*ln((-8*a^2*b
*d*e^3+8*a*b^2*d^3*e-3*(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1/2)*a*b*d^2*e^2-3*(
-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a^2*d*e^3+8*(-a*b)^(
1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*a*d^2*e^2*p-8*ln(e*x+d)*a*b*d*e^3*p*x
+4*b*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e-3*(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1/2
)*a*b*d^2*e^2-3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a^2
*d*e^3+8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*a*d*e^3*p*x+4*b*ln((
-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4-10*(-a*b)^(1/2)*a*b*d^2
e^2+3(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(-a*b)^(1/2)*a^2*d*e^3-8*
(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*a*d*e^3*p*x-4*b*ln((-8*a^2*b*
d*e^3+8*a*b^2*d^3*e-3*(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1/2)*a*b*d^2*e^2-3*(-
a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a^2*d*e^3+8*(-a*b)^(1
/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*(-a*b)^(1/2)*d*e^3*p*x^2+2*I*Pi*a*b*d^
2*e^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+2*I*Pi*a*b*d^2*e^2*csgn(I
c(b*x^2+a)^p)^2*csgn(I*c)+4*ln(e*x+d)*b^2*d^4*p-4*a*d^2*b*p*e^2+4*b*ln((-
8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4-10*(-a*b)^(1/2)*a*b*d^2*
e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(-a*b)^(1/2)*a^2*d*e^3-8*(
-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*(-a*b)^(1/2)*d*e^3*p*x^2-8*b*l
n((-8*a^2*b*d*e^3+8*a*b^2*d^3*e-3*(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1/2)*a*b*
d^2*e^2-3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a^2*d*e^3
+8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*(-a*b)^(1/2)*d^2*e^2*p*x+8
*b*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4-10*(-a*b)^(1/2)*
a*b*d^2*e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(-a*b)^(1/2)*a^2*d
e^3-8(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*(-a*b)^(1/2)*d^2*e^2*p
*x-4*b^2*d^4*p-2*b^2*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e-3*(-a*b)^(1/2)*a^2*e^
4+10*(-a*b)^(1/2)*a*b*d^2*e^2-3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2-8*
(-a*b)^(1/2)*a^2*d*e^3+8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*d^4*
p-2*b^2*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4-10*(-a*b)^(
1/2)*a*b*d^2*e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(-a*b)^(1/2)*
a^2*d*e^3-8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*d^4*p-I*Pi*a^2*e^
4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^2*e^4*csgn(I*c
*(b*x^2+a)^p)^3+I*Pi*b^2*d^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*a*d*p*e^3*
x*b+I*Pi*a^2*e^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*a^2*e^4*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+I*Pi*b^2*d^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2+4*ln(c)*a*b*d^2*e^2+2*ln(c)*b^2*d^4+2*ln(c)*a^2*e^4-4*d^3*p*
x*b^2*e-2*I*Pi*a*b*d^2*e^2*csgn(I*c*(b*x^2+a)^p)^3-I*Pi*b^2*d^4*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b^2*d^4*csgn(I*c*(b*x^2+a)^p)
^3-4*b*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e-3*(-a*b)^(1/2)*a^2*e^4+10*(-a*b)^(1
/2)*a*b*d^2*e^2-3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2-8*(-a*b)^(1/2)*a
^2*d*e^3+8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*(-a*b)^(1/2)*d^3*e
*p+4*b*ln((-8*a^2*b*d*e^3+8*a*b^2*d^3*e+3*(-a*b)^(1/2)*a^2*e^4-10*(-a*b)^(1
/2)*a*b*d^2*e^2+3*(-a*b)^(1/2)*b^2*d^4)*x-10*a^2*b*d^2*e^2+8*(-a*b)^(1/2)*a
^2*d*e^3-8*(-a*b)^(1/2)*a*b*d^3*e+3*a^3*e^4+3*a*b^2*d^4)*(-a*b)^(1/2)*d^3*e

```
*p-4*ln(e*x+d)*a*b*e^4*p*x^2+4*ln(e*x+d)*b^2*d^2*e^2*p*x^2+8*ln(e*x+d)*b^2*
d^3*e*p*x-4*ln(e*x+d)*a*b*d^2*e^2*p)/(e*x+d)^2/(b*d-e*(-a*b)^(1/2))^2/(e*(-
a*b)^(1/2)+b*d)^2/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.02908, size = 1524, normalized size = 8.76

$$\left[\frac{2\left(b^{2}d^{3}e + abde^{3}\right)px + 2\left(bde^{3}px^{2} + 2\,bd^{2}e^{2}px + bd^{3}ep\right)\sqrt{-ab}\log\left(\frac{bx^{2}+2\,\sqrt{-ab}x-a}{bx^{2}+a}\right) + 2\left(b^{2}d^{4} + abd^{2}e^{2}\right)p + \left(\left(b^{2}d^{2}e^{2} - abe^{4}\right)\right.}{2\left(b^{2}d^{6}e + 2\,abd^{4}e^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*d^3*e + a*b*d*e^3)*p*x + 2*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x +
b*d^3*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(b^
2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*
b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4)*p)*log(b*x^2 + a) - 2*((b^2*d^2*e^
2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x + (b^2*d^4 - a*b*d^2*e^2
)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*log(c))/(b^2*d^6*e
+ 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2*d^4*e^3 + 2*a*b*d^2*e^5 + a^2*e^7)*x^2
 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 + a^2*d*e^6)*x), 1/2*(2*(b^2*d^3*e + a*b*
d*e^3)*p*x + 4*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x + b*d^3*e*p)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a) + 2*(b^2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 - a*b*e^4)*
p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4)*p)*log(b*
x^2 + a) - 2*((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x
 + (b^2*d^4 - a*b*d^2*e^2)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2
*e^4)*log(c))/(b^2*d^6*e + 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2*d^4*e^3 + 2*a
*b*d^2*e^5 + a^2*e^7)*x^2 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 + a^2*d*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33172, size = 567, normalized size = 3.26

$$\frac{2\,ab^2dp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2d^4+2\,abd^2e^2+a^2e^4\right)\sqrt{ab}}+\frac{\left(b^2d^2p-abpe^2\right)\log\left(bx^2+a\right)}{2\left(b^2d^4e+2\,abd^2e^3+a^2e^5\right)}-\frac{2\,b^2d^2px^2e^2\log\left(xe+d\right)+4\,b^2d^3pxe\log\left(xe+d\right)-2\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 2*a*b^2*d*p*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*sqrt
(a*b)) + 1/2*(b^2*d^2*p - a*b*p*e^2)*log(b*x^2 + a)/(b^2*d^4*e + 2*a*b*d^2*
e^3 + a^2*e^5) - 1/2*(2*b^2*d^2*p*x^2*e^2*log(x*e + d) + 4*b^2*d^3*p*x*e*lo
g(x*e + d) - 2*b^2*d^3*p*x*e + b^2*d^4*p*log(b*x^2 + a) + 2*b^2*d^4*p*log(x
*e + d) - 2*b^2*d^4*p + 2*a*b*d^2*p*e^2*log(b*x^2 + a) - 2*a*b*p*x^2*e^4*lo
g(x*e + d) - 4*a*b*d*p*x*e^3*log(x*e + d) - 2*a*b*d^2*p*e^2*log(x*e + d) +
b^2*d^4*log(c) - 2*a*b*d*p*x*e^3 - 2*a*b*d^2*p*e^2 + 2*a*b*d^2*e^2*log(c) +
 a^2*p*e^4*log(b*x^2 + a) + a^2*e^4*log(c))/(b^2*d^4*x^2*e^3 + 2*b^2*d^5*x*
e^2 + b^2*d^6*e + 2*a*b*d^2*x^2*e^5 + 4*a*b*d^3*x*e^4 + 2*a*b*d^4*e^3 + a^2
*x^2*e^7 + 2*a^2*d*x*e^6 + a^2*d^2*e^5)
```

3.191 $\int (d+ex)^3 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=320

$$-\frac{\sqrt[3]{a}p\left(-6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}+\frac{\sqrt[3]{a}p\left(-6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}-\sqrt{\cdots}$$

```
[Out] (-3*(4*b*d^3 - a*e^3)*p*x)/(4*b) - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3 - (3*e^3
*p*x^4)/16 - (Sqrt[3]*a^(1/3)*(4*b*d^3 + 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) + (a^(1/3)*
(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*
b^(4/3)) - (a^(1/3)*(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*b^(4/3)) - (d*(b*d^3 - 4*a*e^3)*p
*Log[a + b*x^3])/(4*b*e) + ((d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Rubi [A] time = 0.741717, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {2463, 1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}p\left(-6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}+\frac{\sqrt[3]{a}p\left(-6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}-\sqrt{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```


```
[Out] (-3*(4*b*d^3 - a*e^3)*p*x)/(4*b) - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3 - (3*e^3
*p*x^4)/16 - (Sqrt[3]*a^(1/3)*(4*b*d^3 + 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) + (a^(1/3)*
(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*
b^(4/3)) - (a^(1/3)*(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*b^(4/3)) - (d*(b*d^3 - 4*a*e^3)*p
*Log[a + b*x^3])/(4*b*e) + ((d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```


Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)^3 \log\left(c\left(a+bx^3\right)^p\right) dx = \frac{(d+ex)^4 \log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{(3bp)\int \frac{x^2(d+ex)^4}{a+bx^3}dx}{4e}$$

$$= -\frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{(3p)\int \frac{x^2\left(4bd^4+4e\left(4bd^3-ae^3\right)x+24bd^2e^2x^2+16bde^3x^3}{a+bx^3}}{16e}$$

$$= -de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{p\int \frac{x^2\left(12bd\left(bd^3-4ae^3\right)+12be\left(4bd^3-ae^3\right)x}{a+bx^3}}{16be}$$

$$= -de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{p\int \left(12e\left(4bd^3-ae^3\right) + 72bd^2e^2x\right.}{}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log\left(c\left(a+bx^3\right)^p\right)}{4e} + $$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log\left(c\left(a+bx^3\right)^p\right)}{4e} + $$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{d\left(bd^3-4ae^3\right)p\log\left(a+bx^3\right)}{4be} + $$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p}{4b^{4/3}}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p}{4b^{4/3}}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{\sqrt{3}\sqrt[3]{a}\left(4bd^3+6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right.}{4b^{4/3}}$$

Mathematica [C] time = 0.485492, size = 264, normalized size = 0.82

$$\frac{\frac{\sqrt[3]{a}ep\left(ae^3-4bd^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}ep\left(4bd^3-ae^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}} + (d+ex)^4\log\left(c\left(a+bx^3\right)^p\right) + 9d^2e^2px^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] ((3*e*(-4*b*d^3 + a*e^3)*p*x)/b - 9*d^2*e^2*p*x^2 - 4*d*e^3*p*x^3 - (3*e^4*
p*x^4)/4 + 9*d^2*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + (
a^(1/3)*e*(4*b*d^3 - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(1/3)*
e*(-4*b*d^3 + a*e^3)*p*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2*b^(4/3)) - (d*(b*d
^3 - 4*a*e^3)*p*Log[a + b*x^3])/b + (d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Maple [C] time = 0.783, size = 738, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x^3+a)^p),x)
```

```
[Out] -3*d^3*p*x+3/4/b*e^3*a*p*x-1/2*I*Pi*d^3*x*csgn(I*c*(b*x^3+a)^p)^3-1/8*I*e^3
*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^3+1/4/b^2*p/e*sum((b*d*(4*a*e^3-b*d^3)*_R^2+6
*a*b*d^2*e^2*_R-a^2*e^4+4*a*b*d^3*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))-1/8
*I*e^3*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+1/2*I*e^2
*Pi*d*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+1/2*I*e^2*Pi*d*x^3*cs
gn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x^3+a)^p)*csgn
(I*c*(b*x^3+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-1/
2*I*Pi*d^3*x*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+ln(c)*d^3*
x+1/4*e^3*ln(c)*x^4+1/4*(e*x+d)^4/e*ln((b*x^3+a)^p)+3/2*e*ln(c)*d^2*x^2+e^2
*ln(c)*d*x^3-3/16*e^3*p*x^4-d*e^2*p*x^3-9/4*d^2*e*p*x^2-1/2*I*e^2*Pi*d*x^3*
csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn
(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+1/8*I*e^3*Pi*x^4*csgn(I*c*(
b*x^3+a)^p)^2*csgn(I*c)-1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^3+a)^p)^3-3/4*I*e*
Pi*d^2*x^2*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*Pi*d^3*x*csgn(I*(b*x^3+a)^p)*csgn(
I*c*(b*x^3+a)^p)^2+1/2*I*Pi*d^3*x*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+1/8*I*e
^3*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34227, size = 753, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*a^2*b^3*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 2*sqrt(3
)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^5) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5))*e^3
- 3/4*a*b^2*d^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sq
rt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a*b^4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4))*
e - 1/2*a*b*d^3*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt
(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^2) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) +
1/16*(4*b*p*x^4*e^3*log(b*x^3 + a) + 16*b*d*p*x^3*e^2*log(b*x^3 + a) + 24*b
*d^2*p*x^2*e*log(b*x^3 + a) - 3*b*p*x^4*e^3 - 16*b*d*p*x^3*e^2 - 36*b*d^2*p
*x^2*e + 16*b*d^3*p*x*log(b*x^3 + a) + 4*b*x^4*e^3*log(c) + 16*b*d*x^3*e^2*
log(c) + 24*b*d^2*x^2*e*log(c) - 48*b*d^3*p*x + 16*b*d^3*x*log(c) + 16*a*d*
p*e^2*log(b*x^3 + a) + 12*a*p*x*e^3)/b
```

3.192 $\int (d+ex)^2 \log \left(c \left(a+bx^3\right)^p\right) \, dx$

Optimal. Leaf size=250

$$-\frac{\sqrt[3]{a}dp\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b^{2/3}}+\frac{\sqrt[3]{a}dp\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}dp\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan}{b^{2/3}}$$

```
[Out] -3*d^2*p*x - (3*d*e*p*x^2)/2 - (e^2*p*x^3)/3 - (Sqrt[3]*a^(1/3)*d*(b^(1/3)*
d + a^(1/3)*e)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3)
 + (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) -
 (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(2*b^(2/3)) - ((b*d^3 - a*e^3)*p*Log[a + b*x^3])/(3*b*e) + ((d +
 e*x)^3*Log[c*(a + b*x^3)^p])/(3*e)
```

Rubi [A] time = 0.482857, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {2463, 1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}dp\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b^{2/3}}+\frac{\sqrt[3]{a}dp\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}dp\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```


```
[Out] -3*d^2*p*x - (3*d*e*p*x^2)/2 - (e^2*p*x^3)/3 - (Sqrt[3]*a^(1/3)*d*(b^(1/3)*
d + a^(1/3)*e)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3)
 + (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) -
 (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(2*b^(2/3)) - ((b*d^3 - a*e^3)*p*Log[a + b*x^3])/(3*b*e) + ((d +
 e*x)^3*Log[c*(a + b*x^3)^p])/(3*e)
```


Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)dx = \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{(bp)\int \frac{x^2(d+ex)^3}{a+bx^3}dx}{e}$$

$$= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int \frac{x^2\left(3\left(bd^3-ae^3\right)+9bd^2ex+9bde^2x^2\right)}{a+bx^3}dx}{3e}$$

$$= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int\left(9d^2e+9de^2x-\frac{3\left(3ad^2e+3ade^2x-\left(bd^3-ae\right.\right.}{a+bx^3}\right.}{3e}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} + \frac{p\int\frac{3ad^2e+3ade^2x-\left(bd^3-a\right.}{a+bx^3}}{e}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} + \frac{p\int\frac{3ad^2e+3ade^2x}{a+bx^3}dx}{e} -$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\left(bd^3-ae^3\right)p\log\left(a+bx^3\right)}{3be} + \frac{(d+ex)^3\log\left(c\left(a+b\right.\right.}{3e}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{\left(bd^3-ae^3\right)}{}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d-\right.}{}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\sqrt{3}\sqrt[3]{a}d\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{\sqrt[3]{a}d\left(\sqrt[3]{}\right.}{}$$

Mathematica [C] time = 0.304001, size = 218, normalized size = 0.87

$$\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right) - \frac{p\left(3\sqrt[3]{a}b^{2/3}d^2e\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)-6\sqrt[3]{a}b^{2/3}d^2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\left(bd^3-ae^3\right)\log\left(a+bx\right.\right.}{2b}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-(p*(18*b*d^2*e*x + 9*b*d*e^2*x^2 + 2*b*e^3*x^3 - 9*b*d*e^2*x^2*Hypergeome
tric2F1[2/3, 1, 5/3, -((b*x^3)/a)] - 6*a^(1/3)*b^(2/3)*d^2*e*Log[a^(1/3) +
b^(1/3)*x] + 3*a^(1/3)*b^(2/3)*d^2*e*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]) + 2*(b*d
^3 - a*e^3)*Log[a + b*x^3]))/(2*b) + (d + e*x)^3*Log[c*(a + b*x^3)^p])/(3*e
)
```

Maple [C] time = 0.733, size = 537, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/3*(e*x+d)^3/e*ln((b*x^3+a)^p)+1/2*I*e*Pi*d*x^2*csgn(I*(b*x^3+a)^p)*csgn(I
*c*(b*x^3+a)^p)^2+1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-1/6*I*
e^2*Pi*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+1/6*I*e^2*Pi
*x^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-1/2*I*e*Pi*d*x^2*csgn(I*(b*x^3+a)^p)
*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+1/2*I*Pi*d^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*
(b*x^3+a)^p)^2*x+1/2*I*Pi*d^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*x-1/2*I*e*P
i*d*x^2*csgn(I*c*(b*x^3+a)^p)^3-1/2*I*Pi*d^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(
b*x^3+a)^p)*csgn(I*c)*x-1/2*I*Pi*d^2*csgn(I*c*(b*x^3+a)^p)^3*x+1/6*I*e^2*Pi
*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/6*I*e^2*Pi*x^3*csgn(I*c*
(b*x^3+a)^p)^3+1/3*e^2*ln(c)*x^3-1/3*e^2*p*x^3+e*ln(c)*d*x^2-3/2*d*e*p*x^2+
ln(c)*d^2*x-3*d^2*p*x+1/3*p/e/b*sum(((a*e^3-b*d^3)*_R^2+3*a*d*e^2*_R+3*a*d^
2*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 44.5933, size = 12299, normalized size = 49.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*e^2*p*x^3 + 18*b*d*e*p*x^2 + 36*b*d^2*p*x - 2*(2*a*e^2*p/b - 2*(
1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt
(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d
^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)
^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2
- 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e
^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b*log(1/4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*
(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2
*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 +
a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1
/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*
d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3
)^(1/3)*(I*sqrt(3) + 1))^2*b^2*e + 9*(b^2*d^5 + a*b*d^2*e^3)*p^2*x + 1/2*(3
*b^2*d^3 - 2*a*b*e^3)*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*
b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b
*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3
 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3
/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)
*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))
*p + (15*a*b*d^3*e^2 + a^2*e^5)*p^2) - (6*a*e^2*p - (2*a*e^2*p/b - 2*(1/2)^
(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) +
 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*
p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3
) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(
```

```
9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^
3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b - 3*sqrt(1/3)*b*sqrt((4*(2*a*e^2*p/b - 2*(
1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt
(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d
^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)
^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2
- 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e
^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/b - 2*(1/2)^(2/3
)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/
(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2
+ a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) -
(1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*
b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b
^3)^(1/3)*(I*sqrt(3) + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^4)*p^2)/b^2))*lo
g(-1/4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a
^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a
*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^
6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^
3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 +
(27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))^2*b^2*e + 18*(
b^2*d^5 + a*b*d^2*e^3)*p^2*x - 1/2*(3*b^2*d^3 - 2*a*b*e^3)*(2*a*e^2*p/b - 2
*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sq
rt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b
*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^
3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^
2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3
*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*p - (15*a*b*d^3*e^2 + a^2*e^5)*p^2 +
3/4*sqrt(1/3)*((2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e
*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 +
a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*
a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 +
27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*
p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b^2*e
- 2*(3*b^2*d^3 + a*b*e^3)*p)*sqrt((4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*
p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*
p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p
^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)
*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2
 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(
I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 -
 (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 +
 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2
*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e
^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^
4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3)
 + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^4)*p^2)/b^2)) - (6*a*e^2*p - (2*a*e^
2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^
2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 -
 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^
6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d
^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*
p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b + 3*sqrt(1/3)*b*sqrt((4*(2
*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^
2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/
b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a
^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3
)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2
*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/
b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(
```

```
-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(
9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^
3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p
^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3
+ a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^
4)*p^2)/b^2))*log(-1/4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a
*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(
b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^
3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^
3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2
)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1)
)^2*b^2*e + 18*(b^2*d^5 + a*b*d^2*e^3)*p^2*x - 1/2*(3*b^2*d^3 - 2*a*b*e^3)*
(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*
p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^
3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 +
 a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e
^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b
^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*p - (15*a*b*d^3*e^2 +
 a^2*e^5)*p^2 - 3/4*sqrt(1/3)*((2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^
2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^
3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*
e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^
3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2
*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt
(3) + 1))*b^2*e - 2*(3*b^2*d^3 + a*b*e^3)*p)*sqrt((4*(2*a*e^2*p/b - 2*(1/2)
^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3)
+ 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e
*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/
3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*
(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p
^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/b - 2*(1/2)^(2/3)*(a
^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a
^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^
2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2
)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^
3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^
(1/3)*(I*sqrt(3) + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^4)*p^2)/b^2)) - 4*(b
*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x)*log(b*x^3 + a) - 4*(b*e^2*x^3 + 3
*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x**3+a)**p),x)

[Out] Timed out
```

Giac [A] time = 1.22488, size = 509, normalized size = 2.04

$$-\frac{1}{2}\,ab^2dp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^4}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b^2*d*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sqrt(
3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*
b^4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4))*e -
 1/2*a*b*d^2*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)
*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^
2) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + 1/6
*(2*b*p*x^3*e^2*log(b*x^3 + a) + 6*b*d*p*x^2*e*log(b*x^3 + a) - 2*b*p*x^3*e
^2 - 9*b*d*p*x^2*e + 6*b*d^2*p*x*log(b*x^3 + a) + 2*b*x^3*e^2*log(c) + 6*b*
d*x^2*e*log(c) - 18*b*d^2*p*x + 6*b*d^2*x*log(c) + 2*a*p*e^2*log(b*x^3 + a)
)/b
```

3.193 $\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=229

$$-\frac{\sqrt[3]{a}p\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}+\frac{\sqrt[3]{a}p\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}p\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}}{2b^{2/3}}$$

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 - (Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^(1/3)*e)*p*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)) + (a^(1/3)*(2*b
^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) - (a^(1/3)*(2
*b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
4*b^(2/3)) - (d^2*p*Log[a + b*x^3])/(2*e) + ((d + e*x)^2*Log[c*(a + b*x^3)^
p])/(2*e)
```

Rubi [A] time = 0.31725, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2463, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}p\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}+\frac{\sqrt[3]{a}p\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}p\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 - (Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^(1/3)*e)*p*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)) + (a^(1/3)*(2*b
^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) - (a^(1/3)*(2
*b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
4*b^(2/3)) - (d^2*p*Log[a + b*x^3])/(2*e) + ((d + e*x)^2*Log[c*(a + b*x^3)^
p])/(2*e)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)dx = \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int\frac{x^2(d+ex)^2}{a+bx^3}dx}{2e}$$
$$= \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int\left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade+ae^2x-bd^2x^2}{b(a+bx^3)}\right)dx}{2e}$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{(3p)\int\frac{2ade+ae^2x-bd^2x^2}{a+bx^3}dx}{2e}$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{(3p)\int\frac{2ade+ae^2x}{a+bx^3}dx}{2e} - \frac{\left(3bd^2p\right)\int\frac{x^2}{a+bx^3}}{2e}$$
$$= -3dpx - \frac{3}{4}epx^2 - \frac{d^2p\log\left(a+bx^3\right)}{2e} + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{p\int\frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}de+a^{4/3}\right.}{a^{2/3}-}}{2}$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{d^2p\log\left(a+bx^3\right)}{2e} + \frac{(d+ex)}{}$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(a^{2/3}\right.}{4b^{2/3}}$$
$$= -3dpx - \frac{3}{4}epx^2 - \frac{\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\right.}{2b^{2/3}}$$

Mathematica [C] time = 0.0681739, size = 204, normalized size = 0.89

$$-\frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3}\sqrt[3]{a}dp\tan^{-1}\left(\frac{2b^{2/3}x-\sqrt[3]{a}\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}\right)}{\sqrt[3]{b}} + dx\log\left(c\left(a+bx^3\right)^p\right) + \frac{1}{2}ex^2\log\left(c\left(a+bx^3\right)^p\right) +$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(-(a^(1/3)*b^(1/3))
+ 2*b^(2/3)*x)/(Sqrt[3]*a^(1/3)*b^(1/3))])/b^(1/3) + (3*e*p*x^2*Hypergeomet
ric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (a^(1/3)*d*p*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
2*b^(1/3)) + d*x*Log[c*(a + b*x^3)^p] + (e*x^2*Log[c*(a + b*x^3)^p])/2
```

Maple [C] time = 0.717, size = 335, normalized size = 1.5

$$\left(\frac{ex^2}{2}+dx\right)\ln\left(\left(bx^3+a\right)^p\right) + \frac{i}{4}\pi ex^2\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\left(\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)\right)^2 - \frac{i}{4}\pi ex^2\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x^3+a)^p),x)
```

```
[Out] (1/2*e*x^2+d*x)*ln((b*x^3+a)^p)+1/4*I*Pi*e*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c
*(b*x^3+a)^p)^2-1/4*I*Pi*e*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*cs
```

```
gn(I*c)-1/4*I*Pi*e*x^2*csgn(I*c*(b*x^3+a)^p)^3+1/4*I*Pi*e*x^2*csgn(I*c*(b*x
^3+a)^p)^2*csgn(I*c)+1/2*I*Pi*d*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2
*x-1/2*I*Pi*d*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*x-1/2*I*P
i*d*csgn(I*c*(b*x^3+a)^p)^3*x+1/2*I*Pi*d*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*
x+1/2*ln(c)*e*x^2-3/4*e*p*x^2+ln(c)*d*x-3*d*p*x+1/2/b*a*p*sum((_R*e+2*d)/_R
^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 13.2546, size = 5179, normalized size = 22.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -3/4*e*p*x^2 - 3*d*p*x + 1/4*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8
*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (
1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2
)^(1/3)*(I*sqrt(3) + 1))*log(4*a*d*e^2*p^2 + 2*(4*(1/2)^(2/3)*a*d*e*p^2*(-I
*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)
/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
- a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p + 1/4*(4*(1/2)^(2/3)*a*d
*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^
2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*
b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b*e + (8*b*d^3 + a*e
^3)*p^2*x) - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e
^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*(
(8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*s
qrt(3) + 1) - sqrt(3)*sqrt(-(32*a*d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sq
rt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^
2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a
^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(-2*a*d*e^2*p^2 - (4*(1
/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b
*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^
3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p -
 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^
2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a
*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))
^2*b*e + (8*b*d^3 + a*e^3)*p^2*x + 1/8*sqrt(3)*(8*b*d^2*p - (4*(1/2)^(2/3)*
a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 -
 a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8
*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sqrt(-(32*a*d*
e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3
/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3
+ a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) +
```

```
1))^2*b)/b)) - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a
*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)
*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I
*sqrt(3) + 1) + sqrt(3)*sqrt(-(32*a*d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*
sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/
b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 -
 a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(-2*a*d*e^2*p^2 - (4*
(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a
*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*
p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p
 - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/
b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 +
 a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1
))^2*b*e + (8*b*d^3 + a*e^3)*p^2*x - 1/8*sqrt(3)*(8*b*d^2*p - (4*(1/2)^(2/3
)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 +
(8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sqrt(-(32*a*
d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p
^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^
3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3)
+ 1))^2*b)/b)) + 1/2*(e*p*x^2 + 2*d*p*x)*log(b*x^3 + a) + 1/2*(e*x^2 + 2*d*
x)*log(c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2946, size = 406, normalized size = 1.77

$$-\frac{1}{4}ab^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^4}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^4}\right)e-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*a*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sqrt(3)
*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^
4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4))*e - 1
/2*a*b*d*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + 1/2*p*x
^2*e*log(b*x^3 + a) - 3/4*p*x^2*e + d*p*x*log(b*x^3 + a) + 1/2*x^2*e*log(c)
 - 3*d*p*x + d*x*log(c)
```

3.194 $\int \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=133

$$-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-3px$$

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rubi [A] time = 0.0866253, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2448, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-3px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(c\left(a+bx^3\right)^p\right)dx &= x\log\left(c\left(a+bx^3\right)^p\right)-(3bp)\int\frac{x^3}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+(3ap)\int\frac{1}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+\left(\sqrt[3]{a}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx+\left(\sqrt[3]{a}p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}\left(3a^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{(3\sqrt[3]{}\ldots}{\ldots}\\
&= -3px-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.0387786, size = 129, normalized size = 0.97

$$-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-3px$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

Maple [A] time = 0.072, size = 113, normalized size = 0.9

$$x\ln\left(c\left(bx^3+a\right)^p\right)-3\,px+\frac{ap}{b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{ap}{2\,b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ap\sqrt{3}}{b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p),x)
```

```
[Out] x*ln(c*(b*x^3+a)^p)-3*p*x+1/b*p*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/2/b*p*a/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/b*p*a/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97607, size = 288, normalized size = 2.17

$$px\log\left(bx^3+a\right)+\sqrt{3}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-\frac{1}{2}\,p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [A] time = 177.527, size = 236, normalized size = 1.77

$$\begin{cases} x\log\left(0^p c\right) \\ x\log\left(a^p c\right) \\ px\log\left(b\right)+3px\log\left(x\right)-3px+x\log\left(c\right) \\ -\frac{\sqrt[3]{-1}\sqrt[3]{a}p\log\left(a+bx^3\right)}{b^3\left(\frac{1}{b}\right)^{\frac{8}{3}}}+\frac{3\sqrt[3]{-1}\sqrt[3]{a}p\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{2b^3\left(\frac{1}{b}\right)^{\frac{8}{3}}}+\frac{\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}p\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}x}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b^3\left(\frac{1}{b}\right)^{\frac{8}{3}}}+px\log\left(a+bx^3\right)-3p \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
(p*x*log(b) + 3*p*x*log(x) - 3*p*x + x*log(c), Eq(a, 0)), (-(-1)**(1/3)*a**
(1/3)*p*log(a + b*x**3)/(b**3*(1/b)**(8/3)) + 3*(-1)**(1/3)*a**(1/3)*p*log(
4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3)
 + 4*x**2)/(2*b**3*(1/b)**(8/3)) + (-1)**(1/3)*sqrt(3)*a**(1/3)*p*atan(sqrt
(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x/(3*a**(1/3)*(1/b)**(1/3)))/(b**3*(1/b)**(8/
3)) + p*x*log(a + b*x**3) - 3*p*x + x*log(c), True))
```

Giac [A] time = 1.68589, size = 193, normalized size = 1.45

$$-\frac{1}{2}\,abp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^2}\right)+px$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

3.195 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=308

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^3\right.\right.}{e}$$

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rubi [A] time = 0.51683, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^3\right.\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```


```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```


Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int\left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}\right.}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{\left(\sqrt[3]{b}p\right)\int\frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \ldots}{\ldots}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2}\ldots\right.}{\sqrt[3]{b}d+\sqrt[3]{-1}\ldots}\right.}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2}\ldots\right.}{\sqrt[3]{b}d+\sqrt[3]{-1}\ldots}\right.}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2}\ldots\right.}{\sqrt[3]{b}d+\sqrt[3]{-1}\ldots}\right.}{e}$$

Mathematica [A] time = 0.150079, size = 313, normalized size = 1.02

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e} - \frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
```

```
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Maple [C] time = 0.589, size = 261, normalized size = 0.9

$$\frac{\ln(ex+d)\ln\left(\left(bx^3+a\right)^p\right)}{e}-\frac{p}{e}\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\,bd\_Z^2+3\,bd^2\_Z+ae^3-bd^3\right)}\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_}{\_}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x^3+a)^p)-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-
e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/2*I
*ln(e*x+d)/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/2*I*ln(e*x+d)
/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e
*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^3+a)^p)^2*cs
gn(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

3.196 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=292

$$-\frac{\sqrt[3]{a}\sqrt[3]{b}p\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)}-\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}e^2+\sqrt[3]{a}\sqrt[3]{b}de+b^{2/3}d^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}+\frac{bd^2p\log\left(a\right.}{e\left(bd^3-\right.}$$

```
[Out] -((Sqrt[3]*a^(1/3)*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(b^(2/3)*d^2 + a^(1/3)*b^(1/3)*d*e + a^(2/3)*e^2)) + (a^(1/3)*b^(1/3)*
(b^(1/3)*d + a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(b*d^3 - a*e^3) - (3*b*
d^2*p*Log[d + e*x])/(e*(b*d^3 - a*e^3)) - (a^(1/3)*b^(1/3)*(b^(1/3)*d + a^(
1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*(b*d^3 - a*e^3
)) + (b*d^2*p*Log[a + b*x^3])/(e*(b*d^3 - a*e^3)) - Log[c*(a + b*x^3)^p]/(e
*(d + e*x))
```

Rubi [A] time = 0.549462, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2463, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\sqrt[3]{b}p\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)}-\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}e^2+\sqrt[3]{a}\sqrt[3]{b}de+b^{2/3}d^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}+\frac{bd^2p\log\left(a\right.}{e\left(bd^3-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[3]*a^(1/3)*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(b^(2/3)*d^2 + a^(1/3)*b^(1/3)*d*e + a^(2/3)*e^2)) + (a^(1/3)*b^(1/3)*
(b^(1/3)*d + a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(b*d^3 - a*e^3) - (3*b*
d^2*p*Log[d + e*x])/(e*(b*d^3 - a*e^3)) - (a^(1/3)*b^(1/3)*(b^(1/3)*d + a^(
1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*(b*d^3 - a*e^3
)) + (b*d^2*p*Log[a + b*x^3])/(e*(b*d^3 - a*e^3)) - Log[c*(a + b*x^3)^p]/(e
*(d + e*x))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{x^2}{(d+ex)\left(a+bx^3\right)}\,dx}{e}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \left(-\frac{d^2e}{\left(bd^3-ae^3\right)(d+ex)} + \frac{ade-ae^2x+bd^2x^2}{\left(bd^3-ae^3\right)\left(a+bx^3\right)}\right)dx}{e}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x+bd^2x^2}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)} + \frac{\left(3b^2d^2p\right)\int \frac{x^2}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}de-a^{4/3}e^2\right)\cdots}{a^{2/3}-\sqrt[3]{a}\cdots}}{a^{2/3}e\left(bd^3-\cdots\right.}$$

$$= \frac{\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a\cdots\right.\right.}{e(d+\cdots}$$

$$= \frac{\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(a^{2/3}-\cdots\right.}{2\left(bd^3-ae^3\right)}$$

$$= -\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{bd^3-ae^3} + \frac{\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} - \frac{3bd^2\cdots}{e\left(\cdots\right.}$$

Mathematica [C] time = 0.645978, size = 202, normalized size = 0.69

$$-\frac{\frac{b^{2/3}dp\left(\sqrt[3]{a}e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}d\log\left(a+bx^3\right)-2\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}e\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}d\log(d+ex)\right)+3be^2px^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{2bd^3-2ae^3}\cdots}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -(((3*b*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + b^(2/3)*d*
p*(2*Sqrt[3]*a^(1/3)*e*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*a^(1
/3)*e*Log[a^(1/3) + b^(1/3)*x] + 6*b^(1/3)*d*Log[d + e*x] + a^(1/3)*e*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(1/3)*d*Log[a + b*x^3]))/(2
*b*d^3 - 2*a*e^3) + Log[c*(a + b*x^3)^p]/(d + e*x))/e)
```

Maple [C] time = 0.539, size = 1068, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^2,x)
```

```
[Out] -1/e/(e*x+d)*ln((b*x^3+a)^p)+1/2*(I*Pi*a*e^3*csgn(I*c*(b*x^3+a)^p)^3-I*Pi*b
*d^3*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*a*e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3
+a)^p)*csgn(I*c)-I*Pi*a*e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+I*P
i*b*d^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*b*d^3*csgn(I*(b*x^
3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+I*Pi*b*d^3*csgn(I*c*(b*x^3+a)^p)^2*
csgn(I*c)-I*Pi*a*e^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*sum(_R*ln(((-4*a*e
^7-2*b*d^3*e^4)*_R^3-3*b*d^2*e^3*p*_R^2+8*b*d*e^2*p^2*_R-3*b*e*p^3)*x+(-5*a
*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p^
3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^3
))*a*e^5*x-2*sum(_R*ln(((-4*a*e^7-2*b*d^3*e^4)*_R^3-3*b*d^2*e^3*p*_R^2+8*b*
d*e^2*p^2*_R-3*b*e*p^3)*x+(-5*a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)
*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*
e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^3))*b*d^3*e^2*x+6*ln(-e*x-d)*b*d^2*e*p*x+2*su
m(_R*ln(((-4*a*e^7-2*b*d^3*e^4)*_R^3-3*b*d^2*e^3*p*_R^2+8*b*d*e^2*p^2*_R-3*
b*e*p^3)*x+(-5*a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*b*d^2*e
*p^2*_R-3*b*d*p^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z^2-3*b*
d*e*p^2*_Z+b*p^3))*a*d*e^4-2*sum(_R*ln(((-4*a*e^7-2*b*d^3*e^4)*_R^3-3*b*d^2
*e^3*p*_R^2+8*b*d*e^2*p^2*_R-3*b*e*p^3)*x+(-5*a*d*e^6-b*d^4*e^3)*_R^3+(a*e^
5*p-b*d^3*e^2*p)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p^3),_R=RootOf((a*e^6-b*d^3*e^
3)*_Z^3+3*b*d^2*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^3))*b*d^4*e+6*ln(-e*x-d)*b*d^
3*p-2*ln(c)*a*e^3+2*ln(c)*b*d^3)/(e*x+d)/e/(a*e^3-b*d^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 14.1578, size = 13520, normalized size = 46.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d^4*e - a*d*e^4 + (b*d^3*e^2 - a*e^5)*x)*(2*b*d^2*p/(b*d^3*e - a*
e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*
sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2
 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/
(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p
^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6)
+ 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*log(3/2*(2*b*d^2*p/
(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 -
 a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^
3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) +
 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 -
3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*
e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*
e*p + b*e*p^2*x - 2*b*d*p^2 - 1/4*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e -
 a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(
```

```
-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*
e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p
^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^
3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^
6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2) - 4*(b*d^3 -
a*e^3)*p*log(b*x^3 + a) + (6*b*d^2*e*p*x + 6*b*d^3*p - (b*d^4*e - a*d*e^4 +
 (b*d^3*e^2 - a*e^5)*x)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*
e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(
b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)
) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) -
(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*
d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^
3)^2)^(1/3)*(I*sqrt(3) + 1)) + sqrt(3)*(b*d^4*e - a*d*e^4 + (b*d^3*e^2 - a*
e^5)*x)*sqrt(-((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e
- a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*
(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3
*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*
p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d
^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e
^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*
e - a*b*d^2*e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e
^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e
 - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2
*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^
6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e -
 a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(
1/3)*(I*sqrt(3) + 1))*p + 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a
*b*d^3*e^5 + a^2*e^8)))*log(-3/2*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^
2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*
d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e
 - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)
^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a
*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d
^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*e*p + 2*b*e*p^2*x + 2*b*d*p^2 +
 1/4*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3
*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/
(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4
)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) -
 (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b
*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e
^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 + 1/4*sqrt(3)*(b*d^3*e^2 - a*e^5)*(2*b*d^2*
p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2
 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*
p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6)
 + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3
- 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^
3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*sqrt
(-((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (
b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3)
+ 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5
)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 -
 a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d
^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*
b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*e - a*b*d^2*
e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*
p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3
- 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^
3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3
*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1
```

```
/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt
(3) + 1))*p + 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8))) + (6*b*d^2*e*p*x + 6*b*d^3*p - (b*d^4*e - a*d*e^4 + (b*d^3*e^2
- a*e^5)*x)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2
 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*
e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^
3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3
/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^
4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*
(I*sqrt(3) + 1)) - sqrt(3)*(b*d^4*e - a*d*e^4 + (b*d^3*e^2 - a*e^5)*x)*sqrt
(-((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (
b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3)
+ 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5
)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 -
 a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d
^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*
b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*e - a*b*d^2*
e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*
p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3
 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^
3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3
*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1
/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt
(3) + 1))*p + 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8)))*log(-3/2*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e -
 a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d
^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) +
 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^
3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3
*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^
2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*e*p + 2*b*e*p^2*x + 2*b*d*p^2 + 1/4*(b*d^3*
e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^
2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a
*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p
^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^
3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e
^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)
*(I*sqrt(3) + 1))^2 - 1/4*sqrt(3)*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e -
 a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(
-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*
e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p
^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^
3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^
6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*sqrt(-((b^2*d^6*
e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/
(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^
6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e -
 a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(
1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e
^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3
 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*e - a*b*d^2*e^4)*(2*b*d^
2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e
^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^
3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^
6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^
3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*
d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*p
+ 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)))
- 12*(b*d^2*e*p*x + b*d^3*p)*log(e*x + d) - 4*(b*d^3 - a*e^3)*log(c))/(b*d^
```

```
4*e - a*d*e^4 + (b*d^3*e^2 - a*e^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33915, size = 537, normalized size = 1.84

$$\frac{bd^2p\log\left(\left|bx^3+a\right|\right)}{bd^3e-ae^4}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bp\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2d^2-\left(-ab^2\right)^{\frac{1}{3}}bde+\left(-ab^2\right)^{\frac{2}{3}}e^2}-\frac{\left(ab^3d^4pe^2-ab^3d^3p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^3-a^2b^2dpe^5+a^2b^2p\left(-\frac{a}{b}\right)\right.}{ab^3d^6e^2-2a^2b^2d^3e^5+a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*d^2*p*log(abs(b*x^3 + a))/(b*d^3*e - a*e^4) + sqrt(3)*(-a*b^2)^(1/3)*b*p*
arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(b^2*d^2 - (-a*b^2)^(
1/3)*b*d*e + (-a*b^2)^(2/3)*e^2) - (a*b^3*d^4*p*e^2 - a*b^3*d^3*p*(-a/b)^(1
/3)*e^3 - a^2*b^2*d*p*e^5 + a^2*b^2*p*(-a/b)^(1/3)*e^6)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a*b^3*d^6*e^2 - 2*a^2*b^2*d^3*e^5 + a^3*b*e^8) + 1/2*
((-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*p*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/(b^2*d^3 - a*b*e^3) - (3*b*d^2*p*x*e*log(x*e + d) + b*d^3*p*log(b
*x^3 + a) + 3*b*d^3*p*log(x*e + d) + b*d^3*log(c) - a*p*e^3*log(b*x^3 + a)
- a*e^3*log(c))/(b*d^3*x*e^2 + b*d^4*e - a*x*e^5 - a*d*e^4)
```

3.197 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3} dx$

Optimal. Leaf size=391

$$-\frac{\sqrt[3]{a}b^{2/3}p\left(3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2}+\frac{\sqrt[3]{a}b^{2/3}p\left(3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2}$$

```
[Out] (3*b*d^2*p)/(2*e*(b*d^3 - a*e^3)*(d + e*x)) - (Sqrt[3]*a^(1/3)*b^(2/3)*(2*b
*d^3 - 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(2*(b*d^3 - a*e^3)^2) + (a^(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(
1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*(b*d^3 - a*e^3)^
2) - (3*b*d*(b*d^3 + 2*a*e^3)*p*Log[d + e*x])/(2*e*(b*d^3 - a*e^3)^2) - (a^
(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*(b*d^3 - a*e^3)^2) + (b*d*(b*d^3 + 2*a*
e^3)*p*Log[a + b*x^3])/(2*e*(b*d^3 - a*e^3)^2) - Log[c*(a + b*x^3)^p]/(2*e*
(d + e*x)^2)
```

Rubi [A] time = 0.712848, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2463, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}b^{2/3}p\left(3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2}+\frac{\sqrt[3]{a}b^{2/3}p\left(3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```


```
[Out] (3*b*d^2*p)/(2*e*(b*d^3 - a*e^3)*(d + e*x)) - (Sqrt[3]*a^(1/3)*b^(2/3)*(2*b
*d^3 - 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(2*(b*d^3 - a*e^3)^2) + (a^(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(
1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*(b*d^3 - a*e^3)^
2) - (3*b*d*(b*d^3 + 2*a*e^3)*p*Log[d + e*x])/(2*e*(b*d^3 - a*e^3)^2) - (a^
(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*(b*d^3 - a*e^3)^2) + (b*d*(b*d^3 + 2*a*
e^3)*p*Log[a + b*x^3])/(2*e*(b*d^3 - a*e^3)^2) - Log[c*(a + b*x^3)^p]/(2*e*
(d + e*x)^2)
```


Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3bp)\int \frac{x^2}{(d+ex)^2\left(a+bx^3\right)}\,dx}{2e}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3bp)\int\left(-\frac{d^2e}{\left(bd^3-ae^3\right)(d+ex)^2} - \frac{de\left(bd^3+2ae^3\right)}{\left(bd^3-ae^3\right)^2(d+ex)} + \frac{ae\left(2bd^3+ae^3\right)-3abd^2e^2x+bd\left(bd^3+2a\right.}{\left(bd^3-ae^3\right)^2\left(a+bx^3\right)}\right.}{2e}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3bp)\int \frac{ae\left(2bd^3\right.}{}}{}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3bp)\int \frac{ae\left(2bd^3\right.}{}}{2e\left(bd^3\right.}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} + \frac{bd\left(bd^3+2ae^3\right)p\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2} - \cdots$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} + \frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2} - \frac{3bd\left(bd^3+2a\right.}{2e\left(bd\right.}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} + \frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2} - \frac{3bd\left(bd^3+2a\right.}{2e\left(bd\right.}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{\sqrt{3}\sqrt[3]{a}b^{2/3}\left(2bd^3-3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\left(bd^3-ae^3\right)^2} + \frac{\sqrt[3]{a}b^{2/3}\left(2\right.}{}$$

Mathematica [C] time = 0.706256, size = 303, normalized size = 0.77

$$\frac{\frac{b^{2/3}p(d+ex)\left(-\sqrt[3]{a}e(d+ex)\left(ae^3+2bd^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)-9b^{4/3}d^2e^2x^2(d+ex)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)+2\sqrt[3]{b}d(d+ex)\left(2ae^3+bd^3\right)\log\left(a+bx\right.\cdots}{\left(bd^3-ae^3\right)^2}}{4e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

```
[Out] ((b^(2/3)*p*(d + e*x)*(6*b^(1/3)*d^2*(b*d^3 - a*e^3) - 9*b^(4/3)*d^2*e^2*x^
2*(d + e*x)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + 2*a^(1/3)*e*(2*b
*d^3 + a*e^3)*(d + e*x)*Log[a^(1/3) + b^(1/3)*x] - 6*b^(1/3)*d*(b*d^3 + 2*a
*e^3)*(d + e*x)*Log[d + e*x] - a^(1/3)*e*(2*b*d^3 + a*e^3)*(d + e*x)*(2*Sqr
t[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2]) + 2*b^(1/3)*d*(b*d^3 + 2*a*e^3)*(d + e*x)*Log[a + b
*x^3]))/(b*d^3 - a*e^3)^2 - 2*Log[c*(a + b*x^3)^p])/(4*e*(d + e*x)^2)
```

Maple [C] time = 0.591, size = 4085, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^3,x)

[Out] -1/2/e/(e*x+d)^2*ln((b*x^3+a)^p)+1/4*(6*b^2*d^6*p+2*sum(_R*ln(((-4*a^3*e^13
+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-
4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^
4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+
(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*
a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=R
ootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^
2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a^2*e^9*x^2+2*sum(_R*ln(((-4*a^3*e^1
3+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p
-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e
^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3
+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5
*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=
RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e
^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a^2*d^2*e^7+2*sum(_R*ln(((-4*a^3*e^
13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*
p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*
e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^
3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+
5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R
=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*
e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*b^2*d^8*e-6*ln(e*x+d)*b^2*d^6*p-6*
a*d^3*e^3*b*p-2*I*Pi*a*b*d^3*e^3*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*a^2*e^6*csgn(
I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+2*I*Pi*a*b*d^3*e^3*csgn(I*(b
*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+2*I*Pi*a*b*d^3*e^3*csgn(I*c*(b*x^3+a)^p)
^2*csgn(I*c)-2*ln(c)*b^2*d^6-2*ln(c)*a^2*e^6+6*b^2*d^5*e*p*x+I*Pi*a^2*e^6*c
sgn(I*c*(b*x^3+a)^p)^3+I*Pi*b^2*d^6*csgn(I*c*(b*x^3+a)^p)^3+2*sum(_R*ln(((-
4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*
d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+
3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*
e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*
e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4
*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*
b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*b^2*d^6*e^3*x^2+4*sum(_R*l
n(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a
*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2
)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*
d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a
^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^
3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5
*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a^2*d*e^8*x+4*sum(_R*
ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*
a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^
2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3
*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-
a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b
^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^
5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*b^2*d^7*e^2*x-4*sum(
_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-
10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2
*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-
b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2
+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-
3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d
*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a*b*d^5*e^4+4*ln(
c)*a*b*d^3*e^3-8*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R
^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*
```

```
e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4
*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*
p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*
_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^
6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^
3))*a*b*d^4*e^5*x-6*ln(e*x+d)*b^2*d^4*e^2*p*x^2-12*ln(e*x+d)*b^2*d^5*e*p*x-
12*ln(e*x+d)*a*b*d^3*e^3*p-12*ln(e*x+d)*a*b*d*e^5*p*x^2-24*ln(e*x+d)*a*b*d^
2*e^4*p*x-I*Pi*b^2*d^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*b^2
*d^6*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-6*a*b*d^2*e^4*p*x-2*I*Pi*a*b*d^3*e^3
*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+I*Pi*b^2*d^6*csgn(I*(b
*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a^2*e^6*csgn(I*(b*x^3+a)^p)
*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*a^2*e^6*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-4*s
um(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9
*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*
e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e
^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_
R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p
^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*
b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a*b*d^3*e^6*x^
2)/(e*x+d)^2/(-a*e^3+b*d^3)^2/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 40.5713, size = 26734, normalized size = 68.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*(b^2*d^5*e - a*b*d^2*e^4)*p*x + 2*(b^2*d^8*e - 2*a*b*d^5*e^4 + a^2
*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2
*a*b*d^4*e^5 + a^2*d*e^8)*x)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a
^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^
7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2
*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))
+ 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*
e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d
^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^
3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*
b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9
) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2
*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) +
 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*
log((8*b^2*d^3*e + a*b*e^4)*p^2*x - 3/16*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2
*d^2*e^8)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*
```

p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 + 1/4*(10*b^2*d^6*e + 16*a*b*d^3*e^4 + a^2*e^7)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p - (7*b^2*d^4 + 2*a*b*d*e^3)*p^2) - 8*(b^2*d^6 - 2*a*b*d^3*e^3 + a^2*e^6)*p*log(b*x^3 + a) + 24*(b^2*d^6 - a*b*d^3*e^3)*p + (12*(b^2*d^4*e^2 + 2*a*b*d*e^5)*p*x^2 + 24*(b^2*d^5*e + 2*a*b*d^2*e^4)*p*x - (b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 12*(b^2*d^6 + 2*a*b*d^3*e^3)*p + sqrt(3)*(b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*sqrt(-((b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 - 8*(b^4*d^10*e - 3*a^2*b^2*d^4*e^7 + 2*a^3*b*d*e^10)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^

6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3
- a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 +
a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2
*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))
+ 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*
e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d
^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2
*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p + 16*(b^4*d^8 - 20*a
*b^3*d^5*e^3 - 8*a^2*b^2*d^2*e^6)*p^2)/(b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*
a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)))*log(2*(8*b^2*d^3*e + a*b*e
^4)*p^2*x + 3/16*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*((b^2*d^2*p^2/
(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^
2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p +
2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^
6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6
+ a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2
*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) -
4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*
e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d
^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3
- a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4
+ a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^
6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 - 1/4*(10*b^2*d^6*e + 16*a*b*d^3*e^4 + a^
2*e^7)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p +
2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)
/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e
^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^
6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3
- a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 +
a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2
*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))
+ 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*
e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d
^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2
*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p + (7*b^2*d^4 + 2*a*b
*d*e^3)*p^2 + 1/16*sqrt(3)*(3*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*(
(b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d
*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*
(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2
e^8)(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 -
2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)
^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7
)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2
- 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b
^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b
^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2
*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e
^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 4*(b^2*d^6*e - 2*a*b*d^3*e^4
+ a^2*e^7)*p)*sqrt(-((b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 -
4*a^3*b*d^3*e^11 + a^4*e^14)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e
^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^
2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))
+ 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a
*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*
d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e
^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a
*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^
9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p +

2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) ^2 - 8*(b^4*d^10*e - 3*a^2*b^2*d^4*e^7 + 2*a^3*b*d*e^10)*((b^2*d^2*p^2/(b^2 *d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^ 6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a* b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a ^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4 *p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(- 3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e ^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a *e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^ 2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p + 16*(b^4*d^8 - 20*a*b^3*d^5*e^3 - 8*a^2*b^2* d^2*e^6)*p^2)/(b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b *d^3*e^11 + a^4*e^14))) + (12*(b^2*d^4*e^2 + 2*a*b*d*e^5)*p*x^2 + 24*(b^2*d ^5*e + 2*a*b*d^2*e^4)*p*x - (b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2 *d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2* d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a* b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/ (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/ (b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^ 3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^ 2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2 *e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d ^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3 /(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^ 4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 12*(b^2*d^6 + 2*a*b*d^3*e^3)*p - sqrt(3)*(b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2 *d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*sqrt(-((b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a ^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^ 7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2 *d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a* e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d ^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^ 3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a* b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2 *a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^ 2 - 8*(b^4*d^10*e - 3*a^2*b^2*d^4*e^7 + 2*a^3*b*d*e^10)*((b^2*d^2*p^2/(b^2* d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6 *e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b *d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^ 2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4* p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3 /16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^ 3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a* e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2 *e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p + 16*(b^4*d^8 - 20*a*b^3*d^5*e^3 - 8*a^2*b^2*d

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^2*e^6)*p^2)/(b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*
d^3*e^11 + a^4*e^14)))*log(2*(8*b^2*d^3*e + a*b*e^4)*p^2*x + 3/16*(b^2*d^8*
e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e
^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 +
a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2
/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^
3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/
(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*
b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e
- 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a
^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4
*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqr
t(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7))^2 - 1/4*(10*b^2*d^6*e + 16*a*b*d^3*e^4 + a^2*e^7)*((b^2*d^2*p^2/(b^2*
d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6
*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b
*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e -
 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^
2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*
p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3
/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^
3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*
e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2
*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e -
 2*a*b*d^3*e^4 + a^2*e^7))*p + (7*b^2*d^4 + 2*a*b*d*e^3)*p^2 - 1/16*sqrt(3)
*(3*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*((b^2*d^2*p^2/(b^2*d^6*e^2
- 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a
*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p
)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d
^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) +
 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*
b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2
*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8
)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*
b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 +
 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)
^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d
^3*e^4 + a^2*e^7)) + 4*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)*p)*sqrt(-((b^4
*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^
14)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*
a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-
3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5
+ a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e
^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a
*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^
2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^
6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1
/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3
)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*
e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*
b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 - 8*(b^4*d^10*e - 3*a^2
*b^2*d^4*e^7 + 2*a^3*b*d*e^10)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b
^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)
) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 +
a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2
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*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*
e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*
a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e
^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p +
 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3)
 + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)
)*p + 16*(b^4*d^8 - 20*a*b^3*d^5*e^3 - 8*a^2*b^2*d^2*e^6)*p^2)/(b^4*d^12*e^
2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14))) -
24*((b^2*d^4*e^2 + 2*a*b*d*e^5)*p*x^2 + 2*(b^2*d^5*e + 2*a*b*d^2*e^4)*p*x +
 (b^2*d^6 + 2*a*b*d^3*e^3)*p)*log(e*x + d) - 8*(b^2*d^6 - 2*a*b*d^3*e^3 + a
^2*e^6)*log(c))/(b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2
*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.75418, size = 1079, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^5*d^9*p*e^2 - 3*a*b^5*d^8*p*(-a/b)^(1/3)*e^3 - 3*a^2*b^4*d^6*p*
e^5 + 6*a^2*b^4*d^5*p*(-a/b)^(1/3)*e^6 - 3*a^3*b^3*d^2*p*(-a/b)^(1/3)*e^9 +
 a^4*b^2*p*e^11)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5*d^12*e^2 -
4*a^2*b^4*d^9*e^5 + 6*a^3*b^3*d^6*e^8 - 4*a^4*b^2*d^3*e^11 + a^5*b*e^14) +
1/2*(2*sqrt(3)*(-a*b^2)^(1/3)*a*b^2*d*p - sqrt(3)*(-a*b^2)^(2/3)*a*b*p*e)*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3*d^4 - 2*(-a*b^2
)^(1/3)*a*b^2*d^3*e + 2*a^2*b^2*d*e^3 + 3*(-a*b^2)^(2/3)*a*b*d^2*e^2 - (-a*
b^2)^(1/3)*a^2*b*e^4) + 1/4*(2*(-a*b^2)^(1/3)*b*d^3*p - 3*(-a*b^2)^(2/3)*d^
2*p*e + (-a*b^2)^(1/3)*a*p*e^3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b
^2*d^6 - 2*a*b*d^3*e^3 + a^2*e^6) + 1/2*(b^2*d^4*p + 2*a*b*d*p*e^3)*log(abs
(b*x^3 + a))/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7) - 1/2*(3*b^2*d^4*p*x^2*e
^2*log(x*e + d) + 6*b^2*d^5*p*x*e*log(x*e + d) - 3*b^2*d^5*p*x*e + b^2*d^6*
p*log(b*x^3 + a) + 3*b^2*d^6*p*log(x*e + d) - 3*b^2*d^6*p + b^2*d^6*log(c)
- 2*a*b*d^3*p*e^3*log(b*x^3 + a) + 6*a*b*d*p*x^2*e^5*log(x*e + d) + 12*a*b*
d^2*p*x*e^4*log(x*e + d) + 6*a*b*d^3*p*e^3*log(x*e + d) + 3*a*b*d^2*p*x*e^4
 + 3*a*b*d^3*p*e^3 - 2*a*b*d^3*e^3*log(c) + a^2*p*e^6*log(b*x^3 + a) + a^2*
e^6*log(c))/(b^2*d^6*x^2*e^3 + 2*b^2*d^7*x*e^2 + b^2*d^8*e - 2*a*b*d^3*x^2*
e^6 - 4*a*b*d^4*x*e^5 - 2*a*b*d^5*e^4 + a^2*x^2*e^9 + 2*a^2*d*x*e^8 + a^2*d
^2*e^7)
```

3.198 $\int (d+ex)^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=139

$$\frac{bepx\left(6a^2d^2-4abde+b^2e^2\right)}{4a^3}+\frac{be^2px^2(4ad-be)}{8a^2}-\frac{p(ad-be)^4\log(ax+b)}{4a^4e}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}+\frac{be^3px^3}{12a}+\frac{d^4p\log(x)}{4e}$$

```
[Out] (b*e*(6*a^2*d^2 - 4*a*b*d*e + b^2*e^2)*p*x)/(4*a^3) + (b*e^2*(4*a*d - b*e)*
p*x^2)/(8*a^2) + (b*e^3*p*x^3)/(12*a) + ((d + e*x)^4*Log[c*(a + b/x)^p])/(4
*e) + (d^4*p*Log[x])/(4*e) - ((a*d - b*e)^4*p*Log[b + a*x])/(4*a^4*e)
```

Rubi [A] time = 0.125612, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2463, 514, 72}

$$\frac{bepx\left(6a^2d^2-4abde+b^2e^2\right)}{4a^3}+\frac{be^2px^2(4ad-be)}{8a^2}-\frac{p(ad-be)^4\log(ax+b)}{4a^4e}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}+\frac{be^3px^3}{12a}+\frac{d^4p\log(x)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*(6*a^2*d^2 - 4*a*b*d*e + b^2*e^2)*p*x)/(4*a^3) + (b*e^2*(4*a*d - b*e)*
p*x^2)/(8*a^2) + (b*e^3*p*x^3)/(12*a) + ((d + e*x)^4*Log[c*(a + b/x)^p])/(4
*e) + (d^4*p*Log[x])/(4*e) - ((a*d - b*e)^4*p*Log[b + a*x])/(4*a^4*e)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{\left(a+\frac{b}{x}\right)x^2} dx}{4e}$$

$$= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{x(b+ax)} dx}{4e}$$

$$= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \left(\frac{e^2(6a^2d^2-4abde+b^2e^2)}{a^3} + \frac{d^4}{bx} + \frac{e^3(4ad-be)x}{a^2} + \frac{e^4x^2}{a} - \frac{(a}{a^3 b}\right.}{4e}$$

$$= \frac{be\left(6a^2d^2-4abde+b^2e^2\right)px}{4a^3} + \frac{be^2(4ad-be)px^2}{8a^2} + \frac{be^3px^3}{12a} + \frac{(d+ex)^4 \log\left(c\left(a+\right.\right.}{4e}$$

Mathematica [A] time = 0.140229, size = 114, normalized size = 0.82

$$\frac{\frac{be^2px\left(2a^2\left(18d^2+6dex+e^2x^2\right)-3abe(8d+ex)+6b^2e^2\right)}{6a^3} - \frac{p(ad-be)^4\log(ax+b)}{a^4} + (d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right) + d^4p\log(x)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] ((b*e^2*p*x*(6*b^2*e^2 - 3*a*b*e*(8*d + e*x) + 2*a^2*(18*d^2 + 6*d*e*x + e^
2*x^2)))/(6*a^3) + (d + e*x)^4*Log[c*(a + b/x)^p] + d^4*p*Log[x] - ((a*d -
b*e)^4*p*Log[b + a*x])/a^4)/(4*e)
```

Maple [F] time = 0.582, size = 0, normalized size = 0.

$$\int (ex+d)^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^3*ln(c*(a+b/x)^p),x)
```

Maxima [A] time = 1.05346, size = 224, normalized size = 1.61

$$\frac{1}{24}bp\left(\frac{2a^2e^3x^3 + 3\left(4a^2de^2 - abe^3\right)x^2 + 6\left(6a^2d^2e - 4abde^2 + b^2e^3\right)x}{a^3} + \frac{6\left(4a^3d^3 - 6a^2bd^2e + 4ab^2de^2 - b^3e^3\right)\log(a\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/24*b*p*((2*a^2*e^3*x^3 + 3*(4*a^2*d*e^2 - a*b*e^3)*x^2 + 6*(6*a^2*d^2*e -
 4*a*b*d*e^2 + b^2*e^3)*x)/a^3 + 6*(4*a^3*d^3 - 6*a^2*b*d^2*e + 4*a*b^2*d*e
^2 - b^3*e^3)*log(a*x + b)/a^4) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2
```

```
+ 4*d^3*x)*log((a + b/x)^p*c)
```

Fricas [A] time = 1.68249, size = 498, normalized size = 3.58

$$\frac{2\,a^3be^3px^3 + 3\left(4\,a^3bde^2 - a^2b^2e^3\right)px^2 + 6\left(6\,a^3bd^2e - 4\,a^2b^2de^2 + ab^3e^3\right)px + 6\left(4\,a^3bd^3 - 6\,a^2b^2d^2e + 4\,ab^3de^2 - b^4e^3\right)p\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(2*a^3*b*e^3*p*x^3 + 3*(4*a^3*b*d*e^2 - a^2*b^2*e^3)*p*x^2 + 6*(6*a^3*
b*d^2*e - 4*a^2*b^2*d*e^2 + a*b^3*e^3)*p*x + 6*(4*a^3*b*d^3 - 6*a^2*b^2*d^2
*e + 4*a*b^3*d*e^2 - b^4*e^3)*p*log(a*x + b) + 6*(a^4*e^3*x^4 + 4*a^4*d*e^2
*x^3 + 6*a^4*d^2*e*x^2 + 4*a^4*d^3*x)*log(c) + 6*(a^4*e^3*p*x^4 + 4*a^4*d*e
^2*p*x^3 + 6*a^4*d^2*e*p*x^2 + 4*a^4*d^3*p*x)*log((a*x + b)/x))/a^4
```

Sympy [A] time = 13.9199, size = 484, normalized size = 3.48

$$\begin{cases} d^3px\log\left(a+\frac{b}{x}\right) + d^3x\log(c) + \frac{3d^2epx^2\log\left(a+\frac{b}{x}\right)}{2} + \frac{3d^2ex^2\log(c)}{2} + de^2px^3\log\left(a+\frac{b}{x}\right) + de^2x^3\log(c) + \frac{e^3px^4\log\left(a+\frac{b}{x}\right)}{4} + \frac{e^3x^4\,l}{} \\ d^3px\log(b) - d^3px\log(x) + d^3px + d^3x\log(c) + \frac{3d^2epx^2\log(b)}{2} - \frac{3d^2epx^2\log(x)}{2} + \frac{3d^2epx^2}{4} + \frac{3d^2ex^2\log(c)}{2} + de^2px^3\log(b) - de \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**3*p*x*log(a + b/x) + d**3*x*log(c) + 3*d**2*e*p*x**2*log(a +
b/x)/2 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x**3*log(a + b/x) + d*e**2*x**3*
log(c) + e**3*p*x**4*log(a + b/x)/4 + e**3*x**4*log(c)/4 + b*d**3*p*log(x +
 b/a)/a + 3*b*d**2*e*p*x/(2*a) + b*d*e**2*p*x**2/(2*a) + b*e**3*p*x**3/(12*
a) - 3*b**2*d**2*e*p*log(x + b/a)/(2*a**2) - b**2*d*e**2*p*x/a**2 - b**2*e*
*3*p*x**2/(8*a**2) + b**3*d*e**2*p*log(x + b/a)/a**3 + b**3*e**3*p*x/(4*a**
3) - b**4*e**3*p*log(x + b/a)/(4*a**4), Ne(a, 0)), (d**3*p*x*log(b) - d**3*
p*x*log(x) + d**3*p*x + d**3*x*log(c) + 3*d**2*e*p*x**2*log(b)/2 - 3*d**2*e
*p*x**2*log(x)/2 + 3*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x*
*3*log(b) - d*e**2*p*x**3*log(x) + d*e**2*p*x**3/3 + d*e**2*x**3*log(c) + e
**3*p*x**4*log(b)/4 - e**3*p*x**4*log(x)/4 + e**3*p*x**4/16 + e**3*x**4*log
(c)/4, True))
```

Giac [B] time = 1.23204, size = 437, normalized size = 3.14

$$\frac{6\,a^4px^4e^3\log(ax+b) + 24\,a^4dpx^3e^2\log(ax+b) + 36\,a^4d^2px^2e\log(ax+b) - 6\,a^4px^4e^3\log(x) - 24\,a^4dpx^3e^2\log(x) - 36}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/24*(6*a^4*p*x^4*e^3*log(a*x + b) + 24*a^4*d*p*x^3*e^2*log(a*x + b) + 36*a
^4*d^2*p*x^2*e*log(a*x + b) - 6*a^4*p*x^4*e^3*log(x) - 24*a^4*d*p*x^3*e^2*l
```

```
og(x) - 36*a^4*d^2*p*x^2*e*log(x) + 24*a^4*d^3*p*x*log(a*x + b) + 6*a^4*x^4
*e^3*log(c) + 24*a^4*d*x^3*e^2*log(c) + 36*a^4*d^2*x^2*e*log(c) - 24*a^4*d^
3*p*x*log(x) + 2*a^3*b*p*x^3*e^3 + 12*a^3*b*d*p*x^2*e^2 + 36*a^3*b*d^2*p*x*
e + 24*a^3*b*d^3*p*log(a*x + b) - 36*a^2*b^2*d^2*p*e*log(a*x + b) + 24*a^4*
d^3*x*log(c) - 3*a^2*b^2*p*x^2*e^3 - 24*a^2*b^2*d*p*x*e^2 + 24*a*b^3*d*p*e^
2*log(a*x + b) + 6*a*b^3*p*x*e^3 - 6*b^4*p*e^3*log(a*x + b))/a^4
```

3.199 $\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=102

$$\frac{bepx(3ad-be)}{3a^2}-\frac{p(ad-be)^3\log(ax+b)}{3a^3e}+\frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{be^2px^2}{6a}+\frac{d^3p\log(x)}{3e}$$

```
[Out] (b*e*(3*a*d - b*e)*p*x)/(3*a^2) + (b*e^2*p*x^2)/(6*a) + ((d + e*x)^3*Log[c*
(a + b/x)^p])/(3*e) + (d^3*p*Log[x])/(3*e) - ((a*d - b*e)^3*p*Log[b + a*x])
/(3*a^3*e)
```

Rubi [A] time = 0.093517, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2463, 514, 72}

$$\frac{bepx(3ad-be)}{3a^2}-\frac{p(ad-be)^3\log(ax+b)}{3a^3e}+\frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{be^2px^2}{6a}+\frac{d^3p\log(x)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```


```
[Out] (b*e*(3*a*d - b*e)*p*x)/(3*a^2) + (b*e^2*p*x^2)/(6*a) + ((d + e*x)^3*Log[c*
(a + b/x)^p])/(3*e) + (d^3*p*Log[x])/(3*e) - ((a*d - b*e)^3*p*Log[b + a*x])
/(3*a^3*e)
```


Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{\left(a+\frac{b}{x}\right)x^2} dx}{3e}$$
$$= \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{x(b+ax)} dx}{3e}$$
$$= \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \left(\frac{e^2(3ad-be)}{a^2} + \frac{d^3}{bx} + \frac{e^3x}{a} - \frac{(ad-be)^3}{a^2b(b+ax)}\right) dx}{3e}$$
$$= \frac{be(3ad-be)px}{3a^2} + \frac{be^2px^2}{6a} + \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{d^3p\log(x)}{3e} - \frac{(ad-be)^3p\,\mathrm{lo}}{3a^3}$$

Mathematica [A] time = 0.0813943, size = 86, normalized size = 0.84

$$\frac{2a^3(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + p\left(2a^3d^3\log(x) + abe^2x(6ad+aex-2be) - 2(ad-be)^3\log(ax+b)\right)}{6a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (2*a^3*(d + e*x)^3*Log[c*(a + b/x)^p] + p*(a*b*e^2*x*(6*a*d - 2*b*e + a*e*x
) + 2*a^3*d^3*Log[x] - 2*(a*d - b*e)^3*Log[b + a*x]))/(6*a^3*e)
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int (ex+d)^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^2*ln(c*(a+b/x)^p),x)
```

Maxima [A] time = 1.03793, size = 138, normalized size = 1.35

$$\frac{1}{6}bp\left(\frac{ae^2x^2 + 2\left(3\,ade - be^2\right)x}{a^2} + \frac{2\left(3\,a^2d^2 - 3\,abde + b^2e^2\right)\log(ax+b)}{a^3}\right) + \frac{1}{3}\left(e^2x^3 + 3\,dex^2 + 3\,d^2x\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/6*b*p*((a*e^2*x^2 + 2*(3*a*d*e - b*e^2)*x)/a^2 + 2*(3*a^2*d^2 - 3*a*b*d*e
 + b^2*e^2)*log(a*x + b)/a^3) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((a
+ b/x)^p*c)
```

Fricas [A] time = 1.7112, size = 329, normalized size = 3.23

$$\frac{a^2be^2px^2 + 2\left(3\,a^2bde - ab^2e^2\right)px + 2\left(3\,a^2bd^2 - 3\,ab^2de + b^3e^2\right)p\log\left(ax+b\right) + 2\left(a^3e^2x^3 + 3\,a^3dex^2 + 3\,a^3d^2x\right)\log\left(c\right) + }{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*b*e^2*p*x^2 + 2*(3*a^2*b*d*e - a*b^2*e^2)*p*x + 2*(3*a^2*b*d^2 - 3
*a*b^2*d*e + b^3*e^2)*p*log(a*x + b) + 2*(a^3*e^2*x^3 + 3*a^3*d*e*x^2 + 3*a
^3*d^2*x)*log(c) + 2*(a^3*e^2*p*x^3 + 3*a^3*d*e*p*x^2 + 3*a^3*d^2*p*x)*log(
(a*x + b)/x))/a^3
```

Sympy [A] time = 8.27237, size = 298, normalized size = 2.92

$$\begin{cases} d^2px\log\left(a+\frac{b}{x}\right) + d^2x\log\left(c\right) + depx^2\log\left(a+\frac{b}{x}\right) + dex^2\log\left(c\right) + \frac{e^2px^3\log\left(a+\frac{b}{x}\right)}{3} + \frac{e^2x^3\log\left(c\right)}{3} + \frac{bd^2p\log\left(x+\frac{b}{a}\right)}{a} + \frac{bdepx}{a} + \\ d^2px\log\left(b\right) - d^2px\log\left(x\right) + d^2px + d^2x\log\left(c\right) + depx^2\log\left(b\right) - depx^2\log\left(x\right) + \frac{depx^2}{2} + dex^2\log\left(c\right) + \frac{e^2px^3\log\left(b\right)}{3} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**2*p*x*log(a + b/x) + d**2*x*log(c) + d*e*p*x**2*log(a + b/x)
+ d*e*x**2*log(c) + e**2*p*x**3*log(a + b/x)/3 + e**2*x**3*log(c)/3 + b*d**
2*p*log(x + b/a)/a + b*d*e*p*x/a + b*e**2*p*x**2/(6*a) - b**2*d*e*p*log(x +
 b/a)/a**2 - b**2*e**2*p*x/(3*a**2) + b**3*e**2*p*log(x + b/a)/(3*a**3), Ne
(a, 0)), (d**2*p*x*log(b) - d**2*p*x*log(x) + d**2*p*x + d**2*x*log(c) + d*
e*p*x**2*log(b) - d*e*p*x**2*log(x) + d*e*p*x**2/2 + d*e*x**2*log(c) + e**2
*p*x**3*log(b)/3 - e**2*p*x**3*log(x)/3 + e**2*p*x**3/9 + e**2*x**3*log(c)/
3, True))
```

Giac [B] time = 1.15794, size = 284, normalized size = 2.78

$$2\,a^3px^3e^2\log\left(ax+b\right) + 6\,a^3dpx^2e\log\left(ax+b\right) - 2\,a^3px^3e^2\log\left(x\right) - 6\,a^3dpx^2e\log\left(x\right) + 6\,a^3d^2px\log\left(ax+b\right) + 2\,a^3x^3e^2\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/6*(2*a^3*p*x^3*e^2*log(a*x + b) + 6*a^3*d*p*x^2*e*log(a*x + b) - 2*a^3*p*
x^3*e^2*log(x) - 6*a^3*d*p*x^2*e*log(x) + 6*a^3*d^2*p*x*log(a*x + b) + 2*a^
3*x^3*e^2*log(c) + 6*a^3*d*x^2*e*log(c) - 6*a^3*d^2*p*x*log(x) + a^2*b*p*x^
2*e^2 + 6*a^2*b*d*p*x*e + 6*a^2*b*d^2*p*log(a*x + b) - 6*a*b^2*d*p*e*log(a*
x + b) + 6*a^3*d^2*x*log(c) - 2*a*b^2*p*x*e^2 + 2*b^3*p*e^2*log(a*x + b))/a
^3
```

3.200 $\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal. Leaf size=78

$$-\frac{p(ad-be)^2\log(ax+b)}{2a^2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{bepx}{2a}+\frac{d^2p\log(x)}{2e}$$

```
[Out] (b*e*p*x)/(2*a) + ((d + e*x)^2*Log[c*(a + b/x)^p])/(2*e) + (d^2*p*Log[x])/(
2*e) - ((a*d - b*e)^2*p*Log[b + a*x])/(2*a^2*e)
```

Rubi [A] time = 0.0561594, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2463, 514, 72}

$$-\frac{p(ad-be)^2\log(ax+b)}{2a^2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{bepx}{2a}+\frac{d^2p\log(x)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*p*x)/(2*a) + ((d + e*x)^2*Log[c*(a + b/x)^p])/(2*e) + (d^2*p*Log[x])/(
2*e) - ((a*d - b*e)^2*p*Log[b + a*x])/(2*a^2*e)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx = \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{(bp)\int\frac{(d+ex)^2}{\left(a+\frac{b}{x}\right)x^2}dx}{2e}$$
$$= \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{(bp)\int\frac{(d+ex)^2}{x(b+ax)}dx}{2e}$$
$$= \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{(bp)\int\left(\frac{e^2}{a}+\frac{d^2}{bx}-\frac{(ad-be)^2}{ab(b+ax)}\right)dx}{2e}$$
$$= \frac{bepx}{2a} + \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{d^2p\log(x)}{2e} - \frac{(ad-be)^2p\log(b+ax)}{2a^2e}$$

Mathematica [A] time = 0.0269028, size = 85, normalized size = 1.09

$$\frac{1}{2}bep\left(\frac{x}{a}-\frac{b\log(ax+b)}{a^2}\right)+dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bdp\log\left(a+\frac{b}{x}\right)}{a}+\frac{bdp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*d*p*Log[a + b/x])/a + d*x*Log[c*(a + b/x)^p] + (e*x^2*Log[c*(a + b/x)^p]
)/2 + (b*d*p*Log[x])/a + (b*e*p*(x/a - (b*Log[b + a*x])/a^2))/2
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int (ex+d)\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)*ln(c*(a+b/x)^p),x)
```

Maxima [A] time = 1.02641, size = 74, normalized size = 0.95

$$\frac{1}{2}bp\left(\frac{ex}{a}+\frac{(2ad-be)\log(ax+b)}{a^2}\right)+\frac{1}{2}\left(ex^2+2dx\right)\log\left(\left(a+\frac{b}{x}\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(e*x/a + (2*a*d - b*e)*log(a*x + b)/a^2) + 1/2*(e*x^2 + 2*d*x)*log(
(a + b/x)^p*c)
```

Fricas [A] time = 1.68494, size = 184, normalized size = 2.36

$$\frac{abepx + \left(2\,abd - b^2e\right)p\log\left(ax+b\right) + \left(a^2ex^2 + 2\,a^2dx\right)\log\left(c\right) + \left(a^2epx^2 + 2\,a^2dpx\right)\log\left(\frac{ax+b}{x}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*e*p*x + (2*a*b*d - b^2*e)*p*log(a*x + b) + (a^2*e*x^2 + 2*a^2*d*x)
*log(c) + (a^2*e*p*x^2 + 2*a^2*d*p*x)*log((a*x + b)/x))/a^2
```

Sympy [A] time = 3.9264, size = 156, normalized size = 2.

$$\begin{cases} dpx\log\left(a+\frac{b}{x}\right) + dx\log\left(c\right) + \frac{epx^2\log\left(a+\frac{b}{x}\right)}{2} + \frac{ex^2\log\left(c\right)}{2} + \frac{bdp\log\left(x+\frac{b}{a}\right)}{a} + \frac{bepx}{2a} - \frac{b^2ep\log\left(x+\frac{b}{a}\right)}{2a^2} & \text{for } a \neq 0 \\ dpx\log\left(b\right) - dpx\log\left(x\right) + dpx + dx\log\left(c\right) + \frac{epx^2\log\left(b\right)}{2} - \frac{epx^2\log\left(x\right)}{2} + \frac{epx^2}{4} + \frac{ex^2\log\left(c\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d*p*x*log(a + b/x) + d*x*log(c) + e*p*x**2*log(a + b/x)/2 + e*x*
*2*log(c)/2 + b*d*p*log(x + b/a)/a + b*e*p*x/(2*a) - b**2*e*p*log(x + b/a)/
(2*a**2), Ne(a, 0)), (d*p*x*log(b) - d*p*x*log(x) + d*p*x + d*x*log(c) + e*
p*x**2*log(b)/2 - e*p*x**2*log(x)/2 + e*p*x**2/4 + e*x**2*log(c)/2, True))
```

Giac [A] time = 1.26641, size = 151, normalized size = 1.94

$$\frac{a^2px^2e\log\left(ax+b\right) - a^2px^2e\log\left(x\right) + 2\,a^2dpx\log\left(ax+b\right) + a^2x^2e\log\left(c\right) - 2\,a^2dpx\log\left(x\right) + abpxe + 2\,abdp\log\left(ax+\right.}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*p*x^2*e*log(a*x + b) - a^2*p*x^2*e*log(x) + 2*a^2*d*p*x*log(a*x +
b) + a^2*x^2*e*log(c) - 2*a^2*d*p*x*log(x) + a*b*p*x*e + 2*a*b*d*p*log(a*x
+ b) - b^2*p*e*log(a*x + b) + 2*a^2*d*x*log(c))/a^2
```

3.201 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=113

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\log\left(-\right.}{}$$

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rubi [A] time = 0.157569, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(ap)\int \frac{\log(d+ex)}{b+ax}\,dx}{e}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - p\int$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\mathrm{Li}_2}{}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.0245303, size = 114, normalized size = 1.01

$$-\frac{p\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e} + \frac{p\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} + \frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e} + \frac{p\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e - (p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Maple [F] time = 0.503, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [A] time = 1.08825, size = 215, normalized size = 1.9

$$\frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b}-\frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b}+\frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e}-\frac{p\log(ex+d)\log\left(a+\frac{b}{x}\right)}{e}+\frac{\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x +
 d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e + log
((a + b/x)^p*c)*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^{p}\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

3.202 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2} dx$

Optimal. Leaf size=81

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}+\frac{ap\log(ax+b)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}-\frac{p\log(x)}{de}$$

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Rubi [A] time = 0.0774764, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2463, 514, 72}

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}+\frac{ap\log(ax+b)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}-\frac{p\log(x)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)}\,dx}{e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)}\,dx}{e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \left(\frac{1}{bdx} + \frac{a^2}{b(-ad+be)(b+ax)} + \frac{e^2}{d(ad-be)(d+ex)}\right)dx}{e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{p\log(x)}{de} + \frac{ap\log(b+ax)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)}$$

Mathematica [A] time = 0.0658213, size = 81, normalized size = 1.

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} + \frac{ap\log(ax+b)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)} - \frac{p\log(x)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Maple [F] time = 0.543, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2} \ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^2,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^2,x)
```

Maxima [A] time = 1.01932, size = 115, normalized size = 1.42

$$\frac{bp\left(\frac{a\log(ax+b)}{abd-b^2e} - \frac{e\log(ex+d)}{ad^2-bde} - \frac{\log(x)}{bd}\right)}{e} - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(a*log(a*x + b)/(a*b*d - b^2*e) - e*log(e*x + d)/(a*d^2 - b*d*e) - log(
x)/(b*d))/e - log((a + b/x)^p*c)/((e*x + d)*e)
```

Fricas [A] time = 2.34873, size = 320, normalized size = 3.95

$$-\frac{\left(ad^2 - bde\right)p\log\left(\frac{ax+b}{x}\right) - \left(adepx + ad^2p\right)\log\left(ax+b\right) + \left(be^2px + bdep\right)\log\left(ex+d\right) + \left(ad^2 - bde\right)\log\left(c\right) + \left(\left(ade - be^2\right)\right.}{ad^3e - bd^2e^2 + \left(ad^2e^2 - bde^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -((a*d^2 - b*d*e)*p*log((a*x + b)/x) - (a*d*e*p*x + a*d^2*p)*log(a*x + b) +
 (b*e^2*p*x + b*d*e*p)*log(e*x + d) + (a*d^2 - b*d*e)*log(c) + ((a*d*e - b*
e^2)*p*x + (a*d^2 - b*d*e)*p)*log(x))/(a*d^3*e - b*d^2*e^2 + (a*d^2*e^2 - b
*d*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25694, size = 163, normalized size = 2.01

$$\frac{adpxe\log\left(ax+b\right) - adpxe\log\left(x\right) + bdpe\log\left(ax+b\right) - bpxe^2\log\left(xe+d\right) - bdpe\log\left(xe+d\right) + bpxe^2\log\left(x\right) - ad^2\log\left(c\right.}{ad^2xe^2 + ad^3e - bdxe^3 - bd^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (a*d*p*x*e*log(a*x + b) - a*d*p*x*e*log(x) + b*d*p*e*log(a*x + b) - b*p*x*e
^2*log(x*e + d) - b*d*p*e*log(x*e + d) + b*p*x*e^2*log(x) - a*d^2*log(c) +
b*d*e*log(c))/(a*d^2*x*e^2 + a*d^3*e - b*d*x*e^3 - b*d^2*e^2)
```

3.203 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3} dx$

Optimal. Leaf size=127

$$\frac{a^2p\log(ax+b)}{2e(ad-be)^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{bp(2ad-be)\log(d+ex)}{2d^2(ad-be)^2}+\frac{bp}{2d(d+ex)(ad-be)}-\frac{p\log(x)}{2d^2e}$$

```
[Out] (b*p)/(2*d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(2*e*(d + e*x)^2) -
(p*Log[x])/(2*d^2*e) + (a^2*p*Log[b + a*x])/(2*e*(a*d - b*e)^2) - (b*(2*a*d
 - b*e)*p*Log[d + e*x])/(2*d^2*(a*d - b*e)^2)
```

Rubi [A] time = 0.11854, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2463, 514, 72}

$$\frac{a^2p\log(ax+b)}{2e(ad-be)^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{bp(2ad-be)\log(d+ex)}{2d^2(ad-be)^2}+\frac{bp}{2d(d+ex)(ad-be)}-\frac{p\log(x)}{2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

```
[Out] (b*p)/(2*d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(2*e*(d + e*x)^2) -
(p*Log[x])/(2*d^2*e) + (a^2*p*Log[b + a*x])/(2*e*(a*d - b*e)^2) - (b*(2*a*d
 - b*e)*p*Log[d + e*x])/(2*d^2*(a*d - b*e)^2)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^2}\,dx}{2e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)^2}\,dx}{2e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \left(\frac{1}{bd^2x} - \frac{a^3}{b(-ad+be)^2(b+ax)} + \frac{e^2}{d(ad-be)(d+ex)^2} + \frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)}\right)dx}{2e}$$
$$= \frac{bp}{2d(ad-be)(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{p\log(x)}{2d^2e} + \frac{a^2p\log(b+ax)}{2e(ad-be)^2} - \frac{b(2ad-be)p\log(d+ex)}{2d^2(ad-be)^2}$$

Mathematica [A] time = 0.196341, size = 113, normalized size = 0.89

$$\frac{\frac{a^2p\log(ax+b)}{(ad-be)^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2} + \frac{bep(be-2ad)\log(d+ex)}{d^2(ad-be)^2} + \frac{bep}{d(d+ex)(ad-be)} - \frac{p\log(x)}{d^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

```
[Out] ((b*e*p)/(d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^2 - (p*Lo
g[x])/d^2 + (a^2*p*Log[b + a*x])/(a*d - b*e)^2 + (b*e*(-2*a*d + b*e)*p*Log[
d + e*x])/(d^2*(a*d - b*e)^2))/(2*e)
```

Maple [F] time = 0.544, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^3}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^3,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^3,x)
```

Maxima [A] time = 1.05119, size = 216, normalized size = 1.7

$$\frac{\left(\frac{a^2\log(ax+b)}{a^2bd^2-2\,ab^2de+b^3e^2} - \frac{(2\,ade-be^2)\log(ex+d)}{a^2d^4-2\,abd^3e+b^2d^2e^2} + \frac{e}{ad^3-bd^2e+(ad^2e-bde^2)x} - \frac{\log(x)}{bd^2}\right)bp}{2\,e} - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{2\,(ex+d)^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*log(a*x + b)/(a^2*b*d^2 - 2*a*b^2*d*e + b^3*e^2) - (2*a*d*e - b*e^
2)*log(e*x + d)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2) + e/(a*d^3 - b*d^2*e
```

```
+ (a*d^2*e - b*d*e^2)*x) - log(x)/(b*d^2))*b*p/e - 1/2*log((a + b/x)^p*c)/(
(e*x + d)^2*e)
```

Fricas [B] time = 6.63349, size = 864, normalized size = 6.8

$$\left(abd^2e^2 - b^2de^3\right)px - \left(a^2d^4 - 2\,abd^3e + b^2d^2e^2\right)p\log\left(\frac{ax+b}{x}\right) + \left(abd^3e - b^2d^2e^2\right)p + \left(a^2d^2e^2px^2 + 2\,a^2d^3epx + a^2d^4p\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a*b*d^2*e^2 - b^2*d*e^3)*p*x - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*
p*log((a*x + b)/x) + (a*b*d^3*e - b^2*d^2*e^2)*p + (a^2*d^2*e^2*p*x^2 + 2*a
^2*d^3*e*p*x + a^2*d^4*p)*log(a*x + b) - ((2*a*b*d*e^3 - b^2*e^4)*p*x^2 + 2
*(2*a*b*d^2*e^2 - b^2*d*e^3)*p*x + (2*a*b*d^3*e - b^2*d^2*e^2)*p)*log(e*x +
 d) - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*log(c) - ((a^2*d^2*e^2 - 2*a*b*
d*e^3 + b^2*e^4)*p*x^2 + 2*(a^2*d^3*e - 2*a*b*d^2*e^2 + b^2*d*e^3)*p*x + (a
^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*p)*log(x))/(a^2*d^6*e - 2*a*b*d^5*e^2 +
 b^2*d^4*e^3 + (a^2*d^4*e^3 - 2*a*b*d^3*e^4 + b^2*d^2*e^5)*x^2 + 2*(a^2*d^5
*e^2 - 2*a*b*d^4*e^3 + b^2*d^3*e^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2969, size = 579, normalized size = 4.56

$$a^2d^2px^2e^2\log\left(ax+b\right) + 2\,a^2d^3pxe\log\left(ax+b\right) - a^2d^2px^2e^2\log\left(x\right) - 2\,a^2d^3pxe\log\left(x\right) + 2\,abd^3pe\log\left(ax+b\right) - 2\,abd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*p*x^2*e^2*log(a*x + b) + 2*a^2*d^3*p*x*e*log(a*x + b) - a^2*d^
2*p*x^2*e^2*log(x) - 2*a^2*d^3*p*x*e*log(x) + 2*a*b*d^3*p*e*log(a*x + b) -
2*a*b*d*p*x^2*e^3*log(x*e + d) - 4*a*b*d^2*p*x*e^2*log(x*e + d) - 2*a*b*d^3
*p*e*log(x*e + d) + 2*a*b*d*p*x^2*e^3*log(x) + 4*a*b*d^2*p*x*e^2*log(x) + a
*b*d^2*p*x*e^2 + a*b*d^3*p*e - b^2*d^2*p*e^2*log(a*x + b) + b^2*p*x^2*e^4*l
og(x*e + d) + 2*b^2*d*p*x*e^3*log(x*e + d) + b^2*d^2*p*e^2*log(x*e + d) - a
^2*d^4*log(c) + 2*a*b*d^3*e*log(c) - b^2*p*x^2*e^4*log(x) - 2*b^2*d*p*x*e^3
*log(x) - b^2*d*p*x*e^3 - b^2*d^2*p*e^2 - b^2*d^2*e^2*log(c))/(a^2*d^4*x^2*
e^3 + 2*a^2*d^5*x*e^2 + a^2*d^6*e - 2*a*b*d^3*x^2*e^4 - 4*a*b*d^4*x*e^3 - 2
*a*b*d^5*e^2 + b^2*d^2*x^2*e^5 + 2*b^2*d^3*x*e^4 + b^2*d^4*e^3)
```

3.204 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$

Optimal. Leaf size=175

$$-\frac{bp\left(3a^2d^2-3abde+b^2e^2\right)\log(d+ex)}{3d^3(ad-be)^3}+\frac{a^3p\log(ax+b)}{3e(ad-be)^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}+\frac{bp(2ad-be)}{3d^2(d+ex)(ad-be)^2}+\frac{bp}{6d(d+ex)^2(ad-$$

```
[Out] (b*p)/(6*d*(a*d - b*e)*(d + e*x)^2) + (b*(2*a*d - b*e)*p)/(3*d^2*(a*d - b*e
)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(3*e*(d + e*x)^3) - (p*Log[x])/(3*d^3*e
) + (a^3*p*Log[b + a*x])/(3*e*(a*d - b*e)^3) - (b*(3*a^2*d^2 - 3*a*b*d*e +
b^2*e^2)*p*Log[d + e*x])/(3*d^3*(a*d - b*e)^3)
```

Rubi [A] time = 0.171521, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2463, 514, 72}

$$-\frac{bp\left(3a^2d^2-3abde+b^2e^2\right)\log(d+ex)}{3d^3(ad-be)^3}+\frac{a^3p\log(ax+b)}{3e(ad-be)^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}+\frac{bp(2ad-be)}{3d^2(d+ex)(ad-be)^2}+\frac{bp}{6d(d+ex)^2(ad-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```


```
[Out] (b*p)/(6*d*(a*d - b*e)*(d + e*x)^2) + (b*(2*a*d - b*e)*p)/(3*d^2*(a*d - b*e
)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(3*e*(d + e*x)^3) - (p*Log[x])/(3*d^3*e
) + (a^3*p*Log[b + a*x])/(3*e*(a*d - b*e)^3) - (b*(3*a^2*d^2 - 3*a*b*d*e +
b^2*e^2)*p*Log[d + e*x])/(3*d^3*(a*d - b*e)^3)
```


Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^3}dx}{3e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)^3}dx}{3e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{(bp)\int \left(\frac{1}{bd^3x} + \frac{a^4}{b(-ad+be)^3(b+ax)} + \frac{e^2}{d(ad-be)(d+ex)^3} + \frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)^2} + \frac{e^2(3a^2d^2\cdots}{d^3(ad-\cdots}\right.}{3e}$$
$$= \frac{bp}{6d(ad-be)(d+ex)^2} + \frac{b(2ad-be)p}{3d^2(ad-be)^2(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{p\log(x)}{3d^3e} + \frac{a^3p\log(b+a\cdots}{3e(ad-be)^3}$$

Mathematica [A] time = 0.282941, size = 164, normalized size = 0.94

$$\frac{-\frac{bep(3a^2d^2-3abde+b^2e^2)\log(d+ex)}{d^3(ad-be)^3} + \frac{a^3p\log(ax+b)}{(ad-be)^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3} + \frac{bep(2ad-be)}{d^2(d+ex)(ad-be)^2} + \frac{bep}{2d(d+ex)^2(ad-be)} - \frac{p\log(x)}{d^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

```
[Out] ((b*e*p)/(2*d*(a*d - b*e)*(d + e*x)^2) + (b*e*(2*a*d - b*e)*p)/(d^2*(a*d -
b*e)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^3 - (p*Log[x])/d^3 + (a^3*
p*Log[b + a*x])/(a*d - b*e)^3 - (b*e*(3*a^2*d^2 - 3*a*b*d*e + b^2*e^2)*p*Lo
g[d + e*x])/(d^3*(a*d - b*e)^3))/(3*e)
```

Maple [F] time = 0.539, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^4}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^4,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^4,x)
```

Maxima [A] time = 1.0883, size = 404, normalized size = 2.31

$$\frac{\left(\frac{2a^3\log(ax+b)}{a^3bd^3-3a^2b^2d^2e+3ab^3de^2-b^4e^3} - \frac{2(3a^2d^2e-3abde^2+b^2e^3)\log(ex+d)}{a^3d^6-3a^2bd^5e+3ab^2d^4e^2-b^3d^3e^3} + \frac{5ad^2e-3bde^2+2(2ade^2-be^3)x}{a^2d^6-2abd^5e+b^2d^4e^2+(a^2d^4e^2-2abd^3e^3+b^2d^2e^4)x^2+2(a^2d^5e-2abd^4e^2+b^2d^3e\cdots}\right.}{6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*log(a*x + b)/(a^3*b*d^3 - 3*a^2*b^2*d^2*e + 3*a*b^3*d*e^2 - b^4*
e^3) - 2*(3*a^2*d^2*e - 3*a*b*d*e^2 + b^2*e^3)*log(e*x + d)/(a^3*d^6 - 3*a^
2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) + (5*a*d^2*e - 3*b*d*e^2 + 2*(2*
a*d*e^2 - b*e^3)*x)/(a^2*d^6 - 2*a*b*d^5*e + b^2*d^4*e^2 + (a^2*d^4*e^2 - 2
*a*b*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*(a^2*d^5*e - 2*a*b*d^4*e^2 + b^2*d^3*e^
3)*x) - 2*log(x)/(b*d^3))*b*p/e - 1/3*log((a + b/x)^p*c)/((e*x + d)^3*e)
```

Fricas [B] time = 43.0068, size = 1629, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*d*e^5)*p*x^2 + (9*a^2*b*d^4
*e^2 - 14*a*b^2*d^3*e^3 + 5*b^3*d^2*e^4)*p*x - 2*(a^3*d^6 - 3*a^2*b*d^5*e +
 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*p*log((a*x + b)/x) + (5*a^2*b*d^5*e - 8*a*b
^2*d^4*e^2 + 3*b^3*d^3*e^3)*p + 2*(a^3*d^3*e^3*p*x^3 + 3*a^3*d^4*e^2*p*x^2
+ 3*a^3*d^5*e*p*x + a^3*d^6*p)*log(a*x + b) - 2*((3*a^2*b*d^2*e^4 - 3*a*b^2
*d*e^5 + b^3*e^6)*p*x^3 + 3*(3*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*d*e^5)
*p*x^2 + 3*(3*a^2*b*d^4*e^2 - 3*a*b^2*d^3*e^3 + b^3*d^2*e^4)*p*x + (3*a^2*b
*d^5*e - 3*a*b^2*d^4*e^2 + b^3*d^3*e^3)*p)*log(e*x + d) - 2*(a^3*d^6 - 3*a^
2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*log(c) - 2*((a^3*d^3*e^3 - 3*a^2
*b*d^2*e^4 + 3*a*b^2*d*e^5 - b^3*e^6)*p*x^3 + 3*(a^3*d^4*e^2 - 3*a^2*b*d^3*
e^3 + 3*a*b^2*d^2*e^4 - b^3*d*e^5)*p*x^2 + 3*(a^3*d^5*e - 3*a^2*b*d^4*e^2 +
 3*a*b^2*d^3*e^3 - b^3*d^2*e^4)*p*x + (a^3*d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^
4*e^2 - b^3*d^3*e^3)*p)*log(x))/(a^3*d^9*e - 3*a^2*b*d^8*e^2 + 3*a*b^2*d^7*
e^3 - b^3*d^6*e^4 + (a^3*d^6*e^4 - 3*a^2*b*d^5*e^5 + 3*a*b^2*d^4*e^6 - b^3*
d^3*e^7)*x^3 + 3*(a^3*d^7*e^3 - 3*a^2*b*d^6*e^4 + 3*a*b^2*d^5*e^5 - b^3*d^4
*e^6)*x^2 + 3*(a^3*d^8*e^2 - 3*a^2*b*d^7*e^3 + 3*a*b^2*d^6*e^4 - b^3*d^5*e^
5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2848, size = 1208, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*a^3*d^3*p*x^3*e^3*log(a*x + b) + 6*a^3*d^4*p*x^2*e^2*log(a*x + b) +
6*a^3*d^5*p*x*e*log(a*x + b) - 2*a^3*d^3*p*x^3*e^3*log(x) - 6*a^3*d^4*p*x^2
```

```
*e^2*log(x) - 6*a^3*d^5*p*x*e*log(x) + 6*a^2*b*d^5*p*e*log(a*x + b) - 6*a^2
*b*d^2*p*x^3*e^4*log(x*e + d) - 18*a^2*b*d^3*p*x^2*e^3*log(x*e + d) - 18*a^
2*b*d^4*p*x*e^2*log(x*e + d) - 6*a^2*b*d^5*p*e*log(x*e + d) + 6*a^2*b*d^2*p
*x^3*e^4*log(x) + 18*a^2*b*d^3*p*x^2*e^3*log(x) + 18*a^2*b*d^4*p*x*e^2*log(
x) + 4*a^2*b*d^3*p*x^2*e^3 + 9*a^2*b*d^4*p*x*e^2 + 5*a^2*b*d^5*p*e - 6*a*b^
2*d^4*p*e^2*log(a*x + b) + 6*a*b^2*d*p*x^3*e^5*log(x*e + d) + 18*a*b^2*d^2*
p*x^2*e^4*log(x*e + d) + 18*a*b^2*d^3*p*x*e^3*log(x*e + d) + 6*a*b^2*d^4*p*
e^2*log(x*e + d) - 2*a^3*d^6*log(c) + 6*a^2*b*d^5*e*log(c) - 6*a*b^2*d*p*x^
3*e^5*log(x) - 18*a*b^2*d^2*p*x^2*e^4*log(x) - 18*a*b^2*d^3*p*x*e^3*log(x)
- 6*a*b^2*d^2*p*x^2*e^4 - 14*a*b^2*d^3*p*x*e^3 - 8*a*b^2*d^4*p*e^2 + 2*b^3*
d^3*p*e^3*log(a*x + b) - 2*b^3*p*x^3*e^6*log(x*e + d) - 6*b^3*d*p*x^2*e^5*l
og(x*e + d) - 6*b^3*d^2*p*x*e^4*log(x*e + d) - 2*b^3*d^3*p*e^3*log(x*e + d)
 - 6*a*b^2*d^4*e^2*log(c) + 2*b^3*p*x^3*e^6*log(x) + 6*b^3*d*p*x^2*e^5*log(
x) + 6*b^3*d^2*p*x*e^4*log(x) + 2*b^3*d*p*x^2*e^5 + 5*b^3*d^2*p*x*e^4 + 3*b
^3*d^3*p*e^3 + 2*b^3*d^3*e^3*log(c))/(a^3*d^6*x^3*e^4 + 3*a^3*d^7*x^2*e^3 +
 3*a^3*d^8*x*e^2 + a^3*d^9*e - 3*a^2*b*d^5*x^3*e^5 - 9*a^2*b*d^6*x^2*e^4 -
9*a^2*b*d^7*x*e^3 - 3*a^2*b*d^8*e^2 + 3*a*b^2*d^4*x^3*e^6 + 9*a*b^2*d^5*x^2
*e^5 + 9*a*b^2*d^6*x*e^4 + 3*a*b^2*d^7*e^3 - b^3*d^3*x^3*e^7 - 3*b^3*d^4*x^
2*e^6 - 3*b^3*d^5*x*e^5 - b^3*d^6*e^4)
```

3.205 $\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx$

Optimal. Leaf size=105

$$-\frac{\text{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(ax+b)}{ac-bd}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

```
[Out] (Log[a + b/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(b + a*x))/(a*c - b*d))]*Log[c + d*x])/d - PolyLog[2, (a*(c + d*x))/(a*c
- b*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rubi [A] time = 0.166766, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(ax+b)}{ac-bd}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b/x]/(c + d*x),x]
```

```
[Out] (Log[a + b/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(b + a*x))/(a*c - b*d))]*Log[c + d*x])/d - PolyLog[2, (a*(c + d*x))/(a*c
- b*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int \frac{\log(c+dx)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int \left(\frac{\log(c+dx)}{bx}-\frac{a\log(c+dx)}{b(b+ax)}\right)dx}{d}\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\int \frac{\log(c+dx)}{x}\,dx}{d}-\frac{a\int \frac{\log(c+dx)}{b+ax}\,dx}{d}\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx \cdots\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}+\frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{d}+\cdots\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\frac{\text{Li}_2\left(\frac{a(c+dx)}{ac-bd}\right)}{d}+\cdots
\end{aligned}$$

Mathematica [A] time = 0.0368635, size = 80, normalized size = 0.76

$$\frac{-\text{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)+\text{PolyLog}\left(2,\frac{dx}{c}+1\right)+\log(c+dx)\left(-\log\left(\frac{d(ax+b)}{bd-ac}\right)+\log\left(a+\frac{b}{x}\right)+\log\left(-\frac{dx}{c}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b/x]/(c + d*x),x]
```

```
[Out] ((Log[a + b/x] + Log[-((d*x)/c)] - Log[(d*(b + a*x))/(-(a*c) + b*d)])*Log[c
 + d*x] - PolyLog[2, (a*(c + d*x))/(a*c - b*d)] + PolyLog[2, 1 + (d*x)/c])/
d
```

Maple [A] time = 0.421, size = 114, normalized size = 1.1

$$\frac{1}{d}\text{dilog}\left(\frac{1}{-ac+bd}\left(c\left(a+\frac{b}{x}\right)-ac+bd\right)\right)+\frac{1}{d}\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{1}{-ac+bd}\left(c\left(a+\frac{b}{x}\right)-ac+bd\right)\right)-\frac{1}{d}\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b/x)/(d*x+c),x)
```

```
[Out] 1/d*dilog((c*(a+b/x)-a*c+b*d)/(-a*c+b*d))+1/d*ln(a+b/x)*ln((c*(a+b/x)-a*c+b
*d)/(-a*c+b*d))-1/d*ln(a+b/x)*ln(-b/a/x)-1/d*dilog(-b/a/x)
```

Maxima [A] time = 1.06626, size = 111, normalized size = 1.06

$$-\frac{\log\left(\frac{dx}{c}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d}+\frac{\log(ax+b)\log\left(\frac{adx+bd}{ac-bd}+1\right)+\mathrm{Li}_2\left(-\frac{adx+bd}{ac-bd}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(a*x + b)*log((a*d*x + b*d
)/(a*c - b*d) + 1) + dilog(-(a*d*x + b*d)/(a*c - b*d)))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\frac{ax+b}{x}\right)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((a*x + b)/x)/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b/x)/(d*x+c),x)
```

```
[Out] Integral(log(a + b/x)/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log(a + b/x)/(d*x + c), x)
```

3.206 $\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=301

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^3\right)^p\right)}{e(m+1)}+\frac{\sqrt[3]{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)}+\frac{\sqrt[3]{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\right.}{e(m+1)(m+2)\left(\sqrt[3]{-1}\sqrt[3]{a}e+\right.}$$

```
[Out] (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d - a^(1/3)*e)])/(e*(b^(1/3)*d - a^(1/3)*e)*(1 + m)*(2 +
m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1
/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/(e*(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e)])/(e*(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1
 + m)*Log[c*(a + b*x^3)^p])/(e*(1 + m))
```

Rubi [A] time = 0.755588, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2463, 6725, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^3\right)^p\right)}{e(m+1)}+\frac{\sqrt[3]{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)}+\frac{\sqrt[3]{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\right.}{e(m+1)(m+2)\left(\sqrt[3]{-1}\sqrt[3]{a}e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d - a^(1/3)*e)])/(e*(b^(1/3)*d - a^(1/3)*e)*(1 + m)*(2 +
m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1
/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/(e*(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e)])/(e*(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1
 + m)*Log[c*(a + b*x^3)^p])/(e*(1 + m))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{(3bp)\int \frac{x^2(d+ex)^{1+m}}{a+bx^3}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{(3bp)\int\left(\frac{(d+ex)^{1+m}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(d+ex)^{1+m}}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{}{3b^{2/}}\right.}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{(d+ex)^{1+m}}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e(1+m)} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{(d+ex)^{1+m}}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e(1+m)}$$

$$= \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)(1+m)(2+m)} + \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3\right.}{e\left(\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e\right)(1+}$$

Mathematica [A] time = 0.842994, size = 239, normalized size = 0.79

$$\frac{(d+ex)^{m+1}\left(\log\left(c\left(a+bx^3\right)^p\right) - \frac{\sqrt[3]{b}p(d+ex)\left(-\frac{{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e} - \frac{{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d} - \frac{{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{m+2}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*(-((b^(1/3)*p*(d + e*x)*(-(Hypergeometric2F1[1, 2 + m, 3
 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]/(b^(1/3)*d - a^(1/3)*e))
 - Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)
^(1/3)*a^(1/3)*e)]/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e) - Hypergeometric2F1[1
, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)]/(b^
(1/3)*d - (-1)^(2/3)*a^(1/3)*e)))/(2 + m)) + Log[c*(a + b*x^3)^p]))/(e*(1 +
 m))
```

Maple [F] time = 1.456, size = 0, normalized size = 0.

$$\int (ex+d)^m \ln\left(c\left(bx^3+a\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)^m \log\left(\left(bx^3+a\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^m \log\left(\left(bx^3+a\right)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

3.207 $\int (d+ex)^m \log \left(c \left(a+bx^2\right)^p\right) \, dx$

Optimal. Leaf size=205

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}+\frac{\sqrt{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e(m+1)(m+2)\left(\sqrt{b}d-\sqrt{-a}e\right)}+\frac{\sqrt{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+\right.}{e(m+1)(m+2)\left(\sqrt{-a}e+\right.}$$

```
[Out] (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/(e*(Sqrt[b]*d - Sqrt[-a]*e)*(1 + m)*(2
+ m)) + (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sq
rt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(e*(Sqrt[b]*d + Sqrt[-a]*e)*(1
+ m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a + b*x^2)^p])/(e*(1 + m))
```

Rubi [A] time = 0.247979, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2463, 831, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}+\frac{\sqrt{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e(m+1)(m+2)\left(\sqrt{b}d-\sqrt{-a}e\right)}+\frac{\sqrt{b}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+\right.}{e(m+1)(m+2)\left(\sqrt{-a}e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/(e*(Sqrt[b]*d - Sqrt[-a]*e)*(1 + m)*(2
+ m)) + (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sq
rt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(e*(Sqrt[b]*d + Sqrt[-a]*e)*(1
+ m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a + b*x^2)^p])/(e*(1 + m))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} - \frac{(2bp)\int \frac{x(d+ex)^{1+m}}{a+bx^2}dx}{e(1+m)}$$
$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} - \frac{(2bp)\int\left(-\frac{(d+ex)^{1+m}}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{(d+ex)^{1+m}}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e(1+m)}$$
$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} + \frac{\left(\sqrt{b}p\right)\int \frac{(d+ex)^{1+m}}{\sqrt{-a}-\sqrt{b}x}dx}{e(1+m)} - \frac{\left(\sqrt{b}p\right)\int \frac{(d+ex)^{1+m}}{\sqrt{-a}+\sqrt{b}x}dx}{e(1+m)}$$
$$= \frac{\sqrt{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e\left(\sqrt{b}d-\sqrt{-a}e\right)(1+m)(2+m)} + \frac{\sqrt{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+\right.}{e\left(\sqrt{b}d+\sqrt{-a}e\right)(1+m)(2}$$

Mathematica [A] time = 0.276107, size = 176, normalized size = 0.86

$$\frac{(d+ex)^{m+1}\left(\log\left(c\left(a+bx^2\right)^p\right)+\frac{\sqrt{b}p(d+ex)\left(\left(\sqrt{-a}e+\sqrt{b}d\right)\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\left(\sqrt{b}d-\sqrt{-a}e\right)\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)\right)}{(m+2)\left(ae^2+bd^2\right)}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((Sqrt[b]*p*(d + e*x)*((Sqrt[b]*d + Sqrt[-a]*e)*Hypergeo
metric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] +
(Sqrt[b]*d - Sqrt[-a]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e
*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/((b*d^2 + a*e^2)*(2 + m)) + Log[c*(a + b*x
^2)^p]))/(e*(1 + m))
```

Maple [F] time = 1.411, size = 0, normalized size = 0.

$$\int (ex+d)^m \ln\left(c\left(bx^2+a\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)^m\log\left(\left(bx^2+a\right)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^m\log\left(\left(bx^2+a\right)^pc\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

3.208 $\int (d+ex)^m \log\left(c(a+bx)^p\right)\,dx$

Optimal. Leaf size=89

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx)^p\right)}{e(m+1)}+\frac{bp(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)(m+2)(bd-ae)}$$

```
[Out] (b*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d + e*x))/(b*
d - a*e)])/(e*(b*d - a*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a +
b*x)^p])/(e*(1 + m))
```

Rubi [A] time = 0.045602, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 68}

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx)^p\right)}{e(m+1)}+\frac{bp(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)(m+2)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

```
[Out] (b*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d + e*x))/(b*
d - a*e)])/(e*(b*d - a*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a +
b*x)^p])/(e*(1 + m))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \log\left(c(a+bx)^p\right)\,dx &= \frac{(d+ex)^{1+m}\log\left(c(a+bx)^p\right)}{e(1+m)}-\frac{(bp)\int \frac{(d+ex)^{1+m}}{a+bx}\,dx}{e(1+m)}\\ &= \frac{bp(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{b(d+ex)}{bd-ae}\right)}{e(bd-ae)(1+m)(2+m)}+\frac{(d+ex)^{1+m}\log\left(c(a+bx)^p\right)}{e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0600622, size = 77, normalized size = 0.87

$$\frac{(d+ex)^{m+1}\left(\log\left(c(a+bx)^p\right)+\frac{bp(d+ex)\,{}_2F_1\left(1,m+2;m+3;\frac{b(d+ex)}{bd-ae}\right)}{(m+2)(bd-ae)}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)*(2 + m)) + Log[c*(a + b*x)^p]))/(e*(1 +
 m))
```

Maple [F] time = 1.257, size = 0, normalized size = 0.

$$\int (ex+d)^m \ln \left(c (bx+a)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex+d)^m \log\left((bx+a)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log((b*x + a)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x+a)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^m \log\left((bx+a)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((b*x + a)^p*c), x)
```

3.209 $\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=135

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}+\frac{ap(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{a(d+ex)}{ad-be}\right)}{e(m+1)(m+2)(ad-be)}-\frac{p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)}{de\left(m^2+3m+2\right)}$$

```
[Out] (a*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (a*(d + e*x))/(a*
d - b*e)])/(e*(a*d - b*e)*(1 + m)*(2 + m)) - (p*(d + e*x)^(2 + m)*Hypergeom
etric2F1[1, 2 + m, 3 + m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^
(1 + m)*Log[c*(a + b/x)^p])/(e*(1 + m))
```

Rubi [A] time = 0.0926796, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2463, 514, 86, 65, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}+\frac{ap(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{a(d+ex)}{ad-be}\right)}{e(m+1)(m+2)(ad-be)}-\frac{p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)}{de\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (a*(d + e*x))/(a*
d - b*e)])/(e*(a*d - b*e)*(1 + m)*(2 + m)) - (p*(d + e*x)^(2 + m)*Hypergeom
etric2F1[1, 2 + m, 3 + m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^
(1 + m)*Log[c*(a + b/x)^p])/(e*(1 + m))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{(bp)\int\frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x}\right)x^2}dx}{e(1+m)}\\ &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{(bp)\int\frac{(d+ex)^{1+m}}{x(b+ax)}dx}{e(1+m)}\\ &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{p\int\frac{(d+ex)^{1+m}}{x}dx}{e(1+m)}-\frac{(ap)\int\frac{(d+ex)^{1+m}}{b+ax}dx}{e(1+m)}\\ &= \frac{ap(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{a(d+ex)}{ad-be}\right)}{e(ad-be)(1+m)(2+m)}-\frac{p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0826814, size = 123, normalized size = 0.91

$$\frac{(d+ex)^{m+1}\left((ad-be)\left(p(d+ex)\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)-d(m+2)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-adp(d+ex)\,{}_2F_1\left(1,m+2;m\right.\right.}{de(m+1)(m+2)(be-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(a*d*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (a
*(d + e*x))/(a*d - b*e)]) + (a*d - b*e)*(p*(d + e*x)*Hypergeometric2F1[1, 2
 + m, 3 + m, 1 + (e*x)/d] - d*(2 + m)*Log[c*(a + b/x)^p])))/(d*e*(-(a*d) +
b*e)*(1 + m)*(2 + m))
```

Maple [F] time = 2.635, size = 0, normalized size = 0.

$$\int (ex+d)^m \ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)^m \log\left(c\left(\frac{ax+b}{x}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log(c*((a*x + b)/x)^p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^m \log\left(\left(a+\frac{b}{x}\right)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((a + b/x)^p*c), x)
```

3.210 $\int (d+ex)^m \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=257

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}+\frac{\sqrt{-a}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}d-\sqrt{b}e\right)}+\frac{\sqrt{-a}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+\cdots\right.}{e(m+1)(m+2)\left(\sqrt{-a}d+\cdots\right.}$$

```
[Out] (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/(e*(Sqrt[-a]*d - Sqrt[b]*e)*(1 + m)*(
2 + m)) + (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m,
(Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/(e*(Sqrt[-a]*d + Sqrt[b]*e)
*(1 + m)*(2 + m)) - (2*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^(1 + m)*Log[c*(a + b/x^
2)^p])/(e*(1 + m))
```

Rubi [A] time = 0.532742, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2463, 1570, 961, 65, 831, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}+\frac{\sqrt{-a}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}d-\sqrt{b}e\right)}+\frac{\sqrt{-a}p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+\cdots\right.}{e(m+1)(m+2)\left(\sqrt{-a}d+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/(e*(Sqrt[-a]*d - Sqrt[b]*e)*(1 + m)*(
2 + m)) + (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m,
(Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/(e*(Sqrt[-a]*d + Sqrt[b]*e)
*(1 + m)*(2 + m)) - (2*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^(1 + m)*Log[c*(a + b/x^
2)^p])/(e*(1 + m))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int \frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x^2}\right)x^3}dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int \frac{(d+ex)^{1+m}}{x(b+ax^2)}dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int \left(\frac{(d+ex)^{1+m}}{bx} - \frac{ax(d+ex)^{1+m}}{b(b+ax^2)}\right)dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2p)\int \frac{(d+ex)^{1+m}}{x}dx}{e(1+m)} - \frac{(2ap)\int \frac{x(d+ex)^{1+m}}{b+ax^2}dx}{e(1+m)}$$

$$= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)} + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} - \frac{(2\ldots}{\ldots}$$

$$= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)} + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(\sqrt{\ldots}}{\ldots}$$

$$= \frac{\sqrt{-a}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e\left(\sqrt{-a}d-\sqrt{b}e\right)(1+m)(2+m)} + \frac{\sqrt{-a}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+\ldots\right.}{e\left(\sqrt{-a}d+\sqrt{b}e\right)(1+\ldots}$$

Mathematica [A] time = 0.507459, size = 211, normalized size = 0.82

$$\frac{(d+ex)^{m+1}\left(\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{p(d+ex)\left(-2\left(ad^2+be^2\right){}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)+d\left(ad-\sqrt{-a}\sqrt{b}e\right){}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)+d\left(\sqrt{-a}\sqrt{b}e+ad\right){}_2F_1\left(\ldots\right.}{d(m+2)\left(ad^2+be^2\right)}\right.}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((p*(d + e*x)*(d*(a*d - Sqrt[-a]*Sqrt[b]*e)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] + d*(
a*d + Sqrt[-a]*Sqrt[b]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(d +
 e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*(a*d^2 + b*e^2)*Hypergeometric2F1[1, 2
 + m, 3 + m, 1 + (e*x)/d]))/(d*(a*d^2 + b*e^2)*(2 + m)) + Log[c*(a + b/x^2)
^p]))/(e*(1 + m))
```

Maple [F] time = 3.108, size = 0, normalized size = 0.

$$\int (ex+d)^m \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex+d)^m \log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log(c*((a*x^2 + b)/x^2)^p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^m \log\left(\left(a+\frac{b}{x^2}\right)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((a + b/x^2)^p*c), x)
```

3.211 $\int (f+gx)^m \log \left(c \left(d+e x^n\right)^p\right) \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((f+gx)^m \log \left(c \left(d+e x^n\right)^p\right),x\right)$$

```
[Out] Unintegrable[(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Rubi [A] time = 0.0110441, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (f+gx)^m \log \left(c \left(d+e x^n\right)^p\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]

[Out] Defer[Int][(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Rubi steps

$$\int (f+gx)^m \log \left(c \left(d+e x^n\right)^p\right) \, dx=\int (f+gx)^m \log \left(c \left(d+e x^n\right)^p\right) \, dx$$

Mathematica [A] time = 0.523136, size = 0, normalized size = 0.

$$\int (f+gx)^m \log \left(c \left(d+e x^n\right)^p\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]

[Out] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Maple [A] time = 2.648, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^m \ln \left(c \left(d+e x^n\right)^p\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)

[Out] int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx+f\right)^m\log\left(\left(ex^n+d\right)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x + f)^m*log((e*x^n + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**m*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.212 $\int (f+gx)^3 \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=234

$$\frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right)}{4g}-\frac{3ef^2gnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}-\frac{f^4p\log\left(d+ex^n\right)}{4g}-\frac{ef^3npx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2\right.}{d(n+1)}$$

```
[Out] -((e*f^3*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (3*e*f^2*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n
)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (e*f*g^2*n*p*x^(3 + n)*
Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) - (e*g^
3*n*p*x^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((e*x^n)/d)])
/(4*d*(4 + n)) - (f^4*p*Log[d + e*x^n])/(4*g) + ((f + g*x)^4*Log[c*(d + e*x
^n)^p])/(4*g)
```

Rubi [A] time = 0.232796, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2463, 1844, 260, 364}

$$\frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right)}{4g}-\frac{3ef^2gnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}-\frac{f^4p\log\left(d+ex^n\right)}{4g}-\frac{ef^3npx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2\right.}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```


```
[Out] -((e*f^3*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (3*e*f^2*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n
)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (e*f*g^2*n*p*x^(3 + n)*
Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) - (e*g^
3*n*p*x^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((e*x^n)/d)])
/(4*d*(4 + n)) - (f^4*p*Log[d + e*x^n])/(4*g) + ((f + g*x)^4*Log[c*(d + e*x
^n)^p])/(4*g)
```


Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)\,dx = \frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right)}{4g} - \frac{(enp)\int \frac{x^{-1+n}(f+gx)^4}{d+ex^n}\,dx}{4g}$$

$$= \frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right)}{4g} - \frac{(enp)\int\left(\frac{f^4x^{-1+n}}{d+ex^n} + \frac{4f^3gx^n}{d+ex^n} + \frac{6f^2g^2x^{1+n}}{d+ex^n} + \frac{4fg^3x^{2+n}}{d+ex^n} + \frac{g^4}{d\text{-}}\right.}{4g}$$

$$= \frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right)}{4g} - \left(ef^3np\right)\int\frac{x^n}{d+ex^n}\,dx - \frac{\left(ef^4np\right)\int\frac{x^{-1+n}}{d+ex^n}\,dx}{4g} - \frac{1}{2}\left(3e\right.$$

$$= -\frac{ef^3npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{3ef^2gnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right.}{2d(2+n)}$$

Mathematica [A] time = 0.515633, size = 224, normalized size = 0.96

$$\frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right) - enp\left(\frac{6f^2g^2x^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(n+2)} + \frac{4f^3gx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)} + \frac{f^4\log(d+ex^n)}{en} + \frac{4fg^3x^{n+3}\,{}_2F}{\ }\right.}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (-(e*n*p*((4*f^3*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -
((e*x^n)/d)])/(d*(1 + n)) + (6*f^2*g^2*x^(2 + n)*Hypergeometric2F1[1, (2 +
n)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (4*f*g^3*x^(3 + n)*Hyper
geometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (g^4*x^(4
+ n)*Hypergeometric2F1[1, (4 + n)/n, 2 + 4/n, -((e*x^n)/d)])/(d*(4 + n)) +
(f^4*Log[d + e*x^n])/(e*n))) + (f + g*x)^4*Log[c*(d + e*x^n)^p])/(4*g)
```

Maple [F] time = 2.714, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^3 \ln\left(c\left(d+ex^n\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{16}\left(g^3np - 4g^3\log(c)\right)x^4 - \frac{1}{3}\left(fg^2np - 3fg^2\log(c)\right)x^3 - \frac{3}{4}\left(f^2gnp - 2f^2g\log(c)\right)x^2 - \left(f^3np - f^3\log(c)\right)x + \frac{1}{4}\left(g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/16*(g^3*n*p - 4*g^3*log(c))*x^4 - 1/3*(f*g^2*n*p - 3*f*g^2*log(c))*x^3 -
 3/4*(f^2*g*n*p - 2*f^2*g*log(c))*x^2 - (f^3*n*p - f^3*log(c))*x + 1/4*(g^3
*x^4 + 4*f*g^2*x^3 + 6*f^2*g*x^2 + 4*f^3*x)*log((e*x^n + d)^p) + integrate(
1/4*(d*g^3*n*p*x^3 + 4*d*f*g^2*n*p*x^2 + 6*d*f^2*g*n*p*x + 4*d*f^3*n*p)/(e*
x^n + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g^3x^3 + 3\,fg^2x^2 + 3\,f^2gx + f^3\right)\log\left((ex^n + d)^pc\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*log((e*x^n + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^3 \log\left((ex^n + d)^pc\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*log((e*x^n + d)^p*c), x)
```

3.213 $\int (f+gx)^2 \log \left(c\left(d+ex^n\right)^p\right) \, dx$

Optimal. Leaf size=181

$$\frac{(f+gx)^3 \log \left(c\left(d+ex^n\right)^p\right)}{3g}-\frac{f^3 p \log \left(d+ex^n\right)}{3g}-\frac{ef^2 npx^{n+1} {}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{efgnpx^{n+2} {}_2F_1\left(1,\frac{n+2}{n};2\left(1\right.\right.}{d(n+2)}$$

```
[Out] -((e*f^2*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (e*f*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n,
 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) - (e*g^2*n*p*x^(3 + n)*Hypergeo
metric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(3*d*(3 + n)) - (f^3*p*Log[
d + e*x^n])/(3*g) + ((f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Rubi [A] time = 0.176117, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2463, 1844, 260, 364}

$$\frac{(f+gx)^3 \log \left(c\left(d+ex^n\right)^p\right)}{3g}-\frac{f^3 p \log \left(d+ex^n\right)}{3g}-\frac{ef^2 npx^{n+1} {}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{efgnpx^{n+2} {}_2F_1\left(1,\frac{n+2}{n};2\left(1\right.\right.}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```


```
[Out] -((e*f^2*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (e*f*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n,
 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) - (e*g^2*n*p*x^(3 + n)*Hypergeo
metric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(3*d*(3 + n)) - (f^3*p*Log[
d + e*x^n])/(3*g) + ((f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```


Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (f+gx)^2 \log\left(c\left(d+ex^n\right)^p\right)\,dx = \frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)}{3g} - \frac{(enp)\int \frac{x^{-1+n}(f+gx)^3}{d+ex^n}\,dx}{3g}$$

$$= \frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)}{3g} - \frac{(enp)\int \left(\frac{f^3x^{-1+n}}{d+ex^n}+\frac{3f^2gx^n}{d+ex^n}+\frac{3fg^2x^{1+n}}{d+ex^n}+\frac{g^3x^{2+n}}{d+ex^n}\right)dx}{3g}$$

$$= \frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)}{3g} - \left(ef^2np\right)\int \frac{x^n}{d+ex^n}\,dx - \frac{\left(ef^3np\right)\int \frac{x^{-1+n}}{d+ex^n}\,dx}{3g} - (efgnp)$$

$$= -\frac{ef^2npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{efgnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(2+n)} - \cdots$$

Mathematica [A] time = 0.254249, size = 178, normalized size = 0.98

$$\frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right) - enp\left(\frac{3f^2gx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)} + \frac{f^3\log(d+ex^n)}{en} + \frac{3fg^2x^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(n+2)} + \frac{g^3x^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n}\cdots\right)}{d(n+3)}\cdots\right.}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (-(e*n*p*((3*f^2*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -
((e*x^n)/d)])/(d*(1 + n)) + (3*f*g^2*x^(2 + n)*Hypergeometric2F1[1, (2 + n)
/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (g^3*x^(3 + n)*Hypergeomet
ric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (f^3*Log[d + e*x
^n])/(e*n))) + (f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Maple [F] time = 2.775, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^2 \ln\left(c\left(d+ex^n\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{9}\left(g^2np - 3g^2\log(c)\right)x^3 - \frac{1}{2}\left(fgnp - 2fg\log(c)\right)x^2 - \left(f^2np - f^2\log(c)\right)x + \frac{1}{3}\left(g^2x^3 + 3fgx^2 + 3f^2x\right)\log\left(\left(ex^n+d\right)^p\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/9*(g^2*n*p - 3*g^2*log(c))*x^3 - 1/2*(f*g*n*p - 2*f*g*log(c))*x^2 - (f^2
*n*p - f^2*log(c))*x + 1/3*(g^2*x^3 + 3*f*g*x^2 + 3*f^2*x)*log((e*x^n + d)^
```

```
p) + integrate(1/3*(d*g^2*n*p*x^2 + 3*d*f*g*n*p*x + 3*d*f^2*n*p)/(e*x^n + d
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g^2x^2 + 2\,fgx + f^2\right)\log\left((ex^n + d)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*log((e*x^n + d)^p*c), x)
```

Sympy [C] time = 120.028, size = 284, normalized size = 1.57

$$f^2x\log\left(c\,(d + ex^n)^p\right) + \frac{f^2px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)} + fgx^2\log\left(c\,(d + ex^n)^p\right) + \frac{g^2x^3\log\left(c\,(d + ex^n)^p\right)}{3} - \frac{efgpx^2x^n\Phi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f**2*x*log(c*(d + e*x**n)**p) + f**2*p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)
/e, 1, exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + f*g*x**2*log(c*(d
 + e*x**n)**p) + g**2*x**3*log(c*(d + e*x**n)**p)/3 - e*f*g*p*x**2*x**n*ler
chphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*gamma(2 + 2/n
)) - 2*e*f*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gam
ma(1 + 2/n)/(d*n*gamma(2 + 2/n)) - e*g**2*p*x**3*x**n*lerchphi(e*x**n*exp_p
olar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*d*gamma(2 + 3/n)) - e*g**2*p*x*
*3*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(d*n*
gamma(2 + 3/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^2 \log\left((ex^n + d)^p c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*log((e*x^n + d)^p*c), x)
```

3.214 $\int (f+gx)\log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=132

$$\frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g}-\frac{f^2p\log\left(d+ex^n\right)}{2g}-\frac{efnpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{egnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);\right.}{2d(n+2)}$$

```
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)
/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(
1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (f^2*p*Log[d + e*x^n])/(2*g) +
((f + g*x)^2*Log[c*(d + e*x^n)^p])/(2*g)
```

Rubi [A] time = 0.135616, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2463, 1844, 260, 364}

$$\frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g}-\frac{f^2p\log\left(d+ex^n\right)}{2g}-\frac{efnpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{egnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);\right.}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)
/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(
1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (f^2*p*Log[d + e*x^n])/(2*g) +
((f + g*x)^2*Log[c*(d + e*x^n)^p])/(2*g)
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (f+gx)\log\left(c\left(d+ex^n\right)^p\right)\,dx = \frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g} - \frac{(enp)\int \frac{x^{-1+n}(f+gx)^2}{d+ex^n}\,dx}{2g}$$

$$= \frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g} - \frac{(enp)\int\left(\frac{f^2x^{-1+n}}{d+ex^n}+\frac{2fgx^n}{d+ex^n}+\frac{g^2x^{1+n}}{d+ex^n}\right)dx}{2g}$$

$$= \frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g} - (efnp)\int\frac{x^n}{d+ex^n}\,dx - \frac{\left(ef^2np\right)\int\frac{x^{-1+n}}{d+ex^n}\,dx}{2g} - \frac{1}{2}(egnp$$

$$= -\frac{efnpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{egnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)} - f$$

Mathematica [A] time = 0.123226, size = 130, normalized size = 0.98

$$fx\log\left(c\left(d+ex^n\right)^p\right)+\frac{1}{2}gx^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{efnpx^{n+1}\,{}_2F_1\left(1,\frac{n+1}{n};\frac{n+1}{n}+1;-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{egnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};\frac{n+2}{n}+1;\right.}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/n, 1 + (1 + n)/n, -((e*x^
n)/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 1
 + (2 + n)/n, -((e*x^n)/d)])/(2*d*(2 + n)) + f*x*Log[c*(d + e*x^n)^p] + (g*
x^2*Log[c*(d + e*x^n)^p])/2
```

Maple [F] time = 3.066, size = 0, normalized size = 0.

$$\int (gx+f)\ln\left(c\left(d+ex^n\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)*ln(c*(d+e*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\left(gnp-2g\log(c)\right)x^2-\left(fnp-f\log(c)\right)x+\frac{1}{2}\left(gx^2+2fx\right)\log\left(\left(ex^n+d\right)^p\right)+\int\frac{dgnpx+2dfnp}{2\left(ex^n+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/4*(g*n*p - 2*g*log(c))*x^2 - (f*n*p - f*log(c))*x + 1/2*(g*x^2 + 2*f*x)*
log((e*x^n + d)^p) + integrate(1/2*(d*g*n*p*x + 2*d*f*n*p)/(e*x^n + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx+f\right)\log\left(\left(ex^{n}+d\right)^{p}c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x + f)*log((e*x^n + d)^p*c), x)
```

Sympy [C] time = 15.4403, size = 162, normalized size = 1.23

$$fx\log\left(c\left(d+ex^{n}\right)^{p}\right)+\frac{fpx\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+\frac{gx^{2}\log\left(c\left(d+ex^{n}\right)^{p}\right)}{2}-\frac{egpx^{2}x^{n}\Phi\left(\frac{ex^{n}e^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2d\Gamma\left(2+\frac{2}{n}\right)}-\frac{egpx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f*x*log(c*(d + e*x**n)**p) + f*p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1,
 exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + g*x**2*log(c*(d + e*x**
n)**p)/2 - e*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*g
amma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - e*g*p*x**2*x**n*lerchphi(e*x**n*exp_po
lar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamma(2 + 2/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(gx+f\right)\log\left(\left(ex^{n}+d\right)^{p}c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*log((e*x^n + d)^p*c), x)
```

3.215 $\int \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=54

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rubi [A] time = 0.0168512, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 364}

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \log\left(c\left(d+ex^n\right)^p\right)\,dx &= x\log\left(c\left(d+ex^n\right)^p\right)-(enp)\int \frac{x^n}{d+ex^n}\,dx\\ &= -\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.0288539, size = 52, normalized size = 0.96

$$x\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
```

```
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \ln\left(c\left(d+ex^{n}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$dnp\int \frac{1}{ex^{n}+d}\,dx-\left(np-\log\left(c\right)\right)x+x\log\left(\left(ex^{n}+d\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\left(ex^{n}+d\right)^{p}c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c), x)
```

Sympy [C] time = 2.95144, size = 48, normalized size = 0.89

$$x\log\left(c\left(d+ex^{n}\right)^{p}\right)+\frac{px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p),x)
```

```
[Out] x*log(c*(d + e*x**n)**p) + p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1, exp
_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left((ex^n + d)^p c \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c), x)
```

3.216 $\int \frac{\log(c(d+ex^n)^p)}{f+gx} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Rubi [A] time = 0.0124176, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx} \, dx = \int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx} \, dx$$

Mathematica [A] time = 1.59032, size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Maple [A] time = 2.352, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(d+ex^n\right)^p\right)}{gx+f} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((ex^n + d)^p c\right)}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\,(d + ex^n)^p\right)}{f + gx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f),x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

3.217 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Rubi [A] time = 0.0122881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]

[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx = \int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx$$

Mathematica [A] time = 0.181424, size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]

[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Maple [A] time = 2.131, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(d+ex^n\right)^p\right)}{\left(gx+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-dnp\int \frac{1}{dg^2x^2 + dfgx + (eg^2x^2 + efgx)x^n}\,dx - \frac{np\log(gx+f)}{fg} - \frac{f\log((ex^n+d)^p) + f\log(c) - (gnpx + fnp)\log(}{fg^2x + f^2g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/(d*g^2*x^2 + d*f*g*x + (e*g^2*x^2 + e*f*g*x)*x^n), x) -
n*p*log(g*x + f)/(f*g) - (f*log((e*x^n + d)^p) + f*log(c) - (g*n*p*x + f*n*
p)*log(x))/(f*g^2*x + f^2*g)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log((ex^n+d)^p c)}{g^2x^2 + 2fgx + f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log((ex^n+d)^p c)}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f)^2, x)
```

3.218 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Rubi [A] time = 0.0122042, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3} dx = \int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3} dx$$

Mathematica [A] time = 0.204131, size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Maple [A] time = 2.204, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(d+ex^n\right)^p\right)}{\left(gx+f\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-dnp\int \frac{1}{2\left(dg^3x^3+2dfg^2x^2+df^2gx+\left(eg^3x^3+2efg^2x^2+ef^2gx\right)x^n\right)}\,dx+\frac{fgnpx+f^2np-f^2\log\left(\left(ex^n+d\right)^p\right)-f}{2\left(f^2g^3x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/2/(d*g^3*x^3 + 2*d*f*g^2*x^2 + d*f^2*g*x + (e*g^3*x^3 +
2*e*f*g^2*x^2 + e*f^2*g*x)*x^n), x) + 1/2*(f*g*n*p*x + f^2*n*p - f^2*log((e
*x^n + d)^p) - f^2*log(c) + (g^2*n*p*x^2 + 2*f*g*n*p*x + f^2*n*p)*log(x))/(
f^2*g^3*x^2 + 2*f^3*g^2*x + f^4*g) - 1/2*n*p*log(g*x + f)/(f^2*g)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)}{g^3x^3+3fg^2x^2+3f^2gx+f^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f)^3, x)
```

3.219 $\int \frac{x^3 \log(c(a+bx)^p)}{d+ex} dx$

Optimal. Leaf size=250

$$-\frac{d^3 p \text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^4}+\frac{a^2 dp \log(a+bx)}{2b^2e^2}-\frac{a^2 px}{3b^2 e}+\frac{a^3 p \log(a+bx)}{3b^3 e}+\frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3}-\frac{d^3 \log\left(c(a+b\right.}{e}$$

```
[Out] -((d^2*p*x)/e^3) - (a*d*p*x)/(2*b*e^2) - (a^2*p*x)/(3*b^2*e) + (d*p*x^2)/(4
*e^2) + (a*p*x^2)/(6*b*e) - (p*x^3)/(9*e) + (a^2*d*p*Log[a + b*x])/(2*b^2*e
^2) + (a^3*p*Log[a + b*x])/(3*b^3*e) - (d*x^2*Log[c*(a + b*x)^p])/(2*e^2) +
 (x^3*Log[c*(a + b*x)^p])/(3*e) + (d^2*(a + b*x)*Log[c*(a + b*x)^p])/(b*e^3
) - (d^3*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^4 - (d^3*p*Po
lyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^4
```

Rubi [A] time = 0.245857, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {43, 2416, 2389, 2295, 2395, 2394, 2393, 2391}

$$-\frac{d^3 p \text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^4}+\frac{a^2 dp \log(a+bx)}{2b^2e^2}-\frac{a^2 px}{3b^2 e}+\frac{a^3 p \log(a+bx)}{3b^3 e}+\frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3}-\frac{d^3 \log\left(c(a+b\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((d^2*p*x)/e^3) - (a*d*p*x)/(2*b*e^2) - (a^2*p*x)/(3*b^2*e) + (d*p*x^2)/(4
*e^2) + (a*p*x^2)/(6*b*e) - (p*x^3)/(9*e) + (a^2*d*p*Log[a + b*x])/(2*b^2*e
^2) + (a^3*p*Log[a + b*x])/(3*b^3*e) - (d*x^2*Log[c*(a + b*x)^p])/(2*e^2) +
 (x^3*Log[c*(a + b*x)^p])/(3*e) + (d^2*(a + b*x)*Log[c*(a + b*x)^p])/(b*e^3
) - (d^3*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^4 - (d^3*p*Po
lyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c(a+bx)^p\right)}{e^3} - \frac{dx \log\left(c(a+bx)^p\right)}{e^2} + \frac{x^2 \log\left(c(a+bx)^p\right)}{e} - \frac{d^3 \log\left(c(a+bx)^p\right)}{e^3(d+ex)}\right) d$$

$$= \frac{d^2 \int \log\left(c(a+bx)^p\right)\,dx}{e^3} - \frac{d^3 \int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e^3} - \frac{d \int x \log\left(c(a+bx)^p\right)\,dx}{e^2} + \frac{\int x^2 \log(c(a}{e}$$

$$= -\frac{dx^2 \log\left(c(a+bx)^p\right)}{2e^2} + \frac{x^3 \log\left(c(a+bx)^p\right)}{3e} - \frac{d^3 \log\left(c(a+bx)^p\right) \log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} + \frac{d^2 \operatorname{Subst}(}{}$$

$$= -\frac{d^2px}{e^3} - \frac{dx^2 \log\left(c(a+bx)^p\right)}{2e^2} + \frac{x^3 \log\left(c(a+bx)^p\right)}{3e} + \frac{d^2(a+bx) \log\left(c(a+bx)^p\right)}{be^3} - \frac{d^3 \log(}{}$$

$$= -\frac{d^2px}{e^3} - \frac{adpx}{2be^2} - \frac{a^2px}{3b^2e} + \frac{dpx^2}{4e^2} + \frac{apx^2}{6be} - \frac{px^3}{9e} + \frac{a^2dp \log(a+bx)}{2b^2e^2} + \frac{a^3p \log(a+bx)}{3b^3e} - \frac{dx^2}{}$$

Mathematica [A] time = 0.18436, size = 183, normalized size = 0.73

$$\frac{-36b^3d^3p\text{PolyLog}\left(2, \frac{e(a+bx)}{ae-bd}\right) + b\left(6b \log\left(c(a+bx)^p\right)\left(-6bd^3 \log\left(\frac{b(d+ex)}{bd-ae}\right) + 6ad^2e + bex\left(6d^2 - 3dex + 2e^2x^2\right)\right) - epx}{36b^3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (6*a^2*e^2*(3*b*d + 2*a*e)*p*Log[a + b*x] + b*(-(e*p*x*(12*a^2*e^2 - 6*a*b*
e*(-3*d + e*x) + b^2*(36*d^2 - 9*d*e*x + 4*e^2*x^2))) + 6*b*Log[c*(a + b*x)
^p]*(6*a*d^2*e + b*e*x*(6*d^2 - 3*d*e*x + 2*e^2*x^2) - 6*b*d^3*Log[(b*(d +
e*x))/(b*d - a*e)])) - 36*b^3*d^3*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)
])/(36*b^3*e^4)
```

Maple [C] time = 0.66, size = 919, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] 1/4*d*p*x^2/e^2-1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)/e^
3*x*d^2+1/4*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)/e^2*x^2*d+
1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*d^3/e^4*ln(e*x+d)+
1/3*ln((b*x+a)^p)/e*x^3-ln((b*x+a)^p)*d^3/e^4*ln(e*x+d)-1/2*ln((b*x+a)^p)/e
^2*x^2*d+ln((b*x+a)^p)/e^3*x*d^2+1/3*ln(c)/e*x^3+1/2*I*Pi*csgn(I*c)*csgn(I*
c*(b*x+a)^p)^2/e^3*x*d^2-ln(c)*d^3/e^4*ln(e*x+d)-1/2*ln(c)/e^2*x^2*d+ln(c)/
e^3*x*d^2-49/36*p/e^4*d^3+p/e^4*d^3*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-1/
6*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)/e*x^3+1/4*I*Pi*csgn(
I*c*(b*x+a)^p)^3/e^2*x^2*d-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/e^3*x*d^2+1/6*I*P
i*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2/e*x^3+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d^3/
e^4*ln(e*x+d)+1/6*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e*x^3+1/b*p/
e^3*a*ln(b*(e*x+d)+a*e-b*d)*d^2+1/2/b^2*p/e^2*a^2*ln(b*(e*x+d)+a*e-b*d)*d-1
/6*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x^3+1/3/b^3*p/e*a^3*ln(b*(e*x+d)+a*e-b*d)+p
/e^4*d^3*ln(e*x+d)*ln((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-2/3/b*p/e^3*a*d^2-1/3/
b^2*p/e^2*a^2*d-1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2*d^3/e^4*ln(e*x+d)-
1/4*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2/e^2*x^2*d+1/2*I*Pi*csgn(I*(b*x+a)^
p)*csgn(I*c*(b*x+a)^p)^2/e^3*x*d^2-1/3*a^2*p*x/b^2/e+1/6*a*p*x^2/b/e-1/9*p*
x^3/e-d^2*p*x/e^3-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d^3/e^4*
ln(e*x+d)-1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e^2*x^2*d-1/2*a*
d*p*x/b/e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log\left((bx+a)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \log\left((bx+a)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log\left((bx+a)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

3.220 $\int \frac{x^2 \log(c(a+bx)^p)}{d+ex} dx$

Optimal. Leaf size=159

$$\frac{d^2p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^3}-\frac{a^2p\log(a+bx)}{2b^2e}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{x^2\log(c(a}{2e}$$

```
[Out] (d*p*x)/e^2 + (a*p*x)/(2*b*e) - (p*x^2)/(4*e) - (a^2*p*Log[a + b*x])/(2*b^2
*e) + (x^2*Log[c*(a + b*x)^p])/(2*e) - (d*(a + b*x)*Log[c*(a + b*x)^p])/(b*
e^2) + (d^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^3 + (d^2*p
*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^3
```

Rubi [A] time = 0.16708, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {43, 2416, 2389, 2295, 2395, 2394, 2393, 2391}

$$\frac{d^2p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^3}-\frac{a^2p\log(a+bx)}{2b^2e}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{x^2\log(c(a}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```


```
[Out] (d*p*x)/e^2 + (a*p*x)/(2*b*e) - (p*x^2)/(4*e) - (a^2*p*Log[a + b*x])/(2*b^2
*e) + (x^2*Log[c*(a + b*x)^p])/(2*e) - (d*(a + b*x)*Log[c*(a + b*x)^p])/(b*
e^2) + (d^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^3 + (d^2*p
*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^3
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2 \log(c(a+bx)^p)}{d+ex}\,dx = \int \left(-\frac{d\log(c(a+bx)^p)}{e^2} + \frac{x\log(c(a+bx)^p)}{e} + \frac{d^2\log(c(a+bx)^p)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int \log(c(a+bx)^p)\,dx}{e^2} + \frac{d^2\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e^2} + \frac{\int x\log(c(a+bx)^p)\,dx}{e}$$

$$= \frac{x^2\log(c(a+bx)^p)}{2e} + \frac{d^2\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} - \frac{d\,\mathrm{Subst}\left(\int \log(cx^p)\,dx, x, a+bx\right)}{be^2}$$

$$= \frac{dpx}{e^2} + \frac{x^2\log(c(a+bx)^p)}{2e} - \frac{d(a+bx)\log(c(a+bx)^p)}{be^2} + \frac{d^2\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} -$$

$$= \frac{dpx}{e^2} + \frac{apx}{2be} - \frac{px^2}{4e} - \frac{a^2p\log(a+bx)}{2b^2e} + \frac{x^2\log(c(a+bx)^p)}{2e} - \frac{d(a+bx)\log(c(a+bx)^p)}{be^2} + $$

Mathematica [A] time = 0.0882514, size = 127, normalized size = 0.8

$$\frac{4b^2d^2p\mathrm{PolyLog}\left(2,\frac{e(a+bx)}{ae-bd}\right) - 2a^2e^2p\log(a+bx) + b\log(c(a+bx)^p)\left(4bd^2\log\left(\frac{b(d+ex)}{bd-ae}\right) - 4ade + 2bex(ex-2d)\right) + bep}{4b^2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (b*e*p*x*(4*b*d + 2*a*e - b*e*x) - 2*a^2*e^2*p*Log[a + b*x] + b*Log[c*(a +
b*x)^p]*(-4*a*d*e + 2*b*e*x*(-2*d + e*x) + 4*b*d^2*Log[(b*(d + e*x))/(b*d -
 a*e)]) + 4*b^2*d^2*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(4*b^2*e^3)
```

Maple [C] time = 0.649, size = 666, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] d*p*x/e^2-1/b*p/e^2*a*ln(b*(e*x+d)+a*e-b*d)*d-1/2/b^2*p/e*a^2*ln(b*(e*x+d)+
a*e-b*d)+1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)/e^2*d*x-1
/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*d^2/e^3*ln(e*x+d)+l
n((b*x+a)^p)*d^2/e^3*ln(e*x+d)-ln((b*x+a)^p)/e^2*d*x-ln(c)/e^2*d*x+ln(c)*d^
2/e^3*ln(e*x+d)-p/e^3*d^2*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))+5/4*p/e^3*d^
2+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/e^2*d*x+1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*
c*(b*x+a)^p)^2/e*x^2+1/4*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2/e*x^2-1/2*I*P
i*csgn(I*c*(b*x+a)^p)^3*d^2/e^3*ln(e*x+d)-1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+
a)^p)^2/e^2*d*x+1/2*a*p*x/b/e-p/e^3*d^2*ln(e*x+d)*ln((b*(e*x+d)+a*e-b*d)/(a
*e-b*d))+1/2/b*p/e^2*a*d+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d
^2/e^3*ln(e*x+d)-1/4*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x^2-1/2*I*Pi*csgn(I*(b*x+
a)^p)*csgn(I*c*(b*x+a)^p)^2/e^2*d*x-1/4*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*cs
gn(I*c*(b*x+a)^p)/e*x^2-1/4*p*x^2/e+1/2*ln(c)/e*x^2+1/2*ln((b*x+a)^p)/e*x^2
+1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2*d^2/e^3*ln(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log\left((bx+a)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \log\left((bx+a)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log\left(c\,(a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x**2*log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log \left((bx+a)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

3.221 $\int \frac{x \log(c(a+bx)^p)}{d+ex} dx$

Optimal. Leaf size=91

$$-\frac{dp\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{px}{e}$$

```
[Out] -((p*x)/e) + ((a + b*x)*Log[c*(a + b*x)^p])/(b*e) - (d*Log[c*(a + b*x)^p]*L
og[(b*(d + e*x))/(b*d - a*e)])/e^2 - (d*p*PolyLog[2, -((e*(a + b*x))/(b*d -
 a*e))])/e^2
```

Rubi [A] time = 0.107653, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {43, 2416, 2389, 2295, 2394, 2393, 2391}

$$-\frac{dp\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{px}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((p*x)/e) + ((a + b*x)*Log[c*(a + b*x)^p])/(b*e) - (d*Log[c*(a + b*x)^p]*L
og[(b*(d + e*x))/(b*d - a*e)])/e^2 - (d*p*PolyLog[2, -((e*(a + b*x))/(b*d -
 a*e))])/e^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx &= \int \left(\frac{\log\left(c(a+bx)^p\right)}{e}-\frac{d\log\left(c(a+bx)^p\right)}{e(d+ex)}\right)dx\\
&= \frac{\int \log\left(c(a+bx)^p\right)\,dx}{e}-\frac{d\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e}\\
&= -\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,a+bx\right)}{be}+\frac{(bdp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e^2}\\
&= -\frac{px}{e}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(dp)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\ldots\right)}{e^2}\\
&= -\frac{px}{e}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}-\frac{dp\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e^2}
\end{aligned}
$$

Mathematica [A] time = 0.0327032, size = 79, normalized size = 0.87

$$\frac{-bdp\text{PolyLog}\left(2,\frac{e(a+bx)}{ae-bd}\right)+\log\left(c(a+bx)^p\right)\left(-bd\log\left(\frac{b(d+ex)}{bd-ae}\right)+ae+bex\right)-bepx}{be^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (-(b*e*p*x) + Log[c*(a + b*x)^p]*(a*e + b*e*x - b*d*Log[(b*(d + e*x))/(b*d
- a*e)]) - b*d*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(b*e^2)
```

Maple [C] time = 0.622, size = 427, normalized size = 4.7

$$\frac{\ln\left((bx+a)^p\right)x}{e}-\frac{\ln\left((bx+a)^p\right)d\ln(ex+d)}{e^2}-\frac{px}{e}-\frac{dp}{e^2}+\frac{ap\ln(b(ex+d)+ae-bd)}{be}+\frac{dp}{e^2}\text{dilog}\left(\frac{b(ex+d)+ae-b\ldots}{ae-bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] ln((b*x+a)^p)/e*x-ln((b*x+a)^p)*d/e^2*ln(e*x+d)-p*x/e-p/e^2*d+1/b*p/e*a*ln(
b*(e*x+d)+a*e-b*d)+p/e^2*d*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))+p/e^2*d*ln(
e*x+d)*ln((b*(e*x+d)+a*e-b*d)/(a*e-b*d))+1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^
p)*csgn(I*c*(b*x+a)^p)*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^
p)^2*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x-1/2*I*Pi*csgn(I*c)*
csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)/e*x+1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+
a)^p)^2/e*x+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*
(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x+a)^p)
*csgn(I*c*(b*x+a)^p)^2/e*x+ln(c)/e*x-ln(c)*d/e^2*ln(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \log \left((bx+a)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log \left(c (a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x*log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((b*x + a)^p*c)/(e*x + d), x)
```

3.222 $\int \frac{\log(c(a+bx)^p)}{d+ex} dx$

Optimal. Leaf size=58

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rubi [A] time = 0.0429506, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx,x,a+bx\right)}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}
\end{aligned}$$

Mathematica [A] time = 0.0030189, size = 57, normalized size = 0.98

$$\frac{p\text{PolyLog}\left(2,\frac{e(a+bx)}{ae-bd}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

Maple [C] time = 0.101, size = 242, normalized size = 4.2

$$\frac{\ln(ex+d)\ln\left((bx+a)^p\right)}{e}-\frac{p}{e}\text{dilog}\left(\frac{b(ex+d)+ae-bd}{ae-bd}\right)-\frac{p\ln(ex+d)}{e}\ln\left(\frac{b(ex+d)+ae-bd}{ae-bd}\right)-\frac{\frac{i}{2}\ln(ex+d)\pi\text{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x+a)^p)-1/e*p*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-1/e*p*
ln(e*x+d)*ln((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c)*
csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c)*csgn(I
*c*(b*x+a)^p)^2+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^
2-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^3+ln(e*x+d)/e*ln(c)
```

Maxima [B] time = 1.04569, size = 159, normalized size = 2.74

$$\frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\text{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e}-\frac{p\log(bx+a)\log(ex+d)}{e}+\frac{\log\left((bx+a)^pc\right)\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e*x
 + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((bx+a)^pc\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(c (a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left((bx+a)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

3.223 $\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx$

Optimal. Leaf size=97

$$-\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}$$

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d - (p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (p*P
olyLog[2, 1 + (b*x)/a])/d
```

Rubi [A] time = 0.122699, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d - (p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (p*P
olyLog[2, 1 + (b*x)/a])/d
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx = \int\left(\frac{\log(c(a+bx)^p)}{dx} - \frac{e\log(c(a+bx)^p)}{d(d+ex)}\right)dx$$
$$= \frac{\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d}$$
$$= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(bp)\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx}{d} + \frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}}{d}$$
$$= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{p\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{p\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{}{bc}\right)}{x}\right.}{d}$$
$$= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{p\mathrm{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{p\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d}$$

Mathematica [A] time = 0.0201409, size = 98, normalized size = 1.01

$$-\frac{p\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{p\mathrm{PolyLog}\left(2,\frac{a+bx}{a}\right)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d + (p*PolyLog[2, (a + b*x)/a])/d - (p*PolyLog[2, -((e*(a
 + b*x))/(b*d - a*e))])/d
```

Maple [C] time = 0.592, size = 420, normalized size = 4.3

$$-\frac{\ln\left((bx+a)^p\right)\ln(ex+d)}{d} + \frac{\ln\left((bx+a)^p\right)\ln(x)}{d} - \frac{p}{d}\mathrm{dilog}\left(\frac{bx+a}{a}\right) - \frac{p\ln(x)}{d}\ln\left(\frac{bx+a}{a}\right) + \frac{p}{d}\mathrm{dilog}\left(\frac{b(ex+d)+ae-b}{ae-bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x/(e*x+d),x)
```

```
[Out] -ln((b*x+a)^p)/d*ln(e*x+d)+ln((b*x+a)^p)/d*ln(x)-p/d*dilog(1/a*(b*x+a))-p/d
*ln(x)*ln(1/a*(b*x+a))+p/d*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))+p/d*ln(e*x+
d)*ln((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*c
sgn(I*c*(b*x+a)^p)/d*ln(x)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/d*ln(x)+1/2*I*Pi*
csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d*ln(x)-1/2*I*Pi*csgn(I*c)*csgn(I*c
*(b*x+a)^p)^2/d*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/d*ln(e*x+d)-1/2*I*
Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d*ln(e*x+d)+1/2*I*Pi*csgn(I*c)*c
sgn(I*c*(b*x+a)^p)^2/d*ln(x)+1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*
(b*x+a)^p)/d*ln(e*x+d)-ln(c)/d*ln(e*x+d)+ln(c)/d*ln(x)
```

Maxima [A] time = 1.2386, size = 166, normalized size = 1.71

$$-bp\left(\frac{\log\left(\frac{bx}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{bd}-\frac{\log\left(ex+d\right)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{bd}\right)-\left(\frac{\log\left(ex+d\right)}{d}-\frac{\log\left(x\right)}{d}\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))/(b*d) - (log(e*x + d)*log(-(b
*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/(b*d)) - (
log(e*x + d)/d - log(x)/d)*log((b*x + a)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left((bx+a)^p c\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx\right)^p\right)}{x\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(x*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x), x)
```

3.224 $\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=146

$$\frac{ep\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^2}-\frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2}+\frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}-\frac{\text{lo}}{}$$

```
[Out] (b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
(e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b*(
d + e*x))/(b*d - a*e)])/d^2 + (e*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))]
)/d^2 - (e*p*PolyLog[2, 1 + (b*x)/a])/d^2
```

Rubi [A] time = 0.166364, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^2}-\frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2}+\frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}-\frac{\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
(e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b*(
d + e*x))/(b*d - a*e)])/d^2 + (e*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))]
)/d^2 - (e*p*PolyLog[2, 1 + (b*x)/a])/d^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx &= \int \left(\frac{\log(c(a+bx)^p)}{dx^2} - \frac{e\log(c(a+bx)^p)}{d^2x} + \frac{e^2\log(c(a+bx)^p)}{d^2(d+ex)}\right)dx \\
&= \frac{\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^2} \\
&= -\frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} + \frac{(bp)\int \frac{1}{x(a+bx)}}{d} \\
&= -\frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} - \frac{ep\,\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d^2} \\
&= \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad} - \frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.0482952, size = 139, normalized size = 0.95

$$\frac{aepx\text{PolyLog}\left(2,\frac{e(a+bx)}{ae-bd}\right) - aepx\text{PolyLog}\left(2,\frac{bx}{a}+1\right) + aex\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right) - ad\log(c(a+bx)^p) - aex\log\left(-\right.}{ad^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*d*p*x*Log[x] - b*d*p*x*Log[a + b*x] - a*d*Log[c*(a + b*x)^p] - a*e*x*Log
[-((b*x)/a)]*Log[c*(a + b*x)^p] + a*e*x*Log[c*(a + b*x)^p]*Log[(b*(d + e*x)
)/(b*d - a*e)] + a*e*p*x*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)] - a*e*p*x
*PolyLog[2, 1 + (b*x)/a])/(a*d^2*x)
```

Maple [C] time = 0.595, size = 615, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x^2/(e*x+d),x)
```

```
[Out] ln((b*x+a)^p)*e/d^2*ln(e*x+d)-ln((b*x+a)^p)/d/x-ln((b*x+a)^p)*e/d^2*ln(x)-p
*e/d^2*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-p*e/d^2*ln(e*x+d)*ln((b*(e*x+d)
+a*e-b*d)/(a*e-b*d))+b*p*ln(x)/a/d-b*p*ln(b*x+a)/a/d+p*e/d^2*dilog(1/a*(b*x
+a))+p*e/d^2*ln(x)*ln(1/a*(b*x+a))+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^2*ln(
x)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/d/x-1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*
csgn(I*c*(b*x+a)^p)*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*cs
gn(I*c*(b*x+a)^p)/d/x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e/d^
2*ln(x)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e/d^2*ln(e*x+d)-1/
2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c)*csgn(
I*c*(b*x+a)^p)^2/d/x+1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2*e/d^2*ln(e*x+
d)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d/x+1/2*I*Pi*csgn(I*c)*
csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*x+a)
^p)^3*e/d^2*ln(e*x+d)+ln(c)*e/d^2*ln(e*x+d)-ln(c)/d/x-ln(c)*e/d^2*ln(x)
```

Maxima [A] time = 1.23382, size = 211, normalized size = 1.45

$$bp\left(\frac{\left(\log\left(\frac{bx}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)e}{bd^2}-\frac{\left(\log\left(ex+d\right)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)\right)e}{bd^2}-\frac{\log\left(bx+a\right)}{ad}+\frac{\log\left(x\right)}{ad}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))*e/(b*d^2) - (log(e*x + d)*log(
-(b*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e/(b*d^
2) - log(b*x + a)/(a*d) + log(x)/(a*d)) + (e*log(e*x + d)/d^2 - e*log(x)/d^
2 - 1/(d*x))*log((b*x + a)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx+a\right)^{p}c\right)}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x^2), x)
```

3.225 $\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)} dx$

Optimal. Leaf size=227

$$-\frac{e^2 p \text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3}+\frac{e^2 p \text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^3}-\frac{b^2 p \log(x)}{2a^2 d}+\frac{b^2 p \log(a+bx)}{2a^2 d}+\frac{e^2 \log\left(-\frac{bx}{a}\right) \log\left(c(a+bx)^p\right)}{d^3}$$

```
[Out] -(b*p)/(2*a*d*x) - (b^2*p*Log[x])/(2*a^2*d) - (b*e*p*Log[x])/(a*d^2) + (b^2
*p*Log[a + b*x])/(2*a^2*d) + (b*e*p*Log[a + b*x])/(a*d^2) - Log[c*(a + b*x)
^p]/(2*d*x^2) + (e*Log[c*(a + b*x)^p])/(d^2*x) + (e^2*Log[-((b*x)/a)]*Log[c
*(a + b*x)^p])/d^3 - (e^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)]
)/d^3 - (e^2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d^3 + (e^2*p*PolyL
og[2, 1 + (b*x)/a])/d^3
```

Rubi [A] time = 0.222614, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$-\frac{e^2 p \text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3}+\frac{e^2 p \text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^3}-\frac{b^2 p \log(x)}{2a^2 d}+\frac{b^2 p \log(a+bx)}{2a^2 d}+\frac{e^2 \log\left(-\frac{bx}{a}\right) \log\left(c(a+bx)^p\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -(b*p)/(2*a*d*x) - (b^2*p*Log[x])/(2*a^2*d) - (b*e*p*Log[x])/(a*d^2) + (b^2
*p*Log[a + b*x])/(2*a^2*d) + (b*e*p*Log[a + b*x])/(a*d^2) - Log[c*(a + b*x)
^p]/(2*d*x^2) + (e*Log[c*(a + b*x)^p])/(d^2*x) + (e^2*Log[-((b*x)/a)]*Log[c
*(a + b*x)^p])/d^3 - (e^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)]
)/d^3 - (e^2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d^3 + (e^2*p*PolyL
og[2, 1 + (b*x)/a])/d^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx &= \int \left(\frac{\log(c(a+bx)^p)}{dx^3} - \frac{e\log(c(a+bx)^p)}{d^2x^2} + \frac{e^2\log(c(a+bx)^p)}{d^3x} - \frac{e^3\log(c(a+bx)^p)}{d^3(d+ex)}\right)dx \\
&= \frac{\int \frac{\log(c(a+bx)^p)}{x^3}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^3} \\
&= -\frac{\log(c(a+bx)^p)}{2dx^2} + \frac{e\log(c(a+bx)^p)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^3} - \frac{e^2\log(c(a+bx)^p)\log\left(\frac{b}{}\right.}{d^3} \\
&= -\frac{\log(c(a+bx)^p)}{2dx^2} + \frac{e\log(c(a+bx)^p)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^3} - \frac{e^2\log(c(a+bx)^p)\log\left(\frac{b}{}\right.}{d^3} \\
&= -\frac{bp}{2adx} - \frac{b^2p\log(x)}{2a^2d} - \frac{bep\log(x)}{ad^2} + \frac{b^2p\log(a+bx)}{2a^2d} + \frac{bep\log(a+bx)}{ad^2} - \frac{\log(c(a+bx)^p)}{2dx^2} + \frac{e\log}{}
\end{aligned}$$

Mathematica [A] time = 0.168071, size = 188, normalized size = 0.83

$$-\frac{2e^2p\text{PolyLog}\left(2,\frac{e(a+bx)}{ae-bd}\right) - 2e^2p\text{PolyLog}\left(2,\frac{bx}{a}+1\right) + \frac{bd^2p(-bx\log(a+bx)+a+bx\log(x))}{a^2x} + \frac{d^2\log(c(a+bx)^p)}{x^2} + 2e^2\log(c(a+bx)^p)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -((2*b*d*e*p*(Log[x] - Log[a + b*x]))/a + (b*d^2*p*(a + b*x*Log[x] - b*x*Lo
g[a + b*x]))/(a^2*x) + (d^2*Log[c*(a + b*x)^p])/x^2 - (2*d*e*Log[c*(a + b*x
)^p])/x - 2*e^2*Log[-((b*x)/a)]*Log[c*(a + b*x)^p] + 2*e^2*Log[c*(a + b*x)^
p]*Log[(b*(d + e*x))/(b*d - a*e)] + 2*e^2*p*PolyLog[2, (e*(a + b*x))/(-(b*d
) + a*e)] - 2*e^2*p*PolyLog[2, 1 + (b*x)/a])/(2*d^3)
```

Maple [C] time = 0.608, size = 850, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x^3/(e*x+d),x)
```

```
[Out] -1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d/x^2+1/2*I*Pi*csgn(I*c*(
b*x+a)^p)^3*e^2/d^3*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e^2/d^3*ln(x)-
1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^2/x-1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)
*csgn(I*c*(b*x+a)^p)*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csg
n(I*c*(b*x+a)^p)*e^2/d^3*ln(e*x+d)-1/2*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csg
n(I*c*(b*x+a)^p)*e/d^2/x-1/4*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2/d/x^2+1/4
*I*Pi*csgn(I*c*(b*x+a)^p)^3/d/x^2-p*e^2/d^3*ln(x)*ln(1/a*(b*x+a))+p*e^2/d^3
*ln(e*x+d)*ln((b*(e*x+d)+a*e-b*d)/(a*e-b*d))-1/2*ln((b*x+a)^p)/d/x^2-ln(c)*
e^2/d^3*ln(e*x+d)+ln(c)*e^2/d^3*ln(x)+ln(c)*e/d^2/x-1/2*b^2*p*ln(x)/a^2/d+1
/2*b^2*p*ln(b*x+a)/a^2/d-1/2*b*p/a/x/d-p*e^2/d^3*dilog(1/a*(b*x+a))+p*e^2/d
^3*dilog((b*(e*x+d)+a*e-b*d)/(a*e-b*d))+ln((b*x+a)^p)*e/d^2/x-ln((b*x+a)^p)
*e^2/d^3*ln(e*x+d)+ln((b*x+a)^p)*e^2/d^3*ln(x)-1/2*ln(c)/d/x^2+1/2*I*Pi*csg
n(I*c)*csgn(I*c*(b*x+a)^p)^2*e/d^2/x+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b
*x+a)^p)^2*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2*e^2/d^3*l
n(e*x+d)+1/2*I*Pi*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2*e^2/d^3*ln(x)+1/2*I*Pi*cs
gn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e/d^2/x-b*e*p*ln(x)/a/d^2+b*e*p*ln(b*
x+a)/a/d^2+1/4*I*Pi*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)/d/x^2-1
/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e^2/d^3*ln(e*x+d)
```

Maxima [A] time = 1.23295, size = 292, normalized size = 1.29

$$\frac{1}{2}\left(2\,e\left(\frac{\log(bx+a)}{ad^2}-\frac{\log(x)}{ad^2}\right)-\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)e^2}{bd^3}+\frac{2\left(\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+}{bd-}\right.\right.}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*e*(log(b*x + a)/(a*d^2) - log(x)/(a*d^2)) - 2*(log(b*x/a + 1)*log(x)
 + dilog(-b*x/a))*e^2/(b*d^3) + 2*(log(e*x + d)*log(-(b*e*x + b*d)/(b*d - a
*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e^2/(b*d^3) + b*log(b*x + a)/(
a^2*d) - b*log(x)/(a^2*d) - 1/(a*d*x))*b*p - 1/2*(2*e^2*log(e*x + d)/d^3 -
2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((b*x + a)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((bx+a)^p c\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x^3), x)
```

3.226 $\int \frac{x^3 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=394

$$\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4}+\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^4}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e}-\frac{d^3\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^4}+\cdots$$

```
[Out] (-2*d^2*p*x)/e^3 + (2*a*p*x)/(3*b*e) + (d*p*x^2)/(2*e^2) - (2*p*x^3)/(9*e)
+ (2*Sqrt[a]*d^2*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^3) - (2*a^(3/2)*
p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)*e) + (d^3*p*Log[(e*(Sqrt[-a] - Sq
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sq
rt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/e^4 + (d^2*x*
Log[c*(a + b*x^2)^p])/e^3 + (x^3*Log[c*(a + b*x^2)^p])/(3*e) - (d*(a + b*x^
2)*Log[c*(a + b*x^2)^p])/(2*b*e^2) - (d^3*Log[d + e*x]*Log[c*(a + b*x^2)^p]
)/e^4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^
4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^4
```

Rubi [A] time = 0.426393, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {2466, 2448, 321, 205, 2454, 2389, 2295, 2455, 302, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4}+\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^4}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e}-\frac{d^3\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*d^2*p*x)/e^3 + (2*a*p*x)/(3*b*e) + (d*p*x^2)/(2*e^2) - (2*p*x^3)/(9*e)
+ (2*Sqrt[a]*d^2*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^3) - (2*a^(3/2)*
p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)*e) + (d^3*p*Log[(e*(Sqrt[-a] - Sq
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sq
rt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/e^4 + (d^2*x*
Log[c*(a + b*x^2)^p])/e^3 + (x^3*Log[c*(a + b*x^2)^p])/(3*e) - (d*(a + b*x^
2)*Log[c*(a + b*x^2)^p])/(2*b*e^2) - (d^3*Log[d + e*x]*Log[c*(a + b*x^2)^p]
)/e^4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^
4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^4
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2\log\left(c\left(a+bx^2\right)^p\right)}{e^3} - \frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d^3\log\left(c\left(a+\cdots\right.\right.}{e^3(d+\cdots}\right.$$

$$= \frac{d^2\int\log\left(c\left(a+bx^2\right)^p\right)dx}{e^3} - \frac{d^3\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{e^3} - \frac{d\int x\log\left(c\left(a+bx^2\right)^p\right)dx}{e^2} + \frac{\int x^2 \mathrm{l}\cdots}{\cdots}$$

$$= \frac{d^2x\log\left(c\left(a+bx^2\right)^p\right)}{e^3} + \frac{x^3\log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{d^3\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^4} - \frac{d\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{2d^2px}{e^3} + \frac{d^2x\log\left(c\left(a+bx^2\right)^p\right)}{e^3} + \frac{x^3\log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{d^3\log(d+ex)\log\left(c\left(a+bx^{\cdots}\right.\right.}{e^4}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} + \frac{d^2x\log\left(c\left(a+bx^2\right)^p\right)}{e^3} + \frac{x^3\,\mathrm{l}\cdots}{\cdots}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e} + \frac{d^3p\log\cdots}{\cdots}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e} + \frac{d^3p\log\cdots}{\cdots}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e} + \frac{d^3p\log\cdots}{\cdots}$$

Mathematica [A] time = 0.319472, size = 338, normalized size = 0.86

$$18d^3p\left(\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)+\log(d+ex)\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)+\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{-a}e-\sqrt{b}d}\right)\right)\right)-4e^3p\left(\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-4*e^3*p*((-3*a*x)/b + x^3 + (3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/
2)) - 36*d^2*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + 18*d
^2*e*x*Log[c*(a + b*x^2)^p] + 6*e^3*x^3*Log[c*(a + b*x^2)^p] - 18*d^3*Log[d
 + e*x]*Log[c*(a + b*x^2)^p] + 9*d*e^2*(p*x^2 - ((a + b*x^2)*Log[c*(a + b*x
^2)^p])/b) + 18*d^3*p*((Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d
+ e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog
[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/(18*e^4)
```

Maple [C] time = 0.541, size = 1083, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] -ln((b*x^2+a)^p)*d^3/e^4*ln(e*x+d)-1/2*ln((b*x^2+a)^p)/e^2*x^2*d+2*p/e^3*a/
(a*b)^(1/2)*arctan(1/2*(2*b*(e*x+d)-2*b*d)/e/(a*b)^(1/2))*d^2+1/2*d*p*x^2/e
^2+1/3*ln(c)/e*x^3-ln(c)*d^3/e^4*ln(e*x+d)-1/2*ln(c)/e^2*x^2*d+ln(c)/e^3*x*
d^2-49/18*p/e^4*d^3-1/2/b*p/e^2*a*d*ln(b*(e*x+d)^2-2*(e*x+d)*b*d+a*e^2+b*d^
2)-2/3/b*p/e*a^2/(a*b)^(1/2)*arctan(1/2*(2*b*(e*x+d)-2*b*d)/e/(a*b)^(1/2))+
1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e^2*x^2*d+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3
*d^3/e^4*ln(e*x+d)+1/6*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x
^3-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e^3*x*d^2+p/e^4*d^3*ln(e*x+d)*ln((e*(-a
*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))+p/e^4*d^3*ln(e*x+d)*ln((e*(-
a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+p/e^4*d^3*dilog((e*(-a*b)^(
1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))+p/e^4*d^3*dilog((e*(-a*b)^(1/2)+b
*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+1/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)*csgn(I*c)/e^2*x^2*d+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)*csgn(I*c)*d^3/e^4*ln(e*x+d)-1/6*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x^3+
1/3*ln((b*x^2+a)^p)/e*x^3+ln((b*x^2+a)^p)/e^3*x*d^2+2/3*a*p*x/b/e+1/6*I*Pi*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e*x^3+2/3/b*p/e^2*a*d-2/9*p*x^3/e-1/2*I*P
i*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e^3*x*d^2-2*d^2*p*x/e
^3-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*d^3/e^4*ln(e*x+d)-1/4*I*Pi*cs
gn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e^2*x^2*d+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)^2/e^3*x*d^2+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e^
3*x*d^2-1/6*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e*x^3-
1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*d^3/e^4*ln(e*x+d)-1/4*
I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e^2*x^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \log\left(\left(bx^2+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

3.227 $\int \frac{x^2 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=313

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^3}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+$$

```
[Out] (2*d*p*x)/e^2 - (p*x^2)/(2*e) - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(Sqrt[b]*e^2) - (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*
d - Sqrt[-a]*e))]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 + ((a
+ b*x^2)*Log[c*(a + b*x^2)^p])/(2*b*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^2
)^p])/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
)/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^
3
```

Rubi [A] time = 0.334236, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2466, 2448, 321, 205, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^3}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```


```
[Out] (2*d*p*x)/e^2 - (p*x^2)/(2*e) - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(Sqrt[b]*e^2) - (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*
d - Sqrt[-a]*e))]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 + ((a
+ b*x^2)*Log[c*(a + b*x^2)^p])/(2*b*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^2
)^p])/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
)/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^
3
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+bx^2\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int \log\left(c\left(a+bx^2\right)^p\right)dx}{e^2}+\frac{d^2\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+bx^2\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}+\frac{\text{Subst}\left(\int \log\left(c(a+bx)^p\right)dx,\ldots\right)}{2e}$$

$$= \frac{2dpx}{e^2}-\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}+\frac{\text{Subst}\left(\int \log\left(cx^p\right)dx,\ldots\right)}{2be}$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2be}+\ldots$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\ldots\right)}{\sqrt{b}d-\sqrt{-a}\ldots}\right)\ldots}{e^3}\ldots$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\ldots\right)}{\sqrt{b}d-\sqrt{-a}\ldots}\right)\ldots}{e^3}\ldots$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\ldots\right)}{\sqrt{b}d-\sqrt{-a}\ldots}\right)\ldots}{e^3}\ldots$$

Mathematica [A] time = 0.161251, size = 271, normalized size = 0.87

$$\frac{-2d^2p\left(\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)+\log(d+ex)\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)+\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{-a}e-\sqrt{b}d}\right)\right)\right)+2d^2\log(\ldots}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-(e^2*p*x^2) + 4*d*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])
 - 2*d*e*x*Log[c*(a + b*x^2)^p] + (e^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b
+ 2*d^2*Log[d + e*x]*Log[c*(a + b*x^2)^p] - 2*d^2*p*((Log[(e*(Sqrt[-a] - Sq
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqr
t[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt
[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]
*e)]))/(2*e^3)
```

Maple [C] time = 0.465, size = 825, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] 2*d*p*x/e^2+1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e*x^2+1/4*I*Pi*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x^2-1/4*I*Pi*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e*x^2-ln(c)/e^2*d*x+ln(c)*d^2/e^3*ln(e*x+d)+
5/2*p/e^3*d^2-2*p/e^2*a*d/(a*b)^(1/2)*arctan(1/2*(2*b*(e*x+d)-2*b*d)/e/(a*b
)^(1/2))-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e^2*d*x-1/2*I*Pi*csgn(I
*c*(b*x^2+a)^p)^3*d^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e^2*d*
x+1/2/b*p/e*a*ln(b*(e*x+d)^2-2*(e*x+d)*b*d+a*e^2+b*d^2)+1/2*I*Pi*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*d^2/e^3*ln(e*x+d)+ln((b*x^2+a)^p)*d^2/e^3
*ln(e*x+d)-ln((b*x^2+a)^p)/e^2*d*x-p/e^3*d^2*ln(e*x+d)*ln((e*(-a*b)^(1/2)-b
*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))-p/e^3*d^2*ln(e*x+d)*ln((e*(-a*b)^(1/2)+
b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))-1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x^2
-p/e^3*d^2*dilog((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))-p/e^3
*d^2*dilog((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))-1/2*p*x^2/e
-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*d^2/e^3*ln(e*
x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e^2*d*x+1
/2*ln(c)/e*x^2+1/2*ln((b*x^2+a)^p)/e*x^2+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)*d^2/e^3*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)^2/e^2*d*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \log\left(\left(bx^2+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

3.228 $\int \frac{x \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=256

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e}+\frac{dp}{}$$

```
[Out] (-2*p*x)/e + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e) + (d*p*L
og[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^2 +
 (d*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/e^2 + (x*Log[c*(a + b*x^2)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^2)^p
])/e^2 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^2
 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```

Rubi [A] time = 0.273336, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2466, 2448, 321, 205, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e}+\frac{dp}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```



```
[Out] (-2*p*x)/e + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e) + (d*p*L
og[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^2 +
 (d*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/e^2 + (x*Log[c*(a + b*x^2)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^2)^p
])/e^2 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^2
 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x \log \left(c \left(a+bx^2\right)^p\right)}{d+ex} dx = \int \left(\frac{\log \left(c \left(a+bx^2\right)^p\right)}{e} - \frac{d \log \left(c \left(a+bx^2\right)^p\right)}{e(d+ex)}\right) dx$$

$$= \frac{\int \log \left(c \left(a+bx^2\right)^p\right) dx}{e} - \frac{d \int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex} dx}{e}$$

$$= \frac{x \log \left(c \left(a+bx^2\right)^p\right)}{e} - \frac{d \log(d+ex) \log \left(c \left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp) \int \frac{x \log(d+ex)}{a+bx^2} dx}{e^2} - \frac{(2bp) \int}{}$$

$$= -\frac{2px}{e} + \frac{x \log \left(c \left(a+bx^2\right)^p\right)}{e} - \frac{d \log(d+ex) \log \left(c \left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp) \int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{bx}\right.}\right.}{e^2}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{x \log \left(c \left(a+bx^2\right)^p\right)}{e} - \frac{d \log(d+ex) \log \left(c \left(a+bx^2\right)^p\right)}{e^2} - \frac{(\sqrt{}}{}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right) \log(d+ex)}{e^2} + \frac{dp \log \left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right) \log}{e^2}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right) \log(d+ex)}{e^2} + \frac{dp \log \left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right) \log}{e^2}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right) \log(d+ex)}{e^2} + \frac{dp \log \left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right) \log}{e^2}$$

Mathematica [A] time = 0.11746, size = 225, normalized size = 0.88

$$\frac{dp \left(\text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right) + \text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right) + \log(d+ex) \left(\log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right) + \log \left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{-a}e-\sqrt{b}d}\right)\right)\right) - d \log(d +}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + e*x*Log[c*(a
+ b*x^2)^p] - d*Log[d + e*x]*Log[c*(a + b*x^2)^p] + d*p*((Log[(e*(Sqrt[-a]
- Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-
(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(
Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt
[-a]*e)]))/e^2
```

Maple [C] time = 0.444, size = 576, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] ln((b*x^2+a)^p)/e*x-ln((b*x^2+a)^p)*d/e^2*ln(e*x+d)-2*p*x/e-2*p/e^2*d+2*p/e
*a/(a*b)^(1/2)*arctan(1/2*(2*b*(e*x+d)-2*b*d)/e/(a*b)^(1/2))+p/e^2*d*ln(e*x
+d)*ln((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))+p/e^2*d*ln(e*x+
d)*ln((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+p/e^2*d*dilog((e
*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))+p/e^2*d*dilog((e*(-a*b)^
(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3
*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e*x+1/2*I*Pi*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)^2*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)*csgn(I*c)*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x-1/2*I*
Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*d/e^2*ln(e*x+d)+ln(c)/e*x-ln(c)*d/e^2*
ln(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \log\left(\left(bx^2 + a\right)^p c\right)}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log\left(\left(bx^2 + a\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

3.229 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=201

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}-$$

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rubi [A] time = 0.18499, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx &= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{(2bp)\int\frac{x\log(d+ex)}{a+bx^2}\,dx}{e}\\
&= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{(2bp)\int\left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e}\\
&= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}+\frac{\left(\sqrt{b}p\right)\int\frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e}-\frac{\left(\sqrt{b}p\right)\int\frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e}\\
&= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\right.\right.}{e}\\
&= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\right.\right.}{e}\\
&= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\right.\right.}{e}
\end{aligned}$$

Mathematica [A] time = 0.0255559, size = 201, normalized size = 1.

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Maple [C] time = 0.092, size = 366, normalized size = 1.8

$$\frac{\ln (ex+d)\ln \left(\left(bx^2+a\right)^p\right)}{e}-\frac{p\ln (ex+d)}{e}\ln \left(\left(e\sqrt{-ab}-b(ex+d)+bd\right)\left(e\sqrt{-ab}+bd\right)^{-1}\right)-\frac{p\ln (ex+d)}{e}\ln \left(\left(e\sqrt{-ab}+b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x^2+a)^p)-p/e*ln(e*x+d)*ln((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)
/(e*(-a*b)^(1/2)+b*d))-p/e*ln(e*x+d)*ln((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(
-a*b)^(1/2)-b*d))-p/e*dilog((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+
b*d))-p/e*dilog((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+1/2*I*
ln(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(e*x+d)/
e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*
Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^p)^2*csg
n(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log \left(\left(bx^2+a\right)^p c\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

3.230 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)} dx$

<u>**Optimal**</u>. Leaf size=247

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d}-\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d}+\frac{\log\left(-\dots\right.}{\dots}$$

```
[Out] (p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d
 + (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d) - (Log[d + e*x]*Log
[c*(a + b*x^2)^p])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[
-a]*e)])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d
 + (p*PolyLog[2, 1 + (b*x^2)/a])/(2*d)
```

<u>**Rubi**</u> **[A]** time = 0.302525, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d}-\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d}+\frac{\log\left(-\dots\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```


```
[Out] (p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d
 + (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d) - (Log[d + e*x]*Log
[c*(a + b*x^2)^p])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[
-a]*e)])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d
 + (p*PolyLog[2, 1 + (b*x^2)/a])/(2*d)
```


<u>Rule 2466</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

<u>Rule 2454</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

<u>Rule 2394</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)}dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{d}$$

$$= -\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, x^2\right)}{2d} + \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}dx}{d}$$

$$= \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}dx, x, x\right.}{2d}$$

$$= \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d} - \frac{\left(\sqrt{b}p\right)\int \frac{\log}{\sqrt{-}}}{d}$$

$$= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right.\right.}{2d}$$

$$= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right.\right.}{2d}$$

$$= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right.\right.}{2d}$$

Mathematica [A] time = 0.0707011, size = 232, normalized size = 0.94

$$\frac{p\text{PolyLog}\left(2,\frac{a+bx^2}{a}\right)+\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} + \frac{p\left(\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)+\log(d+ex\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[d + e*x]*Log[c*(a + b*x^2)^p])/d) + (p*(Log[(e*(Sqrt[-a] - Sqrt[b]*x
))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x] + Log[-((e*(Sqrt[-a] + Sqrt[b]*x)
)/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/
(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqr
t[-a]*e)]))/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a +
 b*x^2)/a])/(2*d)
```

Maple [C] time = 0.408, size = 624, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x/(e*x+d),x)
```

```
[Out] -ln((b*x^2+a)^p)/d*ln(e*x+d)+ln((b*x^2+a)^p)/d*ln(x)-p/d*ln(x)*ln((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))-p/d*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p/d*
```

```
dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p/d*dilog((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))+p/d*ln(e*x+d)*ln((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d
))+p/d*ln(e*x+d)*ln((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+p/
d*dilog((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))+p/d*dilog((e*(
-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))-1/2*I*Pi*csgn(I*c*(b*x^2+a
)^p)^3/d*ln(x)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/d*ln(e*
x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/d*ln(x)+1/2*I*Pi*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d*ln(e*x+d)-1/2*I*Pi*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d*ln(x)-1/2*I*Pi*csgn(I*c
*(b*x^2+a)^p)^2*csgn(I*c)/d*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d*ln
(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d*ln(x)-ln(c)/d*ln(e*x+d
)+ln(c)/d*ln(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

3.231 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=306

$$-\frac{ep\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^2}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2}+$$

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d) - (e*p*Log[(e*(Sqrt[-
a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d^2 - (e*p*Log[-((
e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/d^2 - Lo
g[c*(a + b*x^2)^p]/(d*x) - (e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d^
2) + (e*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(
d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(d + e*
x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^2
)
```

Rubi [A] time = 0.350157, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2466, 2455, 205, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{ep\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^2}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```


```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d) - (e*p*Log[(e*(Sqrt[-
a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d^2 - (e*p*Log[-((
e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/d^2 - Lo
g[c*(a + b*x^2)^p]/(d*x) - (e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d^
2) + (e*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(
d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(d + e*
x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^2
)
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+bx^2\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} + \frac{e\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^2} - \frac{e\,\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, x^2\right)}{2d^2} +$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2} + \frac{e\log(d+ex)\log\ldots}{\ldots}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2} + \frac{e\log(d+ex)\log\ldots}{\ldots}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \ldots$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \ldots$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \ldots$$

Mathematica [A] time = 0.191267, size = 268, normalized size = 0.88

$$-\frac{e\left(p\,\mathrm{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)\right)+2ep\left(\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -((-4*Sqrt[b]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (2*d*Log[c*(a + b*
x^2)^p])/x - 2*e*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e*p*((Log[(e*(Sqrt[-
a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))
/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x)
)/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + S
qrt[-a]*e)]) + e*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 +
 (b*x^2)/a]))/(2*d^2)
```

Maple [C] time = 0.413, size = 831, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^2/(e*x+d),x)
```

```
[Out] 1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2*ln(x)-1/
2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2*ln(e*x+d)+
ln(c)*e/d^2*ln(e*x+d)-ln(c)*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*cs
gn(I*c)/d/x-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/d/x+1/2*I*
Pi*csgn(I*c*(b*x^2+a)^p)^3*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e/d
^2*ln(e*x+d)+ln((b*x^2+a)^p)*e/d^2*ln(e*x+d)-ln((b*x^2+a)^p)/d/x-p*e/d^2*di
log((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))-p*e/d^2*dilog((e*(
-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+p*e/d^2*dilog((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))+p*e/d^2*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+2*b*p/
d/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+p*e/d^2*ln(x)*ln((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))+p*e/d^2*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p*e/d^2*ln(
e*x+d)*ln((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(1/2)+b*d))-p*e/d^2*ln(e
*x+d)*ln((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^(1/2)-b*d))+1/2*I*Pi*csgn
(I*c*(b*x^2+a)^p)^3/d/x-ln((b*x^2+a)^p)*e/d^2*ln(x)-ln(c)/d/x-1/2*I*Pi*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^
2+a)^p)^2*csgn(I*c)*e/d^2*ln(x)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)^2*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)*csgn(I*c)/d/x+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e/d^2*ln(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2 + a\right)^p c\right)}{(ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^2), x)
```

3.232 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=371

$$\frac{e^2p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^3}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^3}-$$

```
[Out] (-2*Sqrt[b]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d^2) + (b*p*Log[x])/(
a*d) + (e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[
d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-
a]*e))]*Log[d + e*x])/d^3 - (b*p*Log[a + b*x^2])/(2*a*d) - Log[c*(a + b*x^2
)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^2)^p])/(d^2*x) + (e^2*Log[-((b*x^2)/a)]*
Log[c*(a + b*x^2)^p])/(2*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^3
 + (e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^3 + (
e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^3 + (e^2*
p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^3)
```

Rubi [A] time = 0.394818, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {2466, 2454, 2395, 36, 29, 31, 2455, 205, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{e^2p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^3}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[b]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d^2) + (b*p*Log[x])/(
a*d) + (e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[
d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-
a]*e))]*Log[d + e*x])/d^3 - (b*p*Log[a + b*x^2])/(2*a*d) - Log[c*(a + b*x^2
)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^2)^p])/(d^2*x) + (e^2*Log[-((b*x^2)/a)]*
Log[c*(a + b*x^2)^p])/(2*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^3
 + (e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^3 + (
e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^3 + (e^2*
p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^3)
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+bx^2\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+bx^2\right)^p\right)}{d^3(d+ex)}\right) d$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$= \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} - \frac{e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^3} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x^2}\,dx, x, x^2\right)}{2d} + \frac{e^2}{}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx\right.\right.}{2d^3}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx\right.\right.}{2d^3}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}$$

Mathematica [A] time = 0.211551, size = 320, normalized size = 0.86

$$e^2\left(p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)\right)+2e^2p\left(\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)+\text{lo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-4*Sqrt[b]*d*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*d^2*p*(2*Log[x
] - Log[a + b*x^2]))/a - (d^2*Log[c*(a + b*x^2)^p])/x^2 + (2*d*e*Log[c*(a +
 b*x^2)^p])/x - 2*e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e^2*p*((Log[(e*
(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b
]*d + Sqrt[-a]*e)]) + e^2*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyL
og[2, 1 + (b*x^2)/a]))/(2*d^3)
```

Maple [C] time = 0.358, size = 1071, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^3/(e*x+d),x)
```

```
[Out] -1/2*b*p*ln(b*x^2+a)/a/d-2*b*p/d^2*e/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/
4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d/x^2-1/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2/d/x^2+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csg
n(I*c)*e^2/d^3*ln(e*x+d)-1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d/x^2+l
n((b*x^2+a)^p)*e/d^2/x-ln((b*x^2+a)^p)*e^2/d^3*ln(e*x+d)+ln((b*x^2+a)^p)*e^
2/d^3*ln(x)-p*e^2/d^3*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p*e^2/d^3*dil
og((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p*e^2/d^3*dilog((e*(-a*b)^(1/2)-b*(e*x+
d)+b*d)/(e*(-a*b)^(1/2)+b*d))+p*e^2/d^3*dilog((e*(-a*b)^(1/2)+b*(e*x+d)-b*d
)/(e*(-a*b)^(1/2)-b*d))-1/2*ln((b*x^2+a)^p)/d/x^2-ln(c)*e^2/d^3*ln(e*x+d)+l
n(c)*e^2/d^3*ln(x)+ln(c)*e/d^2/x-p*e^2/d^3*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))+p*e^2/d^3*ln(e*x+d)*ln((e*(-a*b)^(1/2)-b*(e*x+d)+b*d)/(e*(-a*b)^(
1/2)+b*d))+p*e^2/d^3*ln(e*x+d)*ln((e*(-a*b)^(1/2)+b*(e*x+d)-b*d)/(e*(-a*b)^
(1/2)-b*d))+b*p*ln(x)/a/d+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e^2/d^3*ln(e*x+d
)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e/d^2/x-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)*csgn(I*c)*e^2/d^3*ln(x)-1/2*ln(c)/d/x^2-1/2*I*Pi*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2/x-p*e^2/d^3*ln(x)*ln((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e^2/d^3*ln(
x)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e/d^2/x-1/2*I*Pi*cs
gn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e^2/d^3*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)
^p)*csgn(I*c*(b*x^2+a)^p)^2*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*
csgn(I*c)*e/d^2/x-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e^2/
d^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e^2/d^3*ln(x)+1/4*
I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^3), x)
```

3.233 $\int \frac{x^3 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=692

$$\frac{d^3 p \text{PolyLog}\left(2, \frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3 p \text{PolyLog}\left(2, \frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^4} + \frac{d^3 p \text{PolyLog}\left(2, \frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4} - \frac{\sqrt[3]{a}d^2 p \log\left(a^{2/3} - \sqrt[3]{a}\right.}{2\sqrt[3]{b}e^3}$$

```
[Out] (-3*d^2*p*x)/e^3 + (3*d*p*x^2)/(4*e^2) - (p*x^3)/(3*e) - (Sqrt[3]*a^(1/3)*d
^2*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^3) + (Sq
rt[3]*a^(2/3)*d*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(
2/3)*e^2) + (a^(1/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^3) + (a^(2/
3)*d*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)*e^2) + (d^3*p*Log[-((e*(a^(1/3)
 + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((
e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Lo
g[d + e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)
)/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^4 - (a^(1/3)*d^2*p*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e^3) - (a^(2/3)*d*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)*e^2) + (d^2*x*
Log[c*(a + b*x^3)^p])/e^3 - (d*x^2*Log[c*(a + b*x^3)^p])/(2*e^2) + ((a + b*
x^3)*Log[c*(a + b*x^3)^p])/(3*b*e) - (d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p]
)/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^4
 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1
/3)*e)])/e^4
```

Rubi [A] time = 0.891271, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 20, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.87, Rules used = {2466, 2448, 321, 200, 31, 634, 617, 204, 628, 2455, 292, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{d^3 p \text{PolyLog}\left(2, \frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3 p \text{PolyLog}\left(2, \frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^4} + \frac{d^3 p \text{PolyLog}\left(2, \frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4} - \frac{\sqrt[3]{a}d^2 p \log\left(a^{2/3} - \sqrt[3]{a}\right.}{2\sqrt[3]{b}e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*d^2*p*x)/e^3 + (3*d*p*x^2)/(4*e^2) - (p*x^3)/(3*e) - (Sqrt[3]*a^(1/3)*d
^2*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^3) + (Sq
rt[3]*a^(2/3)*d*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(
2/3)*e^2) + (a^(1/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^3) + (a^(2/
3)*d*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)*e^2) + (d^3*p*Log[-((e*(a^(1/3)
 + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((
e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Lo
g[d + e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)
)/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^4 - (a^(1/3)*d^2*p*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e^3) - (a^(2/3)*d*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)*e^2) + (d^2*x*
Log[c*(a + b*x^3)^p])/e^3 - (d*x^2*Log[c*(a + b*x^3)^p])/(2*e^2) + ((a + b*
x^3)*Log[c*(a + b*x^3)^p])/(3*b*e) - (d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p]
)/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^4
 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
```

```
])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1
/3)*e)])/e^4
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \int\left(\frac{d^2\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d^3\log\left(c\left(a+bx\right.\right.}{e^3(d+ex)}\right.$$

$$= \frac{d^2\int\log\left(c\left(a+bx^3\right)^p\right)dx}{e^3} - \frac{d^3\int\frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{e^3} - \frac{d\int x\log\left(c\left(a+bx^3\right)^p\right)dx}{e^2} + \frac{\int x^2\log}{}$$

$$= \frac{d^2x\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+bx^3\right)^p\right)}{2e^2} - \frac{d^3\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^4} + \frac{\text{Subst}}{}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} + \frac{d^2x\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+bx^3\right)^p\right)}{2e^2} - \frac{d^3\log(d+ex)\log\left(c\right.}{e^4}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{d^2x\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+bx^3\right)^p\right)}{2e^2} + \frac{\left(a+bx^3\right)\log\left(\right.}{3b}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3} + \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} + \frac{d^3p\log\left(-\frac{e(}{}\right.}{}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3} + \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} + \frac{d^3p\log\left(-\frac{e(}{}\right.}{}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} - \frac{\sqrt{3}\sqrt[3]{a}d^2p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^3} + \frac{\sqrt{3}a^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e^2} + \frac{\sqrt[3]{a}d}{}$$

Mathematica [C] time = 0.601297, size = 497, normalized size = 0.72

$$-\frac{-12d^3p\left(\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)+\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)+\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)+\log(d+ex)\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\right.}{\sqrt[3]{-1}\sqrt[3]{a}e+}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] -(9*d*e^2*p*x^2*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)]) + (6*d^
2*e*p*(6*b^(1/3)*x - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*(2*Sqrt[3
]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])))/b^(1/3) - 12*d^2*e*x*Log[c*(a + b*x^3)^p] + 6*d*e^2*
x^2*Log[c*(a + b*x^3)^p] + 12*d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p] + (4*e^
3*(b*p*x^3 - (a + b*x^3)*Log[c*(a + b*x^3)^p]))/b - 12*d^3*p*(Log[(e*((-1)^
(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x
] + Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d + e*x]
+ Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*a^(1/
3)*e)]*Log[d + e*x] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e
)] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)]))/(12*e^4
)
```

Maple [C] time = 0.665, size = 912, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^3+a)^p)/(e*x+d),x)
```

```
[Out] 1/6*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x^3+3/4*d*p*x^2/e^2+
1/3*ln(c)/e*x^3-ln(c)*d^3/e^4*ln(e*x+d)-1/2*ln(c)/e^2*x^2*d+ln(c)/e^3*x*d^2
-49/12*p/e^4*d^3+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*d^3/e^4*ln(e*x+d)+1/4*I*P
i*csgn(I*c*(b*x^3+a)^p)^3/e^2*x^2*d-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e^3*x*
d^2+1/6*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e*x^3-ln((b*x^3+a)^p)*d^3/e^
4*ln(e*x+d)-1/2*ln((b*x^3+a)^p)/e^2*x^2*d+ln((b*x^3+a)^p)/e^3*x*d^2-1/6*I*P
i*csgn(I*c*(b*x^3+a)^p)^3/e*x^3+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^
3+a)^p)*csgn(I*c)*d^3/e^4*ln(e*x+d)+1/6/b*p/e*sum((2*_R^2-7*_R*d+11*d^2)/(_
R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b
*d^3))*a+1/3*ln((b*x^3+a)^p)/e*x^3+p/e^4*d^3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/
_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-
b*d^3))+1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^2*x^
2*d-1/3*p*x^3/e-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c
)/e^3*x*d^2-3*d^2*p*x/e^3-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p
)^2*d^3/e^4*ln(e*x+d)-1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/
e^2*x^2*d-1/6*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e*x^
3-1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e^2*x^2*d+1/2*I*Pi*csgn(I*c*(b
*x^3+a)^p)^2*csgn(I*c)/e^3*x*d^2-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)
*d^3/e^4*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e^3
*x*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

3.234 $\int \frac{x^2 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=643

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}+\frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\ldots\right.}{2\sqrt[3]{b}e\ldots}$$

```
[Out] (3*d*p*x)/e^2 - (3*p*x^2)/(4*e) + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^2) - (Sqrt[3]*a^(2/3)*p*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)*e) - (a^(1/3)*d*p*Log[
a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^2) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/
(2*b^(2/3)*e) - (d^2*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)
*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))
/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^
(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e
)]*Log[d + e*x])/e^3 + (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(2*b^(1/3)*e^2) + (a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(4*b^(2/3)*e) - (d*x*Log[c*(a + b*x^3)^p])/e^2 + (x^2*Log[c*(a +
 b*x^3)^p])/(2*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^3 - (d^2*p*Po
lyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^3 - (d^2*p*PolyLog
[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^3 - (d^2*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^3
```

Rubi [A] time = 0.77444, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 17, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {2466, 2448, 321, 200, 31, 634, 617, 204, 628, 2455, 292, 2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}+\frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\ldots\right.}{2\sqrt[3]{b}e\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```


```
[Out] (3*d*p*x)/e^2 - (3*p*x^2)/(4*e) + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^2) - (Sqrt[3]*a^(2/3)*p*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)*e) - (a^(1/3)*d*p*Log[
a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^2) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/
(2*b^(2/3)*e) - (d^2*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)
*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))
/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^
(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e
)]*Log[d + e*x])/e^3 + (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(2*b^(1/3)*e^2) + (a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(4*b^(2/3)*e) - (d*x*Log[c*(a + b*x^3)^p])/e^2 + (x^2*Log[c*(a +
 b*x^3)^p])/(2*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^3 - (d^2*p*Po
lyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^3 - (d^2*p*PolyLog
[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^3 - (d^2*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^3
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \int\left(-\frac{d\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+bx^3\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int\log\left(c\left(a+bx^3\right)^p\right)dx}{e^2}+\frac{d^2\int\frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+bx^3\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^3}-\frac{(3bd^2p}{}$$

$$= \frac{3dpx}{e^2}-\frac{3px^2}{4e}-\frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}+\frac{d^2\log(d+ex)\log\left(c\left(a+b\right.\right.}{e^3}$$

$$= \frac{3dpx}{e^2}-\frac{3px^2}{4e}-\frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}+\frac{d^2\log(d+ex)\log\left(c\left(a+b\right.\right.}{e^3}$$

$$= \frac{3dpx}{e^2}-\frac{3px^2}{4e}-\frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e}-\frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log}{e^3}$$

$$= \frac{3dpx}{e^2}-\frac{3px^2}{4e}-\frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e}-\frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log}{e^3}$$

$$= \frac{3dpx}{e^2}-\frac{3px^2}{4e}+\frac{\sqrt{3}\sqrt[3]{a}dp\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^2}-\frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e}-\frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\right.}{\sqrt[3]{b}e^2}$$

Mathematica [C] time = 0.389401, size = 504, normalized size = 0.78

$$-\frac{4d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)+4d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)+4d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)-\frac{2\sqrt[3]{a}dep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{\sqrt[3]{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] -(-12*d*e*p*x + 3*e^2*p*x^2 - (4*Sqrt[3]*a^(1/3)*d*e*p*ArcTan[(1 - (2*b^(1/
3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 3*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/
3, -((b*x^3)/a)] + (4*a^(1/3)*d*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + 4*d
^2*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) +
 a^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)
)/(-(b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (2*a^(1/3)*d*e*p*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 4*d*e*x*Log[c*(a +
b*x^3)^p] - 2*e^2*x^2*Log[c*(a + b*x^3)^p] - 4*d^2*Log[d + e*x]*Log[c*(a +
b*x^3)^p] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]
 + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e
)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)
*e)])/(4*e^3)
```

Maple [C] time = 0.648, size = 704, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^3+a)^p)/(e*x+d),x)
```

```
[Out] 1/2*ln((b*x^3+a)^p)/e*x^2-ln((b*x^3+a)^p)/e^2*d*x+ln((b*x^3+a)^p)*d^2/e^3*l
n(e*x+d)-p/e^3*d^2*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R
1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-3/4*p*x^2/e+3*d*p*
x/e^2+15/4*p/e^3*d^2+1/2/b*p*sum((_R-3*d)/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R
=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))*a+1/4*I*Pi*csgn(I*c*(b*x
^3+a)^p)^2*csgn(I*c)/e*x^2-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^
p)^2/e^2*d*x-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*d
^2/e^3*ln(e*x+d)+1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x^2
+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^2*d*x-1/4*I
*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e*x^2-1/2*I*Pi*csgn
(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e^2*d*x+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn
(I*c)*d^2/e^3*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*d^2/e^3*ln(e*x+d)-
1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e*x^2+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e^2
*d*x+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*d^2/e^3*ln(e*x+d)
+1/2*ln(c)/e*x^2-ln(c)/e^2*d*x+ln(c)*d^2/e^3*ln(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log \left(\left(bx^3 + a \right)^p c \right)}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

3.235 $\int \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=457

$$\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{2\sqrt[3]{b}e}$$

```
[Out] (-3*p*x)/e - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(b^(1/3)*e) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e) + (d
*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
e^2 + (d*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^2 - (
a^(1/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e) + (
x*Log[c*(a + b*x^3)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^2 + (d*
p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^2 + (d*p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^2 + (d*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^2
```

Rubi [A] time = 0.631522, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2466, 2448, 321, 200, 31, 634, 617, 204, 628, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{2\sqrt[3]{b}e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*p*x)/e - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(b^(1/3)*e) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e) + (d
*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
e^2 + (d*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^2 - (
a^(1/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e) + (
x*Log[c*(a + b*x^3)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^2 + (d*
p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^2 + (d*p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^2 + (d*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^2
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx = \int\left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+bx^3\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{(3bdp)\int \frac{x^2\log(d+ex)}{a+bx^3}dx}{e^2} - \frac{(3bp)\int \frac{\cdots}{a+\cdots}}{e}$$

$$= -\frac{3px}{e} + \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{(3bdp)\int\left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \cdots\right.}{\cdots}$$

$$= -\frac{3px}{e} + \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{\left(\sqrt[3]{b}dp\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{e^2} + \cdots$$

$$= -\frac{3px}{e} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right.}{e^2}$$

$$= -\frac{3px}{e} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right.}{e^2}$$

$$= -\frac{3px}{e} - \frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex}{e^2}$$

Mathematica [A] time = 0.19769, size = 430, normalized size = 0.94

$$\frac{2dp\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right) + 2dp\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right) + 2dp\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right) - \frac{\sqrt[3]{a}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-6*e*p*x - (2*Sqrt[3]*a^(1/3)*e*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/b^(1/3) + (2*a^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + 2*d*p*Log
[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*L
og[d + e*x] + 2*d*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e
)]*Log[d + e*x] + 2*d*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)
*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (a^(1/3)*e*p*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*e*x*Log[c*(a + b*x^3)^p] - 2*d*Lo
g[d + e*x]*Log[c*(a + b*x^3)^p] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(
1/3)*d - a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-
1)^(1/3)*a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-
1)^(2/3)*a^(1/3)*e)])/(2*e^2)
```

Maple [C] time = 0.632, size = 500, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^3+a)^p)/(e*x+d),x)
```

```
[Out] ln((b*x^3+a)^p)/e*x-ln((b*x^3+a)^p)*d/e^2*ln(e*x+d)-3*p*x/e-3*p/e^2*d+1/b*p
*e*sum(1/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*
d^2+a*e^3-b*d^3))*a+p/e^2*d*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+
_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-1/2*I*Pi*
csgn(I*c*(b*x^3+a)^p)^3/e*x+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e*
x+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x-1/2*I*Pi*csgn(I*
(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x
^3+a)^p)^2*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*d/e^2
*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e*x
+ln(c)/e*x-ln(c)*d/e^2*ln(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

3.236 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=308

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^3\right.\right.}{e}$$

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rubi [A] time = 0.387383, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^3\right.\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```



```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```


Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\right.}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int -}{}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2}\right.}{\sqrt[3]{b}d+\sqrt[3]{-1}}\right.}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2}\right.}{\sqrt[3]{b}d+\sqrt[3]{-1}}\right.}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2}\right.}{\sqrt[3]{b}d+\sqrt[3]{-1}}\right.}{e}$$

Mathematica [A] time = 0.0532354, size = 313, normalized size = 1.02

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e} - \frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
```

```
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Maple [C] time = 0.102, size = 261, normalized size = 0.9

$$\frac{\ln(ex+d)\ln\left(\left(bx^3+a\right)^p\right)}{e}-\frac{p}{e}\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bd+3\_Z\,bd^2+ae^3-bd^3\right)}\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+}{\_}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x^3+a)^p)-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-
e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/2*I
*ln(e*x+d)/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/2*I*ln(e*x+d)
/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e
*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^3+a)^p)^2*cs
gn(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(bx^3 + a\right)^p c\right)}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

3.237 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=352

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d}-\text{lo}$$

```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a
^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d + (Log[-((b*x
^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b*x^3)^p])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, 1 + (b*x^3)/a])/(3*d)
```

Rubi [A] time = 0.564933, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d}-\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```


```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a
^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d + (Log[-((b*x
^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b*x^3)^p])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, 1 + (b*x^3)/a])/(3*d)
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{d}$$

$$= -\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, x^3\right)}{3d} + \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}dx}{d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}dx,\right.}{3d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d} + \frac{\left(\sqrt[3]{b}p\right)\int}{}$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-\right.}{\sqrt[3]{b}d+\sqrt[3]{-}}\right.}{}$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-\right.}{\sqrt[3]{b}d+\sqrt[3]{-}}\right.}{}$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-\right.}{\sqrt[3]{b}d+\sqrt[3]{-}}\right.}{}$$

Mathematica [A] time = 0.0534429, size = 358, normalized size = 1.02

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d} + \frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d} + \frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d} + \frac{p\text{PolyLog}\left(2,\frac{a+bx^3}{a}\right)}{3d} - \frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d - (
-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d +
(Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b
*x^3)^p])/d + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d
 + (p*PolyLog[2, (a + b*x^3)/a])/(3*d)
```

Maple [C] time = 0.586, size = 461, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x/(e*x+d),x)
```

```
[Out] -ln((b*x^3+a)^p)/d*ln(e*x+d)+ln((b*x^3+a)^p)/d*ln(x)-p/d*sum(ln(x)*ln((_R1-
x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))+p/d*sum(ln(e*x+d)*ln((-e*x
+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^
2+a*e^3-b*d^3))-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d*ln(e
*x+d)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d*ln(x)+1/2*I*Pi
*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d*ln(e*x+d)-1/2*I*Pi*c
sgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d*ln(x)+1/2*I*Pi*csgn(I*
c*(b*x^3+a)^p)^3/d*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/d*ln(x)-1/2*I
*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/d*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+
a)^p)^2*csgn(I*c)/d*ln(x)-ln(c)/d*ln(e*x+d)+ln(c)/d*ln(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

3.238 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=510

$$-\frac{ep\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2}+$$

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(a^
(1/3)*d)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d) - (e*p*Log[-((
e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p
*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^2 + (b^(1/3)*p*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d) - Log[c*(a +
b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^2) + (e*L
og[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))
/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b
^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^3)/a])/(3
*d^2)
```

Rubi [A] time = 0.677497, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {2466, 2455, 292, 31, 634, 617, 204, 628, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{ep\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(a^
(1/3)*d)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d) - (e*p*Log[-((
e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p
*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^2 + (b^(1/3)*p*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d) - Log[c*(a +
b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^2) + (e*L
og[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))
/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b
^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^3)/a])/(3
*d^2)
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+bx^3\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} + \frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2} - \frac{e\,\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, x^3\right)}{3d^2} +$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2} - $$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2} + \frac{e\log(d+e}{}$$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(}{d^2}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} - \frac{ep}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} - \frac{ep}{}$$

Mathematica [C] time = 0.0606668, size = 424, normalized size = 0.83

$$-\frac{ep\text{PolyLog}\left(2,\frac{a+bx^3}{a}\right)}{3d^2} - \frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2} - \frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a*d) - (e*p*Log
[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 -
(e*p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d - (-1
)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^
2 - Log[c*(a + b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])
/(3*d^2) + (e*Log[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(
1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3
)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a
+ b*x^3)/a])/(3*d^2)
```

Maple [C] time = 0.592, size = 732, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^2/(e*x+d),x)
```

```
[Out] ln((b*x^3+a)^p)*e/d^2*ln(e*x+d)-ln((b*x^3+a)^p)/d/x-ln((b*x^3+a)^p)*e/d^2*l
n(x)-p*e/d^2*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1
=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-p/d/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))+1/2*p/d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+p/d*3^(1/2)
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+p*e/d^2*sum(ln(x)*ln((
_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))+1/2*I*Pi*csgn(I*(b*x^3
+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d/x-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csg
n(I*c*(b*x^3+a)^p)^2*e/d^2*ln(x)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x
^3+a)^p)*csgn(I*c)*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e
/d^2*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(
b*x^3+a)^p)^3*e/d^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/d/
x-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/d^2*ln(e*x
+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d/x+1/2*I*Pi*csgn(
I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b
*x^3+a)^p)^3/d/x+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e/d^2*ln(e*x+d)
+ln(c)*e/d^2*ln(e*x+d)-ln(c)/d/x-ln(c)*e/d^2*ln(x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^2), x)
```

3.239 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=674

$$\frac{e^2p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3}$$

```
[Out] -(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*a
^(2/3)*d) + (Sqrt[3]*b^(1/3)*e*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(a^(1/3)*d^2) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)*d)
 + (b^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d^2) + (e^2*p*Log[-((e*(
a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*
Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^3 - (b^(2/3)*p
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)*d) - (b^(1/3)*e
*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d^2) - Log[c*
(a + b*x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^3)^p])/(d^2*x) + (e^2*Log[-((b
*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^
3)^p])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]
)/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)
*a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, 1 + (b*x^3)/a])/(3*d^3)
```

Rubi [A] time = 0.793271, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 17, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {2466, 2455, 200, 31, 634, 617, 204, 628, 292, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{e^2p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```


```
[Out] -(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*a
^(2/3)*d) + (Sqrt[3]*b^(1/3)*e*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(a^(1/3)*d^2) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)*d)
 + (b^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d^2) + (e^2*p*Log[-((e*(
a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*
Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^3 - (b^(2/3)*p
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)*d) - (b^(1/3)*e
*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d^2) - Log[c*
(a + b*x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^3)^p])/(d^2*x) + (e^2*Log[-((b
*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^
3)^p])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]
)/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)
*a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, 1 + (b*x^3)/a])/(3*d^3)
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3(d+ex)}dx = \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+bx^3\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+bx^3\right)^p\right)}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{d^3}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x} - \frac{e^2\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^3} + \frac{e^2\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^3} - \frac{e^2\log(d+e}{}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\,\mathrm{l}}{}$$

$$= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{b}ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\,\mathrm{l}}{}$$

$$= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{b}ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\,\mathrm{l}}{}$$

Mathematica [C] time = 0.389729, size = 542, normalized size = 0.8

$$\frac{12e^2p\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right) + 12e^2p\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right) + 12e^2p\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right) + 4e^2p\mathrm{PolyLog}\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-6*Sqrt[3]*b^(2/3)*d^2*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(
2/3) - (18*b*d*e*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/a + (6
*b^(2/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + 12*e^2*p*Log[(e*((-1)^(1
/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x]
+ 12*e^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d
+ e*x] + 12*e^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) +
(-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (3*b^(2/3)*d^2*p*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) - (6*d^2*Log[c*(a + b*x^3)^p])/x^2 + (1
2*d*e*Log[c*(a + b*x^3)^p])/x + 4*e^2*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p
] - 12*e^2*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 12*e^2*p*PolyLog[2, (b^(1/3)
*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d +
 e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)] + 4*e^2*p*PolyLog[2, 1 + (b*x^3)
/a])/(12*d^3)
```

Maple [C] time = 0.595, size = 1025, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^3/(e*x+d),x)
```

```
[Out] -1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*e/d^2/x-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn
(I*c*(b*x^3+a)^p)*csgn(I*c)*e^2/d^3*ln(x)-p*e^2/d^3*sum(ln(x)*ln((_R1-x)/_R
1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))+p*e^2/d^3*sum(ln(e*x+d)*ln((-e*
x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d
^2+a*e^3-b*d^3))-1/2*ln((b*x^3+a)^p)/d/x^2-1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csg
n(I*c*(b*x^3+a)^p)^2/d/x^2-p/d^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/d/x^2+1/2*I*
Pi*csgn(I*c*(b*x^3+a)^p)^3*e^2/d^3*ln(e*x+d)+ln((b*x^3+a)^p)*e/d^2/x-ln((b*
x^3+a)^p)*e^2/d^3*ln(e*x+d)+ln((b*x^3+a)^p)*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*c
*(b*x^3+a)^p)^3*e^2/d^3*ln(x)+1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/d/x^2+1/2*p/
d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+p/d^2*e/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))-1/2*p/d^2*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))-ln(c)*e^2/d^3*ln(e*x+d)+ln(c)*e^2/d^3*ln(x)+ln(c)*e/d^2/x+1/2*p/d/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/4*p/d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e^2/d^
3*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/
d^2/x-1/2*ln(c)/d/x^2+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e^2/d^3*ln
(x)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e^2/d^3*ln(e*x+d)-
1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e^2/d^3*ln(e*x+d)+1/2*I*Pi*csgn(
I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e/d^2/x+1/4*I*Pi*csgn(I*(b*x^3+a)^p)
*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d/x^2+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*
c*(b*x^3+a)^p)^2*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e
/d^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^3 + a\right)^p c\right)}{(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^3), x)
```

3.240 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=297

$$\frac{d^3 p \text{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right)}{e^4} - \frac{d^3 p \text{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^4} + \frac{b^2 dp \log(ax+b)}{2a^2e^2} - \frac{b^2 px}{3a^2e} + \frac{b^3 p \log(ax+b)}{3a^3e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}$$

```
[Out] -(b*d*p*x)/(2*a*e^2) - (b^2*p*x)/(3*a^2*e) + (b*p*x^2)/(6*a*e) + (d^2*x*Log
[c*(a + b/x)^p])/e^3 - (d*x^2*Log[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*(a +
 b/x)^p])/(3*e) + (b*d^2*p*Log[b + a*x])/(a*e^3) + (b^2*d*p*Log[b + a*x])/(
2*a^2*e^2) + (b^3*p*Log[b + a*x])/(3*a^3*e) - (d^3*Log[c*(a + b/x)^p]*Log[d
 + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(
b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 + (d^3*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^4 - (d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rubi [A] time = 0.321464, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2466, 2448, 263, 31, 2455, 193, 43, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{d^3 p \text{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right)}{e^4} - \frac{d^3 p \text{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^4} + \frac{b^2 dp \log(ax+b)}{2a^2e^2} - \frac{b^2 px}{3a^2e} + \frac{b^3 p \log(ax+b)}{3a^3e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] -(b*d*p*x)/(2*a*e^2) - (b^2*p*x)/(3*a^2*e) + (b*p*x^2)/(6*a*e) + (d^2*x*Log
[c*(a + b/x)^p])/e^3 - (d*x^2*Log[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*(a +
 b/x)^p])/(3*e) + (b*d^2*p*Log[b + a*x])/(a*e^3) + (b^2*d*p*Log[b + a*x])/(
2*a^2*e^2) + (b^3*p*Log[b + a*x])/(3*a^3*e) - (d^3*Log[c*(a + b/x)^p]*Log[d
 + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(
b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 + (d^3*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^4 - (d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3(d+ex)}\right) dx$$

$$= \frac{d^2 \int \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx}{e^3} - \frac{d \int x \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx}{e^2} + \frac{\int x^2 \log\left(c\left(a+\right.\right.}{e}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) \log(d}{e^4}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) \log(d}{e^4}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{bd^2 p \log(b+ax)}{ae^3} - \frac{d^3 \text{lo}}{}$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2 px}{3a^2 e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} +$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2 px}{3a^2 e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} +$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2 px}{3a^2 e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} +$$

Mathematica [A] time = 0.218275, size = 251, normalized size = 0.85

$$\frac{-6d^3 p \left(-\text{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right) + \text{PolyLog}\left(2, \frac{ex}{d}+1\right) + \log(d+ex)\left(\log\left(-\frac{ex}{d}\right) - \log\left(\frac{e(ax+b)}{be-ad}\right)\right)\right) + \frac{be^3 p\left(2b^2 \log\left(a+\frac{b}{x}\right) + ax(ax-2b)\right.}{a^3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (6*d^2*e*x*Log[c*(a + b/x)^p] - 3*d*e^2*x^2*Log[c*(a + b/x)^p] + 2*e^3*x^3*
Log[c*(a + b/x)^p] + (6*b*d^2*e*p*(Log[a + b/x] + Log[x]))/a + (b*e^3*p*(a*
x*(-2*b + a*x) + 2*b^2*Log[a + b/x] + 2*b^2*Log[x]))/a^3 + (3*b*d*e^2*p*(-(
a*x) + b*Log[b + a*x]))/a^2 - 6*d^3*Log[c*(a + b/x)^p]*Log[d + e*x] - 6*d^3
*p*((Log[-((e*x)/d)] - Log[(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x] - Po
lyLog[2, (a*(d + e*x))/(a*d - b*e)] + PolyLog[2, 1 + (e*x)/d]))/(6*e^4)
```

Maple [F] time = 0.758, size = 0, normalized size = 0.

$$\int \frac{x^3}{ex+d}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

3.241 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=219

$$-\frac{d^2 p \text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}+\frac{d^2 p \text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}-\frac{b^2 p \log(ax+b)}{2a^2e}+\frac{d^2 \log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(\right.\right.}{e^2}$$

```
[Out] (b*p*x)/(2*a*e) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[c*(a + b/x)^p])/(
2*e) - (b*d*p*Log[b + a*x])/(a*e^2) - (b^2*p*Log[b + a*x])/(2*a^2*e) + (d^2
*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-((e*x)/d)]*Log[d + e*x]
)/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^3 - (d^2*p
*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e^3 + (d^2*p*PolyLog[2, 1 + (e*x)/d
])/e^3
```

Rubi [A] time = 0.265552, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2466, 2448, 263, 31, 2455, 193, 43, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{d^2 p \text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}+\frac{d^2 p \text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}-\frac{b^2 p \log(ax+b)}{2a^2e}+\frac{d^2 \log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(\right.\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```


```
[Out] (b*p*x)/(2*a*e) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[c*(a + b/x)^p])/(
2*e) - (b*d*p*Log[b + a*x])/(a*e^2) - (b^2*p*Log[b + a*x])/(2*a^2*e) + (d^2
*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-((e*x)/d)]*Log[d + e*x]
)/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^3 - (d^2*p
*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e^3 + (d^2*p*PolyLog[2, 1 + (e*x)/d
])/e^3
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int\left(-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$=-\frac{d\int\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e^2}+\frac{d^2\int\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e}$$

$$=-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3}+\frac{\left(bd^2p\right)\int\frac{1}{\cdots}}{e\cdots}$$

$$=-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3}+\frac{\left(bd^2p\right)\int(\cdots}{\cdots}$$

$$=-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{bdp\log(b+ax)}{ae^2}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d\cdots}{e^3}$$

$$=\frac{bpx}{2ae}-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{bdp\log(b+ax)}{ae^2}-\frac{b^2p\log(b+ax)}{2a^2e}+\cdots$$

$$=\frac{bpx}{2ae}-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{bdp\log(b+ax)}{ae^2}-\frac{b^2p\log(b+ax)}{2a^2e}+\cdots$$

$$=\frac{bpx}{2ae}-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{bdp\log(b+ax)}{ae^2}-\frac{b^2p\log(b+ax)}{2a^2e}+\cdots$$

Mathematica [A] time = 0.124978, size = 183, normalized size = 0.84

$$\frac{2d^2p\left(-\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)+\text{PolyLog}\left(2,\frac{ex}{d}+1\right)+\log(d+ex)\left(\log\left(-\frac{ex}{d}\right)-\log\left(\frac{e(ax+b)}{be-ad}\right)\right)\right)+\frac{be^2p(ax-b\log(ax+b))}{a^2}+2d\cdots}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (-2*d*e*x*Log[c*(a + b/x)^p] + e^2*x^2*Log[c*(a + b/x)^p] - (2*b*d*e*p*(Log
[a + b/x] + Log[x]))/a + (b*e^2*p*(a*x - b*Log[b + a*x]))/a^2 + 2*d^2*Log[c
*(a + b/x)^p]*Log[d + e*x] + 2*d^2*p*((Log[-((e*x)/d)] - Log[(e*(b + a*x))/
(-(a*d) + b*e)])*Log[d + e*x] - PolyLog[2, (a*(d + e*x))/(a*d - b*e)] + Pol
yLog[2, 1 + (e*x)/d]))/(2*e^3)
```

Maple [F] time = 0.723, size = 0, normalized size = 0.

$$\int \frac{x^2}{ex+d}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

3.242 $\int \frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=151

$$\frac{dp\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}-\frac{dp\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{dp\log(d+ex)\,\mathrm{l}}{e^2}$$

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*Log[b + a*x])/(a*e) - (d*Log[c*(a + b/x)^p]
*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e
*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^2 - (d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rubi [A] time = 0.212287, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2466, 2448, 263, 31, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{dp\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}-\frac{dp\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{dp\log(d+ex)\,\mathrm{l}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*Log[b + a*x])/(a*e) - (d*Log[c*(a + b/x)^p]
*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e
*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^2 - (d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}dx}{e^2} + \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(bdp)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e^2} + \frac{(bp)\ldots}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(dp)\int \frac{\log(d+ex)}{x}dx}{e^2} + \frac{(a\ldots}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex\ldots}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex\ldots}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex\ldots}{e^2}$$

Mathematica [A] time = 0.0571212, size = 149, normalized size = 0.99

$$-\frac{dp\left(-\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)+\text{PolyLog}\left(2,\frac{d+ex}{d}\right)-\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)+\log\left(-\frac{ex}{d}\right)\log(d+ex)\right)}{e^2} - \frac{d\log(d+ex)\log\ldots}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*(Log[a + b/x]/a + Log[x]/a))/e - (d*Log[c*(
a + b/x)^p]*Log[d + e*x])/e^2 - (d*p*(Log[-((e*x)/d)]*Log[d + e*x] - Log[-(
(e*(b + a*x))/(a*d - b*e))]*Log[d + e*x] + PolyLog[2, (d + e*x)/d] - PolyLo
g[2, (a*(d + e*x))/(a*d - b*e)]))/e^2
```

Maple [F] time = 0.742, size = 0, normalized size = 0.

$$\int \frac{x}{ex+d}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(x*ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log \left(\left(a + \frac{b}{x}\right)^{p} c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((a + b/x)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \log \left(c \left(\frac{ax+b}{x}\right)^{p}\right)}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log \left(\left(a + \frac{b}{x}\right)^{p} c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((a + b/x)^p*c)/(e*x + d), x)
```

3.243 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=113

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\log\left(-\right.}{}$$

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rubi [A] time = 0.146883, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int\left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(ap)\int \frac{\log(d+ex)}{b+ax}\,dx}{e}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - p\int$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\mathrm{Li}_2}{}$$
$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.0174174, size = 114, normalized size = 1.01

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e} + \frac{p\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} + \frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e} + \frac{p\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e
*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [A] time = 1.04896, size = 215, normalized size = 1.9

$$\frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b}-\frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b}+\frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e}-\frac{p\log(ex+d)\log\left(a+\frac{b}{x}\right)}{e}+\frac{\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x +
 d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e + log
((a + b/x)^p*c)*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

3.244 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=159

$$\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax}\right)\log\left(c\right.}{d}$$

```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*x
])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d - b
*e))]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x)])/d + (p*PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)])/d - (p*PolyLog[2, 1 + (e*x)/d])/d
```

Rubi [A] time = 0.246079, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax}\right)\log\left(c\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```


```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*x
])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d - b
*e))]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x)])/d + (p*PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)])/d - (p*PolyLog[2, 1 + (e*x)/d])/d
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x}\right)}{d} - \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{(bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{d} +$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d} - \frac{p\int \frac{\log(d+ex)}{x}\,dx}{d} +$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\right.}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\right.}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\right.}{}$$

Mathematica [A] time = 0.0606942, size = 139, normalized size = 0.87

$$-\frac{-p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right) + p\text{PolyLog}\left(2,\frac{b}{ax}+1\right) + p\text{PolyLog}\left(2,\frac{ex}{d}+1\right) + \log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \log\left(-\frac{b}{ax}\right)\log\left(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + Log[c*(a + b/x)^p]*Log[d + e*x] + p
*Log[-((e*x)/d)]*Log[d + e*x] - p*Log[(e*(b + a*x))/(-(a*d) + b*e)]*Log[d +
 e*x] + p*PolyLog[2, 1 + b/(a*x)] - p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)]
 + p*PolyLog[2, 1 + (e*x)/d])/d)
```

Maple [F] time = 0.744, size = 0, normalized size = 0.

$$\int \frac{1}{x\,(ex+d)}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x/(e*x+d),x)
```

Maxima [A] time = 1.21021, size = 242, normalized size = 1.52

$$-\frac{1}{2}\,bp\left(\frac{2\,\log\left(ex+d\right)\log\left(x\right)-\log\left(x\right)^{2}}{bd}+\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_{2}\left(-\frac{ax}{b}\right)\right)}{bd}-\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log\left(x\right)+\mathrm{Li}_{2}\left(-\frac{ex}{d}\right)\right)}{bd}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*((2*log(e*x + d)*log(x) - log(x)^2)/(b*d) + 2*(log(a*x/b + 1)*log(
x) + dilog(-a*x/b))/(b*d) - 2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))/(b*d)
 - 2*(log(e*x + d)*log(-(a*e*x + a*d)/(a*d - b*e) + 1) + dilog((a*e*x + a*d
)/(a*d - b*e)))/(b*d)) - (log(e*x + d)/d - log(x)/d)*log((a + b/x)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^{p}\right)}{ex^{2}+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^2 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x), x)
```

3.245 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=198

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2}+\frac{ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{e\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2}+\frac{e\log(d+ex)}{}$$

```
[Out] p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log[
-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((e*
x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e
*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x)])/d^2 - (e*p*PolyLog[2, (a*(d + e*x)
)/(a*d - b*e)])/d^2 + (e*p*PolyLog[2, 1 + (e*x)/d])/d^2
```

Rubi [A] time = 0.277057, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2466, 2454, 2389, 2295, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2}+\frac{ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{e\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2}+\frac{e\log(d+ex)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```


```
[Out] p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log[
-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((e*
x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e
*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x)])/d^2 - (e*p*PolyLog[2, (a*(d + e*x)
)/(a*d - b*e)])/d^2 + (e*p*PolyLog[2, 1 + (e*x)/d])/d^2
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{d^2}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\text{Subst}\left(\int \log\left(c(a+bx)^p\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx\right.}{d^2}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\text{Subst}\left(\int \log\left(cx^p\right)dx, x, a+\frac{b}{x}\right)}{bd}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} +$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} +$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} +$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} +$$

Mathematica [A] time = 0.0932023, size = 166, normalized size = 0.84

$$\frac{ep\left(-\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)+\text{PolyLog}\left(2,\frac{ex}{d}+1\right)+\log(d+ex)\left(\log\left(-\frac{ex}{d}\right)-\log\left(\frac{e(ax+b)}{be-ad}\right)\right)\right)+ep\text{PolyLog}\left(2,\frac{b}{ax}+1\right)+e\,\text{l}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] ((d*p)/x - (d*(a + b/x)*Log[c*(a + b/x)^p])/b + e*Log[c*(a + b/x)^p]*Log[-(
b/(a*x))] + e*Log[c*(a + b/x)^p]*Log[d + e*x] + e*p*PolyLog[2, 1 + b/(a*x)]
 + e*p*((Log[-((e*x)/d)] - Log[(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x]
- PolyLog[2, (a*(d + e*x))/(a*d - b*e)] + PolyLog[2, 1 + (e*x)/d]))/d^2
```

Maple [F] time = 0.741, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(ex+d)}\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x)
```

Maxima [A] time = 1.23413, size = 311, normalized size = 1.57

$$\frac{1}{2}bp\left(\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e}{bd^2}-\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)e}{bd^2}-\frac{2\left(\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\right.}{bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e/(b*d^2) - 2*(log(e*x/d
 + 1)*log(x) + dilog(-e*x/d))*e/(b*d^2) - 2*(log(e*x + d)*log(-(a*e*x + a*d
)/(a*d - b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e/(b*d^2) - 2*a*log(
a*x + b)/(b^2*d) + 2*a*log(x)/(b^2*d) + (2*e*log(e*x + d)*log(x) - e*log(x)
^2)/(b*d^2) + 2/(b*d*x)) + (e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x))*lo
g((a + b/x)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^{p}\right)}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x^2), x)
```

3.246 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=287

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}-\frac{e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}-\frac{e^2\log\left(-\frac{b}{ax}\right)\log\left(c\left(\right.\right.}{d^3}$$

```
[Out] p/(4*d*x^2) - (a*p)/(2*b*d*x) - (e*p)/(d^2*x) + (a^2*p*Log[a + b/x])/(2*b^2
*d) + (e*(a + b/x)*Log[c*(a + b/x)^p])/(b*d^2) - Log[c*(a + b/x)^p]/(2*d*x^
2) - (e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d^3 - (e^2*Log[c*(a + b/x)^p]
*Log[d + e*x])/d^3 - (e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*Log[
-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(
a*x)])/d^3 + (e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^3 - (e^2*p*Pol
yLog[2, 1 + (e*x)/d])/d^3
```

Rubi [A] time = 0.32949, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2466, 2454, 2395, 43, 2389, 2295, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}-\frac{e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}-\frac{e^2\log\left(-\frac{b}{ax}\right)\log\left(c\left(\right.\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```


```
[Out] p/(4*d*x^2) - (a*p)/(2*b*d*x) - (e*p)/(d^2*x) + (a^2*p*Log[a + b/x])/(2*b^2
*d) + (e*(a + b/x)*Log[c*(a + b/x)^p])/(b*d^2) - Log[c*(a + b/x)^p]/(2*d*x^
2) - (e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d^3 - (e^2*Log[c*(a + b/x)^p]
*Log[d + e*x])/d^3 - (e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*Log[
-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(
a*x)])/d^3 + (e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^3 - (e^2*p*Pol
yLog[2, 1 + (e*x)/d])/d^3
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$= -\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \log\left(c(a+\right.\right.}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} + \frac{e\,\text{Subst}\left(\right.}{}$$

$$= -\frac{ep}{d^2x} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3} - \frac{e^2\log\left(c\right.}{}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log}{}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log}{}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log}{}$$

Mathematica [A] time = 0.21616, size = 241, normalized size = 0.84

$$-\frac{4e^2p\left(-\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)+\text{PolyLog}\left(2,\frac{ex}{d}+1\right)+\log(d+ex)\left(\log\left(-\frac{ex}{d}\right)-\log\left(\frac{e(ax+b)}{be-ad}\right)\right)\right)+4e^2p\text{PolyLog}\left(2,\frac{b}{ax}+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -((4*d*e*p)/x - (d^2*p*(b*(b - 2*a*x) + 2*a^2*x^2*Log[a + b/x]))/(b^2*x^2)
- (4*d*e*(a + b/x)*Log[c*(a + b/x)^p])/b + (2*d^2*Log[c*(a + b/x)^p])/x^2 +
 4*e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + 4*e^2*Log[c*(a + b/x)^p]*Log[d
+ e*x] + 4*e^2*p*PolyLog[2, 1 + b/(a*x)] + 4*e^2*p*((Log[-((e*x)/d)] - Log[
(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x] - PolyLog[2, (a*(d + e*x))/(a*d
 - b*e)] + PolyLog[2, 1 + (e*x)/d]))/(4*d^3)
```

Maple [F] time = 0.733, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (ex+d)} \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x)
```

Maxima [A] time = 1.28857, size = 414, normalized size = 1.44

$$\frac{1}{4}\left(4\,e\left(\frac{a\log\left(ax+b\right)}{b^2d^2}-\frac{a\log\left(x\right)}{b^2d^2}-\frac{1}{bd^2x}\right)-\frac{4\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e^2}{bd^3}+\frac{4\left(\log\left(\frac{ex}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex}{d}\right.\right.}{bd^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(4*e*(a*log(a*x + b)/(b^2*d^2) - a*log(x)/(b^2*d^2) - 1/(b*d^2*x)) - 4*
(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e^2/(b*d^3) + 4*(log(e*x/d + 1)*log
(x) + dilog(-e*x/d))*e^2/(b*d^3) + 4*(log(e*x + d)*log(-(a*e*x + a*d)/(a*d
- b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e^2/(b*d^3) + 2*a^2*log(a*x
 + b)/(b^3*d) - 2*a^2*log(x)/(b^3*d) - 2*(2*e^2*log(e*x + d)*log(x) - e^2*l
og(x)^2)/(b*d^3) - (2*a*x - b)/(b^2*d*x^2))*b*p - 1/2*(2*e^2*log(e*x + d)/d
^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((a + b/x)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x^3), x)
```

3.247 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=421

$$\frac{d^3 p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4}+\frac{d^3 p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4}-\frac{2d^3 p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e}-\frac{d^3\log(d}{}$$

```
[Out] (2*b*p*x)/(3*a*e) + (2*Sqrt[b]*d^2*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*
e^3) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)*e) + (d^2*x*Log
[c*(a + b/x^2)^p])/e^3 - (d*x^2*Log[c*(a + b/x^2)^p])/(2*e^2) + (x^3*Log[c*
(a + b/x^2)^p])/(3*e) - (d^3*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^4 - (2*d^
3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x
))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sqrt[b] +
 Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/e^4 - (b*d*p*Log[b +
 a*x^2])/(2*a*e^2) + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - S
qrt[b]*e)])/e^4 + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt
[b]*e)])/e^4 - (2*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rubi [A] time = 0.585927, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2466, 2448, 263, 205, 2455, 260, 193, 321, 2462, 2416, 2394, 2315, 2393, 2391}

$$\frac{d^3 p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4}+\frac{d^3 p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4}-\frac{2d^3 p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e}-\frac{d^3\log(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```


```
[Out] (2*b*p*x)/(3*a*e) + (2*Sqrt[b]*d^2*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*
e^3) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)*e) + (d^2*x*Log
[c*(a + b/x^2)^p])/e^3 - (d*x^2*Log[c*(a + b/x^2)^p])/(2*e^2) + (x^3*Log[c*
(a + b/x^2)^p])/(3*e) - (d^3*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^4 - (2*d^
3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x
))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sqrt[b] +
 Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/e^4 - (b*d*p*Log[b +
 a*x^2])/(2*a*e^2) + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - S
qrt[b]*e)])/e^4 + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt
[b]*e)])/e^4 - (2*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right.\right.}{e^3(d+ex)}\right.$$

$$= \frac{d^2\int\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e^3} - \frac{d^3\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{e^3} - \frac{d\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e^2} + \frac{\int x^2\log}{}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right.}{e^4}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right.}{e^4}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\right.}{3}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\right.}{2e}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\right.}{2e}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\right.}{2e}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\right.}{2e}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\right.}{2e}$$

Mathematica [C] time = 0.41833, size = 375, normalized size = 0.89

$$-\frac{6d^3p\left(-\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)-\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)+2\text{PolyLog}\left(2,\frac{ex}{d}+1\right)-\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)-\log(d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] -((12*Sqrt[b]*d^2*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] - (4*b*e^3*p*x*H
ypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/a - 6*d^2*e*x*Log[c*(a + b/x^
2)^p] + 3*d*e^2*x^2*Log[c*(a + b/x^2)^p] - 2*e^3*x^3*Log[c*(a + b/x^2)^p] +
 (3*b*d*e^2*p*(Log[a + b/x^2] + 2*Log[x]))/a + 6*d^3*Log[c*(a + b/x^2)^p]*L
og[d + e*x] + 6*d^3*p*(2*Log[-((e*x)/d)]*Log[d + e*x] - Log[(e*(Sqrt[b] - S
qrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - Log[(e*(Sqrt[b] + Sqrt
[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] - PolyLog[2, (Sqrt[-a]*(
d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt
[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2, 1 + (e*x)/d]))/(6*e^4)
```

Maple [F] time = 0.773, size = 0, normalized size = 0.

$$\int \frac{x^3}{ex+d}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.248 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=353

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3}+\frac{2d^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}+\frac{d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}$$

```
[Out] (-2*Sqrt[b]*d*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e^2) - (d*x*Log[c*(a
+ b/x^2)^p])/e^2 + (x^2*Log[c*(a + b/x^2)^p])/(2*e) + (d^2*Log[c*(a + b/x^2
)^p]*Log[d + e*x])/e^3 + (2*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*
p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^
3 - (d^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[
d + e*x])/e^3 + (b*p*Log[b + a*x^2])/(2*a*e) - (d^2*p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e^3 - (d^2*p*PolyLog[2, (Sqrt[-a]*(d
+ e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e^3 + (2*d^2*p*PolyLog[2, 1 + (e*x)/d])/
e^3
```

Rubi [A] time = 0.492004, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2466, 2448, 263, 205, 2455, 260, 2462, 2416, 2394, 2315, 2393, 2391}

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3}+\frac{2d^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}+\frac{d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```



```
[Out] (-2*Sqrt[b]*d*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e^2) - (d*x*Log[c*(a
+ b/x^2)^p])/e^2 + (x^2*Log[c*(a + b/x^2)^p])/(2*e) + (d^2*Log[c*(a + b/x^2
)^p]*Log[d + e*x])/e^3 + (2*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*
p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^
3 - (d^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[
d + e*x])/e^3 + (b*p*Log[b + a*x^2])/(2*a*e) - (d^2*p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e^3 - (d^2*p*PolyLog[2, (Sqrt[-a]*(d
+ e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e^3 + (2*d^2*p*PolyLog[2, 1 + (e*x)/d])/
e^3
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e^2} + \frac{d^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{e^2} + \frac{\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} + \frac{(2bd^2p)\int}{}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} + \frac{(2bd^2p)\int}{}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\mathrm{lo}}{e^3}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\mathrm{lo}}{e^3}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\mathrm{lo}}{e^3}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\mathrm{lo}}{e^3}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\mathrm{lo}}{e^3}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\mathrm{lo}}{e^3}$$

Mathematica [A] time = 0.223409, size = 319, normalized size = 0.9

$$\frac{2d^2p\left(-\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)-\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)+2\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)-\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)-\log(d+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] ((4*Sqrt[b]*d*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] - 2*d*e*x*Log[c*(a +
 b/x^2)^p] + e^2*x^2*Log[c*(a + b/x^2)^p] + (b*e^2*p*(Log[a + b/x^2] + 2*Lo
g[x]))/a + 2*d^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] + 2*d^2*p*(2*Log[-((e*x)
/d)]*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)
]*Log[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)
]*Log[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)]
- PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2,
1 + (e*x)/d]))/(2*e^3)
```

Maple [F] time = 0.741, size = 0, normalized size = 0.

$$\int \frac{x^2}{ex+d}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.249 $\int \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=291

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2}-\frac{2dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+$$

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e) + (x*Log[c*(a + b/x^2
)^p])/e - (d*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^2 - (2*d*p*Log[-((e*x)/d)
]*Log[d + e*x])/e^2 + (d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))
/(Sqrt[-a]*d - Sqrt[b]*e)])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqr
t[-a]*d + Sqrt[b]*e)])/e^2 - (2*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rubi [A] time = 0.43809, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2466, 2448, 263, 205, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2}-\frac{2dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```



```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e) + (x*Log[c*(a + b/x^2
)^p])/e - (d*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^2 - (2*d*p*Log[-((e*x)/d)
]*Log[d + e*x])/e^2 + (d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))
/(Sqrt[-a]*d - Sqrt[b]*e)])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqr
t[-a]*d + Sqrt[b]*e)])/e^2 - (2*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}dx}{e^2} + \frac{(2bp)\int \frac{}{(a+}}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2bdp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{e^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2dp)\int \frac{\log(d+}{x}}{e^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)}{e^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)}{e^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)}{e^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)}{e^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)}{e^2}$$

Mathematica [A] time = 0.159671, size = 271, normalized size = 0.93

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)+dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)-2dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)-d\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+ex}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] ((-2*Sqrt[b]*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + e*x*Log[c*(a + b/x^
2)^p] - d*Log[c*(a + b/x^2)^p]*Log[d + e*x] - 2*d*p*Log[-((e*x)/d)]*Log[d +
 e*x] + d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d
+ e*x] + d*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Lo
g[d + e*x] + d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)]
+ d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*d*p*Pol
yLog[2, 1 + (e*x)/d])/e^2
```

Maple [F] time = 0.763, size = 0, normalized size = 0.

$$\int \frac{x}{ex+d}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((a + b/x^2)^p*c)/(e*x + d), x)
```

3.250 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=241

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}+\frac{2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{p\log(a}{}$$

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/
e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e + (2*p*
PolyLog[2, 1 + (e*x)/d])/e
```

Rubi [A] time = 0.334294, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}+\frac{2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{p\log(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/
e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e + (2*p*
PolyLog[2, 1 + (e*x)/d])/e
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(d+ex)}{b\left(b+ax^2\right)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2p)\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(2ap)\int \frac{x\log(d+ex)}{b+ax^2}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - (2p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx - \frac{(2ap)\int (\cdots}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} + \frac{2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{\left(\sqrt{-a}p\right)\int \frac{\log(\cdots}{\sqrt{b}-\cdots}}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e} - \cdots$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e} - \cdots$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e} - \cdots$$

Mathematica [A] time = 0.0573689, size = 242, normalized size = 1.

$$-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e} + \frac{2p\,\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} + \frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{p\log\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e + (2*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (Sqrt[-a]*(d +
 e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(S
qrt[-a]*d + Sqrt[b]*e)])/e
```

Maple [F] time = 0.763, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d} \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a + \frac{b}{x^2}\right)^p c\right)}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

3.251 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=287

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d}-\frac{2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log}{d}$$

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d) - (Log[c*(a + b/x^2)^p]*Log
[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b] -
Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sqrt[
b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyLog[2
, 1 + b/(a*x^2)])/(2*d) + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d -
Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e
)])/d - (2*p*PolyLog[2, 1 + (e*x)/d])/d
```

Rubi [A] time = 0.460966, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d}-\frac{2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d) - (Log[c*(a + b/x^2)^p]*Log
[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b] -
Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sqrt[
b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyLog[2
, 1 + b/(a*x^2)])/(2*d) + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d -
Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e
)])/d - (2*p*PolyLog[2, 1 + (e*x)/d])/d
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^2}\right)}{2d} - \frac{(2bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} + \frac{(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x^2} \cdots\right.}{2d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d} - \frac{(2p)\int \frac{\log(d+e\cdots}{x}}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} - \frac{p\text{Li}\cdots}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} - \frac{p\text{Li}\cdots}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\,\text{lo}\cdots}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\,\text{lo}\cdots}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\,\text{lo}\cdots}{}$$

Mathematica [A] time = 0.116015, size = 264, normalized size = 0.92

$$-\frac{-2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right) - 2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right) + p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right) + 4p\text{PolyLog}\left(2,\frac{ex}{d}+1\right) + 2\log(d+e\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))] + 2*Log[c*(a + b/x^2)^p]*Log[d + e
*x] + 4*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))
/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - 2*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x)
)/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] + p*PolyLog[2, 1 + b/(a*x^2)] -
 2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - 2*p*PolyLo
g[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 4*p*PolyLog[2, 1 + (e
*x)/d])/(2*d)
```

Maple [F] time = 0.785, size = 0, normalized size = 0.

$$\int \frac{1}{x(ex+d)} \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^2 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

3.252 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=357

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}+\frac{2ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{e\log\left(-\right.}{}$$

```
[Out] (2*p)/(d*x) + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[b]*d) - Log[c
*(a + b/x^2)^p]/(d*x) + (e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^2)
+ (e*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^2 + (2*e*p*Log[-((e*x)/d)]*Log[d
+ e*x])/d^2 - (e*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]
*Log[d + e*x])/d^2 - (e*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sq
rt[b]*e))]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x^2)])/(2*d^2) - (e
*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d^2 - (e*p*Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^2 + (2*e*p*PolyL
og[2, 1 + (e*x)/d])/d^2
```

Rubi [A] time = 0.506859, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2466, 2455, 263, 325, 205, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}+\frac{2ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{e\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```


```
[Out] (2*p)/(d*x) + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[b]*d) - Log[c
*(a + b/x^2)^p]/(d*x) + (e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^2)
+ (e*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^2 + (2*e*p*Log[-((e*x)/d)]*Log[d
+ e*x])/d^2 - (e*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]
*Log[d + e*x])/d^2 - (e*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sq
rt[b]*e))]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x^2)])/(2*d^2) - (e
*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d^2 - (e*p*Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^2 + (2*e*p*PolyL
og[2, 1 + (e*x)/d])/d^2
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} + \frac{e\,\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^2}\right)}{2d^2} - \ldots$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} - \frac{(2bp\ldots}{\ldots}$$

$$= \frac{2p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} + \ldots$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\ldots\right.\right.}{\ldots}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\ldots\right.\right.}{\ldots}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\ldots\right.\right.}{\ldots}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\ldots\right.\right.}{\ldots}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\ldots\right.\right.}{\ldots}$$

Mathematica [A] time = 0.202285, size = 320, normalized size = 0.9

$$e\left(p\,\mathrm{PolyLog}\left(2,\frac{b}{ax^2}+1\right)+\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\right)+2ep\left(-\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)-\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (4*d*p*(x^(-1) - (Sqrt[a]*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b]) - (2*d*Log[
c*(a + b/x^2)^p])/x + 2*e*Log[c*(a + b/x^2)^p]*Log[d + e*x] + e*(Log[c*(a +
 b/x^2)^p]*Log[-(b/(a*x^2))] + p*PolyLog[2, 1 + b/(a*x^2)]) + 2*e*p*(2*Log[
-((e*x)/d)]*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqr
t[b]*e)]*Log[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[
b]*e)] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*Poly
Log[2, 1 + (e*x)/d]))/(2*d^2)
```

Maple [F] time = 0.747, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(ex+d)}\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^2), x)
```

3.253 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=414

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}-\frac{2e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{e^2\log}{}$$

```
[Out] p/(2*d*x^2) - (2*e*p)/(d^2*x) - (2*Sqrt[a]*e*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(Sqrt[b]*d^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b*d) + (e*Log[c*(a +
 b/x^2)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^3)
- (e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^3 - (2*e^2*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^3 + (e^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[
b]*e)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(a*x^2)])/(2
*d^3) + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d
^3 + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^3
- (2*e^2*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rubi [A] time = 0.546981, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {2466, 2454, 2389, 2295, 2455, 263, 325, 205, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}-\frac{2e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{e^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```


```
[Out] p/(2*d*x^2) - (2*e*p)/(d^2*x) - (2*Sqrt[a]*e*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(Sqrt[b]*d^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b*d) + (e*Log[c*(a +
 b/x^2)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^3)
- (e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^3 - (2*e^2*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^3 + (e^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[
b]*e)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(a*x^2)])/(2
*d^3) + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d
^3 + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^3
- (2*e^2*p*PolyLog[2, 1 + (e*x)/d])/d^3
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{d^3}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int \log\left(c(a+bx)^p\right)dx, x, \frac{1}{x^2}\right)}{2d}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3} - \text{Su}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\right.}{2d^3}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\text{lo}}{}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\text{lo}}{}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\text{lo}}{}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\text{lo}}{}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\text{lo}}{}$$

Mathematica [A] time = 0.289016, size = 364, normalized size = 0.88

$$-e^2\left(p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)+\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\right) - 2e^2p\left(-\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right) - \text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (4*d*e*p*(-x^(-1) + (Sqrt[a]*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b]) + (2*d*e
*Log[c*(a + b/x^2)^p])/x + d^2*(p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/
b) - 2*e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] - e^2*(Log[c*(a + b/x^2)^p]*Lo
g[-(b/(a*x^2))] + p*PolyLog[2, 1 + b/(a*x^2)]) - 2*e^2*p*(2*Log[-((e*x)/d)]
*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Lo
g[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Lo
g[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2, 1 +
```

```
(e*x)/d]))/(2*d^3)
```

Maple [F] time = 0.769, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} (ex + d)} \ln\left(c\left(a + \frac{b}{x^{2}}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^{2}+b}{x^{2}}\right)^{p}\right)}{ex^{4} + dx^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^3), x)
```

3.254 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=714

$$\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4}-\frac{3d^3p\text{PolyLog}\left(2,\frac{ex}{d}+1\right.}{e^4}$$

```
[Out] -((Sqrt[3]*b^(1/3)*d^2*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])
/(a^(1/3)*e^3)) + (Sqrt[3]*b^(2/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt
[3]*b^(1/3))])/(2*a^(2/3)*e^2) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x^2*
Log[c*(a + b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b^(1/3)
*d^2*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^3) + (b^(2/3)*d*p*Log[b^(1/3) +
 a^(1/3)*x])/(2*a^(2/3)*e^2) - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^4
- (3*d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1/3) + a
^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*((
-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d
+ e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a
^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (b^(1/3)*d^2*p*Log[b^
(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^3) - (b^(2/3)*d*p*Lo
g[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e^2) + (b*p*Log[b
+ a*x^3])/(3*a*e) + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1
/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3
)*b^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1
)^(2/3)*b^(1/3)*e)])/e^4 - (3*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rubi [A] time = 0.921298, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {2466, 2448, 263, 200, 31, 634, 617, 204, 628, 2455, 292, 260, 2462, 2416, 2394, 2315, 2393, 2391}

$$\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4}-\frac{3d^3p\text{PolyLog}\left(2,\frac{ex}{d}+1\right.}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```


```
[Out] -((Sqrt[3]*b^(1/3)*d^2*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])
/(a^(1/3)*e^3)) + (Sqrt[3]*b^(2/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt
[3]*b^(1/3))])/(2*a^(2/3)*e^2) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x^2*
Log[c*(a + b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b^(1/3)
*d^2*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^3) + (b^(2/3)*d*p*Log[b^(1/3) +
 a^(1/3)*x])/(2*a^(2/3)*e^2) - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^4
- (3*d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1/3) + a
^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*((
-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d
+ e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a
^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (b^(1/3)*d^2*p*Log[b^
(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^3) - (b^(2/3)*d*p*Lo
g[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e^2) + (b*p*Log[b
+ a*x^3])/(3*a*e) + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1
/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3
```

```
)*b^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1
)^(2/3)*b^(1/3)*e)])/e^4 - (3*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right.\right.}{e^3(d+ex)}\right.$$

$$= \frac{d^2\int\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e^3} - \frac{d^3\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e^3} - \frac{d\int x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e^2} + \frac{\int x^2\log}{}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right.}{e^4}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right.}{e^4}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right.}{e^4}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\right.}{\sqrt[3]{a}e^3}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\right.}{\sqrt[3]{a}e^3}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}d^2p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}d^2p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}d^2p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2}{}$$

Mathematica [C] time = 0.400552, size = 505, normalized size = 0.71

$$x^2\left(6ad^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)+6ad^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)+6ad^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)-18ad^3p\text{Poly}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (9*b*d*e^2*p*x*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))] - 9*b*d^2*e*p*H
ypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))] + x^2*(2*b*e^3*p*Log[a + b/x^3]
 + 6*a*d^2*e*x*Log[c*(a + b/x^3)^p] - 3*a*d*e^2*x^2*Log[c*(a + b/x^3)^p] +
2*a*e^3*x^3*Log[c*(a + b/x^3)^p] + 6*b*e^3*p*Log[x] - 6*a*d^3*Log[c*(a + b/
x^3)^p]*Log[d + e*x] - 18*a*d^3*p*Log[-((e*x)/d)]*Log[d + e*x] + 6*a*d^3*p*
Log[(e*((-1)^(1/3)*b^(1/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)
]*Log[d + e*x] + 6*a*d^3*p*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^
(1/3)*e)]*Log[d + e*x] + 6*a*d^3*p*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))
```

```
/(-(a^(1/3)*d) + (-1)^(2/3)*b^(1/3)*e)]*Log[d + e*x] + 6*a*d^3*p*PolyLog[2,
 (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)] + 6*a*d^3*p*PolyLog[2, (a^(1/
3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)] + 6*a*d^3*p*PolyLog[2, (a
^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)] - 18*a*d^3*p*PolyLog[
2, 1 + (e*x)/d]))/(6*a*e^4*x^2)
```

Maple [F] time = 0.919, size = 0, normalized size = 0.

$$\int \frac{x^3}{ex+d} \ln \left(c \left(a+\frac{b}{x^3}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3 \log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log \left(\left(a + \frac{b}{x^3}\right)^p c\right)}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((a + b/x^3)^p*c)/(e*x + d), x)
```

3.255 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=666

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}+\frac{3d^2p\text{PolyLog}\left(2,\frac{ex}{d}+\right.}{e^3}$$

```
[Out] (Sqrt[3]*b^(1/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e^2) - (Sqrt[3]*b^(2/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(
1/3))])/(2*a^(2/3)*e) - (d*x*Log[c*(a + b/x^3)^p])/e^2 + (x^2*Log[c*(a + b/
x^3)^p])/(2*e) - (b^(1/3)*d*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^2) - (b^
(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*a^(2/3)*e) + (d^2*Log[c*(a + b/x^3)^p]
*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*p*Lo
g[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^3 -
 (d^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*
b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(
2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^3 + (b
^(1/3)*d*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^2)
+ (b^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e)
- (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^3 - (d^
2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^3
 - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)
])/e^3 + (3*d^2*p*PolyLog[2, 1 + (e*x)/d])/e^3
```

Rubi [A] time = 0.730573, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {2466, 2448, 263, 200, 31, 634, 617, 204, 628, 2455, 292, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}+\frac{3d^2p\text{PolyLog}\left(2,\frac{ex}{d}+\right.}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```


```
[Out] (Sqrt[3]*b^(1/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e^2) - (Sqrt[3]*b^(2/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(
1/3))])/(2*a^(2/3)*e) - (d*x*Log[c*(a + b/x^3)^p])/e^2 + (x^2*Log[c*(a + b/
x^3)^p])/(2*e) - (b^(1/3)*d*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^2) - (b^
(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*a^(2/3)*e) + (d^2*Log[c*(a + b/x^3)^p]
*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*p*Lo
g[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^3 -
 (d^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*
b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(
2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^3 + (b
^(1/3)*d*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^2)
+ (b^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e)
- (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^3 - (d^
2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^3
 - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)
```

```
])/e^3 + (3*d^2*p*PolyLog[2, 1 + (e*x)/d])/e^3
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx = \int \left(-\frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e^2}+\frac{d^2\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}+\frac{\left(3bd^2p\right.}{}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}+\frac{\left(3bd^2p\right.}{}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}+\frac{\left(3d^2p\right)}{}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2}-\frac{b^{2/3}p\log\left(\sqrt[3]{b}+\right.}{2a^{2/3}e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2}-\frac{b^{2/3}p\log\left(\sqrt[3]{b}+\right.}{2a^{2/3}e}$$

$$= \frac{\sqrt{3}\sqrt[3]{b}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2}-\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(\right.}{}$$

$$= \frac{\sqrt{3}\sqrt[3]{b}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2}-\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(\right.}{}$$

$$= \frac{\sqrt{3}\sqrt[3]{b}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2}-\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(\right.}{}$$

Mathematica [C] time = 0.221526, size = 443, normalized size = 0.67

$$ax^2\left(-2d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)-2d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)-2d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)+6d^2p\text{PolyLo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*e^2*p*x*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))] + 3*b*d*e*p*Hype
rgeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))] + a*x^2*(-2*d*e*x*Log[c*(a + b/x^3
)^p] + e^2*x^2*Log[c*(a + b/x^3)^p] + 2*d^2*Log[c*(a + b/x^3)^p]*Log[d + e*
x] + 6*d^2*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*d^2*p*Log[(e*((-1)^(1/3)*b^(1
/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x] - 2*d^2*
p*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^(1/3)*e)]*Log[d + e*x] -
2*d^2*p*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + (-1)^(2/3)
*b^(1/3)*e)]*Log[d + e*x] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)
```

```
*d - b^(1/3)*e)] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)
^(1/3)*b^(1/3)*e)] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-
1)^(2/3)*b^(1/3)*e)] + 6*d^2*p*PolyLog[2, 1 + (e*x)/d]))/(2*a*e^3*x^2)
```

Maple [F] time = 0.718, size = 0, normalized size = 0.

$$\int \frac{x^2}{ex+d} \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2 \log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log \left(\left(a + \frac{b}{x^3} \right)^p c \right)}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((a + b/x^3)^p*c)/(e*x + d), x)
```

3.256 $\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=488

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}-\frac{3dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-$$

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e)) + (x*Log[c*(a + b/x^3)^p])/e + (b^(1/3)*p*Log[b^(1/3) + a^(1/3)*x
])/(a^(1/3)*e) - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Log[-((
e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*
d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^
(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log
[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^
(1/3)*e)]*Log[d + e*x])/e^2 - (b^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x +
a^(2/3)*x^2])/(2*a^(1/3)*e) + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*
d - b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1
)^(1/3)*b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d -
 (-1)^(2/3)*b^(1/3)*e)])/e^2 - (3*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rubi [A] time = 0.601154, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {2466, 2448, 263, 200, 31, 634, 617, 204, 628, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}-\frac{3dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e)) + (x*Log[c*(a + b/x^3)^p])/e + (b^(1/3)*p*Log[b^(1/3) + a^(1/3)*x
])/(a^(1/3)*e) - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Log[-((
e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*
d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^
(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log
[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^
(1/3)*e)]*Log[d + e*x])/e^2 - (b^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x +
a^(2/3)*x^2])/(2*a^(1/3)*e) + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*
d - b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1
)^(1/3)*b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d -
 (-1)^(2/3)*b^(1/3)*e)])/e^2 - (3*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx = \int\left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e} - \frac{d\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}dx}{e^2} + \frac{(3bp)\int\frac{}{(a+}}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int\left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b\left(b+ax^3\right)}\right)d}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3dp)\int\frac{\log(d+ex)}{x}dx}{e^2} + \frac{(3adp)\int\frac{x^2\log}{b}}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\log\left(-\right.}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\log\left(-\right.}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right.\right.}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right.\right.}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right.\right.}{}$$

Mathematica [C] time = 0.113106, size = 403, normalized size = 0.83

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2} - \frac{3dp\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^2} -$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e*x^2) + (x*Log[
c*(a + b/x^3)^p])/e - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Lo
g[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(
1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-(((-1)^(2/3)*e*(b^(1/3)
 - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x]
)/e^2 + (d*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
 + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^2 - (3*d*p*PolyLog[2, (d + e*x)/d
])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^2
```

```
+ (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/
e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e
)])/e^2
```

Maple [F] time = 0.72, size = 0, normalized size = 0.

$$\int \frac{x}{ex+d}\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x)

[Out] int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(x*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**3)**p)/(e*x+d),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((a + b/x^3)^p*c)/(e*x + d), x)
```

3.257 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=344

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}+\frac{3p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log}{}$$

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/
3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3
)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e - (p*Poly
Log[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e + (3*p*PolyLog[2, 1
+ (e*x)/d])/e
```

Rubi [A] time = 0.409433, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}+\frac{3p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/
3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3
)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e - (p*Poly
Log[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e + (3*p*PolyLog[2, 1
+ (e*x)/d])/e
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b\left(b+ax^3\right)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3p)\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(3ap)\int \frac{x^2\log(d+ex)}{b+ax^3}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - (3p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx - \frac{(3ap)\int \left(\frac{\mathrm{l}}{3a^{2/3}}\right.}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} + \frac{3p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{\left(\sqrt[3]{a}p\right)\int \frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p}{}$$

Mathematica [A] time = 0.0808126, size = 350, normalized size = 1.02

$$-\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e} + \frac{3p\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} + \frac{\log(d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d
 - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3)
+ (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/
e + (3*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^
(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (
-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (
-1)^(2/3)*b^(1/3)*e)])/e
```

Maple [F] time = 0.721, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

3.258 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=388

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d}-3p\text{Po}$$

```
[Out] -(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/(3*d) - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-((e*((-1)
^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e
*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
+ (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x^3)])/(3
*d) + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d - (3*p*
PolyLog[2, 1 + (e*x)/d])/d
```

Rubi [A] time = 0.538067, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d}-3p\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```


```
[Out] -(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/(3*d) - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-((e*((-1)
^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e
*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
+ (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x^3)])/(3
*d) + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d - (3*p*
PolyLog[2, 1 + (e*x)/d])/d
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^3}\right)}{3d} - \frac{(3bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} + \frac{(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x^3}\right.}{3d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} - \frac{(3p)\int \frac{\log(d+e}{x}}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} - \frac{p\text{Li}}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} - \frac{p\text{Li}}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\,\text{lo}}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\,\text{lo}}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\,\text{lo}}{}$$

Mathematica [A] time = 0.0849992, size = 395, normalized size = 1.02

$$\frac{p\text{PolyLog}\left(2, \frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d} + \frac{p\text{PolyLog}\left(2, \frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d} + \frac{p\text{PolyLog}\left(2, \frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d} - \frac{p\text{PolyLog}\left(2, \frac{a+\frac{b}{x^3}}{a}\right)}{3d} - 3p\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] -(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/(3*d) - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-(((-1)^(2
/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))
]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/
(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, (a + b/
x^3)/a])/(3*d) - (3*p*PolyLog[2, (d + e*x)/d])/d + (p*PolyLog[2, (a^(1/3)*(
d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
```

```
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d
```

Maple [F] time = 0.708, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(ex+d\right)}\ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^{3}}\right)^{p}c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax^{3}+b}{x^{3}}\right)^{p}\right)}{ex^{2}+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^2 + d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

3.259 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=557

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}-$$

```
[Out] (3*p)/(d*x) - (Sqrt[3]*a^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^
(1/3))])/(b^(1/3)*d) - Log[c*(a + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^p]
*Log[-(b/(a*x^3))])/(3*d^2) - (a^(1/3)*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)
*d) + (e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3
)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*L
og[d + e*x])/d^2 + (a^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2
])/(2*b^(1/3)*d) + (e*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^2) - (e*p*PolyLog[2
, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1
/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (
a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^2 + (3*e*p*PolyLo
g[2, 1 + (e*x)/d])/d^2
```

Rubi [A] time = 0.668387, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {2466, 2455, 263, 325, 292, 31, 634, 617, 204, 628, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```


```
[Out] (3*p)/(d*x) - (Sqrt[3]*a^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^
(1/3))])/(b^(1/3)*d) - Log[c*(a + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^p]
*Log[-(b/(a*x^3))])/(3*d^2) - (a^(1/3)*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)
*d) + (e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3
)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*L
og[d + e*x])/d^2 + (a^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2
])/(2*b^(1/3)*d) + (e*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^2) - (e*p*PolyLog[2
, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1
/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (
a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^2 + (3*e*p*PolyLo
g[2, 1 + (e*x)/d])/d^2
```


Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{e\,\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, \frac{1}{x^3}\right)}{3d^2} - \frac{(3bp\cdots}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} - \frac{(3bp)\int\cdots}{}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\text{I}\cdots}{}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\cdots}{}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} + \frac{e\log\left(c\left(a+\cdots\right.\right.}{}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} + \frac{e\log\left(c\left(a+\cdots\right.\right.}{}$$

$$= \frac{3p}{dx} - \frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\cdots}{}$$

$$= \frac{3p}{dx} - \frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\cdots}{}$$

Mathematica [C] time = 0.183896, size = 429, normalized size = 0.77

$$\frac{4ax^3\left(-3epx\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)-3epx\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)-3epx\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)+epx\text{PolyLog}\left(\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (9*b*d*p*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))] + 4*a*x^3*(-3*d*Log[c
*(a + b/x^3)^p] + e*x*Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))] + 3*e*x*Log[c*
(a + b/x^3)^p]*Log[d + e*x] + 9*e*p*x*Log[-((e*x)/d)]*Log[d + e*x] - 3*e*p*
x*Log[(e*((-1)^(1/3)*b^(1/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*
e)]*Log[d + e*x] - 3*e*p*x*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^
(1/3)*e)]*Log[d + e*x] - 3*e*p*x*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
-(a^(1/3)*d) + (-1)^(2/3)*b^(1/3)*e)]*Log[d + e*x] + e*p*x*PolyLog[2, 1 + b
/(a*x^3)] - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)]
```

```
- 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e
)] - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)
*e)] + 9*e*p*x*PolyLog[2, 1 + (e*x)/d]))/(12*a*d^2*x^4)
```

Maple [F] time = 0.709, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} (ex + d)} \ln \left(c \left(a + \frac{b}{x^{3}}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x)

[Out] int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^{3}+b}{x^{3}}\right)^{p}\right)}{ex^{3} + dx^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**2/(e*x+d),x)

[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^2), x)
```

3.260 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=737

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\right.}{d^3}$$

```
[Out] (3*p)/(4*d*x^2) - (3*e*p)/(d^2*x) - (Sqrt[3]*a^(2/3)*p*ArcTan[(b^(1/3) - 2*
a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(2*b^(2/3)*d) + (Sqrt[3]*a^(1/3)*e*p*ArcTan[
(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(b^(1/3)*d^2) - Log[c*(a + b/x^
3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^3)
^p]*Log[-(b/(a*x^3))])/(3*d^3) + (a^(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*b^
(2/3)*d) + (a^(1/3)*e*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)*d^2) - (e^2*Log[
c*(a + b/x^3)^p]*Log[d + e*x])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d + e*x])
/d^3 + (e^2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log
[d + e*x])/d^3 + (e^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)
*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d
+ e*x])/d^3 - (a^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4
*b^(2/3)*d) - (a^(1/3)*e*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/
(2*b^(1/3)*d^2) - (e^2*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^3) + (e^2*p*PolyLo
g[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2,
(a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^3 + (e^2*p*PolyL
og[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^3 - (3*e^2
*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rubi [A] time = 0.795149, antiderivative size = 737, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 19, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.826, Rules used = {2466, 2455, 263, 325, 200, 31, 634, 617, 204, 628, 292, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```


```
[Out] (3*p)/(4*d*x^2) - (3*e*p)/(d^2*x) - (Sqrt[3]*a^(2/3)*p*ArcTan[(b^(1/3) - 2*
a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(2*b^(2/3)*d) + (Sqrt[3]*a^(1/3)*e*p*ArcTan[
(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(b^(1/3)*d^2) - Log[c*(a + b/x^
3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^3)
^p]*Log[-(b/(a*x^3))])/(3*d^3) + (a^(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*b^
(2/3)*d) + (a^(1/3)*e*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)*d^2) - (e^2*Log[
c*(a + b/x^3)^p]*Log[d + e*x])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d + e*x])
/d^3 + (e^2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log
[d + e*x])/d^3 + (e^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)
*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d
+ e*x])/d^3 - (a^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4
*b^(2/3)*d) - (a^(1/3)*e*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/
(2*b^(1/3)*d^2) - (e^2*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^3) + (e^2*p*PolyLo
```

```
g[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2,
(a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^3 + (e^2*p*PolyL
og[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^3 - (3*e^2
*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^3}-\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x^2}+\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3x}-\frac{e^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3(d+ex)}\right)dx$$

$$=\frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3}\,dx}{d}-\frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}\,dx}{d^2}+\frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}\,dx}{d^3}-\frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2\,\mathrm{Subst}\left(\int \frac{\log(c(\ldots}{}\right.}{3d^{\ldots}}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right.}{d^3}\ldots$$

$$=\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}-\frac{e^2\log\ldots}{}$$

$$=\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}-\frac{e^2\log\ldots}{}$$

$$=\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}+\frac{a^{2/3}p\ldots}{}$$

$$=\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}+\frac{a^{2/3}p\ldots}{}$$

$$=\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{a}ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\,\mathrm{l}\ldots}{}$$

$$=\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{a}ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\,\mathrm{l}\ldots}{}$$

Mathematica [C] time = 0.274554, size = 520, normalized size = 0.71

$$\frac{-10ax^3\left(-6e^2px^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)-6e^2px^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)-6e^2px^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)+2e^2px\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-45*b*d*e*p*x*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))] + 18*b*d^2*p*Hy
pergeometric2F1[1, 5/3, 8/3, -(b/(a*x^3))] - 10*a*x^3*(3*d^2*Log[c*(a + b/x
^3)^p] - 6*d*e*x*Log[c*(a + b/x^3)^p] + 2*e^2*x^2*Log[c*(a + b/x^3)^p]*Log[
-(b/(a*x^3))] + 6*e^2*x^2*Log[c*(a + b/x^3)^p]*Log[d + e*x] + 18*e^2*p*x^2*
Log[-((e*x)/d)]*Log[d + e*x] - 6*e^2*p*x^2*Log[(e*((-1)^(1/3)*b^(1/3) - a^(
1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x] - 6*e^2*p*x^2*Log
[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^(1/3)*e)]*Log[d + e*x] - 6*e^2
*p*x^2*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + (-1)^(2/3)*
b^(1/3)*e)]*Log[d + e*x] + 2*e^2*p*x^2*PolyLog[2, 1 + b/(a*x^3)] - 6*e^2*p*
x^2*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)] - 6*e^2*p*x^2*P
olyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)] - 6*e^2*p
*x^2*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)] + 1
8*e^2*p*x^2*PolyLog[2, 1 + (e*x)/d]))/(60*a*d^3*x^5)
```

Maple [F] time = 0.704, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\,(ex+d)} \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.261 $\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=749

$$\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[3]{d}\sqrt{g}+i\sqrt[3]{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1+\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}+\sqrt[3]{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2(-1)}{(\sqrt{f}-i}\right.}{2\sqrt{f}}$$

```
[Out] (3*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[((-2*I)
*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)
^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1
)^(2/3)*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f
] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*
(d + e*x^3)^p])/(Sqrt[f]*Sqrt[g]) - (((3*I)/2)*p*PolyLog[2, 1 - (2*Sqrt[f])
/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*S
qrt[f]*Sqrt[g]*(d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + ((
2*I)*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] +
(-1)^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
((I/2)*p*PolyLog[2, 1 - (2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1)^(2/3)
*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rubi [A] time = 0.930986, antiderivative size = 749, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {205, 2470, 12, 260, 6725, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[3]{d}\sqrt{g}+i\sqrt[3]{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1+\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}+\sqrt[3]{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2(-1)}{(\sqrt{f}-i}\right.}{2\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```


```
[Out] (3*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[((-2*I)
*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)
^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1
)^(2/3)*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f
] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*
(d + e*x^3)^p])/(Sqrt[f]*Sqrt[g]) - (((3*I)/2)*p*PolyLog[2, 1 - (2*Sqrt[f])
/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*S
qrt[f]*Sqrt[g]*(d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + ((
2*I)*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] +
(-1)^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
```

```
((I/2)*p*PolyLog[2, 1 - (2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1)^(2/3)
*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}} - (3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+ex^3\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^3}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(3ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}\right.}\right.}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt[3]{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt[3]{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt[3]{e}p\right.}{}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right.}{}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right.}{}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right.}{}$$

Mathematica [A] time = 0.596954, size = 867, normalized size = 1.16

$$\frac{-p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\sqrt[3]{e}\sqrt{-f}+\sqrt[3]{d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right) - p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{e}x-\sqrt[3]{-1}\sqrt[3]{d}\right)}{\sqrt[3]{e}\sqrt{-f}-\sqrt[3]{-1}\sqrt[3]{d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right) - p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{\sqrt[3]{e}\sqrt{-f}+(-1)^{2/3}\sqrt[3]{d}\sqrt{g}}\right)\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

```
[Out] (-(p*Log[(Sqrt[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] + d^(1/3)*Sqrt[g
])]*Log[Sqrt[-f] - Sqrt[g]*x]) - p*Log[(Sqrt[g]*(-((-1)^(1/3)*d^(1/3)) + e^
(1/3)*x))/(e^(1/3)*Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - S
qrt[g]*x] - p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[
-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[-((Sqr
t[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g]))]*Log[Sqrt
[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(-(e^(
1/3)*Sqrt[-f]) + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + p
*Log[((-1)^(1/3)*Sqrt[g]*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/(e^(1/3)*Sqrt[-f
] + (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + Log[Sqrt[-f] -
 Sqrt[g]*x]*Log[c*(d + e*x^3)^p] - Log[Sqrt[-f] + Sqrt[g]*x]*Log[c*(d + e*x
^3)^p] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] +
d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*
```

```
Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] -
 Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])] + p*PolyLog[2
, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g])] +
p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(1/3
)*d^(1/3)*Sqrt[g])] + p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3
)*Sqrt[-f] - (-1)^(2/3)*d^(1/3)*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [C] time = 0.716, size = 327, normalized size = 0.4

$$(\ln\left(\left(ex^3+d\right)^p\right)-p\ln\left(ex^3+d\right))\arctan\left(gx\frac{1}{\sqrt{fg}}\right)\frac{1}{\sqrt{fg}}+\frac{p}{2g}\sum_{\_alpha=\mathrm{RootOf}\left(g\_Z^2+f\right)}\left(\frac{1}{\_alpha}\left(\ln\left(x-\_alpha\right)\ln\left(ex^3+d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x^3+d)^p)-p*ln(e*x^3+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^3+d)-sum(ln(x-_alpha)*ln((_R1-x+_alpha)/
_R1)+dilog((_R1-x+_alpha)/_R1),_R1=RootOf(_Z^3*e*g+3*_Z^2*_alpha*e*g-3*_Z*e
*f-_alpha*e*f+d*g))),_alpha=RootOf(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/
(f*g)^(1/2))*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-1/2*I/(f*g)^(1/
2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csg
n(I*c)-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^3+d)^p)^3
+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(
I*c)+1/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*ln(c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

3.262 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rubi [A] time = 0.509221, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```


```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```


Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+ex^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{}$$

Mathematica [A] time = 0.343537, size = 564, normalized size = 1.06

$$-\frac{i\left(p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)+p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)-p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)-p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-I/2)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-
d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] + Sq
rt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/
Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d]
 + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x
)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*Pol
yLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqr
t[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] +
I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt
[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqr
t[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] time = 0.766, size = 504, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root
Of(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*ln(c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.263 $\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx$

Optimal. Leaf size=229

$$-\frac{p\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[Out] (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x)
)/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((Sqrt[g
]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Rubi [A] time = 0.228681, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2409, 2394, 2393, 2391}

$$-\frac{p\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```


```
[Out] (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x)
)/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((Sqrt[g
]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```


Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx=\int \left(\frac{\sqrt{-f}\log\left(c(d+ex)^p\right)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}\log\left(c(d+ex)^p\right)}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx$$

$$=-\frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2\sqrt{-f}}-\frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2\sqrt{-f}}$$

$$=\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{(ep)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2\sqrt{-f}\sqrt{g}}+$$

$$=\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\right.}{2\sqrt{-f}\sqrt{g}}$$

$$=\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\mathrm{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\mathrm{Li}}{}$$

Mathematica [A] time = 0.110755, size = 178, normalized size = 0.78

$$\frac{-p\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+p\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{d\sqrt{g}+e\sqrt{-f}}\right)+\log\left(c(d+ex)^p\right)\left(\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)-\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x)^p]*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g]
)] - Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) - p*PolyLog[
2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + p*PolyLog[2, (Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [C] time = 0.533, size = 419, normalized size = 1.8

$$\left(\ln\left((ex+d)^p\right)-p\ln(ex+d)\right)\arctan\left(\frac{2g(ex+d)-2dg}{2e}\frac{1}{\sqrt{fg}}\right)\frac{1}{\sqrt{fg}}+\frac{p\ln(ex+d)}{2}\ln\left(\left(e\sqrt{-fg}-g(ex+d)+dg\right)\left(e\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x+d)^p)-p*ln(e*x+d))/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f
*g)^(1/2))+1/2*p*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(
e*(-f*g)^(1/2)+d*g))-1/2*p*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*p/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(
e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*p/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)+
```

```
g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/
2))*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g
/(f*g)^(1/2))*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)*csgn(I*c)-1/2*I/(f*g
)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x+d)^p)^3+1/2*I/(f*g)^(1/2)*
arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*ar
ctan(x*g/(f*g)^(1/2))*ln(c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((ex+d)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\,(d+ex)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x)**p)/(f + g*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex+d)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

3.264 $\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=360

$$\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(dx+e)}{(\sqrt{f}-i\sqrt{g}x)(e\sqrt{g}+id\sqrt{f})}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}}+$$

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x)^p])/(Sqrt[f]*Sqrt[g]) + (p*Arc
Tan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]
*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(e + d*x)
)/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
 ((I/2)*p*PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*
p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*p*PolyLog[2
, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*Po
lyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(e + d*x))/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt
[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rubi [A] time = 0.436601, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {205, 2470, 12, 260, 6688, 4876, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(dx+e)}{(\sqrt{f}-i\sqrt{g}x)(e\sqrt{g}+id\sqrt{f})}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}}+$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x)^p])/(Sqrt[f]*Sqrt[g]) + (p*Arc
Tan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]
*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(e + d*x)
)/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
 ((I/2)*p*PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*
p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*p*PolyLog[2
, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*Po
lyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(e + d*x))/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt
[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + (ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x}\right)x^2}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(e+dx)}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex} - \frac{d\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e(e+dx)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(dp)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e+dx}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+d}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-\right.}\right.}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+d}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-\right.}\right.}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+d}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-\right.}\right.}{\sqrt{f}\sqrt{g}}$$

Mathematica [A] time = 0.224825, size = 373, normalized size = 1.04

$$\frac{-p\text{PolyLog}\left(2,\frac{d\left(\sqrt{-f}-\sqrt{g}x\right)}{d\sqrt{-f}+e\sqrt{g}}\right)+p\text{PolyLog}\left(2,\frac{d\left(\sqrt{-f}+\sqrt{g}x\right)}{d\sqrt{-f}-e\sqrt{g}}\right)-p\text{PolyLog}\left(2,\frac{\sqrt{g}x}{\sqrt{-f}}+1\right)+p\text{PolyLog}\left(2,\frac{f\sqrt{g}x}{(-f)^{3/2}}+1\right)+\log\left(\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[(Sqrt[g]*x)/Sqrt[-f]]
*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(e + d*x))/(d*Sqrt[-f] + e*Sqrt
[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - Log[c*(d + e/x)^p]*Log[Sqrt[-f] + Sqrt[g]
*x] - p*Log[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[-((
Sqrt[g]*(e + d*x))/(d*Sqrt[-f] - e*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p
*PolyLog[2, (d*(Sqrt[-f] - Sqrt[g]*x))/(d*Sqrt[-f] + e*Sqrt[g])] + p*PolyLo
g[2, (d*(Sqrt[-f] + Sqrt[g]*x))/(d*Sqrt[-f] - e*Sqrt[g])] - p*PolyLog[2, 1
+ (Sqrt[g]*x)/Sqrt[-f]] + p*PolyLog[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sq
rt[-f]*Sqrt[g])
```

Maple [F] time = 0.752, size = 0, normalized size = 0.

$$\int \frac{1}{g x^{2}+f} \ln \left(c \left(d +\frac{e}{x}\right)^{p}\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{dx+e}{x}\right)^{p}\right)}{gx^{2}+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log(c*((d*x + e)/x)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{gx^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x)^p)/(g*x^2 + f), x)
```

3.265 $\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=597

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}x+\sqrt{e}\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-$$

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x^2)^p])/(Sqrt[f]*Sqrt[g]) + (2*p
*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqr
t[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqr
t[e] - Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f]*Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*
(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (I*p*PolyLog[2, ((-I)*Sqrt[g
]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/
(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)
])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[e]
- Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt
[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rubi [A] time = 0.83858, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {205, 2470, 12, 260, 6688, 4928, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}x+\sqrt{e}\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```


```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x^2)^p])/(Sqrt[f]*Sqrt[g]) + (2*p
*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqr
t[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqr
t[e] - Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f]*Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*
(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (I*p*PolyLog[2, ((-I)*Sqrt[g
]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/
(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)
])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[e]
- Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt
[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(Sqrt[f]*Sqrt[g])
```


Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + (2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x^2}\right)x^3}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x^2}\right)x^3}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x\left(e+dx^2\right)}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex} - \frac{dx\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e\left(e+dx^2\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(2dp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e+dx^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ip)\int \frac{\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(ip)\int \frac{\log\left(1+\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(2dp)\int \left(-\frac{\sqrt{-d}}{2d}\cdots\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\mathrm{Li}_2\left(\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} + \frac{\left(\sqrt{-d}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx}{\sqrt{f}\sqrt{g}} - \cdots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\cdots}{\left(i\sqrt{-d}\cdots\right.}\right.}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\cdots}{\left(i\sqrt{-d}\cdots\right.}\right.}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\cdots}{\left(i\sqrt{-d}\cdots\right.}\right.}{\sqrt{f}\sqrt{g}}$$

Mathematica [A] time = 0.371281, size = 706, normalized size = 1.18

$$-p\mathrm{PolyLog}\left(2,\frac{\sqrt{-d}\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{-d}\sqrt{-f}-\sqrt{e}\sqrt{g}}\right)-p\mathrm{PolyLog}\left(2,\frac{\sqrt{-d}\left(\sqrt{-f}-\sqrt{g}x\right)}{\sqrt{-d}\sqrt{-f}+\sqrt{e}\sqrt{g}}\right)+p\mathrm{PolyLog}\left(2,\frac{\sqrt{-d}\left(\sqrt{-f}+\sqrt{g}x\right)}{\sqrt{-d}\sqrt{-f}-\sqrt{e}\sqrt{g}}\right)+p\mathrm{PolyLog}\left(2,\frac{\sqrt{-d}\left(\sqrt{-\cdots}\right.}{\sqrt{-d}\sqrt{-f}\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + 2*p*Log[(Sqrt[g]*x)/Sqrt[
-f]]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(-Sqrt[e] + Sqrt[-d]*x))/(S
qrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqr
t[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqr
t[-f] - Sqrt[g]*x] - Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] + Sqrt[g]*x] - 2*p*L
og[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*(Sq
rt[e] - Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] +
Sqrt[g]*x] + p*Log[-((Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] -
Sqrt[e]*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p*PolyLog[2, (Sqrt[-d]*(Sqrt
[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])] - p*PolyLog[2, (S
qrt[-d]*(Sqrt[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])] + p*
PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*S
qrt[g])] + p*PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f
] + Sqrt[e]*Sqrt[g])] - 2*p*PolyLog[2, 1 + (Sqrt[g]*x)/Sqrt[-f]] + 2*p*Poly
Log[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [F] time = 0.743, size = 0, normalized size = 0.

$$\int \frac{1}{g x^2 + f} \ln\left(c\left(d + \frac{e}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(c\left(\frac{dx^2+e}{x^2}\right)^p\right)}{gx^2 + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log(c*((d*x^2 + e)/x^2)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x**2)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

3.266 $\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx$

<u>**Optimal**</u>. Leaf size=541

$$-\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right)}{2\sqrt{-f}\sqrt{g}}\cdots$$

```
[Out] -(Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] - g^(1/4)*Sqrt[x]))/(e*S
qrt[-Sqrt[-f]] + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e*Sqrt[x])
^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(2*
Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] + g^(
1/4)*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (Lo
g[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4)
 - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt
[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4)))])/(2*Sqrt[-f]*Sq
rt[g]) - (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] + d*g^(
1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(
-f)^(1/4) + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

<u>**Rubi**</u> **[A]** time = 0.813215, antiderivative size = 541, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2472, 275, 205, 2416, 260, 2394, 2393, 2391}

$$-\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right)}{2\sqrt{-f}\sqrt{g}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -(Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] - g^(1/4)*Sqrt[x]))/(e*S
qrt[-Sqrt[-f]] + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e*Sqrt[x])
^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(2*
Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] + g^(
1/4)*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (Lo
g[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4)
 - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt
[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4)))])/(2*Sqrt[-f]*Sq
rt[g]) - (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] + d*g^(
1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(
-f)^(1/4) + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

<u>Rule 2472</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
```

```
r, s}, x] && FractionQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = 2\,\text{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{f+gx^4}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(-\frac{\sqrt{g}x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}-gx^2\right)} - \frac{\sqrt{g}x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}+gx^2\right)}\right)dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{g}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-gx^2}\,dx, x, \sqrt{x}\right)}{\sqrt{-f}} - \frac{\sqrt{g}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}+gx^2}\,dx, x, \sqrt{x}\right)}{\sqrt{-f}}$$

$$= -\frac{\sqrt{g}\,\text{Subst}\left(\int \left(-\frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}x\right)} + \frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}x\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{-f}} - \frac{\sqrt{g}\,\text{Subst}\left(\int \left(\frac{\log(c(d+\cdots}{2g^{3/4}\left(\sqrt[4]{-f}\cdots\right.}\right.\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-\sqrt{-f}}-\sqrt[4]{g}x}\,dx, x, \sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{-f}-\sqrt[4]{g}x}\,dx, x, \sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-\sqrt{-f}}+\sqrt[4]{g}x}\,d\cdots\right.}{2\sqrt{-f}\sqrt[4]{g}}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(\cdots\right.\right.}{}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(\cdots\right.\right.}{}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(\cdots\right.\right.}{}$$

Mathematica [C] time = 0.277251, size = 422, normalized size = 0.78

$$\frac{p\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right) - p\text{PolyLog}\left(2,\frac{i\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}+id\sqrt[4]{g}}\right) - p\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+ie\sqrt[4]{-f}}\right) + p\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right) + \log\left(c\left(\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1
/4) + d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - I*g^(1/4)
*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*
((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - I*d*g^(1/4))] + Log[c*(d
+ e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - d*g^
(1/4))] + p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4
)))] - p*PolyLog[2, (I*g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4)
)] - p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(I*e*(-f)^(1/4) + d*g^(1/4))] +
 p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(2*Sqr
t[-f]*Sqrt[g])
```

Maple [F] time = 0.727, size = 0, normalized size = 0.

$$\int \frac{1}{g x^{2}+f} \ln \left(c \left(d +e \sqrt{x}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(e\sqrt{x}+d\right)^{p} c\right)}{g x^{2}+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**(1/2))**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(e\sqrt{x}+d\right)^{p} c\right)}{g x^{2}+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

3.267 $\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=561

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[Out] -(Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4) - Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*S
qrt[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e/Sqrt[x])
^p]*Log[-((e*(g^(1/4) + Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^
(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4)
- (-f)^(1/4)/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) +
(Log[c*(d + e/Sqrt[x])^p]*Log[-((e*(g^(1/4) + (-f)^(1/4)/Sqrt[x]))/(d*(-f)^
(1/4) - e*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]
*(d + e/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) +
(p*PolyLog[2, ((-f)^(1/4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))])/(2*
Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]*(d + e/Sqrt[x]))/(d*Sqrt
[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, ((-f)^(1/4)
*(d + e/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rubi [A] time = 1.10713, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2472, 2475, 263, 275, 205, 2416, 260, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -(Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4) - Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*S
qrt[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e/Sqrt[x])
^p]*Log[-((e*(g^(1/4) + Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^
(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4)
- (-f)^(1/4)/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) +
(Log[c*(d + e/Sqrt[x])^p]*Log[-((e*(g^(1/4) + (-f)^(1/4)/Sqrt[x]))/(d*(-f)^
(1/4) - e*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]
*(d + e/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) +
(p*PolyLog[2, ((-f)^(1/4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))])/(2*
Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]*(d + e/Sqrt[x]))/(d*Sqrt
[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, ((-f)^(1/4)
*(d + e/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 2472

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
```

```
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
r, s}, x] && FractionQ[n]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^4}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{\left(f+\frac{g}{x^4}\right)x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int \left(-\frac{fx\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\sqrt{g}-fx^2\right)}-\frac{fx\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\sqrt{g}+fx^2\right)}\right)dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-fx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}-\frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}+fx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}\\
&= -\frac{\sqrt{-f}\,\text{Subst}\left(\int \left(\frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}-\sqrt{-\sqrt{-f}}x\right)}-\frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}+\sqrt{-\sqrt{-f}}x\right)}\right)dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}-\frac{\sqrt{-f}\,\text{Subst}\left(\int\left(-\frac{\sqrt[4]{-\cdots}}{2\cdots}\right.\right.}{\cdots}\\
&= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}-\sqrt{-\sqrt{-f}}x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-\sqrt{-f}}\sqrt{g}}+\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}+\sqrt{-\sqrt{-f}}x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-\sqrt{-f}}\sqrt{g}}-\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^{\cdots}}{\sqrt[4]{g}-\sqrt[4]{-f}x\cdots}\right.}{2\sqrt[4]{-f}\sqrt{g}\cdots}\\
&= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d\cdots\right.\right.}{\cdots}\\
&= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d\cdots\right.\right.}{\cdots}\\
&= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d\cdots\right.\right.}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.530052, size = 912, normalized size = 1.63

$$
\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\sqrt[4]{g}\sqrt{x}-\sqrt[4]{-f}\right)-p\log\left(-\frac{\sqrt[4]{g}\left(\sqrt{x}d+e\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)\log\left(-\sqrt[4]{g}\sqrt{x}-\sqrt[4]{-f}\right)+p\log\left(\frac{f\sqrt[4]{g}\sqrt{x}}{(-f)^{5/4}}\right)\log\left(-\sqrt[4]{g}\sqrt{x}-\sqrt[4]{-\cdots}\right.}{\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/Sqrt[x])^p]*Log[-(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[-((g^(
1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4)))]*Log[-(-f)^(1/4) - g^(1/4
```

```
)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[(-I)*(-f)^(1/4) - g^(1/4)*Sqrt[x]
] + p*Log[(I*g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))]*Log[(-I
)*(-f)^(1/4) - g^(1/4)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[I*(-f)^(1/4)
 - g^(1/4)*Sqrt[x]] + p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(I*d*(-f)^(1/4) + e*g
^(1/4))]*Log[I*(-f)^(1/4) - g^(1/4)*Sqrt[x]] + Log[c*(d + e/Sqrt[x])^p]*Log
[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1
/4) + e*g^(1/4))]*Log[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[I*(-f)^(1/4) -
g^(1/4)*Sqrt[x]]*Log[((-I)*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*Log[(-I)*(-f)^(
1/4) - g^(1/4)*Sqrt[x]]*Log[(I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[(-f)^(1
/4) - g^(1/4)*Sqrt[x]]*Log[(g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[-(-f)^(1/4
) - g^(1/4)*Sqrt[x]]*Log[(f*g^(1/4)*Sqrt[x])/(-f)^(5/4)] - p*PolyLog[2, (d*
((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))] + p*PolyLog[2,
(d*((-f)^(1/4) - I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))] + p*Poly
Log[2, (d*((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - I*e*g^(1/4))] -
 p*PolyLog[2, (d*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))
] - p*PolyLog[2, 1 - (I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*PolyLog[2, 1 + (I*
g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*PolyLog[2, 1 + (g^(1/4)*Sqrt[x])/(-f)^(1/4
)] + p*PolyLog[2, 1 + (f*g^(1/4)*Sqrt[x])/(-f)^(5/4)])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [F] time = 0.747, size = 0, normalized size = 0.

$$\int \frac{1}{g x^2+f} \ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^p\right)}{g x^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log(c*((d*x + e*sqrt(x))/x)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x**(1/2))**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^{p}\right)}{gx^{2} + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

3.268 $\int (f+gx^2)^3 \log\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=338

$$f^2gx^3\log\left(c\left(d+ex^2\right)^p\right)+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^3x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2gp\tan}{e^{3/2}}$$

```
[Out] -2*f^3*p*x + (2*d*f^2*g*p*x)/e - (6*d^2*f*g^2*p*x)/(5*e^2) + (2*d^3*g^3*p*x
)/(7*e^3) - (2*f^2*g*p*x^3)/3 + (2*d*f*g^2*p*x^3)/(5*e) - (2*d^2*g^3*p*x^3)
/(21*e^2) - (6*f*g^2*p*x^5)/25 + (2*d*g^3*p*x^5)/(35*e) - (2*g^3*p*x^7)/49
+ (2*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*f^2*g*
p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (6*d^(5/2)*f*g^2*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
7*e^(7/2)) + f^3*x*Log[c*(d + e*x^2)^p] + f^2*g*x^3*Log[c*(d + e*x^2)^p] +
(3*f*g^2*x^5*Log[c*(d + e*x^2)^p])/5 + (g^3*x^7*Log[c*(d + e*x^2)^p])/7
```

Rubi [A] time = 0.258966, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$f^2gx^3\log\left(c\left(d+ex^2\right)^p\right)+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^3x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2gp\tan}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```


```
[Out] -2*f^3*p*x + (2*d*f^2*g*p*x)/e - (6*d^2*f*g^2*p*x)/(5*e^2) + (2*d^3*g^3*p*x
)/(7*e^3) - (2*f^2*g*p*x^3)/3 + (2*d*f*g^2*p*x^3)/(5*e) - (2*d^2*g^3*p*x^3)
/(21*e^2) - (6*f*g^2*p*x^5)/25 + (2*d*g^3*p*x^5)/(35*e) - (2*g^3*p*x^7)/49
+ (2*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*f^2*g*
p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (6*d^(5/2)*f*g^2*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
7*e^(7/2)) + f^3*x*Log[c*(d + e*x^2)^p] + f^2*g*x^3*Log[c*(d + e*x^2)^p] +
(3*f*g^2*x^5*Log[c*(d + e*x^2)^p])/5 + (g^3*x^7*Log[c*(d + e*x^2)^p])/7
```


Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^2\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^3\log\left(c\left(d+ex^2\right)^p\right)+3f^2gx^2\log\left(c\left(d+ex^2\right)^p\right)+3fg^2x^4\log\left(c\left(d+ex^2\right)^p\right)\right.$$

$$= f^3\int \log\left(c\left(d+ex^2\right)^p\right)dx+\left(3f^2g\right)\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx+\left(3fg^2\right)\int x^4 \mathrm{l}$$

$$= f^3x\log\left(c\left(d+ex^2\right)^p\right)+f^2gx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5\log\left(c\left(d+ex^2\right)^p\right)+$$

$$= -2f^3px+f^3x\log\left(c\left(d+ex^2\right)^p\right)+f^2gx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5\log\left(c\left(d+\right.\right.$$

$$= -2f^3px+\frac{2df^2gpx}{e}-\frac{6d^2fg^2px}{5e^2}+\frac{2d^3g^3px}{7e^3}-\frac{2}{3}f^2gpx^3+\frac{2dfg^2px^3}{5e}-\frac{2d^2g^3px}{21e^2}$$

$$= -2f^3px+\frac{2df^2gpx}{e}-\frac{6d^2fg^2px}{5e^2}+\frac{2d^3g^3px}{7e^3}-\frac{2}{3}f^2gpx^3+\frac{2dfg^2px^3}{5e}-\frac{2d^2g^3px}{21e^2}$$

Mathematica [A] time = 0.273498, size = 215, normalized size = 0.64

$$\frac{1}{35}x\left(35f^2gx^2+35f^3+21fg^2x^4+5g^3x^6\right)\log\left(c\left(d+ex^2\right)^p\right)-\frac{2px\left(35d^2eg^2\left(63f+5gx^2\right)-525d^3g^3-105de^2g\left(35f^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*p*x*(-525*d^3*g^3 + 35*d^2*e*g^2*(63*f + 5*g*x^2) - 105*d*e^2*g*(35*f^2
 + 7*f*g*x^2 + g^2*x^4) + e^3*(3675*f^3 + 1225*f^2*g*x^2 + 441*f*g^2*x^4 +
75*g^3*x^6)))/(3675*e^3) - (2*Sqrt[d]*(-35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^
2*e*f*g^2 + 5*d^3*g^3)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(35*e^(7/2)) + (x*(35
*f^3 + 35*f^2*g*x^2 + 21*f*g^2*x^4 + 5*g^3*x^6)*Log[c*(d + e*x^2)^p])/35
```

Maple [C] time = 0.546, size = 995, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^3*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -2*f^3*p*x+1/7*ln(c)*g^3*x^7+ln(c)*f^3*x-1/7/e^4*(-d*e)^(1/2)*p*ln((-d*e)^(
1/2)*x-d)*g^3*d^3+1/7/e^4*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*g^3*d^3-3/10
*I*Pi*f*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+(1/7*g^
3*x^7+3/5*f*g^2*x^5+f^2*g*x^3+f^3*x)*ln((e*x^2+d)^p)+2/7*d^3*g^3*p*x/e^3-2/
21*d^2*g^3*p*x^3/e^2+2/35*d*g^3*p*x^5/e-3/5/e^3*(-d*e)^(1/2)*p*ln(-(-d*e)^(
1/2)*x-d)*f*g^2*d^2+1/e^2*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*f^2*g*d+3/5/
e^3*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*f*g^2*d^2-1/e^2*(-d*e)^(1/2)*p*ln((
-d*e)^(1/2)*x-d)*f^2*g*d+3/10*I*Pi*f*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2+3/10*I*Pi*f*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c)*f
^2*g*x^3+3/5*ln(c)*f*g^2*x^5+1/2*I*Pi*f^2*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2+1/2*I*Pi*f^2*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/14*
I*Pi*g^3*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*f
^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x-2/3*f^2*g*p*x^3-6/
25*f*g^2*p*x^5-2/49*g^3*p*x^7-1/e*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*f^3+
1/e*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*f^3-1/14*I*Pi*g^3*x^7*csgn(I*c*(e*x
^2+d)^p)^3-1/2*I*Pi*f^3*csgn(I*c*(e*x^2+d)^p)^3*x-6/5*d^2*f*g^2*p*x/e^2+2/5
*d*f*g^2*p*x^3/e+2*d*f^2*g*p*x/e-1/2*I*Pi*f^2*g*x^3*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)-3/10*I*Pi*f*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^3-1/
2*I*Pi*f^2*g*x^3*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f^3*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)^2*x+1/14*I*Pi*g^3*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)^2+1/2*I*Pi*f^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x+1/14*I*Pi*g^3*
x^7*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10436, size = 1335, normalized size = 3.95

$$\left[-\frac{150\,e^3g^3px^7 + 42\left(21\,e^3fg^2 - 5\,de^2g^3\right)px^5 + 70\left(35\,e^3f^2g - 21\,de^2fg^2 + 5\,d^2eg^3\right)px^3 + 105\left(35\,e^3f^3 - 35\,de^2f^2g + 21\,d^2 \cdots\right.}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/3675*(150*e^3*g^3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(3
5*e^3*f^2*g - 21*d*e^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 + 105*(35*e^3*f^3 - 35*d*
```

```
e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqr
t(-d/e) - d)/(e*x^2 + d)) + 210*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e*f*g
^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g^3*p*x^7 + 21*e^3*f*g^2*p*x^5 + 35*e^3*f^
2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e*x^2 + d) - 105*(5*e^3*g^3*x^7 + 21*e^3*f*
g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*e^3*f^3*x)*log(c))/e^3, -1/3675*(150*e^3*g^
3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(35*e^3*f^2*g - 21*d*e
^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 - 210*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e
*f*g^2 - 5*d^3*g^3)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 210*(35*e^3*f^3 -
 35*d*e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g^3*p*x^7 +
21*e^3*f*g^2*p*x^5 + 35*e^3*f^2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e*x^2 + d) -
105*(5*e^3*g^3*x^7 + 21*e^3*f*g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*e^3*f^3*x)*lo
g(c))/e^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**3*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.78728, size = 417, normalized size = 1.23

$$-\frac{2\left(5\,d^4g^3p-21\,d^3fg^2pe+35\,d^2f^2gpe^2-35\,df^3pe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{35\,\sqrt{d}}+\frac{1}{3675}\left(525\,g^3px^7e^3\log\left(x^2e+d\right)-150\,g^3p\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/35*(5*d^4*g^3*p - 21*d^3*f*g^2*p*e + 35*d^2*f^2*g*p*e^2 - 35*d*f^3*p*e^3
)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/3675*(525*g^3*p*x^7*e^3*lo
g(x^2*e + d) - 150*g^3*p*x^7*e^3 + 525*g^3*x^7*e^3*log(c) + 210*d*g^3*p*x^5
*e^2 + 2205*f*g^2*p*x^5*e^3*log(x^2*e + d) - 882*f*g^2*p*x^5*e^3 - 350*d^2*
g^3*p*x^3*e + 2205*f*g^2*x^5*e^3*log(c) + 1470*d*f*g^2*p*x^3*e^2 + 3675*f^2
*g*p*x^3*e^3*log(x^2*e + d) + 1050*d^3*g^3*p*x - 2450*f^2*g*p*x^3*e^3 - 441
0*d^2*f*g^2*p*x*e + 3675*f^2*g*x^3*e^3*log(c) + 7350*d*f^2*g*p*x*e^2 + 3675
*f^3*p*x*e^3*log(x^2*e + d) - 7350*f^3*p*x*e^3 + 3675*f^3*x*e^3*log(c))*e^(
-3)
```

3.269 $\int \left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=221

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{2d^2g^2px}{5e^2}+\frac{2d^{5/2}g}{}$$

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rubi [A] time = 0.170613, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{2d^2g^2px}{5e^2}+\frac{2d^{5/2}g}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log\left(c\left(d+ex^2\right)^p\right)\right) d$$

$$= f^2\int \log\left(c\left(d+ex^2\right)^p\right) dx + (2fg)\int x^2\log\left(c\left(d+ex^2\right)^p\right) dx + g^2\int x^4\log\left(c\left(\right.\right.$$

$$= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-($$

$$= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+\right.\right.$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}f^2p\tan^{-}}{\sqrt{e}}$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}f^2p\tan^{-}}{\sqrt{e}}$$

Mathematica [A] time = 0.11959, size = 151, normalized size = 0.68

$$\frac{\sqrt{e}x\left(15e^2\left(15f^2+10fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)-2p\left(45d^2g^2-15deg\left(10f+gx^2\right)+e^2\left(225f^2+50fgx^2+9g^2\right.\right.\right.}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2
 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[
c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Maple [C] time = 0.533, size = 686, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -1/5/e^3*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x+d)*g^2*d^2-2/5*d^2*g^2*p*x/e^2+2/
15*d*g^2*p*x^3/e+1/5*ln(c)*g^2*x^5+ln(c)*f^2*x-4/9*f*g*p*x^3+2/3*ln(c)*f*g*
x^3-2*f^2*p*x-2/25*g^2*p*x^5+4/3*d*f*g*p*x/e+(1/5*g^2*x^5+2/3*f*g*x^3+f^2*x
)*ln((e*x^2+d)^p)-1/3*I*Pi*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)-1/e*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x+d)*f^2+1/e*(-d*e)^(1/2)*p*
ln(-(-d*e)^(1/2)*x+d)*f^2-1/10*I*Pi*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*P
i*f^2*csgn(I*c*(e*x^2+d)^p)^3*x+1/5/e^3*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d
)*g^2*d^2+1/10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/1
0*I*Pi*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/3*I*Pi*f*g*x^3*csgn(I*c*
(e*x^2+d)^p)^3+1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1
/2*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x+2/3/e^2*(-d*e)^(1/2)*p*ln((
-d*e)^(1/2)*x+d)*f*g*d-2/3/e^2*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d)*f*g*d-1
/10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/3*I*
Pi*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/3*I*Pi*f*g*x^3*csg
n(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09358, size = 922, normalized size = 4.17

$$\left[-\frac{18\,e^2g^2px^5+10\left(10\,e^2fg-3\,deg^2\right)px^3-15\left(15\,e^2f^2-10\,defg+3\,d^2g^2\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+30\left(15\,e^2f^2-10\,de\right.}{225}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2*
f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) -
d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*
g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g
^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^2*p*x
^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^
2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e*f*g +
3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*l
og(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/
e^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32844, size = 271, normalized size = 1.23

$$\frac{2\left(3\,d^3g^2p-10\,d^2fgpe+15\,df^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}}+\frac{1}{225}\left(45\,g^2px^5e^2\log\left(x^2e+d\right)-18\,g^2px^5e^2+45\,g^2x^5e^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g^2*p - 10*d^2*f*g*p*e + 15*d*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/sqrt(d) + 1/225*(45*g^2*p*x^5*e^2*log(x^2*e + d) - 18*g^2*p*x^
5*e^2 + 45*g^2*x^5*e^2*log(c) + 30*d*g^2*p*x^3*e + 150*f*g*p*x^3*e^2*log(x^
2*e + d) - 100*f*g*p*x^3*e^2 + 150*f*g*x^3*e^2*log(c) - 90*d^2*g^2*p*x + 30
0*d*f*g*p*x*e + 225*f^2*p*x*e^2*log(x^2*e + d) - 450*f^2*p*x*e^2 + 225*f^2*
x*e^2*log(c))*e^(-2)
```

3.270 $\int \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=117

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rubi [A] time = 0.0869666, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```


```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```


Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)dx &= \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^2\log\left(c\left(d+ex^2\right)^p\right)\right)dx\\ &= f\int \log\left(c\left(d+ex^2\right)^p\right)dx+g\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx\\ &= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-(2efp)\int \frac{x^2}{d+ex^2}dx-\frac{1}{3}(2egp)\\ &= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int \frac{1}{d+ex^2}dx\\ &= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\,\mathrm{l}\\ &= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\end{aligned}$$

Mathematica [A] time = 0.0373759, size = 117, normalized size = 1.

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Maple [C] time = 0.519, size = 416, normalized size = 3.6

$$\left(\frac{gx^3}{3}+fx\right)\ln\left(\left(ex^2+d\right)^p\right)+\frac{i}{2}\pi f\left(\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2\mathrm{csgn}\left(ic\right)x+\frac{i}{2}\pi f\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\left(\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/3*g*x^3+f*x)*ln((e*x^2+d)^p)+1/2*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c
)*x+1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1/6*I*Pi*g*x^3
```

```
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x+1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*c
sgn(I*c)-1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*Pi*f*csgn(I*c*(e*x^2+
d)^p)^3*x-1/6*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c
)+1/3*ln(c)*g*x^3-2/9*g*p*x^3+ln(c)*f*x-1/3/e^2*(-d*e)^(1/2)*p*ln((-d*e)^(1
/2)*x-d)*d*g+1/e*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*f+1/3/e^2*(-d*e)^(1/2)
*p*ln(-(-d*e)^(1/2)*x-d)*d*g-1/e*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*f+2/3
*d*g*p*x/e-2*f*p*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07543, size = 504, normalized size = 4.31

$$\left[-\frac{2\,egpx^3+3\left(3\,ef-dg\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+6\left(3\,ef-dg\right)px-3\left(egpx^3+3\,efpx\right)\log\left(ex^2+d\right)-3\left(egx^3+3\,ef\right.}{9\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-
d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*lo
g(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3*e*
f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*(e*g
*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

Sympy [A] time = 40.4016, size = 228, normalized size = 1.95

$$\begin{cases}-\frac{id^{\frac{3}{2}}gp\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{2id^{\frac{3}{2}}gp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{i\sqrt{d}fp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}fp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}}+\frac{2dgpx}{3e}+fpx\log\left(d+ex^2\right)-2fpx+fx \\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-I*d**(3/2)*g*p*log(d + e*x**2)/(3*e**2*sqrt(1/e)) + 2*I*d**(3/2
)*g*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*e**2*sqrt(1/e)) + I*sqrt(d)*f*p*log(
d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*f*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(
e*sqrt(1/e)) + 2*d*g*p*x/(3*e) + f*p*x*log(d + e*x**2) - 2*f*p*x + f*x*log(
```

```
c) + g*p*x**3*log(d + e*x**2)/3 - 2*g*p*x**3/9 + g*x**3*log(c)/3, Ne(e, 0))
, ((f*x + g*x**3/3)*log(c*d**p), True))
```

Giac [A] time = 1.32496, size = 147, normalized size = 1.26

$$-\frac{2\left(d^2gp-3\,dfpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}}+\frac{1}{9}\left(3\,gpx^3e\log\left(x^2e+d\right)-2\,gpx^3e+3\,gx^3e\log\left(c\right)+9\,fpxe\log\left(x^2e+d\right)+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g*p - 3*d*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) + 1/9
*(3*g*p*x^3*e*log(x^2*e + d) - 2*g*p*x^3*e + 3*g*x^3*e*log(c) + 9*f*p*x*e*l
og(x^2*e + d) + 6*d*g*p*x - 18*f*p*x*e + 9*f*x*e*log(c))*e^(-1)
```

3.271 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rubi [A] time = 0.458588, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```


```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```


Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+ex^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{}$$

Mathematica [A] time = 0.252517, size = 564, normalized size = 1.06

$$-\frac{i\left(p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)+p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)-p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)-p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-I/2)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-
d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] + Sq
rt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/
Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d]
 + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x
)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*Pol
yLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqr
t[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] +
I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt
[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqr
t[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] time = 0.074, size = 504, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root
Of(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*ln(c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.272 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=751

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\ldots}\right.}{2f^{3/2}\sqrt{g}}$$

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rubi [A] time = 1.02394, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2471, 2463, 801, 635, 205, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\ldots}\right.}{2f^{3/2}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
```

```
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx = \int \left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx$$

$$= -\frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}dx}{4f} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}dx}{4f} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-fg-g^2x^2}dx}{2f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}} + \frac{(ep)\int \frac{}{(\sqrt{-f}\ldots}}{}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{(ep)\int \left(\frac{}{(ef-d\ldots}\right.}{}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \ldots$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \ldots$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\ldots}{}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\ldots}{}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\ldots}{}$$

Mathematica [A] time = 3.66791, size = 1236, normalized size = 1.65

$$\frac{1}{2}\left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f^{3/2}\sqrt{g}} + \frac{x\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f\left(gx^2+f\right)} + \frac{1}{2}p\left(\frac{i\left(\frac{\log\left(x-\frac{i\sqrt{d}}{\sqrt{e}}\right)}{i\sqrt{g}x+\sqrt{f}} + \frac{\sqrt{e}(\ldots}{}\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f*(f + g*x^2)) + (ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f^(3/2
)*Sqrt[g]) + (p*((I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x
) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(S
```

```
qrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/(f*Sqrt[g]) + (I*(Log[(I*Sqrt[d])/Sqrt[
e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + L
og[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/(f*Sqrt[g
]) + ((-I)*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] + Sqrt[e]*(I*Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*S
qrt[f] + Sqrt[g]*x]))/(f*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Sqrt[g]*(Sqrt[
f] - I*Sqrt[g]*x)) - (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x
)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/
(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/(f*Sqrt[g]) + 2*(x/(f^2 + f*g*x^2) + A
rcTan[(Sqrt[g]*x)/Sqrt[f]]/(f^(3/2)*Sqrt[g]))*(-Log[((-I)*Sqrt[d])/Sqrt[e]
+ x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt
[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqr
t[f] - Sqrt[d]*Sqrt[g]))]))/(f^(3/2)*Sqrt[g]) - (I*(Log[(I*Sqrt[d])/Sqrt[e]
 + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt
[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqr
t[d]*Sqrt[g])]))/(f^(3/2)*Sqrt[g]) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Lo
g[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] +
PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*S
qrt[g]))]))/(f^(3/2)*Sqrt[g]) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyL
og[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
]))/(f^(3/2)*Sqrt[g])))/2)/2
```

Maple [F] time = 1.271, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(ex^2+d\right)^p\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)

[Out] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

3.273 $\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=945

result too large to display

```
[Out] 8*f^2*p^2*x - (64*d*f*g*p^2*x)/(9*e) + (184*d^2*g^2*p^2*x)/(75*e^2) + (16*f
*g*p^2*x^3)/27 - (64*d*g^2*p^2*x^3)/(225*e) + (8*g^2*p^2*x^5)/125 - (8*Sqrt
[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (64*d^(3/2)*f*g*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(9*e^(3/2)) - (184*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(75*e^(5/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]^2)/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)
/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(5/2
) + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x)])/Sqrt[e] - (16*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + (8*d^(5/2)*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(5*e
^(5/2)) - 4*f^2*p*x*Log[c*(d + e*x^2)^p] + (8*d*f*g*p*x*Log[c*(d + e*x^2)^p
])/(3*e) - (4*d^2*g^2*p*x*Log[c*(d + e*x^2)^p])/(5*e^2) - (8*f*g*p*x^3*Log[
c*(d + e*x^2)^p])/9 + (4*d*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(15*e) - (4*g^2*
p*x^5*Log[c*(d + e*x^2)^p])/25 + (4*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d
]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (8*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/(3*e^(3/2)) + (4*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(5*e^(5/2)) + f^2*x*Log[c*(d + e*x^2)^p]^2
 + (2*f*g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (g^2*x^5*Log[c*(d + e*x^2)^p]^2)/
5 + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x)])/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*PolyLog[2, 1 - (2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(5/2)
```

Rubi [A] time = 1.25121, antiderivative size = 945, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2457, 2455, 302}

$$\frac{8}{125}g^2p^2x^5+\frac{1}{5}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^5-\frac{4}{25}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^5-\frac{64dg^2p^2x^3}{225e}+\frac{16}{27}fgp^2x^3+\frac{2}{3}fg\log^2\left(c\left(ex^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```


```
[Out] 8*f^2*p^2*x - (64*d*f*g*p^2*x)/(9*e) + (184*d^2*g^2*p^2*x)/(75*e^2) + (16*f
*g*p^2*x^3)/27 - (64*d*g^2*p^2*x^3)/(225*e) + (8*g^2*p^2*x^5)/125 - (8*Sqrt
[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (64*d^(3/2)*f*g*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(9*e^(3/2)) - (184*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(75*e^(5/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]^2)/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)
/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(5/2
) + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x)])/Sqrt[e] - (16*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + (8*d^(5/2)*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(5*e
^(5/2)) - 4*f^2*p*x*Log[c*(d + e*x^2)^p] + (8*d*f*g*p*x*Log[c*(d + e*x^2)^p
])/(3*e) - (4*d^2*g^2*p*x*Log[c*(d + e*x^2)^p])/(5*e^2) - (8*f*g*p*x^3*Log[
```

```
c*(d + e*x^2)^p])/9 + (4*d*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(15*e) - (4*g^2*
p*x^5*Log[c*(d + e*x^2)^p])/25 + (4*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d
]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (8*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/(3*e^(3/2)) + (4*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(5*e^(5/2)) + f^2*x*Log[c*(d + e*x^2)^p]^2
 + (2*f*g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (g^2*x^5*Log[c*(d + e*x^2)^p]^2)/
5 + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x)])/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*PolyLog[2, 1 - (2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(5/2)
```

<u>Rule 2471</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

<u>Rule 2450</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

<u>Rule 2476</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

<u>Rule 2448</u>

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

<u>Rule 321</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 205</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 2470</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
```

```
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2\log^2\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log^2\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log^2\left(c\left(d+ex^2\right)^p\right)\right)$$

$$= f^2\int \log^2\left(c\left(d+ex^2\right)^p\right) dx + (2fg)\int x^2\log^2\left(c\left(d+ex^2\right)^p\right) dx + g^2\int x^4\log^2\left(c\right.$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right)-$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right)-$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right)-$$

$$= -4f^2px\log\left(c\left(d+ex^2\right)^p\right)+\frac{8dfgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4d^2g^2px\log\left(c\left(d+ex^2\right)\right.}{5e^2}$$

$$= 8f^2p^2x-\frac{16dfgp^2x}{3e}+\frac{8d^2g^2p^2x}{5e^2}-4f^2px\log\left(c\left(d+ex^2\right)^p\right)+\frac{8dfgpx\log\left(c\left(d+\right.\right.}{3e}$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2x^5-\frac{8\sqrt{}}{}$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2x^5-\frac{8\sqrt{}}{}$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2x^5-\frac{8\sqrt{}}{}$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2x^5-\frac{8\sqrt{}}{}$$

Mathematica [A] time = 0.509005, size = 435, normalized size = 0.46

$$900i\sqrt{d}p^2\left(3d^2g^2-10defg+15e^2f^2\right)\text{PolyLog}\left(2,\frac{\sqrt{e}x+i\sqrt{d}}{\sqrt{e}x-i\sqrt{d}}\right)+\sqrt{e}x\left(-60p\left(45d^2g^2-15deg\left(10f+gx^2\right)+e^2\left(225f^2+50\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]^2 + 60*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2*(225*e^2*f^2 -
 200*d*e*f*g + 69*d^2*g^2)*p + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*L
og[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 15*(15*e^2*f^2 - 10*d*e*f*g + 3*d
^2*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*x*(8*p^2*(1035*d^2*g^2 - 120*d*e*g*
(25*f + g*x^2) + e^2*(3375*f^2 + 250*f*g*x^2 + 27*g^2*x^4)) - 60*p*(45*d^2*
g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2 + 50*f*g*x^2 + 9*g^2*x^4))*Log
```

```
[c*(d + e*x^2)^p] + 225*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*
x^2)^p]^2) + (900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*Poly
Log[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(3375*e^(5/2))
```

Maple [F] time = 0.945, size = 0, normalized size = 0.

$$\int \left(g\,x^{2}+f\right)^{2} \left(\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g^{2} x^{4} + 2\, f g x^{2} + f^{2}\right) \log\left(\left(e x^{2} + d\right)^{p} c\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^2 + f\right)^2 \log \left(\left(ex^2 + d\right)^p c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

3.274 $\int \left(f+gx^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=548

$$-\frac{4id^{3/2}gp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}+\frac{4i\sqrt{d}fp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}-\frac{4d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}}+$$

```
[Out] 8*f*p^2*x - (32*d*g*p^2*x)/(9*e) + (8*g*p^2*x^3)/27 - (8*Sqrt[d]*f*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(9*e^(3/2)) + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(3/2) + (
8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sq
rt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) - 4*f*p*x*Log[c*(d + e*x^2)^p]
 + (4*d*g*p*x*Log[c*(d + e*x^2)^p])/(3*e) - (4*g*p*x^3*Log[c*(d + e*x^2)^p]
)/9 + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt
[e] - (4*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(3*e
^(3/2)) + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (
(4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/e^(3/2)
```

Rubi [A] time = 0.719811, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2457, 2455, 302}

$$-\frac{4id^{3/2}gp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}+\frac{4i\sqrt{d}fp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}-\frac{4d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```



```
[Out] 8*f*p^2*x - (32*d*g*p^2*x)/(9*e) + (8*g*p^2*x^3)/27 - (8*Sqrt[d]*f*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(9*e^(3/2)) + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(3/2) + (
8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sq
rt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) - 4*f*p*x*Log[c*(d + e*x^2)^p]
 + (4*d*g*p*x*Log[c*(d + e*x^2)^p])/(3*e) - (4*g*p*x^3*Log[c*(d + e*x^2)^p]
)/9 + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt
[e] - (4*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(3*e
^(3/2)) + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (
(4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/e^(3/2)
```


Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
```

```
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^2\log^2\left(c\left(d+ex^2\right)^p\right)\right)dx$$
$$= f\int \log^2\left(c\left(d+ex^2\right)^p\right)dx + g\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)dx$$
$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4efp)\int \frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}$$
$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4efp)\int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e}-\right.$$
$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4fp)\int \log\left(c\left(d+ex^2\right)^p\right)dx+$$
$$= -4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4}{9}gpx^3\log\left(c\left(d+ex^2\right)^p\right)+$$
$$= 8fp^2x-\frac{8dgp^2x}{3e}-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4}{9}gpx^3\log$$
$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{8d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+$$
$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}+$$
$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}+$$
$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}+$$

<u>Mathematica</u> [A] time = 0.264454, size = 281, normalized size = 0.51

$$-36i\sqrt{d}p^2(dg-3ef)\text{PolyLog}\left(2,\frac{\sqrt{e}x+i\sqrt{d}}{\sqrt{e}x-i\sqrt{d}}\right)+\sqrt{e}x\left(9e\left(3f+gx^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)-12p\left(-3dg+9ef+egx^2\right)\log\left(c\left(d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 - 12*Sqrt
[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(9*e*f - 4*d*g)*p + 6*(-3*e*f + d*g)*p
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + (-9*e*f + 3*d*g)*Log[c*(d + e*x
^2)^p]) + Sqrt[e]*x*(8*p^2*(27*e*f - 12*d*g + e*g*x^2) - 12*p*(9*e*f - 3*d*
g + e*g*x^2)*Log[c*(d + e*x^2)^p] + 9*e*(3*f + g*x^2)*Log[c*(d + e*x^2)^p]^
2) - (36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/(
(-I)*Sqrt[d] + Sqrt[e]*x)])/(27*e^(3/2))
```

Maple [F] time = 1.481, size = 0, normalized size = 0.

$$\int \left(g x^2+f\right) \left(\ln \left(c \left(e x^2+d\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g x^2+f\right) \log \left(\left(e x^2+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(f+g x^2\right) \log \left(c \left(d+e x^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g x^2+f\right) \log \left(\left(e x^2+d\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

3.275 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Rubi [A] time = 0.0287186, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Mathematica [A] time = 2.72782, size = 0, normalized size = 0.

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Maple [A] time = 6.98, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)^2}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)**2/(f + g*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

3.276 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Rubi [A] time = 0.0262376, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Mathematica [A] time = 8.45054, size = 0, normalized size = 0.

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Maple [A] time = 7.441, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^2/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f)^2, x)
```

3.277 $\int \left(f+gx^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=682

$$-\frac{2dp(dg-3ef)\text{Unintegrable}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{e}+\frac{32id^{3/2}gp^3\text{PolyLog}\left(2,-\frac{\sqrt{d}-i\sqrt{e}x}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}-\frac{24i\sqrt{d}fp^3\text{PolyLog}\left(2,-\frac{\sqrt{d}-i}{\sqrt{d}+i}\right.}{\sqrt{e}}$$

```
[Out] -48*f*p^3*x + (208*d*g*p^3*x)/(9*e) - (16*g*p^3*x^3)/27 + (48*Sqrt[d]*f*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (208*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(9*e^(3/2)) - ((24*I)*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]
]^2)/Sqrt[e] + (((32*I)/3)*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(
3/2) - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/Sqrt[e] + (64*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + 24*f*p^2*x*Log[c*(
d + e*x^2)^p] - (32*d*g*p^2*x*Log[c*(d + e*x^2)^p])/(3*e) + (8*g*p^2*x^3*Lo
g[c*(d + e*x^2)^p])/9 - (24*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c
*(d + e*x^2)^p])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Lo
g[c*(d + e*x^2)^p])/(3*e^(3/2)) - 6*f*p*x*Log[c*(d + e*x^2)^p]^2 + (2*d*g*p
*x*Log[c*(d + e*x^2)^p]^2)/e - (2*g*p*x^3*Log[c*(d + e*x^2)^p]^2)/3 + f*x*L
og[c*(d + e*x^2)^p]^3 + (g*x^3*Log[c*(d + e*x^2)^p]^3)/3 - ((24*I)*Sqrt[d]*
f*p^3*PolyLog[2, -((Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[
e] + (((32*I)/3)*d^(3/2)*g*p^3*PolyLog[2, -((Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d
] + I*Sqrt[e]*x))])/e^(3/2) - (2*d*(-3*e*f + d*g)*p*Unintegrable[Log[c*(d +
 e*x^2)^p]^2/(d + e*x^2), x])/e
```

Rubi [A] time = 1.38604, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(f+gx^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (208*d*g*p^3*x)/(9*e) - (16*g*p^3*x^3)/27 + (48*Sqrt[d]*f*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (208*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(9*e^(3/2)) - ((24*I)*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]
]^2)/Sqrt[e] + (((32*I)/3)*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(
3/2) - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/Sqrt[e] + (64*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + 24*f*p^2*x*Log[c*(
d + e*x^2)^p] - (32*d*g*p^2*x*Log[c*(d + e*x^2)^p])/(3*e) + (8*g*p^2*x^3*Lo
g[c*(d + e*x^2)^p])/9 - (24*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c
*(d + e*x^2)^p])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Lo
g[c*(d + e*x^2)^p])/(3*e^(3/2)) - 6*f*p*x*Log[c*(d + e*x^2)^p]^2 + (2*d*g*p
*x*Log[c*(d + e*x^2)^p]^2)/e - (2*g*p*x^3*Log[c*(d + e*x^2)^p]^2)/3 + f*x*L
og[c*(d + e*x^2)^p]^3 + (g*x^3*Log[c*(d + e*x^2)^p]^3)/3 - ((24*I)*Sqrt[d]*
f*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((32*
I)/3)*d^(3/2)*g*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^
(3/2) + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] - (2*d^2*
g*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/e
```

Rubi steps

$$\int (f+gx^2)\log^3\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^2\log^3\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$= f\int \log^3\left(c\left(d+ex^2\right)^p\right)dx + g\int x^2\log^3\left(c\left(d+ex^2\right)^p\right)dx$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6efp)\int \frac{x^2\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6efp)\int \left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{e}\right.$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6fp)\int \log^2\left(c\left(d+ex^2\right)^p\right)dx$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right)$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right)$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right)$$

$$= 24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{32dgp^2x\log\left(c\left(d+ex^2\right)^p\right)}{3e}+\frac{8}{9}gp^2x^3\log\left(c\left(d+ex^2\right)^p\right)$$

$$= -48fp^3x+\frac{64dgp^3x}{3e}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{32dgp^2x\log\left(c\left(d+ex^2\right)^p\right)}{3e}+\frac{8}{9}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{64d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}$$

Mathematica [A] time = 4.48061, size = 1460, normalized size = 2.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*p^3*x*(-18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2
}, (d + e*x^2)/d] + 18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2
, 2}, (d + e*x^2)/d]*Log[d + e*x^2] - 9*(d + e*x^2)*HypergeometricPFQ[{-1/2
, 1, 1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]^2 + 2*d*Log[d + e*x^2]^3 - 2
*d*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 2*(d + e*x^2)*Sqrt[1 - (d + e
*x^2)/d]*Log[d + e*x^2]^3))/(6*e*Sqrt[1 - (d + e*x^2)/d]) + (2*d*g*p*x*(-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/e + (6*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] - (
2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d +
 e*x^2)^p])^2)/e^(3/2) + 3*f*p*x*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p])^2 + g*p*x^3*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(
d + e*x^2)^p])^2 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p
 - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) + (g*x^3*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^2*(-2*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]))/
3 + 3*f*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^
2 - (4*((-I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d
+ e*x^2]) - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)] + Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] +
 Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e]) + 3*g*p^2*(-(p*Log[d + e
*x^2]) + Log[c*(d + e*x^2)^p])*((x^3*Log[d + e*x^2]^2)/3 - (4*((9*I)*d^(3/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 3*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-8
 + 6*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 3*Log[d + e*x^2]) + Sqrt[e]
*x*(24*d - 2*e*x^2 + (-9*d + 3*e*x^2)*Log[d + e*x^2]) + (9*I)*d^(3/2)*PolyL
og[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(27*e^(3/2))) +
 (f*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt
[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*Hype
rgeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqr
t[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d +
e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2)
+ Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*
x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] -
Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)
/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(
1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)
/d]/2])))/(Sqrt[-d]*e*x)
```

Maple [A] time = 19.954, size = 0, normalized size = 0.

$$\int \left(gx^2+f\right)\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

3.278 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Rubi [A] time = 0.0266382, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Mathematica [A] time = 3.98084, size = 0, normalized size = 0.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Maple [A] time = 3.108, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^3}{gx^2+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)^3}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)**3/(f + g*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

3.279 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Rubi [A] time = 0.0259536, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Mathematica [A] time = 17.1991, size = 0, normalized size = 0.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Maple [A] time = 7.325, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^3}{\left(gx^2+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{g^2x^4+2\,fgx^2+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^3/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f)^2, x)
```

3.280 $\int \frac{(f+gx^2)^2}{\log\left(c(d+ex^2)^p\right)} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi [A] time = 0.0244526, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.471791, size = 0, normalized size = 0.

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Maple [A] time = 0.782, size = 0, normalized size = 0.

$$\int \frac{(gx^2+f)^2}{\ln\left(c\left(ex^2+d\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)^2}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{g^2x^4+2\,fgx^2+f^2}{\log\left(\left(ex^2+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f+gx^2\right)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)^2}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

3.281 $\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Rubi [A] time = 0.0143442, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.29989, size = 0, normalized size = 0.

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Maple [A] time = 0.512, size = 0, normalized size = 0.

$$\int \frac{gx^2+f}{\ln\left(c\left(ex^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^2 + f}{\log\left(\left(ex^2 + d\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{gx^2 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx^2}{\log\left(c\left(d + ex^2\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^2 + f}{\log\left(\left(ex^2 + d\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

3.282 $\int \frac{1}{(f+gx^2)\log\left(c(d+ex^2)^p\right)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Rubi [A] time = 0.0278251, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.547717, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Maple [A] time = 1.013, size = 0, normalized size = 0.

$$\int \frac{1}{(gx^2+f)\ln\left(c\left(ex^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

3.283 $\int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi [A] time = 0.0266515, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 2.55348, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Maple [A] time = 1.075, size = 0, normalized size = 0.

$$\int \frac{1}{(gx^2+f)^2 \ln\left(c\left(ex^2+d\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^2+f\right)^2 \log \left(\left(ex^2+d\right)^p c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(g^2x^4+2\,fgx^2+f^2\right)\log \left(\left(ex^2+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^2+f\right)^2 \log \left(\left(ex^2+d\right)^p c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

3.284 $\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [A] time = 0.0240136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.864631, size = 0, normalized size = 0.

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A] time = 3.822, size = 0, normalized size = 0.

$$\int \frac{(gx^2+f)^2}{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{eg^2x^6 + (2\,efg + dg^2)x^4 + df^2 + (ef^2 + 2\,dfg)x^2}{2\left(epx\log\left(\left(ex^2+d\right)^p\right) + epx\log\left(c\right)\right)} + \int \frac{5\,eg^2x^6 + 3\,(2\,efg + dg^2)x^4 - df^2 + (ef^2 + 2\,dfg)x^2}{2\left(epx^2\log\left(\left(ex^2+d\right)^p\right) + epx^2\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g^2*x^6 + (2*e*f*g + d*g^2)*x^4 + d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(e
*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + integrate(1/2*(5*e*g^2*x^6 + 3*(2
*e*f*g + d*g^2)*x^4 - d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(e*p*x^2*log((e*x^2 +
d)^p) + e*p*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{g^2x^4 + 2\,fgx^2 + f^2}{\log\left(\left(ex^2+d\right)^p c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f + gx^2\right)^2}{\log\left(c\left(d + ex^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2 + f\right)^2}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

3.285 $\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [A] time = 0.0133854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]

[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.625663, size = 0, normalized size = 0.

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]

[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A] time = 3.711, size = 0, normalized size = 0.

$$\int \frac{gx^2+f}{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{egx^4+\left(ef+dg\right)x^2+df}{2\left(epx\log\left(\left(ex^2+d\right)^p\right)+epx\log\left(c\right)\right)}+\int\frac{3\,egx^4+\left(ef+dg\right)x^2-df}{2\left(epx^2\log\left(\left(ex^2+d\right)^p\right)+epx^2\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*x^4 + (e*f + d*g)*x^2 + d*f)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*lo
g(c)) + integrate(1/2*(3*e*g*x^4 + (e*f + d*g)*x^2 - d*f)/(e*p*x^2*log((e*x
^2 + d)^p) + e*p*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{gx^2+f}{\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{gx^2+f}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

3.286 $\int \frac{1}{(f+gx^2)\log^2\left(c(d+ex^2)^p\right)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [A] time = 0.0263288, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 4.56145, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A] time = 4.606, size = 0, normalized size = 0.

$$\int \frac{1}{(gx^2+f)\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)

[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^2+d}{2\left(egpx^3\log(c)+efpx\log(c)+\left(egpx^3+efpx\right)\log\left(\left(ex^2+d\right)^p\right)\right)}-\int\frac{egx^4-}{2\left(eg^2px^6\log(c)+2efgpx^4\log(c)+ef^2px^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")

[Out] -1/2*(e*x^2 + d)/(e*g*p*x^3*log(c) + e*f*p*x*log(c) + (e*g*p*x^3 + e*f*p*x)
*log((e*x^2 + d)^p)) - integrate(1/2*(e*g*x^4 - (e*f - 3*d*g)*x^2 + d*f)/(e
*g^2*p*x^6*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x^2*log(c) + (e*g^2*p*x^
6 + 2*e*f*g*p*x^4 + e*f^2*p*x^2)*log((e*x^2 + d)^p)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")

[Out] integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)

[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")

[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

3.287 $\int \frac{1}{(f+gx^2)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [A] time = 0.0247196, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 7.49544, size = 0, normalized size = 0.

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A] time = 4.234, size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^2+f\right)^2 \left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^2+d}{2\left(eg^2px^5\log(c)+2efgpx^3\log(c)+ef^2px\log(c)+\left(eg^2px^5+2efgpx^3+ef^2px\right)\log\left(\left(ex^2+d\right)^p\right)\right)}-\int\frac{}{2\left(eg^3px^8\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g^2*p*x^5*log(c) + 2*e*f*g*p*x^3*log(c) + e*f^2*p*x*log
(c) + (e*g^2*p*x^5 + 2*e*f*g*p*x^3 + e*f^2*p*x)*log((e*x^2 + d)^p)) - integ
rate(1/2*(3*e*g*x^4 - (e*f - 5*d*g)*x^2 + d*f)/(e*g^3*p*x^8*log(c) + 3*e*f*
g^2*p*x^6*log(c) + 3*e*f^2*g*p*x^4*log(c) + e*f^3*p*x^2*log(c) + (e*g^3*p*x
^8 + 3*e*f*g^2*p*x^6 + 3*e*f^2*g*p*x^4 + e*f^3*p*x^2)*log((e*x^2 + d)^p)),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(g^2x^4+2fgx^2+f^2\right)\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(gx^2+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

3.288 $\int \left(f+gx^3\right)^3 \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=366

$$\frac{3}{4}f^2gx^4\log\left(c\left(d+ex^2\right)^p\right)+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{3d^2f^2gp\log}{4}$$

```
[Out] -2*f^3*p*x + (6*d^3*f*g^2*p*x)/(7*e^3) + (3*d*f^2*g*p*x^2)/(4*e) - (d^4*g^3
*p*x^2)/(10*e^4) - (2*d^2*f*g^2*p*x^3)/(7*e^2) - (3*f^2*g*p*x^4)/8 + (d^3*g
^3*p*x^4)/(20*e^3) + (6*d*f*g^2*p*x^5)/(35*e) - (d^2*g^3*p*x^6)/(30*e^2) -
(6*f*g^2*p*x^7)/49 + (d*g^3*p*x^8)/(40*e) - (g^3*p*x^10)/50 + (2*Sqrt[d]*f^
3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (6*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(7*e^(7/2)) - (3*d^2*f^2*g*p*Log[d + e*x^2])/(4*e^2) + (d^5
*g^3*p*Log[d + e*x^2])/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p] + (3*f^2*g*x^4
*Log[c*(d + e*x^2)^p])/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p])/7 + (g^3*x^10
*Log[c*(d + e*x^2)^p])/10
```

Rubi [A] time = 0.307897, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2471, 2448, 321, 205, 2454, 2395, 43, 2455, 302}

$$\frac{3}{4}f^2gx^4\log\left(c\left(d+ex^2\right)^p\right)+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{3d^2f^2gp\log}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^3*p*x + (6*d^3*f*g^2*p*x)/(7*e^3) + (3*d*f^2*g*p*x^2)/(4*e) - (d^4*g^3
*p*x^2)/(10*e^4) - (2*d^2*f*g^2*p*x^3)/(7*e^2) - (3*f^2*g*p*x^4)/8 + (d^3*g
^3*p*x^4)/(20*e^3) + (6*d*f*g^2*p*x^5)/(35*e) - (d^2*g^3*p*x^6)/(30*e^2) -
(6*f*g^2*p*x^7)/49 + (d*g^3*p*x^8)/(40*e) - (g^3*p*x^10)/50 + (2*Sqrt[d]*f^
3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (6*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(7*e^(7/2)) - (3*d^2*f^2*g*p*Log[d + e*x^2])/(4*e^2) + (d^5
*g^3*p*Log[d + e*x^2])/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p] + (3*f^2*g*x^4
*Log[c*(d + e*x^2)^p])/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p])/7 + (g^3*x^10
*Log[c*(d + e*x^2)^p])/10
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^3\right)^3 \log \left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^3 \log \left(c\left(d+ex^2\right)^p\right) + 3f^2gx^3 \log \left(c\left(d+ex^2\right)^p\right) + 3fg^2x^6 \log \left(c\left(d+ex^2\right)^p\right) +$$

$$= f^3 \int \log \left(c\left(d+ex^2\right)^p\right) dx + \left(3f^2g\right) \int x^3 \log \left(c\left(d+ex^2\right)^p\right) dx + \left(3fg^2\right) \int x^6 \log$$

$$= f^3x \log \left(c\left(d+ex^2\right)^p\right) + \frac{3}{7}fg^2x^7 \log \left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}\left(3f^2g\right) \text{Subst}\left(\int x \log (c(d$$

$$= -2f^3px + f^3x \log \left(c\left(d+ex^2\right)^p\right) + \frac{3}{4}f^2gx^4 \log \left(c\left(d+ex^2\right)^p\right) + \frac{3}{7}fg^2x^7 \log \left(c\left(d+\right.\right.$$

$$= -2f^3px + \frac{6d^3fg^2px}{7e^3} - \frac{2d^2fg^2px^3}{7e^2} + \frac{6dfg^2px^5}{35e} - \frac{6}{49}fg^2px^7 + \frac{2\sqrt{d}f^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right.}{\sqrt{e}}$$

$$= -2f^3px + \frac{6d^3fg^2px}{7e^3} + \frac{3dfg^2gpx^2}{4e} - \frac{d^4g^3px^2}{10e^4} - \frac{2d^2fg^2px^3}{7e^2} - \frac{3}{8}f^2gpx^4 + \frac{d^3g^3px^4}{20e^3} -$$

Mathematica [A] time = 0.2487, size = 258, normalized size = 0.7

$$\frac{210e^5x\left(105f^2gx^3+140f^3+60fg^2x^6+14g^3x^9\right)\log\left(c\left(d+ex^2\right)^p\right)-epx\left(140d^2e^2g^2x^2\left(60f+7gx^3\right)-210d^3eg^2\left(120f+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x*(2940*d^4*g^3*x + 140*d^2*e^2*g^2*x^2*(60*f + 7*g*x^3) - 210*d^3*e
*g^2*(120*f + 7*g*x^3) - 105*d*e^3*g*x*(210*f^2 + 48*f*g*x^3 + 7*g^2*x^6) +
 3*e^4*(19600*f^3 + 3675*f^2*g*x^3 + 1200*f*g^2*x^6 + 196*g^3*x^9))) - 8400
*Sqrt[d]*e^(3/2)*f*(-7*e^3*f^2 + 3*d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] +
 1470*d^2*g*(-15*e^3*f^2 + 2*d^3*g^2)*p*Log[d + e*x^2] + 210*e^5*x*(140*f^3
 + 105*f^2*g*x^3 + 60*f*g^2*x^6 + 14*g^3*x^9)*Log[c*(d + e*x^2)^p])/(29400*
e^5)
```

Maple [C] time = 0.874, size = 1311, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -2*f^3*p*x+ln(c)*f^3*x-3/14*I*Pi*f*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)*csgn(I*c)-3/8*I*Pi*f^2*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)+1/10*ln(c)*g^3*x^10-1/10*d^4*g^3*p*x^2/e^4+1/20*d^3*g^3*p*x
^4/e^3-1/30*d^2*g^3*p*x^6/e^2+1/40*d*g^3*p*x^8/e+3/7*ln(c)*f*g^2*x^7+3/4*ln
(c)*f^2*g*x^4+1/7/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3-(-9*d^7*e^3*f^2*g^4+4
2*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4
*g^2-49*d*e^9*f^6)^(1/2)-1/7/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^
3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*(-9*d^7*e^3*f^2*g^4+42*
d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)-1/50*g^3*p*x^10-1/2*I*Pi*f^3*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x-3/8*f^2*g*p*x^4-6/49*f*g^2*p*x^
7+(1/10*g^3*x^10+3/7*f*g^2*x^7+3/4*f^2*g*x^4+f^3*x)*ln((e*x^2+d)^p)+3/8*I*P
i*f^2*g*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/20*I*Pi*g^3*x^10*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*f^3*csgn(I*c*(e*x^2+d)^p
```

```
)^3*x-3/4/e^2*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^3*f^2*g^4+42*d^4*e^
6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^2*f^2*g-3/4/e^2*p*ln(-3*d^4*e*f*g^2+7*d*
e^4*f^3-(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^2*f
^2*g+1/20*I*Pi*g^3*x^10*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/20*I*
Pi*g^3*x^10*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3/14*I*Pi*f*g^2*x^7*csgn(I*c*
(e*x^2+d)^p)^3+6/7*d^3*f*g^2*p*x/e^3+3/4*d*f^2*g*p*x^2/e-2/7*d^2*f*g^2*p*x^
3/e^2+6/35*d*f*g^2*p*x^5/e+1/2*I*Pi*f^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)^2*x+1/2*I*Pi*f^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x+1/10/e^5*p*ln(-
3*d^4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f
^6)^(1/2)*x)*d^5*g^3+1/10/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3-(-9*d^7*e^3*f
^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^5*g^3-1/20*I*Pi*g^3*x^10
*csgn(I*c*(e*x^2+d)^p)^3+3/14*I*Pi*f*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^2*csgn(I
*c)+3/8*I*Pi*f^2*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3/14*I*P
i*f*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-3/8*I*Pi*f^2*g*x^4*
csgn(I*c*(e*x^2+d)^p)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16653, size = 1611, normalized size = 4.4

$$\left[-\frac{588\,e^{5}g^{3}px^{10}-735\,de^{4}g^{3}px^{8}+3600\,e^{5}fg^{2}px^{7}+980\,d^{2}e^{3}g^{3}px^{6}-5040\,de^{4}fg^{2}px^{5}+8400\,d^{2}e^{3}fg^{2}px^{3}+735\left(15\,e^{5}f\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] [-1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*e^5*f*g^2*p*x^7
+ 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^2*e^3*f*g^2*p*x^3
 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*e^4*f^2*g - 2*d^4*
e*g^3)*p*x^2 + 4200*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt(-d/e)*log((e*x^2 -
 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*
x - 210*(14*e^5*g^3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4 + 140
*e^5*f^3*p*x - 7*(15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) - 210*(14
*e^5*g^3*x^10 + 60*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*x)*log(c
))/e^5, -1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*e^5*f*g^2
*p*x^7 + 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^2*e^3*f*g^
2*p*x^3 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*e^4*f^2*g -
 2*d^4*e*g^3)*p*x^2 - 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt(d/e)*arctan
(e*x*sqrt(d/e)/d) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*x - 210*(14*e^5*g^
3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4 + 140*e^5*f^3*p*x - 7*(
15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) - 210*(14*e^5*g^3*x^10 + 60
*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*x)*log(c))/e^5]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30057, size = 478, normalized size = 1.31

$$\frac{1}{20}\left(2\,d^5g^3p-15\,d^2f^2gpe^3\right)e^{(-5)}\log\left(x^2e+d\right)-\frac{2\left(3\,d^4fg^2p-7\,df^3pe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{7\,\sqrt{d}}+\frac{1}{29400}\left(2940\,g^3px^{10}e^4\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/20*(2*d^5*g^3*p - 15*d^2*f^2*g*p*e^3)*e^(-5)*log(x^2*e + d) - 2/7*(3*d^4*
f*g^2*p - 7*d*f^3*p*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/294
00*(2940*g^3*p*x^10*e^4*log(x^2*e + d) - 588*g^3*p*x^10*e^4 + 2940*g^3*x^10
*e^4*log(c) + 735*d*g^3*p*x^8*e^3 - 980*d^2*g^3*p*x^6*e^2 + 12600*f*g^2*p*x
^7*e^4*log(x^2*e + d) - 3600*f*g^2*p*x^7*e^4 + 1470*d^3*g^3*p*x^4*e + 12600
*f*g^2*x^7*e^4*log(c) + 5040*d*f*g^2*p*x^5*e^3 - 2940*d^4*g^3*p*x^2 - 8400*
d^2*f*g^2*p*x^3*e^2 + 22050*f^2*g*p*x^4*e^4*log(x^2*e + d) - 11025*f^2*g*p*
x^4*e^4 + 25200*d^3*f*g^2*p*x*e + 22050*f^2*g*x^4*e^4*log(c) + 22050*d*f^2*
g*p*x^2*e^3 + 29400*f^3*p*x*e^4*log(x^2*e + d) - 58800*f^3*p*x*e^4 + 29400*
f^3*x*e^4*log(c))*e^(-4)
```

3.289 $\int (f+gx^3)^2 \log \left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=231

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{2d^3}{7}$$

```
[Out] -2*f^2*p*x + (2*d^3*g^2*p*x)/(7*e^3) + (d*f*g*p*x^2)/(2*e) - (2*d^2*g^2*p*x
^3)/(21*e^2) - (f*g*p*x^4)/4 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 +
(2*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(7/2)*g^2*p*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d + e*x^2])/(2*e^2)
 + f^2*x*Log[c*(d + e*x^2)^p] + (f*g*x^4*Log[c*(d + e*x^2)^p])/2 + (g^2*x^7
*Log[c*(d + e*x^2)^p])/7
```

Rubi [A] time = 0.176745, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2471, 2448, 321, 205, 2454, 2395, 43, 2455, 302}

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{2d^3}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (2*d^3*g^2*p*x)/(7*e^3) + (d*f*g*p*x^2)/(2*e) - (2*d^2*g^2*p*x
^3)/(21*e^2) - (f*g*p*x^4)/4 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 +
(2*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(7/2)*g^2*p*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d + e*x^2])/(2*e^2)
 + f^2*x*Log[c*(d + e*x^2)^p] + (f*g*x^4*Log[c*(d + e*x^2)^p])/2 + (g^2*x^7
*Log[c*(d + e*x^2)^p])/7
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2 \log \left(c\left(d+ex^2\right)^p\right)+2fgx^3 \log \left(c\left(d+ex^2\right)^p\right)+g^2x^6 \log \left(c\left(d+ex^2\right)^p\right)\right) dx$$

$$= f^2 \int \log \left(c\left(d+ex^2\right)^p\right) dx+(2fg) \int x^3 \log \left(c\left(d+ex^2\right)^p\right) dx+g^2 \int x^6 \log \left(c\left(d+\right.\right.$$

$$= f^2x \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7 \log \left(c\left(d+ex^2\right)^p\right)+(fg)\,\text{Subst}\left(\int x \log (c(d+ex)^{p}\right.$$

$$= -2f^2px+f^2x \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4 \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7 \log \left(c\left(d+ex^{2}\right.\right.$$

$$= -2f^2px+\frac{2d^3g^2px}{7e^3}-\frac{2d^2g^2px^3}{21e^2}+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7+\frac{2\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+f$$

$$= -2f^2px+\frac{2d^3g^2px}{7e^3}+\frac{dfgpx^2}{2e}-\frac{2d^2g^2px^3}{21e^2}-\frac{1}{4}fgpx^4+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7+\frac{2\sqrt{d}}{}$$

Mathematica [A] time = 0.207592, size = 178, normalized size = 0.77

$$\frac{1}{14}x\left(14f^2+7fgx^3+2g^2x^6\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{px\left(-280d^2eg^2x^2+840d^3g^2+42de^2gx\left(35f+4gx^3\right)-15e^3\left(392f^2\right.\right.}{2940e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (p*x*(840*d^3*g^2 - 280*d^2*e*g^2*x^2 + 42*d*e^2*g*x*(35*f + 4*g*x^3) - 15*
e^3*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6)))/(2940*e^3) - (2*Sqrt[d]*(-7*e^3*f^
2 + d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d
+ e*x^2])/(2*e^2) + (x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[c*(d + e*x^2)^p
])/14
```

Maple [C] time = 0.723, size = 869, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/7*ln(c)*g^2*x^7+2/7*d^3*g^2*p*x/e^3-2/21*d^2*g^2*p*x^3/e^2+2/35*d*g^2*p*x
^5/e+ln(c)*f^2*x-1/14*I*Pi*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^3+(1/7*g^2*x^7+1/2
*f*g*x^4+f^2*x)*ln((e*x^2+d)^p)-2/49*g^2*p*x^7-1/4*f*g*p*x^4-2*f^2*p*x+1/2*
ln(c)*f*g*x^4+1/7/e^4*p*ln(-d^4*g^2+7*d*e^3*f^2-(-d^7*e*g^4+14*d^4*e^4*f^2*
g^2-49*d*e^7*f^4)^(1/2)*x)*(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/
2)-1/7/e^4*p*ln(-d^4*g^2+7*d*e^3*f^2+(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^
7*f^4)^(1/2)*x)*(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)-1/14*I*P
i*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*I*Pi*f*g*
x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/4*I*Pi*f*g*x^4*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)^2-1/4*I*Pi*f*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)*csgn(I*c)-1/2*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^3*x+1/2*d*f*g*p*x^2/e+
1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1/2*I*Pi*f^2*csg
n(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x-1/2/e^2*p*ln(-d^4*g^2+7*d*e^3*f^2-(-d^7*e*
g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*d^2*f*g-1/4*I*Pi*f*g*x^4*csgn
(I*c*(e*x^2+d)^p)^3+1/14*I*Pi*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
)^p)^2+1/14*I*Pi*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2/e^2*p*ln(-d^
4*g^2+7*d*e^3*f^2+(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*d^2
*f*g-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02251, size = 1027, normalized size = 4.45

$$\left[-\frac{120\,e^3g^2px^7-168\,de^2g^2px^5+735\,e^3fgpx^4+280\,d^2eg^2px^3-1470\,de^2fgpx^2+420\left(7\,e^3f^2-d^3g^2\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2\,ex\sqrt{-}}{ex^2+d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 735*e^3*f*g*p*x^4 + 280
*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 + 420*(7*e^3*f^2 - d^3*g^2)*p*sqrt(
-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 840*(7*e^3*f^2 - d^
3*g^2)*p*x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x - 7*d^
2*e*f*g*p)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^3*f^2
*x)*log(c))/e^3, -1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 735*e^3
*f*g*p*x^4 + 280*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 - 840*(7*e^3*f^2 -
d^3*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 840*(7*e^3*f^2 - d^3*g^2)*p*
x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x - 7*d^2*e*f*g*p
)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^3*f^2*x)*log(c
))/e^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22128, size = 304, normalized size = 1.32

$$-\frac{1}{2}\,d^2fgpe^{(-2)}\log\left(x^2e+d\right)-\frac{2\left(d^4g^2p-7\,df^2pe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{7\,\sqrt{d}}+\frac{1}{2940}\left(420\,g^2px^7e^3\log\left(x^2e+d\right)-120\,g^2px^7e^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/2*d^2*f*g*p*e^(-2)*log(x^2*e + d) - 2/7*(d^4*g^2*p - 7*d*f^2*p*e^3)*arct
an(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/2940*(420*g^2*p*x^7*e^3*log(x^2*
e + d) - 120*g^2*p*x^7*e^3 + 420*g^2*x^7*e^3*log(c) + 168*d*g^2*p*x^5*e^2 -
 280*d^2*g^2*p*x^3*e + 1470*f*g*p*x^4*e^3*log(x^2*e + d) - 735*f*g*p*x^4*e^
3 + 1470*f*g*x^4*e^3*log(c) + 840*d^3*g^2*p*x + 1470*d*f*g*p*x^2*e^2 + 2940
*f^2*p*x*e^3*log(x^2*e + d) - 5880*f^2*p*x*e^3 + 2940*f^2*x*e^3*log(c))*e^(
-3)
```

3.290 $\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)dx$

Optimal. Leaf size=110

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4$$

```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Rubi [A] time = 0.0974112, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2471, 2448, 321, 205, 2454, 2395, 43}

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```


```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```


Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx &= \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^3\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^3\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\mathrm{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)-(2efp)\int \frac{x^2}{d+ex^{\ldots}}\ldots\\
&= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int \frac{1}{d+ex^2}\,dx-\frac{1}{4}\ldots\\
&= -2fpx+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{4}\ldots\\
&= -2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+fx\log\left(c\left(d\ldots\right.\right.
\end{aligned}
$$

Mathematica [A] time = 0.0492158, size = 110, normalized size = 1.

$$
fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Maple [C] time = 0.565, size = 402, normalized size = 3.7

$$\left(\frac{gx^4}{4}+fx\right)\ln\left(\left(ex^2+d\right)^p\right)-\frac{i}{2}\pi f\,\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\mathrm{csgn}\left(ic\right)x-\frac{i}{8}\pi\,gx^4\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\mathrm{csg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/4*g*x^4+f*x)*ln((e*x^2+d)^p)-1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)*csgn(I*c)*x-1/8*I*Pi*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)*csgn(I*c)+1/8*I*Pi*g*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/2*I*Pi*f*c
sgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x-1/2*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^3*x-1/8
*I*Pi*g*x^4*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2*x+1/8*I*Pi*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2+1/4*ln(c)*g*x^4-1/8*g*p*x^4+1/4*d*g*p*x^2/e+ln(c)*f*x+1/e*p*ln(-(-d*e)^(1
/2)*x+d)*f*(-d*e)^(1/2)-1/4/e^2*p*ln(-(-d*e)^(1/2)*x+d)*d^2*g-1/e*p*ln((-d*
e)^(1/2)*x+d)*f*(-d*e)^(1/2)-1/4/e^2*p*ln((-d*e)^(1/2)*x+d)*d^2*g-2*f*p*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06046, size = 562, normalized size = 5.11

$$\left[-\frac{e^2gpx^4-2\,degpx^2-8\,e^2fp\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+16\,e^2fpx-2\left(e^2gpx^4+4\,e^2fpx-d^2gp\right)\log\left(ex^2+d\right)-2\left(e^2g\right.}{8\,e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/8*(e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 8*e^2*f*p*sqrt(-d/e)*log((e*x^2 + 2*e*
x*sqrt(-d/e) - d)/(e*x^2 + d)) + 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*p*
x - d^2*g*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2, -1/8*(
e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 16*e^2*f*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d)
+ 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*p*x - d^2*g*p)*log(e*x^2 + d) - 2
*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2]
```

Sympy [A] time = 72.9811, size = 175, normalized size = 1.59

$$\begin{cases}\frac{i\sqrt{d}fp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}fp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}}-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{dgpx^2}{4e}+fpx\log\left(d+ex^2\right)-2fpx+fx\log\left(c\right)+\frac{gpx^4\log\left(d+ex^2\right)}{4}\\ \left(fx+\frac{gx^4}{4}\right)\log\left(cd^p\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((I*sqrt(d)*f*p*log(d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*f*p*lo
g(-I*sqrt(d)*sqrt(1/e) + x)/(e*sqrt(1/e)) - d**2*g*p*log(d + e*x**2)/(4*e**
2) + d*g*p*x**2/(4*e) + f*p*x*log(d + e*x**2) - 2*f*p*x + f*x*log(c) + g*p*
x**4*log(d + e*x**2)/4 - g*p*x**4/8 + g*x**4*log(c)/4, Ne(e, 0)), ((f*x + g
*x**4/4)*log(c*d**p), True))
```

Giac [A] time = 1.32964, size = 158, normalized size = 1.44

$$-\frac{1}{4}\,d^2gpe^{(-2)}\log\left(x^2e+d\right)+2\,\sqrt{d}fp\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+\frac{1}{8}\left(2\,gpx^4e\log\left(x^2e+d\right)-gpx^4e+2\,gx^4e\log\left(c\right)+2\,dgpx^2+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/4*d^2*g*p*e^(-2)*log(x^2*e + d) + 2*sqrt(d)*f*p*arctan(x*e^(1/2)/sqrt(d)
)*e^(-1/2) + 1/8*(2*g*p*x^4*e*log(x^2*e + d) - g*p*x^4*e + 2*g*x^4*e*log(c)
 + 2*d*g*p*x^2 + 8*f*p*x*e*log(x^2*e + d) - 16*f*p*x*e + 8*f*x*e*log(c))*e^
(-1)
```

3.291 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal. Leaf size=1165

result too large to display

```
[Out] -(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3
))]*Log[-f^(1/3) - g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - (p*Log[-((g^(1/3)*(Sqr
t[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3) - g
^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[-(((-1)^(1/3)*g^(1/3)*(S
qrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log
[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[
((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*S
qrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3))
 + ((-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f
^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]
)/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] +
 Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3)
 - (-1)^(2/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) + (Log[-f^(1/3) - g^(1/3)*x]*
Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) + ((-1)^(2/3)*Log[-f^(1/3) + (-1)
^(1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) - ((-1)^(1/3)*L
og[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3
)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) - Sqrt[
-d]*g^(1/3))]])/(3*f^(2/3)*g^(1/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1
/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))]])/(3*f^(2/3)*g^(1/3)) - ((-1)^
(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1
/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))]])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Po
lyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)
^(1/3)*Sqrt[-d]*g^(1/3))]])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (
Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqr
t[-d]*g^(1/3))]])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f
^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1
/3))]])/(3*f^(2/3)*g^(1/3))
```

Rubi [A] time = 1.60361, antiderivative size = 1165, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2471, 2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{\log\left(c\left(ex^2+d\right)^p\right)\log\left(-\sqrt[3]{g}x-\right.}{3f^{2/3}\sqrt[3]{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] -(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3
))]*Log[-f^(1/3) - g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - (p*Log[-((g^(1/3)*(Sqr
t[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3) - g
^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[-(((-1)^(1/3)*g^(1/3)*(S
qrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log
[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[
((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*S
qrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3))
 + ((-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f
```

```
^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]
)/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] +
 Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3)
 - (-1)^(2/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) + (Log[-f^(1/3) - g^(1/3)*x]*
Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) + ((-1)^(2/3)*Log[-f^(1/3) + (-1)
^(1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) - ((-1)^(1/3)*L
og[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3
)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) - Sqrt[
-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1
/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1)^
(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1
/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Po
lyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)
^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (
Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqr
t[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f
^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1
/3))])/(3*f^(2/3)*g^(1/3))
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx = \int\left(-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}\right)dx$$

$$= -\frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}-\sqrt[3]{g}x}\,dx}{3f^{2/3}}-\frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}\,dx}{3f^{2/3}}-\frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x}\,dx}{3f^{2/3}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}- \cdots$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}- \cdots$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}- \cdots$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{(-1)^{2}\cdots}{\cdots}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{(-1)^{2}\cdots}{\cdots}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{(-1)^{2}\cdots}{\cdots}$$

Mathematica [A] time = 0.812465, size = 990, normalized size = 0.85

$$\frac{-p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)-p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{-d}\sqrt[3]{g}-\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)+\log\left(c\left(ex^2+d\right)^p\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] (-(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/
3))]*Log[-f^(1/3) - g^(1/3)*x]) - p*Log[(g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-
(Sqrt[e]*f^(1/3)) + Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - g^(1/3)*x] - (-1)^(2/
3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] - (-1)^(
```

```
2/3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (
-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] + (-1)^(1
/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-
1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + (-1)^(1/
3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + Log[-f
^(1/3) - g^(1/3)*x]*Log[c*(d + e*x^2)^p] + (-1)^(2/3)*Log[-f^(1/3) + (-1)^(
1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - (-1)^(1/3)*Log[-f^(1/3) - (-1)^(2/3)
*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)
*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3))] - p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog
[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3
)*Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3
)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3)
- (-1)^(2/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))])
/(3*f^(2/3)*g^(1/3))
```

Maple [C] time = 0.609, size = 1180, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f),x)
```

```
[Out] 1/3*(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*(ln
((e*x^2+d)^p)-p*ln(e*x^2+d))/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3)
)+1/3*(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(f/g)^(1/3)*x-1))+1/3*p/g*sum(1/_alpha^2*(ln(x-_alpha)*ln(e*x^2+d)-l
n(x-_alpha)*(ln((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1)-x+_alpha
)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))+ln((RootOf(_Z^2*e+2*_Z
*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alph
a^2*e+d,index=2)))-dilog((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1)
-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))-dilog((RootOf
(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_a
lpha*e+_alpha^2*e+d,index=2))),_alpha=RootOf(_Z^3*g+f))-1/6*I*Pi*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1
/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g/(f/g)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1))-1/12*I*Pi*csgn(I*(e*x^2+d)^p
)*csgn(I*c*(e*x^2+d)^p)^2/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))+1
/12*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(
2/3))-1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(f/g)^(1/3)*x-1))+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)^2/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))+1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I
*c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/12*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))+1/12*I*Pi*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+
(f/g)^(2/3))-1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3
))+1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g/(f/g)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1))+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)^2/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1))+1/
3*ln(c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*ln(c)/g/(f/g)^(2/3)*ln(x^2-(f/g
)^(1/3)*x+(f/g)^(2/3))+1/3*ln(c)/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(f/g)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^3+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^3+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

3.292 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal. Leaf size=1861

result too large to display

```
[Out] (2*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(e*f^(2/3) + d
*g^(2/3))) + (2*(-1)^(2/3)*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
(1 + (-1)^(1/3))^4*f^(4/3)*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))) + (4*Sqrt[d]
*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(2*e*f^(2/3) - (1 + I*Sq
rt[3])*d*g^(2/3))) - (2*e*p*Log[f^(1/3) + g^(1/3)*x])/(9*f*(e*f^(2/3) + d*g
^(2/3))*g^(1/3)) - (2*p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/
3) + Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) - (2*
p*Log[-((g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3
)))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*e*p*Log[
f^(1/3) - (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2
/3)*d*g^(2/3))*g^(1/3)) + ((2*I)*Sqrt[3]*p*Log[-(((-1)^(1/3)*g^(1/3)*(Sqrt[
-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^
(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + ((2*I
)*Sqrt[3]*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3
) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1
 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (4*(-1)^(1/3)*e*p*Log[f^(1/3) + (-1)^(2
/3)*g^(1/3)*x])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*g^(1/3)) - (
2*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)
^(2/3)*Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + (-1)^(2/3)*g^(1/3)*x])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3)) - (2*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[
e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[f^(1/3) + (-1)
^(2/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f^(5/3)*g^(1/3)) + (e*p*Log[d + e*x^
2])/(9*f*(e*f^(2/3) + d*g^(2/3))*g^(1/3)) - ((-1)^(1/3)*e*p*Log[d + e*x^2])
/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))*g^(1/3)) - (2*(-1
)^(1/3)*e*p*Log[d + e*x^2])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*
g^(1/3)) - Log[c*(d + e*x^2)^p]/(9*f^(4/3)*g^(1/3)*(f^(1/3) + g^(1/3)*x)) -
 Log[c*(d + e*x^2)^p]/((1 + (-1)^(1/3))^4*f^(4/3)*g^(1/3)*((-1)^(2/3)*f^(1/
3) + g^(1/3)*x)) + ((-1)^(1/3)*Log[c*(d + e*x^2)^p])/(9*f^(4/3)*g^(1/3)*(f^
(1/3) + (-1)^(2/3)*g^(1/3)*x)) + (2*Log[f^(1/3) + g^(1/3)*x]*Log[c*(d + e*x
^2)^p])/(9*f^(5/3)*g^(1/3)) - ((2*I)*Sqrt[3]*Log[-f^(1/3) + (-1)^(1/3)*g^(1
/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (2*Log[
f^(1/3) + (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^4*f
^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*
f^(1/3) - Sqrt[-d]*g^(1/3))]])/(9*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e
]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))]])/(9*f^(5/3)*
g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/
3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-1)^(1/3))^
5*f^(5/3)*g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(
1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-
1)^(1/3))^5*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-1)
^(1/3))^4*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)
*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3))
```

Rubi [A] time = 2.88701, antiderivative size = 1863, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.5, Rules used = {2471, 2463, 801, 635, 205, 260, 2462, 2416, 2394, 2393, 2391}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

```
[Out] (2*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(e*f^(2/3) + d
*g^(2/3))) + (2*(-1)^(2/3)*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
(1 + (-1)^(1/3))^4*f^(4/3)*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))) + (4*Sqrt[d]
*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(2*e*f^(2/3) + I*(I - Sq
rt[3])*d*g^(2/3))) - (2*e*p*Log[f^(1/3) + g^(1/3)*x])/(9*f*(e*f^(2/3) + d*g
^(2/3))*g^(1/3)) - (2*p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/
3) + Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) - (2*
p*Log[-((g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3
)))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*e*p*Log[
f^(1/3) - (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2
/3)*d*g^(2/3))*g^(1/3)) + ((2*I)*Sqrt[3]*p*Log[-(((-1)^(1/3)*g^(1/3)*(Sqrt[
-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^
(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + ((2*I
)*Sqrt[3]*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3
) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1
 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (4*(-1)^(1/3)*e*p*Log[f^(1/3) + (-1)^(2
/3)*g^(1/3)*x])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*g^(1/3)) - (
2*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)
^(2/3)*Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + (-1)^(2/3)*g^(1/3)*x])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3)) - (2*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[
e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[f^(1/3) + (-1)
^(2/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f^(5/3)*g^(1/3)) + (e*p*Log[d + e*x^
2])/(9*f*(e*f^(2/3) + d*g^(2/3))*g^(1/3)) - ((-1)^(1/3)*e*p*Log[d + e*x^2])
/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))*g^(1/3)) - (2*(-1
)^(1/3)*e*p*Log[d + e*x^2])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*
g^(1/3)) - Log[c*(d + e*x^2)^p]/(9*f^(4/3)*g^(1/3)*(f^(1/3) + g^(1/3)*x)) -
 Log[c*(d + e*x^2)^p]/((1 + (-1)^(1/3))^4*f^(4/3)*g^(1/3)*((-1)^(2/3)*f^(1/
3) + g^(1/3)*x)) + ((-1)^(1/3)*Log[c*(d + e*x^2)^p])/(9*f^(4/3)*g^(1/3)*(f^
(1/3) + (-1)^(2/3)*g^(1/3)*x)) + (2*Log[f^(1/3) + g^(1/3)*x]*Log[c*(d + e*x
^2)^p])/(9*f^(5/3)*g^(1/3)) - ((2*I)*Sqrt[3]*Log[-f^(1/3) + (-1)^(1/3)*g^(1
/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (2*Log[
f^(1/3) + (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^4*f
^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*
f^(1/3) - Sqrt[-d]*g^(1/3))])/(9*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e
]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(9*f^(5/3)*
g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/
3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)^(1/3))^
5*f^(5/3)*g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(
1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/((1 + (-
1)^(1/3))^5*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)
^(1/3))^4*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)
*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3))
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
```

```
0] && LtQ[r, 0]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)^2}+\frac{2\log\left(c\left(d+ex^2\right)^p\right)}{9f^{5/3}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{(-1)^{2/3}\log\left(c\left(d+ex^2\right)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4 f^{4/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)^2}-\ldots$$

$$=\frac{2\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt[3]{f}+\sqrt[3]{g}x}dx}{9f^{5/3}}+\frac{2\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x}dx}{9f^{5/3}}-\frac{\left(2(-1)^{5/6}\sqrt{3}\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}dx}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}}+\frac{\int\frac{\log\left(c\left(d+ex\ldots\right.\right.}{\left(\sqrt[3]{f}+\sqrt[3]{g}\ldots\right.}}{9f^{4/3}\ldots}$$

$$=-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}+\frac{2\ldots}{\ldots}$$

$$=-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}+\frac{2\ldots}{\ldots}$$

$$=-\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}}-\frac{2(-1)^{2/3}ep\log\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\sqrt[3]{g}}+\frac{2\sqrt[3]{-1}ep\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\ldots\right.}{9f\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)\sqrt[3]{g}}\ldots$$

$$=-\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}}-\frac{2p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f^{5/3}\sqrt[3]{g}}-\frac{2p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\ldots}{9f^{5/3}\sqrt[3]{g}\ldots}$$

$$=\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)}-\frac{2ep\log\ldots}{9f\left(ef^{2/3}\ldots\right.}$$

$$=\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)}-\frac{2ep\log\ldots}{9f\left(ef^{2/3}\ldots\right.}$$

Mathematica [A] time = 7.04227, size = 2168, normalized size = 1.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

```
[Out] (x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(3*f*(f + g*x^3)) + (2*Arc
Tan[(-f^(1/3) + 2*g^(1/3)*x)/(Sqrt[3]*f^(1/3))]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p]))/(3*Sqrt[3]*f^(5/3)*g^(1/3)) + (2*Log[f^(1/3) + g^(1/3)*x
]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(9*f^(5/3)*g^(1/3)) - ((-(p
```

```
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*Log[f^(2/3) - f^(1/3)*g^(1/3)*x +
g^(2/3)*x^2])/(9*f^(5/3)*g^(1/3)) + p*(-((-1 + (-1)^(1/3))*(-(Log[((-I)*Sqr
t[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt
[d] - Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(1/3)) - g^(1/3)*x]))/((-1)^(2/3)*Sqr
t[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3))
- ((-1 + (-1)^(1/3))*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3) +
g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(1/
3)) - g^(1/3)*x]))/((-1)^(2/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*(1
 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[((-I)*Sqrt[d])/Sqrt[
e] + x]/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log
[f^(1/3) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/(3*(1 + (-1
)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(
f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[f^(1/3)
+ g^(1/3)*x]))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*(1 + (-1)^(1/3))^
2*f^(4/3)*g^(1/3)) - (Log[((-I)*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3)*f^(1/3) -
 g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[f^(1/3) + (-1)^(2
/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3)))/(3*(1 +
(-1)^(1/3))^2*f^(4/3)*g^(1/3)) - (Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3)*
f^(1/3) - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + Log[f^(1/3)
+ (-1)^(2/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3)))
/(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-Log[((-I)*Sqrt[d])/Sqrt[e] + x
] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2])*((3*f^(2/3)*x)/(f + g*x^
3) - (2*Sqrt[3]*ArcTan[(1 - (2*g^(1/3)*x)/f^(1/3))/Sqrt[3]])/g^(1/3) + (2*L
og[f^(1/3) + g^(1/3)*x])/g^(1/3) - Log[f^(2/3) - f^(1/3)*g^(1/3)*x + g^(2/3
)*x^2]/g^(1/3)))/(9*f^(5/3)) - (2*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[((-1)^(
2/3)*f^(1/3) + g^(1/3)*x)/((-1)^(2/3)*f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[e]
)] + PolyLog[2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f^(
1/3) - Sqrt[d]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(-1
 + (-1)^(1/3))*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[-((-((-1)^(1/3)*f^(1/3)) +
 g^(1/3)*x)/((-1)^(1/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e]))] + PolyLog[
2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) - Sqrt[d
]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Log[
((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(f^(1/3) + g^(1/3)*x)/(f^(1/3) + (I*Sqrt[d]
*g^(1/3))/Sqrt[e])] + PolyLog[2, (I*g^(1/3)*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[
e]*f^(1/3) + I*Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) -
 (2*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[((-1)^(2/3)*f^(1/3) + g^(1/3)*x)/(
(-1)^(2/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*(S
qrt[d] + I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/(3
*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(-1 + (-1)^(1/3))*(Log[((-I)*Sqrt
[d])/Sqrt[e] + x]*Log[(-((-1)^(1/3)*f^(1/3)) + g^(1/3)*x)/(-((-1)^(1/3)*f^(
1/3)) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*(Sqrt[d] + I*Sq
rt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^(1
/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(
f^(1/3) + g^(1/3)*x)/(f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2,
-((g^(1/3)*(I*Sqrt[d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3)))]
))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)))
```

Maple [F] time = 1.281, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(ex^2+d\right)^p\right)}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^6+2\,fgx^3+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f)^2, x)
```

3.293 $\int (f + gx^3)^3 \log^2 \left(c \left(d + ex^2\right)^p\right) dx$

Optimal. Leaf size=1221

result too large to display

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (3*d*f^2*g*p^2*x^2)/e + (d
^4*g^3*p^2*x^2)/e^4 + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) - (288*d*f*g^2*p^2*
x^5)/(1225*e) + (24*f*g^2*p^2*x^7)/343 + (3*f^2*g*p^2*(d + e*x^2)^2)/(8*e^2
) - (d^3*g^3*p^2*(d + e*x^2)^2)/(2*e^5) + (2*d^2*g^3*p^2*(d + e*x^2)^3)/(9*
e^5) - (d*g^3*p^2*(d + e*x^2)^4)/(16*e^5) + (g^3*p^2*(d + e*x^2)^5)/(125*e^
5) - (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2
)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(245*e^(7/2)) + ((4*I)*Sqrt[d]*f^3
*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (((12*I)/7)*d^(7/2)*f*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e
]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (24*d^(7/
2)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[
e]*x)])/(7*e^(7/2)) - (d^5*g^3*p^2*Log[d + e*x^2]^2)/(10*e^5) - 4*f^3*p*x*L
og[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*
d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(7*e^2) + (12*d*f*g^2*p*x^5*Log[c*(d
+ e*x^2)^p])/(35*e) - (12*f*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (3*d*f^2*g
*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (d^4*g^3*p*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/e^5 - (3*f^2*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) +
 (d^3*g^3*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/e^5 - (2*d^2*g^3*p*(d + e*x
^2)^3*Log[c*(d + e*x^2)^p])/(3*e^5) + (d*g^3*p*(d + e*x^2)^4*Log[c*(d + e*x
^2)^p])/(4*e^5) - (g^3*p*(d + e*x^2)^5*Log[c*(d + e*x^2)^p])/(25*e^5) + (4*
Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (
12*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(
7/2)) + (d^5*g^3*p*Log[d + e*x^2]*Log[c*(d + e*x^2)^p])/(5*e^5) + f^3*x*Log
[c*(d + e*x^2)^p]^2 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 + (g^3*x^10*Lo
g[c*(d + e*x^2)^p]^2)/10 - (3*d*f^2*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(
2*e^2) + (3*f^2*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + ((4*I)*Sq
rt[d]*f^3*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e]
- (((12*I)/7)*d^(7/2)*f*g^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqr
t[e]*x)])/e^(7/2)
```

Rubi [A] time = 1.64153, antiderivative size = 1139, normalized size of antiderivative = 0.93, number of steps used = 55, number of rules used = 29, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.208, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2457, 2455, 302, 2398, 2411, 43, 2334, 14, 2301}

$$\frac{1}{10}g^3 \log^2\left(c\left(ex^2+d\right)^p\right)x^{10} + \frac{24}{343}fg^2p^2x^7 + \frac{3}{7}fg^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7 - \frac{12}{49}fg^2p\log\left(c\left(ex^2+d\right)^p\right)x^7 - \frac{288dfg^2p^2x^5}{1225e}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (3*d*f^2*g*p^2*x^2)/e + (d
^4*g^3*p^2*x^2)/e^4 + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) - (288*d*f*g^2*p^2*
x^5)/(1225*e) + (24*f*g^2*p^2*x^7)/343 + (3*f^2*g*p^2*(d + e*x^2)^2)/(8*e^2
) - (d^3*g^3*p^2*(d + e*x^2)^2)/(2*e^5) + (2*d^2*g^3*p^2*(d + e*x^2)^3)/(9*
e^5) - (d*g^3*p^2*(d + e*x^2)^4)/(16*e^5) + (g^3*p^2*(d + e*x^2)^5)/(125*e^
5) - (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2
```

```
)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(245*e^(7/2)) + ((4*I)*Sqrt[d]*f^3
*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (((12*I)/7)*d^(7/2)*f*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e
]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (24*d^(7/
2)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[
e]*x)])/(7*e^(7/2)) - (d^5*g^3*p^2*Log[d + e*x^2]^2)/(10*e^5) - 4*f^3*p*x*L
og[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*
d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(7*e^2) + (12*d*f*g^2*p*x^5*Log[c*(d
+ e*x^2)^p])/(35*e) - (12*f*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (3*d*f^2*g
*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (3*f^2*g*p*(d + e*x^2)^2*Log[c*(
d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c
*(d + e*x^2)^p])/Sqrt[e] - (12*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[c*(d + e*x^2)^p])/(7*e^(7/2)) - (g^3*p*((300*d^4*(d + e*x^2))/e^5 - (30
0*d^3*(d + e*x^2)^2)/e^5 + (200*d^2*(d + e*x^2)^3)/e^5 - (75*d*(d + e*x^2)^
4)/e^5 + (12*(d + e*x^2)^5)/e^5 - (60*d^5*Log[d + e*x^2])/e^5)*Log[c*(d + e
*x^2)^p])/300 + f^3*x*Log[c*(d + e*x^2)^p]^2 + (3*f*g^2*x^7*Log[c*(d + e*x^
2)^p]^2)/7 + (g^3*x^10*Log[c*(d + e*x^2)^p]^2)/10 - (3*d*f^2*g*(d + e*x^2)*
Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (3*f^2*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^
p]^2)/(4*e^2) + ((4*I)*Sqrt[d]*f^3*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x)])/Sqrt[e] - (((12*I)/7)*d^(7/2)*f*g^2*p^2*PolyLog[2, 1 - (2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^3 \log^2\left(c\left(d+ex^2\right)^p\right)+3f^2gx^3 \log^2\left(c\left(d+ex^2\right)^p\right)+3fg^2x^6 \log^2\left(c\left(d+ex\right.\right.\right.$$

$$= f^3 \int \log^2\left(c\left(d+ex^2\right)^p\right) dx + \left(3f^2g\right) \int x^3 \log^2\left(c\left(d+ex^2\right)^p\right) dx + \left(3fg^2\right) \int$$

$$= f^3 x \log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7} f g^2 x^7 \log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}\left(3f^2g\right) \text{Subst}\left(\int x \log\right.$$

$$= f^3 x \log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7} f g^2 x^7 \log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10} g^3 x^{10} \log^2\left(c\left(d+ex\right.\right.$$

$$= f^3 x \log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7} f g^2 x^7 \log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10} g^3 x^{10} \log^2\left(c\left(d+ex\right.\right.$$

$$= -4f^3px \log\left(c\left(d+ex^2\right)^p\right)+\frac{12d^3fg^2px \log\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{4d^2fg^2px^3 \log\left(c\right.}{7e^2}$$

$$= 8f^3p^2x-\frac{24d^3fg^2p^2x}{7e^3}-4f^3px \log\left(c\left(d+ex^2\right)^p\right)+\frac{12d^3fg^2px \log\left(c\left(d+ex^2\right)\right.}{7e^3}$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{568d^2fg^2p^2x^3}{735e^2}-\frac{288dfg^2p^2x^5}{1225e}+\frac{24}{343}$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{d^4g^3p^2x^2}{e^4}+\frac{568d^2fg^2p^2x^3}{735e^2}-\frac{288dfg}{122}$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{d^4g^3p^2x^2}{e^4}+\frac{568d^2fg^2p^2x^3}{735e^2}-\frac{288dfg}{122}$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{d^4g^3p^2x^2}{e^4}+\frac{568d^2fg^2p^2x^3}{735e^2}-\frac{288dfg}{122}$$

Mathematica [A] time = 0.966562, size = 1020, normalized size = 0.84

$$\frac{1}{125} g^3 p^2 x^{10}+\frac{1}{10} g^3 \log^2\left(c\left(ex^2+d\right)^p\right) x^{10}-\frac{1}{25} g^3 p \log\left(c\left(ex^2+d\right)^p\right) x^{10}-\frac{9dg^3p^2x^8}{400e}+\frac{dg^3p \log\left(c\left(ex^2+d\right)^p\right) x^8}{20e}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (9*d*f^2*g*p^2*x^2)/(4*e)
+ (137*d^4*g^3*p^2*x^2)/(300*e^4) + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) + (3*
f^2*g*p^2*x^4)/8 - (77*d^3*g^3*p^2*x^4)/(600*e^3) - (288*d*f*g^2*p^2*x^5)/(
1225*e) + (47*d^2*g^3*p^2*x^6)/(900*e^2) + (24*f*g^2*p^2*x^7)/343 - (9*d*g^
3*p^2*x^8)/(400*e) + (g^3*p^2*x^10)/125 - (((4*I)/7)*Sqrt[d]*f*(-7*e^3*f^2
```

```
+ 3*d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (3*d^2*f^2*g*p^2*
Log[d + e*x^2])/(4*e^2) - (77*d^5*g^3*p^2*Log[d + e*x^2])/(300*e^5) + (3*d^
2*f^2*g*p*Log[c*(d + e*x^2)^p])/(2*e^2) - (d^5*g^3*p*Log[c*(d + e*x^2)^p])/
(5*e^5) - 4*f^3*p*x*Log[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x
^2)^p])/(7*e^3) + (3*d*f^2*g*p*x^2*Log[c*(d + e*x^2)^p])/(2*e) - (d^4*g^3*p
*x^2*Log[c*(d + e*x^2)^p])/(5*e^4) - (4*d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p
])/(7*e^2) - (3*f^2*g*p*x^4*Log[c*(d + e*x^2)^p])/4 + (d^3*g^3*p*x^4*Log[c*
(d + e*x^2)^p])/(10*e^3) + (12*d*f*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
 (d^2*g^3*p*x^6*Log[c*(d + e*x^2)^p])/(15*e^2) - (12*f*g^2*p*x^7*Log[c*(d +
 e*x^2)^p])/49 + (d*g^3*p*x^8*Log[c*(d + e*x^2)^p])/(20*e) - (g^3*p*x^10*Lo
g[c*(d + e*x^2)^p])/25 - (3*d^2*f^2*g*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + (d^
5*g^3*Log[c*(d + e*x^2)^p]^2)/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p]^2 + (3*
f^2*g*x^4*Log[c*(d + e*x^2)^p]^2)/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/
7 + (g^3*x^10*Log[c*(d + e*x^2)^p]^2)/10 - (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]*(490*e^3*f^2*p - 352*d^3*g^2*p - 70*(7*e^3*f^2 - 3*d^3*g^2)*p*Lo
g[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] - 35*(7*e^3*f^2 - 3*d^3*g^2)*Log[c*(
d + e*x^2)^p]))/(245*e^(7/2)) - (((4*I)/7)*Sqrt[d]*f*(-7*e^3*f^2 + 3*d^3*g^
2)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/e^(7
/2)
```

Maple [F] time = 1.704, size = 0, normalized size = 0.

$$\int \left(gx^3+f\right)^3 \left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g^3x^9+3\,fg^2x^6+3\,f^2gx^3+f^3\right)\log\left(\left(ex^2+d\right)^pc\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^3*x^9 + 3*f*g^2*x^6 + 3*f^2*g*x^3 + f^3)*log((e*x^2 + d)^p*c)^2
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3+f\right)^3 \log\left(\left(ex^2+d\right)^p c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^3*log((e*x^2 + d)^p*c)^2, x)
```

3.294 $\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=835

result too large to display

```
[Out] 8*f^2*p^2*x - (1408*d^3*g^2*p^2*x)/(735*e^3) - (2*d*f*g*p^2*x^2)/e + (568*d
^2*g^2*p^2*x^3)/(2205*e^2) - (96*d*g^2*p^2*x^5)/(1225*e) + (8*g^2*p^2*x^7)/
343 + (f*g*p^2*(d + e*x^2)^2)/(4*e^2) - (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
735*e^(7/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e
] - (((4*I)/7)*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*
Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(7*e^(7/2)) - 4*f^2*p*x*Log[c*(d + e*x^2
)^p] + (4*d^3*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*d^2*g^2*p*x^3*Log[
c*(d + e*x^2)^p])/(21*e^2) + (4*d*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
(4*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (2*d*f*g*p*(d + e*x^2)*Log[c*(d + e
*x^2)^p])/e^2 - (f*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(2*e^2) + (4*Sqr
t[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (4*d
^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(7/2))
+ f^2*x*Log[c*(d + e*x^2)^p]^2 + (g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 - (d*f*
g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 + (f*g*(d + e*x^2)^2*Log[c*(d + e
*x^2)^p]^2)/(2*e^2) + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (((4*I)/7)*d^(7/2)*g^2*p^2*PolyLog[2, 1 -
(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Rubi [A] time = 1.07923, antiderivative size = 835, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 23, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.958, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2457, 2455, 302}

$$\frac{8}{343}g^2p^2x^7+\frac{1}{7}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7-\frac{4}{49}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{96dg^2p^2x^5}{1225e}+\frac{4dg^2p\log\left(c\left(ex^2+d\right)^p\right)x^5}{35e}+\frac{568d}{22}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```


```
[Out] 8*f^2*p^2*x - (1408*d^3*g^2*p^2*x)/(735*e^3) - (2*d*f*g*p^2*x^2)/e + (568*d
^2*g^2*p^2*x^3)/(2205*e^2) - (96*d*g^2*p^2*x^5)/(1225*e) + (8*g^2*p^2*x^7)/
343 + (f*g*p^2*(d + e*x^2)^2)/(4*e^2) - (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
735*e^(7/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e
] - (((4*I)/7)*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*
Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(7*e^(7/2)) - 4*f^2*p*x*Log[c*(d + e*x^2
)^p] + (4*d^3*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*d^2*g^2*p*x^3*Log[
c*(d + e*x^2)^p])/(21*e^2) + (4*d*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
(4*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (2*d*f*g*p*(d + e*x^2)*Log[c*(d + e
*x^2)^p])/e^2 - (f*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(2*e^2) + (4*Sqr
t[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (4*d
^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(7/2))
+ f^2*x*Log[c*(d + e*x^2)^p]^2 + (g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 - (d*f*
```

```
g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 + (f*g*(d + e*x^2)^2*Log[c*(d + e
*x^2)^p]^2)/(2*e^2) + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (((4*I)/7)*d^(7/2)*g^2*p^2*PolyLog[2, 1 -
(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2 \log^2\left(c\left(d+ex^2\right)^p\right) + 2fgx^3 \log^2\left(c\left(d+ex^2\right)^p\right) + g^2x^6 \log^2\left(c\left(d+ex^2\right)^p\right)\right)$$

$$= f^2 \int \log^2\left(c\left(d+ex^2\right)^p\right) dx + (2fg) \int x^3 \log^2\left(c\left(d+ex^2\right)^p\right) dx + g^2 \int x^6 \log^2\left(c\right.$$

$$= f^2 x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7} g^2 x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + (fg)\,\mathrm{Subst}\left(\int x \log^2\left(c(d+\right.\right.$$

$$= f^2 x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7} g^2 x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + (fg)\,\mathrm{Subst}\left(\int \left(-\frac{d \log^2\left(c(d\right.}{e}\right.\right.$$

$$= f^2 x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7} g^2 x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{(fg)\,\mathrm{Subst}\left(\int (d+ex) \log^2\left(\right.\right.}{e}$$

$$= -4f^2 px \log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3 g^2 px \log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2 g^2 px^3 \log\left(c\left(d+ex^2\right.\right.}{21e^2}$$

$$= 8f^2p^2x - \frac{8d^3g^2p^2x}{7e^3} - 4f^2 px \log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3 g^2 px \log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2}{}$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{343} g^2p^2x^7 +$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{343} g^2p^2x^7 +$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{343} g^2p^2x^7 +$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{343} g^2p^2x^7 +$$

Mathematica [A] time = 0.561695, size = 475, normalized size = 0.57

$$\frac{-176400i\sqrt{d}p^2\left(d^3g^2-7e^3f^2\right)\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x+i\sqrt{d}}{\sqrt{e}x-i\sqrt{d}}\right)+\sqrt{e}\left(22050\left(e^3x\left(14f^2+7fgx^3+2g^2x^6\right)-7d^2efg\right)\log^2\left(c\left(d+ex\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
^2 - 1680*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(735*e^3*f^2 - 176*d^3*g
^2)*p - 210*(7*e^3*f^2 - d^3*g^2)*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
] - 105*(7*e^3*f^2 - d^3*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*(p^2*x*(-5913
60*d^3*g^2 + 79520*d^2*e*g^2*x^2 - 378*d*e^2*g*x*(1225*f + 64*g*x^3) + 225*
e^3*(10976*f^2 + 343*f*g*x^3 + 32*g^2*x^6)) + 154350*d^2*e*f*g*p^2*Log[d +
e*x^2] - 210*p*(-840*d^3*g^2*x + 70*d^2*e*g*(-21*f + 4*g*x^3) - 42*d*e^2*g*
```

```
x^2*(35*f + 4*g*x^3) + 15*e^3*x*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6))*Log[c*(
d + e*x^2)^p] + 22050*(-7*d^2*e*f*g + e^3*x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6
))*Log[c*(d + e*x^2)^p]^2) - (176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*
PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(308700*e^(
7/2))
```

Maple [F] time = 1.776, size = 0, normalized size = 0.

$$\int \left(g x^3+f\right)^2 \left(\ln\left(c\left(e x^2+d\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g^2 x^6+2 f g x^3+f^2\right) \log\left(\left(e x^2+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3 + f\right)^2 \log \left(\left(ex^2 + d\right)^p c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

3.295 $\int \left(f+gx^3\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=395

$$\frac{4i\sqrt{d}fp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{g\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2}-\frac{dg\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{gp\left(d+ex^2\right.}{}$$

```
[Out] 8*f*p^2*x - (d*g*p^2*x^2)/e + (g*p^2*(d + e*x^2)^2)/(8*e^2) - (8*Sqrt[d]*f*
p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - 4*f*p*x*Log[c*(d + e*x^
2)^p] + (d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (g*p*(d + e*x^2)^2*L
og[c*(d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*L
og[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 - (d*g*(d + e*x^2
)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2
)/(4*e^2) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e]
```

Rubi [A] time = 0.506887, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{4i\sqrt{d}fp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{g\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2}-\frac{dg\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{gp\left(d+ex^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (d*g*p^2*x^2)/e + (g*p^2*(d + e*x^2)^2)/(8*e^2) - (8*Sqrt[d]*f*
p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - 4*f*p*x*Log[c*(d + e*x^
2)^p] + (d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (g*p*(d + e*x^2)^2*L
og[c*(d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*L
og[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 - (d*g*(d + e*x^2
)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2
)/(4*e^2) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
```

```
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(f+gx^3\right)\log^2\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^3\log^2\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$= f\int \log^2\left(c\left(d+ex^2\right)^p\right)dx + g\int x^3\log^2\left(c\left(d+ex^2\right)^p\right)dx$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)dx,x,x^2\right)-(4efp)\int \frac{x^2 \text{l}}{}$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int\left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\log^2\left(c(d+\right.}{e}\right.\right.$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)dx,x,x^2\right)}{2e}-\frac{(dg)\,\text{Subs}}{}$$

$$= -4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}+fx\log^2\left(c\left(d+\right.\right.$$

$$= 8fp^2x-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}+fx\log^2$$

$$= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}$$

$$= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}$$

$$= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}$$

$$= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}$$

Mathematica [A] time = 0.157656, size = 415, normalized size = 1.05

$$\frac{4i\sqrt{d}fp^2\text{PolyLog}\left(2,-\frac{\sqrt{e}x+i\sqrt{d}}{-\sqrt{e}x+i\sqrt{d}}\right)}{\sqrt{e}}-egp\left(\frac{d^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^3p}+\frac{d\left(px^2-\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}\right)}{2e^2}+\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4e}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*
Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]]*Log[((2*I)*Sqrt[d])/(I*Sqrt[d] - Sqrt[e]*x)])/Sqrt
[e] - 4*f*p*x*Log[c*(d + e*x^2)^p] + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^4*Lo
```

```
g[c*(d + e*x^2)^p]^2)/4 - e*g*p*((p*((2*d*x^2)/e^2 - x^4/e - (2*d^2*Log[d +
 e*x^2])/e^3))/8 + (x^4*Log[c*(d + e*x^2)^p])/(4*e) + (d^2*Log[c*(d + e*x^2
)^p]^2)/(4*e^3*p) + (d*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/(2*e
^2)) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, -((I*Sqrt[d] + Sqrt[e]*x)/(I*Sqrt[d]
 - Sqrt[e]*x))])/Sqrt[e]
```

Maple [F] time = 1.532, size = 0, normalized size = 0.

$$\int (g x^3 + f) \left(\ln \left(c \left(e x^2 + d\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g x^3 + f\right) \log \left(\left(e x^2 + d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(f + g x^3\right) \log \left(c \left(d + e x^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3 + f\right) \log \left(\left(ex^2 + d\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

3.296 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Rubi [A] time = 0.0269754, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx=\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Mathematica [A] time = 13.6456, size = 0, normalized size = 0.

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2}{gx^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^3+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^3+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

3.297 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

Rubi [A] time = 0.0255872, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Mathematica [A] time = 22.5065, size = 0, normalized size = 0.

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{g^2x^6+2\,fgx^3+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^2/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f)^2, x)
```

3.298 $\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=1124

result too large to display

```
[Out] -48*f^2*p^3*x + (351136*d^3*g^2*p^3*x)/(25725*e^3) + (6*d*f*g*p^3*x^2)/e -
(55456*d^2*g^2*p^3*x^3)/(77175*e^2) + (5232*d*g^2*p^3*x^5)/(42875*e) - (48*
g^2*p^3*x^7)/2401 - (3*f*g*p^3*(d + e*x^2)^2)/(8*e^2) + (48*Sqrt[d]*f^2*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (351136*d^(7/2)*g^2*p^3*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(25725*e^(7/2)) - ((24*I)*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]
*x)/Sqrt[d]]^2)/Sqrt[e] + (((1408*I)/245)*d^(7/2)*g^2*p^3*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]^2)/e^(7/2) - (48*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log
[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (2816*d^(7/2)*g^2*p^3*ArcT
an[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(245*e^(7
/2)) + 24*f^2*p^2*x*Log[c*(d + e*x^2)^p] - (1408*d^3*g^2*p^2*x*Log[c*(d + e
*x^2)^p])/(245*e^3) + (568*d^2*g^2*p^2*x^3*Log[c*(d + e*x^2)^p])/(735*e^2)
- (288*d*g^2*p^2*x^5*Log[c*(d + e*x^2)^p])/(1225*e) + (24*g^2*p^2*x^7*Log[c
*(d + e*x^2)^p])/343 - (6*d*f*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 +
 (3*f*g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) - (24*Sqrt[d]*f^2*p
^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + (1408*d^(7/2
)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(245*e^(7/2)) -
 6*f^2*p*x*Log[c*(d + e*x^2)^p]^2 + (6*d^3*g^2*p*x*Log[c*(d + e*x^2)^p]^2)/
(7*e^3) - (2*d^2*g^2*p*x^3*Log[c*(d + e*x^2)^p]^2)/(7*e^2) + (6*d*g^2*p*x^5
*Log[c*(d + e*x^2)^p]^2)/(35*e) - (6*g^2*p*x^7*Log[c*(d + e*x^2)^p]^2)/49 +
 (3*d*f*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 - (3*f*g*p*(d + e*x^2)^
2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + f^2*x*Log[c*(d + e*x^2)^p]^3 + (g^2*x^7
*Log[c*(d + e*x^2)^p]^3)/7 - (d*f*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/e^2
 + (f*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^3)/(2*e^2) - ((24*I)*Sqrt[d]*f^2
*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((1408
*I)/245)*d^(7/2)*g^2*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
])/e^(7/2) + 6*d*f^2*p*Unintegrable[Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x]
- (6*d^4*g^2*p*Unintegrable[Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/(7*e^3)
```

Rubi [A] time = 2.64937, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```


```
[Out] -48*f^2*p^3*x + (351136*d^3*g^2*p^3*x)/(25725*e^3) + (6*d*f*g*p^3*x^2)/e -
(55456*d^2*g^2*p^3*x^3)/(77175*e^2) + (5232*d*g^2*p^3*x^5)/(42875*e) - (48*
g^2*p^3*x^7)/2401 - (3*f*g*p^3*(d + e*x^2)^2)/(8*e^2) + (48*Sqrt[d]*f^2*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (351136*d^(7/2)*g^2*p^3*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(25725*e^(7/2)) - ((24*I)*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]
*x)/Sqrt[d]]^2)/Sqrt[e] + (((1408*I)/245)*d^(7/2)*g^2*p^3*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]^2)/e^(7/2) - (48*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log
[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (2816*d^(7/2)*g^2*p^3*ArcT
an[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(245*e^(7
/2)) + 24*f^2*p^2*x*Log[c*(d + e*x^2)^p] - (1408*d^3*g^2*p^2*x*Log[c*(d + e
*x^2)^p])/(245*e^3) + (568*d^2*g^2*p^2*x^3*Log[c*(d + e*x^2)^p])/(735*e^2)
```

```
- (288*d*g^2*p^2*x^5*Log[c*(d + e*x^2)^p])/(1225*e) + (24*g^2*p^2*x^7*Log[c
*(d + e*x^2)^p])/343 - (6*d*f*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 +
 (3*f*g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) - (24*Sqrt[d]*f^2*p
^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + (1408*d^(7/2
)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(245*e^(7/2)) -
 6*f^2*p*x*Log[c*(d + e*x^2)^p]^2 + (6*d^3*g^2*p*x*Log[c*(d + e*x^2)^p]^2)/
(7*e^3) - (2*d^2*g^2*p*x^3*Log[c*(d + e*x^2)^p]^2)/(7*e^2) + (6*d*g^2*p*x^5
*Log[c*(d + e*x^2)^p]^2)/(35*e) - (6*g^2*p*x^7*Log[c*(d + e*x^2)^p]^2)/49 +
 (3*d*f*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 - (3*f*g*p*(d + e*x^2)^
2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + f^2*x*Log[c*(d + e*x^2)^p]^3 + (g^2*x^7
*Log[c*(d + e*x^2)^p]^3)/7 - (d*f*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/e^2
 + (f*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^3)/(2*e^2) - ((24*I)*Sqrt[d]*f^2
*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((1408
*I)/245)*d^(7/2)*g^2*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
])/e^(7/2) + 6*d*f^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] -
(6*d^4*g^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/(7*e^3)
```

Rubi steps

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2\log^3\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log^3\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log^3\left(c\left(d+ex^2\right)^p\right.\right.$$

$$= f^2\int \log^3\left(c\left(d+ex^2\right)^p\right) dx+(2fg)\int x^3\log^3\left(c\left(d+ex^2\right)^p\right) dx+g^2\int x^6\log$$

$$= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)+(fg)\,\text{Subst}\left(\int x\log^3\left(c(\right.\right.$$

$$= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)+(fg)\,\text{Subst}\left(\int\left(-\frac{d\log^3}{}\right.\right.$$

$$= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{(fg)\,\text{Subst}\left(\int(d+ex)\log\right.}{e}$$

$$= -6f^2px\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{2d^2g^2px^3\log^2\left(c\left(\right.\right.}{7e^2}$$

$$= -6f^2px\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{2d^2g^2px^3\log^2\left(c\left(\right.\right.}{7e^2}$$

$$= -6f^2px\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{2d^2g^2px^3\log^2\left(c\left(\right.\right.}{7e^2}$$

$$= \frac{6dfgp^3x^2}{e}-\frac{3fgp^3\left(d+ex^2\right)^2}{8e^2}+24f^2p^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{1408d^3g^2p^2x\log}{245}$$

$$= -48f^2p^3x+\frac{2816d^3g^2p^3x}{245e^3}+\frac{6dfgp^3x^2}{e}-\frac{3fgp^3\left(d+ex^2\right)^2}{8e^2}+24f^2p^2x\log\left(c\left(\right.\right.$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232dg^2p^3x^5}{42875e}-$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232dg^2p^3x^5}{42875e}-$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232dg^2p^3x^5}{42875e}-$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232dg^2p^3x^5}{42875e}-$$

Mathematica [A] time = 9.21294, size = 2539, normalized size = 2.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (f*g*p^3*(d + e*x^2)*(45*d - 3*e*x^2 + (-42*d + 6*e*x^2)*Log[d + e*x^2] + 6
*(3*d - e*x^2)*Log[d + e*x^2]^2 - 4*(d - e*x^2)*Log[d + e*x^2]^3))/(8*e^2)
+ (g^2*p^3*x*(-280*d^3*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + (e
*x^2)/d] - 280*d^2*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 +
(e*x^2)/d] - 112*d^3*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1
+ (e*x^2)/d] - 112*d^2*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2
, 2}, 1 + (e*x^2)/d] + 280*d^3*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2,
 2, 2}, 1 + (e*x^2)/d] + 280*d^2*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}
, {2, 2, 2, 2}, 1 + (e*x^2)/d] - 210*d^3*HypergeometricPFQ[{-1/2, 1, 1, 1,
1}, {2, 2, 2, 2}, 1 + (e*x^2)/d] - 210*d^2*e*x^2*HypergeometricPFQ[{-1/2, 1
, 1, 1, 1}, {2, 2, 2, 2}, 1 + (e*x^2)/d] + 16*d^3*Log[d + e*x^2] + 16*e^3*x
^6*Sqrt[-((e*x^2)/d)]*Log[d + e*x^2] + 280*d^3*HypergeometricPFQ[{-3/2, 1,
1}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + 280*d^2*e*x^2*HypergeometricPFQ
[{-3/2, 1, 1}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] - 280*d^3*Hypergeometr
icPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] - 280*d^2*e
*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d]*Log[d + e
*x^2] + 210*d^3*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d
]*Log[d + e*x^2] + 210*d^2*e*x^2*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2,
2}, 1 + (e*x^2)/d]*Log[d + e*x^2] - 32*d^3*Log[d + e*x^2]^2 + 28*d*e^2*x^4*
Sqrt[-((e*x^2)/d)]*Log[d + e*x^2]^2 - 4*e^3*x^6*Sqrt[-((e*x^2)/d)]*Log[d +
e*x^2]^2 + 140*d^3*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + (e*x^2)/d]*L
og[d + e*x^2]^2 + 140*d^2*e*x^2*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 +
 (e*x^2)/d]*Log[d + e*x^2]^2 - 105*d^3*HypergeometricPFQ[{-1/2, 1, 1}, {2,
2}, 1 + (e*x^2)/d]*Log[d + e*x^2]^2 - 105*d^2*e*x^2*HypergeometricPFQ[{-1/2
, 1, 1}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2]^2 + 10*d^3*Log[d + e*x^2]^3
+ 10*e^3*x^6*Sqrt[-((e*x^2)/d)]*Log[d + e*x^2]^3 + 56*d^2*(d + e*x^2)*Hyper
geometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d]*(3 + 2*Log[d + e*x^
2]) - 56*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, 1 + (e*x^2
)/d]*(1 + 3*Log[d + e*x^2] + Log[d + e*x^2]^2)))/(70*e^3*Sqrt[-((e*x^2)/d)]
) - (3*f*g*p^2*(e*x^2*(-6*d + e*x^2) + (6*d^2 + 4*d*e*x^2 - 2*e^2*x^4)*Log[
d + e*x^2] - 2*(d^2 - e^2*x^4)*Log[d + e*x^2]^2)*(p*Log[d + e*x^2] - Log[c*
(d + e*x^2)^p]))/(4*e^2) + (3*d*f*g*p*x^2*(-(p*Log[d + e*x^2]) + Log[c*(d +
 e*x^2)^p])^2)/(2*e) - (2*d^2*g^2*p*x^3*(-(p*Log[d + e*x^2]) + Log[c*(d + e
*x^2)^p])^2)/(7*e^2) + (6*d*g^2*p*x^5*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x
^2)^p])^2)/(35*e) - (6*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/
Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(7*e^(7/2)) - (3*d
^2*f*g*p*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*
e^2) + (3*p*x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[d + e*x^2]*(-(p*Log[d +
e*x^2]) + Log[c*(d + e*x^2)^p])^2)/14 - (g^2*x^7*(6*p + 7*p*Log[d + e*x^2]
- 7*Log[c*(d + e*x^2)^p])*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/4
9 - (f*g*x^4*(3*p + 2*p*Log[d + e*x^2] - 2*Log[c*(d + e*x^2)^p])*(-(p*Log[d
 + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/4 + (x*(-(p*Log[d + e*x^2]) + Log[c*(
d + e*x^2)^p])^2*(-42*e^3*f^2*p + 6*d^3*g^2*p + 7*e^3*f^2*(-(p*Log[d + e*x^
2]) + Log[c*(d + e*x^2)^p])))/(7*e^3) - (3*f^2*p^2*(p*Log[d + e*x^2] - Log[
c*(d + e*x^2)^p])*((4*I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 4*Sqrt[d]*
ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
] + Log[d + e*x^2]) + Sqrt[e]*x*(8 - 4*Log[d + e*x^2] + Log[d + e*x^2]^2) +
 (4*I)*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x
)]))/Sqrt[e] + 3*g^2*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*((x^7
*Log[d + e*x^2]^2)/7 - (4*((11025*I)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2
+ 105*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-352 + 210*Log[(2*Sqrt[d])/(Sqrt
[d] + I*Sqrt[e]*x)] + 105*Log[d + e*x^2]) + Sqrt[e]*x*(36960*d^3 - 4970*d^2
*e*x^2 + 1512*d*e^2*x^4 - 450*e^3*x^6 - 105*(105*d^3 - 35*d^2*e*x^2 + 21*d*
e^2*x^4 - 15*e^3*x^6)*Log[d + e*x^2]) + (11025*I)*d^(7/2)*PolyLog[2, (I*Sqr
t[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(77175*e^(7/2))) + (f^2*p^3
*(-48*Sqrt[-d^2]*Sqrt[(e*x^2)/(d + e*x^2)]*Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/S
qrt[d + e*x^2]] + Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2
]^2 + Log[d + e*x^2]^3) - 6*Sqrt[-d^2]*Sqrt[(e*x^2)/(d + e*x^2)]*(8*Sqrt[d]
*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] +
```

```
Log[d + e*x^2]*(4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/
(d + e*x^2)] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^
2])) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] - Log
[1 + (e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[-((e*x^2)/d)]*(Log[1 + (e*x^2)/d]^2 -
4*Log[1 + (e*x^2)/d]*Log[(1 + Sqrt[-((e*x^2)/d)])/2] + 2*Log[(1 + Sqrt[-((e
*x^2)/d)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((e*x^2)/d)]/2])))/(Sqrt[-d]*e*x
)
```

Maple [A] time = 105., size = 0, normalized size = 0.

$$\int \left(g x^3+f\right)^2 \left(\ln\left(c\left(e x^2+d\right)^p\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g^2 x^6 + 2\, f g x^3 + f^2\right) \log\left(\left(e x^2+d\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^3, x)
```

3.299 $\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=517

$$6dfp\text{Unintegrable}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)-\frac{24i\sqrt{d}fp^3\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{3gp^2\left(d+ex^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{8e^2}$$

```
[Out] -48*f*p^3*x + (3*d*g*p^3*x^2)/e - (3*g*p^3*(d + e*x^2)^2)/(16*e^2) + (48*Sq
rt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - ((24*I)*Sqrt[d]*f*p^3*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 24*f*p^2*x*Lo
g[c*(d + e*x^2)^p] - (3*d*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 + (3*
g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(8*e^2) - (24*Sqrt[d]*f*p^2*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - 6*f*p*x*Log[c*(d + e
*x^2)^p]^2 + (3*d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) - (3*g*p*
(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(8*e^2) + f*x*Log[c*(d + e*x^2)^p]^3
- (d*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/(2*e^2) + (g*(d + e*x^2)^2*Log[c
*(d + e*x^2)^p]^3)/(4*e^2) - ((24*I)*Sqrt[d]*f*p^3*PolyLog[2, 1 - (2*Sqrt[d
])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 6*d*f*p*Unintegrable[Log[c*(d + e*x^
2)^p]^2/(d + e*x^2), x]
```

Rubi [A] time = 0.767616, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (3*d*g*p^3*x^2)/e - (3*g*p^3*(d + e*x^2)^2)/(16*e^2) + (48*Sq
rt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - ((24*I)*Sqrt[d]*f*p^3*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 24*f*p^2*x*Lo
g[c*(d + e*x^2)^p] - (3*d*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 + (3*
g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(8*e^2) - (24*Sqrt[d]*f*p^2*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - 6*f*p*x*Log[c*(d + e
*x^2)^p]^2 + (3*d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) - (3*g*p*
(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(8*e^2) + f*x*Log[c*(d + e*x^2)^p]^3
- (d*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/(2*e^2) + (g*(d + e*x^2)^2*Log[c
*(d + e*x^2)^p]^3)/(4*e^2) - ((24*I)*Sqrt[d]*f*p^3*PolyLog[2, 1 - (2*Sqrt[d
])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)
^p]^2/(d + e*x^2), x]
```

Rubi steps

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^3\log^3\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$= f\int \log^3\left(c\left(d+ex^2\right)^p\right)dx + g\int x^3\log^3\left(c\left(d+ex^2\right)^p\right)dx$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int x\log^3\left(c(d+ex)^p\right)dx,x,x^2\right)-(6efp)\int \frac{x^2 \text{l}}{}$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int\left(-\frac{d\log^3\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\log^3\left(c(d+\right.}{e}\right.\right.$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int(d+ex)\log^3\left(c(d+ex)^p\right)dx,x,x^2\right)}{2e}-\frac{(dg)\,\text{Subs}}{}$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int x\log^3\left(cx^p\right)dx,x,\right.}{2e^2}$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)-\frac{dg\left(d+ex^2\right)\log^3\left(c\left(d+ex^2\right)^p\right.}{2e^2}$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3dgp\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{3gp\left(d+ex^2\right)^2\text{log}}{8e}$$

$$= \frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{3dgp^2\left(d+ex^2\right)\log\left(c\left(d\right.\right.}{e^2}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{3dgp^2\left(d+e\right.}{}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{d}fp^3\tan^{-1}}{\sqrt{e}}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{d}fp^3\tan^{-1}}{\sqrt{e}}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{d}fp^3\tan^{-1}}{\sqrt{e}}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{d}fp^3\tan^{-1}}{\sqrt{e}}$$

Mathematica [A] time = 2.38113, size = 1066, normalized size = 2.06

$$-\frac{3}{16}gp^3x^4+\frac{1}{4}g\log^3\left(c\left(ex^2+d\right)^p\right)x^4-\frac{3}{8}gp\log^2\left(c\left(ex^2+d\right)^p\right)x^4+\frac{3}{8}gp^2\log\left(c\left(ex^2+d\right)^p\right)x^4+\frac{21dgp^3x^2}{8e}+\frac{3dgp\log^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (21*d*g*p^3*x^2)/(8*e) - (3*g*p^3*x^4)/16 - (3*d^2*g*p^3*Log[d + e*x^2])/(8
*e^2) - (9*d^2*g*p^2*Log[c*(d + e*x^2)^p])/(4*e^2) - (9*d*g*p^2*x^2*Log[c*(
d + e*x^2)^p])/(4*e) + (3*g*p^2*x^4*Log[c*(d + e*x^2)^p])/8 + (9*d^2*g*p*Lo
g[c*(d + e*x^2)^p]^2)/(8*e^2) + (3*d*g*p*x^2*Log[c*(d + e*x^2)^p]^2)/(4*e)
- (3*g*p*x^4*Log[c*(d + e*x^2)^p]^2)/8 - (d^2*g*Log[c*(d + e*x^2)^p]^3)/(4*
e^2) + (g*x^4*Log[c*(d + e*x^2)^p]^3)/4 + (6*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] + 3*f*p*x
*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + f*x*(-(p*L
og[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p - p*Log[d + e*x^2] + Log[c*(
d + e*x^2)^p]) + 3*f*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Lo
g[d + e*x^2]^2 - (4*((-I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x
*(-2 + Log[d + e*x^2]) - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2
*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[2,
 (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e]) + (f*p^3*(-
48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt[d]/Sqrt[d
 + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*Hypergeometric
PFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqrt[d]*Hyper
geometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d + e*x^2] + S
qrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2) + Sqrt[-d]
*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*x^2]^3) +
24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] - Log[(d + e
*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)/d]^2 - 4*
Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(1 + Sqrt[1
 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)/d]/2])))/
(Sqrt[-d]*e*x)
```

Maple [A] time = 64.717, size = 0, normalized size = 0.

$$\int \left(g x^3+f\right) \left(\ln \left(c \left(e x^2+d\right)^p\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g x^3+f\right) \log\left(\left(e x^2+d\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3 + f\right) \log \left(\left(ex^2 + d\right)^p c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

3.300 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Rubi [A] time = 0.0280148, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Mathematica [A] time = 17.1171, size = 0, normalized size = 0.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^3}{gx^3+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^3+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^3+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

3.301 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Rubi [A] time = 0.0259449, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Mathematica [A] time = 37.3044, size = 0, normalized size = 0.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^3}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{g^2x^6+2\,fgx^3+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^3/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f)^2, x)
```

3.302 $\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi [A] time = 0.0244984, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.372037, size = 0, normalized size = 0.

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Maple [A] time = 0.887, size = 0, normalized size = 0.

$$\int \frac{(gx^3+f)^2}{\ln\left(c\left(ex^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^3+f)^2}{\log\left((ex^2+d)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{g^2x^6+2\,fgx^3+f^2}{\log\left((ex^2+d)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^3+f)^2}{\log\left((ex^2+d)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

3.303 $\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Rubi [A] time = 0.0143791, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.321576, size = 0, normalized size = 0.

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Maple [A] time = 0.534, size = 0, normalized size = 0.

$$\int \frac{gx^3+f}{\ln\left(c\left(ex^2+d\right)^p\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{gx^3 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

3.304 $\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Rubi [A] time = 0.0293051, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 2.52548, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Maple [A] time = 1.066, size = 0, normalized size = 0.

$$\int \frac{1}{(gx^3+f)\ln\left(c\left(ex^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^3 + f\right)\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(gx^3 + f\right)\log\left(\left(ex^2 + d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^3 + f\right)\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

3.305 $\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{(f+gx^3)^2 \log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi [A] time = 0.0267897, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(f+gx^3)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^3)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{1}{(f+gx^3)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 8.74833, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx^3)^2 \log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Maple [A] time = 1.182, size = 0, normalized size = 0.

$$\int \frac{1}{(gx^3+f)^2 \ln\left(c\left(ex^2+d\right)^p\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(g^2x^6+2\,fgx^3+f^2\right)\log\left(\left(ex^2+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

3.306 $\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [A] time = 0.0232582, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.701587, size = 0, normalized size = 0.

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A] time = 3.793, size = 0, normalized size = 0.

$$\int \frac{(gx^3+f)^2}{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{eg^2x^8+dg^2x^6+2\,efgx^5+2\,dfgx^3+ef^2x^2+df^2}{2\left(epx\log\left(\left(ex^2+d\right)^p\right)+epx\log\left(c\right)\right)}+\int\frac{7\,eg^2x^8+5\,dg^2x^6+8\,efgx^5+4\,dfgx^3+ef^2x^2-df^2}{2\left(epx^2\log\left(\left(ex^2+d\right)^p\right)+epx^2\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g^2*x^8 + d*g^2*x^6 + 2*e*f*g*x^5 + 2*d*f*g*x^3 + e*f^2*x^2 + d*f^2
)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + integrate(1/2*(7*e*g^2*x^8 +
5*d*g^2*x^6 + 8*e*f*g*x^5 + 4*d*f*g*x^3 + e*f^2*x^2 - d*f^2)/(e*p*x^2*log((
e*x^2 + d)^p) + e*p*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{g^2x^6+2\,fgx^3+f^2}{\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx^3+f\right)^2}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

3.307 $\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [A] time = 0.0141798, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]

[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.531818, size = 0, normalized size = 0.

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]

[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A] time = 3.905, size = 0, normalized size = 0.

$$\int \frac{gx^3+f}{\left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{egx^5 + dgx^3 + efx^2 + df}{2\left(epx\log\left(\left(ex^2+d\right)^p\right) + epx\log\left(c\right)\right)} + \int \frac{4\,egx^5 + 2\,dgx^3 + efx^2 - df}{2\left(epx^2\log\left(\left(ex^2+d\right)^p\right) + epx^2\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*x^5 + d*g*x^3 + e*f*x^2 + d*f)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*
log(c)) + integrate(1/2*(4*e*g*x^5 + 2*d*g*x^3 + e*f*x^2 - d*f)/(e*p*x^2*lo
g((e*x^2 + d)^p) + e*p*x^2*log(c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{gx^3 + f}{\log\left(\left(ex^2+d\right)^p c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + f}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

3.308 $\int \frac{1}{(f+gx^3)\log^2\left(c(d+ex^2)^p\right)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{(f+gx^3)\log^2\left(c(d+ex^2)^p\right)},x\right)$$

```
[Out] Unintegrable[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [A] time = 0.0284273, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(f+gx^3)\log^2\left(c(d+ex^2)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^3)\log^2\left(c(d+ex^2)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log^2\left(c(d+ex^2)^p\right)}\,dx$$

Mathematica [A] time = 7.92197, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx^3)\log^2\left(c(d+ex^2)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A] time = 4.077, size = 0, normalized size = 0.

$$\int \frac{1}{(gx^3+f)\left(\ln\left(c(ex^2+d)^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^2+d}{2\left(egpx^4\log(c)+efpx\log(c)+\left(egpx^4+efpx\right)\log\left(\left(ex^2+d\right)^p\right)\right)}-\int\frac{2\,egx^5-}{2\left(eg^2px^8\log(c)+2\,efgpx^5\log(c)+ef^2px^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g*p*x^4*log(c) + e*f*p*x*log(c) + (e*g*p*x^4 + e*f*p*x)
*log((e*x^2 + d)^p)) - integrate(1/2*(2*e*g*x^5 + 4*d*g*x^3 - e*f*x^2 + d*f
)/(e*g^2*p*x^8*log(c) + 2*e*f*g*p*x^5*log(c) + e*f^2*p*x^2*log(c) + (e*g^2*
p*x^8 + 2*e*f*g*p*x^5 + e*f^2*p*x^2)*log((e*x^2 + d)^p)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(gx^3+f\right)\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(gx^3+f\right)\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

3.309 $\int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [A] time = 0.0263193, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]

[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A] time = 14.5919, size = 0, normalized size = 0.

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]

[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A] time = 4.97, size = 0, normalized size = 0.

$$\int \frac{1}{\left(gx^3+f\right)^2 \left(\ln\left(c\left(ex^2+d\right)^p\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ex^2+d}{2\left(eg^2px^7\log(c)+2efgpx^4\log(c)+ef^2px\log(c)+\left(eg^2px^7+2efgpx^4+ef^2px\right)\log\left(\left(ex^2+d\right)^p\right)\right)}-\int\frac{}{2\left(eg^3px^{11}\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g^2*p*x^7*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x*log
(c) + (e*g^2*p*x^7 + 2*e*f*g*p*x^4 + e*f^2*p*x)*log((e*x^2 + d)^p)) - integ
rate(1/2*(5*e*g*x^5 + 7*d*g*x^3 - e*f*x^2 + d*f)/(e*g^3*p*x^11*log(c) + 3*e
*f*g^2*p*x^8*log(c) + 3*e*f^2*g*p*x^5*log(c) + e*f^3*p*x^2*log(c) + (e*g^3*
p*x^11 + 3*e*f*g^2*p*x^8 + 3*e*f^2*g*p*x^5 + e*f^3*p*x^2)*log((e*x^2 + d)^p
)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(g^2x^6+2fgx^3+f^2\right)\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(gx^3+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")

[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

3.310 $\int x^5 \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=142

$$\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2px^2(4ef-3dg)}{24e^3}+\frac{d^3p(4ef-3dg)\log\left(d+ex^2\right)}{24e^4}+\frac{dpx^4(4ef-3dg)}{48e^2}$$

```
[Out] -(d^2*(4*e*f - 3*d*g)*p*x^2)/(24*e^3) + (d*(4*e*f - 3*d*g)*p*x^4)/(48*e^2)
- ((4*e*f - 3*d*g)*p*x^6)/(72*e) - (g*p*x^8)/32 + (d^3*(4*e*f - 3*d*g)*p*Lo
g[d + e*x^2])/(24*e^4) + (f*x^6*Log[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d +
 e*x^2)^p])/8
```

Rubi [A] time = 0.22868, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2px^2(4ef-3dg)}{24e^3}+\frac{d^3p(4ef-3dg)\log\left(d+ex^2\right)}{24e^4}+\frac{dpx^4(4ef-3dg)}{48e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -(d^2*(4*e*f - 3*d*g)*p*x^2)/(24*e^3) + (d*(4*e*f - 3*d*g)*p*x^4)/(48*e^2)
- ((4*e*f - 3*d*g)*p*x^6)/(72*e) - (g*p*x^8)/32 + (d^3*(4*e*f - 3*d*g)*p*Lo
g[d + e*x^2])/(24*e^4) + (f*x^6*Log[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d +
 e*x^2)^p])/8
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^5 \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) dx = \frac{1}{2} \text{Subst}\left(\int x^2(f+gx) \log \left(c(d+ex)^p\right) dx, x, x^2\right)$$
$$= \frac{1}{6} f x^6 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{8} g x^8 \log \left(c \left(d+ex^2\right)^p\right) - \frac{1}{2}(ep) \text{Subst}\left(\int \frac{x^3(4f+}{12(d+}\right.$$
$$= \frac{1}{6} f x^6 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{8} g x^8 \log \left(c \left(d+ex^2\right)^p\right) - \frac{1}{24}(ep) \text{Subst}\left(\int \frac{x^3(4f+}{d+}\right.$$
$$= \frac{1}{6} f x^6 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{8} g x^8 \log \left(c \left(d+ex^2\right)^p\right) - \frac{1}{24}(ep) \text{Subst}\left(\int \left(-\frac{d^2(-}{}\right.\right.$$
$$= -\frac{d^2(4ef-3dg)px^2}{24e^3} + \frac{d(4ef-3dg)px^4}{48e^2} - \frac{(4ef-3dg)px^6}{72e} - \frac{1}{32} g p x^8 + \frac{d^3(4ef}{}$$

Mathematica [A] time = 0.0412418, size = 170, normalized size = 1.2

$$\frac{1}{6} f x^6 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{8} g x^8 \log \left(c \left(d+ex^2\right)^p\right) - \frac{d^2 f p x^2}{6e^2} + \frac{d^3 f p \log \left(d+ex^2\right)}{6e^3} - \frac{d^2 g p x^4}{16e^2} + \frac{d^3 g p x^2}{8e^3} - \frac{d^4 g p \log \left(d+\right.}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -(d^2*f*p*x^2)/(6*e^2) + (d^3*g*p*x^2)/(8*e^3) + (d*f*p*x^4)/(12*e) - (d^2*
g*p*x^4)/(16*e^2) - (f*p*x^6)/18 + (d*g*p*x^6)/(24*e) - (g*p*x^8)/32 + (d^3
*f*p*Log[d + e*x^2])/(6*e^3) - (d^4*g*p*Log[d + e*x^2])/(8*e^4) + (f*x^6*Lo
g[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d + e*x^2)^p])/8
```

Maple [C] time = 0.567, size = 413, normalized size = 2.9

$$\left(\frac{gx^8}{8} + \frac{fx^6}{6}\right) \ln \left(\left(ex^2+d\right)^p\right) - \frac{i}{12} \pi f x^6 \text{csgn}\left(i \left(ex^2+d\right)^p\right) \text{csgn}\left(ic \left(ex^2+d\right)^p\right) \text{csgn}\left(ic\right) - \frac{i}{12} \pi f x^6 \left(\text{csgn}\left(ic \left(ex^2+d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/8*g*x^8+1/6*f*x^6)*ln((e*x^2+d)^p)-1/12*I*Pi*f*x^6*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/12*I*Pi*f*x^6*csgn(I*c*(e*x^2+d)^p)^3+1/12
*I*Pi*f*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/16*I*Pi*g*x^8*csg
n(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/16*I*Pi*g*x^8*csgn(I*c*(
e*x^2+d)^p)^3+1/12*I*Pi*f*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/16*I*Pi*g
*x^8*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/16*I*Pi*g*x^8*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)^2+1/8*ln(c)*g*x^8-1/32*g*p*x^8+1/6*ln(c)*f*x^6+1/24/e
*d*g*p*x^6-1/18*f*p*x^6-1/16/e^2*d^2*g*p*x^4+1/12/e*d*f*p*x^4+1/8/e^3*d^3*g
```

```
*p*x^2-1/6/e^2*d^2*f*p*x^2-1/8/e^4*ln(e*x^2+d)*d^4*g*p+1/6/e^3*ln(e*x^2+d)*
d^3*f*p
```

Maxima [A] time = 1.00222, size = 178, normalized size = 1.25

$$-\frac{1}{288}\,ep\left(\frac{9\,e^3gx^8+4\left(4\,e^3f-3\,de^2g\right)x^6-6\left(4\,de^2f-3\,d^2eg\right)x^4+12\left(4\,d^2ef-3\,d^3g\right)x^2}{e^4}-\frac{12\left(4\,d^3ef-3\,d^4g\right)\log\left(ex^2+d\right)}{e^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/288*e*p*((9*e^3*g*x^8 + 4*(4*e^3*f - 3*d*e^2*g)*x^6 - 6*(4*d*e^2*f - 3*d
^2*e*g)*x^4 + 12*(4*d^2*e*f - 3*d^3*g)*x^2)/e^4 - 12*(4*d^3*e*f - 3*d^4*g)*
log(e*x^2 + d)/e^5) + 1/24*(3*g*x^8 + 4*f*x^6)*log((e*x^2 + d)^p*c)
```

Fricas [A] time = 1.98399, size = 339, normalized size = 2.39

$$-\frac{9\,e^4gpx^8+4\left(4\,e^4f-3\,de^3g\right)px^6-6\left(4\,de^3f-3\,d^2e^2g\right)px^4+12\left(4\,d^2e^2f-3\,d^3eg\right)px^2-12\left(3\,e^4gpx^8+4\,e^4fpx^6+\left(4\,d^3\right.\right.}{288\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/288*(9*e^4*g*p*x^8 + 4*(4*e^4*f - 3*d*e^3*g)*p*x^6 - 6*(4*d*e^3*f - 3*d^
2*e^2*g)*p*x^4 + 12*(4*d^2*e^2*f - 3*d^3*e*g)*p*x^2 - 12*(3*e^4*g*p*x^8 + 4
*e^4*f*p*x^6 + (4*d^3*e*f - 3*d^4*g)*p)*log(e*x^2 + d) - 12*(3*e^4*g*x^8 +
4*e^4*f*x^6)*log(c))/e^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17367, size = 416, normalized size = 2.93

$$\frac{1}{288}\left(36\,gx^8e\log\left(c\right)+48\,fx^6e\log\left(c\right)+8\left(6\left(x^2e+d\right)^3e^{(-2)}\log\left(x^2e+d\right)-18\left(x^2e+d\right)^2de^{(-2)}\log\left(x^2e+d\right)+18\left(x^2e+d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/288*(36*g*x^8*e*log(c) + 48*f*x^6*e*log(c) + 8*(6*(x^2*e + d)^3*e^(-2)*lo
g(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*log(x^2*e + d) + 18*(x^2*e + d)*d^
2*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*e^(-2) + 9*(x^2*e + d)^2*d*e^(-2)
 - 18*(x^2*e + d)*d^2*e^(-2))*f*p + 3*(12*(x^2*e + d)^4*e^(-3)*log(x^2*e +
d) - 48*(x^2*e + d)^3*d*e^(-3)*log(x^2*e + d) + 72*(x^2*e + d)^2*d^2*e^(-3)
*log(x^2*e + d) - 48*(x^2*e + d)*d^3*e^(-3)*log(x^2*e + d) - 3*(x^2*e + d)^
4*e^(-3) + 16*(x^2*e + d)^3*d*e^(-3) - 36*(x^2*e + d)^2*d^2*e^(-3) + 48*(x^
2*e + d)*d^3*e^(-3))*g*p)*e^(-1)
```

3.311 $\int x^3 \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=119

$$\frac{1}{4} f x^4 \log \left(c \left(d+e x^2\right)^p\right)+\frac{1}{6} g x^6 \log \left(c \left(d+e x^2\right)^p\right)-\frac{d^2 p (3 e f-2 d g) \log \left(d+e x^2\right)}{12 e^3}+\frac{d p x^2 (3 e f-2 d g)}{12 e^2}-\frac{p x^4 (3 e f-2 d g)}{24 e}-$$

```
[Out] (d*(3*e*f - 2*d*g)*p*x^2)/(12*e^2) - ((3*e*f - 2*d*g)*p*x^4)/(24*e) - (g*p*
x^6)/18 - (d^2*(3*e*f - 2*d*g)*p*Log[d + e*x^2])/(12*e^3) + (f*x^4*Log[c*(d
 + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])/6
```

Rubi [A] time = 0.178547, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$\frac{1}{4} f x^4 \log \left(c \left(d+e x^2\right)^p\right)+\frac{1}{6} g x^6 \log \left(c \left(d+e x^2\right)^p\right)-\frac{d^2 p (3 e f-2 d g) \log \left(d+e x^2\right)}{12 e^3}+\frac{d p x^2 (3 e f-2 d g)}{12 e^2}-\frac{p x^4 (3 e f-2 d g)}{24 e}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(3*e*f - 2*d*g)*p*x^2)/(12*e^2) - ((3*e*f - 2*d*g)*p*x^4)/(24*e) - (g*p*
x^6)/18 - (d^2*(3*e*f - 2*d*g)*p*Log[d + e*x^2])/(12*e^3) + (f*x^4*Log[c*(d
 + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])/6
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^3\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)dx=\frac{1}{2}\text{Subst}\left(\int x(f+gx)\log\left(c(d+ex)^p\right)dx,x,x^2\right)$$
$$=\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{2}(ep)\text{Subst}\left(\int\frac{x^2(3f+}{6(d+e}\right.$$
$$=\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}(ep)\text{Subst}\left(\int\frac{x^2(3f+}{d+}\right.$$
$$=\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}(ep)\text{Subst}\left(\int\left(\frac{d(-3e}{}\right.\right.$$
$$=\frac{d(3ef-2dg)px^2}{12e^2}-\frac{(3ef-2dg)px^4}{24e}-\frac{1}{18}gpx^6-\frac{d^2(3ef-2dg)p\log\left(d+ex^2\right)}{12e^3}$$

Mathematica [A] time = 0.0255616, size = 140, normalized size = 1.18

$$\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2fp\log\left(d+ex^2\right)}{4e^2}-\frac{d^2gpx^2}{6e^2}+\frac{d^3gp\log\left(d+ex^2\right)}{6e^3}+\frac{dfpx^2}{4e}+\frac{dg}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*f*p*x^2)/(4*e) - (d^2*g*p*x^2)/(6*e^2) - (f*p*x^4)/8 + (d*g*p*x^4)/(12*e
) - (g*p*x^6)/18 - (d^2*f*p*Log[d + e*x^2])/(4*e^2) + (d^3*g*p*Log[d + e*x^
2])/(6*e^3) + (f*x^4*Log[c*(d + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])
/6
```

Maple [C] time = 0.573, size = 387, normalized size = 3.3

$$\left(\frac{gx^6}{6}+\frac{fx^4}{4}\right)\ln\left(\left(ex^2+d\right)^p\right)+\frac{i}{12}\pi gx^6\left(\text{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2\text{csgn}(ic)-\frac{i}{12}\pi gx^6\left(\text{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^3-\frac{i}{8}\pi fx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/6*g*x^6+1/4*f*x^4)*ln((e*x^2+d)^p)+1/12*I*Pi*g*x^6*csgn(I*c*(e*x^2+d)^p)
^2*csgn(I*c)-1/12*I*Pi*g*x^6*csgn(I*c*(e*x^2+d)^p)^3-1/8*I*Pi*f*x^4*csgn(I*
c*(e*x^2+d)^p)^3-1/12*I*Pi*g*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*
csgn(I*c)+1/8*I*Pi*f*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/8*I*Pi*f*x^4*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/12*I*Pi*g*x^6*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/8*I*Pi*f*x^4*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)^2+1/6*ln(c)*g*x^6-1/18*g*p*x^6+1/4*ln(c)*f*x^4+1/12/e*d*g
*p*x^4-1/8*f*p*x^4-1/6/e^2*d^2*g*p*x^2+1/4/e*d*f*p*x^2+1/6/e^3*ln(e*x^2+d)*
```

```
d^3*g*p-1/4/e^2*ln(e*x^2+d)*d^2*f*p
```

Maxima [A] time = 1.09469, size = 146, normalized size = 1.23

$$-\frac{1}{72}\,ep\left(\frac{4\,e^2gx^6+3\left(3\,e^2f-2\,deg\right)x^4-6\left(3\,def-2\,d^2g\right)x^2}{e^3}+\frac{6\left(3\,d^2ef-2\,d^3g\right)\log\left(ex^2+d\right)}{e^4}\right)+\frac{1}{12}\left(2\,gx^6+3\,fx^4\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/72*e*p*((4*e^2*g*x^6 + 3*(3*e^2*f - 2*d*e*g)*x^4 - 6*(3*d*e*f - 2*d^2*g)
*x^2)/e^3 + 6*(3*d^2*e*f - 2*d^3*g)*log(e*x^2 + d)/e^4) + 1/12*(2*g*x^6 + 3
*f*x^4)*log((e*x^2 + d)^p*c)
```

Fricas [A] time = 2.03243, size = 282, normalized size = 2.37

$$-\frac{4\,e^3gpx^6+3\left(3\,e^3f-2\,de^2g\right)px^4-6\left(3\,de^2f-2\,d^2eg\right)px^2-6\left(2\,e^3gpx^6+3\,e^3fpx^4-\left(3\,d^2ef-2\,d^3g\right)p\right)\log\left(ex^2+d\right)-}{72\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/72*(4*e^3*g*p*x^6 + 3*(3*e^3*f - 2*d*e^2*g)*p*x^4 - 6*(3*d*e^2*f - 2*d^2
*e*g)*p*x^2 - 6*(2*e^3*g*p*x^6 + 3*e^3*f*p*x^4 - (3*d^2*e*f - 2*d^3*g)*p)*l
og(e*x^2 + d) - 6*(2*e^3*g*x^6 + 3*e^3*f*x^4)*log(c))/e^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14927, size = 317, normalized size = 2.66

$$\frac{1}{72}\left(12\,gx^6e\log\left(c\right)+9\left(2\left(x^2e+d\right)^2\log\left(x^2e+d\right)-4\left(x^2e+d\right)d\log\left(x^2e+d\right)-\left(x^2e+d\right)^2+4\left(x^2e+d\right)d\right)fpe^{(-1)}+18\left(\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/72*(12*g*x^6*e*log(c) + 9*(2*(x^2*e + d)^2*log(x^2*e + d) - 4*(x^2*e + d)
*d*log(x^2*e + d) - (x^2*e + d)^2 + 4*(x^2*e + d)*d)*f*p*e^(-1) + 18*((x^2*
```

```
e + d)^2 - 2*(x^2*e + d)*d)*f*e^(-1)*log(c) + 2*(6*(x^2*e + d)^3*e^(-2)*log
(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*log(x^2*e + d) + 18*(x^2*e + d)*d^2
*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*e^(-2) + 9*(x^2*e + d)^2*d*e^(-2)
- 18*(x^2*e + d)*d^2*e^(-2))*g*p)*e^(-1)
```

3.312 $\int x\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=94

$$\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4e^2g}-\frac{px^2(ef-dg)}{4e}-\frac{p\left(f+gx^2\right)^2}{8g}$$

```
[Out] -((e*f - d*g)*p*x^2)/(4*e) - (p*(f + g*x^2)^2)/(8*g) - ((e*f - d*g)^2*p*Log
[d + e*x^2])/(4*e^2*g) + ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*g)
```

Rubi [A] time = 0.0928483, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2475, 2395, 43}

$$\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4e^2g}-\frac{px^2(ef-dg)}{4e}-\frac{p\left(f+gx^2\right)^2}{8g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -((e*f - d*g)*p*x^2)/(4*e) - (p*(f + g*x^2)^2)/(8*g) - ((e*f - d*g)^2*p*Log
[d + e*x^2])/(4*e^2*g) + ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*g)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)dx=\frac{1}{2}\text{Subst}\left(\int(f+gx)\log\left(c(d+ex)^p\right)dx,x,x^2\right)$$
$$=\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{(ep)\text{Subst}\left(\int\frac{(f+gx)^2}{d+ex}dx,x,x^2\right)}{4g}$$
$$=\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{(ep)\text{Subst}\left(\int\left(\frac{g(ef-dg)}{e^2}+\frac{(ef-dg)^2}{e^2(d+ex)}+\frac{g(f+gx)}{e}\right)dx\right.}{4g}$$
$$=-\frac{(ef-dg)px^2}{4e}-\frac{p\left(f+gx^2\right)^2}{8g}-\frac{(ef-dg)^2p\log\left(d+ex^2\right)}{4e^2g}+\frac{\left(f+gx^2\right)^2\log\left(c\right.}{4g}$$

Mathematica [A] time = 0.0437708, size = 98, normalized size = 1.04

$$\frac{1}{2}f\left(\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}-px^2\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + (g*x^4
*Log[c*(d + e*x^2)^p])/4 + (f*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p]
)/e))/2
```

Maple [C] time = 0.571, size = 361, normalized size = 3.8

$$\left(\frac{gx^4}{4}+\frac{fx^2}{2}\right)\ln\left(\left(ex^2+d\right)^p\right)+\frac{i}{8}\pi gx^4\text{csgn}\left(i\left(ex^2+d\right)^p\right)\left(\text{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2+\frac{i}{4}\pi fx^2\left(\text{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2\text{cs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/4*g*x^4+1/2*f*x^2)*ln((e*x^2+d)^p)+1/8*I*Pi*g*x^4*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2+1/4*I*Pi*f*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/8*
I*Pi*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/4*I*Pi*f*x
^2*csgn(I*c*(e*x^2+d)^p)^3-1/8*I*Pi*g*x^4*csgn(I*c*(e*x^2+d)^p)^3+1/4*I*Pi*
f*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/4*I*Pi*f*x^2*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/8*I*Pi*g*x^4*csgn(I*c*(e*x^2+d)
^p)^2*csgn(I*c)+1/4*ln(c)*g*x^4-1/8*g*p*x^4+1/2*ln(c)*f*x^2+1/4*d*g*p*x^2/e
-1/2*f*p*x^2-1/4*d^2*g*p*ln(e*x^2+d)/e^2+1/2/e*ln(e*x^2+d)*d*f*p
```

Maxima [A] time = 1.03011, size = 134, normalized size = 1.43

$$-\frac{ep\left(\frac{eg^2x^4+2\left(2efg-dg^2\right)x^2}{e^2}+\frac{2\left(e^2f^2-2defg+d^2g^2\right)\log\left(ex^2+d\right)}{e^3}\right)}{8g}+\frac{\left(gx^2+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)}{4g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/8*e*p*((e*g^2*x^4 + 2*(2*e*f*g - d*g^2)*x^2)/e^2 + 2*(e^2*f^2 - 2*d*e*f*
g + d^2*g^2)*log(e*x^2 + d)/e^3)/g + 1/4*(g*x^2 + f)^2*log((e*x^2 + d)^p*c)
/g
```

Fricas [A] time = 1.83286, size = 216, normalized size = 2.3

$$-\frac{e^2gpx^4 + 2\left(2e^2f - deg\right)px^2 - 2\left(e^2gpx^4 + 2e^2fpx^2 + \left(2def - d^2g\right)p\right)\log\left(ex^2 + d\right) - 2\left(e^2gx^4 + 2e^2fx^2\right)\log\left(c\right)}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/8*(e^2*g*p*x^4 + 2*(2*e^2*f - d*e*g)*p*x^2 - 2*(e^2*g*p*x^4 + 2*e^2*f*p*
x^2 + (2*d*e*f - d^2*g)*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 2*e^2*f*x^2)*log
(c))/e^2
```

Sympy [A] time = 67.4428, size = 139, normalized size = 1.48

$$\begin{cases} -\frac{d^2gp\log\left(d+ex^2\right)}{4e^2} + \frac{dfp\log\left(d+ex^2\right)}{2e} + \frac{dgpx^2}{4e} + \frac{fpx^2\log\left(d+ex^2\right)}{2} - \frac{fpx^2}{2} + \frac{fx^2\log\left(c\right)}{2} + \frac{gpx^4\log\left(d+ex^2\right)}{4} - \frac{gpx^4}{8} + \frac{gx^4\log\left(c\right)}{4} & \text{for}\ e \neq 0 \\ \left(\frac{fx^2}{2} + \frac{gx^4}{4}\right)\log\left(cd^p\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-d**2*g*p*log(d + e*x**2)/(4*e**2) + d*f*p*log(d + e*x**2)/(2*e)
 + d*g*p*x**2/(4*e) + f*p*x**2*log(d + e*x**2)/2 - f*p*x**2/2 + f*x**2*log(
c)/2 + g*p*x**4*log(d + e*x**2)/4 - g*p*x**4/8 + g*x**4*log(c)/4, Ne(e, 0))
, ((f*x**2/2 + g*x**4/4)*log(c*d**p), True))
```

Giac [A] time = 1.21401, size = 200, normalized size = 2.13

$$\frac{1}{8}\left(\left(2\left(x^2e + d\right)^2\log\left(x^2e + d\right) - 4\left(x^2e + d\right)d\log\left(x^2e + d\right) - \left(x^2e + d\right)^2 + 4\left(x^2e + d\right)d\right)gpe^{(-1)} + 2\left(\left(x^2e + d\right)^2 - 2\left(x^2e + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/8*((2*(x^2*e + d)^2*log(x^2*e + d) - 4*(x^2*e + d)*d*log(x^2*e + d) - (x^
2*e + d)^2 + 4*(x^2*e + d)*d)*g*p*e^(-1) + 2*((x^2*e + d)^2 - 2*(x^2*e + d)
*d)*g*e^(-1)*log(c) - 4*(x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*f*p + 4*(x
^2*e + d)*f*log(c))*e^(-1)
```

3.313 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}fp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}-\frac{1}{2}gpx^2$$

```
[Out] -(g*p*x^2)/2 + (g*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e) + (f*Log[-((e*x^2
)/d)]*Log[c*(d + e*x^2)^p])/2 + (f*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rubi [A] time = 0.111122, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315}

$$\frac{1}{2}fp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}-\frac{1}{2}gpx^2$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -(g*p*x^2)/2 + (g*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e) + (f*Log[-((e*x^2
)/d)]*Log[c*(d + e*x^2)^p])/2 + (f*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(g\log\left(c(d+ex)^p\right)+\frac{f\log\left(c(d+ex)^p\right)}{x}\right)dx,x,x^2\right)\\ &= \frac{1}{2}f\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)+\frac{1}{2}g\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right)dx,x,x^2\right)\\ &= \frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int \log\left(cx^p\right)dx,x,d+ex^2\right)}{2e}-\frac{1}{2}(efp)\,\text{Su}\\ &= -\frac{1}{2}gpx^2+\frac{g\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fp\text{I}\end{aligned}$$

Mathematica [A] time = 0.0191056, size = 80, normalized size = 0.98

$$\frac{1}{2}f\left(p\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)+\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)\right)+\frac{1}{2}g\left(\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}-px^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (g*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/2 + (f*(Log[-((e*x^2)
/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*x^2)/d]))/2
```

Maple [C] time = 0.567, size = 419, normalized size = 5.1

$$\frac{\ln\left(\left(ex^2+d\right)^p\right)x^2g}{2}+\ln\left(\left(ex^2+d\right)^p\right)f\ln(x)-\frac{gpx^2}{2}+\frac{gpd\ln\left(ex^2+d\right)}{2e}-pf\ln(x)\ln\left(\left(-ex+\sqrt{-de}\right)\frac{1}{\sqrt{-de}}\right)-pf\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x,x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)*x^2*g+ln((e*x^2+d)^p)*f*ln(x)-1/2*g*p*x^2+1/2*p/e*g*d*l
n(e*x^2+d)-p*f*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*f*ln(x)*ln((e*x
+(-d*e)^(1/2))/(-d*e)^(1/2))-p*f*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*
f*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)*csgn(I*c)*f*ln(x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)^2*f*ln(x)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*ln(x)-1/4*I*Pi*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x^2*g+1/4*I*Pi*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x^2*g+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn
(I*c)*x^2*g+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f*ln(x)-1/4*I*Pi*csg
n(I*c*(e*x^2+d)^p)^3*x^2*g+1/2*ln(c)*x^2*g+ln(c)*f*ln(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

3.314 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log(x)}{d}$$

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g*p*PolyLog[2, 1
+ (e*x^2)/d])/2
```

Rubi [A] time = 0.126835, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2475, 43, 2416, 2395, 36, 29, 31, 2394, 2315}

$$\frac{1}{2}gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```


```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g*p*PolyLog[2, 1
+ (e*x^2)/d])/2
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{f\log\left(c(d+ex)^p\right)}{x^2}+\frac{g\log\left(c(d+ex)^p\right)}{x}\right)dx,x,x^2\right)\\
&= \frac{1}{2}f\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2}\,dx,x,x^2\right)+\frac{1}{2}g\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}(efp)\,\text{Subst}\left(\int \frac{1}{x(d+\right.\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}gp\,\text{Li}_2\left(1+\frac{ex^2}{d}\right)+\frac{(ef}{}\\
&= \frac{efp\log(x)}{d}-\frac{efp\log\left(d+ex^2\right)}{2d}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+e\right.\right.
\end{aligned}$$

Mathematica [A] time = 0.0276093, size = 92, normalized size = 0.99

$$\frac{1}{2}g\left(p\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)+\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)\right)-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*
x^2)/d]))/2
```

Maple [C] time = 0.499, size = 421, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^3,x)
```

```
[Out] ln((e*x^2+d)^p)*g*ln(x)-1/2*ln((e*x^2+d)^p)*f/x^2+e*f*p*ln(x)/d-1/2*e*f*p*l
n(e*x^2+d)/d-p*g*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g*ln(x)*ln((e
*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-
p*g*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)*g*ln(x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)*g*ln(x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g*ln(x)-1
/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/x^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)
^p)^3*f/x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/
x^2-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g*ln(x)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)^2*f/x^2+ln(c)*g*ln(x)-1/2*ln(c)*f/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**3,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

3.315 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=93

$$-\frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4}+\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4d^2f}-\frac{ep\log(x)(ef-2dg)}{2d^2}-\frac{efp}{4dx^2}$$

```
[Out] -(e*f*p)/(4*d*x^2) - (e*(e*f - 2*d*g)*p*Log[x])/(2*d^2) + ((e*f - d*g)^2*p*
Log[d + e*x^2])/(4*d^2*f) - ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*f*x^4)
```

Rubi [A] time = 0.137383, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 37, 2414, 12, 88}

$$-\frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4}+\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4d^2f}-\frac{ep\log(x)(ef-2dg)}{2d^2}-\frac{efp}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -(e*f*p)/(4*d*x^2) - (e*(e*f - 2*d*g)*p*Log[x])/(2*d^2) + ((e*f - d*g)^2*p*
Log[d + e*x^2])/(4*d^2*f) - ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*f*x^4)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4} - \frac{1}{2}(ep)\operatorname{Subst}\left(\int -\frac{(f+gx)^2}{2fx^2(d+ex)}\,dx, x, x^2\right)$$

$$= -\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4} + \frac{(ep)\operatorname{Subst}\left(\int \frac{(f+gx)^2}{x^2(d+ex)}\,dx, x, x^2\right)}{4f}$$

$$= -\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4} + \frac{(ep)\operatorname{Subst}\left(\int \left(\frac{f^2}{dx^2} + \frac{f(-ef+2dg)}{d^2x} + \frac{(-ef+dg)^2}{d^2(d+ex)}\right) dx, x,}{4f}$$

$$= -\frac{efp}{4dx^2} - \frac{e(ef-2dg)p\log(x)}{2d^2} + \frac{(ef-dg)^2p\log\left(d+ex^2\right)}{4d^2f} - \frac{\left(f+gx^2\right)^2\log\left(c\left(d+e\right.\right.}{4fx^4}$$

Mathematica [A] time = 0.0378179, size = 105, normalized size = 1.13

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2x^2} + \frac{1}{4}efp\left(\frac{e\log\left(d+ex^2\right)}{d^2} - \frac{2e\log(x)}{d^2} - \frac{1}{dx^2}\right) - \frac{egp\log\left(d+ex^2\right)}{2d} + \frac{egp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^2])/(2*d) + (e*f*p*(-(1/(d*x^2)) - (2
*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 - (f*Log[c*(d + e*x^2)^p])/(4*x
^4) - (g*Log[c*(d + e*x^2)^p])/(2*x^2)
```

Maple [C] time = 0.373, size = 392, normalized size = 4.2

$$-\frac{\left(2gx^2+f\right)\ln\left(\left(ex^2+d\right)^p\right)}{4x^4} - \frac{2i\pi d^2gx^2\operatorname{csgn}\left(i\left(ex^2+d\right)^p\right)\left(\operatorname{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2 - 2i\pi d^2gx^2\operatorname{csgn}\left(i\left(ex^2+d\right)^p\right)\operatorname{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^5,x)
```

```
[Out] -1/4*(2*g*x^2+f)/x^4*ln((e*x^2+d)^p)-1/8*(2*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-2*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)*csgn(I*c)-2*I*Pi*d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+2*I*Pi*d^2*g
*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-8*ln(x)*d*e*g*p*x^4+4*ln(x)*e^2*f*p*
x^4+4*ln(e*x^2+d)*d*e*g*p*x^4-2*ln(e*x^2+d)*e^2*f*p*x^4+I*Pi*d^2*f*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*d^2*f*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)-I*Pi*d^2*f*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d^2*f*csgn(
```

```
I*c*(e*x^2+d)^p)^2*csgn(I*c)+4*ln(c)*d^2*g*x^2+2*d*e*f*p*x^2+2*ln(c)*d^2*f)
/d^2/x^4
```

Maxima [A] time = 1.01961, size = 104, normalized size = 1.12

$$\frac{1}{4}\,ep\left(\frac{\left(ef-2\,dg\right)\log\left(ex^2+d\right)}{d^2}-\frac{\left(ef-2\,dg\right)\log\left(x^2\right)}{d^2}-\frac{f}{dx^2}\right)-\frac{\left(2\,gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*e*p*((e*f - 2*d*g)*log(e*x^2 + d)/d^2 - (e*f - 2*d*g)*log(x^2)/d^2 - f/
(d*x^2)) - 1/4*(2*g*x^2 + f)*log((e*x^2 + d)^p*c)/x^4
```

Fricas [A] time = 2.09917, size = 223, normalized size = 2.4

$$-\frac{2\left(e^2f-2\,deg\right)px^4\log\left(x\right)+defpx^2+\left(2\,d^2gpx^2-\left(e^2f-2\,deg\right)px^4+d^2fp\right)\log\left(ex^2+d\right)+\left(2\,d^2gx^2+d^2f\right)\log\left(c\right)}{4\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(e^2*f - 2*d*e*g)*p*x^4*log(x) + d*e*f*p*x^2 + (2*d^2*g*p*x^2 - (e^
2*f - 2*d*e*g)*p*x^4 + d^2*f*p)*log(e*x^2 + d) + (2*d^2*g*x^2 + d^2*f)*log(
c))/(d^2*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26617, size = 435, normalized size = 4.68

$$-\frac{\left(2\left(x^2e+d\right)^2dgpe^2\log\left(x^2e+d\right)-2\left(x^2e+d\right)d^2gpe^2\log\left(x^2e+d\right)-2\left(x^2e+d\right)^2dgpe^2\log\left(x^2e\right)+4\left(x^2e+d\right)d^2gpe^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*(x^2*e + d)^2*d*g*p*e^2*log(x^2*e + d) - 2*(x^2*e + d)*d^2*g*p*e^2*
log(x^2*e + d) - 2*(x^2*e + d)^2*d*g*p*e^2*log(x^2*e) + 4*(x^2*e + d)*d^2*g
*p*e^2*log(x^2*e) - 2*d^3*g*p*e^2*log(x^2*e) - (x^2*e + d)^2*f*p*e^3*log(x^
2*e + d) + 2*(x^2*e + d)*d*f*p*e^3*log(x^2*e + d) + (x^2*e + d)^2*f*p*e^3*l
og(x^2*e) - 2*(x^2*e + d)*d*f*p*e^3*log(x^2*e) + d^2*f*p*e^3*log(x^2*e) + 2
*(x^2*e + d)*d^2*g*e^2*log(c) - 2*d^3*g*e^2*log(c) + (x^2*e + d)*d*f*p*e^3
- d^2*f*p*e^3 + d^2*f*e^3*log(c))*e^(-1)/((x^2*e + d)^2*d^2 - 2*(x^2*e + d)
*d^3 + d^4)
```

3.316 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=125

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{e^2p(2ef-3dg)\log\left(d+ex^2\right)}{12d^3}+\frac{e^2p\log(x)(2ef-3dg)}{6d^3}+\frac{ep(2ef-3dg)}{12d^2x^2}$$

```
[Out] -(e*f*p)/(12*d*x^4) + (e*(2*e*f - 3*d*g)*p)/(12*d^2*x^2) + (e^2*(2*e*f - 3*
d*g)*p*Log[x])/(6*d^3) - (e^2*(2*e*f - 3*d*g)*p*Log[d + e*x^2])/(12*d^3) -
(f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rubi [A] time = 0.162969, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{e^2p(2ef-3dg)\log\left(d+ex^2\right)}{12d^3}+\frac{e^2p\log(x)(2ef-3dg)}{6d^3}+\frac{ep(2ef-3dg)}{12d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -(e*f*p)/(12*d*x^4) + (e*(2*e*f - 3*d*g)*p)/(12*d^2*x^2) + (e^2*(2*e*f - 3*
d*g)*p*Log[x])/(6*d^3) - (e^2*(2*e*f - 3*d*g)*p*Log[d + e*x^2])/(12*d^3) -
(f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^4}\,dx,x,x^2\right)$$
$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{2}(ep)\operatorname{Subst}\left(\int \frac{-2f-3gx}{6x^3(d+ex)}\,dx,x,x^2\right.$$
$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{12}(ep)\operatorname{Subst}\left(\int \frac{-2f-3gx}{x^3(d+ex)}\,dx,x,x^2\right.$$
$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{12}(ep)\operatorname{Subst}\left(\int\left(-\frac{2f}{dx^3}+\frac{2ef-3dg}{d^2x^2}\right.\right.$$
$$= -\frac{efp}{12dx^4} + \frac{e(2ef-3dg)p}{12d^2x^2} + \frac{e^2(2ef-3dg)p\log(x)}{6d^3} - \frac{e^2(2ef-3dg)p\log\left(d+ex^2\right)}{12d^3} -$$

Mathematica [A] time = 0.0628308, size = 130, normalized size = 1.04

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} + \frac{1}{6}efp\left(-\frac{e^2\log\left(d+ex^2\right)}{d^3} + \frac{2e^2\log(x)}{d^3} + \frac{e}{d^2x^2} - \frac{1}{2dx^4}\right) + \frac{1}{4}egp\left(\frac{e\log\left(d+\right.}{d^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] (e*g*p*(-(1/(d*x^2)) - (2*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 + (e*f
*p*(-1/(2*d*x^4) + e/(d^2*x^2) + (2*e^2*Log[x])/d^3 - (e^2*Log[d + e*x^2])/
d^3))/6 - (f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*x^
4)
```

Maple [C] time = 0.395, size = 428, normalized size = 3.4

$$-\frac{\left(3\,gx^2+2\,f\right)\ln\left(\left(ex^2+d\right)^p\right)}{12\,x^6} - \frac{12\,\ln(x)\,de^2gpx^6 - 8\,\ln(x)\,e^3fpx^6 - 6\,\ln\left(-ex^2-d\right)de^2gpx^6 + 4\,\ln\left(-ex^2-d\right)e^3fpx^6 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^7,x)
```

```
[Out] -1/12*(3*g*x^2+2*f)/x^6*ln((e*x^2+d)^p)-1/24*(12*ln(x)*d*e^2*g*p*x^6-8*ln(x
)*e^3*f*p*x^6-6*ln(-e*x^2-d)*d*e^2*g*p*x^6+4*ln(-e*x^2-d)*e^3*f*p*x^6-3*I*P
i*d^3*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3*I*Pi*d^3*
```

```
g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+2*I*Pi*d^3*f*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-3*I*Pi*d^3*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+2*
I*Pi*d^3*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-2*I*Pi*d^3*f*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-2*I*Pi*d^3*f*csgn(I*c*(e*x^2+d)^p)^3+3*I
*Pi*d^3*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+6*d^2*e*g*p*x^4-4*d*e^2*f*p
*x^4+6*ln(c)*d^3*g*x^2+2*d^2*e*f*p*x^2+4*ln(c)*d^3*f)/d^3/x^6
```

Maxima [A] time = 1.02451, size = 140, normalized size = 1.12

$$-\frac{1}{12}\,ep\left(\frac{\left(2\,e^2f-3\,deg\right)\log\left(ex^2+d\right)}{d^3}-\frac{\left(2\,e^2f-3\,deg\right)\log\left(x^2\right)}{d^3}-\frac{\left(2\,ef-3\,dg\right)x^2-df}{d^2x^4}\right)-\frac{\left(3\,gx^2+2\,f\right)\log\left(\left(ex^2+d\right)\right.}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*e*p*((2*e^2*f - 3*d*e*g)*log(e*x^2 + d)/d^3 - (2*e^2*f - 3*d*e*g)*log
(x^2)/d^3 - ((2*e*f - 3*d*g)*x^2 - d*f)/(d^2*x^4)) - 1/12*(3*g*x^2 + 2*f)*l
og((e*x^2 + d)^p*c)/x^6
```

Fricas [A] time = 2.04722, size = 285, normalized size = 2.28

$$\frac{2\left(2\,e^3f-3\,de^2g\right)px^6\log\left(x\right)-d^2efpx^2+\left(2\,de^2f-3\,d^2eg\right)px^4-\left(\left(2\,e^3f-3\,de^2g\right)px^6+3\,d^3gpx^2+2\,d^3fp\right)\log\left(ex^2+\right.}{12\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*e^3*f - 3*d*e^2*g)*p*x^6*log(x) - d^2*e*f*p*x^2 + (2*d*e^2*f - 3
*d^2*e*g)*p*x^4 - ((2*e^3*f - 3*d*e^2*g)*p*x^6 + 3*d^3*g*p*x^2 + 2*d^3*f*p)
*log(e*x^2 + d) - (3*d^3*g*x^2 + 2*d^3*f)*log(c))/(d^3*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28447, size = 695, normalized size = 5.56

$$\frac{\left(3\left(x^2e+d\right)^3dgpe^3\log\left(x^2e+d\right)-9\left(x^2e+d\right)^2d^2gpe^3\log\left(x^2e+d\right)+6\left(x^2e+d\right)d^3gpe^3\log\left(x^2e+d\right)-3\left(x^2e+d\right)^3dg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*(x^2*e + d)^3*d*g*p*e^3*log(x^2*e + d) - 9*(x^2*e + d)^2*d^2*g*p*e^
3*log(x^2*e + d) + 6*(x^2*e + d)*d^3*g*p*e^3*log(x^2*e + d) - 3*(x^2*e + d)
^3*d*g*p*e^3*log(x^2*e) + 9*(x^2*e + d)^2*d^2*g*p*e^3*log(x^2*e) - 9*(x^2*e
 + d)*d^3*g*p*e^3*log(x^2*e) + 3*d^4*g*p*e^3*log(x^2*e) - 3*(x^2*e + d)^2*d
^2*g*p*e^3 + 6*(x^2*e + d)*d^3*g*p*e^3 - 3*d^4*g*p*e^3 - 2*(x^2*e + d)^3*f*
p*e^4*log(x^2*e + d) + 6*(x^2*e + d)^2*d*f*p*e^4*log(x^2*e + d) - 6*(x^2*e
+ d)*d^2*f*p*e^4*log(x^2*e + d) + 2*(x^2*e + d)^3*f*p*e^4*log(x^2*e) - 6*(x
^2*e + d)^2*d*f*p*e^4*log(x^2*e) + 6*(x^2*e + d)*d^2*f*p*e^4*log(x^2*e) - 2
*d^3*f*p*e^4*log(x^2*e) - 3*(x^2*e + d)*d^3*g*e^3*log(c) + 3*d^4*g*e^3*log(
c) + 2*(x^2*e + d)^2*d*f*p*e^4 - 5*(x^2*e + d)*d^2*f*p*e^4 + 3*d^3*f*p*e^4
- 2*d^3*f*e^4*log(c))*e^(-1)/((x^2*e + d)^3*d^3 - 3*(x^2*e + d)^2*d^4 + 3*(
x^2*e + d)*d^5 - d^6)
```

3.317 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^9}\,dx$

Optimal. Leaf size=148

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{e^2p(3ef-4dg)}{24d^3x^2}+\frac{e^3p(3ef-4dg)\log\left(d+ex^2\right)}{24d^4}-\frac{e^3p\log(x)(3ef-4dg)}{12d^4}$$

```
[Out] -(e*f*p)/(24*d*x^6) + (e*(3*e*f - 4*d*g)*p)/(48*d^2*x^4) - (e^2*(3*e*f - 4*
d*g)*p)/(24*d^3*x^2) - (e^3*(3*e*f - 4*d*g)*p*Log[x])/(12*d^4) + (e^3*(3*e*
f - 4*d*g)*p*Log[d + e*x^2])/(24*d^4) - (f*Log[c*(d + e*x^2)^p])/(8*x^8) -
(g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rubi [A] time = 0.202369, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{e^2p(3ef-4dg)}{24d^3x^2}+\frac{e^3p(3ef-4dg)\log\left(d+ex^2\right)}{24d^4}-\frac{e^3p\log(x)(3ef-4dg)}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -(e*f*p)/(24*d*x^6) + (e*(3*e*f - 4*d*g)*p)/(48*d^2*x^4) - (e^2*(3*e*f - 4*
d*g)*p)/(24*d^3*x^2) - (e^3*(3*e*f - 4*d*g)*p*Log[x])/(12*d^4) + (e^3*(3*e*
f - 4*d*g)*p*Log[d + e*x^2])/(24*d^4) - (f*Log[c*(d + e*x^2)^p])/(8*x^8) -
(g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{2}(ep)\,\text{Subst}\left(\int \frac{-3f-4gx}{12x^4(d+ex)}\,dx,x,x$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{24}(ep)\,\text{Subst}\left(\int \frac{-3f-4gx}{x^4(d+ex)}\,dx,x,x^2$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{24}(ep)\,\text{Subst}\left(\int\left(-\frac{3f}{dx^4}+\frac{3ef-4dg}{d^2x^3}\right.\right.$$

$$= -\frac{efp}{24dx^6} + \frac{e(3ef-4dg)p}{48d^2x^4} - \frac{e^2(3ef-4dg)p}{24d^3x^2} - \frac{e^3(3ef-4dg)p\log(x)}{12d^4} + \frac{e^3(3ef-4dg}{2}$$

Mathematica [A] time = 0.110856, size = 158, normalized size = 1.07

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} + \frac{1}{8}efp\left(-\frac{e^2}{d^3x^2}+\frac{e^3\log\left(d+ex^2\right)}{d^4}-\frac{2e^3\log(x)}{d^4}+\frac{e}{2d^2x^4}-\frac{1}{3dx^6}\right)+\frac{1}{6}egp\left(-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] (e*g*p*(-1/(2*d*x^4) + e/(d^2*x^2) + (2*e^2*Log[x])/d^3 - (e^2*Log[d + e*x^
2])/d^3))/6 + (e*f*p*(-1/(3*d*x^6) + e/(2*d^2*x^4) - e^2/(d^3*x^2) - (2*e^3
*Log[x])/d^4 + (e^3*Log[d + e*x^2])/d^4))/8 - (f*Log[c*(d + e*x^2)^p])/(8*x
^8) - (g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Maple [C] time = 0.388, size = 448, normalized size = 3.

$$-\frac{(4gx^2+3f)\ln\left(\left(ex^2+d\right)^p\right)}{24x^8} - \frac{-16\ln(x)de^3gpx^8+12\ln(x)e^4fpx^8+8\ln\left(ex^2+d\right)de^3gpx^8-6\ln\left(ex^2+d\right)e^4fpx^8 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^9,x)
```

```
[Out] -1/24*(4*g*x^2+3*f)/x^8*ln((e*x^2+d)^p)-1/48*(-16*ln(x)*d*e^3*g*p*x^8+12*ln
(x)*e^4*f*p*x^8+8*ln(e*x^2+d)*d*e^3*g*p*x^8-6*ln(e*x^2+d)*e^4*f*p*x^8-3*I*P
i*d^4*f*csgn(I*c*(e*x^2+d)^p)^3+4*I*Pi*d^4*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*cs
gn(I*c)+4*I*Pi*d^4*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3*I*Pi
*d^4*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-8*d^2*e^2*g*p*x^6+6*d*e^
3*f*p*x^6-4*I*Pi*d^4*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I
*c)+3*I*Pi*d^4*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-4*I*Pi*d^4*g*x^2*csgn(I*
c*(e*x^2+d)^p)^3-3*I*Pi*d^4*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)+4*d^3*e*g*p*x^4-3*d^2*e^2*f*p*x^4+8*ln(c)*d^4*g*x^2+2*d^3*e*f*p*x^2+
6*ln(c)*d^4*f)/d^4/x^8
```

Maxima [A] time = 1.03066, size = 178, normalized size = 1.2

$$\frac{1}{48}\,ep\left(\frac{2\left(3\,e^3f-4\,de^2g\right)\log\left(ex^2+d\right)}{d^4}-\frac{2\left(3\,e^3f-4\,de^2g\right)\log\left(x^2\right)}{d^4}-\frac{2\left(3\,e^2f-4\,deg\right)x^4+2\,d^2f-\left(3\,def-4\,d^2g\right)x^2}{d^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] 1/48*e*p*(2*(3*e^3*f - 4*d*e^2*g)*log(e*x^2 + d)/d^4 - 2*(3*e^3*f - 4*d*e^2
*g)*log(x^2)/d^4 - (2*(3*e^2*f - 4*d*e*g)*x^4 + 2*d^2*f - (3*d*e*f - 4*d^2*
g)*x^2)/(d^3*x^6)) - 1/24*(4*g*x^2 + 3*f)*log((e*x^2 + d)^p*c)/x^8
```

Fricas [A] time = 2.23226, size = 346, normalized size = 2.34

$$-\frac{4\left(3\,e^4f-4\,de^3g\right)px^8\log\left(x\right)+2\,d^3efpx^2+2\left(3\,de^3f-4\,d^2e^2g\right)px^6-\left(3\,d^2e^2f-4\,d^3eg\right)px^4-2\left(\left(3\,e^4f-4\,de^3g\right)px^8\right.}{48\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

```
[Out] -1/48*(4*(3*e^4*f - 4*d*e^3*g)*p*x^8*log(x) + 2*d^3*e*f*p*x^2 + 2*(3*d*e^3*
f - 4*d^2*e^2*g)*p*x^6 - (3*d^2*e^2*f - 4*d^3*e*g)*p*x^4 - 2*((3*e^4*f - 4*
d*e^3*g)*p*x^8 - 4*d^4*g*p*x^2 - 3*d^4*f*p)*log(e*x^2 + d) + 2*(4*d^4*g*x^2
 + 3*d^4*f)*log(c))/(d^4*x^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30204, size = 910, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

```
[Out] -1/48*(8*(x^2*e + d)^4*d*g*p*e^4*log(x^2*e + d) - 32*(x^2*e + d)^3*d^2*g*p*
e^4*log(x^2*e + d) + 48*(x^2*e + d)^2*d^3*g*p*e^4*log(x^2*e + d) - 24*(x^2*
e + d)*d^4*g*p*e^4*log(x^2*e + d) - 8*(x^2*e + d)^4*d*g*p*e^4*log(x^2*e) +
32*(x^2*e + d)^3*d^2*g*p*e^4*log(x^2*e) - 48*(x^2*e + d)^2*d^3*g*p*e^4*log(
x^2*e) + 32*(x^2*e + d)*d^4*g*p*e^4*log(x^2*e) - 8*d^5*g*p*e^4*log(x^2*e) -
 8*(x^2*e + d)^3*d^2*g*p*e^4 + 28*(x^2*e + d)^2*d^3*g*p*e^4 - 32*(x^2*e + d
)*d^4*g*p*e^4 + 12*d^5*g*p*e^4 - 6*(x^2*e + d)^4*f*p*e^5*log(x^2*e + d) + 2
4*(x^2*e + d)^3*d*f*p*e^5*log(x^2*e + d) - 36*(x^2*e + d)^2*d^2*f*p*e^5*log
(x^2*e + d) + 24*(x^2*e + d)*d^3*f*p*e^5*log(x^2*e + d) + 6*(x^2*e + d)^4*f
*p*e^5*log(x^2*e) - 24*(x^2*e + d)^3*d*f*p*e^5*log(x^2*e) + 36*(x^2*e + d)^
2*d^2*f*p*e^5*log(x^2*e) - 24*(x^2*e + d)*d^3*f*p*e^5*log(x^2*e) + 6*d^4*f*
p*e^5*log(x^2*e) + 8*(x^2*e + d)*d^4*g*e^4*log(c) - 8*d^5*g*e^4*log(c) + 6*
(x^2*e + d)^3*d*f*p*e^5 - 21*(x^2*e + d)^2*d^2*f*p*e^5 + 26*(x^2*e + d)*d^3
*f*p*e^5 - 11*d^4*f*p*e^5 + 6*d^4*f*e^5*log(c))*e^(-1)/((x^2*e + d)^4*d^4 -
 4*(x^2*e + d)^3*d^5 + 6*(x^2*e + d)^2*d^6 - 4*(x^2*e + d)*d^7 + d^8)
```

3.318 $\int x^2 \left(f + gx^2\right) \log \left(c \left(d + ex^2\right)^p\right) dx$

Optimal. Leaf size=154

$$\frac{1}{3} f x^3 \log \left(c \left(d + ex^2\right)^p\right) + \frac{1}{5} g x^5 \log \left(c \left(d + ex^2\right)^p\right) - \frac{2 d^{3/2} f p \tan^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{3 e^{3/2}} - \frac{2 d^2 g p x}{5 e^2} + \frac{2 d^{5/2} g p \tan^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{5 e^{5/2}} + \frac{2 d f p x}{3 e} +$$

```
[Out] (2*d*f*p*x)/(3*e) - (2*d^2*g*p*x)/(5*e^2) - (2*f*p*x^3)/9 + (2*d*g*p*x^3)/(
15*e) - (2*g*p*x^5)/25 - (2*d^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(
3/2)) + (2*d^(5/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + (f*x^3*Lo
g[c*(d + e*x^2)^p])/3 + (g*x^5*Log[c*(d + e*x^2)^p])/5
```

Rubi [A] time = 0.129955, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2476, 2455, 302, 205}

$$\frac{1}{3} f x^3 \log \left(c \left(d + ex^2\right)^p\right) + \frac{1}{5} g x^5 \log \left(c \left(d + ex^2\right)^p\right) - \frac{2 d^{3/2} f p \tan^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{3 e^{3/2}} - \frac{2 d^2 g p x}{5 e^2} + \frac{2 d^{5/2} g p \tan^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{5 e^{5/2}} + \frac{2 d f p x}{3 e} +$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f*p*x)/(3*e) - (2*d^2*g*p*x)/(5*e^2) - (2*f*p*x^3)/9 + (2*d*g*p*x^3)/(
15*e) - (2*g*p*x^5)/25 - (2*d^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(
3/2)) + (2*d^(5/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + (f*x^3*Lo
g[c*(d + e*x^2)^p])/3 + (g*x^5*Log[c*(d + e*x^2)^p])/5
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x^2\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)dx=\int\left(fx^2\log\left(c\left(d+ex^2\right)^p\right)+gx^4\log\left(c\left(d+ex^2\right)^p\right)\right)dx$$
$$=f\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx+g\int x^4\log\left(c\left(d+ex^2\right)^p\right)dx$$
$$=\frac{1}{3}fx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}gx^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{3}(2efp)\int\frac{x^4}{d+ex^2}dx-\frac{1}{5}(2\ldots$$
$$=\frac{1}{3}fx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}gx^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{3}(2efp)\int\left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{\ldots}{e^2(\ldots}\right.$$
$$=\frac{2dfpx}{3e}-\frac{2d^2gpx}{5e^2}-\frac{2}{9}fpx^3+\frac{2dgpx^3}{15e}-\frac{2}{25}gpx^5+\frac{1}{3}fx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}gx^{\ldots}$$
$$=\frac{2dfpx}{3e}-\frac{2d^2gpx}{5e^2}-\frac{2}{9}fpx^3+\frac{2dgpx^3}{15e}-\frac{2}{25}gpx^5-\frac{2d^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}gp\ldots}{\ldots}$$

Mathematica [A] time = 0.0562203, size = 118, normalized size = 0.77

$$\frac{\sqrt{e}x\left(15e^2x^2\left(5f+3gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)-2p\left(45d^2g-15de\left(5f+gx^2\right)+e^2x^2\left(25f+9gx^2\right)\right)\right)+30d^{3/2}p(3dg-5ef)\,\mathrm{ta}\ldots}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*d^(3/2)*(-5*e*f + 3*d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[e]*x*(-2*
p*(45*d^2*g - 15*d*e*(5*f + g*x^2) + e^2*x^2*(25*f + 9*g*x^2)) + 15*e^2*x^2
*(5*f + 3*g*x^2)*Log[c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Maple [C] time = 0.586, size = 453, normalized size = 2.9

$$\left(\frac{gx^5}{5}+\frac{fx^3}{3}\right)\ln\left(\left(ex^2+d\right)^p\right)-\frac{i}{10}\pi gx^5\left(\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^3+\frac{i}{10}\pi gx^5\left(\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2\mathrm{csgn}\left(ic\right)-\frac{i}{10}\pi gx^5\mathrm{csg}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/5*g*x^5+1/3*f*x^3)*ln((e*x^2+d)^p)-1/10*I*Pi*g*x^5*csgn(I*c*(e*x^2+d)^p)
^3+1/10*I*Pi*g*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/10*I*Pi*g*x^5*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/10*I*Pi*g*x^5*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/6*I*Pi*f*x^3*csgn(I*c*(e*x^2+d)^p)^3+1/6*I
*Pi*f*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/6*I*Pi*f*x^3*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/6*I*Pi*f*x^3*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)^2+1/5*ln(c)*g*x^5-2/25*g*p*x^5+1/3*ln(c)*f*x^3+2/15*d*g*p
*x^3/e-2/9*f*p*x^3-1/5/e^3*(-d*e)^(1/2)*p*d^2*ln((-d*e)^(1/2)*x+d)*g+1/3/e^
2*(-d*e)^(1/2)*p*d*ln((-d*e)^(1/2)*x+d)*f+1/5/e^3*(-d*e)^(1/2)*p*d^2*ln(-(-
d*e)^(1/2)*x+d)*g-1/3/e^2*(-d*e)^(1/2)*p*d*ln(-(-d*e)^(1/2)*x+d)*f-2/5*d^2*
g*p*x/e^2+2/3*d*f*p*x/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37542, size = 693, normalized size = 4.5

$$\left[-\frac{18\,e^2gpx^5 + 10\left(5\,e^2f - 3\,deg\right)px^3 + 15\left(5\,def - 3\,d^2g\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right) - 30\left(5\,def - 3\,d^2g\right)px - 15\left(3\,e^2gp\right.}{225\,e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g*p*x^5 + 10*(5*e^2*f - 3*d*e*g)*p*x^3 + 15*(5*d*e*f - 3*d^
2*g)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 30*(5*d
*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*x^3)*log(e*x^2 + d) - 1
5*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2, -1/225*(18*e^2*g*p*x^5 + 10*(5*e
^2*f - 3*d*e*g)*p*x^3 + 30*(5*d*e*f - 3*d^2*g)*p*sqrt(d/e)*arctan(e*x*sqrt(
d/e)/d) - 30*(5*d*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*x^3)*l
og(e*x^2 + d) - 15*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48279, size = 186, normalized size = 1.21

$$\frac{2\left(3\,d^3gp - 5\,d^2fpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}} + \frac{1}{225}\left(45\,gpx^5e^2\log\left(x^2e + d\right) - 18\,gpx^5e^2 + 45\,gx^5e^2\log\left(c\right) + 30\,dgpx^3e + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g*p - 5*d^2*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) +
 1/225*(45*g*p*x^5*e^2*log(x^2*e + d) - 18*g*p*x^5*e^2 + 45*g*x^5*e^2*log(c
```

```
) + 30*d*g*p*x^3*e + 75*f*p*x^3*e^2*log(x^2*e + d) - 50*f*p*x^3*e^2 + 75*f*
x^3*e^2*log(c) - 90*d^2*g*p*x + 150*d*f*p*x*e)*e^(-2)
```

3.319 $\int \left(f+gx^2\right)\log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=117

$$fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log \left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan ^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan ^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx$$

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rubi [A] time = 0.0856192, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log \left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan ^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan ^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```


```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```


Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)dx &= \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^2\log\left(c\left(d+ex^2\right)^p\right)\right)dx\\
&= f\int \log\left(c\left(d+ex^2\right)^p\right)dx+g\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx\\
&= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-(2efp)\int \frac{x^2}{d+ex^2}dx-\frac{1}{3}(2egp)\int\\
&= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int \frac{1}{d+ex^2}dx-\frac{1}{3}\\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\left(c\right(
\end{aligned}
$$

Mathematica [A] time = 0.035529, size = 117, normalized size = 1.

$$
fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Maple [C] time = 0.102, size = 416, normalized size = 3.6

$$
\left(\frac{gx^3}{3}+fx\right)\ln\left(\left(ex^2+d\right)^p\right)+\frac{i}{6}\pi gx^3\left(\operatorname{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2\operatorname{csgn}(ic)-\frac{i}{6}\pi gx^3\operatorname{csgn}\left(i\left(ex^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(ex^2+d\right)^p\right)\operatorname{cs}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/3*g*x^3+f*x)*ln((e*x^2+d)^p)+1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn
(I*c)-1/6*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/
```

```
2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x-1/6*I*Pi*g*x^3*csgn(
I*c*(e*x^2+d)^p)^3-1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)*x+1/2*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x+1/6*I*Pi*g*x^3*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^3*
x+1/3*ln(c)*g*x^3-2/9*g*p*x^3-1/3/e^2*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*d
*g+1/e*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*f+1/3/e^2*(-d*e)^(1/2)*p*ln(-(-d
*e)^(1/2)*x-d)*d*g-1/e*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*f+ln(c)*f*x+2/3
*d*g*p*x/e-2*f*p*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26087, size = 504, normalized size = 4.31

$$\left[-\frac{2\,egpx^3+3\left(3\,ef-dg\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+6\left(3\,ef-dg\right)px-3\left(egpx^3+3\,efpx\right)\log\left(ex^2+d\right)-3\left(egx^3+3\right.}{9\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-
d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*lo
g(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3*e*
f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*(e*g
*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

Sympy [A] time = 35.0358, size = 228, normalized size = 1.95

$$\begin{cases} -\frac{id^{\frac{3}{2}}gp\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{2id^{\frac{3}{2}}gp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{i\sqrt{d}fp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}fp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}}+\frac{2dgpx}{3e}+fpx\log\left(d+ex^2\right)-2fpx+ \\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-I*d**(3/2)*g*p*log(d + e*x**2)/(3*e**2*sqrt(1/e)) + 2*I*d**(3/2
)*g*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*e**2*sqrt(1/e)) + I*sqrt(d)*f*p*log(
d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*f*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(
e*sqrt(1/e)) + 2*d*g*p*x/(3*e) + f*p*x*log(d + e*x**2) - 2*f*p*x + f*x*log(
```

```
c) + g*p*x**3*log(d + e*x**2)/3 - 2*g*p*x**3/9 + g*x**3*log(c)/3, Ne(e, 0))
, ((f*x + g*x**3/3)*log(c*d**p), True))
```

Giac [A] time = 1.16656, size = 147, normalized size = 1.26

$$-\frac{2\left(d^{2}gp-3\,dfpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}}+\frac{1}{9}\left(3\,gpx^{3}e\log\left(x^{2}e+d\right)-2\,gpx^{3}e+3\,gx^{3}e\log\left(c\right)+9\,fpxe\log\left(x^{2}e+d\right)+6\,dg\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g*p - 3*d*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) + 1/9
*(3*g*p*x^3*e*log(x^2*e + d) - 2*g*p*x^3*e + 3*g*x^3*e*log(c) + 9*f*p*x*e*l
og(x^2*e + d) + 6*d*g*p*x - 18*f*p*x*e + 9*f*x*e*log(c))*e^(-1)
```

3.320 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)+\frac{2p(dg+ef)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}-2gpx$$

```
[Out] -2*g*p*x + (2*(e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
- (f*Log[c*(d + e*x^2)^p])/x + g*x*Log[c*(d + e*x^2)^p]
```

Rubi [A] time = 0.0838046, antiderivative size = 93, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2476, 2448, 321, 205, 2455}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)+\frac{2\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-2gpx$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -2*g*p*x + (2*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[d]
*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f*Log[c*(d + e*x^2)^p])/x + g*
x*Log[c*(d + e*x^2)^p]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx = \int \left(g\log\left(c\left(d+ex^2\right)^p\right)+\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right)dx$$

$$= f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx + g\int \log\left(c\left(d+ex^2\right)^p\right)dx$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x} + gx\log\left(c\left(d+ex^2\right)^p\right) + (2efp)\int \frac{1}{d+ex^2}\,dx - (2egp)\int \frac{x}{d+}$$

$$= -2gpx + \frac{2\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x} + gx\log\left(c\left(d+ex^2\right)^p\right) + (2dgp)$$

$$= -2gpx + \frac{2\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x} + gx\log$$

Mathematica [A] time = 0.0457904, size = 62, normalized size = 0.86

$$\left(gx-\frac{f}{x}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{2p(dg+ef)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}-2gpx$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -2*g*p*x + (2*(e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
+ (-(f/x) + g*x)*Log[c*(d + e*x^2)^p]
```

Maple [C] time = 0.586, size = 427, normalized size = 5.9

$$-\frac{\left(-gx^2+f\right)\ln\left(\left(ex^2+d\right)^p\right)}{x}+\frac{1}{2dex}\left(i\pi gx^2\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\left(\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2de-i\pi gx^2\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\mathrm{csg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^2,x)
```

```
[Out] -(-g*x^2+f)/x*ln((e*x^2+d)^p)+1/2*(I*Pi*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2*d*e-I*Pi*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)*d*e-I*Pi*g*x^2*csgn(I*c*(e*x^2+d)^p)^3*d*e+I*Pi*g*x^2*csgn(I*c*(e*x^
2+d)^p)^2*csgn(I*c)*d*e-I*Pi*d*e*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)^2+I*Pi*d*e*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*d*e
*f*csgn(I*c*(e*x^2+d)^p)^3-I*Pi*d*e*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*l
n(c)*g*x^2*d*e+2*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d)*g*d*x+2*(-d*e)^(1/2)*
p*ln(-(-d*e)^(1/2)*x+d)*f*e*x-2*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*g*d*x-
```

```
2*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*f*e*x-4*d*g*p*x^2*e-2*ln(c)*d*e*f)/d
/e/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50089, size = 436, normalized size = 6.06

$$\left[-\frac{2\,degpx^2+\sqrt{-de}\left(ef+dg\right)px\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)-\left(degpx^2-defp\right)\log\left(ex^2+d\right)-\left(degx^2-def\right)\log\left(c\right)}{dex},-\frac{2\,degpx^2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [-(2*d*e*g*p*x^2 + sqrt(-d*e)*(e*f + d*g)*p*x*log((e*x^2 - 2*sqrt(-d*e)*x -
 d)/(e*x^2 + d)) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 - d*
e*f)*log(c))/(d*e*x), -(2*d*e*g*p*x^2 - 2*sqrt(d*e)*(e*f + d*g)*p*x*arctan(
sqrt(d*e)*x/d) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 - d*e*
f)*log(c))/(d*e*x)]
```

Sympy [A] time = 62.6268, size = 262, normalized size = 3.64

$$\begin{cases}\left(-\frac{f}{x}+gx\right)\log\left(0^{p}c\right)\\ \left(-\frac{f}{x}+gx\right)\log\left(cd^{p}\right)\\ -\frac{fp\log(e)}{x}-\frac{2fp\log(x)}{x}-\frac{2fp}{x}-\frac{f\log(c)}{x}+gpx\log(e)+2gpx\log(x)-2gpx+gx\log(c)\\ \frac{i\sqrt{d}gp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}gp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}}-\frac{fp\log\left(d+ex^2\right)}{x}-\frac{f\log(c)}{x}+gpx\log\left(d+ex^2\right)-2gpx+gx\log(c)+\frac{ifp\log\left(d+ex^2\right)}{\sqrt{d}\sqrt{\frac{1}{e}}}-\cdots\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Piecewise(((-f/x + g*x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/x + g*x)*lo
g(c*d**p), Eq(e, 0)), (-f*p*log(e)/x - 2*f*p*log(x)/x - 2*f*p/x - f*log(c)/
x + g*p*x*log(e) + 2*g*p*x*log(x) - 2*g*p*x + g*x*log(c), Eq(d, 0)), (I*sqr
t(d)*g*p*log(d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*g*p*log(-I*sqrt(d)*sqr
t(1/e) + x)/(e*sqrt(1/e)) - f*p*log(d + e*x**2)/x - f*log(c)/x + g*p*x*log(
d + e*x**2) - 2*g*p*x + g*x*log(c) + I*f*p*log(d + e*x**2)/(sqrt(d)*sqrt(1/
e)) - 2*I*f*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(sqrt(d)*sqrt(1/e)), True))
```

Giac [A] time = 1.26186, size = 105, normalized size = 1.46

$$\frac{2\left(dgp + fpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}} + \frac{gpx^2\log\left(x^2e + d\right) - 2\,gpx^2 + gx^2\log\left(c\right) - fp\log\left(x^2e + d\right) - f\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*(d*g*p + f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + (g*p*x^2*log
(x^2*e + d) - 2*g*p*x^2 + g*x^2*log(c) - f*p*log(x^2*e + d) - f*log(c))/x
```

3.321 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=108

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x}-\frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2efp}{3dx}+\frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] (-2*e*f*p)/(3*d*x) - (2*e^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)
) + (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (f*Log[c*(d + e*x
^2)^p])/(3*x^3) - (g*Log[c*(d + e*x^2)^p])/x
```

Rubi [A] time = 0.0998344, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2476, 2455, 325, 205}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x}-\frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2efp}{3dx}+\frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f*p)/(3*d*x) - (2*e^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)
) + (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (f*Log[c*(d + e*x
^2)^p])/(3*x^3) - (g*Log[c*(d + e*x^2)^p])/x
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx = \int \left(\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^4}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right)dx$$
$$= f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx + g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx$$
$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x} + \frac{1}{3}(2efp)\int \frac{1}{x^2\left(d+ex^2\right)}\,dx + (2egp)$$
$$= -\frac{2efp}{3dx} + \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x} - (2e^2fp$$
$$= -\frac{2efp}{3dx} - \frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - g\log($$

Mathematica [C] time = 0.0394848, size = 96, normalized size = 0.89

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x} - \frac{2efp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx} + \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f*p*Hypergeometr
ic2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d + e*x^2)^p])/(3*x^
3) - (g*Log[c*(d + e*x^2)^p])/x
```

Maple [C] time = 0.566, size = 430, normalized size = 4.

$$-\frac{(3gx^2+f)\ln\left(\left(ex^2+d\right)^p\right)}{3x^3} + \frac{-3i\pi dgx^2\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\left(\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2 + 3i\pi dgx^2\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\mathrm{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^4,x)
```

```
[Out] -1/3*(3*g*x^2+f)/x^3*ln((e*x^2+d)^p)+1/6*(-3*I*Pi*d*g*x^2*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)^2+3*I*Pi*d*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)*csgn(I*c)+3*I*Pi*d*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d*g*x^2*cs
gn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-I*Pi*d*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)^2+I*Pi*d*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi
*d*f*csgn(I*c*(e*x^2+d)^p)^3-I*Pi*d*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*l
n(c)*d*g*x^2+2*sum(_R*ln((18*d^2*e*g^2*p^2-12*d*e^2*f*g*p^2+2*e^3*f^2*p^2+3
*_R^2*d^3)*x+(-3*d^3*g*p+d^2*e*f*p)*_R),_R=RootOf(9*d^2*e*g^2*p^2-6*d*e^2*f
*g*p^2+e^3*f^2*p^2+_Z^2*d^3))*d*x^3-4*e*f*p*x^2-2*ln(c)*d*f)/d/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40017, size = 441, normalized size = 4.08

$$\left[-\frac{\left(ef-3\,dg\right)px^3\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)+2\,efpx^2+\left(3\,dgpx^2+dfp\right)\log\left(ex^2+d\right)+\left(3\,dgx^2+df\right)\log\left(c\right)}{3\,dx^3},-\frac{2\left(ef\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*((e*f - 3*d*g)*p*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e
*x^2 + d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*x^
2 + d*f)*log(c))/(d*x^3), -1/3*(2*(e*f - 3*d*g)*p*x^3*sqrt(e/d)*arctan(x*sq
rt(e/d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*x^2
+ d*f)*log(c))/(d*x^3)]
```

Sympy [A] time = 131.902, size = 1469, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Piecewise(((-f/(3*x**3) - g/x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (-f*p*log
(e)/(3*x**3) - 2*f*p*log(x)/(3*x**3) - 2*f*p/(9*x**3) - f*log(c)/(3*x**3) -
 g*p*log(e)/x - 2*g*p*log(x)/x - 2*g*p/x - g*log(c)/x, Eq(d, 0)), ((-f/(3*x
**3) - g/x)*log(c*d**p), Eq(e, 0)), (-I*d**(7/2)*f*p*sqrt(1/e)*log(d + e*x*
*2)/(3*I*d**(7/2)*x**3*sqrt(1/e) + 3*I*d**(5/2)*e*x**5*sqrt(1/e)) - I*d**(7
/2)*f*sqrt(1/e)*log(c)/(3*I*d**(7/2)*x**3*sqrt(1/e) + 3*I*d**(5/2)*e*x**5*s
qrt(1/e)) - 3*I*d**(7/2)*g*p*x**2*sqrt(1/e)*log(d + e*x**2)/(3*I*d**(7/2)*x
**3*sqrt(1/e) + 3*I*d**(5/2)*e*x**5*sqrt(1/e)) - 3*I*d**(7/2)*g*x**2*sqrt(1
/e)*log(c)/(3*I*d**(7/2)*x**3*sqrt(1/e) + 3*I*d**(5/2)*e*x**5*sqrt(1/e)) -
I*d**(5/2)*f*p*x**2*sqrt(1/e)*log(d + e*x**2)/(3*I*d**(7/2)*x**3*sqrt(1/e)/
e + 3*I*d**(5/2)*x**5*sqrt(1/e)) - 2*I*d**(5/2)*f*p*x**2*sqrt(1/e)/(3*I*d**
(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) - I*d**(5/2)*f*x**2*s
qrt(1/e)*log(c)/(3*I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e
)) - 3*I*d**(5/2)*g*p*x**4*sqrt(1/e)*log(d + e*x**2)/(3*I*d**(7/2)*x**3*sqr
t(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) - 3*I*d**(5/2)*g*x**4*sqrt(1/e)*log
(c)/(3*I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) - 2*I*d**
(3/2)*e*f*p*x**4*sqrt(1/e)/(3*I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x*
*5*sqrt(1/e)) - 3*d**3*g*p*x**3*log(d + e*x**2)/(3*I*d**(7/2)*x**3*sqrt(1/e
```

```
) + 3*I*d**(5/2)*e*x**5*sqrt(1/e)) + 6*d**3*g*p*x**3*log(-I*sqrt(d)*sqrt(1/
e) + x)/(3*I*d**(7/2)*x**3*sqrt(1/e) + 3*I*d**(5/2)*e*x**5*sqrt(1/e)) - 3*d
**3*g*x**3*log(c)/(3*I*d**(7/2)*x**3*sqrt(1/e) + 3*I*d**(5/2)*e*x**5*sqrt(1
/e)) + d**2*f*p*x**3*log(d + e*x**2)/(3*I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d
**(5/2)*x**5*sqrt(1/e)) - 2*d**2*f*p*x**3*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*
I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) + d**2*f*x**3*lo
g(c)/(3*I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) - 3*d**2
*g*p*x**5*log(d + e*x**2)/(3*I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**
5*sqrt(1/e)) + 6*d**2*g*p*x**5*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*I*d**(7/2)*
x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) - 3*d**2*g*x**5*log(c)/(3*I
*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) + d*e*f*p*x**5*lo
g(d + e*x**2)/(3*I*d**(7/2)*x**3*sqrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e))
 - 2*d*e*f*p*x**5*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*I*d**(7/2)*x**3*sqrt(1/e
)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)) + d*e*f*x**5*log(c)/(3*I*d**(7/2)*x**3*s
qrt(1/e)/e + 3*I*d**(5/2)*x**5*sqrt(1/e)), True))
```

Giac [A] time = 1.24153, size = 124, normalized size = 1.15

$$\frac{2\left(3\,dgpe-fpe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d^{\frac{3}{2}}}-\frac{3\,dgpx^2\log\left(x^2e+d\right)+2\,fpx^2e+3\,dgx^2\log\left(c\right)+dfp\log\left(x^2e+d\right)+df\log\left(c\right)}{3\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*d*g*p*e - f*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) - 1/3*
(3*d*g*p*x^2*log(x^2*e + d) + 2*f*p*x^2*e + 3*d*g*x^2*log(c) + d*f*p*log(x^
2*e + d) + d*f*log(c))/(d*x^3)
```

3.322 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=140

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{2e^2fp}{5d^2x}+\frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2efp}{15dx^3}-\frac{2egp}{3dx}$$

```
[Out] (-2*e*f*p)/(15*d*x^3) + (2*e^2*f*p)/(5*d^2*x) - (2*e*g*p)/(3*d*x) + (2*e^(5
/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f*Log[c*(d + e*x^2)^p])/(5*x^5) - (g*Log
[c*(d + e*x^2)^p])/(3*x^3)
```

Rubi [A] time = 0.121402, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2476, 2455, 325, 205}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{2e^2fp}{5d^2x}+\frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2efp}{15dx^3}-\frac{2egp}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p)/(15*d*x^3) + (2*e^2*f*p)/(5*d^2*x) - (2*e*g*p)/(3*d*x) + (2*e^(5
/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f*Log[c*(d + e*x^2)^p])/(5*x^5) - (g*Log
[c*(d + e*x^2)^p])/(3*x^3)
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx = \int \left(\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^6}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\right)dx$$

$$= f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx + g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} + \frac{1}{5}(2efp)\int \frac{1}{x^4\left(d+ex^2\right)}\,dx + \frac{1}{3}(2egp$$

$$= -\frac{2efp}{15dx^3} - \frac{2egp}{3dx} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{\left(2e^2fp\right)\int \frac{1}{x^2\left(d+ex^2\right)}\,d}{5d}$$

$$= -\frac{2efp}{15dx^3} + \frac{2e^2fp}{5d^2x} - \frac{2egp}{3dx} - \frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g\log\left(c\left(d\right.\right.}{3x^3}$$

$$= -\frac{2efp}{15dx^3} + \frac{2e^2fp}{5d^2x} - \frac{2egp}{3dx} + \frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - \frac{f\log\left(c\left(d\right.\right.}{5x}$$

Mathematica [C] time = 0.0055823, size = 101, normalized size = 0.72

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{2efp\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^2}{d}\right)}{15dx^3} - \frac{2egp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e
*g*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d +
 e*x^2)^p])/(5*x^5) - (g*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Maple [C] time = 0.574, size = 483, normalized size = 3.5

$$-\frac{\left(5gx^2+3f\right)\ln\left(\left(ex^2+d\right)^p\right)}{15x^5} + \frac{-5i\pi d^2gx^2\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\left(\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2 + 5i\pi d^2gx^2\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^6,x)
```

```
[Out] -1/15*(5*g*x^2+3*f)/x^5*ln((e*x^2+d)^p)+1/30*(-5*I*Pi*d^2*g*x^2*csgn(I*(e*x
^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+5*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)*csgn(I*c)+5*I*Pi*d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-5*I*Pi*
d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3*I*Pi*f*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2*d^2+3*I*Pi*d^2*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)+3*I*Pi*d^2*f*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*f*csgn(I*c*(e*x
^2+d)^p)^2*csgn(I*c)*d^2+2*sum(_R*ln((50*d^2*e^3*g^2*p^2-60*d*e^4*f*g*p^2+1
```

```
8*e^5*f^2*p^2+3*_R^2*d^5)*x+(5*d^4*e*g*p-3*d^3*e^2*f*p)*_R),_R=RootOf(25*d^
2*e^3*g^2*p^2-30*d*e^4*f*g*p^2+9*e^5*f^2*p^2+_Z^2*d^5))*d^2*x^5-20*d*e*g*p*
x^4+12*e^2*f*p*x^4-10*ln(c)*d^2*g*x^2-4*d*e*f*p*x^2-6*ln(c)*d^2*f)/d^2/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93368, size = 583, normalized size = 4.16

$$\left[-\frac{\left(3\,e^2 f - 5\,deg\right)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2-2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) + 2\,defpx^2 - 2\left(3\,e^2 f - 5\,deg\right)px^4 + \left(5\,d^2gpx^2 + 3\,d^2fp\right)\log\left(ex^2+d\right) +}{15\,d^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/15*((3*e^2*f - 5*d*e*g)*p*x^5*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d)
- d)/(e*x^2 + d)) + 2*d*e*f*p*x^2 - 2*(3*e^2*f - 5*d*e*g)*p*x^4 + (5*d^2*g*
p*x^2 + 3*d^2*f*p)*log(e*x^2 + d) + (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^
5), 1/15*(2*(3*e^2*f - 5*d*e*g)*p*x^5*sqrt(e/d)*arctan(x*sqrt(e/d)) - 2*d*e
*f*p*x^2 + 2*(3*e^2*f - 5*d*e*g)*p*x^4 - (5*d^2*g*p*x^2 + 3*d^2*f*p)*log(e*
x^2 + d) - (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32981, size = 165, normalized size = 1.18

$$-\frac{2\left(5\,dgpe^2 - 3\,fpe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{15\,d^{\frac{5}{2}}} - \frac{10\,dgpx^4e - 6\,fpx^4e^2 + 5\,d^2gpx^2\log\left(x^2e+d\right) + 2\,dfpx^2e + 5\,d^2gx^2\log\left(c\right.}{15\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] -2/15*(5*d*g*p*e^2 - 3*f*p*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(5/2)
- 1/15*(10*d*g*p*x^4*e - 6*f*p*x^4*e^2 + 5*d^2*g*p*x^2*log(x^2*e + d) + 2*d
*f*p*x^2*e + 5*d^2*g*x^2*log(c) + 3*d^2*f*p*log(x^2*e + d) + 3*d^2*f*log(c)
)/(d^2*x^5)
```

3.323 $\int x^5 \left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=251

$$\frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{p\left(d+ex^2\right)^3\left(6d^2g^2-6defg+e^2f\right.}{18e^5}$$

```
[Out] -(d^2*(e*f - d*g)^2*p*x^2)/(2*e^4) + (d*(e*f - 2*d*g)*(e*f - d*g)*p*(d + e*
x^2)^2)/(4*e^5) - ((e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)*p*(d + e*x^2)^3)/(18*e
^5) - (g*(e*f - 2*d*g)*p*(d + e*x^2)^4)/(16*e^5) - (g^2*p*(d + e*x^2)^5)/(5
0*e^5) + (d^3*(10*e^2*f^2 - 15*d*e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2])/(60*e
^5) + (f^2*x^6*Log[c*(d + e*x^2)^p])/6 + (f*g*x^8*Log[c*(d + e*x^2)^p])/4 +
 (g^2*x^10*Log[c*(d + e*x^2)^p])/10
```

Rubi [A] time = 0.470699, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2475, 43, 2414, 12, 893}

$$\frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{p\left(d+ex^2\right)^3\left(6d^2g^2-6defg+e^2f\right.}{18e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -(d^2*(e*f - d*g)^2*p*x^2)/(2*e^4) + (d*(e*f - 2*d*g)*(e*f - d*g)*p*(d + e*
x^2)^2)/(4*e^5) - ((e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)*p*(d + e*x^2)^3)/(18*e
^5) - (g*(e*f - 2*d*g)*p*(d + e*x^2)^4)/(16*e^5) - (g^2*p*(d + e*x^2)^5)/(5
0*e^5) + (d^3*(10*e^2*f^2 - 15*d*e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2])/(60*e
^5) + (f^2*x^6*Log[c*(d + e*x^2)^p])/6 + (f*g*x^8*Log[c*(d + e*x^2)^p])/4 +
 (g^2*x^10*Log[c*(d + e*x^2)^p])/10
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int x^5 \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right) dx = \frac{1}{2}\text{Subst}\left(\int x^2(f+gx)^2 \log\left(c(d+ex)^p\right) dx, x, x^2\right)$$

$$= \frac{1}{6}f^2x^6 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{4}fgx^8 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{10}g^2x^{10} \log\left(c\left(d+ex^2\right)^p\right.$$

$$= \frac{1}{6}f^2x^6 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{4}fgx^8 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{10}g^2x^{10} \log\left(c\left(d+ex^2\right)^p\right.$$

$$= \frac{1}{6}f^2x^6 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{4}fgx^8 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{10}g^2x^{10} \log\left(c\left(d+ex^2\right)^p\right.$$

$$= -\frac{d^2(ef-dg)^2px^2}{2e^4} + \frac{d(ef-2dg)(ef-dg)p\left(d+ex^2\right)^2}{4e^5} - \frac{\left(e^2f^2-6defg+6d^2g^2\right)}{18e^5}$$

Mathematica [A] time = 0.171382, size = 205, normalized size = 0.82

$$\frac{60e^5x^6\left(10f^2+15fgx^2+6g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right) - epx^2\left(30d^2e^2\left(20f^2+15fgx^2+4g^2x^4\right)-180d^3eg\left(5f+gx^2\right)+360\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x^2*(360*d^4*g^2 - 180*d^3*e*g*(5*f + g*x^2) - 30*d*e^3*x^2*(10*f^2
+ 10*f*g*x^2 + 3*g^2*x^4) + 30*d^2*e^2*(20*f^2 + 15*f*g*x^2 + 4*g^2*x^4) +
e^4*x^4*(200*f^2 + 225*f*g*x^2 + 72*g^2*x^4))) + 60*d^3*(10*e^2*f^2 - 15*d*
e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2] + 60*e^5*x^6*(10*f^2 + 15*f*g*x^2 + 6*g
^2*x^4)*Log[c*(d + e*x^2)^p])/(3600*e^5)
```

Maple [C] time = 0.585, size = 687, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/10*ln(c)*g^2*x^10+1/6*ln(c)*f^2*x^6-1/8*I*Pi*f*g*x^8*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/6/e^3*ln(e*x^2+d)*d^3*f^2*p-1/20*I*Pi*g^2
```

```
*x^10*csgn(I*c*(e*x^2+d)^p)^3-1/18*f^2*p*x^6-1/50*g^2*p*x^10-1/16*f*g*p*x^8
+1/4*ln(c)*f*g*x^8+1/40/e*d*g^2*p*x^8-1/30/e^2*d^2*g^2*p*x^6+1/20/e^3*d^3*g
^2*p*x^4+1/12/e*d*f^2*p*x^4-1/10/e^4*d^4*g^2*p*x^2-1/6/e^2*d^2*f^2*p*x^2+1/
10/e^5*ln(e*x^2+d)*d^5*g^2*p-1/12*I*Pi*f^2*x^6*csgn(I*c*(e*x^2+d)^p)^3+(1/1
0*g^2*x^10+1/4*f*g*x^8+1/6*f^2*x^6)*ln((e*x^2+d)^p)+1/20*I*Pi*g^2*x^10*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/20*I*Pi*g^2*x^10*csgn(I*c*(e*x^2+
d)^p)^2*csgn(I*c)-1/8*I*Pi*f*g*x^8*csgn(I*c*(e*x^2+d)^p)^3+1/12*I*Pi*f^2*x^
6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/12*I*Pi*f^2*x^6*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)-1/20*I*Pi*g^2*x^10*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)*csgn(I*c)+1/8*I*Pi*f*g*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^
p)^2+1/8*I*Pi*f*g*x^8*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/12*I*Pi*f^2*x^6*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/12/e*d*f*g*p*x^6-1/8/e
^2*d^2*f*g*p*x^4+1/4/e^3*d^3*f*g*p*x^2-1/4/e^4*ln(e*x^2+d)*d^4*f*g*p
```

Maxima [A] time = 1.02508, size = 301, normalized size = 1.2

$$-\frac{1}{3600}\,ep\left(\frac{72\,e^4g^2x^{10}+45\left(5\,e^4fg-2\,de^3g^2\right)x^8+20\left(10\,e^4f^2-15\,de^3fg+6\,d^2e^2g^2\right)x^6-30\left(10\,de^3f^2-15\,d^2e^2fg+6\right.}{e^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/3600*e*p*((72*e^4*g^2*x^10 + 45*(5*e^4*f*g - 2*d*e^3*g^2)*x^8 + 20*(10*e
^4*f^2 - 15*d*e^3*f*g + 6*d^2*e^2*g^2)*x^6 - 30*(10*d*e^3*f^2 - 15*d^2*e^2*
f*g + 6*d^3*e*g^2)*x^4 + 60*(10*d^2*e^2*f^2 - 15*d^3*e*f*g + 6*d^4*g^2)*x^2
)/e^5 - 60*(10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*log(e*x^2 + d)/e^6)
+ 1/60*(6*g^2*x^10 + 15*f*g*x^8 + 10*f^2*x^6)*log((e*x^2 + d)^p*c)
```

Fricas [A] time = 2.08771, size = 582, normalized size = 2.32

$$-\frac{72\,e^5g^2px^{10}+45\left(5\,e^5fg-2\,de^4g^2\right)px^8+20\left(10\,e^5f^2-15\,de^4fg+6\,d^2e^3g^2\right)px^6-30\left(10\,de^4f^2-15\,d^2e^3fg+6\,d^3e^2g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3600*(72*e^5*g^2*p*x^10 + 45*(5*e^5*f*g - 2*d*e^4*g^2)*p*x^8 + 20*(10*e^
5*f^2 - 15*d*e^4*f*g + 6*d^2*e^3*g^2)*p*x^6 - 30*(10*d*e^4*f^2 - 15*d^2*e^3
*f*g + 6*d^3*e^2*g^2)*p*x^4 + 60*(10*d^2*e^3*f^2 - 15*d^3*e^2*f*g + 6*d^4*e
*g^2)*p*x^2 - 60*(6*e^5*g^2*p*x^10 + 15*e^5*f*g*p*x^8 + 10*e^5*f^2*p*x^6 +
(10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*p)*log(e*x^2 + d) - 60*(6*e^5*g
^2*x^10 + 15*e^5*f*g*x^8 + 10*e^5*f^2*x^6)*log(c))/e^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3058, size = 721, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/3600*(360*g^2*x^10*e*log(c) + 900*f*g*x^8*e*log(c) + 600*f^2*x^6*e*log(c)
 + 100*(6*(x^2*e + d)^3*e^(-2)*log(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*l
og(x^2*e + d) + 18*(x^2*e + d)*d^2*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*
e^(-2) + 9*(x^2*e + d)^2*d*e^(-2) - 18*(x^2*e + d)*d^2*e^(-2))*f^2*p + 75*(
12*(x^2*e + d)^4*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)^3*d*e^(-3)*log(x^2*
e + d) + 72*(x^2*e + d)^2*d^2*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)*d^3*e^
(-3)*log(x^2*e + d) - 3*(x^2*e + d)^4*e^(-3) + 16*(x^2*e + d)^3*d*e^(-3) -
36*(x^2*e + d)^2*d^2*e^(-3) + 48*(x^2*e + d)*d^3*e^(-3))*f*g*p + 6*(60*(x^2
*e + d)^5*e^(-4)*log(x^2*e + d) - 300*(x^2*e + d)^4*d*e^(-4)*log(x^2*e + d)
 + 600*(x^2*e + d)^3*d^2*e^(-4)*log(x^2*e + d) - 600*(x^2*e + d)^2*d^3*e^(-
4)*log(x^2*e + d) + 300*(x^2*e + d)*d^4*e^(-4)*log(x^2*e + d) - 12*(x^2*e +
 d)^5*e^(-4) + 75*(x^2*e + d)^4*d*e^(-4) - 200*(x^2*e + d)^3*d^2*e^(-4) + 3
00*(x^2*e + d)^2*d^3*e^(-4) - 300*(x^2*e + d)*d^4*e^(-4))*g^2*p)*e^(-1)
```

3.324 $\int x^3 \left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=210

$$\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2p\left(3d^2g^2-8defg+6e^2f^2\right)\log\left(d\right.}{24e^4}$$

```
[Out] (d*(e*f - d*g)^2*p*x^2)/(2*e^3) - ((e*f - 3*d*g)*(e*f - d*g)*p*(d + e*x^2)^
2)/(8*e^4) - (g*(2*e*f - 3*d*g)*p*(d + e*x^2)^3)/(18*e^4) - (g^2*p*(d + e*x
^2)^4)/(32*e^4) - (d^2*(6*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2]
)/(24*e^4) + (f^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*x^6*Log[c*(d + e*x^2)^
p])/3 + (g^2*x^8*Log[c*(d + e*x^2)^p])/8
```

Rubi [A] time = 0.360555, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2475, 43, 2414, 12, 893}

$$\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2p\left(3d^2g^2-8defg+6e^2f^2\right)\log\left(d\right.}{24e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(e*f - d*g)^2*p*x^2)/(2*e^3) - ((e*f - 3*d*g)*(e*f - d*g)*p*(d + e*x^2)^
2)/(8*e^4) - (g*(2*e*f - 3*d*g)*p*(d + e*x^2)^3)/(18*e^4) - (g^2*p*(d + e*x
^2)^4)/(32*e^4) - (d^2*(6*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2]
)/(24*e^4) + (f^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*x^6*Log[c*(d + e*x^2)^
p])/3 + (g^2*x^8*Log[c*(d + e*x^2)^p])/8
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int x^3 \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right) dx = \frac{1}{2}\text{Subst}\left(\int x(f+gx)^2 \log\left(c(d+ex)^p\right) dx, x, x^2\right)$$
$$= \frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right) -$$
$$= \frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right) -$$
$$= \frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right) -$$
$$= \frac{d(ef-dg)^2px^2}{2e^3} - \frac{(ef-3dg)(ef-dg)p\left(d+ex^2\right)^2}{8e^4} - \frac{g(2ef-3dg)p\left(d+ex^2\right)^3}{18e^4} -$$

Mathematica [A] time = 0.133462, size = 173, normalized size = 0.82

$$\frac{12e^4x^4\left(6f^2+8fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)+epx^2\left(-6d^2eg\left(16f+3gx^2\right)+36d^3g^2+12de^2\left(6f^2+4fgx^2+g^2x^4\right)-\right.}{288e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (e*p*x^2*(36*d^3*g^2 - 6*d^2*e*g*(16*f + 3*g*x^2) + 12*d*e^2*(6*f^2 + 4*f*g
*x^2 + g^2*x^4) - e^3*x^2*(36*f^2 + 32*f*g*x^2 + 9*g^2*x^4)) - 12*d^2*(6*e^
2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2] + 12*e^4*x^4*(6*f^2 + 8*f*g
*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(288*e^4)
```

Maple [C] time = 0.585, size = 643, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -1/6*I*Pi*f*g*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/24/
e*d*g^2*p*x^6-1/16/e^2*d^2*g^2*p*x^4+1/8/e^3*d^3*g^2*p*x^2+1/4/e*d*f^2*p*x^
2-1/8/e^4*ln(e*x^2+d)*d^4*g^2*p-1/4/e^2*ln(e*x^2+d)*d^2*f^2*p-1/16*I*Pi*g^2
*x^8*csgn(I*c*(e*x^2+d)^p)^3-1/8*I*Pi*f^2*x^4*csgn(I*c*(e*x^2+d)^p)^3-1/9*f
```

```
*g*p*x^6+1/3*ln(c)*f*g*x^6-1/32*g^2*p*x^8-1/8*f^2*p*x^4+1/8*ln(c)*g^2*x^8+1
/4*ln(c)*f^2*x^4+(1/8*g^2*x^8+1/3*f*g*x^6+1/4*f^2*x^4)*ln((e*x^2+d)^p)+1/6/
e*d*f*g*p*x^4-1/3/e^2*d^2*f*g*p*x^2+1/3/e^3*ln(e*x^2+d)*d^3*f*g*p+1/8*I*Pi*
f^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/8*I*Pi*f^2*x^4*csgn(I
*c*(e*x^2+d)^p)^2*csgn(I*c)+1/16*I*Pi*g^2*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2+1/16*I*Pi*g^2*x^8*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/6*I*Pi
*f*g*x^6*csgn(I*c*(e*x^2+d)^p)^3+1/6*I*Pi*f*g*x^6*csgn(I*c*(e*x^2+d)^p)^2*c
sgn(I*c)-1/8*I*Pi*f^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*
c)-1/16*I*Pi*g^2*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/
6*I*Pi*f*g*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
```

Maxima [A] time = 1.02316, size = 250, normalized size = 1.19

$$-\frac{1}{288}ep\left(\frac{9e^3g^2x^8+4\left(8e^3fg-3de^2g^2\right)x^6+6\left(6e^3f^2-8de^2fg+3d^2eg^2\right)x^4-12\left(6de^2f^2-8d^2efg+3d^3g^2\right)x^2}{e^4}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/288*e*p*((9*e^3*g^2*x^8 + 4*(8*e^3*f*g - 3*d*e^2*g^2)*x^6 + 6*(6*e^3*f^2
 - 8*d*e^2*f*g + 3*d^2*e*g^2)*x^4 - 12*(6*d*e^2*f^2 - 8*d^2*e*f*g + 3*d^3*g
^2)*x^2)/e^4 + 12*(6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*log(e*x^2 + d)/
e^5) + 1/24*(3*g^2*x^8 + 8*f*g*x^6 + 6*f^2*x^4)*log((e*x^2 + d)^p*c)
```

Fricas [A] time = 1.7163, size = 477, normalized size = 2.27

$$-\frac{9e^4g^2px^8+4\left(8e^4fg-3de^3g^2\right)px^6+6\left(6e^4f^2-8de^3fg+3d^2e^2g^2\right)px^4-12\left(6de^3f^2-8d^2e^2fg+3d^3eg^2\right)px^2-1\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/288*(9*e^4*g^2*p*x^8 + 4*(8*e^4*f*g - 3*d*e^3*g^2)*p*x^6 + 6*(6*e^4*f^2
- 8*d*e^3*f*g + 3*d^2*e^2*g^2)*p*x^4 - 12*(6*d*e^3*f^2 - 8*d^2*e^2*f*g + 3*
d^3*e*g^2)*p*x^2 - 12*(3*e^4*g^2*p*x^8 + 8*e^4*f*g*p*x^6 + 6*e^4*f^2*p*x^4
- (6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*p)*log(e*x^2 + d) - 12*(3*e^4*g
^2*x^8 + 8*e^4*f*g*x^6 + 6*e^4*f^2*x^4)*log(c))/e^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31719, size = 564, normalized size = 2.69

$$\frac{1}{288}\left(36\,g^2x^8e\log\left(c\right)+96\,fgx^6e\log\left(c\right)+36\left(2\left(x^2e+d\right)^2\log\left(x^2e+d\right)-4\left(x^2e+d\right)d\log\left(x^2e+d\right)-\left(x^2e+d\right)^2+4\left(x^2e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/288*(36*g^2*x^8*e*log(c) + 96*f*g*x^6*e*log(c) + 36*(2*(x^2*e + d)^2*log(
x^2*e + d) - 4*(x^2*e + d)*d*log(x^2*e + d) - (x^2*e + d)^2 + 4*(x^2*e + d)
*d)*f^2*p*e^(-1) + 72*((x^2*e + d)^2 - 2*(x^2*e + d)*d)*f^2*e^(-1)*log(c) +
 16*(6*(x^2*e + d)^3*e^(-2)*log(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*log(
x^2*e + d) + 18*(x^2*e + d)*d^2*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*e^(
-2) + 9*(x^2*e + d)^2*d*e^(-2) - 18*(x^2*e + d)*d^2*e^(-2))*f*g*p + 3*(12*(
x^2*e + d)^4*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)^3*d*e^(-3)*log(x^2*e +
d) + 72*(x^2*e + d)^2*d^2*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)*d^3*e^(-3)
*log(x^2*e + d) - 3*(x^2*e + d)^4*e^(-3) + 16*(x^2*e + d)^3*d*e^(-3) - 36*(
x^2*e + d)^2*d^2*e^(-3) + 48*(x^2*e + d)*d^3*e^(-3))*g^2*p)*e^(-1)
```

3.325 $\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=124

$$\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{px^2(ef-dg)^2}{6e^2}-\frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6e^3g}-\frac{p\left(f+gx^2\right)^2(ef-dg)}{12eg}-\frac{p\left(f+gx^2\right)^3}{18g}$$

```
[Out] -((e*f - d*g)^2*p*x^2)/(6*e^2) - ((e*f - d*g)*p*(f + g*x^2)^2)/(12*e*g) - (
p*(f + g*x^2)^3)/(18*g) - ((e*f - d*g)^3*p*Log[d + e*x^2])/(6*e^3*g) + ((f
+ g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*g)
```

Rubi [A] time = 0.140923, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2475, 2395, 43}

$$\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{px^2(ef-dg)^2}{6e^2}-\frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6e^3g}-\frac{p\left(f+gx^2\right)^2(ef-dg)}{12eg}-\frac{p\left(f+gx^2\right)^3}{18g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -((e*f - d*g)^2*p*x^2)/(6*e^2) - ((e*f - d*g)*p*(f + g*x^2)^2)/(12*e*g) - (
p*(f + g*x^2)^3)/(18*g) - ((e*f - d*g)^3*p*Log[d + e*x^2])/(6*e^3*g) + ((f
+ g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*g)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)dx=\frac{1}{2}\text{Subst}\left(\int (f+gx)^2\log\left(c(d+ex)^p\right)dx,x,x^2\right)$$
$$=\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{(ep)\,\text{Subst}\left(\int\frac{(f+gx)^3}{d+ex}dx,x,x^2\right)}{6g}$$
$$=\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{(ep)\,\text{Subst}\left(\int\left(\frac{g(ef-dg)^2}{e^3}+\frac{(ef-dg)^3}{e^3(d+ex)}+\frac{g(ef-dg)(f+gx)}{e^2}\right.\right.}{6g}$$
$$=-\frac{(ef-dg)^2px^2}{6e^2}-\frac{(ef-dg)p\left(f+gx^2\right)^2}{12eg}-\frac{p\left(f+gx^2\right)^3}{18g}-\frac{(ef-dg)^3p\log\left(d+ex\right.}{6e^3g}$$

Mathematica [A] time = 0.106705, size = 135, normalized size = 1.09

$$\frac{e\left(6e\left(3df^2+ex^2\left(3f^2+3fgx^2+g^2x^4\right)\right)\log\left(c\left(d+ex^2\right)^p\right)-px^2\left(6d^2g^2-3deg\left(6f+gx^2\right)+e^2\left(18f^2+9fgx^2+2g^2x^4\right)\right)\right.}{36e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (6*d^2*g*(-3*e*f + d*g)*p*Log[d + e*x^2] + e*(-(p*x^2*(6*d^2*g^2 - 3*d*e*g*
(6*f + g*x^2) + e^2*(18*f^2 + 9*f*g*x^2 + 2*g^2*x^4))) + 6*e*(3*d*f^2 + e*x
^2*(3*f^2 + 3*f*g*x^2 + g^2*x^4))*Log[c*(d + e*x^2)^p]))/(36*e^3)
```

Maple [C] time = 0.587, size = 599, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/12/e*d*g^2*p*x^4-1/6/e^2*d^2*g^2*p*x^2+1/6/e^3*ln(e*x^2+d)*d^3*g^2*p+1/2/
e*ln(e*x^2+d)*d*f^2*p-1/12*I*Pi*g^2*x^6*csgn(I*c*(e*x^2+d)^p)^3-1/4*I*Pi*f^
2*x^2*csgn(I*c*(e*x^2+d)^p)^3-1/18*g^2*p*x^6-1/2*f^2*p*x^2-1/2*d^2*f*g*p*ln
(e*x^2+d)/e^2-1/4*f*g*p*x^4+1/6*ln(c)*g^2*x^6+1/2*ln(c)*f^2*x^2+(1/6*g^2*x^
6+1/2*f*g*x^4+1/2*f^2*x^2)*ln((e*x^2+d)^p)+1/2*ln(c)*f*g*x^4+1/4*I*Pi*f*g*x
^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/4*I*Pi*f*g*x^4*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)^2-1/4*I*Pi*f*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)+1/2*d*f*g*p*x^2/e-1/4*I*Pi*f*g*x^4*csgn(I*c*(e*x^2+d)^p)^3
-1/12*I*Pi*g^2*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/4*
I*Pi*f^2*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/12*I*Pi*
g^2*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/4*I*Pi*f^2*x^2*csgn(I
*c*(e*x^2+d)^p)^2*csgn(I*c)+1/4*I*Pi*f^2*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2+1/12*I*Pi*g^2*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
```

Maxima [A] time = 1.01013, size = 205, normalized size = 1.65

$$\frac{\left(gx^2+f\right)^3\log\left(\left(ex^2+d\right)^pc\right)}{6\,g}-\frac{ep\left(\frac{2\,e^2g^3x^6+3\left(3\,e^2fg^2-deg^3\right)x^4+6\left(3\,e^2f^2g-3\,defg^2+d^2g^3\right)x^2}{e^3}+\frac{6\left(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3\right)\log\left(ex^2+d\right)}{e^4}\right)}{36\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/6*(g*x^2 + f)^3*log((e*x^2 + d)^p*c)/g - 1/36*e*p*((2*e^2*g^3*x^6 + 3*(3*
e^2*f*g^2 - d*e*g^3)*x^4 + 6*(3*e^2*f^2*g - 3*d*e*f*g^2 + d^2*g^3)*x^2)/e^3
 + 6*(e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*log(e*x^2 + d)/e^4
)/g
```

Fricas [A] time = 1.7619, size = 379, normalized size = 3.06

$$-\frac{2\,e^3g^2px^6+3\left(3\,e^3fg-de^2g^2\right)px^4+6\left(3\,e^3f^2-3\,de^2fg+d^2eg^2\right)px^2-6\left(e^3g^2px^6+3\,e^3fgpx^4+3\,e^3f^2px^2+\left(3\,de^2f\right.\right.}{36\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/36*(2*e^3*g^2*p*x^6 + 3*(3*e^3*f*g - d*e^2*g^2)*p*x^4 + 6*(3*e^3*f^2 - 3
*d*e^2*f*g + d^2*e*g^2)*p*x^2 - 6*(e^3*g^2*p*x^6 + 3*e^3*f*g*p*x^4 + 3*e^3*
f^2*p*x^2 + (3*d*e^2*f^2 - 3*d^2*e*f*g + d^3*g^2)*p)*log(e*x^2 + d) - 6*(e^
3*g^2*x^6 + 3*e^3*f*g*x^4 + 3*e^3*f^2*x^2)*log(c))/e^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19653, size = 389, normalized size = 3.14

$$\frac{1}{36}\left(6\,g^2x^6e\log\left(c\right)+9\left(2\left(x^2e+d\right)^2\log\left(x^2e+d\right)-4\left(x^2e+d\right)d\log\left(x^2e+d\right)-\left(x^2e+d\right)^2+4\left(x^2e+d\right)d\right)fgpe^{(-1)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/36*(6*g^2*x^6*e*log(c) + 9*(2*(x^2*e + d)^2*log(x^2*e + d) - 4*(x^2*e + d
)*d*log(x^2*e + d) - (x^2*e + d)^2 + 4*(x^2*e + d)*d)*f*g*p*e^(-1) + 18*((x
^2*e + d)^2 - 2*(x^2*e + d)*d)*f*g*e^(-1)*log(c) - 18*(x^2*e - (x^2*e + d)*
log(x^2*e + d) + d)*f^2*p + (6*(x^2*e + d)^3*e^(-2)*log(x^2*e + d) - 18*(x^
2*e + d)^2*d*e^(-2)*log(x^2*e + d) + 18*(x^2*e + d)*d^2*e^(-2)*log(x^2*e +
d) - 2*(x^2*e + d)^3*e^(-2) + 9*(x^2*e + d)^2*d*e^(-2) - 18*(x^2*e + d)*d^2
*e^(-2))*g^2*p + 18*(x^2*e + d)*f^2*log(c))*e^(-1)
```

3.326 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x} dx$

Optimal. Leaf size=153

$$\frac{1}{2}f^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{fg\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right.\right.$$

```
[Out] -(f*g*p*x^2) + (d*g^2*p*x^2)/(4*e) - (g^2*p*x^4)/8 - (d^2*g^2*p*Log[d + e*x
^2])/(4*e^2) + (g^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*(d + e*x^2)*Log[c*(d
 + e*x^2)^p])/e + (f^2*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (f^2*p*P
olyLog[2, 1 + (e*x^2)/d])/2
```

Rubi [A] time = 0.202965, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315, 2395}

$$\frac{1}{2}f^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{fg\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -(f*g*p*x^2) + (d*g^2*p*x^2)/(4*e) - (g^2*p*x^4)/8 - (d^2*g^2*p*Log[d + e*x
^2])/(4*e^2) + (g^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*(d + e*x^2)*Log[c*(d
 + e*x^2)^p])/e + (f^2*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (f^2*p*P
olyLog[2, 1 + (e*x^2)/d])/2
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(2fg\log\left(c(d+ex)^p\right)+\frac{f^2\log\left(c(d+ex)^p\right)}{x}+g^2x\log\left(c(d+ex)^p\right)\right)$$

$$= \frac{1}{2}f^2\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right)+(fg)\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx,$$

$$= \frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{(fg)\,\text{Subst}\left(\int \right.}{}$$

$$= -fgpx^2+\frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{fg\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{1}{2}f^2\log$$

$$= -fgpx^2+\frac{dg^2px^2}{4e}-\frac{1}{8}g^2px^4-\frac{d^2g^2p\log\left(d+ex^2\right)}{4e^2}+\frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right)+$$

Mathematica [A] time = 0.0815272, size = 121, normalized size = 0.79

$$\frac{4e^2f^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+2e\log\left(c\left(d+ex^2\right)^p\right)\left(2ef^2\log\left(-\frac{ex^2}{d}\right)+g\left(4df+4efx^2+egx^4\right)\right)-2d^2g^2p\log\left(d+ex^2\right.}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (-(e*g*p*x^2*(8*e*f - 2*d*g + e*g*x^2)) - 2*d^2*g^2*p*Log[d + e*x^2] + 2*e*
(g*(4*d*f + 4*e*f*x^2 + e*g*x^4) + 2*e*f^2*Log[-((e*x^2)/d)])*Log[c*(d + e*
x^2)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^2)/d])/(8*e^2)
```

Maple [C] time = 0.569, size = 652, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x,x)
```

```
[Out] 1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x^4*g^2+1/2*I*Pi*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2*ln(x)-1/2*I*Pi*csgn(I*c*(e*x^2+d
)^p)^3*x^2*f*g+1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x^4*g^2-1/2*I*Pi*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x^2*f*g+ln((e*x^2+d)^p)
*x^2*f*g+ln(c)*x^2*f*g-p*f^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*f
^2*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+p/e*g*d*ln(e*x^2+d)*f+1/2*I*Pi
*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2*ln(x)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2*ln(x)+ln(c)*f^2*ln(x)-p*f^2*dilog((-e*x+
(-d*e)^(1/2))/(-d*e)^(1/2))-p*f^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/
4*d^2*g^2*p*ln(e*x^2+d)/e^2-f*g*p*x^2-1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*x^4*
g^2-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2*ln(x)-1/8*g^2*p*x^4+1/4*ln(c)*x^4*
g^2+1/4*ln((e*x^2+d)^p)*x^4*g^2+ln((e*x^2+d)^p)*f^2*ln(x)+1/2*I*Pi*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)*x^2*f*g+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)^2*x^2*f*g-1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn
(I*c)*x^4*g^2+1/4*d*g^2*p*x^2/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(g^2x^4+2\,fgx^2+f^2\right)\log\left(\left(ex^2+d\right)^pc\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)^2\log\left(\left(ex^2+d\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x, x)
```

3.327 $\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^3} dx$

Optimal. Leaf size=135

$$fgp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+fg\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g^2\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}-$$

```
[Out] -(g^2*p*x^2)/2 + (e*f^2*p*Log[x])/d - (e*f^2*p*Log[d + e*x^2])/(2*d) - (f^2
*Log[c*(d + e*x^2)^p])/(2*x^2) + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*
e) + f*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + f*g*p*PolyLog[2, 1 + (e*x
^2)/d]
```

Rubi [A] time = 0.193142, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2475, 43, 2416, 2389, 2295, 2395, 36, 29, 31, 2394, 2315}

$$fgp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+fg\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g^2\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}-$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```


```
[Out] -(g^2*p*x^2)/2 + (e*f^2*p*Log[x])/d - (e*f^2*p*Log[d + e*x^2])/(2*d) - (f^2
*Log[c*(d + e*x^2)^p])/(2*x^2) + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*
e) + f*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + f*g*p*PolyLog[2, 1 + (e*x
^2)/d]
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^3} dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(f+gx)^2 \log (c(d+ex)^p)}{x^2} dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(g^2 \log (c(d+ex)^p) + \frac{f^2 \log (c(d+ex)^p)}{x^2} + \frac{2fg \log (c(d+ex)^p)}{x}\right) dx, x$$
$$= \frac{1}{2} f^2 \operatorname{Subst}\left(\int \frac{\log (c(d+ex)^p)}{x^2} dx, x, x^2\right) + (fg)\operatorname{Subst}\left(\int \frac{\log (c(d+ex)^p)}{x} dx, x, x$$
$$= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2x^2} + fg \log \left(-\frac{ex^2}{d}\right) \log \left(c\left(d+ex^2\right)^p\right) + \frac{g^2 \operatorname{Subst}\left(\int \log (cx^p)\right.}{2e}$$
$$= -\frac{1}{2}g^2px^2 - \frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2x^2} + \frac{g^2 \left(d+ex^2\right) \log \left(c\left(d+ex^2\right)^p\right)}{2e} + fg \log \left(-\frac{ex^2}{d}\right.$$
$$= -\frac{1}{2}g^2px^2 + \frac{ef^2p \log(x)}{d} - \frac{ef^2p \log \left(d+ex^2\right)}{2d} - \frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2x^2} + \frac{g^2 \left(d+ex^2\right.}{}$$

Mathematica [A] time = 0.0776067, size = 126, normalized size = 0.93

$$\frac{1}{2}\left(2fg\left(p\text{PolyLog}\left(2, \frac{ex^2}{d}+1\right) + \log \left(-\frac{ex^2}{d}\right) \log \left(c\left(d+ex^2\right)^p\right)\right) - \frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^2} + \frac{g^2 \left(d+ex^2\right) \log \left(c\left(d+ex^2\right)^p\right.}{e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (-(g^2*p*x^2) + (e*f^2*p*(2*Log[x] - Log[d + e*x^2]))/d - (f^2*Log[c*(d + e
*x^2)^p])/x^2 + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e + 2*f*g*(Log[-((e*
x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/2
```

Maple [C] time = 0.583, size = 642, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^3,x)
```

```
[Out] -1/2*g^2*p*x^2+2*ln((e*x^2+d)^p)*f*g*ln(x)-2*p*f*g*ln(x)*ln((-e*x+(-d*e)^(1
/2))/(-d*e)^(1/2))-2*p*f*g*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-2*p*f*
g*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+2*ln(c)*f*g*ln(x)-2*p*f*g*dilog((-
e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)^2*x^2*g^2-I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*g*ln(x)-1/4*I*Pi*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2/x^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3
*f^2/x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*x^2*g^2+e*f^2*p*ln(x)/d-1/2*e*f^2
*p*ln(e*x^2+d)/d+1/2*p/e*d*ln(e*x^2+d)*g^2+1/2*ln((e*x^2+d)^p)*x^2*g^2-1/2*
ln((e*x^2+d)^p)*f^2/x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*
csgn(I*c)*x^2*g^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x^2*g^2-1/4*I*
Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/x^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)*csgn(I*c)*f*g*ln(x)+1/2*ln(c)*x^2*g^2-1/2*ln(c)*f^2/x^2+I*
Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f*g*ln(x)+I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)^2*f*g*ln(x)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
```

```
+d)^p)*csgn(I*c)*f^2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(g^2x^4+2\,fgx^2+f^2\right)\log\left(\left(ex^2+d\right)^p c\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^3, x)
```

3.328 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^5} dx$

Optimal. Leaf size=172

$$\frac{1}{2}g^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2}+\frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{e^2f^2p\,l}{}$$

```
[Out] -(e*f^2*p)/(4*d*x^2) - (e^2*f^2*p*Log[x])/(2*d^2) + (2*e*f*g*p*Log[x])/d +
(e^2*f^2*p*Log[d + e*x^2])/(4*d^2) - (e*f*g*p*Log[d + e*x^2])/d - (f^2*Log[
c*(d + e*x^2)^p])/(4*x^4) - (f*g*Log[c*(d + e*x^2)^p])/x^2 + (g^2*Log[-((e*
x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g^2*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rubi [A] time = 0.223455, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2475, 43, 2416, 2395, 44, 36, 29, 31, 2394, 2315}

$$\frac{1}{2}g^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2}+\frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{e^2f^2p\,l}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -(e*f^2*p)/(4*d*x^2) - (e^2*f^2*p*Log[x])/(2*d^2) + (2*e*f*g*p*Log[x])/d +
(e^2*f^2*p*Log[d + e*x^2])/(4*d^2) - (e*f*g*p*Log[d + e*x^2])/d - (f^2*Log[
c*(d + e*x^2)^p])/(4*x^4) - (f*g*Log[c*(d + e*x^2)^p])/x^2 + (g^2*Log[-((e*
x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g^2*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{f^2\log\left(c(d+ex)^p\right)}{x^3} + \frac{2fg\log\left(c(d+ex)^p\right)}{x^2} + \frac{g^2\log\left(c(d+ex)^p\right)}{x}\right)$$

$$= \frac{1}{2}f^2\operatorname{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^3}\,dx, x, x^2\right) + (fg)\operatorname{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2}\,dx,\right.$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2} + \frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right.\right.$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2} + \frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right.\right.$$

$$= -\frac{ef^2p}{4dx^2} - \frac{e^2f^2p\log(x)}{2d^2} + \frac{2efgp\log(x)}{d} + \frac{e^2f^2p\log\left(d+ex^2\right)}{4d^2} - \frac{efgp\log\left(d+ex^2\right)}{d}$$

Mathematica [A] time = 0.111869, size = 148, normalized size = 0.86

$$\frac{1}{4}\left(2g^2\left(p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)\right)-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^4}-\frac{4fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2}-\frac{ef^2p}{\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] ((4*e*f*g*p*(2*Log[x] - Log[d + e*x^2]))/d - (e*f^2*p*(d + 2*e*x^2*Log[x] -
 e*x^2*Log[d + e*x^2]))/(d^2*x^2) - (f^2*Log[c*(d + e*x^2)^p])/x^4 - (4*f*g
*Log[c*(d + e*x^2)^p])/x^2 + 2*g^2*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p]
+ p*PolyLog[2, 1 + (e*x^2)/d]))/4
```

Maple [C] time = 0.509, size = 663, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^5,x)
```

```
[Out] -ln(c)*f*g/x^2-p*g^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g^2*ln(x)
*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g^2*l
n(x)+ln((e*x^2+d)^p)*g^2*ln(x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)^2*g^2*ln(x)-1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/x^4+1/2*I
*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f*g/x^2-1/4*ln((e*x
^2+d)^p)*f^2/x^4-1/2*e^2*f^2*p*ln(x)/d^2+1/4*e^2*f^2*p*ln(e*x^2+d)/d^2+1/8*
I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/x^4+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*g/x
^2-1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2/x^4-ln((e*x^2+d
)^p)*f*g/x^2+ln(c)*g^2*ln(x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g^2
*ln(x)-1/4*ln(c)*f^2/x^4-p*g^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g^
2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*c
sgn(I*c)*f*g/x^2+2*e*f*g*p*ln(x)/d-e*f*g*p*ln(e*x^2+d)/d+1/8*I*Pi*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/x^4-1/2*I*Pi*csgn(I*(e*x^2+d
)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g^2*ln(x)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)^2*f*g/x^2-1/4*e*f^2*p/d/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^2+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(g^{2}x^{4}+2\,fgx^{2}+f^{2}\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)}{x^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{2}+f\right)^{2}\log\left(\left(ex^{2}+d\right)^{p}c\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^5, x)
```

3.329 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^7} dx$

Optimal. Leaf size=130

$$-\frac{(f+gx^2)^3 \log\left(c\left(d+ex^2\right)^p\right)}{6fx^6}+\frac{ep\log(x)\left(3d^2g^2-3defg+e^2f^2\right)}{3d^3}+\frac{efp(ef-3dg)}{6d^2x^2}-\frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6d^3f}-\frac{ef^2p}{12dx^4}$$

```
[Out] -(e*f^2*p)/(12*d*x^4) + (e*f*(e*f - 3*d*g)*p)/(6*d^2*x^2) + (e*(e^2*f^2 - 3
*d*e*f*g + 3*d^2*g^2)*p*Log[x])/(3*d^3) - ((e*f - d*g)^3*p*Log[d + e*x^2])/
(6*d^3*f) - ((f + g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*f*x^6)
```

Rubi [A] time = 0.208371, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2475, 37, 2414, 12, 88}

$$-\frac{(f+gx^2)^3 \log\left(c\left(d+ex^2\right)^p\right)}{6fx^6}+\frac{ep\log(x)\left(3d^2g^2-3defg+e^2f^2\right)}{3d^3}+\frac{efp(ef-3dg)}{6d^2x^2}-\frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6d^3f}-\frac{ef^2p}{12dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```


```
[Out] -(e*f^2*p)/(12*d*x^4) + (e*f*(e*f - 3*d*g)*p)/(6*d^2*x^2) + (e*(e^2*f^2 - 3
*d*e*f*g + 3*d^2*g^2)*p*Log[x])/(3*d^3) - ((e*f - d*g)^3*p*Log[d + e*x^2])/
(6*d^3*f) - ((f + g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*f*x^6)
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{(f+gx^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{6fx^6} - \frac{1}{2}(ep)\,\text{Subst}\left(\int -\frac{(f+gx)^3}{3fx^3(d+ex)}\,dx, x, x^2\right)$$
$$= -\frac{(f+gx^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{6fx^6} + \frac{(ep)\,\text{Subst}\left(\int \frac{(f+gx)^3}{x^3(d+ex)}\,dx, x, x^2\right)}{6f}$$
$$= -\frac{(f+gx^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{6fx^6} + \frac{(ep)\,\text{Subst}\left(\int \left(\frac{f^3}{dx^3} + \frac{f^2(-ef+3dg)}{d^2x^2} + \frac{f(e^2f^2-3defg}{d^3x}\right.\right.}{6f}$$
$$= -\frac{ef^2p}{12dx^4} + \frac{ef(ef-3dg)p}{6d^2x^2} + \frac{e\left(e^2f^2-3defg+3d^2g^2\right)p\log(x)}{3d^3} - \frac{(ef-dg)^3p\log}{6d^3f}$$

Mathematica [A] time = 0.121187, size = 141, normalized size = 1.08

$$-\frac{2d^3\left(f^2+3fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right) - 4epx^6\log(x)\left(3d^2g^2-3defg+e^2f^2\right) + 2epx^6\left(3d^2g^2-3defg+e^2f^2\right)}{12d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -(d*e*f*p*x^2*(-2*e*f*x^2 + d*(f + 6*g*x^2)) - 4*e*(e^2*f^2 - 3*d*e*f*g + 3
*d^2*g^2)*p*x^6*Log[x] + 2*e*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*p*x^6*Log[d
+ e*x^2] + 2*d^3*(f^2 + 3*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(12*d^
3*x^6)
```

Maple [C] time = 0.403, size = 656, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^7,x)
```

```
[Out] -1/6*(3*g^2*x^4+3*f*g*x^2+f^2)/x^6*ln((e*x^2+d)^p)+1/12*(3*I*Pi*d^3*f*g*x^2
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*d^3*f^2*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3*I*Pi*d^3*g^2*x^4*csgn(I*c*(e*x
^2+d)^p)^3-3*I*Pi*d^3*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+12*ln(x)*d^
2*e*g^2*p*x^6-12*ln(x)*d*e^2*f*g*p*x^6+4*ln(x)*e^3*f^2*p*x^6-6*ln(e*x^2+d)*
d^2*e*g^2*p*x^6+6*ln(e*x^2+d)*d*e^2*f*g*p*x^6-2*ln(e*x^2+d)*e^3*f^2*p*x^6+3
```

```
*I*Pi*d^3*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-3*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+d)^p
)*csgn(I*c*(e*x^2+d)^p)^2-3*I*Pi*d^3*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2-6*ln(c)*d^3*g^2*x^4-I*Pi*d^3*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn
(I*c)-I*Pi*d^3*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+I*Pi*d^3*f^2
*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d^3*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)-6*d^2*e*f*g*p*x^4+2*d*e^2*f^2*p*x^4-6*ln(c)*d^3*f*g*x^2-d^2*e*f^2*p*x^2-
2*ln(c)*d^3*f^2)/d^3/x^6
```

Maxima [A] time = 1.02354, size = 185, normalized size = 1.42

$$-\frac{1}{12}\,ep\left(\frac{2\left(e^2f^2-3\,defg+3\,d^2g^2\right)\log\left(ex^2+d\right)}{d^3}-\frac{2\left(e^2f^2-3\,defg+3\,d^2g^2\right)\log\left(x^2\right)}{d^3}+\frac{df^2-2\left(ef^2-3\,dfg\right)x^2}{d^2x^4}\right)-\frac{(3g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*e*p*(2*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(e*x^2 + d)/d^3 - 2*(e^2*
f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(x^2)/d^3 + (d*f^2 - 2*(e*f^2 - 3*d*f*g)*x^
2)/(d^2*x^4)) - 1/6*(3*g^2*x^4 + 3*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^6
```

Fricas [A] time = 1.80621, size = 393, normalized size = 3.02

$$\frac{4\left(e^3f^2-3\,de^2fg+3\,d^2eg^2\right)px^6\log\left(x\right)-d^2ef^2px^2+2\left(de^2f^2-3\,d^2efg\right)px^4-2\left(3\,d^3g^2px^4+3\,d^3fgpx^2+\left(e^3f^2-3\,de^2\right.\right.}{12\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6*log(x) - d^2*e*f^2*p*x^
2 + 2*(d*e^2*f^2 - 3*d^2*e*f*g)*p*x^4 - 2*(3*d^3*g^2*p*x^4 + 3*d^3*f*g*p*x^
2 + (e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6 + d^3*f^2*p)*log(e*x^2 + d)
 - 2*(3*d^3*g^2*x^4 + 3*d^3*f*g*x^2 + d^3*f^2)*log(c))/(d^3*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33923, size = 1068, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*(x^2*e + d)^3*d^2*g^2*p*e^2*log(x^2*e + d) - 12*(x^2*e + d)^2*d^3*
g^2*p*e^2*log(x^2*e + d) + 6*(x^2*e + d)*d^4*g^2*p*e^2*log(x^2*e + d) - 6*(
x^2*e + d)^3*d^2*g^2*p*e^2*log(x^2*e) + 18*(x^2*e + d)^2*d^3*g^2*p*e^2*log(
x^2*e) - 18*(x^2*e + d)*d^4*g^2*p*e^2*log(x^2*e) + 6*d^5*g^2*p*e^2*log(x^2*
e) - 6*(x^2*e + d)^3*d*f*g*p*e^3*log(x^2*e + d) + 18*(x^2*e + d)^2*d^2*f*g*
p*e^3*log(x^2*e + d) - 12*(x^2*e + d)*d^3*f*g*p*e^3*log(x^2*e + d) + 6*(x^2
*e + d)^3*d*f*g*p*e^3*log(x^2*e) - 18*(x^2*e + d)^2*d^2*f*g*p*e^3*log(x^2*e
) + 18*(x^2*e + d)*d^3*f*g*p*e^3*log(x^2*e) - 6*d^4*f*g*p*e^3*log(x^2*e) +
6*(x^2*e + d)^2*d^3*g^2*e^2*log(c) - 12*(x^2*e + d)*d^4*g^2*e^2*log(c) + 6*
d^5*g^2*e^2*log(c) + 6*(x^2*e + d)^2*d^2*f*g*p*e^3 - 12*(x^2*e + d)*d^3*f*g
*p*e^3 + 6*d^4*f*g*p*e^3 + 2*(x^2*e + d)^3*f^2*p*e^4*log(x^2*e + d) - 6*(x^
2*e + d)^2*d*f^2*p*e^4*log(x^2*e + d) + 6*(x^2*e + d)*d^2*f^2*p*e^4*log(x^2
*e + d) - 2*(x^2*e + d)^3*f^2*p*e^4*log(x^2*e) + 6*(x^2*e + d)^2*d*f^2*p*e^
4*log(x^2*e) - 6*(x^2*e + d)*d^2*f^2*p*e^4*log(x^2*e) + 2*d^3*f^2*p*e^4*log
(x^2*e) + 6*(x^2*e + d)*d^3*f*g*e^3*log(c) - 6*d^4*f*g*e^3*log(c) - 2*(x^2*
e + d)^2*d*f^2*p*e^4 + 5*(x^2*e + d)*d^2*f^2*p*e^4 - 3*d^3*f^2*p*e^4 + 2*d^
3*f^2*e^4*log(c))*e^(-1)/((x^2*e + d)^3*d^3 - 3*(x^2*e + d)^2*d^4 + 3*(x^2*
e + d)*d^5 - d^6)
```

3.330 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9} dx$

Optimal. Leaf size=216

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{ep\left(6d^2g^2-8defg+3e^2f^2\right)}{24d^3x^2}+\frac{e^2p\left(6d^2g^2-8def\right.}{}$$

```
[Out] -(e*f^2*p)/(24*d*x^6) + (e*f*(3*e*f - 8*d*g)*p)/(48*d^2*x^4) - (e*(3*e^2*f^
2 - 8*d*e*f*g + 6*d^2*g^2)*p)/(24*d^3*x^2) - (e^2*(3*e^2*f^2 - 8*d*e*f*g +
6*d^2*g^2)*p*Log[x])/(12*d^4) + (e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*
Log[d + e*x^2])/(24*d^4) - (f^2*Log[c*(d + e*x^2)^p])/(8*x^8) - (f*g*Log[c*
(d + e*x^2)^p])/(3*x^6) - (g^2*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rubi [A] time = 0.289703, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2475, 43, 2414, 12, 893}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{ep\left(6d^2g^2-8defg+3e^2f^2\right)}{24d^3x^2}+\frac{e^2p\left(6d^2g^2-8def\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -(e*f^2*p)/(24*d*x^6) + (e*f*(3*e*f - 8*d*g)*p)/(48*d^2*x^4) - (e*(3*e^2*f^
2 - 8*d*e*f*g + 6*d^2*g^2)*p)/(24*d^3*x^2) - (e^2*(3*e^2*f^2 - 8*d*e*f*g +
6*d^2*g^2)*p*Log[x])/(12*d^4) + (e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*
Log[d + e*x^2])/(24*d^4) - (f^2*Log[c*(d + e*x^2)^p])/(8*x^8) - (f*g*Log[c*
(d + e*x^2)^p])/(3*x^6) - (g^2*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{2}(ep)\,\text{S}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{24}(ep)$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{24}(ep)$$

$$= -\frac{ef^2p}{24dx^6} + \frac{ef(3ef-8dg)p}{48d^2x^4} - \frac{e\left(3e^2f^2-8defg+6d^2g^2\right)p}{24d^3x^2} - \frac{e^2\left(3e^2f^2-8defg\right.}{12d}$$

Mathematica [A] time = 0.160982, size = 184, normalized size = 0.85

$$-\frac{2d^4\left(3f^2+8fgx^2+6g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right) + depx^2\left(2d^2\left(f^2+4fgx^2+6g^2x^4\right) - defx^2\left(3f+16gx^2\right) + 6e^2f^2x^4\right)}{48d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -(d*e*p*x^2*(6*e^2*f^2*x^4 - d*e*f*x^2*(3*f + 16*g*x^2) + 2*d^2*(f^2 + 4*f*
g*x^2 + 6*g^2*x^4)) + 4*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*Log[x
] - 2*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*Log[d + e*x^2] + 2*d^4*
(3*f^2 + 8*f*g*x^2 + 6*g^2*x^4)*Log[c*(d + e*x^2)^p])/(48*d^4*x^8)
```

Maple [C] time = 0.408, size = 713, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^9,x)
```

```
[Out] -1/24*(6*g^2*x^4+8*f*g*x^2+3*f^2)/x^8*ln((e*x^2+d)^p)-1/48*(6*ln(c)*d^4*f^2
+12*ln(c)*d^4*g^2*x^4+6*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)^2+6*I*Pi*d^4*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*I*Pi*d^4*g
^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+8*I*Pi*d^4*f*g*x
^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+8*I*Pi*d^4*f*g*x^2*csgn(I*c*
(e*x^2+d)^p)^2*csgn(I*c)-8*I*Pi*d^4*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)+12*d^3*e*g^2*p*x^6+6*d*e^3*f^2*p*x^6-3*d^2*e^2*f^2*p*x
^4+2*d^3*e*f^2*p*x^2-6*ln(-e*x^2-d)*e^4*f^2*p*x^8+12*ln(x)*e^4*f^2*p*x^8+16
*ln(c)*d^4*f*g*x^2-3*I*Pi*d^4*f^2*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d^4*f^2*cs
gn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+16*ln(-e*x^2-d)*d*e^3*f*g*p*x^8-16*d^2*e^2*
f*g*p*x^6+8*d^3*e*f*g*p*x^4-8*I*Pi*d^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-12*l
n(-e*x^2-d)*d^2*e^2*g^2*p*x^8+24*ln(x)*d^2*e^2*g^2*p*x^8-6*I*Pi*d^4*g^2*x^4
*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)^2-32*ln(x)*d*e^3*f*g*p*x^8-3*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)*csgn(I*c))/d^4/x^8
```

Maxima [A] time = 1.02201, size = 247, normalized size = 1.14

$$\frac{1}{48}\,ep\left(\frac{2\left(3\,e^3f^2-8\,de^2fg+6\,d^2eg^2\right)\log\left(ex^2+d\right)}{d^4}-\frac{2\left(3\,e^3f^2-8\,de^2fg+6\,d^2eg^2\right)\log\left(x^2\right)}{d^4}-\frac{2\left(3\,e^2f^2-8\,defg+6\,d^2g^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] 1/48*e*p*(2*(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(e*x^2 + d)/d^4 - 2*
(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(x^2)/d^4 - (2*(3*e^2*f^2 - 8*d*
e*f*g + 6*d^2*g^2)*x^4 + 2*d^2*f^2 - (3*d*e*f^2 - 8*d^2*f*g)*x^2)/(d^3*x^6)
) - 1/24*(6*g^2*x^4 + 8*f*g*x^2 + 3*f^2)*log((e*x^2 + d)^p*c)/x^8
```

Fricas [A] time = 1.85303, size = 489, normalized size = 2.26

$$-\frac{4\left(3\,e^4f^2-8\,de^3fg+6\,d^2e^2g^2\right)px^8\log\left(x\right)+2\,d^3ef^2px^2+2\left(3\,de^3f^2-8\,d^2e^2fg+6\,d^3eg^2\right)px^6-\left(3\,d^2e^2f^2-8\,d^3efg\right)px}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

```
[Out] -1/48*(4*(3*e^4*f^2 - 8*d*e^3*f*g + 6*d^2*e^2*g^2)*p*x^8*log(x) + 2*d^3*e*f
^2*p*x^2 + 2*(3*d*e^3*f^2 - 8*d^2*e^2*f*g + 6*d^3*e*g^2)*p*x^6 - (3*d^2*e^2
*f^2 - 8*d^3*e*f*g)*p*x^4 + 2*(6*d^4*g^2*p*x^4 - (3*e^4*f^2 - 8*d*e^3*f*g +
 6*d^2*e^2*g^2)*p*x^8 + 8*d^4*f*g*p*x^2 + 3*d^4*f^2*p)*log(e*x^2 + d) + 2*(
6*d^4*g^2*x^4 + 8*d^4*f*g*x^2 + 3*d^4*f^2)*log(c))/(d^4*x^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35522, size = 1470, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

```
[Out] 1/48*(12*(x^2*e + d)^4*d^2*g^2*p*e^3*log(x^2*e + d) - 48*(x^2*e + d)^3*d^3*
g^2*p*e^3*log(x^2*e + d) + 60*(x^2*e + d)^2*d^4*g^2*p*e^3*log(x^2*e + d) -
24*(x^2*e + d)*d^5*g^2*p*e^3*log(x^2*e + d) - 12*(x^2*e + d)^4*d^2*g^2*p*e^
3*log(x^2*e) + 48*(x^2*e + d)^3*d^3*g^2*p*e^3*log(x^2*e) - 72*(x^2*e + d)^2
*d^4*g^2*p*e^3*log(x^2*e) + 48*(x^2*e + d)*d^5*g^2*p*e^3*log(x^2*e) - 12*d^
6*g^2*p*e^3*log(x^2*e) - 12*(x^2*e + d)^3*d^3*g^2*p*e^3 + 36*(x^2*e + d)^2*
d^4*g^2*p*e^3 - 36*(x^2*e + d)*d^5*g^2*p*e^3 + 12*d^6*g^2*p*e^3 - 16*(x^2*e
 + d)^4*d*f*g*p*e^4*log(x^2*e + d) + 64*(x^2*e + d)^3*d^2*f*g*p*e^4*log(x^2
*e + d) - 96*(x^2*e + d)^2*d^3*f*g*p*e^4*log(x^2*e + d) + 48*(x^2*e + d)*d^
4*f*g*p*e^4*log(x^2*e + d) + 16*(x^2*e + d)^4*d*f*g*p*e^4*log(x^2*e) - 64*(
x^2*e + d)^3*d^2*f*g*p*e^4*log(x^2*e) + 96*(x^2*e + d)^2*d^3*f*g*p*e^4*log(
x^2*e) - 64*(x^2*e + d)*d^4*f*g*p*e^4*log(x^2*e) + 16*d^5*f*g*p*e^4*log(x^2
*e) - 12*(x^2*e + d)^2*d^4*g^2*e^3*log(c) + 24*(x^2*e + d)*d^5*g^2*e^3*log(
c) - 12*d^6*g^2*e^3*log(c) + 16*(x^2*e + d)^3*d^2*f*g*p*e^4 - 56*(x^2*e + d
)^2*d^3*f*g*p*e^4 + 64*(x^2*e + d)*d^4*f*g*p*e^4 - 24*d^5*f*g*p*e^4 + 6*(x^
2*e + d)^4*f^2*p*e^5*log(x^2*e + d) - 24*(x^2*e + d)^3*d*f^2*p*e^5*log(x^2*
e + d) + 36*(x^2*e + d)^2*d^2*f^2*p*e^5*log(x^2*e + d) - 24*(x^2*e + d)*d^3
*f^2*p*e^5*log(x^2*e + d) - 6*(x^2*e + d)^4*f^2*p*e^5*log(x^2*e) + 24*(x^2*
e + d)^3*d*f^2*p*e^5*log(x^2*e) - 36*(x^2*e + d)^2*d^2*f^2*p*e^5*log(x^2*e)
 + 24*(x^2*e + d)*d^3*f^2*p*e^5*log(x^2*e) - 6*d^4*f^2*p*e^5*log(x^2*e) - 1
6*(x^2*e + d)*d^4*f*g*e^4*log(c) + 16*d^5*f*g*e^4*log(c) - 6*(x^2*e + d)^3*
d*f^2*p*e^5 + 21*(x^2*e + d)^2*d^2*f^2*p*e^5 - 26*(x^2*e + d)*d^3*f^2*p*e^5
 + 11*d^4*f^2*p*e^5 - 6*d^4*f^2*e^5*log(c))*e^(-1)/((x^2*e + d)^4*d^4 - 4*(
x^2*e + d)^3*d^5 + 6*(x^2*e + d)^2*d^6 - 4*(x^2*e + d)*d^7 + d^8)
```

3.331 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^{11}} dx$

Optimal. Leaf size=253

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{6x^6} + \frac{e^2p\left(10d^2g^2-15defg+6e^2f^2\right)}{60d^4x^2} - \frac{ep\left(10d^2g^2-1\right.}{120}$$

```
[Out] -(e*f^2*p)/(40*d*x^8) + (e*f*(2*e*f - 5*d*g)*p)/(60*d^2*x^6) - (e*(6*e^2*f^
2 - 15*d*e*f*g + 10*d^2*g^2)*p)/(120*d^3*x^4) + (e^2*(6*e^2*f^2 - 15*d*e*f*
g + 10*d^2*g^2)*p)/(60*d^4*x^2) + (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2
)*p*Log[x])/(30*d^5) - (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*Log[d +
 e*x^2])/(60*d^5) - (f^2*Log[c*(d + e*x^2)^p])/(10*x^10) - (f*g*Log[c*(d +
e*x^2)^p])/(4*x^8) - (g^2*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rubi [A] time = 0.333418, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2475, 43, 2414, 12, 893}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{6x^6} + \frac{e^2p\left(10d^2g^2-15defg+6e^2f^2\right)}{60d^4x^2} - \frac{ep\left(10d^2g^2-1\right.}{120}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```


```
[Out] -(e*f^2*p)/(40*d*x^8) + (e*f*(2*e*f - 5*d*g)*p)/(60*d^2*x^6) - (e*(6*e^2*f^
2 - 15*d*e*f*g + 10*d^2*g^2)*p)/(120*d^3*x^4) + (e^2*(6*e^2*f^2 - 15*d*e*f*
g + 10*d^2*g^2)*p)/(60*d^4*x^2) + (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2
)*p*Log[x])/(30*d^5) - (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*Log[d +
 e*x^2])/(60*d^5) - (f^2*Log[c*(d + e*x^2)^p])/(10*x^10) - (f*g*Log[c*(d +
e*x^2)^p])/(4*x^8) - (g^2*Log[c*(d + e*x^2)^p])/(6*x^6)
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
```

```
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^{11}} dx = \frac{1}{2} \text{Subst}\left(\int \frac{(f+gx)^2 \log (c(d+ex)^p)}{x^6} dx, x, x^2\right)$$

$$= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg \log \left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2 \log \left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{2}(ep)\,\text{S}$$

$$= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg \log \left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2 \log \left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{60}(ep)$$

$$= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg \log \left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2 \log \left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{60}(ep)$$

$$= -\frac{ef^2p}{40dx^8} + \frac{ef(2ef-5dg)p}{60d^2x^6} - \frac{e\left(6e^2f^2-15defg+10d^2g^2\right)p}{120d^3x^4} + \frac{e^2\left(6e^2f^2-15de\right.}{60d^4}$$

Mathematica [A] time = 0.219371, size = 215, normalized size = 0.85

$$-\frac{2d^5\left(6f^2+15fgx^2+10g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)+depx^2\left(-d^2ex^2\left(4f^2+15fgx^2+20g^2x^4\right)+d^3\left(3f^2+10fgx^2+10g\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

```
[Out] -(d*e*p*x^2*(-12*e^3*f^2*x^6 + 6*d*e^2*f*x^4*(f + 5*g*x^2) + d^3*(3*f^2 + 1
0*f*g*x^2 + 10*g^2*x^4) - d^2*e*x^2*(4*f^2 + 15*f*g*x^2 + 20*g^2*x^4)) - 4*
e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[x] + 2*e^3*(6*e^2*f^2
- 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[d + e*x^2] + 2*d^5*(6*f^2 + 15*f*g*x^
2 + 10*g^2*x^4)*Log[c*(d + e*x^2)^p])/(120*d^5*x^10)
```

Maple [C] time = 0.415, size = 748, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^11,x)
```

```
[Out] -1/60*(10*g^2*x^4+15*f*g*x^2+6*f^2)/x^10*ln((e*x^2+d)^p)+1/120*(-15*I*Pi*d^
5*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-12*ln(c)*d^5*f^2-15*I
*Pi*d^5*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-20*ln(c)*d^5*g^2*x^4+10*I
*Pi*d^5*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-60*ln(x
)*d*e^4*f*g*p*x^10+30*ln(e*x^2+d)*d*e^4*f*g*p*x^10-10*I*Pi*d^5*g^2*x^4*csgn
(I*c*(e*x^2+d)^p)^2*csgn(I*c)-10*I*Pi*d^5*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)^2+24*ln(x)*e^5*f^2*p*x^10-12*ln(e*x^2+d)*e^5*f^2*p*x^10-30
*ln(c)*d^5*f*g*x^2+15*I*Pi*d^5*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)-6*I*Pi*d^5*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+15*I*Pi*d^
5*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+20*d^3*e^2*g^2*p*x^8+12*d*e^4*f^2*p*x^8-1
0*d^4*e*g^2*p*x^6-6*d^2*e^3*f^2*p*x^6+4*d^3*e^2*f^2*p*x^4-3*d^4*e*f^2*p*x^2
+6*I*Pi*d^5*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+40*ln(x
)*d^2*e^3*g^2*p*x^10-20*ln(e*x^2+d)*d^2*e^3*g^2*p*x^10-30*d^2*e^3*f*g*p*x^8
+15*d^3*e^2*f*g*p*x^6-10*d^4*e*f*g*p*x^4+10*I*Pi*d^5*g^2*x^4*csgn(I*c*(e*x^
2+d)^p)^3-6*I*Pi*d^5*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+6*I*Pi
*d^5*f^2*csgn(I*c*(e*x^2+d)^p)^3)/d^5/x^10
```

Maxima [A] time = 1.03691, size = 301, normalized size = 1.19

$$-\frac{1}{120}\,ep\Bigg(\frac{2\left(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2\right)\log\left(ex^2+d\right)}{d^5}-\frac{2\left(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2\right)\log\left(x^2\right)}{d^5}-\frac{2\left(6\,e^3f^2-15\,de^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="maxima")
```

```
[Out] -1/120*e*p*(2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(e*x^2 + d)/d^
5 - 2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(x^2)/d^5 - (2*(6*e^3*
f^2 - 15*d*e^2*f*g + 10*d^2*e*g^2)*x^6 - 3*d^3*f^2 - (6*d*e^2*f^2 - 15*d^2*
e*f*g + 10*d^3*g^2)*x^4 + 2*(2*d^2*e*f^2 - 5*d^3*f*g)*x^2)/(d^4*x^8)) - 1/6
0*(10*g^2*x^4 + 15*f*g*x^2 + 6*f^2)*log((e*x^2 + d)^p*c)/x^10
```

Fricas [A] time = 2.07507, size = 587, normalized size = 2.32

$$\frac{4\left(6\,e^5f^2-15\,de^4fg+10\,d^2e^3g^2\right)px^{10}\log\left(x\right)-3\,d^4ef^2px^2+2\left(6\,de^4f^2-15\,d^2e^3fg+10\,d^3e^2g^2\right)px^8-\left(6\,d^2e^3f^2-15\,d^3e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(4*(6*e^5*f^2 - 15*d*e^4*f*g + 10*d^2*e^3*g^2)*p*x^10*log(x) - 3*d^4*
e*f^2*p*x^2 + 2*(6*d*e^4*f^2 - 15*d^2*e^3*f*g + 10*d^3*e^2*g^2)*p*x^8 - (6*
d^2*e^3*f^2 - 15*d^3*e^2*f*g + 10*d^4*e*g^2)*p*x^6 + 2*(2*d^3*e^2*f^2 - 5*d
^4*e*f*g)*p*x^4 - 2*(10*d^5*g^2*p*x^4 + (6*e^5*f^2 - 15*d*e^4*f*g + 10*d^2*
e^3*g^2)*p*x^10 + 15*d^5*f*g*p*x^2 + 6*d^5*f^2*p)*log(e*x^2 + d) - 2*(10*d^
5*g^2*x^4 + 15*d^5*f*g*x^2 + 6*d^5*f^2)*log(c))/(d^5*x^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**11,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20026, size = 1809, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="giac")
```

```
[Out] -1/120*(20*(x^2*e + d)^5*d^2*g^2*p*e^4*log(x^2*e + d) - 100*(x^2*e + d)^4*d
^3*g^2*p*e^4*log(x^2*e + d) + 200*(x^2*e + d)^3*d^4*g^2*p*e^4*log(x^2*e + d
) - 180*(x^2*e + d)^2*d^5*g^2*p*e^4*log(x^2*e + d) + 60*(x^2*e + d)*d^6*g^2
*p*e^4*log(x^2*e + d) - 20*(x^2*e + d)^5*d^2*g^2*p*e^4*log(x^2*e) + 100*(x^
2*e + d)^4*d^3*g^2*p*e^4*log(x^2*e) - 200*(x^2*e + d)^3*d^4*g^2*p*e^4*log(x
^2*e) + 200*(x^2*e + d)^2*d^5*g^2*p*e^4*log(x^2*e) - 100*(x^2*e + d)*d^6*g^
2*p*e^4*log(x^2*e) + 20*d^7*g^2*p*e^4*log(x^2*e) - 20*(x^2*e + d)^4*d^3*g^2
*p*e^4 + 90*(x^2*e + d)^3*d^4*g^2*p*e^4 - 150*(x^2*e + d)^2*d^5*g^2*p*e^4 +
 110*(x^2*e + d)*d^6*g^2*p*e^4 - 30*d^7*g^2*p*e^4 - 30*(x^2*e + d)^5*d*f*g*
p*e^5*log(x^2*e + d) + 150*(x^2*e + d)^4*d^2*f*g*p*e^5*log(x^2*e + d) - 300
*(x^2*e + d)^3*d^3*f*g*p*e^5*log(x^2*e + d) + 300*(x^2*e + d)^2*d^4*f*g*p*e
^5*log(x^2*e + d) - 120*(x^2*e + d)*d^5*f*g*p*e^5*log(x^2*e + d) + 30*(x^2*
e + d)^5*d*f*g*p*e^5*log(x^2*e) - 150*(x^2*e + d)^4*d^2*f*g*p*e^5*log(x^2*e
) + 300*(x^2*e + d)^3*d^3*f*g*p*e^5*log(x^2*e) - 300*(x^2*e + d)^2*d^4*f*g*
p*e^5*log(x^2*e) + 150*(x^2*e + d)*d^5*f*g*p*e^5*log(x^2*e) - 30*d^6*f*g*p*
e^5*log(x^2*e) + 20*(x^2*e + d)^2*d^5*g^2*e^4*log(c) - 40*(x^2*e + d)*d^6*g
^2*e^4*log(c) + 20*d^7*g^2*e^4*log(c) + 30*(x^2*e + d)^4*d^2*f*g*p*e^5 - 13
5*(x^2*e + d)^3*d^3*f*g*p*e^5 + 235*(x^2*e + d)^2*d^4*f*g*p*e^5 - 185*(x^2*
e + d)*d^5*f*g*p*e^5 + 55*d^6*f*g*p*e^5 + 12*(x^2*e + d)^5*f^2*p*e^6*log(x^
2*e + d) - 60*(x^2*e + d)^4*d*f^2*p*e^6*log(x^2*e + d) + 120*(x^2*e + d)^3*
d^2*f^2*p*e^6*log(x^2*e + d) - 120*(x^2*e + d)^2*d^3*f^2*p*e^6*log(x^2*e +
d) + 60*(x^2*e + d)*d^4*f^2*p*e^6*log(x^2*e + d) - 12*(x^2*e + d)^5*f^2*p*e
^6*log(x^2*e) + 60*(x^2*e + d)^4*d*f^2*p*e^6*log(x^2*e) - 120*(x^2*e + d)^3
*d^2*f^2*p*e^6*log(x^2*e) + 120*(x^2*e + d)^2*d^3*f^2*p*e^6*log(x^2*e) - 60
*(x^2*e + d)*d^4*f^2*p*e^6*log(x^2*e) + 12*d^5*f^2*p*e^6*log(x^2*e) + 30*(x
^2*e + d)*d^5*f*g*e^5*log(c) - 30*d^6*f*g*e^5*log(c) - 12*(x^2*e + d)^4*d*f
^2*p*e^6 + 54*(x^2*e + d)^3*d^2*f^2*p*e^6 - 94*(x^2*e + d)^2*d^3*f^2*p*e^6
+ 77*(x^2*e + d)*d^4*f^2*p*e^6 - 25*d^5*f^2*p*e^6 + 12*d^5*f^2*e^6*log(c))*
e^(-1)/((x^2*e + d)^5*d^5 - 5*(x^2*e + d)^4*d^6 + 10*(x^2*e + d)^3*d^7 - 10
*(x^2*e + d)^2*d^8 + 5*(x^2*e + d)*d^9 - d^10)
```

3.332 $\int x^2\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=278

$$\frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{4d^2fgpx}{5e^2}+4d\cdots$$

```
[Out] (2*d*f^2*p*x)/(3*e) - (4*d^2*f*g*p*x)/(5*e^2) + (2*d^3*g^2*p*x)/(7*e^3) - (
2*f^2*p*x^3)/9 + (4*d*f*g*p*x^3)/(15*e) - (2*d^2*g^2*p*x^3)/(21*e^2) - (4*f
*g*p*x^5)/25 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 - (2*d^(3/2)*f^2*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) + (4*d^(5/2)*f*g*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(7*e^(7/2)) + (f^2*x^3*Log[c*(d + e*x^2)^p])/3 + (2*f*g*x^5*Log[c*(d + e*x^
2)^p])/5 + (g^2*x^7*Log[c*(d + e*x^2)^p])/7
```

Rubi [A] time = 0.236472, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2476, 2455, 302, 205}

$$\frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{4d^2fgpx}{5e^2}+4d\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f^2*p*x)/(3*e) - (4*d^2*f*g*p*x)/(5*e^2) + (2*d^3*g^2*p*x)/(7*e^3) - (
2*f^2*p*x^3)/9 + (4*d*f*g*p*x^3)/(15*e) - (2*d^2*g^2*p*x^3)/(21*e^2) - (4*f
*g*p*x^5)/25 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 - (2*d^(3/2)*f^2*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) + (4*d^(5/2)*f*g*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(7*e^(7/2)) + (f^2*x^3*Log[c*(d + e*x^2)^p])/3 + (2*f*g*x^5*Log[c*(d + e*x^
2)^p])/5 + (g^2*x^7*Log[c*(d + e*x^2)^p])/7
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x^2 \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2x^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^4\log\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log\left(c\left(d+ex^2\right)\right.\right.$$

$$= f^2\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx+(2fg)\int x^4\log\left(c\left(d+ex^2\right)^p\right)dx+g^2\int x^6 \mathrm{lc}$$

$$= \frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)\right.$$

$$= \frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)\right.$$

$$= \frac{2df^2px}{3e}-\frac{4d^2fgpx}{5e^2}+\frac{2d^3g^2px}{7e^3}-\frac{2}{9}f^2px^3+\frac{4dfgpx^3}{15e}-\frac{2d^2g^2px^3}{21e^2}-\frac{4}{25}fgpx$$

$$= \frac{2df^2px}{3e}-\frac{4d^2fgpx}{5e^2}+\frac{2d^3g^2px}{7e^3}-\frac{2}{9}f^2px^3+\frac{4dfgpx^3}{15e}-\frac{2d^2g^2px^3}{21e^2}-\frac{4}{25}fgpx$$

Mathematica [A] time = 0.165823, size = 188, normalized size = 0.68

$$\frac{\sqrt{e}x\left(105e^3x^2\left(35f^2+42fgx^2+15g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)+2p\left(-105d^2eg\left(42f+5gx^2\right)+1575d^3g^2+105de^2\left(35f^2\right.\right.\right.}{11025e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-210*d^(3/2)*(35*e^2*f^2 - 42*d*e*f*g + 15*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/S
qrt[d]] + Sqrt[e]*x*(2*p*(1575*d^3*g^2 - 105*d^2*e*g*(42*f + 5*g*x^2) + 105
*d*e^2*(35*f^2 + 14*f*g*x^2 + 3*g^2*x^4) - e^3*x^2*(1225*f^2 + 882*f*g*x^2
+ 225*g^2*x^4)) + 105*e^3*x^2*(35*f^2 + 42*f*g*x^2 + 15*g^2*x^4)*Log[c*(d +
 e*x^2)^p]))/(11025*e^(7/2))
```

Maple [C] time = 0.602, size = 761, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -2/9*f^2*p*x^3+1/7*ln(c)*g^2*x^7+1/3*ln(c)*f^2*x^3+(1/7*g^2*x^7+2/5*f*g*x^5
+1/3*f^2*x^3)*ln((e*x^2+d)^p)+2/7*d^3*g^2*p*x/e^3-2/21*d^2*g^2*p*x^3/e^2+2/
35*d*g^2*p*x^5/e+2/3*d*f^2*p*x/e+2/5*ln(c)*f*g*x^5-1/14*I*Pi*g^2*x^7*csgn(I
*c*(e*x^2+d)^p)^3-2/49*g^2*p*x^7-4/25*f*g*p*x^5-1/5*I*Pi*f*g*x^5*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/14*I*Pi*g^2*x^7*csgn(I*(e*x^2+d
)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/6*I*Pi*f^2*x^3*csgn(I*c*(e*x^2+d)^p)
^3-2/5/e^3*(-d*e)^(1/2)*p*d^2*ln(-(-d*e)^(1/2)*x-d)*f*g+2/5/e^3*(-d*e)^(1/2
)*p*d^2*ln((-d*e)^(1/2)*x-d)*f*g+1/5*I*Pi*f*g*x^5*csgn(I*c*(e*x^2+d)^p)^2*c
sgn(I*c)+1/5*I*Pi*f*g*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/14*
```

```
I*Pi*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/14*I*Pi*g^2*x^7*
csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-4/5*d^2*f*g*p*x/e^2-1/6*I*Pi*f^2*x^3*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+4/15*d*f*g*p*x^3/e+1/7/e^4*
(-d*e)^(1/2)*p*d^3*ln(-(-d*e)^(1/2)*x-d)*g^2-1/7/e^4*(-d*e)^(1/2)*p*d^3*ln(
(-d*e)^(1/2)*x-d)*g^2+1/3/e^2*(-d*e)^(1/2)*p*d*ln(-(-d*e)^(1/2)*x-d)*f^2-1/
3/e^2*(-d*e)^(1/2)*p*d*ln((-d*e)^(1/2)*x-d)*f^2-1/5*I*Pi*f*g*x^5*csgn(I*c*(
e*x^2+d)^p)^3+1/6*I*Pi*f^2*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+
1/6*I*Pi*f^2*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68206, size = 1138, normalized size = 4.09

$$\left[-\frac{450\,e^3g^2px^7+126\left(14\,e^3fg-5\,de^2g^2\right)px^5+70\left(35\,e^3f^2-42\,de^2fg+15\,d^2eg^2\right)px^3-105\left(35\,de^2f^2-42\,d^2efg+15\,d^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/11025*(450*e^3*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(3
5*e^3*f^2 - 42*d*e^2*f*g + 15*d^2*e*g^2)*p*x^3 - 105*(35*d*e^2*f^2 - 42*d^2
*e*f*g + 15*d^3*g^2)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2
 + d)) - 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 15*d^3*g^2)*p*x - 105*(15*e^3*g
^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*e^3*f^2*p*x^3)*log(e*x^2 + d) - 105*(15*e^
3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*e^3*f^2*x^3)*log(c))/e^3, -1/11025*(450*e^3
*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(35*e^3*f^2 - 42*d*e
^2*f*g + 15*d^2*e*g^2)*p*x^3 + 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 15*d^3*g^
2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) - 210*(35*d*e^2*f^2 - 42*d^2*e*f*g +
 15*d^3*g^2)*p*x - 105*(15*e^3*g^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*e^3*f^2*p*
x^3)*log(e*x^2 + d) - 105*(15*e^3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*e^3*f^2*x^3
)*log(c))/e^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

[Out] Timed out

Giac [A] time = 1.27028, size = 332, normalized size = 1.19

$$-\frac{2\left(15\,d^4g^2p-42\,d^3fgpe+35\,d^2f^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{105\,\sqrt{d}}+\frac{1}{11025}\left(1575\,g^2px^7e^3\log\left(x^2e+d\right)-450\,g^2px^7e^3+157\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/105*(15*d^4*g^2*p - 42*d^3*f*g*p*e + 35*d^2*f^2*p*e^2)*arctan(x*e^(1/2)/
sqrt(d))*e^(-7/2)/sqrt(d) + 1/11025*(1575*g^2*p*x^7*e^3*log(x^2*e + d) - 45
0*g^2*p*x^7*e^3 + 1575*g^2*x^7*e^3*log(c) + 630*d*g^2*p*x^5*e^2 + 4410*f*g*
p*x^5*e^3*log(x^2*e + d) - 1764*f*g*p*x^5*e^3 - 1050*d^2*g^2*p*x^3*e + 4410
*f*g*x^5*e^3*log(c) + 2940*d*f*g*p*x^3*e^2 + 3675*f^2*p*x^3*e^3*log(x^2*e +
 d) + 3150*d^3*g^2*p*x - 2450*f^2*p*x^3*e^3 - 8820*d^2*f*g*p*x*e + 3675*f^2
*x^3*e^3*log(c) + 7350*d*f^2*p*x*e^2)*e^(-3)
```

3.333 $\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=221

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{2d^2g^2px}{5e^2}+\frac{2d^{5/2}g\cdots}{}$$

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rubi [A] time = 0.162879, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{2d^2g^2px}{5e^2}+\frac{2d^{5/2}g\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log\left(c\left(d+ex^2\right)^p\right)\right)\,d$$

$$= f^2\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int x^4\log\left(c\left(\right.\right.$$

$$= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\left(\right.$$

$$= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+\right.\right.$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}f^2p\tan^{-}}{\sqrt{e}}$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}f^2p\tan^{-}}{\sqrt{e}}$$

Mathematica [A] time = 0.116807, size = 151, normalized size = 0.68

$$\frac{\sqrt{e}x\left(15e^2\left(15f^2+10fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)-2p\left(45d^2g^2-15deg\left(10f+gx^2\right)+e^2\left(225f^2+50fgx^2+9g^2\right.\right.\right.}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2
 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[
c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Maple [C] time = 0.105, size = 686, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -1/5/e^3*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x+d)*g^2*d^2-2/5*d^2*g^2*p*x/e^2+2/
15*d*g^2*p*x^3/e+1/5*ln(c)*g^2*x^5+ln(c)*f^2*x-4/9*f*g*p*x^3+2/3*ln(c)*f*g*
x^3-2*f^2*p*x-2/25*g^2*p*x^5+4/3*d*f*g*p*x/e+(1/5*g^2*x^5+2/3*f*g*x^3+f^2*x
)*ln((e*x^2+d)^p)-1/3*I*Pi*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)-1/e*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x+d)*f^2+1/e*(-d*e)^(1/2)*p*
ln(-(-d*e)^(1/2)*x+d)*f^2-1/10*I*Pi*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*P
i*f^2*csgn(I*c*(e*x^2+d)^p)^3*x+1/5/e^3*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d
)*g^2*d^2+1/10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/1
0*I*Pi*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/3*I*Pi*f*g*x^3*csgn(I*c*
(e*x^2+d)^p)^3+1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1
/2*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x+2/3/e^2*(-d*e)^(1/2)*p*ln((
-d*e)^(1/2)*x+d)*f*g*d-2/3/e^2*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d)*f*g*d-1
/10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/3*I*
Pi*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/3*I*Pi*f*g*x^3*csg
n(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73103, size = 922, normalized size = 4.17

$$\left[-\frac{18\,e^2g^2px^5+10\left(10\,e^2fg-3\,deg^2\right)px^3-15\left(15\,e^2f^2-10\,defg+3\,d^2g^2\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+30\left(15\,e^2f^2-10\,de\cdots\right.}{225\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2*
f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) -
d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*
g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g
^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^2*p*x
^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^
2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e*f*g +
3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*l
og(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/
e^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33595, size = 271, normalized size = 1.23

$$\frac{2\left(3\,d^3g^2p-10\,d^2fgpe+15\,df^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}}+\frac{1}{225}\left(45\,g^2px^5e^2\log\left(x^2e+d\right)-18\,g^2px^5e^2+45\,g^2x^5e^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g^2*p - 10*d^2*f*g*p*e + 15*d*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/sqrt(d) + 1/225*(45*g^2*p*x^5*e^2*log(x^2*e + d) - 18*g^2*p*x^
5*e^2 + 45*g^2*x^5*e^2*log(c) + 30*d*g^2*p*x^3*e + 150*f*g*p*x^3*e^2*log(x^
2*e + d) - 100*f*g*p*x^3*e^2 + 150*f*g*x^3*e^2*log(c) - 90*d^2*g^2*p*x + 30
0*d*f*g*p*x*e + 225*f^2*p*x*e^2*log(x^2*e + d) - 450*f^2*p*x*e^2 + 225*f^2*
x*e^2*log(c))*e^(-2)
```

3.334 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^2} dx$

Optimal. Leaf size=178

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -4*f*g*p*x + (2*d*g^2*p*x)/(3*e) - (2*g^2*p*x^3)/9 + (2*Sqrt[e]*f^2*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (4*Sqrt[d]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/Sqrt[e] - (2*d^(3/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) -
(f^2*Log[c*(d + e*x^2)^p])/x + 2*f*g*x*Log[c*(d + e*x^2)^p] + (g^2*x^3*Log[
c*(d + e*x^2)^p])/3
```

Rubi [A] time = 0.155254, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2476, 2448, 321, 205, 2455, 302}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -4*f*g*p*x + (2*d*g^2*p*x)/(3*e) - (2*g^2*p*x^3)/9 + (2*Sqrt[e]*f^2*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (4*Sqrt[d]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/Sqrt[e] - (2*d^(3/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) -
(f^2*Log[c*(d + e*x^2)^p])/x + 2*f*g*x*Log[c*(d + e*x^2)^p] + (g^2*x^3*Log[
c*(d + e*x^2)^p])/3
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx = \int \left(2fg\log\left(c\left(d+ex^2\right)^p\right)+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^2}+g^2x^2\log\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$= f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx+(2fg)\int \log\left(c\left(d+ex^2\right)^p\right)dx+g^2\int x^2\log\left(c\left(d\right.\right.$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)+(\ldots$$

$$= -4fgpx+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^{\ldots}\right.$$

$$= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\ldots$$

$$= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\ldots$$

Mathematica [A] time = 0.13557, size = 112, normalized size = 0.63

$$\frac{1}{9}\left(\left(-\frac{9f^2}{x}+18fgx+3g^2x^3\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{6p\left(-d^2g^2+6defg+3e^2f^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{2gpx\left(-3dg+18ef+eg\ldots\right.}{e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] ((-2*g*p*x*(18*e*f - 3*d*g + e*g*x^2))/e + (6*(3*e^2*f^2 + 6*d*e*f*g - d^2*
g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2)) + ((-9*f^2)/x + 18*f*
g*x + 3*g^2*x^3)*Log[c*(d + e*x^2)^p])/9
```

Maple [C] time = 0.606, size = 742, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^2,x)
```

```
[Out] -1/3*(-g^2*x^4-6*f*g*x^2+3*f^2)/x*ln((e*x^2+d)^p)-1/18*(-18*I*Pi*f*g*csgn(I
*c*(e*x^2+d)^p)^2*csgn(I*c)*x^2*e^2*d-3*I*Pi*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^
2*csgn(I*c)*e^2*d-3*I*Pi*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2*e^2*d+9*I*Pi*d*e^2*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+18*I*Pi*f*g*csgn
(I*c*(e*x^2+d)^p)^3*x^2*e^2*d+3*I*Pi*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3*e^2*d-
18*I*Pi*f*g*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x^2*e^2*d-9*I*Pi*d*
e^2*f^2*csgn(I*c*(e*x^2+d)^p)^3-6*ln(c)*g^2*x^4*e^2*d+18*I*Pi*f*g*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x^2*e^2*d+3*I*Pi*g^2*x^4*csgn(I*
(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*e^2*d+9*I*Pi*d*e^2*f^2*csgn(I*
(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-9*I*Pi*d*e^2*f^2*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)*csgn(I*c)+4*d*e^2*g^2*p*x^4-36*ln(c)*f*g*x^2*e^2*d-6*(
-d*e)^(1/2)*d^2*p*ln(-(-d*e)^(1/2)*x-d)*g^2*x+36*(-d*e)^(1/2)*p*ln(-(-d*e)^
(1/2)*x-d)*f*g*e*d*x+18*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*f^2*e^2*x+6*(-
d*e)^(1/2)*d^2*p*ln(-(-d*e)^(1/2)*x+d)*g^2*x-36*(-d*e)^(1/2)*p*ln(-(-d*e)^(
1/2)*x+d)*f*g*e*d*x-18*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d)*f^2*e^2*x-12*d^
2*e*g^2*p*x^2+72*d*e^2*f*g*p*x^2+18*ln(c)*d*e^2*f^2)/e^2/d/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77989, size = 790, normalized size = 4.44

$$\left[-\frac{2\,de^2g^2px^4-3\left(3\,e^2f^2+6\,defg-d^2g^2\right)\sqrt{-de}px\log\left(\frac{ex^2+2\,\sqrt{-de}x-d}{ex^2+d}\right)+6\left(6\,de^2fg-d^2eg^2\right)px^2-3\left(de^2g^2px^4+6\,de^2fgp\cdots\right.}{9\,de^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/9*(2*d*e^2*g^2*p*x^4 - 3*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt(-d*e)*p
*x*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(6*d*e^2*f*g - d^2*e*g
^2)*p*x^2 - 3*(d*e^2*g^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(e*x
^2 + d) - 3*(d*e^2*g^2*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*e^2*
x), -1/9*(2*d*e^2*g^2*p*x^4 - 6*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt(d*e)
*p*x*arctan(sqrt(d*e)*x/d) + 6*(6*d*e^2*f*g - d^2*e*g^2)*p*x^2 - 3*(d*e^2*g
^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(e*x^2 + d) - 3*(d*e^2*g^2
*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*e^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2542, size = 227, normalized size = 1.28

$$-\frac{2\left(d^2g^2p-6\,dfgpe-3\,f^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}}+\frac{\left(3\,g^2px^4e\log\left(x^2e+d\right)-2\,g^2px^4e+3\,g^2x^4e\log\left(c\right)+18\,fgpx^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g^2*p - 6*d*f*g*p*e - 3*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-
3/2)/sqrt(d) + 1/9*(3*g^2*p*x^4*e*log(x^2*e + d) - 2*g^2*p*x^4*e + 3*g^2*x^
4*e*log(c) + 18*f*g*p*x^2*e*log(x^2*e + d) + 6*d*g^2*p*x^2 - 36*f*g*p*x^2*e
 + 18*f*g*x^2*e*log(c) - 9*f^2*p*e*log(x^2*e + d) - 9*f^2*e*log(c))*e^(-1)/
x
```

3.335 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^4} \, dx$

Optimal. Leaf size=169

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}+g^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{2e^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2ef^2p}{3dx}+\frac{4\sqrt{e}fgp\tan^{-1}}{\sqrt{d}}$$

```
[Out] (-2*e*f^2*p)/(3*d*x) - 2*g^2*p*x - (2*e^(3/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*d^(3/2)) + (4*Sqrt[e]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 (2*Sqrt[d]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f^2*Log[c*(d + e*
x^2)^p])/(3*x^3) - (2*f*g*Log[c*(d + e*x^2)^p])/x + g^2*x*Log[c*(d + e*x^2)
^p]
```

Rubi [A] time = 0.146754, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2476, 2448, 321, 205, 2455, 325}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}+g^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{2e^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2ef^2p}{3dx}+\frac{4\sqrt{e}fgp\tan^{-1}}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f^2*p)/(3*d*x) - 2*g^2*p*x - (2*e^(3/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*d^(3/2)) + (4*Sqrt[e]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 (2*Sqrt[d]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f^2*Log[c*(d + e*
x^2)^p])/(3*x^3) - (2*f*g*Log[c*(d + e*x^2)^p])/x + g^2*x*Log[c*(d + e*x^2)
^p]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx &= \int \left(g^2\log\left(c\left(d+ex^2\right)^p\right)+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^4}+\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right)dx \\
&= f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx+(2fg)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx+g^2\int \log\left(c\left(d+e\right.\right. \\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}+g^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}\left(2e\right. \\
&= -\frac{2ef^2p}{3dx}-2g^2px+\frac{4\sqrt{e}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d\right.\right.}{x} \\
&= -\frac{2ef^2p}{3dx}-2g^2px-\frac{2e^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{4\sqrt{e}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}g^2p\tan}{\sqrt{e}}
\end{aligned}$$

Mathematica [C] time = 0.133533, size = 113, normalized size = 0.67

$$-\frac{\left(f^2+6fgx^2-3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2ef^2p\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx}+\frac{2gp(dg+2ef)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}-2g^2px$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] -2*g^2*p*x + (2*g*(2*e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqr
t[e]) - (2*e*f^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) -
 ((f^2 + 6*f*g*x^2 - 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Maple [C] time = 0.805, size = 700, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^4,x)
```

```
[Out] -1/3*(-3*g^2*x^4+6*f*g*x^2+f^2)/x^3*ln((e*x^2+d)^p)+1/6*(6*I*Pi*d*f*g*x^2*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-6*I*Pi*d*f*g*x^2*csgn(I*
(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*d*f^2*csgn(I*c*(e*x^2+d)^p)^2*csg
n(I*c)+6*I*Pi*d*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d*g^2*x^4*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*d*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2+3*I*Pi*d*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*I*Pi*d*f
*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+6*ln(c)*d*g^2*x^4-3*I*Pi*d*g^2*x^4
*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)*csgn(I*c)+I*Pi*d*f^2*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d*f^2*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-12*d*g^2*p*x^4-12*ln(c)*d*f*g*x^2
-4*e*f^2*p*x^2+2*sum(_R*ln((18*d^4*g^4*p^2+72*d^3*e*f*g^3*p^2+60*d^2*e^2*f^
2*g^2*p^2-24*d*e^3*f^3*g*p^2+2*e^4*f^4*p^2+3*_R^2*d^3*e)*x+(-3*d^4*g^2*p-6*
d^3*e*f*g*p+d^2*e^2*f^2*p)*_R),_R=RootOf(9*d^4*g^4*p^2+36*d^3*e*f*g^3*p^2+3
0*d^2*e^2*f^2*g^2*p^2-12*d*e^3*f^3*g*p^2+e^4*f^4*p^2+_Z^2*d^3*e))*d*x^3-2*l
n(c)*d*f^2)/d/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67833, size = 744, normalized size = 4.4

$$\left[-\frac{6\,d^2eg^2px^4 + 2\,de^2f^2px^2 - \left(e^2f^2 - 6\,defg - 3\,d^2g^2\right)\sqrt{-de}px^3\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right) - \left(3\,d^2eg^2px^4 - 6\,d^2efgpx^2 - d^2ef^2p\right)}{3\,d^2ex^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(6*d^2*e*g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 - (e^2*f^2 - 6*d*e*f*g - 3*d^2
*g^2)*sqrt(-d*e)*p*x^3*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - (3*d
^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x^2 - d^2*e*f^2*p)*log(e*x^2 + d) - (3*d^2*e
*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^2*e*f^2)*log(c))/(d^2*e*x^3), -1/3*(6*d^2*e*
g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 + 2*(e^2*f^2 - 6*d*e*f*g - 3*d^2*g^2)*sqrt(d*
e)*p*x^3*arctan(sqrt(d*e)*x/d) - (3*d^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x^2 - d
^2*e*f^2*p)*log(e*x^2 + d) - (3*d^2*e*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^2*e*f^2
)*log(c))/(d^2*e*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35648, size = 208, normalized size = 1.23

$$\frac{2\left(3\,d^2g^2p+6\,dfgpe-f^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d^{\frac{3}{2}}}+\frac{3\,dg^2px^4\log\left(x^2e+d\right)-6\,dg^2px^4+3\,dg^2x^4\log\left(c\right)-6\,dfgpx^2\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*d^2*g^2*p + 6*d*f*g*p*e - f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1
/2)/d^(3/2) + 1/3*(3*d*g^2*p*x^4*log(x^2*e + d) - 6*d*g^2*p*x^4 + 3*d*g^2*x
^4*log(c) - 6*d*f*g*p*x^2*log(x^2*e + d) - 2*f^2*p*x^2*e - 6*d*f*g*x^2*log(
c) - d*f^2*p*log(x^2*e + d) - d*f^2*log(c))/(d*x^3)
```

3.336 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6} dx$

Optimal. Leaf size=200

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{2fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+\frac{2e^2f^2p}{5d^2x}+\frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fgp\tan}{3d^{3/2}}$$

```
[Out] (-2*e*f^2*p)/(15*d*x^3) + (2*e^2*f^2*p)/(5*d^2*x) - (4*e*f*g*p)/(3*d*x) + (
2*e^(5/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (4*e^(3/2)*f*g*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) + (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/Sqrt[d] - (f^2*Log[c*(d + e*x^2)^p])/(5*x^5) - (2*f*g*Log[c*
(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Rubi [A] time = 0.170722, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2476, 2455, 325, 205}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{2fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+\frac{2e^2f^2p}{5d^2x}+\frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fgp\tan}{3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f^2*p)/(15*d*x^3) + (2*e^2*f^2*p)/(5*d^2*x) - (4*e*f*g*p)/(3*d*x) + (
2*e^(5/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (4*e^(3/2)*f*g*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) + (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/Sqrt[d] - (f^2*Log[c*(d + e*x^2)^p])/(5*x^5) - (2*f*g*Log[c*
(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^6} dx = \int \left(\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^6} + \frac{2fg \log \left(c\left(d+ex^2\right)^p\right)}{x^4} + \frac{g^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^2}\right) dx$$

$$= f^2 \int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^6} dx + (2fg) \int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^4} dx + g^2 \int \frac{\log \left(c\left(d+\right.\right.}{x^2}$$

$$= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{2fg \log \left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g^2 \log \left(c\left(d+ex^2\right)^p\right)}{x} + \frac{1}{5}(2e$$

$$= -\frac{2ef^2p}{15dx^3} - \frac{4efgp}{3dx} + \frac{2\sqrt{e}g^2p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{2fg \log \left(c\left(d\right.\right.}{3x^3}$$

$$= -\frac{2ef^2p}{15dx^3} + \frac{2e^2f^2p}{5d^2x} - \frac{4efgp}{3dx} - \frac{4e^{3/2}fgp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}g^2p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f^2}{}$$

$$= -\frac{2ef^2p}{15dx^3} + \frac{2e^2f^2p}{5d^2x} - \frac{4efgp}{3dx} + \frac{2e^{5/2}f^2p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{4e^{3/2}fgp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2}{}$$

Mathematica [C] time = 0.063699, size = 156, normalized size = 0.78

$$-\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{2fg \log \left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g^2 \log \left(c\left(d+ex^2\right)^p\right)}{x} - \frac{2ef^2p\, {}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^2}{d}\right)}{15dx^3} - \frac{4efgp\, {}_2F_1\left(-\frac{1}{2}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f^2*p*Hypergeo
metric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (4*e*f*g*p*Hypergeomet
ric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(d + e*x^2)^p])/(5
*x^5) - (2*f*g*Log[c*(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Maple [C] time = 0.594, size = 753, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^6,x)
```

```
[Out] -1/15*(15*g^2*x^4+10*f*g*x^2+3*f^2)/x^5*ln((e*x^2+d)^p)+1/30*(15*I*Pi*d^2*g
^2*x^4*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)^2*d^2+3*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^3*d^2+10*I*Pi*d^2*f*g*x^2*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-15*I*Pi*d^2*g^2*x^4*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+10*I*Pi*d^2*f*g*x^2*csgn(I*c*(e*x^2
+d)^p)^3-10*I*Pi*d^2*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-10*I*Pi*d^2*
```

```
f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-30*ln(c)*d^2*g^2*x^4-3*
I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*d^2+15*I*Pi*d^2*g^2*x^4*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-15*I*Pi*d^2*g^2*x^4*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)+3*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
*csgn(I*c)*d^2-40*d*e*f*g*p*x^4+12*e^2*f^2*p*x^4+2*sum(_R*ln((450*d^4*e*g^4
*p^2-600*d^3*e^2*f*g^3*p^2+380*d^2*e^3*f^2*g^2*p^2-120*d*e^4*f^3*g*p^2+18*e
^5*f^4*p^2+3*_R^2*d^5)*x+(-15*d^5*g^2*p+10*d^4*e*f*g*p-3*d^3*e^2*f^2*p)*_R)
,_R=RootOf(225*d^4*e*g^4*p^2-300*d^3*e^2*f*g^3*p^2+190*d^2*e^3*f^2*g^2*p^2-
60*d*e^4*f^3*g*p^2+9*e^5*f^4*p^2+_Z^2*d^5))*d^2*x^5-20*ln(c)*d^2*f*g*x^2-4*
d*e*f^2*p*x^2-6*ln(c)*f^2*d^2)/d^2/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88557, size = 774, normalized size = 3.87

$$\left[\frac{\left(3e^2f^2 - 10defg + 15d^2g^2\right)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) - 2def^2px^2 + 2\left(3e^2f^2 - 10defg\right)px^4 - \left(15d^2g^2px^4 + 10d^2fg\cdots\right.}{15d^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*((3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(-e/d)*log((e*x^2 +
2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*d*
e*f*g)*p*x^4 - (15*d^2*g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e*x^
2 + d) - (15*d^2*g^2*x^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5), 1
/15*(2*(3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(e/d)*arctan(x*sqrt(
e/d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*d*e*f*g)*p*x^4 - (15*d^2*g^2*p*
x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e*x^2 + d) - (15*d^2*g^2*x^4 + 10
*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31353, size = 244, normalized size = 1.22

$$\frac{2\left(15\,d^2g^2pe-10\,dfgpe^2+3\,f^2pe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{15\,d^{\frac{5}{2}}}-\frac{15\,d^2g^2px^4\log\left(x^2e+d\right)+20\,dfgpx^4e+15\,d^2g^2x^4\log\left(c\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(15*d^2*g^2*p*e - 10*d*f*g*p*e^2 + 3*f^2*p*e^3)*arctan(x*e^(1/2)/sqrt(
d))*e^(-1/2)/d^(5/2) - 1/15*(15*d^2*g^2*p*x^4*log(x^2*e + d) + 20*d*f*g*p*x
^4*e + 15*d^2*g^2*x^4*log(c) - 6*f^2*p*x^4*e^2 + 10*d^2*f*g*p*x^2*log(x^2*e
 + d) + 2*d*f^2*p*x^2*e + 10*d^2*f*g*x^2*log(c) + 3*d^2*f^2*p*log(x^2*e + d
) + 3*d^2*f^2*log(c))/(d^2*x^5)
```

3.337 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^8} dx$

Optimal. Leaf size=252

$$-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{2e^2f^2p}{21d^2x^3}-\frac{2e^3f^2p}{7d^3x}-\frac{2e^{7/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^2}{5\ldots}$$

```
[Out] (-2*e*f^2*p)/(35*d*x^5) + (2*e^2*f^2*p)/(21*d^2*x^3) - (4*e*f*g*p)/(15*d*x^
3) - (2*e^3*f^2*p)/(7*d^3*x) + (4*e^2*f*g*p)/(5*d^2*x) - (2*e*g^2*p)/(3*d*x
) - (2*e^(7/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*d^(7/2)) + (4*e^(5/2)*
f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g^2*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f^2*Log[c*(d + e*x^2)^p])/(7*x^7) - (2*f
*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Rubi [A] time = 0.206453, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2476, 2455, 325, 205}

$$-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{2e^2f^2p}{21d^2x^3}-\frac{2e^3f^2p}{7d^3x}-\frac{2e^{7/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^2}{5\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```


```
[Out] (-2*e*f^2*p)/(35*d*x^5) + (2*e^2*f^2*p)/(21*d^2*x^3) - (4*e*f*g*p)/(15*d*x^
3) - (2*e^3*f^2*p)/(7*d^3*x) + (4*e^2*f*g*p)/(5*d^2*x) - (2*e*g^2*p)/(3*d*x
) - (2*e^(7/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*d^(7/2)) + (4*e^(5/2)*
f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g^2*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f^2*Log[c*(d + e*x^2)^p])/(7*x^7) - (2*f
*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*(d + e*x^2)^p])/(3*x^3)
```


Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^8}\,dx = \int\left(\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^8}+\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x^6}+\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\right)dx$$

$$= f^2\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^8}\,dx+(2fg)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx+g^2\int\frac{\log\left(c\left(d+\right.\right.}{x^4}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{1}{7}(2e$$

$$= -\frac{2ef^2p}{35dx^5}-\frac{4efgp}{15dx^3}-\frac{2eg^2p}{3dx}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g^2}{}$$

$$= -\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}-\frac{2e^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f^2\log}{}$$

$$= -\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}-\frac{2e^3f^2p}{7d^3x}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}+\frac{4e^{5/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}$$

$$= -\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}-\frac{2e^3f^2p}{7d^3x}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}-\frac{2e^{7/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}$$

Mathematica [C] time = 0.0293723, size = 161, normalized size = 0.64

$$-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2ef^2p\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{ex^2}{d}\right)}{35dx^5}-\frac{4efgp\,{}_2F_1\left(-\frac{3}{2}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

```
[Out] (-2*e*f^2*p*Hypergeometric2F1[-5/2, 1, -3/2, -((e*x^2)/d)])/(35*d*x^5) - (4
*e*f*g*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e*
g^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(
d + e*x^2)^p])/(7*x^7) - (2*f*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*
(d + e*x^2)^p])/(3*x^3)
```

Maple [C] time = 0.424, size = 784, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^8,x)
```

```
[Out] -1/105*(35*g^2*x^4+42*f*g*x^2+15*f^2)/x^7*ln((e*x^2+d)^p)-1/210*(-42*I*Pi*d
^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-15*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn
```

```
(I*c*(e*x^2+d)^p)*csgn(I*c)+30*ln(c)*d^4*f^2+84*(-d*e)^(1/2)*p*e^2*ln(-e*x+
(-d*e)^(1/2))*f*g*d*x^7-84*(-d*e)^(1/2)*p*e^2*ln(-e*x-(-d*e)^(1/2))*f*g*d*x
^7-15*I*Pi*d^4*f^2*csgn(I*c*(e*x^2+d)^p)^3+70*ln(c)*d^4*g^2*x^4+42*I*Pi*d^4
*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-35*I*Pi*d^4*g^2*x^4*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+42*I*Pi*d^4*f*g*x^2*csgn(
I*c*(e*x^2+d)^p)^2*csgn(I*c)+140*d^3*e*g^2*p*x^6+60*d*e^3*f^2*p*x^6-20*d^2*
e^2*f^2*p*x^4+12*d^3*e*f^2*p*x^2+35*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2+35*I*Pi*d^4*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
-70*(-d*e)^(1/2)*p*e*ln(-e*x+(-d*e)^(1/2))*g^2*d^2*x^7+70*(-d*e)^(1/2)*p*e*
ln(-e*x-(-d*e)^(1/2))*g^2*d^2*x^7+84*ln(c)*d^4*f*g*x^2+15*I*Pi*d^4*f^2*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+15*I*Pi*d^4*f^2*csgn(I*c*(e*x^2+d)^
p)^2*csgn(I*c)-35*I*Pi*d^4*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3-30*(-d*e)^(1/2)*
p*e^3*ln(-e*x+(-d*e)^(1/2))*f^2*x^7+30*(-d*e)^(1/2)*p*e^3*ln(-e*x-(-d*e)^(1
/2))*f^2*x^7-168*d^2*e^2*f*g*p*x^6+56*d^3*e*f*g*p*x^4-42*I*Pi*d^4*f*g*x^2*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c))/d^4/x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81863, size = 956, normalized size = 3.79

$$\left[\frac{\left(15\,e^{3}f^{2}-42\,de^{2}fg+35\,d^{2}eg^{2}\right)px^{7}\sqrt{-\frac{e}{d}}\log\left(\frac{ex^{2}-2\,dx\sqrt{-\frac{e}{d}}-d}{ex^{2}+d}\right)-6\,d^{2}ef^{2}px^{2}-2\left(15\,e^{3}f^{2}-42\,de^{2}fg+35\,d^{2}eg^{2}\right)px^{6}+2\,(5}{105}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="fricas")
```

```
[Out] [1/105*((15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(-e/d)*log((e*
x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 6*d^2*e*f^2*p*x^2 - 2*(15*e^3*f^
2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^6 + 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*p*x
^4 - (35*d^3*g^2*p*x^4 + 42*d^3*f*g*p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d) -
(35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 15*d^3*f^2)*log(c))/(d^3*x^7), -1/105*(2
*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(e/d)*arctan(x*sqrt(e
/d)) + 6*d^2*e*f^2*p*x^2 + 2*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x
^6 - 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*p*x^4 + (35*d^3*g^2*p*x^4 + 42*d^3*f*g*
p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d) + (35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 1
5*d^3*f^2)*log(c))/(d^3*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36621, size = 300, normalized size = 1.19

$$-\frac{2\left(35\,d^{2}g^{2}pe^{2}-42\,dfgpe^{3}+15\,f^{2}pe^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{105\,d^{\frac{7}{2}}}-\frac{70\,d^{2}g^{2}px^{6}e-84\,dfgpx^{6}e^{2}+35\,d^{3}g^{2}px^{4}\log\left(x^{2}e+d\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="giac")
```

```
[Out] -2/105*(35*d^2*g^2*p*e^2 - 42*d*f*g*p*e^3 + 15*f^2*p*e^4)*arctan(x*e^(1/2)/
sqrt(d))*e^(-1/2)/d^(7/2) - 1/105*(70*d^2*g^2*p*x^6*e - 84*d*f*g*p*x^6*e^2
+ 35*d^3*g^2*p*x^4*log(x^2*e + d) + 30*f^2*p*x^6*e^3 + 28*d^2*f*g*p*x^4*e +
 35*d^3*g^2*x^4*log(c) - 10*d*f^2*p*x^4*e^2 + 42*d^3*f*g*p*x^2*log(x^2*e +
d) + 6*d^2*f^2*p*x^2*e + 42*d^3*f*g*x^2*log(c) + 15*d^3*f^2*p*log(x^2*e + d
) + 15*d^3*f^2*log(c))/(d^3*x^7)
```

3.338 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=188

$$\frac{f^2 p \text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^3}+\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3}-\frac{f\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+\frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{4g}$$

```
[Out] (f*p*x^2)/(2*g^2) + (d*p*x^2)/(4*e*g) - (p*x^4)/(8*g) - (d^2*p*Log[d + e*x^
2])/(4*e^2*g) + (x^4*Log[c*(d + e*x^2)^p])/(4*g) - (f*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/(2*e*g^2) + (f^2*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f
 - d*g)])/(2*g^3) + (f^2*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g
^3)
```

Rubi [A] time = 0.275934, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2475, 43, 2416, 2389, 2295, 2395, 2394, 2393, 2391}

$$\frac{f^2 p \text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^3}+\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3}-\frac{f\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+\frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{4g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (f*p*x^2)/(2*g^2) + (d*p*x^2)/(4*e*g) - (p*x^4)/(8*g) - (d^2*p*Log[d + e*x^
2])/(4*e^2*g) + (x^4*Log[c*(d + e*x^2)^p])/(4*g) - (f*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/(2*e*g^2) + (f^2*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f
 - d*g)])/(2*g^3) + (f^2*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g
^3)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2 \log\left(c(d+ex)^p\right)}{f+gx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{f\log\left(c(d+ex)^p\right)}{g^2} + \frac{x\log\left(c(d+ex)^p\right)}{g} + \frac{f^2\log\left(c(d+ex)^p\right)}{g^2(f+gx)}\right) dx, x, x^2\right) \\
&= -\frac{f\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2g^2} + \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g^2} + \frac{\text{Subst}\left(\int x \text{l}\right.}{} \\
&= \frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4g} + \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} - \frac{f\,\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+\right.}{2eg^2} \\
&= \frac{fpx^2}{2g^2} + \frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4g} - \frac{f\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2} + \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e}{-}\right.}{2g^3} \\
&= \frac{fpx^2}{2g^2} + \frac{dpx^2}{4eg} - \frac{px^4}{8g} - \frac{d^2p\log\left(d+ex^2\right)}{4e^2g} + \frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4g} - \frac{f\left(d+ex^2\right)\log\left(c\left(d+e\right.\right.}{2eg^2}
\end{aligned}
$$

Mathematica [A] time = 0.123798, size = 143, normalized size = 0.76

$$
\frac{4e^2f^2p\text{PolyLog}\left(2, \frac{g(d+ex^2)}{dg-ef}\right) + e\log\left(c\left(d+ex^2\right)^p\right)\left(4ef^2\log\left(\frac{e(f+gx^2)}{ef-dg}\right) + 2g\left(-2df - 2efx^2 + egx^4\right)\right) - 2d^2g^2p\log\left(d+e\right.}{8e^2g^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (e*g*p*x^2*(4*e*f + 2*d*g - e*g*x^2) - 2*d^2*g^2*p*Log[d + e*x^2] + e*Log[c
*(d + e*x^2)^p]*(2*g*(-2*d*f - 2*e*f*x^2 + e*g*x^4) + 4*e*f^2*Log[(e*(f + g
*x^2))/(e*f - d*g)]) + 4*e^2*f^2*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g
)])/(8*e^2*g^3)
```

Maple [C] time = 0.849, size = 902, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] 1/4*ln((e*x^2+d)^p)/g*x^4-1/2*ln((e*x^2+d)^p)/g^2*f*x^2+1/2*ln((e*x^2+d)^p)
*f^2/g^3*ln(g*x^2+f)-1/2*p*f^2/g^3*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha
)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z
^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=
2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*
f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/8*p*x^4/g+1/4*d*p*x^2/e/g+1/2*f*p*x^
2/g^2-1/4*d^2*p*ln(e*x^2+d)/e^2/g-1/2*p/e/g^2*d*ln(e*x^2+d)*f+1/4*I*Pi*csgn
```

```
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2/g^3*ln(g*x^2+f)+1/8*I*Pi*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g*x^4-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^
3*f^2/g^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*cs
gn(I*c)/g^2*f*x^2+1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*x^4-1/4*I*Pi
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g^2*f*x^2+1/4*I*Pi*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)*f^2/g^3*ln(g*x^2+f)-1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)/g*x^4-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)*f^2/g^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g
^2*f*x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*f*x^2-1/8*I*Pi*csgn
(I*c*(e*x^2+d)^p)^3/g*x^4+1/4*ln(c)/g*x^4-1/2*ln(c)/g^2*f*x^2+1/2*ln(c)*f^2
/g^3*ln(g*x^2+f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.339 $\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=112

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2g^2}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg}-\frac{px^2}{2g}$$

```
[Out] -(p*x^2)/(2*g) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e*g) - (f*Log[c*(d +
 e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) - (f*p*PolyLog[2, -((g
*(d + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rubi [A] time = 0.187701, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2393, 2391}

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2g^2}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg}-\frac{px^2}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -(p*x^2)/(2*g) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e*g) - (f*Log[c*(d +
 e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) - (f*p*PolyLog[2, -((g
*(d + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{f+gx}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{g} - \frac{f\log\left(c(d+ex)^p\right)}{g(f+gx)}\right)dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \log\left(c(d+ex)^p\right)dx, x, x^2\right)}{2g} - \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g}$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2} + \frac{\text{Subst}\left(\int \log\left(cx^p\right)dx, x, d+ex^2\right)}{2eg} + \frac{(efp)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{px^2}{2g} + \frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2} + \frac{(fp)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{px^2}{2g} + \frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2} - \frac{fp\text{Li}_2\left(-\frac{g\left(d+ex^2\cdots\right.}{ef-dg}\right.}{2g^2}$$

Mathematica [A] time = 0.0411701, size = 91, normalized size = 0.81

$$-\frac{efp\,\text{PolyLog}\left(2,\frac{g\left(d+ex^2\right)}{dg-ef}\right)-\log\left(c\left(d+ex^2\right)^p\right)\left(-ef\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)+dg+egx^2\right)+egpx^2}{2eg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -(e*g*p*x^2 - Log[c*(d + e*x^2)^p]*(d*g + e*g*x^2 - e*f*Log[(e*(f + g*x^2))
/(e*f - d*g)]) + e*f*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(2*e*g^2
)
```

Maple [C] time = 0.68, size = 672, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g*x^2-1/2*ln((e*x^2+d)^p)*f/g^2*ln(g*x^2+f)-1/2*p*x^2/g
+1/2*p/e/g*d*ln(e*x^2+d)+1/2*p*f/g^2*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alp
ha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,inde
x=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+
e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2/g*x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
*f/g^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(
I*c)/g*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/g
^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g*x^2+1/4*I*Pi*csgn(I*c*(e*
x^2+d)^p)^3*f/g^2*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*
x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^2*ln(g*x^2+f)+1/2*ln(c)/
g*x^2-1/2*ln(c)*f/g^2*ln(g*x^2+f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log \left(\left(ex^2+d \right)^p c \right)}{gx^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.340 $\int \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=70

$$\frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g}$$

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g) + (p*PolyLog[
2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g)
```

Rubi [A] time = 0.09544, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2475, 2394, 2393, 2391}

$$\frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g) + (p*PolyLog[
2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x \log \left(c \left(d+e x^2\right)^p\right)}{f+g x^2} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{\log \left(c (d+e x)^p\right)}{f+g x} \, dx,x,x^2\right)$$
$$= \frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g}-\frac{(e p) \text{Subst}\left(\int \frac{\log \left(\frac{e (f+g x)}{e f-d g}\right)}{d+e x} \, dx,x,x^2\right)}{2 g}$$
$$= \frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g}-\frac{p \text{Subst}\left(\int \frac{\log \left(1+\frac{g x}{e f-d g}\right)}{x} \, dx,x,d+e x^2\right)}{2 g}$$
$$= \frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g}+\frac{p \text{Li}_2\left(-\frac{g \left(d+e x^2\right)}{e f-d g}\right)}{2 g}$$

Mathematica [A] time = 0.0069087, size = 64, normalized size = 0.91

$$\frac{p \text{PolyLog}\left(2,\frac{g \left(d+e x^2\right)}{d g-e f}\right)+\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d
 + e*x^2))/(-(e*f) + d*g)])/(2*g)
```

Maple [C] time = 0.649, size = 472, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] 1/2/g*ln(g*x^2+f)*ln((e*x^2+d)^p)-1/2/g*p*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x
-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_a
lpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f
,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alph
a)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g
-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*c*(e*x^2+
d)^p)^3+1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/2/g*ln(g
*x^2+f)*ln(c)
```

Maxima [B] time = 1.01482, size = 186, normalized size = 2.66

$$\frac{ep\left(\frac{\log(ex^2+d)\log(gx^2+f)}{e}-\frac{\log(gx^2+f)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{e}\right)}{2g}-\frac{p\log\left(ex^2+d\right)\log\left(gx^2+f\right)}{2g}+\frac{\log\left(gx^2+f\right)\log\left(\left(ex^2\right.\right.}{2g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*e*p*(log(e*x^2 + d)*log(g*x^2 + f)/e - (log(g*x^2 + f)*log(-(e*g*x^2 +
e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))/e)/g - 1/2*p*lo
g(e*x^2 + d)*log(g*x^2 + f)/g + 1/2*log(g*x^2 + f)*log((e*x^2 + d)^p*c)/g
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.341 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=119

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f}+\frac{p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f}$$

```
[Out] (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f) - (Log[c*(d + e*x^2)^p]*Log[
(e*(f + g*x^2))/(e*f - d*g)])/(2*f) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f
- d*g))])/(2*f) + (p*PolyLog[2, 1 + (e*x^2)/d])/(2*f)
```

Rubi [A] time = 0.206326, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f}+\frac{p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f) - (Log[c*(d + e*x^2)^p]*Log[
(e*(f + g*x^2))/(e*f - d*g)])/(2*f) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f
- d*g))])/(2*f) + (p*PolyLog[2, 1 + (e*x^2)/d])/(2*f)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{g\log(c(d+ex)^p)}{f(f+gx)}\right) dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f} - \frac{g\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2f}$$

$$= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f} - \frac{(ep)\operatorname{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx \ldots\right.}{2f}$$

$$= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f} + \frac{p\operatorname{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f} + \frac{p\operatorname{Subs}\ldots}{}$$

$$= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f} - \frac{p\operatorname{Li}_2\left(-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f} + \frac{p\operatorname{Li}_2 \ldots}{}$$

Mathematica [A] time = 0.0356858, size = 92, normalized size = 0.77

$$\frac{-p\operatorname{PolyLog}\left(2,\frac{g\left(d+ex^2\right)}{dg-ef}\right)+p\operatorname{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\log\left(c\left(d+ex^2\right)^p\right)\left(\log\left(-\frac{ex^2}{d}\right)-\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*(Log[-((e*x^2)/d)] - Log[(e*(f + g*x^2))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)
/d])/(2*f)
```

Maple [C] time = 0.665, size = 732, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f),x)
```

```
[Out] -1/2*ln((e*x^2+d)^p)/f*ln(g*x^2+f)+ln((e*x^2+d)^p)/f*ln(x)-p/f*ln(x)*ln((-e
*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p/f*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)
)-p/f*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p/f*dilog((e*x+(-d*e)^(1/2))/
(-d*e)^(1/2))+1/2*p/f*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf
(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,ind
ex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((R
ootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_
alpha=RootOf(_Z^2*e+d))+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f*ln(x)-
1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f*ln(x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)^2/f*ln(x)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f*ln(g*x^2+f)-1
/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*ln(g*x^2+f)-1/4*I*Pi*
csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f*ln(g*x^2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p
)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f*ln(x)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)*csgn(I*c)/f*ln(g*x^2+f)-1/2*ln(c)/f*ln(g*x^2+f)+1/f*ln(c)
*ln(x)
```

Maxima [A] time = 1.72022, size = 189, normalized size = 1.59

$$-\frac{1}{2}ep\left(\frac{2\log\left(\frac{ex^2}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)}{ef}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{ef}\right)-\frac{1}{2}\left(\frac{\log\left(gx^2+f\right)}{f}-\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] -1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f) - (log(g*x^2
+ f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d
*g)))/(e*f)) - 1/2*(log(g*x^2 + f)/f - log(x^2)/f)*log((e*x^2 + d)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^pc\right)}{gx^3+fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^3 + f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(ex^2+d\right)^pc\right)}{\left(gx^2+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x), x)
```

3.342 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=176

$$\frac{gp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f^2}-\frac{gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}$$

```
[Out] (e*p*Log[x])/(d*f) - (e*p*Log[d + e*x^2])/(2*d*f) - Log[c*(d + e*x^2)^p]/(2
*f*x^2) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (g*Log[c*(d
+ e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*f^2) + (g*p*PolyLog[2, -((
g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) - (g*p*PolyLog[2, 1 + (e*x^2)/d])/(2*
f^2)
```

Rubi [A] time = 0.26662, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2475, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{gp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f^2}-\frac{gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] (e*p*Log[x])/(d*f) - (e*p*Log[d + e*x^2])/(2*d*f) - Log[c*(d + e*x^2)^p]/(2
*f*x^2) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (g*Log[c*(d
+ e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*f^2) + (g*p*PolyLog[2, -((
g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) - (g*p*PolyLog[2, 1 + (e*x^2)/d])/(2*
f^2)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{fx^2} - \frac{g\log\left(c(d+ex)^p\right)}{f^2x} + \frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^2\right)}{2f} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f^2} + \frac{g^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,d\right.}{2f^2}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} + \frac{(e}{}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} - \frac{gp}{}$$

$$= \frac{ep\log(x)}{df} - \frac{ep\log\left(d+ex^2\right)}{2df} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} + \frac{g\log\left(c\left(\right.\right.}{}$$

Mathematica [A] time = 0.0727305, size = 147, normalized size = 0.84

$$\frac{-g\left(p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)\right)+gp\text{PolyLog}\left(2,\frac{g(d+ex^2)}{dg-ef}\right)+g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right.}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] ((e*f*p*(2*Log[x] - Log[d + e*x^2]))/d - (f*Log[c*(d + e*x^2)^p])/x^2 + g*L
og[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + g*p*PolyLog[2, (g*(d
 + e*x^2))/(-(e*f) + d*g)] - g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*
PolyLog[2, 1 + (e*x^2)/d]))/(2*f^2)
```

Maple [C] time = 0.686, size = 942, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x)
```

```
[Out] -1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/x^2-1/4*I*Pi*csgn(I
*c*(e*x^2+d)^p)^3*g/f^2*ln(g*x^2+f)+p*g/f^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-
d*e)^(1/2))+p*g/f^2*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*I*Pi*csgn
(I*c*(e*x^2+d)^p)^3*g/f^2*ln(x)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/
f/x^2+e*p*ln(x)/d/f-1/2*e*p*ln(e*x^2+d)/d/f+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)*csgn(I*c)*g/f^2*ln(x)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)*csgn(I*c)*g/f^2*ln(g*x^2+f)+1/2*ln(c)*g/f^2*ln(g*x^2+f)-l
n(c)*g/f^2*ln(x)+1/2*ln((e*x^2+d)^p)*g/f^2*ln(g*x^2+f)-ln((e*x^2+d)^p)*g/f^
2*ln(x)-1/2*p*g/f^2*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_
Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_al
```

```
pha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index
=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((Roo
tOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e
*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_al
pha=RootOf(_Z^2*e+d))+p*g/f^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+p*g/f
^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*ln(c)/f/x^2-1/2*ln((e*x^2+d)^
p)/f/x^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p
)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/x^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*cs
gn(I*c)*g/f^2*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)^2*g/f^2*ln(g*x^2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*
g/f^2*ln(x)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^2*ln(x)
```

Maxima [A] time = 1.92295, size = 240, normalized size = 1.36

$$\frac{1}{2}ep\left(\frac{\left(2\log\left(\frac{ex^2}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)\right)g}{ef^2}-\frac{\left(\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)\right)g}{ef^2}-\frac{\log\left(ex^2+d\right)}{df}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))*g/(e*f^2) - (log(g*x
^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f
- d*g)))*g/(e*f^2) - log(e*x^2 + d)/(d*f) + 2*log(x)/(d*f)) + 1/2*(g*log(g*
x^2 + f)/f^2 - g*log(x^2)/f^2 - 1/(f*x^2))*log((e*x^2 + d)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^5+fx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^5 + f*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^3), x)
```

3.343 $\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=667

$$\frac{i f^{3/2} p \text{PolyLog}\left(2, 1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}+\frac{i f^{3/2} p \text{PolyLog}\left(2, 1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}-\frac{i f^{3/2} p \text{PolyLog}\left(\right.}{g^{5/2}}$$

```
[Out] (2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*e^(3/2)*g) + (2*f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[
f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sq
rt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f*x*Log
[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g) + (f^(3/2)*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - (I*f^(3/2)*p*PolyLog[
2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) + ((I/2)*f^(3/2)*p*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
- Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) + ((I/2)*f^(3/2)*p*P
olyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f
] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rubi [A] time = 0.717945, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2476, 2448, 321, 205, 2455, 302, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{i f^{3/2} p \text{PolyLog}\left(2, 1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}+\frac{i f^{3/2} p \text{PolyLog}\left(2, 1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}-\frac{i f^{3/2} p \text{PolyLog}\left(\right.}{g^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```


```
[Out] (2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*e^(3/2)*g) + (2*f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[
f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sq
rt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f*x*Log
[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g) + (f^(3/2)*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - (I*f^(3/2)*p*PolyLog[
2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) + ((I/2)*f^(3/2)*p*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
- Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) + ((I/2)*f^(3/2)*p*P
olyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f
] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```


Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \int \left(-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{g}+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{g^2\left(f+gx^2\right)}\right)dx$$

$$= -\frac{f\int \log\left(c\left(d+ex^2\right)^p\right)dx}{g^2}+\frac{f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx}{g^2}+\frac{\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx}{g}$$

$$= -\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}}+\frac{(2ef}{}$$

$$= \frac{2fpx}{g^2}-\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}-\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}-$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log}{g^{5/2}}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log}{g^{5/2}}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log}{g^{5/2}}$$

Mathematica [A] time = 0.548642, size = 691, normalized size = 1.04

$$-\frac{if^{3/2}p\left(\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{gx}\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-2*p*x^3)/(9*g) + (2*d*p*(Sqrt[e]*x - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
)/(3*e^(3/2)*g) + (2*f*p*(x - (Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
))/g^2 - (f*x*Log[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g)
+ (f^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - ((I/
2)*f^(3/2)*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqr
t[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*(Sqrt[-d] + S
qrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)
/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] +
```

```
Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/S
qrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I
*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*
x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f]
+ I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/g^(5/2)
```

Maple [C] time = 0.631, size = 1011, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] 1/3*(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g*x^3-p*f/g^2*x*ln(e*x^2+d)-1/6*I*Pi*cs
gn(I*c*(e*x^2+d)^p)^3/g*x^3+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)^2/g*x^3+1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*x^3+p*Sum(1/2*(ln(
x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alph
a*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*
g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)
-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))*f^2/g^3/_
alpha,_alpha=RootOf(_Z^2*g+f))-2/9*p*x^3/g+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)^2*f^2/g^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-2/3*p/g*d^
2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-2*p*f/g^2*d/(d*e)^(1/2)*arctan(x*e/
(d*e)^(1/2))-(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g^2*f*x+2/3*d*p*x/e/g-ln(c)/g^
2*f*x+ln(c)*f^2/g^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/3*p/g*x^3*ln(e*x^
2+d)+1/3*ln(c)/g*x^3+2*f*p*x/g^2+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g^2*f*x+1
/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*f*x+1/2*I*P
i*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/g^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1
/2))+(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*f^2/g^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(
1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/g^2/
(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2/g^2*f*x-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*cs
gn(I*c)/g*x^3-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*f*x-1/2*I*Pi*c
sgn(I*c*(e*x^2+d)^p)^3*f^2/g^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.344 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=585

$$-\frac{i\sqrt{f}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}-\frac{i\sqrt{f}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}+\frac{i\sqrt{f}p\text{PolyLog}\left(2 \ldots\right.}{g^{3/2}}$$

```
[Out] (-2*p*x)/g + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g) - (2*Sqr
t[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqr
t[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) + (x*Log[c*(d + e*x^2)^p])/g - (Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(3/2) + (I*Sqrt[f]*p*
PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(3/2) - ((I/2)*Sqrt[
f]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*
Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) - ((I/2)*Sqr
t[f]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2)
```

Rubi [A] time = 0.594126, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2476, 2448, 321, 205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{i\sqrt{f}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}-\frac{i\sqrt{f}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}+\frac{i\sqrt{f}p\text{PolyLog}\left(2 \ldots\right.}{g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```


```
[Out] (-2*p*x)/g + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g) - (2*Sqr
t[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqr
t[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) + (x*Log[c*(d + e*x^2)^p])/g - (Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(3/2) + (I*Sqrt[f]*p*
PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(3/2) - ((I/2)*Sqrt[
f]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*
Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) - ((I/2)*Sqr
t[f]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2)
```


Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \log\left(c\left(d+ex^2\right)^p\right)dx}{g} - \frac{f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx}{g}$$

$$= \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}} - \frac{(2ep)\int \frac{x^2}{d+ex^2}\,dx}{g} + \frac{(2efp)\int}{}$$

$$= -\frac{2px}{g} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}} + \frac{\left(2e\sqrt{f}p\right)\int \frac{x\tan^{-1}(\ldots)}{d+ex^2}}{g^{3/2}}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}} -$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}} -$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} + \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log(}{}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} + \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log(}{}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} + \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log(}{}$$

Mathematica [A] time = 0.292048, size = 680, normalized size = 1.16

$$\frac{i\sqrt{f}p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right) + i\sqrt{f}p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right) - i\sqrt{f}p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right) - i\sqrt{f}p\text{PolyLog}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-4*Sqrt[g]*p*x + (4*Sqrt[d]*Sqrt[g]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
 + I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + I*Sqrt[f]*p*Log[(Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 -
(I*Sqrt[g]*x)/Sqrt[f]] - I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(
(-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] -
I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[
-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + 2*Sqrt[g]*x*Log[c*(d + e*x^2
)^p] - 2*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + I*Sqrt[
f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt
[-d]*Sqrt[g])] + I*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(
Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(S
qrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[f]*
p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d
]*Sqrt[g])])/(2*g^(3/2))
```

Maple [C] time = 0.555, size = 746, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g*x-(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*f/g/(f*
g)^(1/2)*arctan(x*g/(f*g)^(1/2))+p/g*x*ln(e*x^2+d)-2*p*x/g+2*p/g*d/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))+p*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*l
n(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-
e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootO
f(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*
_alpha*e*g+d*g-e*f,index=2)))/e))*f/g^2/_alpha,_alpha=RootOf(_Z^2*g+f))-1/2
*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g/(f*g)^(1/2)*arctan(x*
g/(f*g)^(1/2))+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*x+1/2*I*Pi*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/g/(f*g)^(1/2)*arctan(x*g/
(f*g)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g
*x+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g*x-1/2*I*Pi*csgn(I
*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-1/2*I*P
i*csgn(I*c*(e*x^2+d)^p)^3/g*x+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f/g/(f*g)^(1
/2)*arctan(x*g/(f*g)^(1/2))+ln(c)/g*x-ln(c)*f/g/(f*g)^(1/2)*arctan(x*g/(f*g
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.345 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rubi [A] time = 0.425435, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+ex^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\cdots}{\cdots}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\cdots}{\cdots}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\cdots}{\cdots}$$

Mathematica [A] time = 0.158188, size = 564, normalized size = 1.06

$$-\frac{i\left(p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)+p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)-p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)-p\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\cdots\right.}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\cdots}\right.\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-I/2)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-
d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] + Sq
rt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/
Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d]
 + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x
)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*Pol
yLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqr
t[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] +
I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt
[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqr
t[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] time = 0.082, size = 504, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root
Of(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*ln(c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

3.346 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=581

$$-\frac{i\sqrt{g}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}}-\frac{i\sqrt{g}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}}+\frac{i\sqrt{g}p\text{PolyLog}\left(2\right.}{f^{3/2}}$$

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f) - (2*Sqrt[g]*p*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) + (
Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - S
qrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))
])/f^(3/2) + (Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(3/2) - Log[c*(d + e*x^2)^p]/(f*x) - (Sqrt[g]*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/f^(3/2) + (I*Sqrt[g]*p*PolyLog[2, 1
- (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLog[
2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqr
t[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLo
g[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2)
```

Rubi [A] time = 0.596958, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2476, 2455, 205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{i\sqrt{g}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}}-\frac{i\sqrt{g}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}}+\frac{i\sqrt{g}p\text{PolyLog}\left(2\right.}{f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f) - (2*Sqrt[g]*p*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) + (
Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - S
qrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))
])/f^(3/2) + (Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(3/2) - Log[c*(d + e*x^2)^p]/(f*x) - (Sqrt[g]*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/f^(3/2) + (I*Sqrt[g]*p*PolyLog[2, 1
- (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLog[
2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqr
t[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLo
g[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2)
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx}{f} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx}{f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} + \frac{(2ep)\int \frac{1}{d+ex^2}\,dx}{f} + \frac{(2egp)\int \frac{x\,\mathrm{ta}}{\sqrt{f}}}{f}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} + \frac{\left(2e\sqrt{g}p\right)\int}{f}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} + \frac{\left(2e\sqrt{g}p\right)\int}{}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} - \frac{\left(\sqrt{e}\sqrt{g}p\right)\int}{f}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}}\right.}{f^{3/2}}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}}\right.}{f^{3/2}}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}}\right.}{f^{3/2}}$$

Mathematica [A] time = 0.298604, size = 673, normalized size = 1.16

$$i\sqrt{g}p\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)+i\sqrt{g}p\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)-i\sqrt{g}p\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)-i\sqrt{g}p\mathrm{PolyLog}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

```
[Out] ((4*Sqrt[e]*Sqrt[f]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + I*Sqrt[g]*p*Lo
g[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*
Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + I*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[
e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqr
t[f]] - I*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt
[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - I*Sqrt[g]*p*Log[(
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log
```

```
[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (2*Sqrt[f]*Log[c*(d + e*x^2)^p])/x - 2*Sqrt[g
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + I*Sqrt[g]*p*PolyLog[2,
 (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]
+ I*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
 + I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt
[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[g]*p*PolyLog[2, (S
qrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/(2
*f^(3/2))
```

Maple [C] time = 0.652, size = 755, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x)
```

```
[Out] -(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-(l
n((e*x^2+d)^p)-p*ln(e*x^2+d))/f/x+p*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*
(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_
alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*
g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,
index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog(
(RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))/f/_alpha,_alpha=RootOf(_Z^2*g+f))-p
/f/x*ln(e*x^2+d)+2*p/f*e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/2*I*Pi*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/
2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/x+1/2*I*Pi*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f*g/(f*g)^(1/2)*arctan(x*g/(
f*g)^(1/2))+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/
x+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+
1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)
^2*csgn(I*c)/f/x-ln(c)/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-ln(c)/f/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^4+fx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^4 + f*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^2), x)
```

3.347 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=651

$$\frac{ig^{3/2}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}+\frac{ig^{3/2}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}-\frac{ig^{3/2}p\text{PolyLog}\left(2,1\right.}{f^{5/2}}$$

```
[Out] (-2*e*p)/(3*d*f*x) - (2*e^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)*f
) - (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) + (2*g^(3/2)*
p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(
5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/f^(5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sq
rt[f] - I*Sqrt[g]*x))])/f^(5/2) - Log[c*(d + e*x^2)^p]/(3*f*x^3) + (g*Log[c
*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d +
e*x^2)^p])/f^(5/2) - (I*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*S
qrt[g]*x)])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(5/2)
```

Rubi [A] time = 0.649823, antiderivative size = 651, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2476, 2455, 325, 205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ig^{3/2}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}+\frac{ig^{3/2}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}-\frac{ig^{3/2}p\text{PolyLog}\left(2,1\right.}{f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*e*p)/(3*d*f*x) - (2*e^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)*f
) - (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) + (2*g^(3/2)*
p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(
5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/f^(5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sq
rt[f] - I*Sqrt[g]*x))])/f^(5/2) - Log[c*(d + e*x^2)^p]/(3*f*x^3) + (g*Log[c
*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d +
e*x^2)^p])/f^(5/2) - (I*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*S
qrt[g]*x)])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(5/2)
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4\left(f+gx^2\right)}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx^4} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x^2} + \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx}{f} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx}{f^2} + \frac{g^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx}{f^2}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}} + \frac{(2ep)\int \frac{}{x^2(d}}{3f}$$

$$= -\frac{2ep}{3dfx} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\text{lo}}{f^{5/}}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} - \frac{g^{3/2}}{}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} - \frac{g^{3/2}}{}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} - \frac{g^{3/2}}{}$$

Mathematica [C] time = 0.26156, size = 754, normalized size = 1.16

$$-\frac{2eg^{3/2}p\left(\frac{i\left(\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(\sqrt{f}-i\sqrt{g}x)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)}{\sqrt{e}}+\frac{\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}}\right)}{\sqrt{e}}\right)}{4\sqrt{e}}+\frac{i\left(\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(\sqrt{f}-i\sqrt{g}x)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)}{\sqrt{e}}+\frac{\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}}\right)}{\sqrt{e}}\right)}{4\sqrt{e}}-\frac{i\left(\text{PolyLog}\right.}{}\right.}{f^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (2*e*p*Hyperge
ometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*f*x) - Log[c*(d + e*x^2)^p]/(3
*f*x^3) + (g*Log[c*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/f^(5/2) - (2*e*g^(3/2)*p*(((I/4)*((Log[(Sqrt[g
]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (
I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x
))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqrt[e] + ((I/4)*((Log
[-((Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g]))
]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sqrt[f] -
I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqrt[e] - (
(I/4)*((Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*
Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sq
rt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqr
t[e] - ((I/4)*((Log[-((Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] -
 Sqrt[-d]*Sqrt[g]))]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (
Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]/Sq
rt[e]))/Sqrt[e]))/f^(5/2)
```

Maple [C] time = 0.651, size = 1005, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))*g^2/f^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))
-1/3*(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/f/x^3+(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*
g/f^2/x+p*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,i
ndex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*
(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_
Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alph
a*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=2)))/e))*g/f^2/_alpha,_alpha=RootOf(_Z^2*g+f))-1/3*p/f/x^3*ln(e*x^2+d)-
2/3*p/f*e^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-2/3*e*p/d/f/x+p*g/f^2/x*l
n(e*x^2+d)-2*p*g/f^2*e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/2*I*Pi*csgn(I*
c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^2/x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^2/x
+1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/x^3-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)*csgn(I*c)*g/f^2/x+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c
)*g^2/f^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*
```

```
csgn(I*c*(e*x^2+d)^p)^2*g/f^2/x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g^2/f^2/(f
*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)*csgn(I*c)/f/x^3-1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f/x^3+
1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g^2/f^2/(f*g)^(1/2)*ar
ctan(x*g/(f*g)^(1/2))-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/
f/x^3-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g^2/f^2/
(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+ln(c)*g^2/f^2/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))-1/3*ln(c)/f/x^3+ln(c)*g/f^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^6+fx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^6 + f*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**4/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="giac")

[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^4), x)
```

3.348 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=199

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{g^3}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^3}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+$$

```
[Out] -(p*x^2)/(2*g^2) + (e*f^2*p*Log[d + e*x^2])/(2*g^3*(e*f - d*g)) + ((d + e*x
^2)*Log[c*(d + e*x^2)^p])/(2*e*g^2) - (f^2*Log[c*(d + e*x^2)^p])/(2*g^3*(f
+ g*x^2)) - (e*f^2*p*Log[f + g*x^2])/(2*g^3*(e*f - d*g)) - (f*Log[c*(d + e*
x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^3 - (f*p*PolyLog[2, -((g*(d + e
*x^2))/(e*f - d*g))])/g^3
```

Rubi [A] time = 0.283417, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2475, 43, 2416, 2389, 2295, 2395, 36, 31, 2394, 2393, 2391}

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{g^3}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^3}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```


```
[Out] -(p*x^2)/(2*g^2) + (e*f^2*p*Log[d + e*x^2])/(2*g^3*(e*f - d*g)) + ((d + e*x
^2)*Log[c*(d + e*x^2)^p])/(2*e*g^2) - (f^2*Log[c*(d + e*x^2)^p])/(2*g^3*(f
+ g*x^2)) - (e*f^2*p*Log[f + g*x^2])/(2*g^3*(e*f - d*g)) - (f*Log[c*(d + e*
x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^3 - (f*p*PolyLog[2, -((g*(d + e
*x^2))/(e*f - d*g))])/g^3
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^5 \log \left(c \left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx = \frac{1}{2} \text{Subst}\left(\int \frac{x^2 \log (c(d+ex)^p)}{(f+gx)^2} dx, x, x^2\right)$$

$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{\log (c(d+ex)^p)}{g^2} + \frac{f^2 \log (c(d+ex)^p)}{g^2(f+gx)^2} - \frac{2f \log (c(d+ex)^p)}{g^2(f+gx)}\right) dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \log (c(d+ex)^p) \, dx, x, x^2\right)}{2g^2} - \frac{f \, \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx} dx, x, x^2\right)}{g^2} + \frac{f^2 \, \text{Subst}\left(\int \frac{\log(c}{(f}\right.}{2g}$$

$$= -\frac{f^2 \log \left(c \left(d+ex^2\right)^p\right)}{2g^3 \left(f+gx^2\right)} - \frac{f \log \left(c \left(d+ex^2\right)^p\right) \log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^3} + \frac{\text{Subst}\left(\int \log (cx^p) \, dx, x, d+\right.}{2eg^2}$$

$$= -\frac{px^2}{2g^2} + \frac{\left(d+ex^2\right) \log \left(c \left(d+ex^2\right)^p\right)}{2eg^2} - \frac{f^2 \log \left(c \left(d+ex^2\right)^p\right)}{2g^3 \left(f+gx^2\right)} - \frac{f \log \left(c \left(d+ex^2\right)^p\right) \log \left(\frac{e(f+}{ef-}\right.}{g^3}$$

$$= -\frac{px^2}{2g^2} + \frac{ef^2 p \log \left(d+ex^2\right)}{2g^3(ef-dg)} + \frac{\left(d+ex^2\right) \log \left(c \left(d+ex^2\right)^p\right)}{2eg^2} - \frac{f^2 \log \left(c \left(d+ex^2\right)^p\right)}{2g^3 \left(f+gx^2\right)} - \frac{ef^2 p \log}{2g^3(}$$

Mathematica [A] time = 0.221841, size = 166, normalized size = 0.83

$$-\frac{\frac{2f\left(p\text{PolyLog}\left(2,\frac{g(d+ex^2)}{dg-ef}\right)+\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)\right)}{g} + \frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)} - \frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e} + \frac{ef^2 p\left(\log\left(d+ex^2\right)-\log\left(f+gx^2\right)\right)}{g(dg-ef)} + px^2}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e + (f^2*Log[c*(d + e*x^2)^p])
/(g*(f + g*x^2)) + (e*f^2*p*(Log[d + e*x^2] - Log[f + g*x^2]))/(g*(-(e*f) +
 d*g)) + (2*f*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + p*Po
lyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]))/g)/(2*g^2)
```

Maple [C] time = 0.713, size = 985, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g^2*x^2-ln((e*x^2+d)^p)*f/g^3*ln(g*x^2+f)-1/2*ln((e*x^2
+d)^p)*f^2/g^3/(g*x^2+f)-1/2*p*x^2/g^2+1/2*p*e/g^3*f^2/(d*g-e*f)*ln(g*x^2+f
)+1/2*p/e/g/(d*g-e*f)*ln(e*x^2+d)*d^2-1/2*p/g^2/(d*g-e*f)*ln(e*x^2+d)*f*d-1
/2*p*e/g^3/(d*g-e*f)*ln(e*x^2+d)*f^2+p*f/g^3*sum(ln(x-_alpha)*ln(g*x^2+f)-l
n(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+
e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_a
lpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*
```

```
e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/2*I*Pi*csgn(I*c*(e*x^2+d)^
p)^3*f/g^3*ln(g*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^3*ln(
g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2/g^3/(g*x^
2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*x^2-1/4*I*Pi*csgn(I*c*(
e*x^2+d)^p)^3/g^2*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/
g^2*x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/g^3/(g*x^2+f)-1/2*I*
Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g^3*ln(g*x^2+f)+1/4*I*Pi*c
sgn(I*c*(e*x^2+d)^p)^3*f^2/g^3/(g*x^2+f)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)*csgn(I*c)*f/g^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/g^3/(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*x^2+1/2*ln(c)/g^2*x^2-ln(c)*f/g^3*ln
(g*x^2+f)-1/2*ln(c)*f^2/g^3/(g*x^2+f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5 \log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \log \left(\left(ex^2 + d \right)^p c \right)}{\left(gx^2 + f \right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

3.349 $\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=155

$$\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2g^2}+\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}-\frac{efp\log\left(d+ex^2\right)}{2g^2(ef-dg)}+\frac{efp\log\left(f+\right.}{2g^2(ef-\right.}$$

```
[Out] -(e*f*p*Log[d + e*x^2])/(2*g^2*(e*f - d*g)) + (f*Log[c*(d + e*x^2)^p])/(2*g
^2*(f + g*x^2)) + (e*f*p*Log[f + g*x^2])/(2*g^2*(e*f - d*g)) + (Log[c*(d +
e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) + (p*PolyLog[2, -((g*(d
 + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rubi [A] time = 0.221205, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2475, 43, 2416, 2395, 36, 31, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2g^2}+\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}-\frac{efp\log\left(d+ex^2\right)}{2g^2(ef-dg)}+\frac{efp\log\left(f+\right.}{2g^2(ef-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -(e*f*p*Log[d + e*x^2])/(2*g^2*(e*f - d*g)) + (f*Log[c*(d + e*x^2)^p])/(2*g
^2*(f + g*x^2)) + (e*f*p*Log[f + g*x^2])/(2*g^2*(e*f - d*g)) + (Log[c*(d +
e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) + (p*PolyLog[2, -((g*(d
 + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{(f+gx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{f\log\left(c(d+ex)^p\right)}{g(f+gx)^2}+\frac{\log\left(c(d+ex)^p\right)}{g(f+gx)}\right)dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g} - \frac{f\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}\,dx, x, x^2\right)}{2g}$$
$$= \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2} - \frac{(ep)\operatorname{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x,\right.}{2g^2}$$
$$= \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2} - \frac{p\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d\right.}{2g^2}$$
$$= -\frac{efp\log\left(d+ex^2\right)}{2g^2(ef-dg)} + \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{efp\log\left(f+gx^2\right)}{2g^2(ef-dg)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\right.}{2g^2}$$

Mathematica [A] time = 0.103261, size = 131, normalized size = 0.85

$$\frac{p\operatorname{PolyLog}\left(2,\frac{g\left(d+ex^2\right)}{dg-ef}\right)+\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}+\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)+\frac{efp\log\left(d+ex^2\right)}{dg-ef}+\frac{efp\log\left(f+gx^2\right)}{ef-dg}}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + (e*f*p*Log[f + g*x^2])/(e*f - d*g) + Log[c*(d + e*x^2)^p]*Log[(e*(f +
g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(2*g^2
)
```

Maple [C] time = 0.671, size = 732, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g^2*ln(g*x^2+f)+1/2*ln((e*x^2+d)^p)*f/g^2/(g*x^2+f)-1/2
*p/g^2*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*
_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e
*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+
2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-
d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z
^2*e+d))-1/2*p*e*f/g^2/(d*g-e*f)*ln(g*x^2+f)+1/2*p*e*f/g^2/(d*g-e*f)*ln(e*x
```

```
^2+d)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g^2*ln(g*x^2+f)+
1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g^2/(g*x^2+f)-1/4*I*
Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*ln(g*x^2+f)-1/4*
I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/g^2/(g*x^2+f)-1/
4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^
p)^3*f/g^2/(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*ln(g*x^
2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^2/(g*x^2+f)+1/2*ln(c)/g
^2*ln(g*x^2+f)+1/2*ln(c)*f/g^2/(g*x^2+f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log \left(\left(ex^2 + d \right)^p c \right)}{\left(gx^2 + f \right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \log \left(\left(ex^2 + d \right)^p c \right)}{g^2 x^4 + 2\, fgx^2 + f^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log \left(\left(ex^2 + d \right)^p c \right)}{\left(gx^2 + f \right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

3.350 $\int \frac{x \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=83

$$-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)}+\frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)}-\frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

```
[Out] (e*p*Log[d + e*x^2])/(2*g*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*g*(f + g*x
^2)) - (e*p*Log[f + g*x^2])/(2*g*(e*f - d*g))
```

Rubi [A] time = 0.074348, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2475, 2395, 36, 31}

$$-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)}+\frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)}-\frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (e*p*Log[d + e*x^2])/(2*g*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*g*(f + g*x
^2)) - (e*p*Log[f + g*x^2])/(2*g*(e*f - d*g))
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{(f+gx)^2}\,dx,x,x^2\right)$$
$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} + \frac{(ep)\,\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}\,dx,x,x^2\right)}{2g}$$
$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} - \frac{(ep)\,\text{Subst}\left(\int \frac{1}{f+gx}\,dx,x,x^2\right)}{2(ef-dg)} + \frac{\left(e^2p\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx,x,x^2\right)}{2g(ef-dg)}$$
$$= \frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} - \frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

Mathematica [A] time = 0.051894, size = 63, normalized size = 0.76

$$\frac{\frac{ep\left(\log\left(d+ex^2\right)-\log\left(f+gx^2\right)\right)}{ef-dg} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (-(Log[c*(d + e*x^2)^p]/(f + g*x^2)) + (e*p*(Log[d + e*x^2] - Log[f + g*x^2
]))/(e*f - d*g))/(2*g)
```

Maple [C] time = 0.382, size = 371, normalized size = 4.5

$$-\frac{\ln\left(\left(ex^2+d\right)^p\right)}{2g\left(gx^2+f\right)} - \frac{i\pi dg\,\text{csgn}\left(i\left(ex^2+d\right)^p\right)\left(\text{csgn}\left(ic\left(ex^2+d\right)^p\right)\right)^2 - i\pi dg\,\text{csgn}\left(i\left(ex^2+d\right)^p\right)\text{csgn}\left(ic\left(ex^2+d\right)^p\right)\text{csg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] -1/2/g/(g*x^2+f)*ln((e*x^2+d)^p)-1/4*(I*Pi*d*g*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2-I*Pi*d*g*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c
)-I*Pi*d*g*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d*g*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)-I*Pi*e*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+I*Pi*e*f*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*e*f*csgn(I*c*(e*x^2+d)^p)^3-
I*Pi*e*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(-e*x^2-d)*e*g*p*x^2-2*ln(g*
x^2+f)*e*g*p*x^2+2*ln(-e*x^2-d)*e*f*p-2*ln(g*x^2+f)*e*f*p+2*ln(c)*d*g-2*ln(
c)*e*f)/g/(g*x^2+f)/(d*g-e*f)
```

Maxima [A] time = 1.0145, size = 100, normalized size = 1.2

$$\frac{ep\left(\frac{\log\left(ex^2+d\right)}{ef-dg} - \frac{\log\left(gx^2+f\right)}{ef-dg}\right)}{2g} - \frac{\log\left(\left(ex^2+d\right)^p c\right)}{2\left(gx^2+f\right)g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] 1/2*e*p*(log(e*x^2 + d)/(e*f - d*g) - log(g*x^2 + f)/(e*f - d*g))/g - 1/2*l
og((e*x^2 + d)^p*c)/((g*x^2 + f)*g)
```

Fricas [A] time = 2.06877, size = 194, normalized size = 2.34

$$\frac{\left(egpx^2+dgp\right)\log\left(ex^2+d\right)-\left(egpx^2+efp\right)\log\left(gx^2+f\right)-\left(ef-dg\right)\log\left(c\right)}{2\left(ef^2g-dfg^2+\left(efg^2-dg^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((e*g*p*x^2 + d*g*p)*log(e*x^2 + d) - (e*g*p*x^2 + e*f*p)*log(g*x^2 + f
) - (e*f - d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 - d*g^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16787, size = 246, normalized size = 2.96

$$-\frac{\left(x^2e+d\right)gpe\log\left(x^2e+d\right)-\left(x^2e+d\right)gpe\log\left(\left(x^2e+d\right)g-dg+fe\right)+dgpe\log\left(\left(x^2e+d\right)g-dg+fe\right)-fpe^2\log\left(\left(x^2e\right.\right.}{2\left(\left(x^2e+d\right)dg^3-d^2g^3-\left(x^2e+d\right)fg^2e+2\,dfg^2e-f^2ge^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] -1/2*((x^2*e + d)*g*p*e*log(x^2*e + d) - (x^2*e + d)*g*p*e*log((x^2*e + d)*
g - d*g + f*e) + d*g*p*e*log((x^2*e + d)*g - d*g + f*e) - f*p*e^2*log((x^2*
e + d)*g - d*g + f*e) + d*g*e*log(c) - f*e^2*log(c))/((x^2*e + d)*d*g^3 - d
^2*g^3 - (x^2*e + d)*f*g^2*e + 2*d*f*g^2*e - f^2*g*e^2)
```

3.351 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=201

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f^2}+\frac{p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}$$

```
[Out] -(e*p*Log[d + e*x^2])/(2*f*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(2*f*(f + g*
x^2)) + (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (e*p*Log[f + g*x
^2])/(2*f*(e*f - d*g)) - (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)])/(2*f^2) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) + (p
*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

Rubi [A] time = 0.282518, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2475, 44, 2416, 2394, 2315, 2395, 36, 31, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f^2}+\frac{p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```


```
[Out] -(e*p*Log[d + e*x^2])/(2*f*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(2*f*(f + g*
x^2)) + (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (e*p*Log[f + g*x
^2])/(2*f*(e*f - d*g)) - (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)])/(2*f^2) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) + (p
*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x(f+gx)^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{f^2x} - \frac{g\log\left(c(d+ex)^p\right)}{f(f+gx)^2} - \frac{g\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)dx,x,x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^2\right)}{2f^2} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx,x,x^2\right)}{2f^2} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p}{(f+gx)^2}\right.}{2f}$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2} - \frac{(ep}{}$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2} + \frac{p\text{L}}{}$$

$$= -\frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} + \frac{ep\log\left(f+gx^2\right)}{2f(ef-dg)}$$

Mathematica [A] time = 0.121042, size = 170, normalized size = 0.85

$$\frac{-p\text{PolyLog}\left(2,\frac{g\left(d+ex^2\right)}{dg-ef}\right) + p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right) + \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} - \log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right) + \log\left(-\frac{ex^2}{d}\right)\log}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + (e*f*p*Log[f + g*x^2])/(e*f -
 d*g) - Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

Maple [C] time = 0.708, size = 984, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x)
```

```
[Out] -1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/(g*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p
)^3/f^2*ln(x)+1/2*ln((e*x^2+d)^p)/f/(g*x^2+f)+ln((e*x^2+d)^p)/f^2*ln(x)-1/2
*ln((e*x^2+d)^p)/f^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
/f^2*ln(g*x^2+f)-p/f^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p/f^2*ln(
x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csg
n(I*c)/f/(g*x^2+f)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2
*ln(x)+1/2*ln(c)/f/(g*x^2+f)+ln(c)/f^2*ln(x)-p/f^2*dilog((-e*x+(-d*e)^(1/2)
)/(-d*e)^(1/2))-p/f^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*p/f^2*sum(
ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-
```

```
d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)
)+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,ind
ex=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/
2*p*e/f/(d*g-e*f)*ln(g*x^2+f)+1/2*p*e/f/(d*g-e*f)*ln(e*x^2+d)-1/2*ln(c)/f^2
*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/(g*x^2+
f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2*ln(g*x^2+f)+1/2
*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2*ln(x)+1/4*I*Pi*csgn(I*c*(e*x^2+
d)^p)^3/f^2*ln(g*x^2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*
csgn(I*c)/f^2*ln(x)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn
(I*c)/f/(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I
*c)/f^2*ln(g*x^2+f)
```

Maxima [A] time = 1.75911, size = 266, normalized size = 1.32

$$-\frac{1}{2}\,ep\left(\frac{\log\left(ex^2+d\right)}{ef^2-dfg}-\frac{\log\left(gx^2+f\right)}{ef^2-dfg}+\frac{2\,\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)}{ef^2}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2}{ef}\right.}{ef^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e*p*(log(e*x^2 + d)/(e*f^2 - d*f*g) - log(g*x^2 + f)/(e*f^2 - d*f*g) +
 (2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f^2) - (log(g*x^2 + f)*lo
g(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))/(
e*f^2)) + 1/2*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + log(x^2)/f^2)*log((
e*x^2 + d)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^5+2\,fgx^3+f^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x), x)
```

3.352 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=251

$$\frac{gp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{f^3}-\frac{gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{f^3}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)}+\frac{g\log\left(c\left(\right.\right.}{}$$

```
[Out] (e*p*Log[x])/(d*f^2) - (e*p*Log[d + e*x^2])/(2*d*f^2) + (e*g*p*Log[d + e*x^
2])/(2*f^2*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*f^2*x^2) - (g*Log[c*(d +
e*x^2)^p])/(2*f^2*(f + g*x^2)) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])
/f^3 - (e*g*p*Log[f + g*x^2])/(2*f^2*(e*f - d*g)) + (g*Log[c*(d + e*x^2)^p]
*Log[(e*(f + g*x^2))/(e*f - d*g)])/f^3 + (g*p*PolyLog[2, -((g*(d + e*x^2))/
(e*f - d*g))])/f^3 - (g*p*PolyLog[2, 1 + (e*x^2)/d])/f^3
```

Rubi [A] time = 0.340879, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2475, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{gp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{f^3}-\frac{gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{f^3}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)}+\frac{g\log\left(c\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] (e*p*Log[x])/(d*f^2) - (e*p*Log[d + e*x^2])/(2*d*f^2) + (e*g*p*Log[d + e*x^
2])/(2*f^2*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*f^2*x^2) - (g*Log[c*(d +
e*x^2)^p])/(2*f^2*(f + g*x^2)) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])
/f^3 - (e*g*p*Log[f + g*x^2])/(2*f^2*(e*f - d*g)) + (g*Log[c*(d + e*x^2)^p]
*Log[(e*(f + g*x^2))/(e*f - d*g)])/f^3 + (g*p*PolyLog[2, -((g*(d + e*x^2))/
(e*f - d*g))])/f^3 - (g*p*PolyLog[2, 1 + (e*x^2)/d])/f^3
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{\log\left(c(d+ex)^p\right)}{f^2x^2} - \frac{2g\log\left(c(d+ex)^p\right)}{f^3x} + \frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)^2} + \frac{2g^2\log\left(c(d+ex)^p\right)}{f^3(f+gx)}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx,x,x^2\right)}{2f^2} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^2\right)}{f^3} + \frac{g^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\right.}{f^3}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3} + \frac{g\log\left(c\left(d+ex^2\right.\right.}{}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3} + \frac{g\log\left(c\left(d+ex^2\right.\right.}{}$$

$$= \frac{ep\log(x)}{df^2} - \frac{ep\log\left(d+ex^2\right)}{2df^2} + \frac{egp\log\left(d+ex^2\right)}{2f^2(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)} -$$

Mathematica [A] time = 0.174308, size = 208, normalized size = 0.83

$$\frac{2g\left(p\text{PolyLog}\left(2,\frac{g\left(d+ex^2\right)}{dg-ef}\right)+\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)\right)-2g\left(p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right.\right.}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] ((e*f*p*(2*Log[x] - Log[d + e*x^2]))/d - (f*Log[c*(d + e*x^2)^p])/x^2 - (f*
g*Log[c*(d + e*x^2)^p])/(f + g*x^2) + (e*f*g*p*(Log[d + e*x^2] - Log[f + g*
x^2]))/(e*f - d*g) + 2*g*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]) - 2*g*(Log[-((e*x^2)/d
)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/(2*f^3)
```

Maple [C] time = 0.724, size = 1216, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x)
```

```
[Out] -I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^3*ln(x)-1/4*I*Pi*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g/f^2/(g*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p
)^3*g/f^3*ln(g*x^2+f)-1/2*ln((e*x^2+d)^p)/f^2/x^2-1/4*I*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2/x^2+e*p*ln(x)/d/f^2+2*p*g/f^3*dilog((-e*x+(
-d*e)^(1/2))/(-d*e)^(1/2))+2*p*g/f^3*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))
-p*g/f^3*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alph
```

```
a)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*
g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)
-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(
_Z^2*e+d))+ln(c)*g/f^3*ln(g*x^2+f)-1/2*ln(c)*g/f^2/(g*x^2+f)-2*ln(c)*g/f^3*
ln(x)+ln((e*x^2+d)^p)*g/f^3*ln(g*x^2+f)-1/2*ln((e*x^2+d)^p)*g/f^2/(g*x^2+f)
-2*ln((e*x^2+d)^p)*g/f^3*ln(x)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f
^2/x^2-1/2*ln(c)/f^2/x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
*csgn(I*c)*g/f^2/(g*x^2+f)+I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*c
sgn(I*c)*g/f^3*ln(x)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)*g/f^3*ln(g*x^2+f)-p*e/f^2/(d*g-e*f)*ln(e*x^2+d)*g+1/2*p*e^2/f/d/(d*g
-e*f)*ln(e*x^2+d)+1/2*p*e/f^2*g/(d*g-e*f)*ln(g*x^2+f)+1/2*I*Pi*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g/f^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+
d)^p)^3*g/f^2/(g*x^2+f)+I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^3*ln(x)+2*p*g/f^3*
ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+2*p*g/f^3*ln(x)*ln((e*x+(-d*e)^(
1/2))/(-d*e)^(1/2))+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn
(I*c)/f^2/x^2+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^3*ln(g*x^2+f)-
I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g/f^3*ln(x)-1/4*I*Pi*csgn(
I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^2/(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)
^3/f^2/x^2
```

Maxima [A] time = 1.26513, size = 398, normalized size = 1.59

$$-\frac{1}{2}\left(f\left(\frac{e\log\left(ex^2+d\right)}{def^3-d^2f^2g}-\frac{g\log\left(gx^2+f\right)}{ef^4-df^3g}-\frac{\log\left(x^2\right)}{df^3}\right)-2g\left(\frac{\log\left(ex^2+d\right)}{ef^3-df^2g}-\frac{\log\left(gx^2+f\right)}{ef^3-df^2g}\right)-\frac{2\left(2\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)\right.}{ef^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(f*(e*log(e*x^2 + d)/(d*e*f^3 - d^2*f^2*g) - g*log(g*x^2 + f)/(e*f^4 -
 d*f^3*g) - log(x^2)/(d*f^3)) - 2*g*(log(e*x^2 + d)/(e*f^3 - d*f^2*g) - log
(g*x^2 + f)/(e*f^3 - d*f^2*g)) - 2*(2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^
2/d))*g/(e*f^3) + 2*(log(g*x^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) +
 dilog((e*g*x^2 + e*f)/(e*f - d*g)))*g/(e*f^3))*e*p - 1/2*((2*g*x^2 + f)/(f
^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 2*g*log(x^2)/f^3)*log((e*x^2
 + d)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^7+2\,fgx^5+f^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^3), x)
```

3.353 $\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=802

$$-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}+\frac{ep\log\left(\sqrt{g}x+\sqrt{-f}\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}-\frac{2px}{g^2}+\frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{3}{}$$

```
[Out] (-2*p*x)/g^2 + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) + (S
qrt[d]*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g^2*(e*f - d*g)) - (e*(-f)
^(3/2)*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) - (3*Sqrt[f]*p*
ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/
2) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(2*g^(5/2)) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sq
rt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g
])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*g^(5/2)) + (e*(-f)^(3/2)*p*Log[Sqrt[-f] +
Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) + (x*Log[c*(d + e*x^2)^p])/g^2 - (f*Log
[c*(d + e*x^2)^p])/(4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*Log[c*(d + e*x^2
)^p])/(4*g^(5/2)*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/(2*g^(5/2)) + (((3*I)/2)*Sqrt[f]*p*PolyLog[2,
1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (((3*I)/4)*Sqrt[f]*p*Po
lyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
 - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (((3*I)/4)*Sqrt[f
]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*S
qrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rubi [A] time = 1.68756, antiderivative size = 802, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {2476, 2448, 321, 205, 2471, 2463, 801, 635, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}+\frac{ep\log\left(\sqrt{g}x+\sqrt{-f}\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}-\frac{2px}{g^2}+\frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```


```
[Out] (-2*p*x)/g^2 + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) + (S
qrt[d]*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g^2*(e*f - d*g)) - (e*(-f)
^(3/2)*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) - (3*Sqrt[f]*p*
ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/
2) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(2*g^(5/2)) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sq
rt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g
])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*g^(5/2)) + (e*(-f)^(3/2)*p*Log[Sqrt[-f] +
Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) + (x*Log[c*(d + e*x^2)^p])/g^2 - (f*Log
[c*(d + e*x^2)^p])/(4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*Log[c*(d + e*x^2
)^p])/(4*g^(5/2)*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/(2*g^(5/2)) + (((3*I)/2)*Sqrt[f]*p*PolyLog[2,
1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (((3*I)/4)*Sqrt[f]*p*Po
```

```
lyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
 - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (((3*I)/4)*Sqrt[f
]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*S
qrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx=\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{g^2\left(f+gx^2\right)^2}-\frac{2f\log\left(c\left(d+ex^2\right)^p\right)}{g^2\left(f+gx^2\right)}\right)dx$$

$$=\frac{\int\log\left(c\left(d+ex^2\right)^p\right)dx}{g^2}-\frac{(2f)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx}{g^2}+\frac{f^2\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx}{g^2}$$

$$=\frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2}-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}}+\frac{f^2\int\left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log(c(\cdots}{4f(\sqrt{-f}\cdots}\right.}{g^2}$$

$$=-\frac{2px}{g^2}+\frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2}-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}}-\frac{f\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}dx}{4g}-\cdots$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f\log\left(c\left(d+ex^2\cdots\right.\right.}{4g^{5/2}\left(\sqrt{-f}+\sqrt{\cdots}\right.}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f\log\left(c\left(d+ex^2\cdots\right.\right.}{4g^{5/2}\left(\sqrt{-f}+\sqrt{\cdots}\right.}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)}-\frac{4\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\cdots\right.}{g^{5/2}}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)}-\frac{4\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\cdots\right.}{g^{5/2}}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}-\frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)}-\frac{3\sqrt{f}p\cdots}{\cdots}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}-\frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)}-\frac{3\sqrt{f}p\cdots}{\cdots}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}-\frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)}-\frac{3\sqrt{f}p\cdots}{\cdots}$$

Mathematica [A] time = 4.11136, size = 1349, normalized size = 1.68

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((6*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(p*Log[d + e*x^2] - Log[c*(d + e*x^
2)^p]))/g^(5/2) + (4*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/g^2 +
(2*f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(g^2*(f + g*x^2)) + p*
((4*(((-I)*Sqrt[d])/Sqrt[e] + x)*(-1 + Log[((-I)*Sqrt[d])/Sqrt[e] + x]))/g^
2 + (4*((I*Sqrt[d])/Sqrt[e] + x)*(-1 + Log[(I*Sqrt[d])/Sqrt[e] + x]))/g^2 +
 (I*f*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(
-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f]
 - Sqrt[d]*Sqrt[g])))/g^(5/2) + (I*f*(Log[(I*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f]
 + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + Log[I*Sqrt[f] - S
qrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/g^(5/2) + (f*((-I)*(Sqrt[
e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + Sqrt[e]*(I*
Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*
x])))/((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*g^(5/2)*(Sqrt[f] - I*Sqrt[g]*x))
 - (f*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x)) - (I*Sqrt[e]
*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f
] - Sqrt[d]*Sqrt[g])))/g^(5/2) + 4*((x*(2 + f/(f + g*x^2)))/(2*g^2) - (3*Sq
rt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*g^(5/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e
] + x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) - ((3*I)*Sqrt[f]*(L
og[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*
Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x)
)/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))]))/g^(5/2) + ((3*I)*Sqrt[f]*(Log[(I*
Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f
] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[
e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/g^(5/2) + ((3*I)*Sqrt[f]*(Log[((-I)*Sqrt[d
])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sq
rt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*S
qrt[f] - Sqrt[d]*Sqrt[g]))]))/g^(5/2) - ((3*I)*Sqrt[f]*(Log[((-I)*Sqrt[d])/
Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[
d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f
] + Sqrt[d]*Sqrt[g])]))/g^(5/2)))/4
```

Maple [F] time = 1.505, size = 0, normalized size = 0.

$$\int \frac{x^4 \ln \left(c \left(e x^2+d\right)^p\right)}{\left(g x^2+f\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4 \log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

3.354 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=746

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}g^{3/2}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}g^{3/2}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\ldots}\right.}{2\sqrt{f}g^{3/2}}$$

```
[Out] -((Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g*(e*f - d*g))) - (e*Sqr
t[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(3/2)*(e*f - d*g)) + (p*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2)
) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt
[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/
(2*Sqrt[f]*g^(3/2)) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]
*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
 I*Sqrt[g]*x))])/(2*Sqrt[f]*g^(3/2)) + (e*Sqrt[-f]*p*Log[Sqrt[-f] + Sqrt[g]
*x])/(2*g^(3/2)*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] -
Sqrt[g]*x)) - Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] + Sqrt[g]*x)) + (Ar
cTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*Sqrt[f]*g^(3/2)) - ((I/2
)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2))
+ ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sq
rt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g^(3/
2)) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((
I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g
^(3/2))
```

Rubi [A] time = 1.51254, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {2476, 2471, 2463, 801, 635, 205, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}g^{3/2}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}g^{3/2}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\ldots}\right.}{2\sqrt{f}g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g*(e*f - d*g))) - (e*Sqr
t[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(3/2)*(e*f - d*g)) + (p*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2)
) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt
[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/
(2*Sqrt[f]*g^(3/2)) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]
*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
 I*Sqrt[g]*x))])/(2*Sqrt[f]*g^(3/2)) + (e*Sqrt[-f]*p*Log[Sqrt[-f] + Sqrt[g]
*x])/(2*g^(3/2)*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] -
Sqrt[g]*x)) - Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] + Sqrt[g]*x)) + (Ar
cTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*Sqrt[f]*g^(3/2)) - ((I/2
)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2))
+ ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sq
rt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g^(3/
2)) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((
```

```
I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g
^(3/2))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \left(-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)^2}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx}{g} - \frac{f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}g^{3/2}} - \frac{f\int \left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}g^{3/2}} + \frac{1}{4}\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\,dx + \frac{1}{4}\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\,dx +$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2\sqrt{f}g^{3/2}} + (ep)\int \frac{x\,\mathrm{ta}}{\sqrt{f}g^3}$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2\sqrt{f}g^{3/2}} + \frac{\left(\sqrt{e}p\right)\int \frac{\tan}{\sqrt{-}}}{\sqrt{f}g^3}$$

$$= -\frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}}{\left(i\sqrt{e}\sqrt{f}-\right.}\right.}{\sqrt{f}g^{3/2}}$$

$$= -\frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}}{\left(i\sqrt{e}\sqrt{f}-\right.}\right.}{\sqrt{f}g^{3/2}}$$

$$= -\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}}{}$$

$$= -\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}}{}$$

$$= -\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}}{}$$

<u>Mathematica</u> [A] time = 3.30286, size = 1231, normalized size = 1.65

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{2\sqrt{f}g^{3/2}} + \frac{px\log\left(ex^2+d\right)-x\log\left(c\left(ex^2+d\right)^p\right)}{2g^2x^2+2fg} + \frac{1}{4}p\left(-\frac{i\left(\frac{\log\left(x-\frac{i\sqrt{d}}{\sqrt{e}}\right)}{i\sqrt{g}x+\sqrt{f}}+\frac{\sqrt{e}(\log(}{}\right.}{g}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/
(2*Sqrt[f]*g^(3/2)) + (p*x*Log[d + e*x^2] - x*Log[c*(d + e*x^2)^p])/(2*f*g
+ 2*g^2*x^2) + (p*(((-I)*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt
[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]
))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/g^(3/2) - (I*(Log[(I*Sqrt[d])/Sqrt
[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] +
Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/g^(3/2)
+ (-((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] + x]) +
 Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*Sqrt[f
] + Sqrt[g]*x]))/((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*g^(3/2)*(I*Sqrt[f] +
Sqrt[g]*x)) + (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x)) - (I
*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/(Sqrt[e
]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/g^(3/2) + 4*(-x/(2*g*(f + g*x^2)) + ArcTan[(S
qrt[g]*x)/Sqrt[f]]/(2*Sqrt[f]*g^(3/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e] + x] -
 Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])/Sqrt[
e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sq
rt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] -
 Sqrt[d]*Sqrt[g]))]))/(Sqrt[f]*g^(3/2)) - (I*(Log[(I*Sqrt[d])/Sqrt[e] + x]*
Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]
+ PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*S
qrt[g])]))/(Sqrt[f]*g^(3/2)) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqr
t[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLo
g[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]
))]))/(Sqrt[f]*g^(3/2)) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*
(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2,
(Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/(S
qrt[f]*g^(3/2))))/4
```

Maple [F] time = 1.414, size = 0, normalized size = 0.

$$\int \frac{x^2 \ln\left(c\left(ex^2+d\right)^p\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\log\left(\left(ex^2+d\right)^pc\right)}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\log\left(\left(ex^2+d\right)^pc\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

3.355 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=751

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}}$$

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rubi [A] time = 0.824598, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2471, 2463, 801, 635, 205, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
```

```
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx = \int\left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx$$

$$= -\frac{g\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}dx}{4f}-\frac{g\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}dx}{4f}-\frac{g\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{-fg-g^2x^2}dx}{2f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}+\frac{(ep)\int\frac{}{\left(\sqrt{-f}\sqrt{\right.}}}{}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}-\frac{(ep)\int\left(\frac{}{(ef-d}\right.}{}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\mathrm{lo}}{}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\mathrm{lo}}{}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\mathrm{lo}}{}$$

Mathematica [A] time = 3.14, size = 1236, normalized size = 1.65

$$\frac{1}{2}\left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f^{3/2}\sqrt{g}}+\frac{x\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f\left(gx^2+f\right)}+\frac{1}{2}p\left(\frac{i\left(\frac{\log\left(x-\frac{i\sqrt{d}}{\sqrt{e}}\right)}{i\sqrt{g}x+\sqrt{f}}+\frac{\sqrt{e}(1}{}\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f*(f + g*x^2)) + (ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f^(3/2
)*Sqrt[g]) + (p*((I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x
) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(S
```

```
qrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/(f*Sqrt[g]) + (I*(Log[(I*Sqrt[d])/Sqrt[
e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + L
og[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/(f*Sqrt[g
]) + ((-I)*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] + Sqrt[e]*(I*Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*S
qrt[f] + Sqrt[g]*x]))/(f*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Sqrt[g]*(Sqrt[
f] - I*Sqrt[g]*x)) - (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x
)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/
(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/(f*Sqrt[g]) + 2*(x/(f^2 + f*g*x^2) + A
rcTan[(Sqrt[g]*x)/Sqrt[f]]/(f^(3/2)*Sqrt[g]))*(-Log[((-I)*Sqrt[d])/Sqrt[e]
+ x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt
[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqr
t[f] - Sqrt[d]*Sqrt[g]))]))/(f^(3/2)*Sqrt[g]) - (I*(Log[(I*Sqrt[d])/Sqrt[e]
 + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt
[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqr
t[d]*Sqrt[g])]))/(f^(3/2)*Sqrt[g]) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Lo
g[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] +
PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*S
qrt[g]))]))/(f^(3/2)*Sqrt[g]) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyL
og[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
]))/(f^(3/2)*Sqrt[g])))/2)/2
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(ex^2+d\right)^p\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

3.356 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=803

$$-\frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)}+\frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2}-\frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}+3\sqrt{g}p\,\mathrm{ta}$$

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (Sqrt[d]*Sqrt[e]*
g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f^2*(e*f - d*g)) - (e*Sqrt[g]*p*Log[Sqrt[
-f] - Sqrt[g]*x])/(2*(-f)^(3/2)*(e*f - d*g)) - (3*Sqrt[g]*p*ArcTan[(Sqrt[g]
*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) + (3*Sqrt[g]
*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*
x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*f
^(5/2)) + (3*Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I
*Sqrt[g]*x))])/(2*f^(5/2)) + (e*Sqrt[g]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*(-f
)^(3/2)*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(f^2*x) + (Sqrt[g]*Log[c*(d + e
*x^2)^p])/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sqrt[g]*Log[c*(d + e*x^2)^p])/(
4*f^2*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
c*(d + e*x^2)^p])/(2*f^(5/2)) + (((3*I)/2)*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt
[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog[2, 1
+ (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]
*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog
[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2)
```

Rubi [A] time = 1.54166, antiderivative size = 803, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2476, 2455, 205, 2471, 2463, 801, 635, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)}+\frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2}-\frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}+3\sqrt{g}p\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```


```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (Sqrt[d]*Sqrt[e]*
g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f^2*(e*f - d*g)) - (e*Sqrt[g]*p*Log[Sqrt[
-f] - Sqrt[g]*x])/(2*(-f)^(3/2)*(e*f - d*g)) - (3*Sqrt[g]*p*ArcTan[(Sqrt[g]
*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) + (3*Sqrt[g]
*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*
x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*f
^(5/2)) + (3*Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I
*Sqrt[g]*x))])/(2*f^(5/2)) + (e*Sqrt[g]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*(-f
)^(3/2)*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(f^2*x) + (Sqrt[g]*Log[c*(d + e
*x^2)^p])/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sqrt[g]*Log[c*(d + e*x^2)^p])/(
4*f^2*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
c*(d + e*x^2)^p])/(2*f^(5/2)) + (((3*I)/2)*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt
[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog[2, 1
```

```
+ (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]
*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog
[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2)
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f\left(f+gx^2\right)^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx}{f^2} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx}{f^2} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx}{f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}} - \frac{g\int \left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right.}\right.}{f}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}} + \frac{g^2\int \frac{\log\left(c\left(d+ex^2\right)\right.}{\left(\sqrt{-f}\sqrt{g}-gx\right.}}{4f^2}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - 3\sqrt{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - 3\sqrt{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \frac{\sqrt{g}p\tan^{-1}\ldots}{}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \frac{\sqrt{g}p\tan^{-1}\ldots}{}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\ldots}{f^{5/2}}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\ldots}{f^{5/2}}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\ldots}{f^{5/2}}$$

Mathematica [A] time = 4.50928, size = 1438, normalized size = 1.79

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

```
[Out] ((4*p*Log[d + e*x^2] - 4*Log[c*(d + e*x^2)^p])/(f^2*x) + (2*g*x*(p*Log[d +
e*x^2] - Log[c*(d + e*x^2)^p]))/(f^2*(f + g*x^2)) + (6*Sqrt[g]*ArcTan[(Sqrt
[g]*x)/Sqrt[f]]*(p*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/f^(5/2) + p*(((4
*I)*(Sqrt[e]*x*Log[x] + I*Sqrt[d]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Sqrt[e]
*x*Log[I*Sqrt[d] - Sqrt[e]*x]))/(Sqrt[d]*f^2*x) - (4*(I*Sqrt[e]*x*Log[x] +
Sqrt[d]*Log[(I*Sqrt[d])/Sqrt[e] + x] - I*Sqrt[e]*x*Log[I*Sqrt[d] + Sqrt[e]*
x]))/(Sqrt[d]*f^2*x) - (I*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f]
 + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - S
qrt[g]*x]))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/f^2 - (I*Sqrt[g]*(Log[(I*
Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] +
Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g
])))/f^2 + (Sqrt[g]*(-((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d
])/Sqrt[e] + x]) + Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]
*x] - Log[I*Sqrt[f] + Sqrt[g]*x])))/(f^2*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g]
)*(I*Sqrt[f] + Sqrt[g]*x)) + (Sqrt[g]*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sq
rt[f] + Sqrt[g]*x)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f
] + Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/f^2 + 4*(-(2 + (g*x^
2)/(f + g*x^2))/(2*f^2*x) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*f^(5
/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Lo
g[d + e*x^2]) - ((3*I)*Sqrt[g]*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(
Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2, -
((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))]))/
f^(5/2) + ((3*I)*Sqrt[g]*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f
] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt[g
]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/f^(5/2) +
 ((3*I)*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] + I*
Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(S
qrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))]))/f^(5/2) - ((
3*I)*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqr
t[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d
] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/f^(5/2)))/4
```

Maple [F] time = 1.51, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(ex^2+d\right)^p\right)}{x^2\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^6+2\,fgx^4+f^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^2), x)
```

3.357 $\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2}\,dx$

Optimal. Leaf size=163

$$\frac{in\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}+\frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (2*n*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^n])/(Sqrt[a]*Sqrt[b]) + (I*
n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.145471, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{in\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}+\frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```


```
[Out] (I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (2*n*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^n])/(Sqrt[a]*Sqrt[b]) + (I*
n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```


Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - (2bn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - \frac{\left(2\sqrt{b}n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{\sqrt{a}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx}{a}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - \frac{(2n)\int -\ldots}{\ldots}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{(2in)\,\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{in\mathrm{Li}_2\left(1\ldots\right.}{\sqrt{\ldots}}$$

Mathematica [A] time = 0.0423675, size = 128, normalized size = 0.79

$$\frac{in\mathrm{PolyLog}\left(2,\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)+\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\log\left(c\left(a+bx^2\right)^n\right)+2n\log\left(\frac{2i}{-\frac{\sqrt{b}x}{\sqrt{a}}+i}\right)+in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*n*Log[(2*
I)/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^n]) + I*n*PolyLog[2, (I*S
```

```
qrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Maple [F] time = 1.373, size = 0, normalized size = 0.

$$\int \frac{\ln\left(c\left(bx^2+a\right)^n\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x)
```

```
[Out] int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^n c\right)}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**n)/(b*x**2+a),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**n)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(bx^2+a\right)^n c\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

3.358 $\int \frac{\log(1-x^2)}{2-x^2} \, dx$

Optimal. Leaf size=239

$$-\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)}{\sqrt{2}}+\frac{\text{PolyLog}\left(2,\frac{4(1-x)}{(2-\sqrt{2})(x+\sqrt{2})}+1\right)}{2\sqrt{2}}+\frac{\text{PolyLog}\left(2,1-\frac{4(x+1)}{(2+\sqrt{2})(x+\sqrt{2})}\right)}{2\sqrt{2}}+\frac{\log\left(1-x^2\right)\tanh^{-1}\left(\frac{x}{\sqrt{\ldots}}\right.}{\sqrt{2}}$$

```
[Out] Sqrt[2]*ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt
[2]]*Log[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] - (ArcTanh[x/
Sqrt[2]]*Log[(4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] + (ArcTanh
[x/Sqrt[2]]*Log[1 - x^2])/Sqrt[2] - PolyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + x
)]/Sqrt[2] + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]/(2*S
qrt[2]) + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/(2*Sqrt
[2])
```

Rubi [A] time = 0.28966, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {206, 2470, 12, 5992, 5920, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)}{\sqrt{2}}+\frac{\text{PolyLog}\left(2,\frac{4(1-x)}{(2-\sqrt{2})(x+\sqrt{2})}+1\right)}{2\sqrt{2}}+\frac{\text{PolyLog}\left(2,1-\frac{4(x+1)}{(2+\sqrt{2})(x+\sqrt{2})}\right)}{2\sqrt{2}}+\frac{\log\left(1-x^2\right)\tanh^{-1}\left(\frac{x}{\sqrt{\ldots}}\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - x^2]/(2 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt
[2]]*Log[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] - (ArcTanh[x/
Sqrt[2]]*Log[(4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] + (ArcTanh
[x/Sqrt[2]]*Log[1 - x^2])/Sqrt[2] - PolyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + x
)]/Sqrt[2] + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]/(2*S
qrt[2]) + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/(2*Sqrt
[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} + 2\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}(1-x^2)}\,dx$$
$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} + \sqrt{2}\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx$$
$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} + \sqrt{2}\int \left(-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(-1+x)} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(1+x)}\right)dx$$
$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{-1+x}\,dx}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1+x}\,dx}{\sqrt{2}}$$
$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{(2-\sqrt{2})(\sqrt{2}+x)}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{(2+\sqrt{2})(\sqrt{}\right.}}{\sqrt{2}}$$
$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{(2-\sqrt{2})(\sqrt{2}+x)}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{(2+\sqrt{2})(\sqrt{}\right.}}{\sqrt{2}}$$
$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{(2-\sqrt{2})(\sqrt{2}+x)}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{(2+\sqrt{2})(\sqrt{}\right.}}{\sqrt{2}}$$

Mathematica [A] time = 0.119758, size = 248, normalized size = 1.04

$$\frac{\text{PolyLog}\left(2,\frac{x-1}{-1-\sqrt{2}}\right)+\log\left(1-\frac{x-1}{-1-\sqrt{2}}\right)\log(x-1)}{2\sqrt{2}} - \frac{\text{PolyLog}\left(2,\frac{x-1}{\sqrt{2}-1}\right)+\log\left(1-\frac{x-1}{\sqrt{2}-1}\right)\log(x-1)}{2\sqrt{2}} + \frac{\text{PolyLog}\left(2,\frac{x-}{1-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - x^2]/(2 - x^2),x]
```

```
[Out] -((Log[Sqrt[2] - x] - Log[Sqrt[2] + x])*(-Log[-1 + x] - Log[1 + x] + Log[1
- x^2]))/(2*Sqrt[2]) + (Log[1 - (-1 + x)/(-1 - Sqrt[2])]*Log[-1 + x] + Poly
Log[2, (-1 + x)/(-1 - Sqrt[2])])/(2*Sqrt[2]) - (Log[1 - (-1 + x)/(-1 + Sqrt
[2])]*Log[-1 + x] + PolyLog[2, (-1 + x)/(-1 + Sqrt[2])])/(2*Sqrt[2]) + (Log
[1 + x]*Log[1 - (1 + x)/(1 - Sqrt[2])] + PolyLog[2, (1 + x)/(1 - Sqrt[2])])
/(2*Sqrt[2]) - (Log[1 + x]*Log[1 - (1 + x)/(1 + Sqrt[2])] + PolyLog[2, (1 +
 x)/(1 + Sqrt[2])])/(2*Sqrt[2])
```

Maple [A] time = 0.293, size = 214, normalized size = 0.9

$$-\frac{\sqrt{2}\ln(-x^2+1)\ln(x-\sqrt{2})}{4} + \frac{\sqrt{2}\ln(x-\sqrt{2})}{4}\ln\left(\frac{1+x}{1+\sqrt{2}}\right) + \frac{\sqrt{2}\ln(x-\sqrt{2})}{4}\ln\left(\frac{x-1}{\sqrt{2}-1}\right) + \frac{\sqrt{2}}{4}\text{dilog}\left(\frac{1+x}{1+\sqrt{2}}\right) + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-x^2+1)/(-x^2+2),x)
```

```
[Out] -1/4*2^(1/2)*ln(-x^2+1)*ln(x-2^(1/2))+1/4*2^(1/2)*ln(x-2^(1/2))*ln((1+x)/(1
+2^(1/2)))+1/4*2^(1/2)*ln(x-2^(1/2))*ln((x-1)/(2^(1/2)-1))+1/4*2^(1/2)*dilo
g((1+x)/(1+2^(1/2)))+1/4*2^(1/2)*dilog((x-1)/(2^(1/2)-1))+1/4*2^(1/2)*ln(-x
^2+1)*ln(x+2^(1/2))-1/4*2^(1/2)*ln(x+2^(1/2))*ln((1+x)/(-2^(1/2)+1))-1/4*2^
(1/2)*ln(x+2^(1/2))*ln((x-1)/(-1-2^(1/2)))-1/4*2^(1/2)*dilog((x-1)/(-1-2^(1
/2)))-1/4*2^(1/2)*dilog((1+x)/(-2^(1/2)+1))
```

Maxima [A] time = 1.58357, size = 281, normalized size = 1.18

$$\frac{1}{4}\sqrt{2}\left(\left(\log\left(2x+2\sqrt{2}\right)-\log\left(2x-2\sqrt{2}\right)\right)\log\left(-x^2+1\right)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}+1}+1\right)+\log\left(x-\sqrt{2}\right)\log\left(\frac{x-\sqrt{2}}{\sqrt{2}+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(-x^2 + 1) -
log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - sqrt(2))*l
og((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(
sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) - 1) + 1) -
 dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2) + 1)) -
 dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2) - 1)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\log\left(-x^2+1\right)}{x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] integral(-log(-x^2 + 1)/(x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\log\left(1-x^2\right)}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-x**2+1)/(-x**2+2),x)
```

```
[Out] -Integral(log(1 - x**2)/(x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(-x^2+1\right)}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(-log(-x^2 + 1)/(x^2 - 2), x)
```

3.359 $\int \frac{\log(d+ex^2)}{1-x^2}\,dx$

Optimal. Leaf size=217

$$\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}-\sqrt{e}x\right)}{(x+1)\left(\sqrt{-d}-\sqrt{e}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}+\sqrt{e}x\right)}{(x+1)\left(\sqrt{-d}+\sqrt{e}\right)}\right)-\text{PolyLog}\left(2,1-\frac{2}{x+1}\right)+\tanh^{-1}(x)$$

```
[Out] 2*ArcTanh[x]*Log[2/(1 + x)] - ArcTanh[x]*Log[(2*(Sqrt[-d] - Sqrt[e]*x))/((S
qrt[-d] - Sqrt[e])*(1 + x))] - ArcTanh[x]*Log[(2*(Sqrt[-d] + Sqrt[e]*x))/((
Sqrt[-d] + Sqrt[e])*(1 + x))] + ArcTanh[x]*Log[d + e*x^2] - PolyLog[2, 1 -
2/(1 + x)] + PolyLog[2, 1 - (2*(Sqrt[-d] - Sqrt[e]*x))/((Sqrt[-d] - Sqrt[e]
)*(1 + x))]/2 + PolyLog[2, 1 - (2*(Sqrt[-d] + Sqrt[e]*x))/((Sqrt[-d] + Sqrt
[e])*(1 + x))]/2
```

Rubi [A] time = 0.253017, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {206, 2470, 5992, 5920, 2402, 2315, 2447}

$$\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}-\sqrt{e}x\right)}{(x+1)\left(\sqrt{-d}-\sqrt{e}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}+\sqrt{e}x\right)}{(x+1)\left(\sqrt{-d}+\sqrt{e}\right)}\right)-\text{PolyLog}\left(2,1-\frac{2}{x+1}\right)+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d + e*x^2]/(1 - x^2),x]
```


```
[Out] 2*ArcTanh[x]*Log[2/(1 + x)] - ArcTanh[x]*Log[(2*(Sqrt[-d] - Sqrt[e]*x))/((S
qrt[-d] - Sqrt[e])*(1 + x))] - ArcTanh[x]*Log[(2*(Sqrt[-d] + Sqrt[e]*x))/((
Sqrt[-d] + Sqrt[e])*(1 + x))] + ArcTanh[x]*Log[d + e*x^2] - PolyLog[2, 1 -
2/(1 + x)] + PolyLog[2, 1 - (2*(Sqrt[-d] - Sqrt[e]*x))/((Sqrt[-d] - Sqrt[e]
)*(1 + x))]/2 + PolyLog[2, 1 - (2*(Sqrt[-d] + Sqrt[e]*x))/((Sqrt[-d] + Sqrt
[e])*(1 + x))]/2
```


Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(d+ex^2\right)}{1-x^2}\,dx = \tanh^{-1}(x)\log\left(d+ex^2\right)-(2e)\int \frac{x\tanh^{-1}(x)}{d+ex^2}\,dx$$

$$= \tanh^{-1}(x)\log\left(d+ex^2\right)-(2e)\int \left(-\frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= \tanh^{-1}(x)\log\left(d+ex^2\right)+\sqrt{e}\int \frac{\tanh^{-1}(x)}{\sqrt{-d}-\sqrt{e}x}\,dx-\sqrt{e}\int \frac{\tanh^{-1}(x)}{\sqrt{-d}+\sqrt{e}x}\,dx$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}+\sqrt{\right.}}{\left(\sqrt{-d}+\sqrt{e}\right)(\right.}\right.$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}+\sqrt{\right.}}{\left(\sqrt{-d}+\sqrt{e}\right)(\right.}\right.$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}+\sqrt{\right.}}{\left(\sqrt{-d}+\sqrt{e}\right)(\right.}\right.$$

Mathematica [C] time = 0.123114, size = 468, normalized size = 2.16

$$\frac{1}{2}\left(-\text{PolyLog}\left(2,\frac{\sqrt{d}-i\sqrt{e}x}{\sqrt{d}-i\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}-i\sqrt{e}x}{\sqrt{d}+i\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}+i\sqrt{e}x}{\sqrt{d}-i\sqrt{e}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{d}+i\sqrt{e}x}{\sqrt{d}+i\sqrt{e}}\right)-\text{l}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d + e*x^2]/(1 - x^2),x]
```

```
[Out] (Log[1 - x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[(Sqrt[e]*(-1 + x))/(I*Sqr
t[d] - Sqrt[e])]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[1 + x]*Log[((-I)*Sqr
```

```
t[d])/Sqrt[e] + x] + Log[((-I)*Sqrt[e]*(1 + x))/(Sqrt[d] - I*Sqrt[e])]*Log[
((-I)*Sqrt[d])/Sqrt[e] + x] + Log[1 - x]*Log[(I*Sqrt[d])/Sqrt[e] + x] - Log
[(Sqrt[e]*(-1 + x))/((-I)*Sqrt[d] - Sqrt[e])]*Log[(I*Sqrt[d])/Sqrt[e] + x]
- Log[1 + x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[(I*Sqrt[e]*(1 + x))/(Sqrt[d
] + I*Sqrt[e])]*Log[(I*Sqrt[d])/Sqrt[e] + x] - Log[1 - x]*Log[d + e*x^2] +
Log[1 + x]*Log[d + e*x^2] - PolyLog[2, (Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d] - I
*Sqrt[e])] + PolyLog[2, (Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d] + I*Sqrt[e])] + Po
lyLog[2, (Sqrt[d] + I*Sqrt[e]*x)/(Sqrt[d] - I*Sqrt[e])] - PolyLog[2, (Sqrt[
d] + I*Sqrt[e]*x)/(Sqrt[d] + I*Sqrt[e])])/2
```

Maple [A] time = 0.096, size = 282, normalized size = 1.3

$$-\frac{\ln\left(x-1\right)\ln\left(ex^{2}+d\right)}{2}+\frac{\ln\left(x-1\right)}{2}\ln\left(\left(-\left(x-1\right)e+\sqrt{-de}-e\right)\left(-e+\sqrt{-de}\right)^{-1}\right)+\frac{\ln\left(x-1\right)}{2}\ln\left(\left(\left(x-1\right)e+\sqrt{-de}+e\right)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*x^2+d)/(-x^2+1),x)
```

```
[Out] -1/2*ln(x-1)*ln(e*x^2+d)+1/2*ln(x-1)*ln((-(x-1)*e+(-d*e)^(1/2)-e)/(-e+(-d*e
)^(1/2)))+1/2*ln(x-1)*ln(((x-1)*e+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))+1/2*dil
og((-(x-1)*e+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))+1/2*dilog(((x-1)*e+(-d*e)^(
1/2)+e)/(e+(-d*e)^(1/2)))+1/2*ln(1+x)*ln(e*x^2+d)-1/2*ln(1+x)*ln((-(1+x)*e+
(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))-1/2*ln(1+x)*ln(((1+x)*e+(-d*e)^(1/2)-e)/(
-e+(-d*e)^(1/2)))-1/2*dilog((-(1+x)*e+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))-1/2
*dilog(((1+x)*e+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\log\left(ex^{2}+d\right)}{x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(log(e*x^2 + d)/(x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\log\left(ex^{2}+d\right)}{x^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-log(e*x^2 + d)/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\log\left(d+ex^2\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*x**2+d)/(-x**2+1),x)
```

```
[Out] -Integral(log(d + e*x**2)/(x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(ex^2+d\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-log(e*x^2 + d)/(x^2 - 1), x)
```

3.360 $\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=144

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}-\frac{d^2gpx^n}{3e^2n}+\frac{d^3gp\log\left(d+ex^n\right)}{3e^3n}+\frac{dgp}{6e}$$

```
[Out] -(d^2*g*p*x^n)/(3*e^2*n) + (d*g*p*x^(2*n))/(6*e*n) - (g*p*x^(3*n))/(9*n) +
(d^3*g*p*Log[d + e*x^n])/(3*e^3*n) + (g*x^(3*n)*Log[c*(d + e*x^n)^p])/(3*n)
 + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^
n)/d])/n
```

Rubi [A] time = 0.170998, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2475, 14, 2416, 2394, 2315, 2395, 43}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}-\frac{d^2gpx^n}{3e^2n}+\frac{d^3gp\log\left(d+ex^n\right)}{3e^3n}+\frac{dgp}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(d^2*g*p*x^n)/(3*e^2*n) + (d*g*p*x^(2*n))/(6*e*n) - (g*p*x^(3*n))/(9*n) +
(d^3*g*p*Log[d + e*x^n])/(3*e^3*n) + (g*x^(3*n)*Log[c*(d + e*x^n)^p])/(3*n)
 + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^
n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{3n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+gx^3\right)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x}+gx^2\log\left(c(d+ex)^p\right)\right)dx, x, x^n\right)}{n}$$
$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{g\,\text{Subst}\left(\int x^2\log\left(c(d+ex)^p\right)dx, x, x^n\right)}{n}$$
$$= \frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(efp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{e}{}\right.}{d+ex}\right.}{n}$$
$$= \frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{(eg}{}$$
$$= -\frac{d^2gpx^n}{3e^2n} + \frac{dgpx^{2n}}{6en} - \frac{gpx^{3n}}{9n} + \frac{d^3gp\log\left(d+ex^n\right)}{3e^3n} + \frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n} + \frac{f}{}$$

Mathematica [A] time = 0.169468, size = 118, normalized size = 0.82

$$\frac{18f\left(p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)\right)+6gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)-\frac{gp\left(ex^n\left(6d^2-3dex^n+2e^2x^{2n}\right)-6d^3\log(d\right.}{e^3}}{18n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-((g*p*(e*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n)) - 6*d^3*Log[d + e*x^n]))
/e^3) + 6*g*x^(3*n)*Log[c*(d + e*x^n)^p] + 18*f*(Log[-((e*x^n)/d)]*Log[c*(d
 + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/(18*n)
```

Maple [C] time = 4.184, size = 428, normalized size = 3.

$$\frac{\left(g\left(x^{n}\right)^{3}+3 f \ln(x) n\right) \ln\left(\left(d+e x^{n}\right)^{p}\right)}{3 n}-\frac{i}{2} \pi\left(\operatorname{csgn}\left(i c\left(d+e x^{n}\right)^{p}\right)\right)^{3} f \ln(x)+\frac{i}{2} \pi \operatorname{csgn}\left(i\left(d+e x^{n}\right)^{p}\right)\left(\operatorname{csgn}\left(i c\left(d+e x^{n}\right)^{p}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(3*n))*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] 1/3*(g*(x^n)^3+3*f*ln(x)*n)/n*ln((d+e*x^n)^p)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p
)^3*f*ln(x)+1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f*ln(x)-1/
6*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g*(x^n)^3/n+1/2*
I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*ln(x)+1/6*I*Pi*csgn(I*c*(d+e*x^n)^
p)^2*csgn(I*c)*g*(x^n)^3/n-1/6*I*Pi*csgn(I*c*(d+e*x^n)^p)^3*g*(x^n)^3/n+1/6
*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*g*(x^n)^3/n-1/2*I*Pi*csgn
(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x)+ln(c)*f*ln(x)+1/3*l
n(c)*g*(x^n)^3/n+1/3*d^3*g*p*ln(d+e*x^n)/e^3/n-1/9*p/n*g*(x^n)^3+1/6*p/e/n*
g*(x^n)^2*d-1/3*d^2*g*p*x^n/e^2/n-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+
e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9 e^{3} f n^{2} p \log(x)^{2}-3 d e^{2} g p x^{2 n}+6 d^{2} e g p x^{n}+2\left(e^{3} g p-3 e^{3} g \log(c)\right) x^{3 n}-6\left(3 e^{3} f n \log(x)+e^{3} g x^{3 n}\right) \log\left(\left(e x^{n}+d\right)^{p}\right)-}{18 e^{3} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/18*(9*e^3*f*n^2*p*log(x)^2 - 3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 2*(
e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*log(x) + e^3*g*x^(3*n))*lo
g((e*x^n + d)^p) - 6*(d^3*g*n*p + 3*e^3*f*n*log(c))*log(x))/(e^3*n) + integ
rate(1/3*(3*d*e^3*f*n*p*log(x) - d^4*g*p)/(e^4*x*x^n + d*e^3*x), x)
```

Fricas [A] time = 2.1244, size = 363, normalized size = 2.52

$$-\frac{18 e^{3} f n p \log(x) \log\left(\frac{e x^{n}+d}{d}\right)-18 e^{3} f n \log(c) \log(x)-3 d e^{2} g p x^{2 n}+6 d^{2} e g p x^{n}+18 e^{3} f p \operatorname{Li}_{2}\left(-\frac{e x^{n}+d}{d}+1\right)+2\left(e^{3} g p-3 e\right.}{18 e^{3} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/18*(18*e^3*f*n*p*log(x)*log((e*x^n + d)/d) - 18*e^3*f*n*log(c)*log(x) -
3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 18*e^3*f*p*dilog(-(e*x^n + d)/d + 1
) + 2*(e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*p*log(x) + e^3*g*p*
x^(3*n) + d^3*g*p)*log(e*x^n + d))/(e^3*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(3*n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^{3n} + f)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(3*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

3.361 $\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=124

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{dgpx^n}{2en}-\frac{gpx^2}{4n}$$

```
[Out] (d*g*p*x^n)/(2*e*n) - (g*p*x^(2*n))/(4*n) - (d^2*g*p*Log[d + e*x^n])/(2*e^2
*n) + (g*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (f*Log[-((e*x^n)/d)]*Log[c*(
d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.142098, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2475, 14, 2416, 2394, 2315, 2395, 43}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{dgpx^n}{2en}-\frac{gpx^2}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*g*p*x^n)/(2*e*n) - (g*p*x^(2*n))/(4*n) - (d^2*g*p*Log[d + e*x^n])/(2*e^2
*n) + (g*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (f*Log[-((e*x^n)/d)]*Log[c*(
d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{2n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+gx^2\right)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x}+gx\log\left(c(d+ex)^p\right)\right)dx,x,x^n\right)}{n}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\,\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)dx,x,x^n\right)}{n}$$

$$= \frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(efp)\,\text{Subst}\left(\int \frac{\log(-\ldots}{d+ex\ldots}\right.}{n}$$

$$= \frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}-\ldots$$

$$= \frac{dgpx^n}{2en}-\frac{gpx^{2n}}{4n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\ldots}{\ldots}$$

Mathematica [A] time = 0.111868, size = 100, normalized size = 0.81

$$\frac{4e^2fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+2e^2\log\left(c\left(d+ex^n\right)^p\right)\left(2f\log\left(-\frac{ex^n}{d}\right)+gx^{2n}\right)-2d^2gp\log\left(d+ex^n\right)-egpx^n\left(ex^n-2d\right)}{4e^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-2*d + e*x^n)) - 2*d^2*g*p*Log[d + e*x^n] + 2*e^2*(g*x^(2*n)
+ 2*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p] + 4*e^2*f*p*PolyLog[2, 1 + (e
*x^n)/d])/(4*e^2*n)
```

Maple [C] time = 4.243, size = 410, normalized size = 3.3

$$\frac{\left(2 f \ln \left(x\right) n+g \left(x^{n}\right)^{2}\right) \ln \left(\left(d+e x^{n}\right)^{p}\right)}{2 n}-\frac{i}{2}\pi \left(\mathrm{csgn}\left(i c \left(d+e x^{n}\right)^{p}\right)\right)^{3} f \ln \left(x\right)-\frac{\frac{i}{4}\pi \left(\mathrm{csgn}\left(i c \left(d+e x^{n}\right)^{p}\right)\right)^{3} g \left(x^{n}\right)^{2}}{n}+\frac{i}{2}\pi \,\mathrm{cs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] 1/2*(2*f*ln(x)*n+g*(x^n)^2)/n*ln((d+e*x^n)^p)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p
)^3*f*ln(x)-1/4*I*Pi*csgn(I*c*(d+e*x^n)^p)^3*g*(x^n)^2/n+1/2*I*Pi*csgn(I*(d
+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f*ln(x)-1/4*I*Pi*csgn(I*(d+e*x^n)^p)*csg
n(I*c*(d+e*x^n)^p)*csgn(I*c)*g*(x^n)^2/n+1/4*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*c
sgn(I*c)*g*(x^n)^2/n+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*ln(x)+1/4
*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*g*(x^n)^2/n-1/2*I*Pi*csgn
(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x)+ln(c)*f*ln(x)+1/2*l
n(c)*g*(x^n)^2/n-1/4*p/n*g*(x^n)^2+1/2*d*g*p*x^n/e/n-1/2*d^2*g*p*ln(d+e*x^n
)/e^2/n-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2 e^{2} f n^{2} p \log \left(x\right)^{2}-2 d e g p x^{n}+\left(e^{2} g p-2 e^{2} g \log \left(c\right)\right) x^{2 n}-2 \left(2 e^{2} f n \log \left(x\right)+e^{2} g x^{2 n}\right) \log \left(\left(e x^{n}+d\right)^{p}\right)+2 \left(d^{2} g n p-2 e^{2}\right.}{4 e^{2} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*e^2*f*n^2*p*log(x)^2 - 2*d*e*g*p*x^n + (e^2*g*p - 2*e^2*g*log(c))*x
^(2*n) - 2*(2*e^2*f*n*log(x) + e^2*g*x^(2*n))*log((e*x^n + d)^p) + 2*(d^2*g
*n*p - 2*e^2*f*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e^2*f*n*p*log
(x) + d^3*g*p)/(e^3*x*x^n + d*e^2*x), x)
```

Fricas [A] time = 2.08481, size = 323, normalized size = 2.6

$$-\frac{4 e^{2} f n p \log \left(x\right) \log \left(\frac{e x^{n}+d}{d}\right)-4 e^{2} f n \log \left(c\right) \log \left(x\right)-2 d e g p x^{n}+4 e^{2} f p \mathrm{Li}_{2}\left(-\frac{e x^{n}+d}{d}+1\right)+\left(e^{2} g p-2 e^{2} g \log \left(c\right)\right) x^{2 n}-2 \left(\right.}{4 e^{2} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(4*e^2*f*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f*n*log(c)*log(x) - 2*d
*e*g*p*x^n + 4*e^2*f*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g*p - 2*e^2*g*log(c
))*x^(2*n) - 2*(2*e^2*f*n*p*log(x) + e^2*g*p*x^(2*n) - d^2*g*p)*log(e*x^n +
 d))/(e^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{2n}+f\right)\log\left((ex^{n}+d)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

3.362 $\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=83

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}-\frac{gpx^n}{n}$$

```
[Out] -((g*p*x^n)/n) + (g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f*Log[-((e*x
^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.10986, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}-\frac{gpx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((g*p*x^n)/n) + (g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f*Log[-((e*x
^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(f+gx^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(f+gx)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(g\log\left(c(d+ex)^p\right)+\frac{f\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}$$
$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{g\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right)dx, x, x^n\right)}{n}$$
$$= \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{g\,\text{Subst}\left(\int \log\left(cx^p\right)dx, x, d+ex^n\right)}{en} - \frac{(efp)\,\text{Su}}{}$$
$$= -\frac{gpx^n}{n} + \frac{g\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{fp\text{Li}_2}{}$$

Mathematica [A] time = 0.0577596, size = 68, normalized size = 0.82

$$\frac{efp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\log\left(c\left(d+ex^n\right)^p\right)\left(ef\log\left(-\frac{ex^n}{d}\right)+dg+egx^n\right)-egpx^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n) + (d*g + e*g*x^n + e*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p
] + e*f*p*PolyLog[2, 1 + (e*x^n)/d])/(e*n)
```

Maple [C] time = 4.306, size = 376, normalized size = 4.5

$$\frac{\left(f\ln(x)\,n+gx^n\right)\ln\left(\left(d+ex^n\right)^p\right)}{n} + \frac{i}{2}\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\left(\text{csgn}\left(ic\left(d+ex^n\right)^p\right)\right)^2 f\ln(x) + \frac{\frac{i}{2}\pi\,\text{csgn}\left(i\left(d+ex^n\right)^p\right)\left(\text{cs}\right.}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] (f*ln(x)*n+g*x^n)/n*ln((d+e*x^n)^p)+1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(
d+e*x^n)^p)^2*f*ln(x)+1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*
g*x^n/n-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x
)-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g*x^n/n-1/2*
I*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*ln(x)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3*g*x^n
/n+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*ln(x)+1/2*I*Pi*csgn(I*c*(d+
e*x^n)^p)^2*csgn(I*c)*g*x^n/n+ln(c)*f*ln(x)+ln(c)*g*x^n/n-g*p*x^n/n+p/e/n*g
*d*ln(d+e*x^n)-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{efn^2p\log(x)^2 + 2\left(egp - eg\log(c)\right)x^n - 2\left(efn\log(x) + egx^n\right)\log\left((ex^n+d)^p\right) - 2\left(dgnp + efn\log(c)\right)\log(x)}{2\,en} + \int \frac{de$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(e*f*n^2*p*log(x)^2 + 2*(e*g*p - e*g*log(c))*x^n - 2*(e*f*n*log(x) + e
*g*x^n)*log((e*x^n + d)^p) - 2*(d*g*n*p + e*f*n*log(c))*log(x))/(e*n) + int
egrate((d*e*f*n*p*log(x) - d^2*g*p)/(e^2*x*x^n + d*e*x), x)
```

Fricas [A] time = 2.10273, size = 244, normalized size = 2.94

$$-\frac{efnp\log(x)\log\left(\frac{ex^n+d}{d}\right) - efn\log(c)\log(x) + efp\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + \left(egp - eg\log(c)\right)x^n - \left(efnp\log(x) + egpx^n + dg}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -(e*f*n*p*log(x)*log((e*x^n + d)/d) - e*f*n*log(c)*log(x) + e*f*p*dilog(-(e
*x^n + d)/d + 1) + (e*g*p - e*g*log(c))*x^n - (e*f*n*p*log(x) + e*g*p*x^n +
 d*g*p)*log(e*x^n + d))/(e*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f + gx^n\right)\log\left(c\left(d + ex^n\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**n)*log(c*(d + e*x**n)**p)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^n + f\right)\log\left(\left(ex^n + d\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^n + f)*log((e*x^n + d)^p*c)/x, x)
```

3.363 $\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=97

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{egp\log\left(d+ex^n\right)}{dn}+\frac{egp\log(x)}{d}$$

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^n])/(d*n) - (g*Log[c*(d + e*x^n)^p])/
(n*x^n) + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1
+ (e*x^n)/d])/n
```

Rubi [A] time = 0.140534, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2475, 14, 2416, 2395, 36, 29, 31, 2394, 2315}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{egp\log\left(d+ex^n\right)}{dn}+\frac{egp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^n])/(d*n) - (g*Log[c*(d + e*x^n)^p])/
(n*x^n) + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1
+ (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(f+gx^{-n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{g\log(c(d+ex)^p)}{x^2}+\frac{f\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}$$
$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}+\frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^n\right)}{n}$$
$$= -\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(efp)\,\text{Subst}\left(\int \frac{\log(}{d+}\right.}{n}$$
$$= -\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}+$$
$$= \frac{egp\log(x)}{d}-\frac{egp\log\left(d+ex^n\right)}{dn}-\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d\right.}{n}$$

Mathematica [A] time = 0.0934804, size = 87, normalized size = 0.9

$$\frac{f\left(p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)\right)-gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)+\frac{egp(n\log(x)-\log(d+ex^n))}{d}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] ((e*g*p*(n*Log[x] - Log[d + e*x^n]))/d - (g*Log[c*(d + e*x^n)^p])/x^n + f*(
Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

Maple [C] time = 4.279, size = 423, normalized size = 4.4

$$\frac{\left(f\ln(x)\,nx^{n}-g\right)\ln\left(\left(d+ex^{n}\right)^{p}\right)}{x^{n}n}+\frac{\frac{i}{2}\pi\,\mathrm{csgn}\left(i\left(d+ex^{n}\right)^{p}\right)\left(\mathrm{csgn}\left(ic\left(d+ex^{n}\right)^{p}\right)\right)^{2}f\ln\left(x^{n}\right)}{n}-\frac{\frac{i}{2}\pi\,\mathrm{csgn}\left(i\left(d+ex^{n}\right)^{p}\right)\left(\mathrm{csgn}\right.}{x^{n}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] (f*ln(x)*n*x^n-g)/n/(x^n)*ln((d+e*x^n)^p)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)^2*f*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+
e*x^n)^p)^2*g/(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*cs
gn(I*c)*f*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn
(I*c)*g/(x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*ln(x^n)+1/2*I/n*Pi*csgn(
I*c*(d+e*x^n)^p)^3*g/(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*l
n(x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*g/(x^n)+1/n*ln(c)*f*ln(
x^n)-1/n*ln(c)*g/(x^n)-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)-e
*g*p*ln(d+e*x^n)/d/n+p*e/n*g/d*ln(x^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(fn^{2}p\log(x)^{2}-2\,fn\log(c)\log(x)\right)x^{n}-2\left(fnx^{n}\log(x)-g\right)\log\left(\left(ex^{n}+d\right)^{p}\right)+2\,g\log(c)}{2\,nx^{n}}+\int\frac{dfnp\log(x)+egp}{exx^{n}+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((f*n^2*p*log(x)^2 - 2*f*n*log(c)*log(x))*x^n - 2*(f*n*x^n*log(x) - g)
*log((e*x^n + d)^p) + 2*g*log(c))/(n*x^n) + integrate((d*f*n*p*log(x) + e*g
*p)/(e*x*x^n + d*x), x)
```

Fricas [A] time = 2.1328, size = 266, normalized size = 2.74

$$-\frac{dfnpx^{n}\log(x)\log\left(\frac{ex^{n}+d}{d}\right)+dfpx^{n}\mathrm{Li}_{2}\left(-\frac{ex^{n}+d}{d}+1\right)+dg\log(c)-\left(egnp+dfn\log(c)\right)x^{n}\log(x)+\left(dgp-\left(dfnp\log(x\right.\right.}{dnx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -(d*f*n*p*x^n*log(x)*log((e*x^n + d)/d) + d*f*p*x^n*dilog(-(e*x^n + d)/d +
1) + d*g*log(c) - (e*g*n*p + d*f*n*log(c))*x^n*log(x) + (d*g*p - (d*f*n*p*l
og(x) - e*g*p)*x^n)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f + \frac{g}{x^n}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^n)*log((e*x^n + d)^p*c)/x, x)
```

3.364 $\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=126

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2gp\log\left(d+ex^n\right)}{2d^2n}-\frac{e^2gp\log(x)}{2d^2}-$$

```
[Out] -(e*g*p)/(2*d*n*x^n) - (e^2*g*p*Log[x])/(2*d^2) + (e^2*g*p*Log[d + e*x^n])/
(2*d^2*n) - (g*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) + (f*Log[-((e*x^n)/d)]*L
og[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.169208, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2475, 14, 2416, 2395, 44, 2394, 2315}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2gp\log\left(d+ex^n\right)}{2d^2n}-\frac{e^2gp\log(x)}{2d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(e*g*p)/(2*d*n*x^n) - (e^2*g*p*Log[x])/(2*d^2) + (e^2*g*p*Log[d + e*x^n])/
(2*d^2*n) - (g*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) + (f*Log[-((e*x^n)/d)]*L
og[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(f+gx^{-2n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{g\log(c(d+ex)^p)}{x^3}+\frac{f\log(c(d+ex)^p)}{x}\right)dx,x,x^n\right)}{n}\\ &= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx,x,x^n\right)}{n}\\ &= -\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(efp)\,\text{Subst}\left(\int \frac{\text{lo}}{}\right.}{n}\\ &= -\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}+\\ &= -\frac{egpx^{-n}}{2dn}-\frac{e^2gp\log(x)}{2d^2}+\frac{e^2gp\log\left(d+ex^n\right)}{2d^2n}-\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log}{}\end{aligned}$$

Mathematica [A] time = 0.197987, size = 104, normalized size = 0.83

$$-\frac{-2f\left(p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)\right)+gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)+\frac{egpx^{-n}\left(-ex^n\log\left(d+ex^n\right)+d+enx^n\text{lo}\right.}{d^2}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((e*g*p*(d + e*n*x^n*Log[x] - e*x^n*Log[d + e*x^n]))/(d^2*x^n) + (g*Log[c*
(d + e*x^n)^p])/x^(2*n) - 2*f*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*P
olyLog[2, 1 + (e*x^n)/d]))/(2*n)
```

Maple [C] time = 4.392, size = 448, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] 1/2*(2*f*ln(x)*n*(x^n)^2-g)/n/(x^n)^2*ln((d+e*x^n)^p)-1/2*I/n*Pi*csgn(I*c*(
d+e*x^n)^p)^3*f*ln(x^n)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p
)*csgn(I*c)*g/(x^n)^2-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^
2*g/(x^n)^2-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*g/(x^n)^2-1/2*I/n*
Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x^n)+1/4*I/n*Pi
*csgn(I*c*(d+e*x^n)^p)^3*g/(x^n)^2+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*
(d+e*x^n)^p)^2*f*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*ln(
x^n)+1/n*ln(c)*f*ln(x^n)-1/2/n*ln(c)*g/(x^n)^2+1/2*e^2*g*p*ln(d+e*x^n)/d^2/
n-1/2*e*g*p/d/n/(x^n)-1/2*p*e^2/n*g/d^2*ln(x^n)-p/n*f*dilog((d+e*x^n)/d)-p*
f*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{egpx^n + dg\log(c) + \left(dfn^2p\log(x)^2 - 2\,dfn\log(c)\log(x)\right)x^{2\,n} - \left(2\,dfnx^{2\,n}\log(x) - dg\right)\log\left((ex^n+d)^p\right)}{2\,dnx^{2\,n}} + \int \frac{2\,d^2fnp}{2\,(de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*p*x^n + d*g*log(c) + (d*f*n^2*p*log(x)^2 - 2*d*f*n*log(c)*log(x))
*x^(2*n) - (2*d*f*n*x^(2*n)*log(x) - d*g)*log((e*x^n + d)^p))/(d*n*x^(2*n))
 + integrate(1/2*(2*d^2*f*n*p*log(x) - e^2*g*p)/(d*e*x*x^n + d^2*x), x)
```

Fricas [A] time = 2.1054, size = 352, normalized size = 2.79

$$-\frac{2\,d^2fnpx^{2\,n}\log(x)\log\left(\frac{ex^n+d}{d}\right) + 2\,d^2fpx^{2\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + degpx^n + d^2g\log(c) + \left(e^2gnp - 2\,d^2fn\log(c)\right)x^{2\,n}\log(x}{2\,d^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f*p*x^(2*n)*dil
og(-(e*x^n + d)/d + 1) + d*e*g*p*x^n + d^2*g*log(c) + (e^2*g*n*p - 2*d^2*f*
n*log(c))*x^(2*n)*log(x) + (d^2*g*p - (2*d^2*f*n*p*log(x) + e^2*g*p)*x^(2*n
))*log(e*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f + \frac{g}{x^{2n}}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))*log((e*x^n + d)^p*c)/x, x)
```

3.365 $\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=327

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}+\frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n}-\frac{2d^2}{3}$$

```
[Out] (-2*d^2*f*g*p*x^n)/(3*e^2*n) + (d^5*g^2*p*x^n)/(6*e^5*n) + (d*f*g*p*x^(2*n)
)/(3*e*n) - (d^4*g^2*p*x^(2*n))/(12*e^4*n) - (2*f*g*p*x^(3*n))/(9*n) + (d^3
*g^2*p*x^(3*n))/(18*e^3*n) - (d^2*g^2*p*x^(4*n))/(24*e^2*n) + (d*g^2*p*x^(5
*n))/(30*e*n) - (g^2*p*x^(6*n))/(36*n) + (2*d^3*f*g*p*Log[d + e*x^n])/(3*e^
3*n) - (d^6*g^2*p*Log[d + e*x^n])/(6*e^6*n) + (2*f*g*x^(3*n)*Log[c*(d + e*x
^n)^p])/(3*n) + (g^2*x^(6*n)*Log[c*(d + e*x^n)^p])/(6*n) + (f^2*Log[-((e*x^
n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.325745, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2475, 266, 43, 2416, 2394, 2315, 2395}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}+\frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n}-\frac{2d^2}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```



```
[Out] (-2*d^2*f*g*p*x^n)/(3*e^2*n) + (d^5*g^2*p*x^n)/(6*e^5*n) + (d*f*g*p*x^(2*n)
)/(3*e*n) - (d^4*g^2*p*x^(2*n))/(12*e^4*n) - (2*f*g*p*x^(3*n))/(9*n) + (d^3
*g^2*p*x^(3*n))/(18*e^3*n) - (d^2*g^2*p*x^(4*n))/(24*e^2*n) + (d*g^2*p*x^(5
*n))/(30*e*n) - (g^2*p*x^(6*n))/(36*n) + (2*d^3*f*g*p*Log[d + e*x^n])/(3*e^
3*n) - (d^6*g^2*p*Log[d + e*x^n])/(6*e^6*n) + (2*f*g*x^(3*n)*Log[c*(d + e*x
^n)^p])/(3*n) + (g^2*x^(6*n)*Log[c*(d + e*x^n)^p])/(6*n) + (f^2*Log[-((e*x^
n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(f+gx^{3n}\right)^2 \log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+gx^3\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{f^2 \log(c(d+ex)^p)}{x} + 2fgx^2\log(c(d+ex)^p) + g^2x^5\log(c(d+ex)^p)\right) dx, x$$

$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\,\text{Subst}\left(\int x^2\log(c(d+ex)^p)\,dx, x, x$$

$$= \frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n} + \frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{n}$$

$$= \frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n} + \frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{n}$$

$$= -\frac{2d^2fgpx^n}{3e^2n} + \frac{d^5g^2px^n}{6e^5n} + \frac{dfgpx^{2n}}{3en} - \frac{d^4g^2px^{2n}}{12e^4n} - \frac{2fgpx^{3n}}{9n} + \frac{d^3g^2px^{3n}}{18e^3n} - \frac{d^2g^2}{24e}$$

Mathematica [A] time = 0.374937, size = 209, normalized size = 0.64

$$\frac{360e^6f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right) + 60e^6\log\left(c\left(d+ex^n\right)^p\right)\left(6f^2\log\left(-\frac{ex^n}{d}\right)+gx^{3n}\left(4f+gx^{3n}\right)\right) - egpx^n\left(15d^2e^3\left(16f+g\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-60*d^5*g + 30*d^4*e*g*x^n - 20*d^3*e^2*g*x^(2*n) + 10*e^5*x^
(2*n)*(8*f + g*x^(3*n)) - 12*d*e^4*x^n*(10*f + g*x^(3*n)) + 15*d^2*e^3*(16*
f + g*x^(3*n)))) - 60*d^3*g*(-4*e^3*f + d^3*g)*p*Log[d + e*x^n] + 60*e^6*(g
*x^(3*n)*(4*f + g*x^(3*n)) + 6*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p]
+ 360*e^6*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(360*e^6*n)
```

Maple [C] time = 5.224, size = 795, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(3*n))^2*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] -1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)-p/n*f^2*dilog((d+e*x^n)/d)+
2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n+1/3*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+
e*x^n)^p)^2*(x^n)^3*f*g+1/6*(g^2*(x^n)^6+4*f*g*(x^n)^3+6*f^2*ln(x)*n)/n*ln(
(d+e*x^n)^p)-1/3*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*(x^n)^3*f*g+1/12*I/n*Pi*csg
n(I*c*(d+e*x^n)^p)^2*csgn(I*c)*(x^n)^6*g^2-1/24*p/e^2/n*g^2*d^2*(x^n)^4+1/1
8*p/e^3/n*g^2*d^3*(x^n)^3-1/12*p/e^4/n*g^2*(x^n)^2*d^4+1/30*p/e/n*g^2*(x^n)
^5*d-1/12*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*(x^n)^
6*g^2-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln
(x^n)+1/3*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*(x^n)^3*f*g-1/6*d^6*g^2*
p*ln(d+e*x^n)/e^6/n-p*f^2*ln(x)*ln((d+e*x^n)/d)+1/6*d^5*g^2*p*x^n/e^5/n-1/3
6*p/n*g^2*(x^n)^6-1/12*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*(x^n)^6*g^2+1/3*p/e/n
*f*g*(x^n)^2*d+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f^2*l
n(x^n)+1/12*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*(x^n)^6*g^2+
1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^2*ln(x^n)-2/3*d^2*f*g*p*x^n/
e^2/n-2/9*p/n*f*g*(x^n)^3+2/3/n*ln(c)*(x^n)^3*f*g+1/6/n*ln(c)*(x^n)^6*g^2+1
/n*ln(c)*f^2*ln(x^n)-1/3*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*c
sgn(I*c)*(x^n)^3*f*g
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{180\, e^{6} f^{2} n^{2} p \log\left(x\right)^{2} - 12\, d e^{5} g^{2} p x^{5\, n} + 15\, d^{2} e^{4} g^{2} p x^{4\, n} + 10\, \left(e^{6} g^{2} p - 6\, e^{6} g^{2} \log\left(c\right)\right) x^{6\, n} + 20\, \left(4\, e^{6} f g p - d^{3} e^{3} g^{2} p - 12\, e^{6} f g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/360*(180*e^6*f^2*n^2*p*log(x)^2 - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g^
2*p*x^(4*n) + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6*n) + 20*(4*e^6*f*g*p -
 d^3*e^3*g^2*p - 12*e^6*f*g*log(c))*x^(3*n) - 30*(4*d*e^5*f*g*p - d^4*e^2*g
^2*p)*x^(2*n) + 60*(4*d^2*e^4*f*g*p - d^5*e*g^2*p)*x^n - 60*(6*e^6*f^2*n*lo
g(x) + e^6*g^2*x^(6*n) + 4*e^6*f*g*x^(3*n))*log((e*x^n + d)^p) - 60*(4*d^3*
e^3*f*g*n*p - d^6*g^2*n*p + 6*e^6*f^2*n*log(c))*log(x))/(e^6*n) + integrate
(1/6*(6*d*e^6*f^2*n*p*log(x) - 4*d^4*e^3*f*g*p + d^7*g^2*p)/(e^7*x*x^n + d*
e^6*x), x)
```

Fricas [A] time = 2.2259, size = 659, normalized size = 2.02

$$-\frac{360\,e^{6}f^{2}np\log\left(x\right)\log\left(\frac{ex^{n}+d}{d}\right)-360\,e^{6}f^{2}n\log\left(c\right)\log\left(x\right)-12\,de^{5}g^{2}px^{5\,n}+15\,d^{2}e^{4}g^{2}px^{4\,n}+360\,e^{6}f^{2}p\mathrm{Li}_2\left(-\frac{ex^{n}+d}{d}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/360*(360*e^6*f^2*n*p*log(x)*log((e*x^n + d)/d) - 360*e^6*f^2*n*log(c)*lo
g(x) - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g^2*p*x^(4*n) + 360*e^6*f^2*p*di
log(-(e*x^n + d)/d + 1) - 30*(4*d*e^5*f*g - d^4*e^2*g^2)*p*x^(2*n) + 60*(4*
d^2*e^4*f*g - d^5*e*g^2)*p*x^n + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6*n)
- 20*(12*e^6*f*g*log(c) - (4*e^6*f*g - d^3*e^3*g^2)*p)*x^(3*n) - 60*(6*e^6*
f^2*n*p*log(x) + e^6*g^2*p*x^(6*n) + 4*e^6*f*g*p*x^(3*n) + (4*d^3*e^3*f*g -
 d^6*g^2)*p)*log(e*x^n + d))/(e^6*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(3*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{3\,n}+f\right)^{2}\log\left(\left(ex^{n}+d\right)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(3*n) + f)^2*log((e*x^n + d)^p*c)/x, x)
```

3.366 $\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=254

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n} + \frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{fgx^{2n} \log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{g^2 x^{4n} \log\left(c\left(d+ex^n\right)^p\right)}{4n} - \frac{d^2 fg}{}$$

```
[Out] (d*f*g*p*x^n)/(e*n) + (d^3*g^2*p*x^n)/(4*e^3*n) - (f*g*p*x^(2*n))/(2*n) - (
d^2*g^2*p*x^(2*n))/(8*e^2*n) + (d*g^2*p*x^(3*n))/(12*e*n) - (g^2*p*x^(4*n))
/(16*n) - (d^2*f*g*p*Log[d + e*x^n])/(e^2*n) - (d^4*g^2*p*Log[d + e*x^n])/(
4*e^4*n) + (f*g*x^(2*n)*Log[c*(d + e*x^n)^p])/n + (g^2*x^(4*n)*Log[c*(d + e
*x^n)^p])/(4*n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*P
olyLog[2, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.267391, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2475, 266, 43, 2416, 2394, 2315, 2395}

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n} + \frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{fgx^{2n} \log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{g^2 x^{4n} \log\left(c\left(d+ex^n\right)^p\right)}{4n} - \frac{d^2 fg}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*f*g*p*x^n)/(e*n) + (d^3*g^2*p*x^n)/(4*e^3*n) - (f*g*p*x^(2*n))/(2*n) - (
d^2*g^2*p*x^(2*n))/(8*e^2*n) + (d*g^2*p*x^(3*n))/(12*e*n) - (g^2*p*x^(4*n))
/(16*n) - (d^2*f*g*p*Log[d + e*x^n])/(e^2*n) - (d^4*g^2*p*Log[d + e*x^n])/(
4*e^4*n) + (f*g*x^(2*n)*Log[c*(d + e*x^n)^p])/n + (g^2*x^(4*n)*Log[c*(d + e
*x^n)^p])/(4*n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*P
olyLog[2, 1 + (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(f+gx^{2n}\right)^2\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+gx^2\right)^2\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{f^2\log(c(d+ex)^p)}{x}+2fgx\log\left(c(d+ex)^p\right)+g^2x^3\log\left(c(d+ex)^p\right)\right)dx,x,\right.}{n}$$

$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{(2fg)\,\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)dx,x,x^n\right.}{n}$$

$$= \frac{fgx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{n}$$

$$= \frac{fgx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{n}$$

$$= \frac{dfgpx^n}{en}+\frac{d^3g^2px^n}{4e^3n}-\frac{fgpx^{2n}}{2n}-\frac{d^2g^2px^{2n}}{8e^2n}+\frac{dg^2px^{3n}}{12en}-\frac{g^2px^{4n}}{16n}-\frac{d^2fgp\log(d}{e^2n}$$

Mathematica [A] time = 0.259248, size = 171, normalized size = 0.67

$$\frac{48e^4f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+12e^4\log\left(c\left(d+ex^n\right)^p\right)\left(4f^2\log\left(-\frac{ex^n}{d}\right)+gx^{2n}\left(4f+gx^{2n}\right)\right)-egpx^n\left(6d^2egx^n-12d^3g\right.}{48e^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-12*d^3*g + 6*d^2*e*g*x^n + 3*e^3*x^n*(8*f + g*x^(2*n)) - 4*d
*e^2*(12*f + g*x^(2*n)))) - 12*d^2*g*(4*e^2*f + d^2*g)*p*Log[d + e*x^n] + 1
2*e^4*(g*x^(2*n)*(4*f + g*x^(2*n)) + 4*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*
x^n)^p] + 48*e^4*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(48*e^4*n)
```

Maple [C] time = 5.136, size = 734, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] -1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)-d^2*f*g*p*ln(d+e*x^n)/e^2/n
-p/n*f^2*dilog((d+e*x^n)/d)-1/16*p/n*g^2*(x^n)^4+1/4/n*ln(c)*(x^n)^4*g^2+1/
4*(g^2*(x^n)^4+4*f^2*ln(x)*n+4*f*g*(x^n)^2)/n*ln((d+e*x^n)^p)-1/2*I/n*Pi*cs
gn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(x^n)+1/2*I/n*Pi*cs
gn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*(x^n)^2*f*g-1/8*I/n*Pi*csgn(I*(d+
e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*(x^n)^4*g^2+1/2*I/n*Pi*csgn(I*c*(
d+e*x^n)^p)^2*csgn(I*c)*(x^n)^2*f*g-1/8*p/e^2/n*g^2*(x^n)^2*d^2-1/4*d^4*g^2
*p*ln(d+e*x^n)/e^4/n-p*f^2*ln(x)*ln((d+e*x^n)/d)+1/4*d^3*g^2*p*x^n/e^3/n-1/
2*p/n*f*g*(x^n)^2+1/n*ln(c)*(x^n)^2*f*g-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*
(x^n)^2*f*g+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f^2*ln(x
^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^2*ln(x^n)+d*f*g*p*x^n/e/
n+1/n*ln(c)*f^2*ln(x^n)-1/8*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*(x^n)^4*g^2+1/8*
I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*(x^n)^4*g^2+1/12*p/e/n*g
^2*d*(x^n)^3+1/8*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*(x^n)^4*g^2-1/2*I
/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*(x^n)^2*f*g
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{24\,e^4 f^2 n^2 p \log(x)^2 - 4\,de^3 g^2 p x^{3\,n} + 3\left(e^4 g^2 p - 4\,e^4 g^2 \log(c)\right)x^{4\,n} + 6\left(4\,e^4 fgp + d^2 e^2 g^2 p - 8\,e^4 fg \log(c)\right)x^{2\,n} - 12\left(4\,de^{3}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/48*(24*e^4*f^2*n^2*p*log(x)^2 - 4*d*e^3*g^2*p*x^(3*n) + 3*(e^4*g^2*p - 4
*e^4*g^2*log(c))*x^(4*n) + 6*(4*e^4*f*g*p + d^2*e^2*g^2*p - 8*e^4*f*g*log(c
))*x^(2*n) - 12*(4*d*e^3*f*g*p + d^3*e*g^2*p)*x^n - 12*(4*e^4*f^2*n*log(x)
+ e^4*g^2*x^(4*n) + 4*e^4*f*g*x^(2*n))*log((e*x^n + d)^p) + 12*(4*d^2*e^2*f
*g*n*p + d^4*g^2*n*p - 4*e^4*f^2*n*log(c))*log(x))/(e^4*n) + integrate(1/4*
(4*d*e^4*f^2*n*p*log(x) + 4*d^3*e^2*f*g*p + d^5*g^2*p)/(e^5*x*x^n + d*e^4*x
), x)
```

Fricas [A] time = 2.08815, size = 551, normalized size = 2.17

$$-\frac{48\,e^4 f^2 np \log(x) \log\left(\frac{ex^n+d}{d}\right) - 48\,e^4 f^2 n \log(c) \log(x) - 4\,de^3 g^2 p x^{3\,n} + 48\,e^4 f^2 p \mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) - 12\left(4\,de^3 fg + d^3 eg^2\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/48*(48*e^4*f^2*n*p*log(x)*log((e*x^n + d)/d) - 48*e^4*f^2*n*log(c)*log(x
) - 4*d*e^3*g^2*p*x^(3*n) + 48*e^4*f^2*p*dilog(-(e*x^n + d)/d + 1) - 12*(4*
d*e^3*f*g + d^3*e*g^2)*p*x^n + 3*(e^4*g^2*p - 4*e^4*g^2*log(c))*x^(4*n) - 6
*(8*e^4*f*g*log(c) - (4*e^4*f*g + d^2*e^2*g^2)*p)*x^(2*n) - 12*(4*e^4*f^2*n
*p*log(x) + e^4*g^2*p*x^(4*n) + 4*e^4*f*g*p*x^(2*n) - (4*d^2*e^2*f*g + d^4*
g^2)*p)*log(e*x^n + d))/(e^4*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{2n}+f\right)^{2}\log\left(\left(ex^{n}+d\right)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)/x, x)
```

3.367 $\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=176

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}+\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fg\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}$$

```
[Out] (-2*f*g*p*x^n)/n + (d*g^2*p*x^n)/(2*e*n) - (g^2*p*x^(2*n))/(4*n) - (d^2*g^2
*p*Log[d + e*x^n])/(2*e^2*n) + (g^2*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (
2*f*g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f^2*Log[-((e*x^n)/d)]*Log[
c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.196758, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315, 2395}

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}+\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fg\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-2*f*g*p*x^n)/n + (d*g^2*p*x^n)/(2*e*n) - (g^2*p*x^(2*n))/(4*n) - (d^2*g^2
*p*Log[d + e*x^n])/(2*e^2*n) + (g^2*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (
2*f*g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f^2*Log[-((e*x^n)/d)]*Log[
c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(f+gx^n\right)^2 \log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(2fg\log\left(c(d+ex)^p\right)+\frac{f^2\log(c(d+ex)^p)}{x}+g^2x\log\left(c(d+ex)^p\right)\right)dx,x,x^n\right)}{n}$$

$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{(2fg)\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right)dx,x,x^n\right)}{n}$$

$$= \frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{(2fg)\,\text{Subst}\left(\int \log\right.}{}$$

$$= -\frac{2fgpx^n}{n}+\frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{2fg\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{f^2\log}{}$$

$$= -\frac{2fgpx^n}{n}+\frac{dg^2px^n}{2en}-\frac{g^2px^{2n}}{4n}-\frac{d^2g^2p\log\left(d+ex^n\right)}{2e^2n}+\frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}$$

Mathematica [A] time = 0.189758, size = 124, normalized size = 0.7

$$\frac{4e^2f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+2e\log\left(c\left(d+ex^n\right)^p\right)\left(2ef^2\log\left(-\frac{ex^n}{d}\right)+4dfg+egx^n\left(4f+gx^n\right)\right)-2d^2g^2p\log\left(d+ex^n\right.}{4e^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(8*e*f - 2*d*g + e*g*x^n)) - 2*d^2*g^2*p*Log[d + e*x^n] + 2*e*
(4*d*f*g + e*g*x^n*(4*f + g*x^n) + 2*e*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*
x^n)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(4*e^2*n)
```

Maple [C] time = 5.079, size = 665, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] 1/2*(2*f^2*ln(x)*n+g^2*(x^n)^2+4*f*g*x^n)/n*ln((d+e*x^n)^p)-1/4*I/n*Pi*csgn
(I*c*(d+e*x^n)^p)^3*(x^n)^2*g^2+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c
)*f^2*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c
)*f^2*ln(x^n)+I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*x^n*g*f-1/4*I/n*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*(x^n)^2*g^2+1/4*I/n*Pi*c
sgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*(x^n)^2*g^2+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)^2*(x^n)^2*g^2-I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d
+e*x^n)^p)*csgn(I*c)*x^n*g*f-I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*x^n*g*f-1/2*I/n
*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn
(I*c*(d+e*x^n)^p)^2*f^2*ln(x^n)+I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^
n)^p)^2*x^n*g*f+1/2/n*ln(c)*(x^n)^2*g^2+2/n*ln(c)*x^n*g*f+1/n*ln(c)*f^2*ln(
x^n)-1/4*p/n*g^2*(x^n)^2+1/2*d*g^2*p*x^n/e/n-1/2*d^2*g^2*p*ln(d+e*x^n)/e^2/
n-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)-2*f*g*p*x^n/n+2*p/
e/n*f*g*d*ln(d+e*x^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,e^2f^2n^2p\log(x)^2+\left(e^2g^2p-2\,e^2g^2\log(c)\right)x^{2\,n}+2\left(4\,e^2fgp-deg^2p-4\,e^2fg\log(c)\right)x^n-2\left(2\,e^2f^2n\log(x)+e^2g^2x^{2\,n}\right.}{4\,e^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*e^2*f^2*n^2*p*log(x)^2 + (e^2*g^2*p - 2*e^2*g^2*log(c))*x^(2*n) + 2
*(4*e^2*f*g*p - d*e*g^2*p - 4*e^2*f*g*log(c))*x^n - 2*(2*e^2*f^2*n*log(x) +
 e^2*g^2*x^(2*n) + 4*e^2*f*g*x^n)*log((e*x^n + d)^p) - 2*(4*d*e*f*g*n*p - d
^2*g^2*n*p + 2*e^2*f^2*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e^2*f
^2*n*p*log(x) - 4*d^2*e*f*g*p + d^3*g^2*p)/(e^3*x*x^n + d*e^2*x), x)
```

Fricas [A] time = 2.15976, size = 437, normalized size = 2.48

$$-\frac{4\,e^2f^2np\log(x)\log\left(\frac{ex^n+d}{d}\right)-4\,e^2f^2n\log(c)\log(x)+4\,e^2f^2p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+\left(e^2g^2p-2\,e^2g^2\log(c)\right)x^{2\,n}-2\left(4\,e^2fg\right.}{4\,e^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(4*e^2*f^2*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f^2*n*log(c)*log(x) +
 4*e^2*f^2*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g^2*p - 2*e^2*g^2*log(c))*x^(
2*n) - 2*(4*e^2*f*g*log(c) - (4*e^2*f*g - d*e*g^2)*p)*x^n - 2*(2*e^2*f^2*n*
p*log(x) + e^2*g^2*p*x^(2*n) + 4*e^2*f*g*p*x^n + (4*d*e*f*g - d^2*g^2)*p)*l
og(e*x^n + d))/(e^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^n + f)^2 \log\left((ex^n + d)^p c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)/x, x)
```

3.368 $\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=193

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}+\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2g}{}$$

```
[Out] -(e*g^2*p)/(2*d*n*x^n) + (2*e*f*g*p*Log[x])/d - (e^2*g^2*p*Log[x])/(2*d^2)
 - (2*e*f*g*p*Log[d + e*x^n])/(d*n) + (e^2*g^2*p*Log[d + e*x^n])/(2*d^2*n) -
 (g^2*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) - (2*f*g*Log[c*(d + e*x^n)^p])/(n
*x^n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2,
1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.242307, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2475, 263, 43, 2416, 2395, 44, 36, 29, 31, 2394, 2315}

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}+\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2g}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(e*g^2*p)/(2*d*n*x^n) + (2*e*f*g*p*Log[x])/d - (e^2*g^2*p*Log[x])/(2*d^2)
 - (2*e*f*g*p*Log[d + e*x^n])/(d*n) + (e^2*g^2*p*Log[d + e*x^n])/(2*d^2*n) -
 (g^2*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) - (2*f*g*Log[c*(d + e*x^n)^p])/(n
*x^n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2,
1 + (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(f+gx^{-n}\right)^2 \log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{g^2\log(c(d+ex)^p)}{x^3} + \frac{2fg\log(c(d+ex)^p)}{x^2} + \frac{f^2\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}$$

$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^n\right)}{n} + \frac{g^2\,\text{Su}}{}$$

$$= -\frac{g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n} - \frac{2fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{n}$$

$$= -\frac{g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n} - \frac{2fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+\right.\right.}{n}$$

$$= -\frac{eg^2px^{-n}}{2dn} + \frac{2efgp\log(x)}{d} - \frac{e^2g^2p\log(x)}{2d^2} - \frac{2efgp\log\left(d+ex^n\right)}{dn} + \frac{e^2g^2p\log\left(d+e\right.}{2d^2n}$$

Mathematica [A] time = 0.375687, size = 150, normalized size = 0.78

$$-\frac{-2f^2\left(p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)\right)+4fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)+g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)+}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((-4*e*f*g*p*(n*Log[x] - Log[d + e*x^n]))/d + (e*g^2*p*(d/x^n + e*n*Log[x]
 - e*Log[d + e*x^n]))/d^2 + (g^2*Log[c*(d + e*x^n)^p])/x^(2*n) + (4*f*g*Log
[c*(d + e*x^n)^p])/x^n - 2*f^2*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*
PolyLog[2, 1 + (e*x^n)/d]))/(2*n)
```

Maple [C] time = 5.204, size = 693, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] 1/2*(2*f^2*ln(x)*n*(x^n)^2-4*f*g*x^n-g^2)/n/(x^n)^2*ln((d+e*x^n)^p)-1/4*I/n
*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*g^2/(x^n)^2-I/n*Pi*csgn(I*(
d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f*g/(x^n)+I/n*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*g/(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn
(I*c*(d+e*x^n)^p)^2*f^2*ln(x^n)+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*g^2/(x^n
)^2+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g^2/(x^n
)^2-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(x
^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^2*ln(x^n)-1/2*I/n*Pi*csg
n(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)-I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f
*g/(x^n)+I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*g/(x^n)-1/4*I/n*Pi*csgn(I*c*(d+e*
x^n)^p)^2*csgn(I*c)*g^2/(x^n)^2+1/n*ln(c)*f^2*ln(x^n)-2/n*ln(c)*f*g/(x^n)-1
/2/n*ln(c)*g^2/(x^n)^2-2*e*f*g*p*ln(d+e*x^n)/d/n+2*p*e/n*f*g/d*ln(x^n)+1/2*
e^2*g^2*p*ln(d+e*x^n)/d^2/n-1/2*e*g^2*p/d/n/(x^n)-1/2*p*e^2/n*g^2/d^2*ln(x^
```

```
n)-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{dg^2\log(c) + \left(df^2n^2p\log(x)^2 - 2\,df^2n\log(c)\log(x)\right)x^{2n} + \left(eg^2p + 4\,dfg\log(c)\right)x^n - \left(2\,df^2nx^{2n}\log(x) - 4\,dfgx^n\right.}{2\,dnx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(d*g^2*log(c) + (d*f^2*n^2*p*log(x)^2 - 2*d*f^2*n*log(c)*log(x))*x^(2*
n) + (e*g^2*p + 4*d*f*g*log(c))*x^n - (2*d*f^2*n*x^(2*n)*log(x) - 4*d*f*g*x
^n - d*g^2)*log((e*x^n + d)^p))/(d*n*x^(2*n)) + integrate(1/2*(2*d^2*f^2*n*
p*log(x) + 4*d*e*f*g*p - e^2*g^2*p)/(d*e*x*x^n + d^2*x), x)
```

Fricas [A] time = 2.15334, size = 467, normalized size = 2.42

$$-\frac{2\,d^2f^2npx^{2n}\log(x)\log\left(\frac{ex^n+d}{d}\right) + 2\,d^2f^2px^{2n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + d^2g^2\log(c) - \left(2\,d^2f^2n\log(c) + \left(4\,defg - e^2g^2\right)np\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f^2*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f^2*p*x^(2*n)
*dilog(-(e*x^n + d)/d + 1) + d^2*g^2*log(c) - (2*d^2*f^2*n*log(c) + (4*d*e*
f*g - e^2*g^2)*n*p)*x^(2*n)*log(x) + (d*e*g^2*p + 4*d^2*f*g*log(c))*x^n + (
4*d^2*f*g*p*x^n + d^2*g^2*p - (2*d^2*f^2*n*p*log(x) - (4*d*e*f*g - e^2*g^2)
*p)*x^(2*n))*log(e*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f + \frac{g}{x^n}\right)^2 \log\left((ex^n + d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)/x, x)
```

3.369 $\int \frac{\left(f+gx^{-2n}\right)^2 \log\left(c(d+ex^n)^p\right)}{x} \, dx$

Optimal. Leaf size=257

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}+\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+$$

```
[Out] -(e*g^2*p)/(12*d*n*x^(3*n)) + (e^2*g^2*p)/(8*d^2*n*x^(2*n)) - (e*f*g*p)/(d*
n*x^n) - (e^3*g^2*p)/(4*d^3*n*x^n) - (e^2*f*g*p*Log[x])/d^2 - (e^4*g^2*p*Lo
g[x])/(4*d^4) + (e^2*f*g*p*Log[d + e*x^n])/(d^2*n) + (e^4*g^2*p*Log[d + e*x
^n])/(4*d^4*n) - (g^2*Log[c*(d + e*x^n)^p])/(4*n*x^(4*n)) - (f*g*Log[c*(d +
 e*x^n)^p])/(n*x^(2*n)) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n +
(f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rubi [A] time = 0.317214, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2475, 263, 266, 43, 2416, 2395, 44, 2394, 2315}

$$\frac{f^2 p \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}+\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(e*g^2*p)/(12*d*n*x^(3*n)) + (e^2*g^2*p)/(8*d^2*n*x^(2*n)) - (e*f*g*p)/(d*
n*x^n) - (e^3*g^2*p)/(4*d^3*n*x^n) - (e^2*f*g*p*Log[x])/d^2 - (e^4*g^2*p*Lo
g[x])/(4*d^4) + (e^2*f*g*p*Log[d + e*x^n])/(d^2*n) + (e^4*g^2*p*Log[d + e*x
^n])/(4*d^4*n) - (g^2*Log[c*(d + e*x^n)^p])/(4*n*x^(4*n)) - (f*g*Log[c*(d +
 e*x^n)^p])/(n*x^(2*n)) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n +
(f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log \left(c\left(d+ex^n\right)^p\right)}{x} dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)^2 \log(c(d+ex)^p)}{x} dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{g^2 \log(c(d+ex)^p)}{x^5} + \frac{2fg \log(c(d+ex)^p)}{x^3} + \frac{f^2 \log(c(d+ex)^p)}{x}\right) dx, x, x^n\right)}{n}$$

$$= \frac{f^2 \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x} dx, x, x^n\right)}{n} + \frac{(2fg) \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3} dx, x, x^n\right)}{n} + -$$

$$= -\frac{g^2x^{-4n} \log\left(c\left(d+ex^n\right)^p\right)}{4n} - \frac{fgx^{-2n} \log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log\left(c\left(\right.\right.}{n}$$

$$= -\frac{g^2x^{-4n} \log\left(c\left(d+ex^n\right)^p\right)}{4n} - \frac{fgx^{-2n} \log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log\left(c\left(\right.\right.}{n}$$

$$= -\frac{eg^2px^{-3n}}{12dn} + \frac{e^2g^2px^{-2n}}{8d^2n} - \frac{efgpx^{-n}}{dn} - \frac{e^3g^2px^{-n}}{4d^3n} - \frac{e^2fgp \log(x)}{d^2} - \frac{e^4g^2p \log(x}{4d^4}$$

Mathematica [A] time = 0.569967, size = 188, normalized size = 0.73

$$-\frac{-24f^2\left(p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)\right)+24fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)+6g^2x^{-4n}\log\left(c\left(d+ex\right.\right.}{24n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((24*e*f*g*p*(d/x^n + e*n*Log[x] - e*Log[d + e*x^n]))/d^2 + (e*g^2*p*((d*(
2*d^2 - 3*d*e*x^n + 6*e^2*x^(2*n)))/x^(3*n) + 6*e^3*n*Log[x] - 6*e^3*Log[d
+ e*x^n]))/d^4 + (6*g^2*Log[c*(d + e*x^n)^p])/x^(4*n) + (24*f*g*Log[c*(d +
e*x^n)^p])/x^(2*n) - 24*f^2*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*Pol
yLog[2, 1 + (e*x^n)/d]))/(24*n)
```

Maple [C] time = 5.238, size = 755, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)/x,x)
```

```
[Out] 1/4*(4*f^2*ln(x)*n*(x^n)^4-4*f*g*(x^n)^2-g^2)/n/(x^n)^4*ln((d+e*x^n)^p)-1/4
/n*ln(c)*g^2/(x^n)^4-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)+e^2*f*g
*p*ln(d+e*x^n)/d^2/n-p/n*f^2*dilog((d+e*x^n)/d)-1/4*e^3*g^2*p/d^3/n/(x^n)-1
/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f*g/(x^n)^2-1/2*I/n*P
i*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(x^n)+1/4*e^4*g
^2*p*ln(d+e*x^n)/d^4/n-p*f^2*ln(x)*ln((d+e*x^n)/d)+1/2*I/n*Pi*csgn(I*c*(d+e
*x^n)^p)^3*f*g/(x^n)^2+1/8*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)
*csgn(I*c)*g^2/(x^n)^4-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*g/(x^
n)^2+1/8*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*g^2/(x^n)^4+1/8*p*e^2/n*g^2/d^2/(x^
n)^2-1/4*p*e^4/n*g^2/d^4*ln(x^n)-1/12*p*e/n*g^2/d/(x^n)^3-p*e^2/n*f*g/d^2*l
n(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f^2*ln(x^n)+1
```

```
/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^2*ln(x^n)-e*f*g*p/d/n/(x^n)+1
/n*ln(c)*f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*c
sgn(I*c)*f*g/(x^n)^2-1/8*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*g^2/(x^n)
^4-1/8*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*g^2/(x^n)^4-1/n*l
n(c)*f*g/(x^n)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,d^2eg^2px^n + 6\,d^3g^2\log{(c)} + 12\left(d^3f^2n^2p\log{(x)}^2 - 2\,d^3f^2n\log{(c)}\log{(x)}\right)x^{4\,n} + 6\left(4\,d^2efgp + e^3g^2p\right)x^{3\,n} - 3\left(de^2g^2p -\right.}{24\,d^3nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/24*(2*d^2*e*g^2*p*x^n + 6*d^3*g^2*log(c) + 12*(d^3*f^2*n^2*p*log(x)^2 -
2*d^3*f^2*n*log(c)*log(x))*x^(4*n) + 6*(4*d^2*e*f*g*p + e^3*g^2*p)*x^(3*n)
- 3*(d*e^2*g^2*p - 8*d^3*f*g*log(c))*x^(2*n) - 6*(4*d^3*f^2*n*x^(4*n)*log(x
) - 4*d^3*f*g*x^(2*n) - d^3*g^2)*log((e*x^n + d)^p))/(d^3*n*x^(4*n)) + inte
grate(1/4*(4*d^4*f^2*n*p*log(x) - 4*d^2*e^2*f*g*p - e^4*g^2*p)/(d^3*e*x*x^n
 + d^4*x), x)
```

Fricas [A] time = 2.18595, size = 590, normalized size = 2.3

$$-\frac{24\,d^4f^2npx^{4\,n}\log{(x)}\log{\left(\frac{ex^n+d}{d}\right)} + 24\,d^4f^2px^{4\,n}\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + 2\,d^3eg^2px^n + 6\,d^4g^2\log{(c)} + 6\left(4\,d^3efg + de^3g^2\right)px^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/24*(24*d^4*f^2*n*p*x^(4*n)*log(x)*log((e*x^n + d)/d) + 24*d^4*f^2*p*x^(4
*n)*dilog(-(e*x^n + d)/d + 1) + 2*d^3*e*g^2*p*x^n + 6*d^4*g^2*log(c) + 6*(4
*d^3*e*f*g + d*e^3*g^2)*p*x^(3*n) - 6*(4*d^4*f^2*n*log(c) - (4*d^2*e^2*f*g
+ e^4*g^2)*n*p)*x^(4*n)*log(x) - 3*(d^2*e^2*g^2*p - 8*d^4*f*g*log(c))*x^(2*
n) + 6*(4*d^4*f*g*p*x^(2*n) + d^4*g^2*p - (4*d^4*f^2*n*p*log(x) + (4*d^2*e^
2*f*g + e^4*g^2)*p)*x^(4*n))*log(e*x^n + d))/(d^4*n*x^(4*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f + \frac{g}{x^{2n}}\right)^2 \log\left((ex^n + d)^p c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)/x, x)
```

3.370 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$

Optimal. Leaf size=266

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}-$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*n) - (Log[c*(d
 + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/(e*Sqrt[-f] - d*Sqrt[g])])/(2
*f*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
2*f*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*
f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rubi [A] time = 0.423664, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2475, 266, 36, 29, 31, 2416, 2394, 2315, 260, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
```


```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*n) - (Log[c*(d
 + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/(e*Sqrt[-f] - d*Sqrt[g])])/(2
*f*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
2*f*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*
f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx^2)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{gx\log(c(d+ex)^p)}{f(f+gx^2)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} - \frac{g\operatorname{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2}\,dx, x, x^n\right)}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{g\operatorname{Subst}\left(\int \left(-\frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx, x, x^n\right)}{fn} - \frac{(ep)\,\mathrm{S}}{}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} + \frac{p\operatorname{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} + \frac{\sqrt{g}\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}x}\,dx, x, x^n\right)}{2fn} - \frac{\sqrt{g}\,\mathrm{Sub}}{}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e(\cdot}{e}\right.}{2fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e(\cdot}{e}\right.}{2fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e(\cdot}{e}\right.}{2fn}$$

Mathematica [F] time = 4.93591, size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))), x]
```

Maple [C] time = 1.066, size = 695, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x)
```

```
[Out] -1/2/n*ln((d+e*x^n)^p)/f*ln(f+g*(x^n)^2)+1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/f*
p/n*dilog((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/2/n*p/f*ln(d+e*x^n
)*ln(f+g*(x^n)^2)-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-(d+e*x^n)*g+d*g)
/(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+(d+e*x^n)*g
```

```
-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2/n*p/f*dilog((e*(-f*g)^(1/2)-(d+e*x^n)*g+d*g
)/(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f*dilog((e*(-f*g)^(1/2)+(d+e*x^n)*g-d*g)/(e
*(-f*g)^(1/2)-d*g))-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f*ln(x^n)+1/4*I/n*Pi
*csgn(I*c*(d+e*x^n)^p)^3/f*ln(f+g*(x^n)^2)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)
^2*csgn(I*c)/f*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)
^2/f*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)
/f*ln(x^n)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f*ln(f+g*
(x^n)^2)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f*l
n(f+g*(x^n)^2)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f*ln(f+g*(x^n)^
2)-1/2/n*ln(c)/f*ln(f+g*(x^n)^2)+1/n*ln(c)/f*ln(x^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{(gx^{2n} + f)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((ex^n + d)^p c\right)}{gxx^{2n} + fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g*x*x^(2*n) + f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{(gx^{2n} + f)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

3.371 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$

Optimal. Leaf size=121

$$-\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{fn}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(f + g*x^n))/(e*f - d*g)])/(f*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f
- d*g))])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rubi [A] time = 0.195171, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{fn}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(f + g*x^n))/(e*f - d*g)])/(f*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f
- d*g))])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{g\log(c(d+ex)^p)}{f(f+gx)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^n\right)}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \ldots\right.}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} + \frac{p\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn} - \frac{p\text{Li}_2\left(-\frac{g\left(d+ex^n\right)}{ef-dg}\right)}{fn} + \frac{p\text{Li}_2\left(1+\ldots\right.}{fn}$$

Mathematica [A] time = 0.0749918, size = 92, normalized size = 0.76

$$\frac{-p\text{PolyLog}\left(2,\frac{g\left(d+ex^n\right)}{dg-ef}\right)+p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\log\left(c\left(d+ex^n\right)^p\right)\left(\log\left(-\frac{ex^n}{d}\right)-\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*(Log[-((e*x^n)/d)] - Log[(e*(f + g*x^n))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)
/d])/(f*n)
```

Maple [C] time = 1.197, size = 532, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n),x)
```

```
[Out] -1/n*ln((d+e*x^n)^p)/f*ln(f+g*x^n)+1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/f*p/n*di
log((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/n*p/f*dilog((e*(f+g*x^n)
+d*g-f*e)/(d*g-e*f))+1/n*p/f*ln(f+g*x^n)*ln((e*(f+g*x^n)+d*g-f*e)/(d*g-e*f)
)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n
)^p)^3/f*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f*ln(x^n)
-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f*ln(f+g*x^n)-1/2*I
/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*(d+
e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*(
d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p
)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f*ln(x^n)-1/n*ln(c)/f*ln(f+g*x^n)+1/n*ln(
c)/f*ln(x^n)
```

Maxima [A] time = 1.43952, size = 208, normalized size = 1.72

$$-enp\left(\frac{\log(x^n)\log\left(\frac{ex^n}{d}+1\right)+\mathrm{Li}_2\left(-\frac{ex^n}{d}\right)}{efn^2}-\frac{\log\left(gx^n+f\right)\log\left(-\frac{egx^n+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^n+ef}{ef-dg}\right)}{efn^2}\right)-\left(\frac{\log\left(gx^n+f\right)}{fn}-\frac{\log(}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="maxima")
```

```
[Out] -e*n*p*((log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f*n^2) - (log(g*x^
n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^n + e*f)/(e*f -
 d*g)))/(e*f*n^2)) - (log(g*x^n + f)/(f*n) - log(x^n)/(f*n))*log((e*x^n + d
)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)}{gxx^n+fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g*x*x^n + f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(gx^n + f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^n + f)*x), x)
```

3.372 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$

Optimal. Leaf size=70

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{fn}$$

```
[Out] (Log[c*(d + e*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f*n) + (p*PolyL
og[2, (f*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

Rubi [A] time = 0.163625, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2475, 2412, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```


```
[Out] (Log[c*(d + e*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f*n) + (p*PolyL
og[2, (f*(d + e*x^n))/(d*f - e*g)])/(f*n)
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2412

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx,x,x^n\right)}{n}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right)}{fn} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\,dx,x,x^n\right)}{fn}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right)}{fn} - \frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\,dx,x,d+ex^n\right)}{fn}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right)}{fn} + \frac{p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

Mathematica [A] time = 0.0227354, size = 64, normalized size = 0.91

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)+\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(fx^n+g\right)}{eg-df}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(g + f*x^n))/(-(d*f) + e*g)] + p*PolyLog[2, (f
*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

Maple [C] time = 1.198, size = 298, normalized size = 4.3

$$\frac{\ln\left(g+fx^n\right)\ln\left(\left(d+ex^n\right)^p\right)}{nf} - \frac{p}{nf}\text{dilog}\left(\frac{\left(g+fx^n\right)e+df-eg}{df-eg}\right) - \frac{p\ln\left(g+fx^n\right)}{nf}\ln\left(\frac{\left(g+fx^n\right)e+df-eg}{df-eg}\right) + \frac{\frac{i}{2}\ln\left(g+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x)
```

```
[Out] 1/n*ln(g+f*x^n)/f*ln((d+e*x^n)^p)-1/n/f*p*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-
e*g))-1/n/f*p*ln(g+f*x^n)*ln(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))+1/2*I/n*ln(g+
f*x^n)/f*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I/n*ln(g+f*x^n)
/f*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I/n*ln(g+f*x^
n)/f*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I/n*ln(g+f*x^n)/f*Pi*csgn(I*c*(d+e*x^n)
^p)^2*csgn(I*c)+1/n*ln(g+f*x^n)/f*ln(c)
```

Maxima [A] time = 1.40701, size = 151, normalized size = 2.16

$$\left(\frac{\log\left(f+\frac{g}{x^n}\right)}{fn}-\frac{\log\left(\frac{1}{x^n}\right)}{fn}\right)\log\left((ex^n+d)^pc\right)-\frac{\left(\log\left(fx^n+g\right)\log\left(\frac{efx^n+eg}{df-eg}+1\right)+\mathrm{Li}_2\left(-\frac{efx^n+eg}{df-eg}\right)\right)p}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="maxima")
```

```
[Out] (log(f + g/x^n)/(f*n) - log(1/(x^n))/(f*n))*log((e*x^n + d)^p*c) - (log(f*x
^n + g)*log((e*f*x^n + e*g)/(d*f - e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*f
- e*g)))*p/(f*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^n\log\left((ex^n+d)^pc\right)}{fxx^n+gx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="fricas")
```

```
[Out] integral(x^n*log((e*x^n + d)^p*c)/(f*x*x^n + g*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left((ex^n+d)^pc\right)}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^n)*x), x)
```

3.373 $\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$

Optimal. Leaf size=221

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\right.}{2fn}$$

```
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] - Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt
[g])])/(2*f*n) + (Log[c*(d + e*x^n)^p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(
d*Sqrt[-f] - e*Sqrt[g]))])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(
d*Sqrt[-f] - e*Sqrt[g])])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(d
*Sqrt[-f] + e*Sqrt[g])])/(2*f*n)
```

Rubi [A] time = 0.41202, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2475, 263, 260, 2416, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\right.}{2fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] - Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt
[g])])/(2*f*n) + (Log[c*(d + e*x^n)^p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(
d*Sqrt[-f] - e*Sqrt[g]))])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(
d*Sqrt[-f] - e*Sqrt[g])])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(d
*Sqrt[-f] + e*Sqrt[g])])/(2*f*n)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}x\right)}+\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}x\right)}\right)dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}x}\,dx,x,x^n\right)}{2\sqrt{-f}n} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}x}\,dx,x,x^n\right)}{2\sqrt{-f}n}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} - \frac{(ep)\,\text{Subst}\Bigg(}{}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} - \frac{p\,\text{Subst}\Bigg(\int}{}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} + \frac{p\text{Li}_2\left(\frac{\sqrt{-f}(d+}{d\sqrt{-f}-}\right.}{2fn}$$

Mathematica [F] time = 1.42749, size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))), x]
```

Maple [C] time = 1.067, size = 461, normalized size = 2.1

$$\frac{\ln\left(f\left(x^{n}\right)^{2}+g\right)\ln\left(\left(d+e x^{n}\right)^{p}\right)}{2 n f}-\frac{p\ln\left(d+e x^{n}\right)\ln\left(f\left(x^{n}\right)^{2}+g\right)}{2 n f}+\frac{p\ln\left(d+e x^{n}\right)}{2 n f}\ln\left(\left(e\sqrt{-f g}-f\left(d+e x^{n}\right)+d f\right)\left(e\sqrt{-f g}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x)
```

```
[Out] 1/2/n/f*ln(f*(x^n)^2+g)*ln((d+e*x^n)^p)-1/2/n/f*p*ln(d+e*x^n)*ln(f*(x^n)^2+
g)+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2
)+d*f))+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)
^(1/2)-d*f))+1/2/n/f*p*dilog((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/
2)+d*f))+1/2/n/f*p*dilog((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d
*f))+1/4*I/n/f*ln(f*(x^n)^2+g)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)
^2-1/4*I/n/f*ln(f*(x^n)^2+g)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*c
sgn(I*c)-1/4*I/n/f*ln(f*(x^n)^2+g)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/4*I/n/f*ln(
f*(x^n)^2+g)*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+1/2/n/f*ln(f*(x^n)^2+g)*l
n(c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(e x^{n}+d\right)^{p} c\right)}{\left(f+\frac{g}{x^{2 n}}\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{2 n}\log\left(\left(e x^{n}+d\right)^{p} c\right)}{f x x^{2 n}+g x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="fricas")
```

```
[Out] integral(x^(2*n)*log((e*x^n + d)^p*c)/(f*x*x^(2*n) + g*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n))),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(f + \frac{g}{x^{2n}}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

3.374 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx$

Optimal. Leaf size=419

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}-$$

```
[Out] -(d*e*Sqrt[g]*p*ArcTan[(Sqrt[g]*x^n)/Sqrt[f]])/(2*f^(3/2)*(e^2*f + d^2*g)*n
) - (e^2*p*Log[d + e*x^n])/(2*f*(e^2*f + d^2*g)*n) + Log[c*(d + e*x^n)^p]/(
2*f*n*(f + g*x^(2*n))) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) -
 (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*f^2*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/
(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2*n) + (e^2*p*Log[f + g*x^(2*n)])/(4*f*(e^2
*f + d^2*g)*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt
[g]))])/(2*f^2*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*f^2*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rubi [A] time = 0.565102, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {2475, 266, 44, 2416, 2394, 2315, 2413, 706, 31, 635, 205, 260, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

```
[Out] -(d*e*Sqrt[g]*p*ArcTan[(Sqrt[g]*x^n)/Sqrt[f]])/(2*f^(3/2)*(e^2*f + d^2*g)*n
) - (e^2*p*Log[d + e*x^n])/(2*f*(e^2*f + d^2*g)*n) + Log[c*(d + e*x^n)^p]/(
2*f*n*(f + g*x^(2*n))) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) -
 (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*f^2*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/
(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2*n) + (e^2*p*Log[f + g*x^(2*n)])/(4*f*(e^2
*f + d^2*g)*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt
[g]))])/(2*f^2*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*f^2*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[((f + g*x^r)^(q + 1)*(a
+ b*Log[c*(d + e*x)^n])^p)/(g*r*(q + 1)), x] - Dist[(b*e*n*p)/(g*r*(q + 1))
, Int[((f + g*x^r)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x\left(f+gx^2\right)^2}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x}-\frac{gx\log(c(d+ex)^p)}{f\left(f+gx^2\right)^2}-\frac{gx\log(c(d+ex)^p)}{f^2\left(f+gx^2\right)}\right)dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{f^2n}-\frac{g\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2}\,dx,x,x^n\right)}{f^2n}-\frac{g\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\left(f+gx^2\right)^2}\right.}{fn}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}-\frac{g\,\text{Subst}\left(\int\left(-\frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right.}\right.\right.}{f^2n}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n}+\frac{\sqrt{g}\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}x}\right.}{2f^2n}$$

$$= -\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n}+\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log}{2f^2n}$$

$$= -\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n}-\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n}+\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}$$

$$= -\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n}-\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n}+\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}$$

Mathematica [F] time = 7.43512, size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2), x]
```

Maple [C] time = 1.095, size = 1036, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x)
```

```
[Out] -1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(x^n)+1/4/n*p*e^2/f/(d^2*g+e^2*f)
*ln(f+g*(x^n)^2)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(
I*c)/f/(f+g*(x^n)^2)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*c
sgn(I*c)/f^2*ln(x^n)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*c
sgn(I*c)/f^2*ln(f+g*(x^n)^2)-1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)+(d+e*x^n)*g-
d*g)/(e*(-f*g)^(1/2)-d*g))-1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)-(d+e*x^n)*g+d*
g)/(e*(-f*g)^(1/2)+d*g))-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f/(f+g*(x^n)^2)
+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(f+g*(x^n)^2)-1/2/n*p/f^2*ln(d+e*
x^n)*ln((e*(-f*g)^(1/2)-(d+e*x^n)*g+d*g)/(e*(-f*g)^(1/2)+d*g))+1/n*ln((d+e*
x^n)^p)/f^2*ln(x^n)-1/n*p/f^2*dilog((d+e*x^n)/d)-1/2/n*ln((d+e*x^n)^p)/f^2*
ln(f+g*(x^n)^2)+1/2/n*ln((d+e*x^n)^p)/f/(f+g*(x^n)^2)+1/2/n*ln(c)/f/(f+g*(x
^n)^2)+1/n*ln(c)/f^2*ln(x^n)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x
^n)^p)^2/f/(f+g*(x^n)^2)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f^2*l
n(x^n)-1/2*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f)/n+1/2*I/n*Pi*csgn(I*(d+e*x^n)^
p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(x^n)-1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)
^(1/2)+(d+e*x^n)*g-d*g)/(e*(-f*g)^(1/2)-d*g))+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)
^p)^2*csgn(I*c)/f/(f+g*(x^n)^2)+1/2/n*p/f^2*ln(d+e*x^n)*ln(f+g*(x^n)^2)-1/2
/n*ln(c)/f^2*ln(f+g*(x^n)^2)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f
^2*ln(f+g*(x^n)^2)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f
^2*ln(f+g*(x^n)^2)-1/n*p/f^2*ln(x^n)*ln((d+e*x^n)/d)-1/2/n*p*e/f*g/(d^2*g+e
^2*f)*d/(f*g)^(1/2)*arctan(x^n*g/(f*g)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(gx^{2n} + f\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left((ex^n + d)^p c\right)}{g^2 x x^{4n} + 2 fgx x^{2n} + f^2 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x*x^(4*n) + 2*f*g*x*x^(2*n) + f^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(gx^{2n} + f\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

3.375 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2} dx$

Optimal. Leaf size=204

$$-\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}$$

```
[Out] -((e*p*Log[d + e*x^n])/(f*(e*f - d*g)*n)) + Log[c*(d + e*x^n)^p]/(f*n*(f +
g*x^n)) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) + (e*p*Log[f + g
*x^n])/(f*(e*f - d*g)*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f -
 d*g)])/(f^2*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f - d*g))])/(f^2*n) +
(p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rubi [A] time = 0.268145, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2475, 44, 2416, 2394, 2315, 2395, 36, 31, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```


```
[Out] -((e*p*Log[d + e*x^n])/(f*(e*f - d*g)*n)) + Log[c*(d + e*x^n)^p]/(f*n*(f +
g*x^n)) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) + (e*p*Log[f + g
*x^n])/(f*(e*f - d*g)*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f -
 d*g)])/(f^2*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f - d*g))])/(f^2*n) +
(p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)^2}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x} - \frac{g\log(c(d+ex)^p)}{f(f+gx)^2} - \frac{g\log(c(d+ex)^p)}{f^2(f+gx)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{f^2n} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^n\right)}{f^2n} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p}{(f+gx)^2}\right.}{fn}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{fn\left(f+gx^n\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{f^2n} - \frac{(ep)}{}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{fn\left(f+gx^n\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{f^2n} + \frac{p\text{Li}_2}{}$$

$$= -\frac{ep\log\left(d+ex^n\right)}{f(ef-dg)n} + \frac{\log\left(c\left(d+ex^n\right)^p\right)}{fn\left(f+gx^n\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} + \frac{ep\log\left(f+gx^n\right)}{f(ef-dg)n} -$$

Mathematica [A] time = 0.183618, size = 171, normalized size = 0.84

$$\frac{-p\text{PolyLog}\left(2, \frac{g(d+ex^n)}{dg-ef}\right) + p\text{PolyLog}\left(2, \frac{ex^n}{d}+1\right) + \frac{f\log\left(c\left(d+ex^n\right)^p\right)}{f+gx^n} - \log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right) + \log\left(-\frac{ex^n}{d}\right)\log\left(\right.}{f^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

```
[Out] (-((e*f*p*Log[d + e*x^n])/(e*f - d*g)) + (f*Log[c*(d + e*x^n)^p])/(f + g*x^
n) + Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + (e*f*p*Log[f + g*x^n])/(e*f -
 d*g) - Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Maple [C] time = 1.217, size = 805, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x)
```

```
[Out] -1/n*ln((d+e*x^n)^p)/f^2*ln(f+g*x^n)+1/n*ln((d+e*x^n)^p)/f/(f+g*x^n)+1/n*ln
((d+e*x^n)^p)/f^2*ln(x^n)+1/n*p*e/f/(d*g-e*f)*ln(d+e*x^n)-1/n*p*e/f/(d*g-e*
f)*ln(f+g*x^n)-1/n*p/f^2*dilog((d+e*x^n)/d)-1/n*p/f^2*ln(x^n)*ln((d+e*x^n)/
d)+1/n*p/f^2*dilog((e*(f+g*x^n)+d*g-f*e)/(d*g-e*f))+1/n*p/f^2*ln(f+g*x^n)*l
n((e*(f+g*x^n)+d*g-f*e)/(d*g-e*f))-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(
I*c)/f^2*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f/(f+g*x^
n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f/(f+g*x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^
n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p
)*csgn(I*c*(d+e*x^n)^p)^2/f/(f+g*x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^
```

```
2*ln(f+g*x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(x^n)-1/2*I/n*Pi*csg
n(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f/(f+g*x^n)+1/2*I/n*Pi*csg
n(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e
*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(d
+e*x^n)^p)^2*csgn(I*c)/f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*
(d+e*x^n)^p)*csgn(I*c)/f^2*ln(f+g*x^n)-1/n*ln(c)/f^2*ln(f+g*x^n)+1/n*ln(c)/
f/(f+g*x^n)+1/n*ln(c)/f^2*ln(x^n)
```

Maxima [A] time = 1.43476, size = 315, normalized size = 1.54

$$-enp\left(\frac{\log\left(\frac{ex^n+d}{e}\right)}{ef^2n^2-dfgn^2}-\frac{\log\left(\frac{gx^n+f}{g}\right)}{ef^2n^2-dfgn^2}+\frac{\log\left(x^n\right)\log\left(\frac{ex^n}{d}+1\right)+\mathrm{Li}_2\left(-\frac{ex^n}{d}\right)}{ef^2n^2}-\frac{\log\left(gx^n+f\right)\log\left(-\frac{egx^n+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{eg}{e}\right.}{ef^2n^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="maxima")
```

```
[Out] -e*n*p*(log((e*x^n + d)/e)/(e*f^2*n^2 - d*f*g*n^2) - log((g*x^n + f)/g)/(e*
f^2*n^2 - d*f*g*n^2) + (log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f^2
*n^2) - (log(g*x^n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e*g*
x^n + e*f)/(e*f - d*g)))/(e*f^2*n^2)) + (1/(f*g*n*x^n + f^2*n) - log(g*x^n
+ f)/(f^2*n) + log(x^n)/(f^2*n))*log((e*x^n + d)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)}{g^2xx^{2n}+2fgxx^n+f^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x*x^(2*n) + 2*f*g*x*x^n + f^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(ex^n+d\right)^p c\right)}{\left(gx^n+f\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^n + f)^2*x), x)
```

3.376 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx$

Optimal. Leaf size=156

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}+\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{f^2n\left(fx^n+g\right)}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(fx^n+g\right)}{df-eg}\right)}{f^2n}+\frac{egp\log\left(d+ex^n\right)}{f^2n(df-eg)}-\frac{egp\log\left(fx^n+g\right)}{f^2n(df-eg)}$$

```
[Out] (e*g*p*Log[d + e*x^n])/(f^2*(d*f - e*g)*n) + (g*Log[c*(d + e*x^n)^p])/(f^2*
n*(g + f*x^n)) - (e*g*p*Log[g + f*x^n])/(f^2*(d*f - e*g)*n) + (Log[c*(d + e
*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f^2*n) + (p*PolyLog[2, (f*(d
 + e*x^n))/(d*f - e*g)])/(f^2*n)
```

Rubi [A] time = 0.277577, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2475, 263, 43, 2416, 2395, 36, 31, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}+\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{f^2n\left(fx^n+g\right)}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(fx^n+g\right)}{df-eg}\right)}{f^2n}+\frac{egp\log\left(d+ex^n\right)}{f^2n(df-eg)}-\frac{egp\log\left(fx^n+g\right)}{f^2n(df-eg)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (e*g*p*Log[d + e*x^n])/(f^2*(d*f - e*g)*n) + (g*Log[c*(d + e*x^n)^p])/(f^2*
n*(g + f*x^n)) - (e*g*p*Log[g + f*x^n])/(f^2*(d*f - e*g)*n) + (Log[c*(d + e
*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f^2*n) + (p*PolyLog[2, (f*(d
 + e*x^n))/(d*f - e*g)])/(f^2*n)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)^2 x}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{g\log(c(d+ex)^p)}{f(g+fx)^2}+\frac{\log(c(d+ex)^p)}{f(g+fx)}\right)dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx,x,x^n\right)}{fn} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(g+fx)^2}\,dx,x,x^n\right)}{fn}$$
$$= \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right)}{f^2n} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\,dx,x,x^n\right)}{f^2n} -$$
$$= \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right)}{f^2n} - \frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\,dx,x,d+ex\right.}{f^2n}$$
$$= \frac{egp\log\left(d+ex^n\right)}{f^2(df-eg)n} + \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{f^2n\left(g+fx^n\right)} - \frac{egp\log\left(g+fx^n\right)}{f^2(df-eg)n} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right.}{f^2n}$$

Mathematica [B] time = 1.49929, size = 433, normalized size = 2.78

$$\frac{p\left(fx^n+g\right)\text{PolyLog}\left(2,-\frac{fx^n}{g}\right)+g\log\left(f-fx^{-n}\right)\log\left(c\left(d+ex^n\right)^p\right)-fx^n\log\left(c\left(d+ex^n\right)^p\right)+fx^n\log\left(f-fx^{-n}\right)\log\left(c\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (g*p*Log[f - f/x^n] + f*p*x^n*Log[f - f/x^n] - g*n*p*Log[x]*Log[f - f/x^n]
- f*n*p*x^n*Log[x]*Log[f - f/x^n] - p*Log[e + d/x^n]*(-(f*x^n) + (g + f*x^n
)*Log[f - f/x^n]) - f*x^n*Log[c*(d + e*x^n)^p] + g*Log[f - f/x^n]*Log[c*(d
+ e*x^n)^p] + f*x^n*Log[f - f/x^n]*Log[c*(d + e*x^n)^p] + g*n*p*Log[x]*Log[
1 + (f*x^n)/g] + f*n*p*x^n*Log[x]*Log[1 + (f*x^n)/g] + p*(g + f*x^n)*PolyLo
g[2, -((f*x^n)/g)])/(f^2*n*(g + f*x^n)) - (p*(-((d*f*Log[e + d/x^n])/(d*f -
 e*g)) + (f*x^n*Log[e + d/x^n])/(g + f*x^n) + Log[-(d/(e*x^n))]*Log[e + d/x
^n] + (d*f*Log[f + g/x^n])/(d*f - e*g) - Log[e + d/x^n]*Log[(d*(f + g/x^n))
/(d*f - e*g)] - PolyLog[2, -((g*(e + d/x^n))/(d*f - e*g))] + PolyLog[2, 1 +
 d/(e*x^n)]))/(f^2*n)
```

Maple [C] time = 1.22, size = 589, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x)
```

```
[Out] 1/n*ln((d+e*x^n)^p)/f^2*ln(g+f*x^n)+1/n*ln((d+e*x^n)^p)*g/f^2/(g+f*x^n)-1/n
*p/f^2*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))-1/n*p/f^2*ln(g+f*x^n)*ln(((g+
```

```
f*x^n)*e+d*f-e*g)/(d*f-e*g))-e*g*p*ln(g+f*x^n)/f^2/(d*f-e*g)/n+1/n*p*e/f^2*
g/(d*f-e*g)*ln((g+f*x^n)*e+d*f-e*g)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c
*(d+e*x^n)^p)*csgn(I*c)*g/f^2/(g+f*x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn
(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(g+f*x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*
csgn(I*c*(d+e*x^n)^p)^2*g/f^2/(g+f*x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/
f^2*ln(g+f*x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*g/f^2/(g+f*x^n)+1/2*I/n*
Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(g+f*x^n)+1/2*I/n*Pi*c
sgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f^2*ln(g+f*x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x
^n)^p)^2*csgn(I*c)*g/f^2/(g+f*x^n)+1/n*ln(c)/f^2*ln(g+f*x^n)+1/n*ln(c)*g/f^
2/(g+f*x^n)
```

Maxima [A] time = 1.45836, size = 282, normalized size = 1.81

$$enp\left(\frac{d\log\left(\frac{ex^n+d}{e}\right)}{def^2n^2-e^2fgn^2}-\frac{g\log\left(\frac{fx^n+g}{f}\right)}{df^3n^2-ef^2gn^2}-\frac{\log\left(fx^n+g\right)\log\left(\frac{efx^n+eg}{df-eg}+1\right)+\mathrm{Li}_2\left(-\frac{efx^n+eg}{df-eg}\right)}{ef^2n^2}\right)-\left(\frac{1}{f^2n+\frac{fgn}{x^n}}-\frac{\log\left(f+\right.}{f^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="maxima")
```

```
[Out] e*n*p*(d*log((e*x^n + d)/e)/(d*e*f^2*n^2 - e^2*f*g*n^2) - g*log((f*x^n + g)
/f)/(d*f^3*n^2 - e*f^2*g*n^2) - (log(f*x^n + g)*log((e*f*x^n + e*g)/(d*f -
e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*f - e*g)))/(e*f^2*n^2)) - (1/(f^2*n +
 f*g*n/x^n) - log(f + g/x^n)/(f^2*n) + log(1/(x^n))/(f^2*n))*log((e*x^n + d
)^p*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((ex^n+d)^pc\right)}{f^2x+\frac{2fgxx^n}{x^{2n}}+\frac{g^2x}{x^{2n}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^n/x^(2*n) + g^2*x/x^(2*n))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^n}\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^n)^2*x), x)
```

3.377 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx$

Optimal. Leaf size=377

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(fx^{2n}+g\right)}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+$$

```
[Out] -(d*e*Sqrt[g]*p*ArcTan[(Sqrt[f]*x^n)/Sqrt[g]])/(2*f^(3/2)*(d^2*f + e^2*g)*n
) - (e^2*g*p*Log[d + e*x^n])/(2*f^2*(d^2*f + e^2*g)*n) + (g*Log[c*(d + e*x^
n)^p])/(2*f^2*n*(g + f*x^(2*n))) + (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] -
Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n) + (Log[c*(d + e*x^n)^p]
*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(d*Sqrt[-f] - e*Sqrt[g]))])/(2*f^2*n) +
 (e^2*g*p*Log[g + f*x^(2*n)])/(4*f^2*(d^2*f + e^2*g)*n) + (p*PolyLog[2, (Sq
rt[-f]*(d + e*x^n))/(d*Sqrt[-f] - e*Sqrt[g])])/(2*f^2*n) + (p*PolyLog[2, (S
qrt[-f]*(d + e*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n)
```

Rubi [A] time = 0.608658, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {2475, 263, 266, 43, 2416, 2413, 706, 31, 635, 205, 260, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(fx^{2n}+g\right)}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```


```
[Out] -(d*e*Sqrt[g]*p*ArcTan[(Sqrt[f]*x^n)/Sqrt[g]])/(2*f^(3/2)*(d^2*f + e^2*g)*n
) - (e^2*g*p*Log[d + e*x^n])/(2*f^2*(d^2*f + e^2*g)*n) + (g*Log[c*(d + e*x^
n)^p])/(2*f^2*n*(g + f*x^(2*n))) + (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] -
Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n) + (Log[c*(d + e*x^n)^p]
*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(d*Sqrt[-f] - e*Sqrt[g]))])/(2*f^2*n) +
 (e^2*g*p*Log[g + f*x^(2*n)])/(4*f^2*(d^2*f + e^2*g)*n) + (p*PolyLog[2, (Sq
rt[-f]*(d + e*x^n))/(d*Sqrt[-f] - e*Sqrt[g])])/(2*f^2*n) + (p*PolyLog[2, (S
qrt[-f]*(d + e*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n)
```


Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[((f + g*x^r)^(q + 1)*(a
+ b*Log[c*(d + e*x)^n])^p)/(g*r*(q + 1)), x] - Dist[(b*e*n*p)/(g*r*(q + 1))
, Int[((f + g*x^r)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)^2 x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{gx\log(c(d+ex)^p)}{f\left(g+fx^2\right)^2}+\frac{x\log(c(d+ex)^p)}{f\left(g+fx^2\right)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{g+fx^2}\,dx, x, x^n\right)}{fn} - \frac{g\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\left(g+fx^2\right)^2}\,dx, x, x^n\right)}{fn}$$

$$= \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}x\right)}+\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}x\right)}\right)dx, x, x^n\right)}{fn} - \frac{(egp)\,\text{Sub}}{}$$

$$= \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}x}\,dx, x, x^n\right)}{2(-f)^{3/2}n} + \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}x}\,dx, x, x^n\right)}{2(-f)^{3/2}n} - \frac{(}{}$$

$$= -\frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n} + \frac{\log\left(c\left(d\right.\right.}{}$$

$$= -\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}\left(d^2f+e^2g\right)n} - \frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\right.}{2f^2n}$$

$$= -\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}\left(d^2f+e^2g\right)n} - \frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\right.}{2f^2n}$$

Mathematica [F] time = 1.2449, size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2), x]
```

Maple [C] time = 1.082, size = 810, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x)
```

```
[Out] 1/2/n*ln((d+e*x^n)^p)/f^2*ln(f*(x^n)^2+g)+1/2/n*ln((d+e*x^n)^p)*g/f^2/(f*(x
^n)^2+g)-1/2/n*p/f^2*ln(d+e*x^n)*ln(f*(x^n)^2+g)+1/2/n*p/f^2*ln(d+e*x^n)*ln
((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n*p/f^2*ln(d+e*
x^n)*ln((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))+1/2/n*p/f^2*
dilog((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n*p/f^2*di
log((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))-1/2*e^2*g*p*ln(d
+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/4/n*p*e^2*g/f^2/(d^2*f+e^2*g)*ln(f*(x^n)^2+g)
-1/2/n*p*e*g/f/(d^2*f+e^2*g)*d/(f*g)^(1/2)*arctan(f*x^n/(f*g)^(1/2))-1/4*I/
n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(f*(x^n)^2+g)-1/4*I/n*Pi*csgn(I*c*(d+e*x
^n)^p)^3*g/f^2/(f*(x^n)^2+g)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x
^n)^p)*csgn(I*c)*g/f^2/(f*(x^n)^2+g)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*
c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(f*(x^n)^2+g)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(f*(x^n)^2+g)+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^
p)^2*csgn(I*c)*g/f^2/(f*(x^n)^2+g)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*
(d+e*x^n)^p)^2*g/f^2/(f*(x^n)^2+g)+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(
I*c)/f^2*ln(f*(x^n)^2+g)+1/2/n*ln(c)/f^2*ln(f*(x^n)^2+g)+1/2/n*ln(c)*g/f^2/
(f*(x^n)^2+g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(f + \frac{g}{x^{2n}}\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((ex^n + d)^p c\right)}{f^2 x + \frac{2 fgx x^{2n}}{x^{4n}} + \frac{g^2 x}{x^{4n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^(2*n)/x^(4*n) + g^2*x/x^(4
*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n)))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(f + \frac{g}{x^{2n}}\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

3.378 $\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}\,dx$

Optimal. Leaf size=25

$$-\frac{\text{PolyLog}\left(2,1-c\left(d+ex^n\right)\right)}{cen}$$

```
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rubi [A] time = 0.157073, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2475, 2412, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,1-c\left(d+ex^n\right)\right)}{cen}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
```

```
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2412

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^n\right)}{cen}$$
$$= -\frac{\text{Li}_2(1-c(d+ex^n))}{cen}$$

Mathematica [A] time = 0.0686126, size = 26, normalized size = 1.04

$$-\frac{\text{PolyLog}(2,-cd-cex^n+1)}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
```

```
[Out] -(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Maple [A] time = 0.083, size = 23, normalized size = 0.9

$$-\frac{\text{dilog}(cex^n+cd)}{nce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x)
```

```
[Out] -1/n/c/e*dilog(c*e*x^n+c*d)
```

Maxima [B] time = 1.40197, size = 143, normalized size = 5.72

$$\left(\frac{\log\left(ce+\frac{cd-1}{x^n}\right)}{cen}-\frac{\log\left(\frac{1}{x^n}\right)}{cen}\right)\log\left((ex^n+d)c\right)-\frac{\log(cex^n+cd)\log(cex^n+cd-1)+\text{Li}_2(-cex^n-cd+1)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="maxima")
```

```
[Out] (log(c*e + (c*d - 1)/x^n)/(c*e*n) - log(1/(x^n))/(c*e*n))*log((e*x^n + d)*c
) - (log(c*e*x^n + c*d)*log(c*e*x^n + c*d - 1) + dilog(-c*e*x^n - c*d + 1))
/(c*e*n)
```

Fricas [A] time = 1.69567, size = 49, normalized size = 1.96

$$-\frac{\text{Li}_2\left(-cex^n-cd+1\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="fricas")
```

```
[Out] -dilog(-c*e*x^n - c*d + 1)/(c*e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n))/x/(c*e+(c*d-1)/(x**n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n+d)c\right)}{\left(ce+\frac{cd-1}{x^n}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)*c)/((c*e + (c*d - 1)/x^n)*x), x)
```

3.379 $\int \frac{x^{-1+n}\log(c(d+ex^n))}{-1+cd+cex^n}\,dx$

Optimal. Leaf size=25

$$-\frac{\text{PolyLog}(2,1-c(d+ex^n))}{cen}$$

```
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rubi [A] time = 0.0985, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2475, 2393, 2391}

$$-\frac{\text{PolyLog}(2,1-c(d+ex^n))}{cen}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
```

```
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\log(c(d+ex^n))}{-1+cd+cex^n}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,-1+cd+cex^n\right)}{cen}\\ &= -\frac{\text{Li}_2(1-c(d+ex^n))}{cen}\end{aligned}$$

Mathematica [A] time = 0.0212684, size = 26, normalized size = 1.04

$$-\frac{\text{PolyLog}\left(2,-cd-cex^n+1\right)}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
```

```
[Out] -(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Maple [A] time = 0.388, size = 23, normalized size = 0.9

$$-\frac{\text{dilog}\left(cex^n+cd\right)}{nce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*ln(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x)
```

```
[Out] -1/n/c/e*dilog(c*e*x^n+c*d)
```

Maxima [B] time = 1.0575, size = 147, normalized size = 5.88

$$\frac{\log\left(cex^n+cd-1\right)\log\left((ex^n+d)c\right)}{cen}-\frac{\log\left(cex^n+cd-1\right)\log\left(ex^n+d\right)}{cen}+\frac{\log\left(-cex^n-cd+1\right)\log\left(ex^n+d\right)+\text{Li}_2\left(cex^n+\right.}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="maxima")
```

```
[Out] log(c*e*x^n + c*d - 1)*log((e*x^n + d)*c)/(c*e*n) - log(c*e*x^n + c*d - 1)*
log(e*x^n + d)/(c*e*n) + (log(-c*e*x^n - c*d + 1)*log(e*x^n + d) + dilog(c*
e*x^n + c*d))/(c*e*n)
```

Fricas [A] time = 1.70788, size = 49, normalized size = 1.96

$$-\frac{\text{Li}_2\left(-cex^n-cd+1\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="fricas")
```

```
[Out] -dilog(-c*e*x^n - c*d + 1)/(c*e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*ln(c*(d+e*x**n))/(-1+c*d+c*e*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-1} \log \left((ex^n + d)c\right)}{cex^n + cd - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)*log((e*x^n + d)*c)/(c*e*x^n + c*d - 1), x)
```

3.380 $\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx$

Optimal. Leaf size=26

$$\frac{\text{PolyLog}\left(2,1-c\left(d+ex^{-n}\right)\right)}{cen}$$

```
[Out] PolyLog[2, 1 - c*(d + e/x^n)]/(c*e*n)
```

Rubi [A] time = 0.155378, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2475, 2412, 2393, 2391}

$$\frac{\text{PolyLog}\left(2,1-c\left(d+ex^{-n}\right)\right)}{cen}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
```

```
[Out] PolyLog[2, 1 - c*(d + e/x^n)]/(c*e*n)
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2412

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx = -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^{-n}\right)}{cen}$$
$$= \frac{\text{Li}_2(1-cd-cex^{-n})}{cen}$$

Mathematica [A] time = 0.068676, size = 34, normalized size = 1.31

$$\frac{\text{PolyLog}(2, -x^{-n}(cdx^n+ce-x^n))}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
```

```
[Out] PolyLog[2, -((c*e - x^n + c*d*x^n)/x^n)]/(c*e*n)
```

Maple [A] time = 0.088, size = 24, normalized size = 0.9

$$\frac{1}{cen}\text{dilog}\left(cd+\frac{ce}{x^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x)
```

```
[Out] 1/n/c/e*dilog(c*d+c*e/(x^n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$n\int \frac{\log(x)}{cdxx^n+cex}\,dx + \frac{\log(dx^n+e)\log(x)+\log(c)\log(x)-\log(x)\log(x^n)}{ce} - \frac{\log(c)\log\left(\frac{ce+(cd-1)x^n}{cd-1}\right)}{cen} - \frac{\log(dx^n+e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="maxima")
```

```
[Out] n*integrate(log(x)/(c*d*x*x^n + c*e*x), x) + (log(d*x^n + e)*log(x) + log(c
)*log(x) - log(x)*log(x^n))/(c*e) - log(c)*log((c*e + (c*d - 1)*x^n)/(c*d -
 1))/(c*e*n) - (log(d*x^n + e)*log((c*d*e + (c*d^2 - d)*x^n - e)/e + 1) + d
ilog(-(c*d*e + (c*d^2 - d)*x^n - e)/e))/(c*e*n) + (log(x^n)*log((c*d - 1)*x
^n/(c*e) + 1) + dilog(-(c*d - 1)*x^n/(c*e)))/(c*e*n)
```

Fricas [A] time = 1.75127, size = 55, normalized size = 2.12

$$\frac{\text{Li}_2\left(-\frac{cdx^n+ce}{x^n}+1\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="fricas")
```

```
[Out] dilog(-(c*d*x^n + c*e)/x^n + 1)/(c*e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(x**n)))/x/(c*e-(-c*d+1)*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+\frac{e}{x^n}\right)\right)}{(ce+(cd-1)x^n)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x^n))/((c*e + (c*d - 1)*x^n)*x), x)
```

3.381 $\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=607

$$\frac{3d^2g^2 2^{-q-1}(d+ex^n)^2 \left(c(d+ex^n)^p\right)^{-2/p} \log^q\left(c(d+ex^n)^p\right)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q} \text{Gamma}\left(q+1,-\frac{2\log(c(d+ex^n)^p)}{p}\right)}{e^4 n} - \frac{d^3g^2 (}{}$$

```
[Out] (4^(-1 - q)*g^2*(d + e*x^n)^4*Gamma[1 + q, (-4*Log[c*(d + e*x^n)^p])/p]*Log
[c*(d + e*x^n)^p]^q)/(e^4*n*(c*(d + e*x^n)^p)^(4/p)*(-(Log[c*(d + e*x^n)^p]
/p))^q) - (d*g^2*(d + e*x^n)^3*Gamma[1 + q, (-3*Log[c*(d + e*x^n)^p])/p]*Lo
g[c*(d + e*x^n)^p]^q)/(3^q*e^4*n*(c*(d + e*x^n)^p)^(3/p)*(-(Log[c*(d + e*x^
n)^p]/p))^q) + (f*g*(d + e*x^n)^2*Gamma[1 + q, (-2*Log[c*(d + e*x^n)^p])/p]
*Log[c*(d + e*x^n)^p]^q)/(2^q*e^2*n*(c*(d + e*x^n)^p)^(2/p)*(-(Log[c*(d + e
*x^n)^p]/p))^q) + (3*2^(-1 - q)*d^2*g^2*(d + e*x^n)^2*Gamma[1 + q, (-2*Log[
c*(d + e*x^n)^p])/p]*Log[c*(d + e*x^n)^p]^q)/(e^4*n*(c*(d + e*x^n)^p)^(2/p)
*(-(Log[c*(d + e*x^n)^p]/p))^q) - (2*d*f*g*(d + e*x^n)*Gamma[1 + q, -(Log[c
*(d + e*x^n)^p]/p)]*Log[c*(d + e*x^n)^p]^q)/(e^2*n*(c*(d + e*x^n)^p)^p^(-1)
*(-(Log[c*(d + e*x^n)^p]/p))^q) - (d^3*g^2*(d + e*x^n)*Gamma[1 + q, -(Log[c
*(d + e*x^n)^p]/p)]*Log[c*(d + e*x^n)^p]^q)/(e^4*n*(c*(d + e*x^n)^p)^p^(-1)
*(-(Log[c*(d + e*x^n)^p]/p))^q) + f^2*Unintegrable[Log[c*(d + e*x^n)^p]^q/x
, x]
```

Rubi [A] time = 0.0845421, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Mathematica [A] time = 0.416753, size = 0, normalized size = 0.

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A] time = 29.864, size = 0, normalized size = 0.

$$\int \frac{\left(f+g x^{2 n}\right)^{2} \left(\ln \left(c \left(d+e x^{n}\right)^{p}\right)\right)^{q}}{x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

```
[Out] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(g^{2} x^{4 n} + 2 f g x^{2 n} + f^{2}\right) \log \left(\left(e x^{n} + d\right)^{p} c\right)^{q}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^(4*n) + 2*f*g*x^(2*n) + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g x^{2 n} + f\right)^{2} \log \left(\left(e x^{n} + d\right)^{p} c\right)^{q}}{x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

3.382 $\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=306

$$\frac{g^2 2^{-q-1} (d+ex^n)^2 \left(c (d+ex^n)^p\right)^{-2/p} \log^q\left(c (d+ex^n)^p\right) \left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q} \text{Gamma}\left(q+1,-\frac{2\log(c(d+ex^n)^p)}{p}\right)}{e^2 n} - \frac{dg^2 (d+ex^n)}{}$$

```
[Out] (2^(-1 - q)*g^2*(d + e*x^n)^2*Gamma[1 + q, (-2*Log[c*(d + e*x^n)^p])/p]*Log
[c*(d + e*x^n)^p]^q)/(e^2*n*(c*(d + e*x^n)^p)^(2/p)*(-(Log[c*(d + e*x^n)^p]
/p))^q) + (2*f*g*(d + e*x^n)*Gamma[1 + q, -(Log[c*(d + e*x^n)^p]/p)]*Log[c*
(d + e*x^n)^p]^q)/(e*n*(c*(d + e*x^n)^p)^p^(-1)*(-(Log[c*(d + e*x^n)^p]/p))
^q) - (d*g^2*(d + e*x^n)*Gamma[1 + q, -(Log[c*(d + e*x^n)^p]/p)]*Log[c*(d +
 e*x^n)^p]^q)/(e^2*n*(c*(d + e*x^n)^p)^p^(-1)*(-(Log[c*(d + e*x^n)^p]/p))^q
) + f^2*Unintegrable[Log[c*(d + e*x^n)^p]^q/x, x]
```

Rubi [A] time = 0.0791144, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(f+gx^n)^2 \log^q\left(c (d+ex^n)^p\right)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^n)^2 \log^q\left(c (d+ex^n)^p\right)}{x} \, dx = \int \frac{(f+gx^n)^2 \log^q\left(c (d+ex^n)^p\right)}{x} \, dx$$

Mathematica [A] time = 0.297022, size = 0, normalized size = 0.

$$\int \frac{(f+gx^n)^2 \log^q\left(c (d+ex^n)^p\right)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A] time = 29.012, size = 0, normalized size = 0.

$$\int \frac{(f+gx^n)^2 \left(\ln\left(c (d+ex^n)^p\right)\right)^q}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

```
[Out] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(g^{2}x^{2n}+2\,fgx^{n}+f^{2}\right)\log\left(\left(ex^{n}+d\right)^{p}c\right)^{q}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^(2*n) + 2*f*g*x^n + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx^{n}+f\right)^{2}\log\left(\left(ex^{n}+d\right)^{p}c\right)^{q}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

3.383 $\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x},x\right)$$

```
[Out] Unintegrable[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi [A] time = 0.107068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx = \int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Mathematica [A] time = 0.435311, size = 0, normalized size = 0.

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A] time = 28.329, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^q}{x}\left(f+\frac{g}{x^n}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

```
[Out] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(f^2x^{2n}+2\,fgx^n+g^2\right)\log\left(\left(ex^n+d\right)^pc\right)^q}{xx^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((f^2*x^(2*n) + 2*f*g*x^n + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(2*n))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(f+\frac{g}{x^n}\right)^2\log\left(\left(ex^n+d\right)^pc\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)^q/x, x)
```

3.384 $\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c(d+ex^n)^p\right)}{x} dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x},x\right)$$

```
[Out] Unintegrable[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi [A] time = 0.109634, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx = \int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Mathematica [A] time = 0.368483, size = 0, normalized size = 0.

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A] time = 27.697, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^q}{x}\left(f+\frac{g}{x^{2n}}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

```
[Out] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(f^{2}x^{4n}+2\,fgx^{2n}+g^{2}\right)\log\left(\left(ex^{n}+d\right)^{p}c\right)^{q}}{xx^{4n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((f^2*x^(4*n) + 2*f*g*x^(2*n) + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(4
*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f+\frac{g}{x^{2n}}\right)^{2}\log\left(\left(ex^{n}+d\right)^{p}c\right)^{q}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)^q/x, x)
```

3.385 $\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)} dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Rubi [A] time = 0.0905777, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx = \int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx$$

Mathematica [A] time = 2.41722, size = 0, normalized size = 0.

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Maple [A] time = 15.357, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^q}{x\left(f+gx^{2n}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((ex^n+d)^p c\right)^q}{gxx^{2n}+fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^q/(g*x*x^(2*n) + f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n+d)^p c\right)^q}{\left(gx^{2n}+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((g*x^(2*n) + f)*x), x)
```

3.386 $\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)} dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Rubi [A] time = 0.0861844, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)} dx = \int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)} dx$$

Mathematica [A] time = 1.75876, size = 0, normalized size = 0.

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Maple [A] time = 19.992, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^q}{x\left(f+gx^n\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left((ex^n+d)^p c\right)^q}{gxx^n+fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^q/(g*x*x^n + f*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)^q}{x\left(f+gx^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**n),x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**q/(x*(f + g*x**n)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n+d)^p c\right)^q}{\left(gx^n+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((g*x^n + f)*x), x)
```

3.387 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})} dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Rubi [A] time = 0.10548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)} dx = \int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)} dx$$

Mathematica [A] time = 1.80261, size = 0, normalized size = 0.

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Maple [A] time = 21.518, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^q}{x}\left(f+\frac{g}{x^n}\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^n \log\left((ex^n + d)^p c\right)^q}{fxx^n + gx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="fricas")
```

```
[Out] integral(x^n*log((e*x^n + d)^p*c)^q/(f*x*x^n + g*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((ex^n + d)^p c\right)^q}{\left(f + \frac{g}{x^n}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((f + g/x^n)*x), x)
```

3.388 $\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)},x\right)$$

```
[Out] Unintegrable[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Rubi [A] time = 0.1102, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx=\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Mathematica [A] time = 0.237651, size = 0, normalized size = 0.

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Maple [A] time = 18.854, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(c\left(d+ex^n\right)^p\right)\right)^q}{x}\left(f+\frac{g}{x^{2n}}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{2n}\log\left(\left(ex^n+d\right)^p c\right)^q}{fxx^{2n}+gx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="fricas")
```

```
[Out] integral(x^(2*n)*log((e*x^n + d)^p*c)^q/(f*x*x^(2*n) + g*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**(2*n))),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)^q}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((f + g/x^(2*n))*x), x)
```

3.389 $\int \frac{\log(x)\log(d+ex^m)}{x}\,dx$

Optimal. Leaf size=69

$$\frac{\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(x)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(\frac{ex^m}{d}+1\right)$$

```
[Out] (Log[x]^2*Log[d + e*x^m])/2 - (Log[x]^2*Log[1 + (e*x^m)/d])/2 - (Log[x]*Pol
yLog[2, -((e*x^m)/d)])/m + PolyLog[3, -((e*x^m)/d)]/m^2
```

Rubi [A] time = 0.124492, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2375, 2337, 2374, 6589}

$$\frac{\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(x)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(\frac{ex^m}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*Log[d + e*x^m])/x,x]
```

```
[Out] (Log[x]^2*Log[d + e*x^m])/2 - (Log[x]^2*Log[1 + (e*x^m)/d])/2 - (Log[x]*Pol
yLog[2, -((e*x^m)/d)])/m + PolyLog[3, -((e*x^m)/d)]/m^2
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}(em)\int \frac{x^{-1+m}\log^2(x)}{d+ex^m}\,dx$$
$$= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) + \int \frac{\log(x)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx$$
$$= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{\int \frac{\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{m}$$
$$= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}$$

Mathematica [A] time = 0.0579992, size = 75, normalized size = 1.09

$$\frac{\mathrm{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2} + \frac{\log(x)\mathrm{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} - \frac{1}{6}\log^2(x)\left(3\log\left(\frac{dx^{-m}}{e}+1\right) - 3\log(d+ex^m) + m\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*Log[d + e*x^m])/x,x]
```

```
[Out] -(Log[x]^2*(m*Log[x] + 3*Log[1 + d/(e*x^m)] - 3*Log[d + e*x^m]))/6 + (Log[x
]*PolyLog[2, -(d/(e*x^m))])/m + PolyLog[3, -(d/(e*x^m))]/m^2
```

Maple [A] time = 2.256, size = 66, normalized size = 1.

$$\frac{(\ln(x))^2\ln(d+ex^m)}{2} - \frac{(\ln(x))^2}{2}\ln\left(1+\frac{ex^m}{d}\right) - \frac{\ln(x)}{m}\mathrm{polylog}\left(2,-\frac{ex^m}{d}\right) + \frac{1}{m^2}\mathrm{polylog}\left(3,-\frac{ex^m}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln(d+e*x^m)/x,x)
```

```
[Out] 1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d)
/m+polylog(3,-e*x^m/d)/m^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}m\log(x)^3 + dm\int \frac{\log(x)^2}{2(exx^m+dx)}\,dx + \frac{1}{2}\log(ex^m+d)\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="maxima")
```

```
[Out] -1/6*m*log(x)^3 + d*m*integrate(1/2*log(x)^2/(e*x*x^m + d*x), x) + 1/2*log(
e*x^m + d)*log(x)^2
```

Fricas [C] time = 1.72426, size = 185, normalized size = 2.68

$$\frac{m^2 \log(ex^m + d)\log(x)^2 - m^2\log(x)^2\log\left(\frac{ex^m+d}{d}\right) - 2\,m\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)\log(x) + 2\,\mathrm{polylog}\left(3, -\frac{ex^m}{d}\right)}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(m^2*log(e*x^m + d)*log(x)^2 - m^2*log(x)^2*log((e*x^m + d)/d) - 2*m*di
log(-(e*x^m + d)/d + 1)*log(x) + 2*polylog(3, -e*x^m/d))/m^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln(d+e*x**m)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ex^m + d)\log(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^m + d)*log(x)/x, x)
```

3.390 $\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx$

Optimal. Leaf size=8

$$\text{PolyLog}\left(2,-\frac{a}{x}\right)$$

```
[Out] PolyLog[2, -(a/x)]
```

Rubi [A] time = 0.0095414, antiderivative size = 12, normalized size of antiderivative = 1.5, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2447}

$$\text{PolyLog}\left(2,1-\frac{a+x}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + x)/x]/x,x]
```

```
[Out] PolyLog[2, 1 - (a + x)/x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx = \text{Li}_2\left(1-\frac{a+x}{x}\right)$$

Mathematica [B] time = 0.0028248, size = 34, normalized size = 4.25

$$-\text{PolyLog}\left(2,-\frac{-a-x}{x}\right)-\log\left(-\frac{a}{x}\right)\log\left(\frac{a+x}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + x)/x]/x,x]
```

```
[Out] -(Log[-(a/x)]*Log[(a + x)/x]) - PolyLog[2, -((-a - x)/x)]
```

Maple [A] time = 0.081, size = 9, normalized size = 1.1

$$\text{dilog}\left(1+\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+x)/x)/x,x)
```

```
[Out] dilog(1+a/x)
```

Maxima [B] time = 1.03128, size = 80, normalized size = 10.

$$-(\log(a+x)-\log(x))\log(x)+\log(a+x)\log(x)-\frac{1}{2}\log(x)^2+\log(x)\log\left(\frac{a+x}{x}\right)-\log(x)\log\left(\frac{x}{a}+1\right)-\mathrm{Li}_2\left(-\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(a + x) - log(x))*log(x) + log(a + x)*log(x) - 1/2*log(x)^2 + log(x)*l
og((a + x)/x) - log(x)*log(x/a + 1) - dilog(-x/a)
```

Fricas [A] time = 1.43906, size = 31, normalized size = 3.88

$$\mathrm{Li}_2\left(-\frac{a+x}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x)/x)/x,x, algorithm="fricas")
```

```
[Out] dilog(-(a + x)/x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{a}{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+x)/x)/x,x)
```

```
[Out] Integral(log(a/x + 1)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x)/x)/x,x, algorithm="giac")
```

```
[Out] integrate(log((a + x)/x)/x, x)
```

3.391 $\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

```
[Out] PolyLog[2, -(a/x^2)]/2
```

Rubi [A] time = 0.0184693, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2461, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + x^2)/x^2]/x,x]

[Out] PolyLog[2, -(a/x^2)]/2
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx &= \int \frac{\log\left(1+\frac{a}{x^2}\right)}{x}\,dx\\ &= \frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.002811, size = 12, normalized size = 1.

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + x^2)/x^2]/x,x]

[Out] PolyLog[2, -(a/x^2)]/2
```

Maple [B] time = 0.139, size = 76, normalized size = 6.3

$$-\ln\left(x^{-1}\right)\ln\left(1+\frac{a}{x^{2}}\right)+\ln\left(x^{-1}\right)\ln\left(1-\frac{1}{x}\sqrt{-a}\right)+\ln\left(x^{-1}\right)\ln\left(1+\frac{1}{x}\sqrt{-a}\right)+\operatorname{dilog}\left(1+\frac{1}{x}\sqrt{-a}\right)+\operatorname{dilog}\left(1-\frac{1}{x}\sqrt{-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((x^2+a)/x^2)/x,x)
```

```
[Out] -ln(1/x)*ln(1+a/x^2)+ln(1/x)*ln(1-1/x*(-a)^(1/2))+ln(1/x)*ln(1+1/x*(-a)^(1/
2))+dilog(1+1/x*(-a)^(1/2))+dilog(1-1/x*(-a)^(1/2))
```

Maxima [B] time = 1.06037, size = 93, normalized size = 7.75

$$-\left(\log\left(x^{2}+a\right)-2\,\log\left(x\right)\right)\log\left(x\right)+\log\left(x^{2}+a\right)\log\left(x\right)-\log\left(x\right)^{2}-\log\left(x\right)\log\left(\frac{x^{2}}{a}+1\right)+\log\left(x\right)\log\left(\frac{x^{2}+a}{x^{2}}\right)-\frac{1}{2}\,\mathrm{Li}_{2}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="maxima")
```

```
[Out] -(log(x^2 + a) - 2*log(x))*log(x) + log(x^2 + a)*log(x) - log(x)^2 - log(x)
*log(x^2/a + 1) + log(x)*log((x^2 + a)/x^2) - 1/2*dilog(-x^2/a)
```

Fricas [A] time = 1.59569, size = 42, normalized size = 3.5

$$\frac{1}{2}\,\mathrm{Li}_{2}\left(-\frac{x^{2}+a}{x^{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*dilog(-(x^2 + a)/x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\frac{a}{x^{2}}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((x**2+a)/x**2)/x,x)
```

```
[Out] Integral(log(a/x**2 + 1)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{x^2+a}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(log((x^2 + a)/x^2)/x, x)
```

3.392 $\int \frac{\log(x^{-n}(a+x^n))}{x} \, dx$

Optimal. Leaf size=14

$$\frac{\text{PolyLog}(2,-ax^{-n})}{n}$$

```
[Out] PolyLog[2, -(a/x^n)]/n
```

Rubi [A] time = 0.0188247, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2461, 2391}

$$\frac{\text{PolyLog}(2,-ax^{-n})}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + x^n)/x^n]/x,x]

[Out] PolyLog[2, -(a/x^n)]/n
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(x^{-n}\left(a+x^n\right)\right)}{x} \, dx &= \int \frac{\log\left(1+ax^{-n}\right)}{x} \, dx\\ &= \frac{\text{Li}_2\left(-ax^{-n}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0041753, size = 14, normalized size = 1.

$$\frac{\text{PolyLog}(2,-ax^{-n})}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + x^n)/x^n]/x,x]

[Out] PolyLog[2, -(a/x^n)]/n
```

Maple [A] time = 0.086, size = 15, normalized size = 1.1

$$\frac{1}{n}\mathrm{dilog}\left(1+\frac{a}{x^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+x^n)/(x^n))/x,x)
```

```
[Out] 1/n*dilog(1+a/(x^n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$an\int\frac{\log(x)}{ax+xx^n}\,dx+\log(a+x^n)\log(x)-\log(x)\log(x^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(log(x)/(a*x + x*x^n), x) + log(a + x^n)*log(x) - log(x)*log(x
^n)
```

Fricas [B] time = 1.64604, size = 151, normalized size = 10.79

$$\frac{n^2\log(x)^2-2\,n\log(x)\log\left(\frac{a+x^n}{a}\right)+2\,n\log(x)\log\left(\frac{a+x^n}{x^n}\right)-2\,\mathrm{Li}_2\left(-\frac{a+x^n}{a}+1\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(n^2*log(x)^2 - 2*n*log(x)*log((a + x^n)/a) + 2*n*log(x)*log((a + x^n)/
x^n) - 2*dilog(-(a + x^n)/a + 1))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log(ax^{-n}+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+x**n)/(x**n))/x,x)
```

```
[Out] Integral(log(a*x**(-n) + 1)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((a + x^n)/x^n)/x, x)
```

3.393 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx$

Optimal. Leaf size=35

$$-\text{PolyLog}\left(2,\frac{a}{bx}+1\right)-\log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

```
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, 1 + a/(b*x)]
```

Rubi [A] time = 0.055865, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2461, 2454, 2394, 2315}

$$-\text{PolyLog}\left(2,\frac{a}{bx}+1\right)-\log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x)/x]/x,x]
```

```
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, 1 + a/(b*x)]
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = \int \frac{\log\left(b+\frac{a}{x}\right)}{x}\,dx$$
$$= -\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x}\right)$$
$$= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) + a\,\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x}\right)$$
$$= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) - \text{Li}_2\left(1+\frac{a}{bx}\right)$$

Mathematica [A] time = 0.0037919, size = 36, normalized size = 1.03

$$-\text{PolyLog}\left(2, \frac{\frac{a}{x}+b}{b}\right) - \log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x)/x]/x,x]
```

```
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, (b + a/x)/b]
```

Maple [A] time = 0.084, size = 34, normalized size = 1.

$$-\text{dilog}\left(-\frac{a}{bx}\right) - \ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x+a)/x)/x,x)
```

```
[Out] -dilog(-a/b/x)-ln(b+a/x)*ln(-a/b/x)
```

Maxima [A] time = 1.05408, size = 90, normalized size = 2.57

$$-(\log(bx+a) - \log(x))\log(x) + \log(bx+a)\log(x) - \log\left(\frac{bx}{a}+1\right)\log(x) - \frac{1}{2}\log(x)^2 + \log(x)\log\left(\frac{bx+a}{x}\right) - \text{Li}_2\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x + a) - log(x))*log(x) + log(b*x + a)*log(x) - log(b*x/a + 1)*log(
x) - 1/2*log(x)^2 + log(x)*log((b*x + a)/x) - dilog(-b*x/a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bx+a}{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)/x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{a}{x}+b\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x+a)/x)/x,x)
```

```
[Out] Integral(log(a/x + b)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx+a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)/x)/x, x)
```

3.394 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{a}{bx^2}+1\right)-\frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)$$

```
[Out] -(Log[b + a/x^2]*Log[-(a/(b*x^2))])/2 - PolyLog[2, 1 + a/(b*x^2)]/2
```

Rubi [A] time = 0.0470272, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2461, 2454, 2394, 2315}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{a}{bx^2}+1\right)-\frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^2)/x^2]/x,x]
```

```
[Out] -(Log[b + a/x^2]*Log[-(a/(b*x^2))])/2 - PolyLog[2, 1 + a/(b*x^2)]/2
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(b+\frac{a}{x^2}\right)}{x}\,dx$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right) - \frac{1}{2}\text{Li}_2\left(1+\frac{a}{bx^2}\right)$$

Mathematica [A] time = 0.0040833, size = 40, normalized size = 1.03

$$-\frac{1}{2}\text{PolyLog}\left(2, \frac{\frac{a}{x^2}+b}{b}\right) - \frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^2)/x^2]/x,x]
```

```
[Out] -(Log[b + a/x^2]*Log[-(a/(b*x^2))])/2 - PolyLog[2, (b + a/x^2)/b]/2
```

Maple [B] time = 0.093, size = 108, normalized size = 2.8

$$-\ln\left(x^{-1}\right)\ln\left(b+\frac{a}{x^2}\right) + \ln\left(x^{-1}\right)\ln\left(\left(-\frac{a}{x}+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\right) + \ln\left(x^{-1}\right)\ln\left(\left(\frac{a}{x}+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\right) + \text{dilog}\left(\left(-\frac{a}{x}+\sqrt{-ab}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^2+a)/x^2)/x,x)
```

```
[Out] -ln(1/x)*ln(b+a/x^2)+ln(1/x)*ln((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln(1/x)*l
n((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+
dilog((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

Maxima [B] time = 1.02814, size = 104, normalized size = 2.67

$$-\left(\log\left(bx^2+a\right) - 2\log(x)\right)\log(x) + \log\left(bx^2+a\right)\log(x) - \log\left(\frac{bx^2}{a}+1\right)\log(x) - \log(x)^2 + \log(x)\log\left(\frac{bx^2+a}{x^2}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x^2 + a) - 2*log(x))*log(x) + log(b*x^2 + a)*log(x) - log(b*x^2/a +
 1)*log(x) - log(x)^2 + log(x)*log((b*x^2 + a)/x^2) - 1/2*dilog(-b*x^2/a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)/x^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{a}{x^2} + b\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x**2+a)/x**2)/x,x)
```

```
[Out] Integral(log(a/x**2 + b)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)/x^2)/x, x)
```

3.395 $\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{\text{PolyLog}\left(2,\frac{ax^{-n}}{b}+1\right)}{n}-\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(ax^{-n}+b\right)}{n}$$

```
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n])/n) - PolyLog[2, 1 + a/(b*x^n)]/n
```

Rubi [A] time = 0.0503694, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2461, 2454, 2394, 2315}

$$-\frac{\text{PolyLog}\left(2,\frac{ax^{-n}}{b}+1\right)}{n}-\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(ax^{-n}+b\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^n)/x^n]/x,x]
```

```
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n])/n) - PolyLog[2, 1 + a/(b*x^n)]/n
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = \int \frac{\log(b+ax^{-n})}{x}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log(b+ax^{-n})}{n} + \frac{a\,\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log(b+ax^{-n})}{n} - \frac{\text{Li}_2\left(1+\frac{ax^{-n}}{b}\right)}{n}$$

Mathematica [A] time = 0.0171986, size = 44, normalized size = 0.94

$$-\frac{\text{PolyLog}\left(2, \frac{ax^{-n}+b}{b}\right) + \log\left(-\frac{ax^{-n}}{b}\right)\log(ax^{-n}+b)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^n)/x^n]/x,x]
```

```
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n] + PolyLog[2, (b + a/x^n)/b])/n)
```

Maple [A] time = 0.085, size = 46, normalized size = 1.

$$-\frac{1}{n}\ln\left(-\frac{a}{bx^n}\right)\ln\left(b+\frac{a}{x^n}\right) - \frac{1}{n}\text{dilog}\left(-\frac{a}{bx^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+b*x^n)/(x^n))/x,x)
```

```
[Out] -ln(-a/b/(x^n))*ln(b+a/(x^n))/n-1/n*dilog(-a/b/(x^n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$an\int \frac{\log(x)}{bxx^n + ax}\,dx + \log(bx^n + a)\log(x) - \log(x)\log(x^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(log(x)/(b*x*x^n + a*x), x) + log(b*x^n + a)*log(x) - log(x)*l
og(x^n)
```

Fricas [A] time = 1.69493, size = 159, normalized size = 3.38

$$\frac{n^2\log(x)^2 - 2\,n\log(x)\log\left(\frac{bx^n+a}{a}\right) + 2\,n\log(x)\log\left(\frac{bx^n+a}{x^n}\right) - 2\,\text{Li}_2\left(-\frac{bx^n+a}{a}+1\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(n^2*log(x)^2 - 2*n*log(x)*log((b*x^n + a)/a) + 2*n*log(x)*log((b*x^n +
 a)/x^n) - 2*dilog(-(b*x^n + a)/a + 1))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(ax^{-n}+b\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+b*x**n)/(x**n))/x,x)
```

```
[Out] Integral(log(a*x**(-n) + b)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^n + a)/x^n)/x, x)
```

3.396 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx$

Optimal. Leaf size=105

$$-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

```
[Out] (Log[b + a/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - PolyLog[2, (b*(c + d*x))/(b*c
- a*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rubi [A] time = 0.167946, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2465, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] (Log[b + a/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - PolyLog[2, (b*(c + d*x))/(b*c
- a*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx &= \int \frac{\log\left(b+\frac{a}{x}\right)}{c+dx}\,dx\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{a\int \frac{\log(c+dx)}{\left(b+\frac{a}{x}\right)x^2}\,dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{a\int \left(\frac{\log(c+dx)}{ax} - \frac{b\log(c+dx)}{a(a+bx)}\right)dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\int \frac{\log(c+dx)}{x}\,dx}{d} - \frac{b\int \frac{\log(c+dx)}{a+bx}\,dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx + \\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} + \frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{d} + \frac{\text{Su}}{}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \frac{\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{d} + \frac{\text{Li}}{}
\end{aligned}$$

Mathematica [A] time = 0.0286185, size = 80, normalized size = 0.76

$$\frac{-\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\text{PolyLog}\left(2,\frac{dx}{c}+1\right)+\log(c+dx)\left(-\log\left(\frac{d(a+bx)}{ad-bc}\right)+\log\left(\frac{a}{x}+b\right)+\log\left(-\frac{dx}{c}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] ((Log[b + a/x] + Log[-((d*x)/c)] - Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c
 + d*x] - PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + PolyLog[2, 1 + (d*x)/c])/
```

d

Maple [A] time = 0.411, size = 114, normalized size = 1.1

$$-\frac{1}{d}\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)-\frac{1}{d}\mathrm{dilog}\left(-\frac{a}{bx}\right)+\frac{1}{d}\mathrm{dilog}\left(\frac{1}{ad-bc}\left(c\left(b+\frac{a}{x}\right)+ad-bc\right)\right)+\frac{1}{d}\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{1}{ad-bc}\left(c\left(b+\frac{a}{x}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x+a)/x)/(d*x+c),x)
```

```
[Out] -1/d*ln(b+a/x)*ln(-a/b/x)-1/d*dilog(-a/b/x)+1/d*dilog((c*(b+a/x)+a*d-b*c)/(
a*d-b*c))+1/d*ln(b+a/x)*ln((c*(b+a/x)+a*d-b*c)/(a*d-b*c))
```

Maxima [A] time = 1.0664, size = 167, normalized size = 1.59

$$-\frac{(\log(bx+a)-\log(x))\log(dx+c)}{d}+\frac{\log(dx+c)\log\left(\frac{bx+a}{x}\right)}{d}-\frac{\log\left(\frac{dx}{c}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d}+\frac{\log(bx+a)\log\left(\frac{bdx}{bc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(log(b*x + a) - log(x))*log(d*x + c)/d + log(d*x + c)*log((b*x + a)/x)/d -
 (log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(b*x + a)*log((b*d*x + a*d
)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\frac{bx+a}{x}\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)/x)/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\frac{a}{x}+b\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x+a)/x)/(d*x+c),x)
```

```
[Out] Integral(log(a/x + b)/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)/x)/(d*x + c), x)
```

3.397 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx$

Optimal. Leaf size=227

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}+\frac{2\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log}{d}$$

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d -
 PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)]/d - PolyLog[2, (S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]/d + (2*PolyLog[2, 1 + (d*x)/c])
/d
```

Rubi [A] time = 0.378362, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2465, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}+\frac{2\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```


```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d -
 PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)]/d - PolyLog[2, (S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]/d + (2*PolyLog[2, 1 + (d*x)/c])
/d
```


Rule 2465

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(b+\frac{a}{x^2}\right)}{c+dx}\,dx$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{(2a)\int \frac{\log(c+dx)}{\left(b+\frac{a}{x^2}\right)x^3}\,dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{(2a)\int \left(\frac{\log(c+dx)}{ax} - \frac{bx\log(c+dx)}{a(a+bx^2)}\right)dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\int \frac{\log(c+dx)}{x}\,dx}{d} - \frac{(2b)\int \frac{x\log(c+dx)}{a+bx^2}\,dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - 2\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx - \frac{(2b)\int \left(-\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \cdots\right)}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} + \frac{2\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d} + \frac{\sqrt{b}\int \frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{d} - \frac{\sqrt{b}\int \frac{\log(c\cdots}{\sqrt{-a}+\cdots}}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}\cdots\right)}{\sqrt{b}c-\sqrt{-a}d}\cdots\right)}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}\cdots\right)}{\sqrt{b}c-\sqrt{-a}d}\cdots\right)}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}\cdots\right)}{\sqrt{b}c-\sqrt{-a}d}\cdots\right)}{d}$$

Mathematica [A] time = 0.106279, size = 228, normalized size = 1.

$$-\frac{\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d} - \frac{\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d} + \frac{2\mathrm{PolyLog}\left(2,\frac{c+dx}{c}\right)}{d} + \frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d} - \frac{\log(c+dx)\log\left(\cdots\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d +
 (2*PolyLog[2, (c + d*x)/c])/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c
- Sqrt[-a]*d)]/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]
/d
```

Maple [A] time = 0.112, size = 335, normalized size = 1.5

$$\frac{\ln\left(x^{-1}\right)}{d}\ln\left(\left(-\frac{a}{x}+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\right) + \frac{\ln\left(x^{-1}\right)}{d}\ln\left(\left(\frac{a}{x}+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\right) - \frac{\ln\left(x^{-1}\right)}{d}\ln\left(b+\frac{a}{x^2}\right) + \frac{1}{d}\mathrm{dilog}\left(\left(-\frac{a}{x}+\sqrt{-ab}\right)\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^2+a)/x^2)/(d*x+c),x)
```

```
[Out] 1/d*ln(1/x)*ln((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/d*ln(1/x)*ln((a/x+(-a*b)
^(1/2))/(-a*b)^(1/2))-1/d*ln(1/x)*ln(b+a/x^2)+1/d*dilog((-a/x+(-a*b)^(1/2))
/(-a*b)^(1/2))+1/d*dilog((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/d*ln(c/x+d)*ln(
b+a/x^2)-1/d*ln(c/x+d)*ln((c*(-a*b)^(1/2)-a*(c/x+d)+a*d)/(c*(-a*b)^(1/2)+a*
d))-1/d*ln(c/x+d)*ln((c*(-a*b)^(1/2)+a*(c/x+d)-a*d)/(c*(-a*b)^(1/2)-a*d))-1
/d*dilog((c*(-a*b)^(1/2)-a*(c/x+d)+a*d)/(c*(-a*b)^(1/2)+a*d))-1/d*dilog((c*
(-a*b)^(1/2)+a*(c/x+d)-a*d)/(c*(-a*b)^(1/2)-a*d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{a}{x^2}+b\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x**2+a)/x**2)/(d*x+c),x)
```

```
[Out] Integral(log(a/x**2 + b)/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

3.398 $\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\log\left(ax^{-n}+b\right)}{c+dx},x\right)$$

```
[Out] Unintegrable[Log[b + a/x^n]/(c + d*x), x]
```

Rubi [A] time = 0.0257867, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(x^{-n}\left(a+bx^n\right)\right)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[(a + b*x^n)/x^n]/(c + d*x),x]
```

```
[Out] Defer[Int][Log[b + a/x^n]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\log\left(x^{-n}\left(a+bx^n\right)\right)}{c+dx} \, dx = \int \frac{\log\left(b+ax^{-n}\right)}{c+dx} \, dx$$

Mathematica [A] time = 0.470659, size = 0, normalized size = 0.

$$\int \frac{\log\left(x^{-n}\left(a+bx^n\right)\right)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x),x]
```

```
[Out] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x), x]
```

Maple [A] time = 1.703, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c} \ln\left(\frac{a+bx^n}{x^n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
```

```
[Out] int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+b*x**n)/(x**n))/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

3.399 $\int (fx)^q \left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right) \, dx$

Optimal. Leaf size=92

$$\frac{(fx)^{q+1}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(q+1)}-\frac{bemnx^{m+1}(fx)^q\,{}_2F_1\left(1,\frac{m+q+1}{m};\frac{2m+q+1}{m};-\frac{ex^m}{d}\right)}{d(q+1)(m+q+1)}$$

```
[Out] -((b*e*m*n*x^(1 + m)*(f*x)^q*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m +
 q)/m, -((e*x^m)/d)])/(d*(1 + q)*(1 + m + q))) + ((f*x)^(1 + q)*(a + b*Log[
c*(d + e*x^m)^n]))/(f*(1 + q))
```

Rubi [A] time = 0.0499879, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2455, 20, 364}

$$\frac{(fx)^{q+1}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(q+1)}-\frac{bemnx^{m+1}(fx)^q\,{}_2F_1\left(1,\frac{m+q+1}{m};\frac{2m+q+1}{m};-\frac{ex^m}{d}\right)}{d(q+1)(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```


```
[Out] -((b*e*m*n*x^(1 + m)*(f*x)^q*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m +
 q)/m, -((e*x^m)/d)])/(d*(1 + q)*(1 + m + q))) + ((f*x)^(1 + q)*(a + b*Log[
c*(d + e*x^m)^n]))/(f*(1 + q))
```


Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^q \left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)dx = \frac{(fx)^{1+q}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(1+q)} - \frac{(bemn)\int \frac{x^{-1+m}(fx)^{1+q}}{d+ex^m}dx}{f(1+q)}$$
$$= \frac{(fx)^{1+q}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(1+q)} - \frac{\left(bemnx^{-q}(fx)^q\right)\int \frac{x^{m+q}}{d+ex^m}dx}{1+q}$$
$$= -\frac{bemnx^{1+m}(fx)^q\,{}_2F_1\left(1,\frac{1+m+q}{m};\frac{1+2m+q}{m};-\frac{ex^m}{d}\right)}{d(1+q)(1+m+q)} + \frac{(fx)^{1+q}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(1+q)}$$

Mathematica [A] time = 0.0548306, size = 82, normalized size = 0.89

$$\frac{x(fx)^q\left(d(m+q+1)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)-bemnx^m\,{}_2F_1\left(1,\frac{m+q+1}{m};\frac{2m+q+1}{m};-\frac{ex^m}{d}\right)\right)}{d(q+1)(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

```
[Out] (x*(f*x)^q*(-(b*e*m*n*x^m*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m + q)
/m, -((e*x^m)/d)]) + d*(1 + m + q)*(a + b*Log[c*(d + e*x^m)^n])))/(d*(1 + q
)*(1 + m + q))
```

Maple [F] time = 2.73, size = 0, normalized size = 0.

$$\int (fx)^q\left(a+b\ln\left(c\left(d+ex^m\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
```

```
[Out] int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx\right)^q b\log\left(\left(ex^m+d\right)^n c\right)+\left(fx\right)^q a, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="fricas")
```

```
[Out] integral((f*x)^q*b*log((e*x^m + d)^n*c) + (f*x)^q*a, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^q \left(a + b\log\left(c\left(d + ex^m\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**q*(a+b*ln(c*(d+e*x**m)**n)),x)
```

```
[Out] Integral((f*x)**q*(a + b*log(c*(d + e*x**m)**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^m + d\right)^n c\right) + a\right)(fx)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*(f*x)^q, x)
```

3.400 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=166

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}+$$

```
[Out] (b*d^7*n*Sqrt[x])/(4*e^7) - (b*d^6*n*x)/(8*e^6) + (b*d^5*n*x^(3/2))/(12*e^5
) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3) - (b*d^2*n*x^3)/(24
*e^2) + (b*d*n*x^(7/2))/(28*e) - (b*n*x^4)/32 - (b*d^8*n*Log[d + e*Sqrt[x]]
)/(4*e^8) + (x^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/4
```

Rubi [A] time = 0.136079, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^7*n*Sqrt[x])/(4*e^7) - (b*d^6*n*x)/(8*e^6) + (b*d^5*n*x^(3/2))/(12*e^5
) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3) - (b*d^2*n*x^3)/(24
*e^2) + (b*d*n*x^(7/2))/(28*e) - (b*n*x^4)/32 - (b*d^8*n*Log[d + e*Sqrt[x]]
)/(4*e^8) + (x^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx = 2\,\text{Subst}\left(\int x^7\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \sqrt{x}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{4}(ben)\,\text{Subst}\left(\int \frac{x^8}{d+ex}dx, x, \sqrt{x}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{4}(ben)\,\text{Subst}\left(\int\left(-\frac{d^7}{e^8}+\frac{d^6x}{e^7}-\frac{d^5x^2}{e^6}+\frac{d^4x^3}{e^5}\right.\right.$$
$$= \frac{bd^7n\sqrt{x}}{4e^7} - \frac{bd^6nx}{8e^6} + \frac{bd^5nx^{3/2}}{12e^5} - \frac{bd^4nx^2}{16e^4} + \frac{bd^3nx^{5/2}}{20e^3} - \frac{bd^2nx^3}{24e^2} + \frac{bdnx^{7/2}}{28e} - \frac{1}{32}b$$

Mathematica [A] time = 0.130716, size = 159, normalized size = 0.96

$$\frac{ax^4}{4} + \frac{1}{4}bx^4\log\left(c\left(d+e\sqrt{x}\right)^n\right) - \frac{1}{4}ben\left(-\frac{d^5x^{3/2}}{3e^6} + \frac{d^4x^2}{4e^5} - \frac{d^3x^{5/2}}{5e^4} + \frac{d^2x^3}{6e^3} - \frac{d^7\sqrt{x}}{e^8} + \frac{d^6x}{2e^7} + \frac{d^8\log\left(d+e\sqrt{x}\right)}{e^9} - \frac{dx^{7/2}}{7e^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^7*Sqrt[x])/e^8) + (d^6*x)/(2*e^7) - (d^5*x^(3/2))/
(3*e^6) + (d^4*x^2)/(4*e^5) - (d^3*x^(5/2))/(5*e^4) + (d^2*x^3)/(6*e^3) - (
d*x^(7/2))/(7*e^2) + x^4/(8*e) + (d^8*Log[d + e*Sqrt[x]])/e^9))/4 + (b*x^4*
Log[c*(d + e*Sqrt[x])^n])/4
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Maxima [A] time = 1.04502, size = 173, normalized size = 1.04

$$\frac{1}{4}bx^4\log\left(\left(e\sqrt{x}+d\right)^n c\right) + \frac{1}{4}ax^4 - \frac{1}{3360}ben\left(\frac{840\,d^8\log\left(e\sqrt{x}+d\right)}{e^9} + \frac{105\,e^7x^4 - 120\,de^6x^{\frac{7}{2}} + 140\,d^2e^5x^3 - 168\,d^3e^4x^{\frac{5}{2}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((e*sqrt(x) + d)^n*c) + 1/4*a*x^4 - 1/3360*b*e*n*(840*d^8*log(
e*sqrt(x) + d)/e^9 + (105*e^7*x^4 - 120*d*e^6*x^(7/2) + 140*d^2*e^5*x^3 - 1
68*d^3*e^4*x^(5/2) + 210*d^4*e^3*x^2 - 280*d^5*e^2*x^(3/2) + 420*d^6*e*x -
840*d^7*sqrt(x))/e^8)
```

Fricas [A] time = 1.82234, size = 358, normalized size = 2.16

$$\frac{840\,be^{8}x^{4}\log(c)-140\,bd^{2}e^{6}nx^{3}-210\,bd^{4}e^{4}nx^{2}-420\,bd^{6}e^{2}nx-105\left(be^{8}n-8\,ae^{8}\right)x^{4}+840\left(be^{8}nx^{4}-bd^{8}n\right)\log\left(e\sqrt{x}+\right.}{3360\,e^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/3360*(840*b*e^8*x^4*log(c) - 140*b*d^2*e^6*n*x^3 - 210*b*d^4*e^4*n*x^2 -
420*b*d^6*e^2*n*x - 105*(b*e^8*n - 8*a*e^8)*x^4 + 840*(b*e^8*n*x^4 - b*d^8*
n)*log(e*sqrt(x) + d) + 8*(15*b*d*e^7*n*x^3 + 21*b*d^3*e^5*n*x^2 + 35*b*d^5
*e^3*n*x + 105*b*d^7*e*n)*sqrt(x))/e^8
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37637, size = 779, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/3360*((840*(sqrt(x)*e + d)^8*e^(-6)*log(sqrt(x)*e + d) - 6720*(sqrt(x)*e
+ d)^7*d*e^(-6)*log(sqrt(x)*e + d) + 23520*(sqrt(x)*e + d)^6*d^2*e^(-6)*log
(sqrt(x)*e + d) - 47040*(sqrt(x)*e + d)^5*d^3*e^(-6)*log(sqrt(x)*e + d) + 5
8800*(sqrt(x)*e + d)^4*d^4*e^(-6)*log(sqrt(x)*e + d) - 47040*(sqrt(x)*e + d
)^3*d^5*e^(-6)*log(sqrt(x)*e + d) + 23520*(sqrt(x)*e + d)^2*d^6*e^(-6)*log(
sqrt(x)*e + d) - 6720*(sqrt(x)*e + d)*d^7*e^(-6)*log(sqrt(x)*e + d) - 105*(
sqrt(x)*e + d)^8*e^(-6) + 960*(sqrt(x)*e + d)^7*d*e^(-6) - 3920*(sqrt(x)*e
+ d)^6*d^2*e^(-6) + 9408*(sqrt(x)*e + d)^5*d^3*e^(-6) - 14700*(sqrt(x)*e +
d)^4*d^4*e^(-6) + 15680*(sqrt(x)*e + d)^3*d^5*e^(-6) - 11760*(sqrt(x)*e + d
)^2*d^6*e^(-6) + 6720*(sqrt(x)*e + d)*d^7*e^(-6))*b*n*e^(-1) + 840*((sqrt(x
)*e + d)^8 - 8*(sqrt(x)*e + d)^7*d + 28*(sqrt(x)*e + d)^6*d^2 - 56*(sqrt(x)
*e + d)^5*d^3 + 70*(sqrt(x)*e + d)^4*d^4 - 56*(sqrt(x)*e + d)^3*d^5 + 28*(s
qrt(x)*e + d)^2*d^6 - 8*(sqrt(x)*e + d)*d^7)*b*e^(-7)*log(c) + 840*((sqrt(x
)*e + d)^8 - 8*(sqrt(x)*e + d)^7*d + 28*(sqrt(x)*e + d)^6*d^2 - 56*(sqrt(x)
*e + d)^5*d^3 + 70*(sqrt(x)*e + d)^4*d^4 - 56*(sqrt(x)*e + d)^3*d^5 + 28*(s
qrt(x)*e + d)^2*d^6 - 8*(sqrt(x)*e + d)*d^7)*a*e^(-7))*e^(-1)
```

3.401 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=134

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^3nx^{3/2}}{9e^3}-\frac{bd^2nx^2}{12e^2}+\frac{bd^5n\sqrt{x}}{3e^5}-\frac{bd^4nx}{6e^4}-\frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}+\frac{bdnx^{5/2}}{15e}-\frac{1}{18}bnx^3$$

```
[Out] (b*d^5*n*Sqrt[x])/(3*e^5) - (b*d^4*n*x)/(6*e^4) + (b*d^3*n*x^(3/2))/(9*e^3)
 - (b*d^2*n*x^2)/(12*e^2) + (b*d*n*x^(5/2))/(15*e) - (b*n*x^3)/18 - (b*d^6*
n*Log[d + e*Sqrt[x]])/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/3
```

Rubi [A] time = 0.0993512, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^3nx^{3/2}}{9e^3}-\frac{bd^2nx^2}{12e^2}+\frac{bd^5n\sqrt{x}}{3e^5}-\frac{bd^4nx}{6e^4}-\frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}+\frac{bdnx^{5/2}}{15e}-\frac{1}{18}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^5*n*Sqrt[x])/(3*e^5) - (b*d^4*n*x)/(6*e^4) + (b*d^3*n*x^(3/2))/(9*e^3)
 - (b*d^2*n*x^2)/(12*e^2) + (b*d*n*x^(5/2))/(15*e) - (b*n*x^3)/18 - (b*d^6*
n*Log[d + e*Sqrt[x]])/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx = 2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{x^6}{d+ex}dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{3}(ben)\,\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}-\right.\right.$$
$$= \frac{bd^5n\sqrt{x}}{3e^5} - \frac{bd^4nx}{6e^4} + \frac{bd^3nx^{3/2}}{9e^3} - \frac{bd^2nx^2}{12e^2} + \frac{bdnx^{5/2}}{15e} - \frac{1}{18}bnx^3 - \frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}$$

Mathematica [A] time = 0.0903966, size = 131, normalized size = 0.98

$$\frac{ax^3}{3} + \frac{1}{3}bx^3\log\left(c\left(d+e\sqrt{x}\right)^n\right) - \frac{1}{3}ben\left(-\frac{d^3x^{3/2}}{3e^4} + \frac{d^2x^2}{4e^3} - \frac{d^5\sqrt{x}}{e^6} + \frac{d^4x}{2e^5} + \frac{d^6\log\left(d+e\sqrt{x}\right)}{e^7} - \frac{dx^{5/2}}{5e^2} + \frac{x^3}{6e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (a*x^3)/3 - (b*e*n*(-((d^5*Sqrt[x])/e^6) + (d^4*x)/(2*e^5) - (d^3*x^(3/2))/
(3*e^4) + (d^2*x^2)/(4*e^3) - (d*x^(5/2))/(5*e^2) + x^3/(6*e) + (d^6*Log[d
+ e*Sqrt[x]])/e^7))/3 + (b*x^3*Log[c*(d + e*Sqrt[x])^n])/3
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Maxima [A] time = 1.06794, size = 143, normalized size = 1.07

$$\frac{1}{3}bx^3\log\left(\left(e\sqrt{x}+d\right)^nc\right) + \frac{1}{3}ax^3 - \frac{1}{180}ben\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7} + \frac{10\,e^5x^3 - 12\,de^4x^{\frac{5}{2}} + 15\,d^2e^3x^2 - 20\,d^3e^2x^{\frac{3}{2}} + 30\,d^4ex}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a*x^3 - 1/180*b*e*n*(60*d^6*log(e*
sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3
*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)
```

Fricas [A] time = 1.69733, size = 288, normalized size = 2.15

$$\frac{60\,be^6x^3\log(c) - 15\,bd^2e^4nx^2 - 30\,bd^4e^2nx - 10\left(be^6n - 6\,ae^6\right)x^3 + 60\left(be^6nx^3 - bd^6n\right)\log\left(e\sqrt{x}+d\right) + 4\left(3\,bde^5nx^2 + 5\right.}{180\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/180*(60*b*e^6*x^3*log(c) - 15*b*d^2*e^4*n*x^2 - 30*b*d^4*e^2*n*x - 10*(b*
e^6*n - 6*a*e^6)*x^3 + 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x) + d) + 4*(3
*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 15*b*d^5*e*n)*sqrt(x))/e^6
```

Sympy [A] time = 29.4452, size = 128, normalized size = 0.96

$$\frac{ax^3}{3} + b\left(-\frac{en\left(\frac{2d^6\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e = 0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6} - \frac{2d^5\sqrt{x}}{e^6} + \frac{d^4x}{e^5} - \frac{2d^3x^{\frac{3}{2}}}{3e^4} + \frac{d^2x^2}{2e^3} - \frac{2dx^{\frac{5}{2}}}{5e^2} + \frac{x^3}{3e}\right)}{6} + \frac{x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**3/3 + b*(-e*n*(2*d**6*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**6 - 2*d**5*sqrt(x)/e**6 + d**4*x/e**5 - 2*d**3*x**(3/2)/(
3*e**4) + d**2*x**2/(2*e**3) - 2*d*x**(5/2)/(5*e**2) + x**3/(3*e))/6 + x**3
*log(c*(d + e*sqrt(x))**n)/3)
```

Giac [B] time = 1.29028, size = 582, normalized size = 4.34

$$\frac{1}{180}\left(\left(60\left(\sqrt{x}e+d\right)^6 e^{(-4)}\log\left(\sqrt{x}e+d\right) - 360\left(\sqrt{x}e+d\right)^5 de^{(-4)}\log\left(\sqrt{x}e+d\right) + 900\left(\sqrt{x}e+d\right)^4 d^2e^{(-4)}\log\left(\sqrt{x}e+d\right) -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/180*((60*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d
)^5*d*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt
(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d) + 900*(sq
rt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-
4)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-4) + 72*(sqrt(x)*e + d)^5*
d*e^(-4) - 225*(sqrt(x)*e + d)^4*d^2*e^(-4) + 400*(sqrt(x)*e + d)^3*d^3*e^(
-4) - 450*(sqrt(x)*e + d)^2*d^4*e^(-4) + 360*(sqrt(x)*e + d)*d^5*e^(-4))*b*
n*e^(-1) + 60*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*e +
d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqrt(x)
*e + d)*d^5)*b*e^(-5)*log(c) + 60*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*
d + 15*(sqrt(x)*e + d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d
)^2*d^4 - 6*(sqrt(x)*e + d)*d^5)*a*e^(-5))*e^(-1)
```

3.402 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2$$

```
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (
b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (x^2*(a + b*Log[c*(d +
e*Sqrt[x])^n]))/2
```

Rubi [A] time = 0.0721291, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2454, 2395, 43}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (
b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (x^2*(a + b*Log[c*(d +
e*Sqrt[x])^n]))/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = 2\,\mathrm{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{2}(ben)\,\mathrm{Subst}\left(\int \frac{x^4}{d+ex}\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{2}(ben)\,\mathrm{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{}{e^4}\right.\right.$$
$$= \frac{bd^3n\sqrt{x}}{2e^3} - \frac{bd^2nx}{4e^2} + \frac{bdnx^{3/2}}{6e} - \frac{1}{8}bnx^2 - \frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4} + \frac{1}{2}x^2\left(a+b\log\left(c\right.\right.$$

Mathematica [A] time = 0.0329987, size = 107, normalized size = 1.05

$$\frac{ax^2}{2} + \frac{1}{2}bx^2\log\left(c\left(d+e\sqrt{x}\right)^n\right) + \frac{bd^3n\sqrt{x}}{2e^3} - \frac{bd^2nx}{4e^2} - \frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4} + \frac{bdnx^{3/2}}{6e} - \frac{1}{8}bnx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) + (
a*x^2)/2 - (b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (b*x^2*Log[
c*(d + e*Sqrt[x])^n])/2
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Maxima [A] time = 1.04189, size = 113, normalized size = 1.11

$$-\frac{1}{24}\,ben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5} + \frac{3\,e^3x^2 - 4\,de^2x^{\frac{3}{2}} + 6\,d^2ex - 12\,d^3\sqrt{x}}{e^4}\right) + \frac{1}{2}\,bx^2\log\left(\left(e\sqrt{x}+d\right)^nc\right) + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] -1/24*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) +
 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 1/2*b*x^2*log((e*sqrt(x) + d)^n*c) + 1/
2*a*x^2
```

Fricas [A] time = 1.74638, size = 224, normalized size = 2.2

$$\frac{12\,be^4x^2\log\left(c\right) - 6\,bd^2e^2nx - 3\left(be^4n - 4\,ae^4\right)x^2 + 12\left(be^4nx^2 - bd^4n\right)\log\left(e\sqrt{x}+d\right) + 4\left(bde^3nx + 3\,bd^3en\right)\sqrt{x}}{24\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b*e^4*x^2*log(c) - 6*b*d^2*e^2*n*x - 3*(b*e^4*n - 4*a*e^4)*x^2 + 1
2*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 4*(b*d*e^3*n*x + 3*b*d^3*e*n
)*sqrt(x))/e^4
```

Sympy [A] time = 4.7292, size = 100, normalized size = 0.98

$$\frac{ax^2}{2} + b\left(-\frac{en\left(\frac{2d^4\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e = 0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^4} - \frac{2d^3\sqrt{x}}{e^4} + \frac{d^2x}{e^3} - \frac{2dx^{\frac{3}{2}}}{3e^2} + \frac{x^2}{2e}\right)}{4} + \frac{x^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(2*d**4*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**4 - 2*d**3*sqrt(x)/e**4 + d**2*x/e**3 - 2*d*x**(3/2)/(3*e
**2) + x**2/(2*e))/4 + x**2*log(c*(d + e*sqrt(x))**n)/2)
```

Giac [B] time = 1.24899, size = 385, normalized size = 3.77

$$\frac{1}{24}\left(\left(12\left(\sqrt{x}e+d\right)^4e^{(-2)}\log\left(\sqrt{x}e+d\right) - 48\left(\sqrt{x}e+d\right)^3de^{(-2)}\log\left(\sqrt{x}e+d\right) + 72\left(\sqrt{x}e+d\right)^2d^2e^{(-2)}\log\left(\sqrt{x}e+d\right) - 48\left(\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/24*((12*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)^
3*d*e^(-2)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)
*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e +
 d)^4*e^(-2) + 16*(sqrt(x)*e + d)^3*d*e^(-2) - 36*(sqrt(x)*e + d)^2*d^2*e^(
-2) + 48*(sqrt(x)*e + d)*d^3*e^(-2))*b*n*e^(-1) + 12*((sqrt(x)*e + d)^4 - 4
*(sqrt(x)*e + d)^3*d + 6*(sqrt(x)*e + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*b*e
^(-3)*log(c) + 12*((sqrt(x)*e + d)^4 - 4*(sqrt(x)*e + d)^3*d + 6*(sqrt(x)*e
 + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*a*e^(-3))*e^(-1)
```

3.403 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=60

$$ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Rubi [A] time = 0.040151, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 266, 43}

$$ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx &= ax+b\int \log\left(c\left(d+e\sqrt{x}\right)^n\right)dx\\ &= ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{1}{2}(ben)\int \frac{\sqrt{x}}{d+e\sqrt{x}}dx\\ &= ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \frac{x^2}{d+ex}dx,x,\sqrt{x}\right)\\ &= ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx,x,\sqrt{x}\right)\\ &= \frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)\end{aligned}$$

Mathematica [A] time = 0.0289376, size = 60, normalized size = 1.

$$ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Maple [A] time = 0.084, size = 53, normalized size = 0.9

$$ax-\frac{bnx}{2}-\frac{bd^2n}{e^2}\ln\left(d+e\sqrt{x}\right)+bx\ln\left(c\left(d+e\sqrt{x}\right)^n\right)+\frac{bdn}{e}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e*x^(1/2))^n),x)
```

```
[Out] a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^(1/2))^n)+b*d*n*x
^(1/2)/e
```

Maxima [A] time = 1.01856, size = 77, normalized size = 1.28

$$-\frac{1}{2}\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="maxima")
```

```
[Out] -1/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*lo
g((e*sqrt(x) + d)^n*c))*b + a*x
```

Fricas [A] time = 1.90945, size = 158, normalized size = 2.63

$$\frac{2\,be^{2}x\log\left(c\right)+2\,bden\sqrt{x}-\left(be^{2}n-2\,ae^{2}\right)x+2\,\left(be^{2}nx-bd^{2}n\right)\log\left(e\sqrt{x}+d\right)}{2\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*e^2*x*log(c) + 2*b*d*e*n*sqrt(x) - (b*e^2*n - 2*a*e^2)*x + 2*(b*e^
2*n*x - b*d^2*n)*log(e*sqrt(x) + d))/e^2
```

Sympy [A] time = 1.45738, size = 66, normalized size = 1.1

$$ax+b\left(-\frac{en\left(\frac{2d^{2}\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{2}}-\frac{2d\sqrt{x}}{e^{2}}+\frac{x}{e}\right)}{2}+x\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(1/2))**n),x)
```

```
[Out] a*x + b*(-e*n*(2*d**2*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt(x))/
e, True))/e**2 - 2*d*sqrt(x)/e**2 + x/e)/2 + x*log(c*(d + e*sqrt(x))**n))
```

Giac [B] time = 1.30731, size = 144, normalized size = 2.4

$$\frac{1}{2}\left(\left(2\left(\sqrt{x}e+d\right)^{2}\log\left(\sqrt{x}e+d\right)-4\left(\sqrt{x}e+d\right)d\log\left(\sqrt{x}e+d\right)-\left(\sqrt{x}e+d\right)^{2}+4\left(\sqrt{x}e+d\right)d\right)ne^{(-1)}+2\left(\left(\sqrt{x}e+d\right)^{2}-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="giac")
```

```
[Out] 1/2*((2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(sqrt
(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*n*e^(-1) + 2*((sqrt(x
)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*e^(-1)*log(c))*b*e^(-1) + a*x
```

3.404 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$2bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)] + 2*b*n*PolyLog[2,
 1 + (e*Sqrt[x])/d]
```

Rubi [A] time = 0.0484987, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$2bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)] + 2*b*n*PolyLog[2,
 1 + (e*Sqrt[x])/d]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)-(2ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,\sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+2bn\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.0032828, size = 53, normalized size = 1.04

$$2bn\text{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)+a\log(x)+2b\log\left(-\frac{e\sqrt{x}}{d}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*b*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] + a*Log[x] + 2*b*n*PolyL
og[2, (d + e*Sqrt[x])/d]
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int\frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
```

Maxima [B] time = 1.63072, size = 146, normalized size = 2.86

$$-2\left(\log\left(\frac{e\sqrt{x}}{d}+1\right)\log\left(\sqrt{x}\right)+\text{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)bn+\frac{2\left(ben\sqrt{x}\log\left(\sqrt{x}\right)-ben\sqrt{x}\right)}{d}+\frac{bd\log\left(\left(e\sqrt{x}+d\right)^n\right)\log(x)+(bd\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="maxima")
```

```
[Out] -2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b*n + 2*(b*e*n
*sqrt(x)*log(sqrt(x)) - b*e*n*sqrt(x))/d + (b*d*log((e*sqrt(x) + d)^n)*log(
x) + (b*d*log(c) + a*d)*log(x) - (b*e*n*x*log(x) - 2*b*e*n*x)/sqrt(x))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \log \left(\left(e\sqrt{x} + d\right)^n c\right) + a}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

3.405 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{be^2n\log(x)}{2d^2}-\frac{ben}{d\sqrt{x}}$$

```
[Out] -((b*e*n)/(d*Sqrt[x])) + (b*e^2*n*Log[d + e*Sqrt[x]])/d^2 - (a + b*Log[c*(d
 + e*Sqrt[x])^n])/x - (b*e^2*n*Log[x])/(2*d^2)
```

Rubi [A] time = 0.0577774, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{be^2n\log(x)}{2d^2}-\frac{ben}{d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

```
[Out] -((b*e*n)/(d*Sqrt[x])) + (b*e^2*n*Log[d + e*Sqrt[x]])/d^2 - (a + b*Log[c*(d
 + e*Sqrt[x])^n])/x - (b*e^2*n*Log[x])/(2*d^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^2} - \frac{e}{d^2x} + \frac{e^2}{d^2(d+ex)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{ben}{d\sqrt{x}} + \frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} - \frac{be^2n\log(x)}{2d^2}$$

Mathematica [A] time = 0.0448363, size = 67, normalized size = 0.96

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + ben\left(\frac{e\log\left(d+e\sqrt{x}\right)}{d^2} - \frac{e\log(x)}{2d^2} - \frac{1}{d\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

```
[Out] -(a/x) - (b*Log[c*(d + e*Sqrt[x])^n])/x + b*e*n*(-(1/(d*Sqrt[x])) + (e*Log[
d + e*Sqrt[x]])/d^2 - (e*Log[x])/(2*d^2))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)
```

Maxima [A] time = 1.025, size = 82, normalized size = 1.17

$$\frac{1}{2}ben\left(\frac{2\,e\log\left(e\sqrt{x}+d\right)}{d^2} - \frac{e\log(x)}{d^2} - \frac{2}{d\sqrt{x}}\right) - \frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(2*e*log(e*sqrt(x) + d)/d^2 - e*log(x)/d^2 - 2/(d*sqrt(x))) - b*l
og((e*sqrt(x) + d)^n*c)/x - a/x
```

Fricas [A] time = 2.07748, size = 161, normalized size = 2.3

$$-\frac{be^2nx\log\left(\sqrt{x}\right)+bden\sqrt{x}+bd^2\log\left(c\right)+ad^2-\left(be^2nx-bd^2n\right)\log\left(e\sqrt{x}+d\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*e^2*n*x*log(sqrt(x)) + b*d*e*n*sqrt(x) + b*d^2*log(c) + a*d^2 - (b*e^2*
n*x - b*d^2*n)*log(e*sqrt(x) + d))/(d^2*x)
```

Sympy [A] time = 96.1634, size = 493, normalized size = 7.04

$$\begin{cases} -\frac{ad^3\sqrt{x}}{d^3x^{\frac{3}{2}}+d^2ex^2}-\frac{ad^2ex}{d^3x^{\frac{3}{2}}+d^2ex^2}-\frac{bd^3n\sqrt{x}\log\left(d+e\sqrt{x}\right)}{d^3x^{\frac{3}{2}}+d^2ex^2}-\frac{bd^3\sqrt{x}\log\left(c\right)}{d^3x^{\frac{3}{2}}+d^2ex^2}-\frac{bd^2enx\log\left(d+e\sqrt{x}\right)}{d^3x^{\frac{3}{2}}+d^2ex^2}-\frac{bd^2enx}{d^3x^{\frac{3}{2}}+d^2ex^2}-\frac{bd^2ex\log\left(c\right)}{d^3x^{\frac{3}{2}}+d^2ex^2}-\frac{bde^2nx^{\frac{3}{2}}\log\left(x\right)}{2\left(d^3x^{\frac{3}{2}}+d^2ex^2\right)}+ \\ -\frac{a}{x}-\frac{bn\log\left(e\right)}{x}-\frac{bn\log\left(x\right)}{2x}-\frac{bn}{2x}-\frac{b\log\left(c\right)}{x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**2,x)
```

```
[Out] Piecewise((-a*d**3*sqrt(x)/(d**3*x**(3/2) + d**2*e*x**2) - a*d**2*e*x/(d**3
*x**(3/2) + d**2*e*x**2) - b*d**3*n*sqrt(x)*log(d + e*sqrt(x))/(d**3*x**(3/
2) + d**2*e*x**2) - b*d**3*sqrt(x)*log(c)/(d**3*x**(3/2) + d**2*e*x**2) - b
*d**2*e*n*x*log(d + e*sqrt(x))/(d**3*x**(3/2) + d**2*e*x**2) - b*d**2*e*n*x
/(d**3*x**(3/2) + d**2*e*x**2) - b*d**2*e*x*log(c)/(d**3*x**(3/2) + d**2*e*
x**2) - b*d*e**2*n*x**(3/2)*log(x)/(2*(d**3*x**(3/2) + d**2*e*x**2)) + b*d*
e**2*n*x**(3/2)*log(d + e*sqrt(x))/(d**3*x**(3/2) + d**2*e*x**2) - b*d*e**2
*n*x**(3/2)/(d**3*x**(3/2) + d**2*e*x**2) + b*d*e**2*x**(3/2)*log(c)/(d**3*
x**(3/2) + d**2*e*x**2) - b*e**3*n*x**2*log(x)/(2*(d**3*x**(3/2) + d**2*e*x
**2)) + b*e**3*n*x**2*log(d + e*sqrt(x))/(d**3*x**(3/2) + d**2*e*x**2) + b*
e**3*x**2*log(c)/(d**3*x**(3/2) + d**2*e*x**2), Ne(d, 0)), (-a/x - b*n*log(
e)/x - b*n*log(x)/(2*x) - b*n/(2*x) - b*log(c)/x, True))
```

Giac [B] time = 1.29019, size = 252, normalized size = 3.6

$$\frac{\left(\left(\sqrt{x}e+d\right)^2bne^3\log\left(\sqrt{x}e+d\right)-2\left(\sqrt{x}e+d\right)bdne^3\log\left(\sqrt{x}e+d\right)-\left(\sqrt{x}e+d\right)^2bne^3\log\left(\sqrt{x}e\right)+2\left(\sqrt{x}e+d\right)bdne^3\log\right.}{\left(\sqrt{x}e+d\right)^2d^2-2\left(\sqrt{x}e+d\right)d^3+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] ((sqrt(x)*e + d)^2*b*n*e^3*log(sqrt(x)*e + d) - 2*(sqrt(x)*e + d)*b*d*n*e^3
*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2*b*n*e^3*log(sqrt(x)*e) + 2*(sqrt(x)
*e + d)*b*d*n*e^3*log(sqrt(x)*e) - b*d^2*n*e^3*log(sqrt(x)*e) - (sqrt(x)*e
+ d)*b*d*n*e^3 + b*d^2*n*e^3 - b*d^2*e^3*log(c) - a*d^2*e^3)*e^(-1)/((sqrt(
x)*e + d)^2*d^2 - 2*(sqrt(x)*e + d)*d^3 + d^4)
```

3.406 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^2n}{4d^2x}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}-\frac{ben}{6dx^{3/2}}$$

```
[Out] -(b*e*n)/(6*d*x^(3/2)) + (b*e^2*n)/(4*d^2*x) - (b*e^3*n)/(2*d^3*Sqrt[x]) +
(b*e^4*n*Log[d + e*Sqrt[x]])/(2*d^4) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(2*
x^2) - (b*e^4*n*Log[x])/(4*d^4)
```

Rubi [A] time = 0.0742414, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^2n}{4d^2x}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}-\frac{ben}{6dx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

```
[Out] -(b*e*n)/(6*d*x^(3/2)) + (b*e^2*n)/(4*d^2*x) - (b*e^3*n)/(2*d^3*Sqrt[x]) +
(b*e^4*n*Log[d + e*Sqrt[x]])/(2*d^4) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(2*
x^2) - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^5}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{1}{x^4(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^4} - \frac{e}{d^2x^3} + \frac{e^2}{d^3x^2} - \frac{e^3}{d^4x} + \frac{e^4}{d^4(d+\right.\right.$$
$$= -\frac{ben}{6dx^{3/2}} + \frac{be^2n}{4d^2x} - \frac{be^3n}{2d^3\sqrt{x}} + \frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} - \frac{be^4n\,\mathrm{l}}{4c}$$

Mathematica [A] time = 0.0365354, size = 104, normalized size = 0.95

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}ben\left(-\frac{e^2}{d^3\sqrt{x}} + \frac{e^3\log\left(d+e\sqrt{x}\right)}{d^4} - \frac{e^3\log(x)}{2d^4} + \frac{e}{2d^2x} - \frac{1}{3dx^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*Log[c*(d + e*Sqrt[x])^n])/(2*x^2) + (b*e*n*(-1/(3*d*x^(3/2)
) + e/(2*d^2*x) - e^2/(d^3*Sqrt[x]) + (e^3*Log[d + e*Sqrt[x]])/d^4 - (e^3*L
og[x])/(2*d^4)))/2
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
```

Maxima [A] time = 1.03812, size = 113, normalized size = 1.04

$$\frac{1}{12}ben\left(\frac{6e^3\log\left(e\sqrt{x}+d\right)}{d^4} - \frac{3e^3\log(x)}{d^4} - \frac{6e^2x - 3de\sqrt{x} + 2d^2}{d^3x^{\frac{3}{2}}}\right) - \frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)}{2x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b*e*n*(6*e^3*log(e*sqrt(x) + d)/d^4 - 3*e^3*log(x)/d^4 - (6*e^2*x - 3*
d*e*sqrt(x) + 2*d^2)/(d^3*x^(3/2))) - 1/2*b*log((e*sqrt(x) + d)^n*c)/x^2 -
1/2*a/x^2
```

Fricas [A] time = 2.13139, size = 240, normalized size = 2.2

$$-\frac{6\,be^4nx^2\log\left(\sqrt{x}\right)-3\,bd^2e^2nx+6\,bd^4\log\left(c\right)+6\,ad^4-6\left(be^4nx^2-bd^4n\right)\log\left(e\sqrt{x}+d\right)+2\left(3\,bde^3nx+bd^3en\right)\sqrt{x}}{12\,d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b*e^4*n*x^2*log(sqrt(x)) - 3*b*d^2*e^2*n*x + 6*b*d^4*log(c) + 6*a*
d^4 - 6*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 2*(3*b*d*e^3*n*x + b*d
^3*e*n)*sqrt(x))/(d^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28607, size = 494, normalized size = 4.53

$$\left(6\left(\sqrt{x}e+d\right)^4bne^5\log\left(\sqrt{x}e+d\right)-24\left(\sqrt{x}e+d\right)^3bdne^5\log\left(\sqrt{x}e+d\right)+36\left(\sqrt{x}e+d\right)^2bd^2ne^5\log\left(\sqrt{x}e+d\right)-24\left(\sqrt{x}e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/12*(6*(sqrt(x)*e + d)^4*b*n*e^5*log(sqrt(x)*e + d) - 24*(sqrt(x)*e + d)^3
*b*d*n*e^5*log(sqrt(x)*e + d) + 36*(sqrt(x)*e + d)^2*b*d^2*n*e^5*log(sqrt(x
)*e + d) - 24*(sqrt(x)*e + d)*b*d^3*n*e^5*log(sqrt(x)*e + d) - 6*(sqrt(x)*e
 + d)^4*b*n*e^5*log(sqrt(x)*e) + 24*(sqrt(x)*e + d)^3*b*d*n*e^5*log(sqrt(x)
*e) - 36*(sqrt(x)*e + d)^2*b*d^2*n*e^5*log(sqrt(x)*e) + 24*(sqrt(x)*e + d)*
b*d^3*n*e^5*log(sqrt(x)*e) - 6*b*d^4*n*e^5*log(sqrt(x)*e) - 6*(sqrt(x)*e +
d)^3*b*d*n*e^5 + 21*(sqrt(x)*e + d)^2*b*d^2*n*e^5 - 26*(sqrt(x)*e + d)*b*d^
3*n*e^5 + 11*b*d^4*n*e^5 - 6*b*d^4*e^5*log(c) - 6*a*d^4*e^5)*e^(-1)/((sqrt(
x)*e + d)^4*d^4 - 4*(sqrt(x)*e + d)^3*d^5 + 6*(sqrt(x)*e + d)^2*d^6 - 4*(sq
rt(x)*e + d)*d^7 + d^8)
```

3.407 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=141

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^2n}{12d^2x^2}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^4n}{6d^4x}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{ben}{15dx^{5/2}}$$

```
[Out] -(b*e*n)/(15*d*x^(5/2)) + (b*e^2*n)/(12*d^2*x^2) - (b*e^3*n)/(9*d^3*x^(3/2)
) + (b*e^4*n)/(6*d^4*x) - (b*e^5*n)/(3*d^5*Sqrt[x]) + (b*e^6*n*Log[d + e*Sq
rt[x]])/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) - (b*e^6*n*Log[x
])/(6*d^6)
```

Rubi [A] time = 0.0921506, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^2n}{12d^2x^2}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^4n}{6d^4x}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{ben}{15dx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -(b*e*n)/(15*d*x^(5/2)) + (b*e^2*n)/(12*d^2*x^2) - (b*e^3*n)/(9*d^3*x^(3/2)
) + (b*e^4*n)/(6*d^4*x) - (b*e^5*n)/(3*d^5*Sqrt[x]) + (b*e^6*n*Log[d + e*Sq
rt[x]])/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) - (b*e^6*n*Log[x
])/(6*d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = 2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^6} - \frac{e}{d^2x^5} + \frac{e^2}{d^3x^4} - \frac{e^3}{d^4x^3} + \frac{e^4}{d^5x^2} - \frac{e}{d}\right.\right.$$
$$= -\frac{ben}{15dx^{5/2}} + \frac{be^2n}{12d^2x^2} - \frac{be^3n}{9d^3x^{3/2}} + \frac{be^4n}{6d^4x} - \frac{be^5n}{3d^5\sqrt{x}} + \frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6} - \frac{a+b\log\left(c\left(d\right.\right.}{3x^3}$$

Mathematica [A] time = 0.131117, size = 132, normalized size = 0.94

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}ben\left(-\frac{e^2}{3d^3x^{3/2}} - \frac{e^4}{d^5\sqrt{x}} + \frac{e^3}{2d^4x} + \frac{e^5\log\left(d+e\sqrt{x}\right)}{d^6} - \frac{e^5\log(x)}{2d^6} + \frac{e}{4d^2x^2} - \frac{1}{5dx^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) + (b*e*n*(-1/(5*d*x^(5/2)
) + e/(4*d^2*x^2) - e^2/(3*d^3*x^(3/2)) + e^3/(2*d^4*x) - e^4/(d^5*Sqrt[x])
 + (e^5*Log[d + e*Sqrt[x]])/d^6 - (e^5*Log[x])/(2*d^6)))/3
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

Maxima [A] time = 1.06341, size = 143, normalized size = 1.01

$$\frac{1}{180}ben\left(\frac{60\,e^5\log\left(e\sqrt{x}+d\right)}{d^6} - \frac{30\,e^5\log(x)}{d^6} - \frac{60\,e^4x^2 - 30\,de^3x^{\frac{3}{2}} + 20\,d^2e^2x - 15\,d^3e\sqrt{x} + 12\,d^4}{d^5x^{\frac{5}{2}}}\right) - \frac{b\log\left(\left(e\sqrt{x}+d\right)^nc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*e*n*(60*e^5*log(e*sqrt(x) + d)/d^6 - 30*e^5*log(x)/d^6 - (60*e^4*x^
2 - 30*d*e^3*x^(3/2) + 20*d^2*e^2*x - 15*d^3*e*sqrt(x) + 12*d^4)/(d^5*x^(5/
2))) - 1/3*b*log((e*sqrt(x) + d)^n*c)/x^3 - 1/3*a/x^3
```

Fricas [A] time = 2.08661, size = 308, normalized size = 2.18

$$-\frac{60\,be^6nx^3\log\left(\sqrt{x}\right)-30\,bd^2e^4nx^2-15\,bd^4e^2nx+60\,bd^6\log\left(c\right)+60\,ad^6-60\,\left(be^6nx^3-bd^6n\right)\log\left(e\sqrt{x}+d\right)+4\,(15}{180\,d^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/180*(60*b*e^6*n*x^3*log(sqrt(x)) - 30*b*d^2*e^4*n*x^2 - 15*b*d^4*e^2*n*x
 + 60*b*d^6*log(c) + 60*a*d^6 - 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x) +
d) + 4*(15*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d^5*e*n)*sqrt(x))/(d^6*x^3
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3381, size = 732, normalized size = 5.19

$$\frac{\left(60\left(\sqrt{x}e+d\right)^6bne^7\log\left(\sqrt{x}e+d\right)-360\left(\sqrt{x}e+d\right)^5bdne^7\log\left(\sqrt{x}e+d\right)+900\left(\sqrt{x}e+d\right)^4bd^2ne^7\log\left(\sqrt{x}e+d\right)-120}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/180*(60*(sqrt(x)*e + d)^6*b*n*e^7*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d
)^5*b*d*n*e^7*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*b*d^2*n*e^7*log(sq
rt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*b*d^3*n*e^7*log(sqrt(x)*e + d) + 900*
(sqrt(x)*e + d)^2*b*d^4*n*e^7*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*b*d^
5*n*e^7*log(sqrt(x)*e + d) - 60*(sqrt(x)*e + d)^6*b*n*e^7*log(sqrt(x)*e) +
360*(sqrt(x)*e + d)^5*b*d*n*e^7*log(sqrt(x)*e) - 900*(sqrt(x)*e + d)^4*b*d^
2*n*e^7*log(sqrt(x)*e) + 1200*(sqrt(x)*e + d)^3*b*d^3*n*e^7*log(sqrt(x)*e)
- 900*(sqrt(x)*e + d)^2*b*d^4*n*e^7*log(sqrt(x)*e) + 360*(sqrt(x)*e + d)*b*
d^5*n*e^7*log(sqrt(x)*e) - 60*b*d^6*n*e^7*log(sqrt(x)*e) - 60*(sqrt(x)*e +
d)^5*b*d*n*e^7 + 330*(sqrt(x)*e + d)^4*b*d^2*n*e^7 - 740*(sqrt(x)*e + d)^3*
b*d^3*n*e^7 + 855*(sqrt(x)*e + d)^2*b*d^4*n*e^7 - 522*(sqrt(x)*e + d)*b*d^5
*n*e^7 + 137*b*d^6*n*e^7 - 60*b*d^6*e^7*log(c) - 60*a*d^6*e^7)*e^(-1)/((sqr
t(x)*e + d)^6*d^6 - 6*(sqrt(x)*e + d)^5*d^7 + 15*(sqrt(x)*e + d)^4*d^8 - 20
*(sqrt(x)*e + d)^3*d^9 + 15*(sqrt(x)*e + d)^2*d^10 - 6*(sqrt(x)*e + d)*d^11
 + d^12)
```

3.408 $\int x^2 \left(a+b \log \left(c \left(d+e\sqrt{x}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=480

$$-\frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6}+\frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}-\frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+\cdots\right.}{e^{\cdots}}$$

```
[Out] (5*b^2*d^4*n^2*(d + e*Sqrt[x])^2)/(2*e^6) - (40*b^2*d^3*n^2*(d + e*Sqrt[x])
^3)/(27*e^6) + (5*b^2*d^2*n^2*(d + e*Sqrt[x])^4)/(8*e^6) - (4*b^2*d*n^2*(d
+ e*Sqrt[x])^5)/(25*e^6) + (b^2*n^2*(d + e*Sqrt[x])^6)/(54*e^6) - (4*b^2*d^
5*n^2*Sqrt[x])/e^5 + (b^2*d^6*n^2*Log[d + e*Sqrt[x]]^2)/(3*e^6) + (4*b*d^5*
n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^6 - (5*b*d^4*n*(d + e
*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^6 + (40*b*d^3*n*(d + e*Sqrt
[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) - (5*b*d^2*n*(d + e*Sqrt[x
])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) + (4*b*d*n*(d + e*Sqrt[x])^5
*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(5*e^6) - (b*n*(d + e*Sqrt[x])^6*(a + b*
Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) - (2*b*d^6*n*Log[d + e*Sqrt[x]]*(a + b*L
og[c*(d + e*Sqrt[x])^n]))/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2
)/3
```

Rubi [A] time = 0.476373, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6}-\frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6}+\frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}-\frac{225d^2\left(d+e\sqrt{x}\right)^4}{e^6}-\frac{60d^6\log\left(d+e\sqrt{x}\right)}{e^6}+\frac{72d\left(d+\cdots\right.}{e^6}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (5*b^2*d^4*n^2*(d + e*Sqrt[x])^2)/(2*e^6) - (40*b^2*d^3*n^2*(d + e*Sqrt[x])
^3)/(27*e^6) + (5*b^2*d^2*n^2*(d + e*Sqrt[x])^4)/(8*e^6) - (4*b^2*d*n^2*(d
+ e*Sqrt[x])^5)/(25*e^6) + (b^2*n^2*(d + e*Sqrt[x])^6)/(54*e^6) - (4*b^2*d^
5*n^2*Sqrt[x])/e^5 + (b^2*d^6*n^2*Log[d + e*Sqrt[x]]^2)/(3*e^6) + (b*n*((36
0*d^5*(d + e*Sqrt[x]))/e^6 - (450*d^4*(d + e*Sqrt[x])^2)/e^6 + (400*d^3*(d
+ e*Sqrt[x])^3)/e^6 - (225*d^2*(d + e*Sqrt[x])^4)/e^6 + (72*d*(d + e*Sqrt[x
])^5)/e^6 - (10*(d + e*Sqrt[x])^6)/e^6 - (60*d^6*Log[d + e*Sqrt[x]])/e^6)*(
a + b*Log[c*(d + e*Sqrt[x])^n]))/90 + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n])
^2)/3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2dx = 2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2dx,x,\sqrt{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{x^6\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}d\right.$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{1}{3}(2bn)\,\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}d\right.$$
$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6}-\frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6}+\frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}-\frac{225d^2\left(d+e\right.}{e^6}\right.$$
$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6}-\frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6}+\frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}-\frac{225d^2\left(d+e\right.}{e^6}\right.$$
$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6}-\frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6}+\frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}-\frac{225d^2\left(d+e\right.}{e^6}\right.$$
$$= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6}-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6}+\frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}-\frac{4b^2dn^2\left(d+\right.}{25e^6}$$
$$= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6}-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6}+\frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}-\frac{4b^2dn^2\left(d+\right.}{25e^6}$$

Mathematica [A] time = 0.316471, size = 295, normalized size = 0.61

$$\frac{e\sqrt{x}\left(1800a^2e^5x^{5/2}+60abn\left(-15d^2e^3x^{3/2}+20d^3e^2x-30d^4e\sqrt{x}+60d^5+12de^4x^2-10e^5x^{5/2}\right)+b^2n^2\left(555d^2e^3x^{3/2}-1140d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (e*Sqrt[x]*(1800*a^2*e^5*x^(5/2) + 60*a*b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20
*d^3*e^2*x - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)) + b^2*n^2*
(-8820*d^5 + 2610*d^4*e*Sqrt[x] - 1140*d^3*e^2*x + 555*d^2*e^3*x^(3/2) - 26
4*d*e^4*x^2 + 100*e^5*x^(5/2))) - 60*b*(60*a*(d^6 - e^6*x^3) + b*n*(-147*d^
6 - 60*d^5*e*Sqrt[x] + 30*d^4*e^2*x - 20*d^3*e^3*x^(3/2) + 15*d^2*e^4*x^2 -
 12*d*e^5*x^(5/2) + 10*e^6*x^3))*Log[c*(d + e*Sqrt[x])^n] - 1800*b^2*(d^6 -
 e^6*x^3)*Log[c*(d + e*Sqrt[x])^n]^2)/(5400*e^6)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Maxima [A] time = 1.07361, size = 437, normalized size = 0.91

$$\frac{1}{3} b^2x^3 \log\left(\left(e\sqrt{x}+d\right)^n c\right)^2 + \frac{2}{3} abx^3 \log\left(\left(e\sqrt{x}+d\right)^n c\right) + \frac{1}{3} a^2x^3 - \frac{1}{90} aben\left(\frac{60\, d^6 \log\left(e\sqrt{x}+d\right)}{e^7} + \frac{10\, e^5x^3 - 12\, de^4x^{\frac{5}{2}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((e*sqrt(x) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*sqrt(x) + d)^n*
c) + 1/3*a^2*x^3 - 1/90*a*b*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^
3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 6
0*d^5*sqrt(x))/e^6) - 1/5400*(60*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e
^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*
x - 60*d^5*sqrt(x))/e^6)*log((e*sqrt(x) + d)^n*c) - (100*e^6*x^3 - 264*d*e^
5*x^(5/2) + 555*d^2*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*
x^(3/2) + 2610*d^4*e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x)
)*n^2/e^6)*b^2
```

Fricas [A] time = 2.25006, size = 1081, normalized size = 2.25

$$\frac{1800\, b^2e^6x^3 \log\left(c\right)^2 + 100\left(b^2e^6n^2 - 6\, abe^6n + 18\, a^2e^6\right)x^3 + 15\left(37\, b^2d^2e^4n^2 - 60\, abd^2e^4n\right)x^2 + 1800\left(b^2e^6n^2x^3 - b^2d^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/5400*(1800*b^2*e^6*x^3*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2
*e^6)*x^3 + 15*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n)*x^2 + 1800*(b^2*e^6*
n^2*x^3 - b^2*d^6*n^2)*log(e*sqrt(x) + d)^2 + 90*(29*b^2*d^4*e^2*n^2 - 20*a
*b*d^4*e^2*n)*x - 60*(15*b^2*d^2*e^4*n^2*x^2 + 30*b^2*d^4*e^2*n^2*x - 147*b
^2*d^6*n^2 + 60*a*b*d^6*n + 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^3 - 60*(b^2*e^
6*n*x^3 - b^2*d^6*n)*log(c) - 4*(3*b^2*d*e^5*n^2*x^2 + 5*b^2*d^3*e^3*n^2*x
+ 15*b^2*d^5*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) - 300*(3*b^2*d^2*e^4*n*x^2
+ 6*b^2*d^4*e^2*n*x + 2*(b^2*e^6*n - 6*a*b*e^6)*x^3)*log(c) - 12*(735*b^2*d
^5*e*n^2 - 300*a*b*d^5*e*n + 2*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x^2 + 5*
(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(3*b^2*d*e^5*n*x^2 + 5*b^2*d
^3*e^3*n*x + 15*b^2*d^5*e*n)*log(c))*sqrt(x))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36998, size = 1619, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/5400*((1800*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x
)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^4*d^2*e^(-
4)*log(sqrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e
+ d)^2 + 27000*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d)^2 - 10800*(s
qrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e + d)^6*e^(-4
)*log(sqrt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) -
 13500*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) + 24000*(sqrt(x)*e +
 d)^3*d^3*e^(-4)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d^4*e^(-4)*lo
g(sqrt(x)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) + 10
0*(sqrt(x)*e + d)^6*e^(-4) - 864*(sqrt(x)*e + d)^5*d*e^(-4) + 3375*(sqrt(x)
*e + d)^4*d^2*e^(-4) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-4) + 13500*(sqrt(x)*e
 + d)^2*d^4*e^(-4) - 21600*(sqrt(x)*e + d)*d^5*e^(-4))*b^2*n^2*e^(-1) + 60*
(60*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e
^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e +
 d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e
 + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-4)*log(
sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-4) + 72*(sqrt(x)*e + d)^5*d*e^(-4
) - 225*(sqrt(x)*e + d)^4*d^2*e^(-4) + 400*(sqrt(x)*e + d)^3*d^3*e^(-4) - 4
50*(sqrt(x)*e + d)^2*d^4*e^(-4) + 360*(sqrt(x)*e + d)*d^5*e^(-4))*b^2*n*e^(
-1)*log(c) + 1800*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*
e + d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqr
t(x)*e + d)*d^5)*b^2*e^(-5)*log(c)^2 + 60*(60*(sqrt(x)*e + d)^6*e^(-4)*log(
sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) + 900*(s
qrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*
e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e
+ d) - 360*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e +
d)^6*e^(-4) + 72*(sqrt(x)*e + d)^5*d*e^(-4) - 225*(sqrt(x)*e + d)^4*d^2*e^(
-4) + 400*(sqrt(x)*e + d)^3*d^3*e^(-4) - 450*(sqrt(x)*e + d)^2*d^4*e^(-4) +
 360*(sqrt(x)*e + d)*d^5*e^(-4))*a*b*n*e^(-1) + 3600*((sqrt(x)*e + d)^6 - 6
*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*e + d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3
+ 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqrt(x)*e + d)*d^5)*a*b*e^(-5)*log(c) + 180
0*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*e + d)^4*d^2 - 2
0*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqrt(x)*e + d)*d^5)
*a^2*e^(-5))*e^(-1)
```

3.409 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=342

$$-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a\right.}{}$$

```
[Out] (3*b^2*d^2*n^2*(d + e*Sqrt[x])^2)/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3)
/(9*e^4) + (b^2*n^2*(d + e*Sqrt[x])^4)/(16*e^4) - (4*b^2*d^3*n^2*Sqrt[x])/e
^3 + (b^2*d^4*n^2*Log[d + e*Sqrt[x]]^2)/(2*e^4) + (4*b*d^3*n*(d + e*Sqrt[x]
)*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^4 - (3*b*d^2*n*(d + e*Sqrt[x])^2*(a +
 b*Log[c*(d + e*Sqrt[x])^n]))/e^4 + (4*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c
*(d + e*Sqrt[x])^n]))/(3*e^4) - (b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*
Sqrt[x])^n]))/(4*e^4) - (b*d^4*n*Log[d + e*Sqrt[x]]*(a + b*Log[c*(d + e*Sqr
t[x])^n]))/e^4 + (x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/2
```

Rubi [A] time = 0.361032, antiderivative size = 263, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4}-\frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4}-\frac{12d^4\log\left(d+e\sqrt{x}\right)}{e^4}+\frac{16d\left(d+e\sqrt{x}\right)^3}{e^4}-\frac{3\left(d+e\sqrt{x}\right)^4}{e^4}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (3*b^2*d^2*n^2*(d + e*Sqrt[x])^2)/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3)
/(9*e^4) + (b^2*n^2*(d + e*Sqrt[x])^4)/(16*e^4) - (4*b^2*d^3*n^2*Sqrt[x])/e
^3 + (b^2*d^4*n^2*Log[d + e*Sqrt[x]]^2)/(2*e^4) + (b*n*((48*d^3*(d + e*Sqrt
[x]))/e^4 - (36*d^2*(d + e*Sqrt[x])^2)/e^4 + (16*d*(d + e*Sqrt[x])^3)/e^4 -
 (3*(d + e*Sqrt[x])^4)/e^4 - (12*d^4*Log[d + e*Sqrt[x]])/e^4)*(a + b*Log[c*
(d + e*Sqrt[x])^n]))/12 + (x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx = 2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2 dx, x, \sqrt{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - (ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex} dx\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4\left(a+b\log\left(cx^n\right)\right)}{x} dx\right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4} - \frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4} + \frac{16d\left(d+e\sqrt{x}\right)^3}{e^4} - \frac{3\left(d+e\sqrt{x}\right)^4}{e^4} - \right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4} - \frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4} + \frac{16d\left(d+e\sqrt{x}\right)^3}{e^4} - \frac{3\left(d+e\sqrt{x}\right)^4}{e^4} - \right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4} - \frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4} + \frac{16d\left(d+e\sqrt{x}\right)^3}{e^4} - \frac{3\left(d+e\sqrt{x}\right)^4}{e^4} - \right.$$

$$= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4} - \frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4} + \frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4} - \frac{4b^2d^3n^2\sqrt{x}}{e^3} + \cdots$$

$$= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4} - \frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4} + \frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4} - \frac{4b^2d^3n^2\sqrt{x}}{e^3} + \cdots$$

Mathematica [A] time = 0.192595, size = 223, normalized size = 0.65

$$\frac{e\sqrt{x}\left(72a^2e^3x^{3/2}+12abn\left(-6d^2e\sqrt{x}+12d^3+4de^2x-3e^3x^{3/2}\right)+b^2n^2\left(78d^2e\sqrt{x}-300d^3-28de^2x+9e^3x^{3/2}\right)\right)-12b\left(1\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (e*Sqrt[x]*(72*a^2*e^3*x^(3/2) + 12*a*b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*e
^2*x - 3*e^3*x^(3/2)) + b^2*n^2*(-300*d^3 + 78*d^2*e*Sqrt[x] - 28*d*e^2*x +
 9*e^3*x^(3/2))) - 12*b*(12*a*(d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*Sqr
t[x] + 6*d^2*e^2*x - 4*d*e^3*x^(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x])^n]
 - 72*b^2*(d^4 - e^4*x^2)*Log[c*(d + e*Sqrt[x])^n]^2)/(144*e^4)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Maxima [A] time = 1.06406, size = 347, normalized size = 1.01

$$\frac{1}{2}b^2x^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-\frac{1}{12}aben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)+abx^2\log\left(\left(e\sqrt{x}+d\right)^{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*sqrt(x) + d)^n*c)^2 - 1/12*a*b*e*n*(12*d^4*log(e*sqrt(x)
 + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)
 + a*b*x^2*log((e*sqrt(x) + d)^n*c) + 1/2*a^2*x^2 - 1/144*(12*e*n*(12*d^4*l
og(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*s
qrt(x))/e^4)*log((e*sqrt(x) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt(x) +
 d)^2 - 28*d*e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*
d^3*e*sqrt(x))*n^2/e^4)*b^2
```

Fricas [A] time = 2.24195, size = 788, normalized size = 2.3

$$\frac{72\,b^2e^4x^2\log\left(c\right)^2+9\left(b^2e^4n^2-4\,abe^4n+8\,a^2e^4\right)x^2+72\left(b^2e^4n^2x^2-b^2d^4n^2\right)\log\left(e\sqrt{x}+d\right)^2+6\left(13\,b^2d^2e^2n^2-12\,abd^2e^{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(72*b^2*e^4*x^2*log(c)^2 + 9*(b^2*e^4*n^2 - 4*a*b*e^4*n + 8*a^2*e^4)*
x^2 + 72*(b^2*e^4*n^2*x^2 - b^2*d^4*n^2)*log(e*sqrt(x) + d)^2 + 6*(13*b^2*d
^2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 12*(6*b^2*d^2*e^2*n^2*x - 25*b^2*d^4*n^2
 + 12*a*b*d^4*n + 3*(b^2*e^4*n^2 - 4*a*b*e^4*n)*x^2 - 12*(b^2*e^4*n*x^2 - b
^2*d^4*n)*log(c) - 4*(b^2*d*e^3*n^2*x + 3*b^2*d^3*e*n^2)*sqrt(x))*log(e*sqr
t(x) + d) - 36*(2*b^2*d^2*e^2*n*x + (b^2*e^4*n - 4*a*b*e^4)*x^2)*log(c) - 4
*(75*b^2*d^3*e*n^2 - 36*a*b*d^3*e*n + (7*b^2*d*e^3*n^2 - 12*a*b*d*e^3*n)*x
- 12*(b^2*d*e^3*n*x + 3*b^2*d^3*e*n)*log(c))*sqrt(x))/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Giac [B] time = 1.2789, size = 1073, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/144*((72*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e +
 d)^3*d*e^(-2)*log(sqrt(x)*e + d)^2 + 432*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(
sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d)^2 - 36
*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d) + 192*(sqrt(x)*e + d)^3*d*e^(-
2)*log(sqrt(x)*e + d) - 432*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d)
 + 576*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d) + 9*(sqrt(x)*e + d)^4*
e^(-2) - 64*(sqrt(x)*e + d)^3*d*e^(-2) + 216*(sqrt(x)*e + d)^2*d^2*e^(-2) -
 576*(sqrt(x)*e + d)*d^3*e^(-2))*b^2*n^2*e^(-1) + 12*(12*(sqrt(x)*e + d)^4*
e^(-2)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-2)*log(sqrt(x)*e + d
) + 72*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)
*d^3*e^(-2)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-2) + 16*(sqrt(x)*e
 + d)^3*d*e^(-2) - 36*(sqrt(x)*e + d)^2*d^2*e^(-2) + 48*(sqrt(x)*e + d)*d^3
*e^(-2))*b^2*n*e^(-1)*log(c) + 72*((sqrt(x)*e + d)^4 - 4*(sqrt(x)*e + d)^3*
d + 6*(sqrt(x)*e + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*b^2*e^(-3)*log(c)^2 +
12*(12*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)^3*d
*e^(-2)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e
+ d) - 48*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)
^4*e^(-2) + 16*(sqrt(x)*e + d)^3*d*e^(-2) - 36*(sqrt(x)*e + d)^2*d^2*e^(-2)
 + 48*(sqrt(x)*e + d)*d^3*e^(-2))*a*b*n*e^(-1) + 144*((sqrt(x)*e + d)^4 - 4
*(sqrt(x)*e + d)^3*d + 6*(sqrt(x)*e + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*a*b
*e^(-3)*log(c) + 72*((sqrt(x)*e + d)^4 - 4*(sqrt(x)*e + d)^3*d + 6*(sqrt(x)
*e + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*a^2*e^(-3))*e^(-1)
```

3.410 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=195

$$-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}$$

```
[Out] (b^2*n^2*(d + e*Sqrt[x])^2)/(2*e^2) + (4*a*b*d*n*Sqrt[x])/e - (4*b^2*d*n^2*
Sqrt[x])/e + (4*b^2*d*n*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^2 - (b*
n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^2 - (2*d*(d + e*Sqr
t[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 + ((d + e*Sqrt[x])^2*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2)/e^2
```

Rubi [A] time = 0.183552, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2451, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```


```
[Out] (b^2*n^2*(d + e*Sqrt[x])^2)/(2*e^2) + (4*a*b*d*n*Sqrt[x])/e - (4*b^2*d*n^2*
Sqrt[x])/e + (4*b^2*d*n*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^2 - (b*
n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^2 - (2*d*(d + e*Sqr
t[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 + ((d + e*Sqrt[x])^2*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2)/e^2
```


Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(-\frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}\right) dx,$$

$$= \frac{2\,\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\,\text{Subst}\left(\int (a+b\log\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx, x, d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\,\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^2\right.}{e^2}$$

$$= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right.\right.}{e^2}$$

$$= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2} + \frac{4abdn\sqrt{x}}{e} - \frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2} - \frac{2d\left(d\right.}{}$$

$$= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2} + \frac{4abdn\sqrt{x}}{e} - \frac{4b^2dn^2\sqrt{x}}{e} + \frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}$$

Mathematica [A] time = 0.0820895, size = 150, normalized size = 0.77

$$\frac{-2a^2\left(d^2-e^2x\right)+2b\left(d+e\sqrt{x}\right)\left(-2ad+2ae\sqrt{x}+3bdn-ben\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)-2abn\left(d-e\sqrt{x}\right)^2-2b^2\left(d^2-e^2\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (-2*a*b*n*(d - e*Sqrt[x])^2 + b^2*e*n^2*(-6*d + e*Sqrt[x])*Sqrt[x] - 2*a^2*
(d^2 - e^2*x) + 2*b*(d + e*Sqrt[x])*(-2*a*d + 3*b*d*n + 2*a*e*Sqrt[x] - b*e
*n*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n] - 2*b^2*(d^2 - e^2*x)*Log[c*(d + e*Sqr
t[x])^n]^2)/(2*e^2)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Maxima [A] time = 1.05363, size = 242, normalized size = 1.24

$$-\left(en\left(\frac{2\,d^{2}\log\left(e\sqrt{x}+d\right)}{e^{3}}+\frac{ex-2\,d\sqrt{x}}{e^{2}}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)\right)ab-\frac{1}{2}\left(2\,en\left(\frac{2\,d^{2}\log\left(e\sqrt{x}+d\right)}{e^{3}}+\frac{ex-2\,d\sqrt{x}}{e^{2}}\right)\log\left(\left(e\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*log((e
*sqrt(x) + d)^n*c))*a*b - 1/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x -
 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) + d)^n*c)^
2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*d*e*
sqrt(x))*n^2/e^2)*b^2 + a^2*x
```

Fricas [A] time = 1.84546, size = 509, normalized size = 2.61

$$\frac{2\,b^{2}e^{2}x\log\left(c\right)^{2}+2\left(b^{2}e^{2}n^{2}x-b^{2}d^{2}n^{2}\right)\log\left(e\sqrt{x}+d\right)^{2}-2\left(b^{2}e^{2}n-2\,abe^{2}\right)x\log\left(c\right)+\left(b^{2}e^{2}n^{2}-2\,abe^{2}n+2\,a^{2}e^{2}\right)x+2\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*e^2*x*log(c)^2 + 2*(b^2*e^2*n^2*x - b^2*d^2*n^2)*log(e*sqrt(x) +
 d)^2 - 2*(b^2*e^2*n - 2*a*b*e^2)*x*log(c) + (b^2*e^2*n^2 - 2*a*b*e^2*n + 2
*a^2*e^2)*x + 2*(2*b^2*d*e*n^2*sqrt(x) + 3*b^2*d^2*n^2 - 2*a*b*d^2*n - (b^2
*e^2*n^2 - 2*a*b*e^2*n)*x + 2*(b^2*e^2*n*x - b^2*d^2*n)*log(c))*log(e*sqrt(
x) + d) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sqrt(x))/e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Giac [B] time = 1.33501, size = 487, normalized size = 2.5

$$\frac{1}{2}\Big(\Big(2\big(\sqrt{x}e+d\big)^2\log\big(\sqrt{x}e+d\big)^2-4\big(\sqrt{x}e+d\big)d\log\big(\sqrt{x}e+d\big)^2-2\big(\sqrt{x}e+d\big)^2\log\big(\sqrt{x}e+d\big)+8\big(\sqrt{x}e+d\big)d\log\big(\sqrt{x}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(sqrt(x)*e + d)*d*log(sq
rt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) + 8*(sqrt(x)*e + d)
*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x)*e + d)*d)*b^2*n^2*e^
(-1) + 2*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(
sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*b^2*n*e^(-1)*log(
c) + 2*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*b^2*e^(-1)*log(c)^2 + 2*(2
*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e +
 d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*a*b*n*e^(-1) + 4*((sqrt(x)*e
 + d)^2 - 2*(sqrt(x)*e + d)*d)*a*b*e^(-1)*log(c) + 2*((sqrt(x)*e + d)^2 - 2
*(sqrt(x)*e + d)*d)*a^2*e^(-1))*e^(-1)
```

3.411 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$4bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-4b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)] + 4*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d] - 4*b^2*n^2*PolyLog[
3, 1 + (e*Sqrt[x])/d]
```

Rubi [A] time = 0.130325, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$4bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-4b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)] + 4*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d] - 4*b^2*n^2*PolyLog[
3, 1 + (e*Sqrt[x])/d]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
```

```
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) - (4ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log}{d+ex}\right.$$
$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) - (4bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log}{x}\right.$$
$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) + 4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\text{Li}_2\left(1\right.$$
$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) + 4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\text{Li}_2\left(1\right.$$

Mathematica [B] time = 0.120154, size = 195, normalized size = 2.1

$$2bn\left(\log(x)\left(\log\left(d+e\sqrt{x}\right)-\log\left(\frac{e\sqrt{x}}{d}+1\right)\right)-2\text{PolyLog}\left(2,-\frac{e\sqrt{x}}{d}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)-bn\log\left(d+e\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((Log[d + e*Sqrt[
x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) + 2*
b^2*n^2*(Log[d + e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*
PolyLog[2, 1 + (e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d])
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2 \log \left(\left(e\sqrt{x}+d\right)^n\right)^2 \log(x) + \int \frac{\left(b^2 e \log(c)^2 + 2\,abe \log(c) + a^2 e\right)x - \left(b^2 enx \log(x) - 2\left(b^2 e \log(c) + abe\right)x - 2\left(b^2 d \log\right.\right.}{ex^2 + dx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((e*sqrt(x) + d)^n)^2*log(x) + integrate(((b^2*e*log(c)^2 + 2*a*b*e*
log(c) + a^2*e)*x - (b^2*e*n*x*log(x) - 2*(b^2*e*log(c) + a*b*e)*x - 2*(b^2
*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + (b^2*d*log(c)^2 + 2*a*
b*d*log(c) + a^2*d)*sqrt(x))/(e*x^2 + d*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \log \left(\left(e\sqrt{x}+d\right)^n c\right)^2 + 2\,ab \log \left(\left(e\sqrt{x}+d\right)^n c\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \log \left(c\left(d + e\sqrt{x}\right)^n\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(\left(e\sqrt{x}+d\right)^n c\right) + a\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x, x)
```

3.412 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=155

$$\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^2}-\frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}-\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}}$$

```
[Out] (-2*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^2*Sqrt[x]) -
 (2*b*e^2*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/d^
2 - (a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + (b^2*e^2*n^2*Log[x])/d^2 + (2*b^
2*e^2*n^2*PolyLog[2, d/(d + e*Sqrt[x])])/d^2
```

Rubi [A] time = 0.35124, antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$-\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^2}+\frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2}-\frac{2be^2n\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}-\frac{2be}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] (-2*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^2*Sqrt[x]) +
 (e^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/d^2 - (a + b*Log[c*(d + e*Sqrt[x]
)^n])^2/x - (2*b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d
)])/d^2 + (b^2*e^2*n^2*Log[x])/d^2 - (2*b^2*e^2*n^2*PolyLog[2, 1 + (e*Sqrt[
x])/d])/d^2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3}dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^2(d+ex)}dx, x, \sqrt{x}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+e\sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+e\sqrt{x}\right)}{d} - \frac{(2be}{}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} - \frac{(2be}{}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2} - \frac{(}{}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2} - \frac{(}{}$$

Mathematica [A] time = 0.163453, size = 188, normalized size = 1.21

$$2\left(ben\left(-\frac{ben\text{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)}{d^2} + \frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2bd^2n} - \frac{e\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} - \frac{a+b\log}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] 2*(-(a + b*Log[c*(d + e*Sqrt[x])^n])^2/(2*x) + b*e*n*(-((a + b*Log[c*(d + e
*Sqrt[x])^n])/(d*Sqrt[x])) + (e*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*b*d^
2*n) - (e*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 + (b*
e*n*(-(Log[d + e*Sqrt[x]]/d) + Log[x]/(2*d)))/d - (b*e*n*PolyLog[2, (d + e*
Sqrt[x])/d])/d^2))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\log\left(\frac{e\sqrt{x}}{d}+1\right)\log\left(\sqrt{x}\right)+\mathrm{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)b^2e^2n^2}{d^2}+\frac{2\left(abe^2n-\left(e^2n^2-e^2n\log\left(c\right)\right)b^2\right)\log\left(e\sqrt{x}+d\right)}{d^2}-\frac{2\left(b^2e^2n\log\left(c\right)+ab\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b^2*e^2*n^2/d^2
 + 2*(a*b*e^2*n - (e^2*n^2 - e^2*n*log(c))*b^2)*log(e*sqrt(x) + d)/d^2 - 2*
(b^2*e^2*n*log(c) + a*b*e^2*n)*log(sqrt(x))/d^2 + integrate((b^2*e^4*n^2*x
+ b^2*d^2*e^2*n^2)/x, x)/d^4 + 1/3*(2*b^2*e^5*n^2*x^(3/2) - 6*b^2*d^2*e^3*n
^2*sqrt(x)*log(sqrt(x)) - 3*b^2*d*e^4*n^2*x + 12*b^2*d^2*e^3*n^2*sqrt(x))/d
^5 - 1/3*(3*b^2*d^3*e^2*n^2*x^(3/2)*log(e*sqrt(x) + d)^2 + 2*b^2*e^5*n^2*x^
3 - 3*b^2*d^2*e^3*n^2*x^2*log(x) + 12*b^2*d^2*e^3*n^2*x^2 + 3*b^2*d^5*sqrt(
x)*log((e*sqrt(x) + d)^n)^2 + 6*(b^2*d^4*e*n*log(c) + a*b*d^4*e*n)*x - 3*(2
*b^2*d^3*e^2*n*x^(3/2)*log(e*sqrt(x) + d) - 2*b^2*d^4*e*n*x - (b^2*d^3*e^2*
n*x*log(x) + 2*b^2*d^5*log(c) + 2*a*b*d^5)*sqrt(x))*log((e*sqrt(x) + d)^n))
/(d^5*x^(3/2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2+2\,ab\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^2, x)
```

3.413 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=293

$$\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^4}-\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}}$$

```
[Out] -(b^2*e^2*n^2)/(6*d^2*x) + (5*b^2*e^3*n^2)/(6*d^3*Sqrt[x]) - (5*b^2*e^4*n^2
*Log[d + e*Sqrt[x]])/(6*d^4) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(3*
d*x^(3/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) - (b*e^3*
n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^4*Sqrt[x]) - (b*e^4*
n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/d^4 - (a + b
*Log[c*(d + e*Sqrt[x])^n])^2/(2*x^2) + (11*b^2*e^4*n^2*Log[x])/(12*d^4) + (
b^2*e^4*n^2*PolyLog[2, d/(d + e*Sqrt[x])])/d^4
```

Rubi [A] time = 0.667209, antiderivative size = 318, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^4}+\frac{e^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4}-\frac{be^4n\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{be^3n\left(d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -(b^2*e^2*n^2)/(6*d^2*x) + (5*b^2*e^3*n^2)/(6*d^3*Sqrt[x]) - (5*b^2*e^4*n^2
*Log[d + e*Sqrt[x]])/(6*d^4) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(3*
d*x^(3/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) - (b*e^3*
n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^4*Sqrt[x]) + (e^4*(a
 + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4) - (a + b*Log[c*(d + e*Sqrt[x])^n]
)^2/(2*x^2) - (b*e^4*n*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d
)])/d^4 + (11*b^2*e^4*n^2*Log[x])/(12*d^4) - (b^2*e^4*n^2*PolyLog[2, 1 + (e
*Sqrt[x])/d])/d^4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^5}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^4(d+ex)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{d} - \frac{(ben)\,\text{Su}}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} - \frac{(ben)\,\text{Subst}\left(\int \frac{a+b}{(-}\right.}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{\left(a+b\log\left(c\left(d\right.\right.\right.}{2x^2}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{b^2e^3n^2}{3d^3\sqrt{x}} - \frac{b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{3d^4} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} + \frac{be^2n\left(\right.}{}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{5b^2e^3n^2}{6d^3\sqrt{x}} - \frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} + \frac{be^2}{}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{5b^2e^3n^2}{6d^3\sqrt{x}} - \frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} + \frac{be^2}{}$$

Mathematica [A] time = 0.32548, size = 353, normalized size = 1.2

$$-\frac{e\sqrt{x}\left(12b^2e^3n^2x^{3/2}\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)+4bd^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6bd^2en\sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6e^3x^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+12be^3nx^{3/2}\log\left(-\frac{e\sqrt{x}}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -(6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2 + (e*Sqrt[x]*(4*b*d^3*n*(a + b*Log[c
*(d + e*Sqrt[x])^n]) - 6*b*d^2*e*n*Sqrt[x]*(a + b*Log[c*(d + e*Sqrt[x])^n])
 + 12*b*d*e^2*n*x*(a + b*Log[c*(d + e*Sqrt[x])^n]) - 6*e^3*x^(3/2)*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2 + 12*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e*Sqrt[x]
)^n])*Log[-((e*Sqrt[x])/d)] + 6*b^2*e^3*n^2*x^(3/2)*(2*Log[d + e*Sqrt[x]] -
 Log[x]) - 3*b^2*e^2*n^2*x*(2*d - 2*e*Sqrt[x]*Log[d + e*Sqrt[x]] + e*Sqrt[x
```

```
]*Log[x]) + 2*b^2*e*n^2*Sqrt[x]*(d*(d - 2*e*Sqrt[x]) + 2*e^2*x*Log[d + e*Sq
rt[x]] - e^2*x*Log[x]) + 12*b^2*e^3*n^2*x^(3/2)*PolyLog[2, 1 + (e*Sqrt[x])/
d]))/d^4)/(12*x^2)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(\left(e\sqrt{x}+d\right)^n\right)^2}{2\,x^2}+\int\frac{2\left(b^2e\log\left(c\right)^2+2\,abe\log\left(c\right)+a^2e\right)x+\left(b^2enx+4\left(b^2e\log\left(c\right)+abe\right)x+4\left(b^2d\log\left(c\right)+a\right.\right.}{2\left(ex^4+dx^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((e*sqrt(x) + d)^n)^2/x^2 + integrate(1/2*(2*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 4*(b^2*e*log(c) + a*b*e)*x + 4*(b^
2*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 2*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^4 + d*x^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(\left(e\sqrt{x}+d\right)^n c\right)^2+2\,ab\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^3, x)
```

3.414 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=408

$$\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{3d^6}-\frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2}-\frac{2be^6n\log\left(1-\frac{d}{d+\ldots}\right.}{}$$

```
[Out] -(b^2*e^2*n^2)/(30*d^2*x^2) + (b^2*e^3*n^2)/(10*d^3*x^(3/2)) - (47*b^2*e^4*
n^2)/(180*d^4*x) + (77*b^2*e^5*n^2)/(90*d^5*Sqrt[x]) - (77*b^2*e^6*n^2*Log[
d + e*Sqrt[x]])/(90*d^6) - (2*b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(15*d
*x^(5/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(6*d^2*x^2) - (2*b*e
^3*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*d^3*x^(3/2)) + (b*e^4*n*(a + b*Lo
g[c*(d + e*Sqrt[x])^n]))/(3*d^4*x) - (2*b*e^5*n*(d + e*Sqrt[x])*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*d^6*Sqrt[x]) - (2*b*e^6*n*Log[1 - d/(d + e*Sqrt[x
])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x]
)^n])^2/(3*x^3) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (2*b^2*e^6*n^2*PolyL
og[2, d/(d + e*Sqrt[x])])/(3*d^6)
```

Rubi [A] time = 1.03361, antiderivative size = 432, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{3d^6}-\frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2}+\frac{e^6\left(a+b\log\left(\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

```
[Out] -(b^2*e^2*n^2)/(30*d^2*x^2) + (b^2*e^3*n^2)/(10*d^3*x^(3/2)) - (47*b^2*e^4*
n^2)/(180*d^4*x) + (77*b^2*e^5*n^2)/(90*d^5*Sqrt[x]) - (77*b^2*e^6*n^2*Log[
d + e*Sqrt[x]])/(90*d^6) - (2*b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(15*d
*x^(5/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(6*d^2*x^2) - (2*b*e
^3*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*d^3*x^(3/2)) + (b*e^4*n*(a + b*Lo
g[c*(d + e*Sqrt[x])^n]))/(3*d^4*x) - (2*b*e^5*n*(d + e*Sqrt[x])*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*d^6*Sqrt[x]) + (e^6*(a + b*Log[c*(d + e*Sqrt[x])^
n])^2)/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x])^n])^2/(3*x^3) - (2*b*e^6*n*(a
 + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/(3*d^6) + (137*b^2*e^
6*n^2*Log[x])/(180*d^6) - (2*b^2*e^6*n^2*PolyLog[2, 1 + (e*Sqrt[x])/d])/(3*
d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, \sqrt{\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt{x}\right)}{3d} - \frac{(2be}{}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} - \frac{(2ben)\,\text{Subst}}{}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} - \frac{\left(a+b\log\right.}{}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{2b^2e^3n^2}{45d^3x^{3/2}} - \frac{b^2e^4n^2}{15d^4x} + \frac{2b^2e^5n^2}{15d^5\sqrt{x}} - \frac{2b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{15d^6} - \frac{2ben\left(a+b\right.}{}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{3b^2e^4n^2}{20d^4x} + \frac{3b^2e^5n^2}{10d^5\sqrt{x}} - \frac{3b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{10d^6} - \frac{2ben\left(a+\right.}{}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{47b^2e^5n^2}{90d^5\sqrt{x}} - \frac{47b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} - \frac{2ben\left(\right.}{}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} - \frac{2ben\left(\right.}{}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} - \frac{2ben\left(\right.}{}$$

Mathematica [A] time = 0.279603, size = 538, normalized size = 1.32

$$-\frac{120b^2e^6n^2x^3\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)+60a^2d^6-60a^2e^6x^3+120abd^6\log\left(c\left(d+e\sqrt{x}\right)^n\right)-120abe^6x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

```
[Out] -(60*a^2*d^6 + 24*a*b*d^5*e*n*Sqrt[x] - 30*a*b*d^4*e^2*n*x + 6*b^2*d^4*e^2*
n^2*x + 40*a*b*d^3*e^3*n*x^(3/2) - 18*b^2*d^3*e^3*n^2*x^(3/2) - 60*a*b*d^2*
e^4*n*x^2 + 47*b^2*d^2*e^4*n^2*x^2 + 120*a*b*d*e^5*n*x^(5/2) - 154*b^2*d*e^
5*n^2*x^(5/2) - 60*a^2*e^6*x^3 + 274*b^2*e^6*n^2*x^3*Log[d + e*Sqrt[x]] + 1
20*a*b*d^6*Log[c*(d + e*Sqrt[x])^n] + 24*b^2*d^5*e*n*Sqrt[x]*Log[c*(d + e*S
qrt[x])^n] - 30*b^2*d^4*e^2*n*x*Log[c*(d + e*Sqrt[x])^n] + 40*b^2*d^3*e^3*n
*x^(3/2)*Log[c*(d + e*Sqrt[x])^n] - 60*b^2*d^2*e^4*n*x^2*Log[c*(d + e*Sqrt[
x])^n] + 120*b^2*d*e^5*n*x^(5/2)*Log[c*(d + e*Sqrt[x])^n] - 120*a*b*e^6*x^3
*Log[c*(d + e*Sqrt[x])^n] + 60*b^2*d^6*Log[c*(d + e*Sqrt[x])^n]^2 - 60*b^2*
e^6*x^3*Log[c*(d + e*Sqrt[x])^n]^2 + 120*a*b*e^6*n*x^3*Log[-((e*Sqrt[x])/d)
] + 120*b^2*e^6*n*x^3*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] - 137*
b^2*e^6*n^2*x^3*Log[x] + 120*b^2*e^6*n^2*x^3*PolyLog[2, 1 + (e*Sqrt[x])/d])
/(180*d^6*x^3)
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(\left(e\sqrt{x}+d\right)^n\right)^2}{3\,x^3}+\int\frac{3\left(b^2e\log\left(c\right)^2+2\,abe\log\left(c\right)+a^2e\right)x+\left(b^2enx+6\left(b^2e\log\left(c\right)+abe\right)x+6\left(b^2d\log\left(c\right)+abd\right)\cdots\right)\cdots}{3\left(ex^5+dx^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log((e*sqrt(x) + d)^n)^2/x^3 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(b^
2*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 3*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^5 + d*x^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \log\left(\left(e\sqrt{x}+d\right)^n c\right)^2 + 2\,ab \log\left(\left(e\sqrt{x}+d\right)^n c\right) + a^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(\left(e\sqrt{x}+d\right)^n c\right) + a\right)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^4, x)
```

3.415 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=907

result too large to display

```
[Out] (-15*b^3*d^4*n^3*(d + e*Sqrt[x])^2)/(4*e^6) + (40*b^3*d^3*n^3*(d + e*Sqrt[x
])^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*Sqrt[x])^4)/(32*e^6) + (12*b^3*d*n^
3*(d + e*Sqrt[x])^5)/(125*e^6) - (b^3*n^3*(d + e*Sqrt[x])^6)/(108*e^6) - (1
2*a*b^2*d^5*n^2*Sqrt[x])/e^5 + (12*b^3*d^5*n^3*Sqrt[x])/e^5 - (12*b^3*d^5*n
^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^6 + (15*b^2*d^4*n^2*(d + e*S
qrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) - (40*b^2*d^3*n^2*(d +
e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) + (15*b^2*d^2*n^2*(d
 + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(8*e^6) - (12*b^2*d*n^2*(
d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(25*e^6) + (b^2*n^2*(d +
 e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(18*e^6) + (6*b*d^5*n*(d +
e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^6 - (15*b*d^4*n*(d + e*Sqr
t[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^6) + (20*b*d^3*n*(d + e*Sq
rt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(3*e^6) - (15*b*d^2*n*(d + e*S
qrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*e^6) + (6*b*d*n*(d + e*Sqr
t[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(5*e^6) - (b*n*(d + e*Sqrt[x])^
6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(6*e^6) - (2*d^5*(d + e*Sqrt[x])*(a +
 b*Log[c*(d + e*Sqrt[x])^n])^3)/e^6 + (5*d^4*(d + e*Sqrt[x])^2*(a + b*Log[c
*(d + e*Sqrt[x])^n])^3)/e^6 - (20*d^3*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e
*Sqrt[x])^n])^3)/(3*e^6) + (5*d^2*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqr
t[x])^n])^3)/e^6 - (2*d*(d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^
3)/e^6 + ((d + e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(3*e^6)
```

Rubi [A] time = 1.00708, antiderivative size = 907, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{108e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^6}{3e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^6}{6e^6}+\frac{b^2n^2\left(a+b\,\mathrm{l}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```


```
[Out] (-15*b^3*d^4*n^3*(d + e*Sqrt[x])^2)/(4*e^6) + (40*b^3*d^3*n^3*(d + e*Sqrt[x
])^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*Sqrt[x])^4)/(32*e^6) + (12*b^3*d*n^
3*(d + e*Sqrt[x])^5)/(125*e^6) - (b^3*n^3*(d + e*Sqrt[x])^6)/(108*e^6) - (1
2*a*b^2*d^5*n^2*Sqrt[x])/e^5 + (12*b^3*d^5*n^3*Sqrt[x])/e^5 - (12*b^3*d^5*n
^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^6 + (15*b^2*d^4*n^2*(d + e*S
qrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) - (40*b^2*d^3*n^2*(d +
e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) + (15*b^2*d^2*n^2*(d
 + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(8*e^6) - (12*b^2*d*n^2*(
d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(25*e^6) + (b^2*n^2*(d +
 e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(18*e^6) + (6*b*d^5*n*(d +
e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^6 - (15*b*d^4*n*(d + e*Sqr
t[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^6) + (20*b*d^3*n*(d + e*Sq
rt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(3*e^6) - (15*b*d^2*n*(d + e*S
qrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*e^6) + (6*b*d*n*(d + e*Sqr
t[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(5*e^6) - (b*n*(d + e*Sqrt[x])^
6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(6*e^6) - (2*d^5*(d + e*Sqrt[x])*(a +
```

```
 b*Log[c*(d + e*Sqrt[x])^n])^3)/e^6 + (5*d^4*(d + e*Sqrt[x])^2*(a + b*Log[c
*(d + e*Sqrt[x])^n])^3)/e^6 - (20*d^3*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e
*Sqrt[x])^n])^3)/(3*e^6) + (5*d^2*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqr
t[x])^n])^3)/e^6 - (2*d*(d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^
3)/e^6 + ((d + e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(3*e^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = 2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}\right.\right.$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e^5}-\frac{(10d)\,\text{Subst}\left(\int(d+ex)\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt{x}\right)}{e^6}-\frac{(10d)\,\text{Subst}\left(\int x^4\left(a+b\log(cx\right.\right.}{e^6}$$
$$= -\frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}+\frac{5d^4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{e^6}$$
$$= \frac{6bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^6}-\frac{15bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d\right.\right.\right.}{2e^6}$$
$$= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6}+\frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6}-\frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6}+\frac{12b^3dn^3}{1}$$
$$= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6}+\frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6}-\frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6}+\frac{12b^3dn^3}{1}$$

Mathematica [A] time = 0.47715, size = 577, normalized size = 0.64

$$\frac{-60b\left(1800a^2\left(d^6-e^6x^3\right)-60abn\left(20d^3e^3x^{3/2}-15d^2e^4x^2-30d^4e^2x+60d^5e\sqrt{x}+147d^6+12de^5x^{5/2}-10e^6x^3\right)+b^2n^2\left(1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e*n^3*Sqrt[x]*(809340*d^5 - 140070*d^4*e*Sqrt[x] + 41180*d^3*e^2*x - 1
3785*d^2*e^3*x^(3/2) + 4368*d*e^4*x^2 - 1000*e^5*x^(5/2)) + 1800*a^2*b*n*(1
47*d^6 + 60*d^5*e*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e^4*
x^2 + 12*d*e^5*x^(5/2) - 10*e^6*x^3) - 36000*a^3*(d^6 - e^6*x^3) + 60*a*b^2
*n^2*(8111*d^6 - 8820*d^5*e*Sqrt[x] + 2610*d^4*e^2*x - 1140*d^3*e^3*x^(3/2)
 + 555*d^2*e^4*x^2 - 264*d*e^5*x^(5/2) + 100*e^6*x^3) - 60*b*(b^2*n^2*(1348
9*d^6 + 8820*d^5*e*Sqrt[x] - 2610*d^4*e^2*x + 1140*d^3*e^3*x^(3/2) - 555*d^
2*e^4*x^2 + 264*d*e^5*x^(5/2) - 100*e^6*x^3) - 60*a*b*n*(147*d^6 + 60*d^5*e
*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e^4*x^2 + 12*d*e^5*x^
(5/2) - 10*e^6*x^3) + 1800*a^2*(d^6 - e^6*x^3))*Log[c*(d + e*Sqrt[x])^n] -
1800*b^2*(60*a*(d^6 - e^6*x^3) + b*n*(-147*d^6 - 60*d^5*e*Sqrt[x] + 30*d^4*
e^2*x - 20*d^3*e^3*x^(3/2) + 15*d^2*e^4*x^2 - 12*d*e^5*x^(5/2) + 10*e^6*x^3
))*Log[c*(d + e*Sqrt[x])^n]^2 - 36000*b^3*(d^6 - e^6*x^3)*Log[c*(d + e*Sqrt
[x])^n]^3)/(108000*e^6)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Maxima [A] time = 1.09476, size = 899, normalized size = 0.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log((e*sqrt(x) + d)^n*c)^3 + a*b^2*x^3*log((e*sqrt(x) + d)^n*c)
^2 + a^2*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a^3*x^3 - 1/60*a^2*b*e*n*(60*
d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^
2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6) - 1/1800*(60*e*n
*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e
^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)*log((e*sqrt
(x) + d)^n*c) - (100*e^6*x^3 - 264*d*e^5*x^(5/2) + 555*d^2*e^4*x^2 + 1800*d
^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(3/2) + 2610*d^4*e^2*x + 8820*d^6*
log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*n^2/e^6)*a*b^2 - 1/108000*(1800*e*
n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*
e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)*log((e*sqr
t(x) + d)^n*c)^2 + e*n*((1000*e^6*x^3 + 36000*d^6*log(e*sqrt(x) + d)^3 - 43
68*d*e^5*x^(5/2) + 13785*d^2*e^4*x^2 + 264600*d^6*log(e*sqrt(x) + d)^2 - 41
180*d^3*e^3*x^(3/2) + 140070*d^4*e^2*x + 809340*d^6*log(e*sqrt(x) + d) - 80
9340*d^5*e*sqrt(x))*n^2/e^7 - 60*(100*e^6*x^3 - 264*d*e^5*x^(5/2) + 555*d^2
*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(3/2) + 2610*d^4*
e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*n*log((e*sqrt(x)
+ d)^n*c)/e^7))*b^3
```

Fricas [A] time = 2.47634, size = 2655, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/108000*(36000*b^3*e^6*x^3*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
+ 18*a^2*b*e^6*n - 36*a^3*e^6)*x^3 + 36000*(b^3*e^6*n^3*x^3 - b^3*d^6*n^3)*
log(e*sqrt(x) + d)^3 - 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2 + 1
800*a^2*b*d^2*e^4*n)*x^2 - 1800*(15*b^3*d^2*e^4*n^3*x^2 + 30*b^3*d^4*e^2*n^
3*x - 147*b^3*d^6*n^3 + 60*a*b^2*d^6*n^2 + 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^
2)*x^3 - 60*(b^3*e^6*n^2*x^3 - b^3*d^6*n^2)*log(c) - 4*(3*b^3*d*e^5*n^3*x^2
 + 5*b^3*d^3*e^3*n^3*x + 15*b^3*d^5*e*n^3)*sqrt(x))*log(e*sqrt(x) + d)^2 -
9000*(3*b^3*d^2*e^4*n*x^2 + 6*b^3*d^4*e^2*n*x + 2*(b^3*e^6*n - 6*a*b^2*e^6)
*x^3)*log(c)^2 - 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n^2 + 1800*a
^2*b*d^4*e^2*n)*x - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b
*d^6*n - 100*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^3 - 15*(37*
b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 - 1800*(b^3*e^6*n*x^3 - b^3*d^6
*n)*log(c)^2 - 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x + 60*(15*b^
3*d^2*e^4*n^2*x^2 + 30*b^3*d^4*e^2*n^2*x - 147*b^3*d^6*n^2 + 60*a*b^2*d^6*n
 + 10*(b^3*e^6*n^2 - 6*a*b^2*e^6*n)*x^3)*log(c) + 12*(735*b^3*d^5*e*n^3 - 3
00*a*b^2*d^5*e*n^2 + 2*(11*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x^2 + 5*(19*
b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 20*(3*b^3*d*e^5*n^2*x^2 + 5*b^3
*d^3*e^3*n^2*x + 15*b^3*d^5*e*n^2)*log(c))*sqrt(x))*log(e*sqrt(x) + d) + 30
```

```
0*(20*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^3 + 3*(37*b^3*d^2*e^4*
n^2 - 60*a*b^2*d^2*e^4*n)*x^2 + 18*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n
)*x)*log(c) + 4*(202335*b^3*d^5*e*n^3 - 132300*a*b^2*d^5*e*n^2 + 27000*a^2*
b*d^5*e*n + 12*(91*b^3*d*e^5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)
*x^2 + 1800*(3*b^3*d*e^5*n*x^2 + 5*b^3*d^3*e^3*n*x + 15*b^3*d^5*e*n)*log(c)
^2 + 5*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*
n)*x - 180*(735*b^3*d^5*e*n^2 - 300*a*b^2*d^5*e*n + 2*(11*b^3*d*e^5*n^2 - 3
0*a*b^2*d*e^5*n)*x^2 + 5*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x)*log(c
))*sqrt(x))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4142, size = 3444, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/108000*((36000*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d)^3 - 216000*(sq
rt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d)^3 + 540000*(sqrt(x)*e + d)^4*d^2
*e^(-4)*log(sqrt(x)*e + d)^3 - 720000*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt
(x)*e + d)^3 + 540000*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d)^3 - 2
16000*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d)^3 - 18000*(sqrt(x)*e +
d)^6*e^(-4)*log(sqrt(x)*e + d)^2 + 129600*(sqrt(x)*e + d)^5*d*e^(-4)*log(sq
rt(x)*e + d)^2 - 405000*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d)^2 +
 720000*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d)^2 - 810000*(sqrt(x)
*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d)^2 + 648000*(sqrt(x)*e + d)*d^5*e^(-
4)*log(sqrt(x)*e + d)^2 + 6000*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d)
- 51840*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) + 202500*(sqrt(x)*e +
 d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) - 480000*(sqrt(x)*e + d)^3*d^3*e^(-4)*l
og(sqrt(x)*e + d) + 810000*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d)
- 1296000*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) - 1000*(sqrt(x)*e +
 d)^6*e^(-4) + 10368*(sqrt(x)*e + d)^5*d*e^(-4) - 50625*(sqrt(x)*e + d)^4*d
^2*e^(-4) + 160000*(sqrt(x)*e + d)^3*d^3*e^(-4) - 405000*(sqrt(x)*e + d)^2*
d^4*e^(-4) + 1296000*(sqrt(x)*e + d)*d^5*e^(-4))*b^3*n^3*e^(-1) + 60*(1800*
(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e + d)^5*d*e
^(-4)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)
*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d)^2 + 27000
*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e + d)*
d^5*e^(-4)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*
e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) - 13500*(sqrt(x
)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) + 24000*(sqrt(x)*e + d)^3*d^3*e^(-
4)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e +
d) + 21600*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) + 100*(sqrt(x)*e +
```

d)^6*e^(-4) - 864*(sqrt(x)*e + d)^5*d*e^(-4) + 3375*(sqrt(x)*e + d)^4*d^2*e^(-4) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-4) + 13500*(sqrt(x)*e + d)^2*d^4*e^(-4) - 21600*(sqrt(x)*e + d)*d^5*e^(-4))*b^3*n^2*e^(-1)*log(c) + 1800*(60*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-4) + 72*(sqrt(x)*e + d)^5*d*e^(-4) - 225*(sqrt(x)*e + d)^4*d^2*e^(-4) + 400*(sqrt(x)*e + d)^3*d^3*e^(-4) - 450*(sqrt(x)*e + d)^2*d^4*e^(-4) + 360*(sqrt(x)*e + d)*d^5*e^(-4))*b^3*n*e^(-1)*log(c)^2 + 36000*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*e + d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqrt(x)*e + d)*d^5)*b^3*e^(-5)*log(c)^3 + 60*(1800*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) - 13500*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) + 24000*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) + 100*(sqrt(x)*e + d)^6*e^(-4) - 864*(sqrt(x)*e + d)^5*d*e^(-4) + 3375*(sqrt(x)*e + d)^4*d^2*e^(-4) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-4) + 13500*(sqrt(x)*e + d)^2*d^4*e^(-4) - 21600*(sqrt(x)*e + d)*d^5*e^(-4))*a*b^2*n^2*e^(-1) + 3600*(60*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-4) + 72*(sqrt(x)*e + d)^5*d*e^(-4) - 225*(sqrt(x)*e + d)^4*d^2*e^(-4) + 400*(sqrt(x)*e + d)^3*d^3*e^(-4) - 450*(sqrt(x)*e + d)^2*d^4*e^(-4) + 360*(sqrt(x)*e + d)*d^5*e^(-4))*a*b^2*n*e^(-1)*log(c) + 108000*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*e + d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqrt(x)*e + d)*d^5)*a*b^2*e^(-5)*log(c)^2 + 1800*(60*(sqrt(x)*e + d)^6*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-4)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-4)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-4)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-4)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-4) + 72*(sqrt(x)*e + d)^5*d*e^(-4) - 225*(sqrt(x)*e + d)^4*d^2*e^(-4) + 400*(sqrt(x)*e + d)^3*d^3*e^(-4) - 450*(sqrt(x)*e + d)^2*d^4*e^(-4) + 360*(sqrt(x)*e + d)*d^5*e^(-4))*a^2*b*n*e^(-1) + 108000*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*e + d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqrt(x)*e + d)*d^5)*a^2*b*e^(-5)*log(c) + 36000*((sqrt(x)*e + d)^6 - 6*(sqrt(x)*e + d)^5*d + 15*(sqrt(x)*e + d)^4*d^2 - 20*(sqrt(x)*e + d)^3*d^3 + 15*(sqrt(x)*e + d)^2*d^4 - 6*(sqrt(x)*e + d)*d^5)*a^3*e^(-5))*e^(-1)

3.416 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=595

$$\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3\left(a+\cdots\right.}{3e^4\cdots}$$

```
[Out] (-9*b^3*d^2*n^3*(d + e*Sqrt[x])^2)/(4*e^4) + (4*b^3*d*n^3*(d + e*Sqrt[x])^3
)/(9*e^4) - (3*b^3*n^3*(d + e*Sqrt[x])^4)/(64*e^4) - (12*a*b^2*d^3*n^2*Sqrt
[x])/e^3 + (12*b^3*d^3*n^3*Sqrt[x])/e^3 - (12*b^3*d^3*n^2*(d + e*Sqrt[x])*L
og[c*(d + e*Sqrt[x])^n])/e^4 + (9*b^2*d^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*e^4) + (3*b^2*n^2*(d + e*Sqrt[x])^4*(a + b*Log[c*
(d + e*Sqrt[x])^n]))/(16*e^4) + (6*b*d^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d
+ e*Sqrt[x])^n])^2)/e^4 - (9*b*d^2*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*
Sqrt[x])^n])^2)/(2*e^4) + (2*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sq
rt[x])^n])^2)/e^4 - (3*b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(8*e^4) - (2*d^3*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/
e^4 + (3*d^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 - (2
*d*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 + ((d + e*Sqrt
[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^4)
```

Rubi [A] time = 0.619151, antiderivative size = 595, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3\left(a+\cdots\right.}{3e^4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-9*b^3*d^2*n^3*(d + e*Sqrt[x])^2)/(4*e^4) + (4*b^3*d*n^3*(d + e*Sqrt[x])^3
)/(9*e^4) - (3*b^3*n^3*(d + e*Sqrt[x])^4)/(64*e^4) - (12*a*b^2*d^3*n^2*Sqrt
[x])/e^3 + (12*b^3*d^3*n^3*Sqrt[x])/e^3 - (12*b^3*d^3*n^2*(d + e*Sqrt[x])*L
og[c*(d + e*Sqrt[x])^n])/e^4 + (9*b^2*d^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*e^4) + (3*b^2*n^2*(d + e*Sqrt[x])^4*(a + b*Log[c*
(d + e*Sqrt[x])^n]))/(16*e^4) + (6*b*d^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d
+ e*Sqrt[x])^n])^2)/e^4 - (9*b*d^2*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*
Sqrt[x])^n])^2)/(2*e^4) + (2*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sq
rt[x])^n])^2)/e^4 - (3*b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(8*e^4) - (2*d^3*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/
e^4 + (3*d^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 - (2
*d*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 + ((d + e*Sqrt
[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^4)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = 2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}+\frac{3d^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}-\right.\right.$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e^3}-\frac{(6d)\,\text{Subst}\left(\int(d+ex)^2\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x^3\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt{x}\right)}{e^4}-\frac{(6d)\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)\right.}{e^4}$$
$$= -\frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}+\frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{e^4}$$
$$= \frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}-\frac{9bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{2e^4}$$
$$= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-\frac{12ab^2d^3n^2\sqrt{x}}{e^3}+$$
$$= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-\frac{12ab^2d^3n^2\sqrt{x}}{e^3}+$$

Mathematica [A] time = 0.297533, size = 433, normalized size = 0.73

$$\frac{-12b\left(72a^2\left(d^4-e^4x^2\right)-12abn\left(-6d^2e^2x+12d^3e\sqrt{x}+25d^4+4de^3x^{3/2}-3e^4x^2\right)+b^2n^2\left(-78d^2e^2x+300d^3e\sqrt{x}+415d^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e*n^3*Sqrt[x]*(4980*d^3 - 690*d^2*e*Sqrt[x] + 148*d*e^2*x - 27*e^3*x^(
3/2)) + 72*a^2*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(3/
2) - 3*e^4*x^2) - 288*a^3*(d^4 - e^4*x^2) + 12*a*b^2*n^2*(161*d^4 - 300*d^3
*e*Sqrt[x] + 78*d^2*e^2*x - 28*d*e^3*x^(3/2) + 9*e^4*x^2) - 12*b*(b^2*n^2*(
415*d^4 + 300*d^3*e*Sqrt[x] - 78*d^2*e^2*x + 28*d*e^3*x^(3/2) - 9*e^4*x^2)
- 12*a*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(3/2) - 3*e
^4*x^2) + 72*a^2*(d^4 - e^4*x^2))*Log[c*(d + e*Sqrt[x])^n] - 72*b^2*(12*a*(
d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*Sqrt[x] + 6*d^2*e^2*x - 4*d*e^3*x^
(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x])^n]^2 - 288*b^3*(d^4 - e^4*x^2)*Lo
g[c*(d + e*Sqrt[x])^n]^3)/(576*e^4)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Maxima [A] time = 1.12213, size = 724, normalized size = 1.22

$$\frac{1}{2}b^3x^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^3+\frac{3}{2}ab^2x^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-\frac{1}{8}a^2ben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-}{e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*sqrt(x) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*sqrt(x) + d)^
n*c)^2 - 1/8*a^2*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^
2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 3/2*a^2*b*x^2*log((e*sqrt(x)
 + d)^n*c) + 1/2*a^3*x^2 - 1/48*(12*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3
*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*sqrt(x
) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt(x) + d)^2 - 28*d*e^3*x^(3/2) +
 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*d^3*e*sqrt(x))*n^2/e^4)*a*
b^2 - 1/576*(72*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x
^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*sqrt(x) + d)^n*c)^2 + e*n*
((288*d^4*log(e*sqrt(x) + d)^3 + 27*e^4*x^2 + 1800*d^4*log(e*sqrt(x) + d)^2
 - 148*d*e^3*x^(3/2) + 690*d^2*e^2*x + 4980*d^4*log(e*sqrt(x) + d) - 4980*d
^3*e*sqrt(x))*n^2/e^5 - 12*(9*e^4*x^2 + 72*d^4*log(e*sqrt(x) + d)^2 - 28*d*
e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*d^3*e*sqrt(x)
)*n*log((e*sqrt(x) + d)^n*c)/e^5))*b^3
```

Fricas [A] time = 2.26034, size = 1875, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/576*(288*b^3*e^4*x^2*log(c)^3 + 288*(b^3*e^4*n^3*x^2 - b^3*d^4*n^3)*log(e
*sqrt(x) + d)^3 - 9*(3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 + 24*a^2*b*e^4*n - 32
*a^3*e^4)*x^2 - 72*(6*b^3*d^2*e^2*n^3*x - 25*b^3*d^4*n^3 + 12*a*b^2*d^4*n^2
 + 3*(b^3*e^4*n^3 - 4*a*b^2*e^4*n^2)*x^2 - 12*(b^3*e^4*n^2*x^2 - b^3*d^4*n^
2)*log(c) - 4*(b^3*d*e^3*n^3*x + 3*b^3*d^3*e*n^3)*sqrt(x))*log(e*sqrt(x) +
d)^2 - 216*(2*b^3*d^2*e^2*n*x + (b^3*e^4*n - 4*a*b^2*e^4)*x^2)*log(c)^2 - 6
*(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*x - 12*
(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n - 9*(b^3*e^4*n^3 - 4*
a*b^2*e^4*n^2 + 8*a^2*b*e^4*n)*x^2 - 72*(b^3*e^4*n*x^2 - b^3*d^4*n)*log(c)^
2 - 6*(13*b^3*d^2*e^2*n^3 - 12*a*b^2*d^2*e^2*n^2)*x + 12*(6*b^3*d^2*e^2*n^2
*x - 25*b^3*d^4*n^2 + 12*a*b^2*d^4*n + 3*(b^3*e^4*n^2 - 4*a*b^2*e^4*n)*x^2)
*log(c) + 4*(75*b^3*d^3*e*n^3 - 36*a*b^2*d^3*e*n^2 + (7*b^3*d*e^3*n^3 - 12*
a*b^2*d*e^3*n^2)*x - 12*(b^3*d*e^3*n^2*x + 3*b^3*d^3*e*n^2)*log(c))*sqrt(x)
)*log(e*sqrt(x) + d) + 36*(3*(b^3*e^4*n^2 - 4*a*b^2*e^4*n + 8*a^2*b*e^4)*x^
2 + 2*(13*b^3*d^2*e^2*n^2 - 12*a*b^2*d^2*e^2*n)*x)*log(c) + 4*(1245*b^3*d^3
*e*n^3 - 900*a*b^2*d^3*e*n^2 + 216*a^2*b*d^3*e*n + 72*(b^3*d*e^3*n*x + 3*b^
3*d^3*e*n)*log(c)^2 + (37*b^3*d*e^3*n^3 - 84*a*b^2*d*e^3*n^2 + 72*a^2*b*d*e
^3*n)*x - 12*(75*b^3*d^3*e*n^2 - 36*a*b^2*d^3*e*n + (7*b^3*d*e^3*n^2 - 12*a
*b^2*d*e^3*n)*x)*log(c))*sqrt(x))/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Giac [B] time = 1.26749, size = 2283, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/576*((288*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d)^3 - 1152*(sqrt(x)*e
 + d)^3*d*e^(-2)*log(sqrt(x)*e + d)^3 + 1728*(sqrt(x)*e + d)^2*d^2*e^(-2)*l
og(sqrt(x)*e + d)^3 - 1152*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d)^3
- 216*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d)^2 + 1152*(sqrt(x)*e + d)^
3*d*e^(-2)*log(sqrt(x)*e + d)^2 - 2592*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqr
t(x)*e + d)^2 + 3456*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d)^2 + 108*
(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d) - 768*(sqrt(x)*e + d)^3*d*e^(-2
)*log(sqrt(x)*e + d) + 2592*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d)
 - 6912*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d) - 27*(sqrt(x)*e + d)^
4*e^(-2) + 256*(sqrt(x)*e + d)^3*d*e^(-2) - 1296*(sqrt(x)*e + d)^2*d^2*e^(-
2) + 6912*(sqrt(x)*e + d)*d^3*e^(-2))*b^3*n^3*e^(-1) + 12*(72*(sqrt(x)*e +
d)^4*e^(-2)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)^3*d*e^(-2)*log(sqrt(
x)*e + d)^2 + 432*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d)^2 - 288*(
sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d)^2 - 36*(sqrt(x)*e + d)^4*e^(-2
)*log(sqrt(x)*e + d) + 192*(sqrt(x)*e + d)^3*d*e^(-2)*log(sqrt(x)*e + d) -
432*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d) + 576*(sqrt(x)*e + d)*d
^3*e^(-2)*log(sqrt(x)*e + d) + 9*(sqrt(x)*e + d)^4*e^(-2) - 64*(sqrt(x)*e +
 d)^3*d*e^(-2) + 216*(sqrt(x)*e + d)^2*d^2*e^(-2) - 576*(sqrt(x)*e + d)*d^3
*e^(-2))*b^3*n^2*e^(-1)*log(c) + 72*(12*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt(x
)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-2)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e
 + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-2)*log(s
qrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-2) + 16*(sqrt(x)*e + d)^3*d*e^(-2)
- 36*(sqrt(x)*e + d)^2*d^2*e^(-2) + 48*(sqrt(x)*e + d)*d^3*e^(-2))*b^3*n*e^
(-1)*log(c)^2 + 288*((sqrt(x)*e + d)^4 - 4*(sqrt(x)*e + d)^3*d + 6*(sqrt(x)
*e + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*b^3*e^(-3)*log(c)^3 + 12*(72*(sqrt(x
)*e + d)^4*e^(-2)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)^3*d*e^(-2)*log
(sqrt(x)*e + d)^2 + 432*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d)^2 -
 288*(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d)^2 - 36*(sqrt(x)*e + d)^4
*e^(-2)*log(sqrt(x)*e + d) + 192*(sqrt(x)*e + d)^3*d*e^(-2)*log(sqrt(x)*e +
 d) - 432*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d) + 576*(sqrt(x)*e
+ d)*d^3*e^(-2)*log(sqrt(x)*e + d) + 9*(sqrt(x)*e + d)^4*e^(-2) - 64*(sqrt(
x)*e + d)^3*d*e^(-2) + 216*(sqrt(x)*e + d)^2*d^2*e^(-2) - 576*(sqrt(x)*e +
d)*d^3*e^(-2))*a*b^2*n^2*e^(-1) + 144*(12*(sqrt(x)*e + d)^4*e^(-2)*log(sqrt
(x)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-2)*log(sqrt(x)*e + d) + 72*(sqrt(x)
*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-2)*log
(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-2) + 16*(sqrt(x)*e + d)^3*d*e^(-2
) - 36*(sqrt(x)*e + d)^2*d^2*e^(-2) + 48*(sqrt(x)*e + d)*d^3*e^(-2))*a*b^2*
n*e^(-1)*log(c) + 864*((sqrt(x)*e + d)^4 - 4*(sqrt(x)*e + d)^3*d + 6*(sqrt(
```

```
x)*e + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*a*b^2*e^(-3)*log(c)^2 + 72*(12*(sq
rt(x)*e + d)^4*e^(-2)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-2)*lo
g(sqrt(x)*e + d) + 72*(sqrt(x)*e + d)^2*d^2*e^(-2)*log(sqrt(x)*e + d) - 48*
(sqrt(x)*e + d)*d^3*e^(-2)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-2)
+ 16*(sqrt(x)*e + d)^3*d*e^(-2) - 36*(sqrt(x)*e + d)^2*d^2*e^(-2) + 48*(sqr
t(x)*e + d)*d^3*e^(-2))*a^2*b*n*e^(-1) + 864*((sqrt(x)*e + d)^4 - 4*(sqrt(x
)*e + d)^3*d + 6*(sqrt(x)*e + d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*a^2*b*e^(-3
)*log(c) + 288*((sqrt(x)*e + d)^4 - 4*(sqrt(x)*e + d)^3*d + 6*(sqrt(x)*e +
d)^2*d^2 - 4*(sqrt(x)*e + d)*d^3)*a^3*e^(-3))*e^(-1)
```

3.417 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=284

$$\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}+\frac{6bdn\left(d+e\right.}{}$$

```
[Out] (-3*b^3*n^3*(d + e*Sqrt[x])^2)/(4*e^2) - (12*a*b^2*d*n^2*Sqrt[x])/e + (12*b
^3*d*n^3*Sqrt[x])/e - (12*b^3*d*n^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n
])/e^2 + (3*b^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*
e^2) + (6*b*d*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 - (
3*b*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^2) - (2*d*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2 + ((d + e*Sqrt[x])^
2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2
```

Rubi [A] time = 0.251127, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2451, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}+\frac{6bdn\left(d+e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```


```
[Out] (-3*b^3*n^3*(d + e*Sqrt[x])^2)/(4*e^2) - (12*a*b^2*d*n^2*Sqrt[x])/e + (12*b
^3*d*n^3*Sqrt[x])/e - (12*b^3*d*n^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n
])/e^2 + (3*b^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*
e^2) + (6*b*d*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 - (
3*b*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^2) - (2*d*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2 + ((d + e*Sqrt[x])^
2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2
```


Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}\right)dx,$$

$$= \frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e}-\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\right.\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt{x}\right)}{e^2}-\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^3\right.}{e^2}$$

$$= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right.\right.}{e^2}$$

$$= \frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{2e^2}$$

$$= -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right.}{2e^2}$$

$$= -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}+\frac{12b^3dn^3\sqrt{x}}{e}-\frac{12b^3dn^2\left(d+e\sqrt{x}\right)\log\left(c\left(d\right.\right.}{e^2}$$

Mathematica [A] time = 0.202161, size = 241, normalized size = 0.85

$$4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3-8d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3+24bdn\left(\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-8*d*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 4*(d + e*Sqrt[x]
)^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 24*b*d*n*((d + e*Sqrt[x])*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2 - 2*b*n*(e*(a - b*n)*Sqrt[x] + b*(d + e*Sqrt[x])
*Log[c*(d + e*Sqrt[x])^n])) - 3*b*n*(2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d +
e*Sqrt[x])^n])^2 + b*n*(b*e*n*(2*d*Sqrt[x] + e*x) - 2*(d + e*Sqrt[x])^2*(a
+ b*Log[c*(d + e*Sqrt[x])^n]))))/(4*e^2)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Maxima [A] time = 1.12399, size = 514, normalized size = 1.81

$$-\frac{3}{2}\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)\right)a^2b-\frac{3}{2}\left(2\,en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] -3/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*lo
g((e*sqrt(x) + d)^n*c))*a^2*b - 3/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 +
(e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) + d)
^n*c)^2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) -
6*d*e*sqrt(x))*n^2/e^2)*a*b^2 - 1/4*(6*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 +
(e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c)^2 - 4*x*log((e*sqrt(x) +
d)^n*c)^3 + e*n*((4*d^2*log(e*sqrt(x) + d)^3 + 18*d^2*log(e*sqrt(x) + d)^2
+ 3*e^2*x + 42*d^2*log(e*sqrt(x) + d) - 42*d*e*sqrt(x))*n^2/e^3 - 6*(2*d^2*
log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*d*e*sqrt(x))*n*
log((e*sqrt(x) + d)^n*c)/e^3))*b^3 + a^3*x
```

Fricas [B] time = 1.91796, size = 1146, normalized size = 4.04

$$4\,b^3e^2x\log\left(c\right)^3+4\left(b^3e^2n^3x-b^3d^2n^3\right)\log\left(e\sqrt{x}+d\right)^3-6\left(b^3e^2n-2\,ab^2e^2\right)x\log\left(c\right)^2+6\left(2\,b^3den^3\sqrt{x}+3\,b^3d^2n^3-2\,ab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*e^2*x*log(c)^3 + 4*(b^3*e^2*n^3*x - b^3*d^2*n^3)*log(e*sqrt(x) +
 d)^3 - 6*(b^3*e^2*n - 2*a*b^2*e^2)*x*log(c)^2 + 6*(2*b^3*d*e*n^3*sqrt(x) +
 3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2 - (b^3*e^2*n^3 - 2*a*b^2*e^2*n^2)*x + 2*(b
^3*e^2*n^2*x - b^3*d^2*n^2)*log(c))*log(e*sqrt(x) + d)^2 + 6*(b^3*e^2*n^2 -
 2*a*b^2*e^2*n + 2*a^2*b*e^2)*x*log(c) - (3*b^3*e^2*n^3 - 6*a*b^2*e^2*n^2 +
 6*a^2*b*e^2*n - 4*a^3*e^2)*x - 6*(7*b^3*d^2*n^3 - 6*a*b^2*d^2*n^2 + 2*a^2*
b*d^2*n - 2*(b^3*e^2*n*x - b^3*d^2*n)*log(c)^2 - (b^3*e^2*n^3 - 2*a*b^2*e^2
*n^2 + 2*a^2*b*e^2*n)*x - 2*(3*b^3*d^2*n^2 - 2*a*b^2*d^2*n - (b^3*e^2*n^2 -
 2*a*b^2*e^2*n)*x)*log(c) + 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log(c) - 2*a*b
^2*d*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) + 6*(7*b^3*d*e*n^3 + 2*b^3*d*e*n*lo
g(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*n^2 - 2*a*b^2*d*e*n
)*log(c))*sqrt(x))/e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^{n}\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Giac [B] time = 1.17528, size = 1030, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*((4*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^3 - 8*(sqrt(x)*e + d)*d*log(sq
rt(x)*e + d)^3 - 6*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 + 24*(sqrt(x)*e +
 d)*d*log(sqrt(x)*e + d)^2 + 6*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 48*(s
qrt(x)*e + d)*d*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^2 + 48*(sqrt(x)*e +
d)*d)*b^3*n^3*e^(-1) + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(sqr
t(x)*e + d)*d*log(sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)
 + 8*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x)*
e + d)*d)*b^3*n^2*e^(-1)*log(c) + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)
 - 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*
e + d)*d)*b^3*n*e^(-1)*log(c)^2 + 4*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*
d)*b^3*e^(-1)*log(c)^3 + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(s
qrt(x)*e + d)*d*log(sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e +
d) + 8*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x
)*e + d)*d)*a*b^2*n^2*e^(-1) + 12*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) -
 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e
+ d)*d)*a*b^2*n*e^(-1)*log(c) + 12*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d
)*a*b^2*e^(-1)*log(c)^2 + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sq
rt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d
```

```
)*a^2*b*n*e^(-1) + 12*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*a^2*b*e^(-1
)*log(c) + 4*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*a^3*e^(-1))*e^(-1)
```

3.418 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$-12b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+6bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+$$

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[-((e*Sqrt[x])/d)] + 6*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2*PolyLog[2, 1 + (e*Sqrt[x])/d] - 12*b^2*n^2*(a +
b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[3, 1 + (e*Sqrt[x])/d] + 12*b^3*n^3*Poly
Log[4, 1 + (e*Sqrt[x])/d]
```

Rubi [A] time = 0.196425, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-12b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+6bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```


```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[-((e*Sqrt[x])/d)] + 6*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2*PolyLog[2, 1 + (e*Sqrt[x])/d] - 12*b^2*n^2*(a +
b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[3, 1 + (e*Sqrt[x])/d] + 12*b^3*n^3*Poly
Log[4, 1 + (e*Sqrt[x])/d]
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \sqrt{x}\right)$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) - (6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) - (6bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(-\right.}{x}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\text{Li}_2\left(1+\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\text{Li}_2\left(1+\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\text{Li}_2\left(1+\right.$$

Mathematica [B] time = 0.154776, size = 333, normalized size = 2.47

$$6b^2n^2\left(-2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(d+e\sqrt{x}\right)\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)+\log\left(-\frac{e\sqrt{x}}{d}\right)\log^2\left(d+e\sqrt{x}\right)\right)\left(a+b\log\left(c\left(d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*((Log[d + e*Sqr
t[x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) +
6*b^2*n^2*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(Log[d
+ e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*PolyLog[2, 1 +
(e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d]) + 2*b^3*n^3*(Log[d + e*Sq
rt[x]]^3*Log[-((e*Sqrt[x])/d)] + 3*Log[d + e*Sqrt[x]]^2*PolyLog[2, 1 + (e*S
qrt[x])/d] - 6*Log[d + e*Sqrt[x]]*PolyLog[3, 1 + (e*Sqrt[x])/d] + 6*PolyLog
```

```
[4, 1 + (e*Sqrt[x])/d])
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)

[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^{3}\log\left(\left(e\sqrt{x}+d\right)^{n}\right)^{3}\log\left(x\right)+\int-\frac{3\left(b^{3}enx\log\left(x\right)-2\left(b^{3}e\log\left(c\right)+ab^{2}e\right)x-2\left(b^{3}d\log\left(c\right)+ab^{2}d\right)\sqrt{x}\right)\log\left(\left(e\sqrt{x}+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="maxima")

[Out] b^3*log((e*sqrt(x) + d)^n)^3*log(x) + integrate(-1/2*(3*(b^3*e*n*x*log(x) -
 2*(b^3*e*log(c) + a*b^2*e)*x - 2*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*log((e*
sqrt(x) + d)^n)^2 - 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(
c) + a^3*e)*x - 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d
*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) - 2
*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/
(e*x^2 + d*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{3}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{3}+3\,ab^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{2}+3\,a^{2}b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="fricas")

[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x, x)
```

3.419 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=263

$$\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^2}$$

```
[Out] (-3*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(d^2*Sqrt[x])
 - (3*b*e^2*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n])^2
)/d^2 - (a + b*Log[c*(d + e*Sqrt[x])^n])^3/x + (6*b^2*e^2*n^2*(a + b*Log[c*
(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 + (6*b^2*e^2*n^2*(a + b*Log[
c*(d + e*Sqrt[x])^n])*PolyLog[2, d/(d + e*Sqrt[x])])/d^2 + (6*b^3*e^2*n^3*P
olyLog[2, 1 + (e*Sqrt[x])/d])/d^2 + (6*b^3*e^2*n^3*PolyLog[3, d/(d + e*Sqrt
[x])])/d^2
```

Rubi [A] time = 0.594744, antiderivative size = 283, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391}

$$-\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(d^2*Sqrt[x])
 + (e^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/d^2 - (a + b*Log[c*(d + e*Sqrt[
x])^n])^3/x + (6*b^2*e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt
[x])/d)])/d^2 - (3*b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt
[x])/d)])/d^2 + (6*b^3*e^2*n^3*PolyLog[2, 1 + (e*Sqrt[x])/d])/d^2 - (6*b^2*
e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d])/d^2
 + (6*b^3*e^2*n^3*PolyLog[3, 1 + (e*Sqrt[x])/d])/d^2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + (3ben)\,\mathrm{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(d+ex)}\,dx, x,$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + (3bn)\,\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + \frac{(3bn)\,\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{d} - \frac{(3$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} - \frac{(3b$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + \frac{6b$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{d^2} -$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{d^2} -$$

Mathematica [B] time = 0.712587, size = 536, normalized size = 2.04

$$\frac{3b^2n^2\left(-2e^2x\,\mathrm{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)-2e^2x\left(\log\left(d+e\sqrt{x}\right)-1\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+\left(d+e\sqrt{x}\right)\log\left(d+e\sqrt{x}\right)\left(\left(e\sqrt{x}-d\right)\log\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n
])^2 - 3*b*d^2*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(
d + e*Sqrt[x])^n])^2 + 3*b*e^2*n*x*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sq
rt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - d^2*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^3 - (3*b*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x])/2 + 3*b^2*n^2*(a - b*n*Log[d + e*Sqrt
[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]*(-2*
e*Sqrt[x] + (-d + e*Sqrt[x])*Log[d + e*Sqrt[x]]) - 2*e^2*x*(-1 + Log[d + e*
Sqrt[x]])*Log[-((e*Sqrt[x])/d)] - 2*e^2*x*PolyLog[2, 1 + (e*Sqrt[x])/d]) +
b^3*n^3*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]^2*(-3*e*Sqrt[x] + (-d + e*Sqrt[
x])*Log[d + e*Sqrt[x]]) - 3*e^2*x*(-2 + Log[d + e*Sqrt[x]])*Log[d + e*Sqrt[
```

```
x]]*Log[-((e*Sqrt[x])/d)] - 6*e^2*x*(-1 + Log[d + e*Sqrt[x]])*PolyLog[2, 1
+ (e*Sqrt[x])/d] + 6*e^2*x*PolyLog[3, 1 + (e*Sqrt[x])/d]))/(d^2*x)
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,b^3d^2\sqrt{x}\log\left(\left(e\sqrt{x}+d\right)^{n}\right)^{3}-3\left(2\,b^3e^2nx^{\frac{3}{2}}\log\left(e\sqrt{x}+d\right)-2\,b^3denx-\left(b^3e^2nx\log\left(x\right)+2\,b^3d^2\log\left(c\right)+2\,ab^2d^2\right)\sqrt{x}\right)\mathrm{lo}}{2\,d^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*d^2*sqrt(x)*log((e*sqrt(x) + d)^n)^3 - 3*(2*b^3*e^2*n*x^(3/2)*l
og(e*sqrt(x) + d) - 2*b^3*d*e*n*x - (b^3*e^2*n*x*log(x) + 2*b^3*d^2*log(c)
+ 2*a*b^2*d^2)*sqrt(x))*log((e*sqrt(x) + d)^n)^2)/(d^2*x^(3/2)) - integrate
(-1/2*(2*(b^3*d^2*e*log(c)^3 + 3*a*b^2*d^2*e*log(c)^2 + 3*a^2*b*d^2*e*log(c
) + a^3*d^2*e)*x^(3/2) + 2*(b^3*d^3*log(c)^3 + 3*a*b^2*d^3*log(c)^2 + 3*a^2
*b*d^3*log(c) + a^3*d^3)*x - 3*(2*b^3*e^3*n^2*x^(5/2)*log(e*sqrt(x) + d) -
2*b^3*d*e^2*n^2*x^2 - 2*(b^3*d^2*e*log(c)^2 + 2*a*b^2*d^2*e*log(c) + a^2*b*
d^2*e)*x^(3/2) - 2*(b^3*d^3*log(c)^2 + 2*a*b^2*d^3*log(c) + a^2*b*d^3)*x -
(b^3*e^3*n^2*x^2*log(x) + 2*(b^3*d^2*e*n*log(c) + a*b^2*d^2*e*n)*x)*sqrt(x)
)*log((e*sqrt(x) + d)^n))/(d^2*e*x^(7/2) + d^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{3}+3\,ab^2\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{2}+3\,a^2b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^2, x)
```

3.420 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=573

$$\frac{3b^2e^4n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{5b^3e^4n^3\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{2d^4}+\frac{3b^3e^4n^3\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^4}+$$

```
[Out] -(b^3*e^3*n^3)/(2*d^3*Sqrt[x]) + (b^3*e^4*n^3*Log[d + e*Sqrt[x]])/(2*d^4) -
 (b^2*e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) + (5*b^2*e^3*n^2*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^4*Sqrt[x]) + (5*b^2*
e^4*n^2*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^4
) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d*x^(3/2)) + (3*b*e^2*n*(
a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*d^2*x) - (3*b*e^3*n*(d + e*Sqrt[x])*(
a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4*Sqrt[x]) - (3*b*e^4*n*Log[1 - d/(
d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4) - (a + b*Log[c*
(d + e*Sqrt[x])^n])^3/(2*x^2) + (3*b^2*e^4*n^2*(a + b*Log[c*(d + e*Sqrt[x])
^n])*Log[-((e*Sqrt[x])/d)])/d^4 - (3*b^3*e^4*n^3*Log[x])/(2*d^4) - (5*b^3*e
^4*n^3*PolyLog[2, d/(d + e*Sqrt[x])])/(2*d^4) + (3*b^2*e^4*n^2*(a + b*Log[c
*(d + e*Sqrt[x])^n])*PolyLog[2, d/(d + e*Sqrt[x])])/d^4 + (3*b^3*e^4*n^3*Po
lyLog[2, 1 + (e*Sqrt[x])/d])/d^4 + (3*b^3*e^4*n^3*PolyLog[3, d/(d + e*Sqrt[
x])])/d^4
```

Rubi [A] time = 1.49695, antiderivative size = 550, normalized size of antiderivative = 0.96, number of steps used = 35, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$-\frac{3b^2e^4n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}+\frac{11b^3e^4n^3\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{2d^4}+\frac{3b^3e^4n^3\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+\right.}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -(b^3*e^3*n^3)/(2*d^3*Sqrt[x]) + (b^3*e^4*n^3*Log[d + e*Sqrt[x]])/(2*d^4) -
 (b^2*e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) + (5*b^2*e^3*n^2*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^4*Sqrt[x]) - (5*b*e^
4*n*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*d^4) - (b*e*n*(a + b*Log[c*(d +
e*Sqrt[x])^n])^2)/(2*d*x^(3/2)) + (3*b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(4*d^2*x) - (3*b*e^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(2*d^4*Sqrt[x]) + (e^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*d^4) -
(a + b*Log[c*(d + e*Sqrt[x])^n])^3/(2*x^2) + (11*b^2*e^4*n^2*(a + b*Log[c*(
d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/(2*d^4) - (3*b*e^4*n*(a + b*Log[c
*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)])/(2*d^4) - (3*b^3*e^4*n^3*Log[
x])/(2*d^4) + (11*b^3*e^4*n^3*PolyLog[2, 1 + (e*Sqrt[x])/d])/(2*d^4) - (3*b
^2*e^4*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d])/
d^4 + (3*b^3*e^4*n^3*PolyLog[3, 1 + (e*Sqrt[x])/d])/d^4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^5}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(d+ex)}\,dx, x\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{2d} - \frac{(3}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} - \frac{(3ben)\,\text{Subst}}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} - \frac{\left(a+b\,\text{lo}\right.}{}$$

$$= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a\right.}{}$$

$$= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} + \frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right.\right.}{2d^4\sqrt{x}}$$

$$= -\frac{b^3e^3n^3}{2d^3\sqrt{x}} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} + \frac{5b^2e^3n^2\left(\right.}{}$$

$$= -\frac{b^3e^3n^3}{2d^3\sqrt{x}} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} + \frac{5b^2e^3n^2\left(\right.}{}$$

Mathematica [A] time = 1.05605, size = 841, normalized size = 1.47

$$-\frac{2\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3d^4+6bn\log\left(d+e\sqrt{x}\right)\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -(2*b*d^3*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])
^n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[
x])^n])^2 + 6*b*d*e^3*n*x^(3/2)*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d +
e*Sqrt[x])^n])^2 + 6*b*d^4*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt[x]]
 + b*Log[c*(d + e*Sqrt[x])^n])^2 - 6*b*e^4*n*x^2*Log[d + e*Sqrt[x]]*(a - b*
n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 + 2*d^4*(a - b*n*Log[d
 + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3 + 3*b*e^4*n*x^2*(a - b*n*Log[
d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] - 2*b^2*n^2*(a - b*n*
Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(-3*(d^4 - e^4*x^2)*Log[d
```

```
+ e*Sqrt[x]]^2 + e^2*x*(-d^2 + 5*d*e*Sqrt[x] + 11*e^2*x*Log[-((e*Sqrt[x])/d
)]) - Log[d + e*Sqrt[x]]*(2*d^3*e*Sqrt[x] - 3*d^2*e^2*x + 6*d*e^3*x^(3/2) +
 11*e^4*x^2 + 6*e^4*x^2*Log[-((e*Sqrt[x])/d)]) - 6*e^4*x^2*PolyLog[2, 1 + (
e*Sqrt[x])/d]) + b^3*n^3*(d^2*e^2*x*(2 - 3*Log[d + e*Sqrt[x]])*Log[d + e*Sq
rt[x]] + 2*d^3*e*Sqrt[x]*Log[d + e*Sqrt[x]]^2 + 2*d^4*Log[d + e*Sqrt[x]]^3
+ 2*d*e^3*x^(3/2)*(1 - 5*Log[d + e*Sqrt[x]] + 3*Log[d + e*Sqrt[x]]^2) + 12*
e^4*x^2*(-Log[d + e*Sqrt[x]] + Log[-((e*Sqrt[x])/d)]) + 11*e^4*x^2*(Log[d +
 e*Sqrt[x]]*(Log[d + e*Sqrt[x]] - 2*Log[-((e*Sqrt[x])/d)]) - 2*PolyLog[2, 1
 + (e*Sqrt[x])/d]) - 2*e^4*x^2*(Log[d + e*Sqrt[x]]^2*(Log[d + e*Sqrt[x]] -
3*Log[-((e*Sqrt[x])/d)]) - 6*Log[d + e*Sqrt[x]]*PolyLog[2, 1 + (e*Sqrt[x])/
d] + 6*PolyLog[3, 1 + (e*Sqrt[x])/d])))/(4*d^4*x^2)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3\log\left(\left(e\sqrt{x}+d\right)^n\right)^3}{2\,x^2}+\int\frac{3\left(b^3enx+4\left(b^3e\log\left(c\right)+ab^2e\right)x+4\left(b^3d\log\left(c\right)+ab^2d\right)\sqrt{x}\right)\log\left(\left(e\sqrt{x}+d\right)^n\right)^2+4\left(b^3e\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log((e*sqrt(x) + d)^n)^3/x^2 + integrate(1/4*(3*(b^3*e*n*x + 4*(b^
3*e*log(c) + a*b^2*e)*x + 4*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*log((e*sqrt(x
) + d)^n)^2 + 4*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a
^3*e)*x + 12*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(
c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 4*(b^3
*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/(e*x^
4 + d*x^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)^3+3\,ab^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2+3\,a^2b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^3, x)
```

3.421 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) dx$

Optimal. Leaf size=171

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}-be^8n$$

```
[Out] (b*e^7*n*Sqrt[x])/(4*d^7) - (b*e^6*n*x)/(8*d^6) + (b*e^5*n*x^(3/2))/(12*d^5
) - (b*e^4*n*x^2)/(16*d^4) + (b*e^3*n*x^(5/2))/(20*d^3) - (b*e^2*n*x^3)/(24
*d^2) + (b*e*n*x^(7/2))/(28*d) - (b*e^8*n*Log[d + e/Sqrt[x]])/(4*d^8) + (x^
4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/4 - (b*e^8*n*Log[x])/(8*d^8)
```

Rubi [A] time = 0.131568, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}-be^8n$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^7*n*Sqrt[x])/(4*d^7) - (b*e^6*n*x)/(8*d^6) + (b*e^5*n*x^(3/2))/(12*d^5
) - (b*e^4*n*x^2)/(16*d^4) + (b*e^3*n*x^(5/2))/(20*d^3) - (b*e^2*n*x^3)/(24
*d^2) + (b*e*n*x^(7/2))/(28*d) - (b*e^8*n*Log[d + e/Sqrt[x]])/(4*d^8) + (x^
4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/4 - (b*e^8*n*Log[x])/(8*d^8)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx=-\left(2\,\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^9}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\frac{1}{x^8(d+ex)}dx,x,\frac{1}{\sqrt{x}}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^8}-\frac{e}{d^2x^7}+\frac{e^2}{d^3x^6}-\frac{e^3}{d^4x^5}\right.\right.$$
$$=\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{benx^{7/2}}{28d}-\frac{be^8n\,\mathrm{lo}}{}$$

Mathematica [A] time = 0.132673, size = 158, normalized size = 0.92

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{4}ben\left(-\frac{e^4x^{3/2}}{3d^5}+\frac{e^3x^2}{4d^4}-\frac{e^2x^{5/2}}{5d^3}-\frac{e^6\sqrt{x}}{d^7}+\frac{e^5x}{2d^6}+\frac{e^7\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^8}+\frac{e^7\log(x)}{2d^8}+\frac{ex^3}{6d^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/Sqrt[x])^n])/4 - (b*e*n*(-((e^6*Sqrt[x])/d^
7) + (e^5*x)/(2*d^6) - (e^4*x^(3/2))/(3*d^5) + (e^3*x^2)/(4*d^4) - (e^2*x^(
5/2))/(5*d^3) + (e*x^3)/(6*d^2) - x^(7/2)/(7*d) + (e^7*Log[d + e/Sqrt[x]])/
d^8 + (e^7*Log[x])/(2*d^8)))/4
```

Maple [F] time = 0.42, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Maxima [A] time = 1.05057, size = 159, normalized size = 0.93

$$\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{4}ax^4-\frac{1}{1680}ben\left(\frac{420\,e^7\log\left(d\sqrt{x}+e\right)}{d^8}-\frac{60\,d^6x^{\frac{7}{2}}-70\,d^5ex^3+84\,d^4e^2x^{\frac{5}{2}}-105\,d^3e^3x^2+1}{d^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/sqrt(x))^n) + 1/4*a*x^4 - 1/1680*b*e*n*(420*e^7*log(
d*sqrt(x) + e)/d^8 - (60*d^6*x^(7/2) - 70*d^5*e*x^3 + 84*d^4*e^2*x^(5/2) -
105*d^3*e^3*x^2 + 140*d^2*e^4*x^(3/2) - 210*d*e^5*x + 420*e^6*sqrt(x))/d^7)
```

Fricas [A] time = 1.78585, size = 440, normalized size = 2.57

$$\frac{420\,bd^8x^4\log(c) - 70\,bd^6e^2nx^3 + 420\,ad^8x^4 - 105\,bd^4e^4nx^2 - 210\,bd^2e^6nx - 420\,bd^8n\log\left(\sqrt{x}\right) + 420\left(bd^8 - be^8\right)n\log\left(d\right.}{1680\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/1680*(420*b*d^8*x^4*log(c) - 70*b*d^6*e^2*n*x^3 + 420*a*d^8*x^4 - 105*b*d
^4*e^4*n*x^2 - 210*b*d^2*e^6*n*x - 420*b*d^8*n*log(sqrt(x)) + 420*(b*d^8 -
b*e^8)*n*log(d*sqrt(x) + e) + 420*(b*d^8*n*x^4 - b*d^8*n)*log((d*x + e*sqrt
(x))/x) + 4*(15*b*d^7*e*n*x^3 + 21*b*d^5*e^3*n*x^2 + 35*b*d^3*e^5*n*x + 105
*b*d*e^7*n)*sqrt(x))/d^8
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31369, size = 163, normalized size = 0.95

$$\frac{1}{4}\,bx^4\log(c) + \frac{1}{4}\,ax^4 + \frac{1}{1680}\left(420\,x^4\log\left(d + \frac{e}{\sqrt{x}}\right) + \left(\frac{60\,d^6x^{\frac{7}{2}} - 70\,d^5x^3e + 84\,d^4x^{\frac{5}{2}}e^2 - 105\,d^3x^2e^3 + 140\,d^2x^{\frac{3}{2}}e^4 - 210\,d\right.}{d^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/1680*(420*x^4*log(d + e/sqrt(x)) + ((60*d^
6*x^(7/2) - 70*d^5*x^3*e + 84*d^4*x^(5/2)*e^2 - 105*d^3*x^2*e^3 + 140*d^2*x
^(3/2)*e^4 - 210*d*x*e^5 + 420*sqrt(x)*e^6)/d^7 - 420*e^7*log(abs(d*sqrt(x)
 + e))/d^8)*e)*b*n
```

3.422 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=139

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{benx^{5/2}}{15d}$$

```
[Out] (b*e^5*n*Sqrt[x])/(3*d^5) - (b*e^4*n*x)/(6*d^4) + (b*e^3*n*x^(3/2))/(9*d^3)
 - (b*e^2*n*x^2)/(12*d^2) + (b*e*n*x^(5/2))/(15*d) - (b*e^6*n*Log[d + e/Sqr
t[x]])/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/3 - (b*e^6*n*Log[x]
)/(6*d^6)
```

Rubi [A] time = 0.0962967, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{benx^{5/2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^5*n*Sqrt[x])/(3*d^5) - (b*e^4*n*x)/(6*d^4) + (b*e^3*n*x^(3/2))/(9*d^3)
 - (b*e^2*n*x^2)/(12*d^2) + (b*e*n*x^(5/2))/(15*d) - (b*e^6*n*Log[d + e/Sqr
t[x]])/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/3 - (b*e^6*n*Log[x]
)/(6*d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = -\left(2\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^7}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\frac{1}{x^6(d+ex)}dx,x,\frac{1}{\sqrt{x}}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}+\right.\right.$$
$$=\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}+\frac{1}{3}x^3\left(a+b\right.$$

Mathematica [A] time = 0.0862148, size = 130, normalized size = 0.94

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{3}ben\left(-\frac{e^2x^{3/2}}{3d^3}-\frac{e^4\sqrt{x}}{d^5}+\frac{e^3x}{2d^4}+\frac{e^5\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^6}+\frac{e^5\log(x)}{2d^6}+\frac{ex^2}{4d^2}-\frac{x^{5/2}}{5d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/Sqrt[x])^n])/3 - (b*e*n*(-((e^4*Sqrt[x])/d^
5) + (e^3*x)/(2*d^4) - (e^2*x^(3/2))/(3*d^3) + (e*x^2)/(4*d^2) - x^(5/2)/(5
*d) + (e^5*Log[d + e/Sqrt[x]])/d^6 + (e^5*Log[x])/(2*d^6)))/3
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Maxima [A] time = 1.029, size = 130, normalized size = 0.94

$$\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{3}ax^3-\frac{1}{180}ben\left(\frac{60e^5\log\left(d\sqrt{x}+e\right)}{d^6}-\frac{12d^4x^{\frac{5}{2}}-15d^3ex^2+20d^2e^2x^{\frac{3}{2}}-30de^3x+60e^4\sqrt{x}}{d^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/sqrt(x))^n) + 1/3*a*x^3 - 1/180*b*e*n*(60*e^5*log(d*
sqrt(x) + e)/d^6 - (12*d^4*x^(5/2) - 15*d^3*e*x^2 + 20*d^2*e^2*x^(3/2) - 30
*d*e^3*x + 60*e^4*sqrt(x))/d^5)
```

Fricas [A] time = 1.93807, size = 369, normalized size = 2.65

$$\frac{60\,bd^6x^3\log(c) - 15\,bd^4e^2nx^2 + 60\,ad^6x^3 - 30\,bd^2e^4nx - 60\,bd^6n\log\left(\sqrt{x}\right) + 60\left(bd^6 - be^6\right)n\log\left(d\sqrt{x} + e\right) + 60\left(bd^6\right.}{180\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/180*(60*b*d^6*x^3*log(c) - 15*b*d^4*e^2*n*x^2 + 60*a*d^6*x^3 - 30*b*d^2*e
^4*n*x - 60*b*d^6*n*log(sqrt(x)) + 60*(b*d^6 - b*e^6)*n*log(d*sqrt(x) + e)
+ 60*(b*d^6*n*x^3 - b*d^6*n)*log((d*x + e*sqrt(x))/x) + 4*(3*b*d^5*e*n*x^2
+ 5*b*d^3*e^3*n*x + 15*b*d*e^5*n)*sqrt(x))/d^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29016, size = 136, normalized size = 0.98

$$\frac{1}{3}\,bx^3\log(c) + \frac{1}{3}\,ax^3 + \frac{1}{180}\left(60\,x^3\log\left(d + \frac{e}{\sqrt{x}}\right) + \left(\frac{12\,d^4x^{\frac{5}{2}} - 15\,d^3x^2e + 20\,d^2x^{\frac{3}{2}}e^2 - 30\,dxe^3 + 60\,\sqrt{x}e^4}{d^5} - \frac{60\,e^5\log\left(\right.}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/180*(60*x^3*log(d + e/sqrt(x)) + ((12*d^4*
x^(5/2) - 15*d^3*x^2*e + 20*d^2*x^(3/2)*e^2 - 30*d*x*e^3 + 60*sqrt(x)*e^4)/
d^5 - 60*e^5*log(abs(d*sqrt(x) + e))/d^6)*e)*b*n
```

3.423 $\int x \left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right) dx$

Optimal. Leaf size=107

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}+\frac{benx^{3/2}}{6d}$$

```
[Out] (b*e^3*n*Sqrt[x])/(2*d^3) - (b*e^2*n*x)/(4*d^2) + (b*e*n*x^(3/2))/(6*d) - (
b*e^4*n*Log[d + e/Sqrt[x]])/(2*d^4) + (x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])
)/2 - (b*e^4*n*Log[x])/(4*d^4)
```

Rubi [A] time = 0.0727471, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2454, 2395, 44}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}+\frac{benx^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^3*n*Sqrt[x])/(2*d^3) - (b*e^2*n*x)/(4*d^2) + (b*e*n*x^(3/2))/(6*d) - (
b*e^4*n*Log[d + e/Sqrt[x]])/(2*d^4) + (x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])
)/2 - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx=-\left(2\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^5}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\frac{1}{x^4(d+ex)}dx,x,\frac{1}{\sqrt{x}}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^4}-\frac{e}{d^2x^3}+\frac{e^2}{d^3x^2}-\frac{e^3}{d^4x}+\right.\right.$$
$$=\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right.$$

Mathematica [A] time = 0.0280822, size = 102, normalized size = 0.95

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{2}ben\left(-\frac{e^2\sqrt{x}}{d^3}+\frac{e^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}+\frac{e^3\log(x)}{2d^4}+\frac{ex}{2d^2}-\frac{x^{3/2}}{3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/Sqrt[x])^n])/2 - (b*e*n*(-((e^2*Sqrt[x])/d^
3) + (e*x)/(2*d^2) - x^(3/2)/(3*d) + (e^3*Log[d + e/Sqrt[x]])/d^4 + (e^3*Lo
g[x])/(2*d^4)))/2
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Maxima [A] time = 1.05956, size = 100, normalized size = 0.93

$$-\frac{1}{12}ben\left(\frac{6e^3\log\left(d\sqrt{x}+e\right)}{d^4}-\frac{2d^2x^{\frac{3}{2}}-3dex+6e^2\sqrt{x}}{d^3}\right)+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] -1/12*b*e*n*(6*e^3*log(d*sqrt(x) + e)/d^4 - (2*d^2*x^(3/2) - 3*d*e*x + 6*e^
2*sqrt(x))/d^3) + 1/2*b*x^2*log(c*(d + e/sqrt(x))^n) + 1/2*a*x^2
```

Fricas [A] time = 1.79551, size = 300, normalized size = 2.8

$$\frac{6\,bd^4x^2\log\left(c\right) - 3\,bd^2e^2nx + 6\,ad^4x^2 - 6\,bd^4n\log\left(\sqrt{x}\right) + 6\,\left(bd^4 - be^4\right)n\log\left(d\sqrt{x} + e\right) + 6\,\left(bd^4nx^2 - bd^4n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*d^4*x^2*log(c) - 3*b*d^2*e^2*n*x + 6*a*d^4*x^2 - 6*b*d^4*n*log(sq
rt(x)) + 6*(b*d^4 - b*e^4)*n*log(d*sqrt(x) + e) + 6*(b*d^4*n*x^2 - b*d^4*n)
*log((d*x + e*sqrt(x))/x) + 2*(b*d^3*e*n*x + 3*b*d*e^3*n)*sqrt(x))/d^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35307, size = 109, normalized size = 1.02

$$\frac{1}{2}\,bx^2\log\left(c\right) + \frac{1}{12}\left(6\,x^2\log\left(d + \frac{e}{\sqrt{x}}\right) + \left(\frac{2\,d^2x^{\frac{3}{2}} - 3\,dxe + 6\,\sqrt{x}e^2}{d^3} - \frac{6\,e^3\log\left(\left|d\sqrt{x} + e\right|\right)}{d^4}\right)e\right)bn + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(d + e/sqrt(x)) + ((2*d^2*x^(3/2) - 3*d*x
*e + 6*sqrt(x)*e^2)/d^3 - 6*e^3*log(abs(d*sqrt(x) + e))/d^4)*e)*b*n + 1/2*a
*x^2
```

3.424 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=53

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(d\sqrt{x}+e\right)}{d^2}+\frac{ben\sqrt{x}}{d}$$

```
[Out] (b*e*n*Sqrt[x])/d + a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - (b*e^2*n*Log[e + d
*Sqrt[x]])/d^2
```

Rubi [A] time = 0.0343658, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2448, 263, 190, 43}

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(d\sqrt{x}+e\right)}{d^2}+\frac{ben\sqrt{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

```
[Out] (b*e*n*Sqrt[x])/d + a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - (b*e^2*n*Log[e + d
*Sqrt[x]])/d^2
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = ax+b\int \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}(ben)\int \frac{1}{e+d\sqrt{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+(ben)\,\text{Subst}\left(\int \frac{x}{e+dx}dx,x,\sqrt{x}\right)$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+(ben)\,\text{Subst}\left(\int \left(\frac{1}{d}-\frac{e}{d(e+dx)}\right)dx,x,\sqrt{x}\right)$$
$$= \frac{ben\sqrt{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(e+d\sqrt{x}\right)}{d^2}$$

Mathematica [A] time = 0.0314587, size = 62, normalized size = 1.17

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-ben\left(\frac{e\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}+\frac{e\log(x)}{2d^2}-\frac{\sqrt{x}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

```
[Out] a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - b*e*n*(-(Sqrt[x]/d) + (e*Log[d + e/Sqr
t[x]])/d^2 + (e*Log[x])/(2*d^2))
```

Maple [A] time = 0.093, size = 94, normalized size = 1.8

$$ax+xb\ln\left(c\left(\left(e+d\sqrt{x}\right)\frac{1}{\sqrt{x}}\right)^n\right)+\frac{enb}{d}\sqrt{x}+\frac{be^2n}{2\,d^2}\ln\left(d\sqrt{x}-e\right)-\frac{be^2n}{2\,d^2}\ln\left(e+d\sqrt{x}\right)-\frac{be^2n\ln\left(xd^2-e^2\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/x^(1/2))^n),x)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(1/2))/x^(1/2))^n)+b*e*n*x^(1/2)/d+1/2*b*e^2*n/d^2*ln(
d*x^(1/2)-e)-1/2*b*e^2*n*ln(e+d*x^(1/2))/d^2-1/2*b*e^2*n*ln(d^2*x-e^2)/d^2
```

Maxima [A] time = 1.0404, size = 65, normalized size = 1.23

$$-\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^2}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="maxima")
```

```
[Out] -(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n))*
b + a*x
```

Fricas [A] time = 1.79058, size = 217, normalized size = 4.09

$$\frac{bd^2x\log(c) - bd^2n\log\left(\sqrt{x}\right) + bden\sqrt{x} + ad^2x + \left(bd^2 - be^2\right)n\log\left(d\sqrt{x} + e\right) + \left(bd^2nx - bd^2n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] (b*d^2*x*log(c) - b*d^2*n*log(sqrt(x)) + b*d*e*n*sqrt(x) + a*d^2*x + (b*d^2
 - b*e^2)*n*log(d*sqrt(x) + e) + (b*d^2*n*x - b*d^2*n)*log((d*x + e*sqrt(x)
)/x))/d^2
```

Sympy [A] time = 16.9777, size = 76, normalized size = 1.43

$$ax + b\left(\frac{en\left(\frac{2\sqrt{x}}{d} - \frac{2e^2\left(\begin{cases}\frac{1}{d\sqrt{x}} & \text{for}\ e = 0\\ \frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^2} + \frac{2e\log\left(\frac{1}{\sqrt{x}}\right)}{d^2}\right)}{2} + x\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(1/2))**n),x)
```

```
[Out] a*x + b*(e*n*(2*sqrt(x)/d - 2*e**2*Piecewise((1/(d*sqrt(x)), Eq(e, 0)), (lo
g(d + e/sqrt(x))/e, True))/d**2 + 2*e*log(1/sqrt(x))/d**2)/2 + x*log(c*(d +
 e/sqrt(x))**n))
```

Giac [A] time = 1.34533, size = 76, normalized size = 1.43

$$-\left(\left(\left(\frac{e\log\left(\left|d\sqrt{x} + e\right|\right)}{d^2} - \frac{\sqrt{x}}{d}\right)e - x\log\left(d + \frac{e}{\sqrt{x}}\right)\right)n - x\log(c)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="giac")
```

```
[Out] -(((e*log(abs(d*sqrt(x) + e))/d^2 - sqrt(x)/d)*e - x*log(d + e/sqrt(x)))*n
- x*log(c))*b + a*x
```

3.425 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$-2bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 2*b*n*PolyLog[2
, 1 + e/(d*Sqrt[x])]
```

Rubi [A] time = 0.0502189, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$-2bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 2*b*n*PolyLog[2
, 1 + e/(d*Sqrt[x])]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx = -\left(2\,\mathrm{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right) + (2ben)\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right) - 2bn\mathrm{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)$$

Mathematica [A] time = 0.0029514, size = 53, normalized size = 1.04

$$-2bn\mathrm{PolyLog}\left(2, \frac{d+\frac{e}{\sqrt{x}}}{d}\right) + a\log(x) - 2b\log\left(-\frac{e}{d\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*b*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + a*Log[x] - 2*b*n*Poly
Log[2, (d + e/Sqrt[x])/d]
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)
```

Maxima [B] time = 2.36046, size = 167, normalized size = 3.27

$$-2\left(\log\left(\frac{d\sqrt{x}}{e}+1\right)\log\left(\sqrt{x}\right) + \mathrm{Li}_2\left(-\frac{d\sqrt{x}}{e}\right)\right)bn + \frac{ben\log(x)^2 + 4\,bdn\sqrt{x}\log(x) + 4\,be\log\left(\left(d\sqrt{x}+e\right)^n\right)\log(x) - 4\,be}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="maxima")
```

```
[Out] -2*(log(d*sqrt(x)/e + 1)*log(sqrt(x)) + dilog(-d*sqrt(x)/e))*b*n + 1/4*(b*e
*n*log(x)^2 + 4*b*d*n*sqrt(x)*log(x) + 4*b*e*log((d*sqrt(x) + e)^n)*log(x)
- 4*b*e*log(x)*log(x^(1/2*n)) - 8*b*d*n*sqrt(x) + 4*(b*e*log(c) + a*e)*log(
x) - 4*(b*d*n*x*log(x) - 2*b*d*n*x)/sqrt(x))/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{n}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + e*sqrt(x))/x)^n) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)/x, x)
```

3.426 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{bdn}{e\sqrt{x}}+\frac{bn}{2x}$$

```
[Out] (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e^2 - (a +
 b*Log[c*(d + e/Sqrt[x])^n])/x
```

Rubi [A] time = 0.051102, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{bdn}{e\sqrt{x}}+\frac{bn}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e^2 - (a +
 b*Log[c*(d + e/Sqrt[x])^n])/x
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \frac{x^2}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= \frac{bn}{2x} - \frac{bdn}{e\sqrt{x}} + \frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2} - \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

Mathematica [A] time = 0.0331369, size = 68, normalized size = 1.05

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + \frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2} - \frac{bdn}{e\sqrt{x}} + \frac{bn}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e
^2 - (b*Log[c*(d + e/Sqrt[x])^n])/x
```

Maple [A] time = 0.103, size = 63, normalized size = 1.

$$-\frac{a}{x} - \frac{b}{x}\ln\left(ce^{n\ln\left(d+e\frac{1}{\sqrt{x}}\right)}\right) + \frac{bn}{2x} + \frac{bd^2n}{e^2}\ln\left(d+e\frac{1}{\sqrt{x}}\right) - \frac{bdn}{e}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^2,x)
```

```
[Out] -a/x-1/x*b*ln(c*exp(n*ln(d+e/x^(1/2))))+1/2*b*n/x+b*d^2*n*ln(d+e/x^(1/2))/e
^2-b*d*n/e/x^(1/2)
```

Maxima [A] time = 1.06207, size = 101, normalized size = 1.55

$$\frac{1}{2}ben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3} - \frac{d^2\log(x)}{e^3} - \frac{2\,d\sqrt{x}-e}{e^2x}\right) - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e
)/(e^2*x)) - b*log(c*(d + e/sqrt(x))^n)/x - a/x
```

Fricas [A] time = 1.93247, size = 165, normalized size = 2.54

$$-\frac{2\,bden\sqrt{x} - be^2n + 2\,be^2\log\left(c\right) + 2\,ae^2 - 2\left(bd^2nx - be^2n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)}{2\,e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*e*n*sqrt(x) - b*e^2*n + 2*b*e^2*log(c) + 2*a*e^2 - 2*(b*d^2*n*x
 - b*e^2*n)*log((d*x + e*sqrt(x))/x))/(e^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31088, size = 120, normalized size = 1.85

$$\frac{\left(2\,bd^2nx\log\left(d\sqrt{x}+e\right) - 2\,bd^2nx\log\left(\sqrt{x}\right) - 2\,bdn\sqrt{x}e - 2\,bne^2\log\left(d\sqrt{x}+e\right) + 2\,bne^2\log\left(\sqrt{x}\right) + bne^2 - 2\,be^2\log\left(c\right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*d^2*n*x*log(d*sqrt(x) + e) - 2*b*d^2*n*x*log(sqrt(x)) - 2*b*d*n*sq
rt(x)*e - 2*b*n*e^2*log(d*sqrt(x) + e) + 2*b*n*e^2*log(sqrt(x)) + b*n*e^2 -
 2*b*e^2*log(c) - 2*a*e^2)*e^(-2)/x
```

3.427 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^2n}{4e^2x}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{bdn}{6ex^{3/2}}+\frac{bn}{8x^2}$$

```
[Out] (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) - (b*d^3*n)/(2*
e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (a + b*Log[c*(d + e/S
qrt[x])^n])/(2*x^2)
```

Rubi [A] time = 0.0757875, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^2n}{4e^2x}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{bdn}{6ex^{3/2}}+\frac{bn}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```



```
[Out] (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) - (b*d^3*n)/(2*
e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (a + b*Log[c*(d + e/S
qrt[x])^n])/(2*x^2)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = -\left(2\,\mathrm{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\mathrm{Subst}\left(\int \frac{x^4}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\mathrm{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{d^4}{e^4(d+ex)}\right)\right)$$

$$= \frac{bn}{8x^2} - \frac{bdn}{6ex^{3/2}} + \frac{bd^2n}{4e^2x} - \frac{bd^3n}{2e^3\sqrt{x}} + \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4} - \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}$$

Mathematica [A] time = 0.0672797, size = 109, normalized size = 1.05

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2} - \frac{bd^3n}{2e^3\sqrt{x}} + \frac{bd^2n}{4e^2x} + \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4} - \frac{bdn}{6ex^{3/2}} + \frac{bn}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) -
(b*d^3*n)/(2*e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (b*Log[c
*(d + e/Sqrt[x])^n])/(2*x^2)
```

Maple [F] time = 0.33, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

Maxima [A] time = 1.04361, size = 128, normalized size = 1.23

$$\frac{1}{24}ben\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5} - \frac{6\,d^4\log(x)}{e^5} - \frac{12\,d^3x^{\frac{3}{2}} - 6\,d^2ex + 4\,de^2\sqrt{x} - 3\,e^3}{e^4x^2}\right) - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/24*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3
/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) - 1/2*b*log(c*(d + e/
```

```
sqrt(x))^n)/x^2 - 1/2*a/x^2
```

Fricas [A] time = 1.91319, size = 228, normalized size = 2.19

$$\frac{6\,bd^2e^2nx+3\,be^4n-12\,be^4\log\left(c\right)-12\,ae^4+12\left(bd^4nx^2-be^4n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\left(3\,bd^3enx+bde^3n\right)\sqrt{x}}{24\,e^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b*d^2*e^2*n*x + 3*b*e^4*n - 12*b*e^4*log(c) - 12*a*e^4 + 12*(b*d^4*
n*x^2 - b*e^4*n)*log((d*x + e*sqrt(x))/x) - 4*(3*b*d^3*e*n*x + b*d*e^3*n)*s
qrt(x))/(e^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41518, size = 157, normalized size = 1.51

$$\frac{\left(12\,bd^4nx^2\log\left(d\sqrt{x}+e\right)-12\,bd^4nx^2\log\left(\sqrt{x}\right)-12\,bd^3nx^{\frac{3}{2}}e+6\,bd^2nxe^2-4\,bdn\sqrt{x}e^3-12\,bne^4\log\left(d\sqrt{x}+e\right)+12\,bne^4}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*b*d^4*n*x^2*log(d*sqrt(x) + e) - 12*b*d^4*n*x^2*log(sqrt(x)) - 12*
b*d^3*n*x^(3/2)*e + 6*b*d^2*n*x*e^2 - 4*b*d*n*sqrt(x)*e^3 - 12*b*n*e^4*log(
d*sqrt(x) + e) + 12*b*n*e^4*log(sqrt(x)) + 3*b*n*e^4 - 12*b*e^4*log(c) - 12
*a*e^4)*e^(-4)/x^2
```

3.428 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=136

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^4n}{6e^4x}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{bdn}{15ex^{5/2}}+\frac{bn}{18x^3}$$

```
[Out] (b*n)/(18*x^3) - (b*d*n)/(15*e*x^(5/2)) + (b*d^2*n)/(12*e^2*x^2) - (b*d^3*n
)/(9*e^3*x^(3/2)) + (b*d^4*n)/(6*e^4*x) - (b*d^5*n)/(3*e^5*Sqrt[x]) + (b*d^
6*n*Log[d + e/Sqrt[x]])/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Rubi [A] time = 0.0970842, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^4n}{6e^4x}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{bdn}{15ex^{5/2}}+\frac{bn}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] (b*n)/(18*x^3) - (b*d*n)/(15*e*x^(5/2)) + (b*d^2*n)/(12*e^2*x^2) - (b*d^3*n
)/(9*e^3*x^(3/2)) + (b*d^4*n)/(6*e^4*x) - (b*d^5*n)/(3*e^5*Sqrt[x]) + (b*d^
6*n*Log[d + e/Sqrt[x]])/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx = -\left(2\,\text{Subst}\left(\int x^5\,(a+b\log\left(c(d+ex)^n\right))\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\,\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx,x,\frac{1}{\sqrt{x}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\,\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}-\frac{dx^4}{e^2}+\frac{x^5}{e}+\right.\right.$$
$$= \frac{bn}{18x^3}-\frac{bdn}{15ex^{5/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^4n}{6e^4x}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{a+b\log\left(c\right.}{3\ldots}$$

Mathematica [A] time = 0.0857543, size = 133, normalized size = 0.98

$$-\frac{a}{3x^3}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}ben\left(-\frac{d^3}{3e^4x^{3/2}}+\frac{d^2}{4e^3x^2}-\frac{d^5}{e^6\sqrt{x}}+\frac{d^4}{2e^5x}+\frac{d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^7}-\frac{d}{5e^2x^{5/2}}+\frac{1}{6ex^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] -a/(3*x^3) + (b*e*n*(1/(6*e*x^3) - d/(5*e^2*x^(5/2)) + d^2/(4*e^3*x^2) - d^
3/(3*e^4*x^(3/2)) + d^4/(2*e^5*x) - d^5/(e^6*Sqrt[x]) + (d^6*Log[d + e/Sqrt
[x]])/e^7))/3 - (b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

Maxima [A] time = 1.0244, size = 158, normalized size = 1.16

$$\frac{1}{180}\,ben\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7}-\frac{30\,d^6\log\left(x\right)}{e^7}-\frac{60\,d^5x^{\frac{5}{2}}-30\,d^4ex^2+20\,d^3e^2x^{\frac{3}{2}}-15\,d^2e^3x+12\,de^4\sqrt{x}-10\,e^5}{e^6x^3}\right)-\frac{b\log\left(c\right.}{3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x^
(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x)
 - 10*e^5)/(e^6*x^3)) - 1/3*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a/x^3
```

Fricas [A] time = 1.88629, size = 292, normalized size = 2.15

$$\frac{30\,bd^4e^2nx^2+15\,bd^2e^4nx+10\,be^6n-60\,be^6\log\left(c\right)-60\,ae^6+60\left(bd^6nx^3-be^6n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\left(15\,bd^5enx^2+5\,bd\right.}{180\,e^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(30*b*d^4*e^2*n*x^2 + 15*b*d^2*e^4*n*x + 10*b*e^6*n - 60*b*e^6*log(c)
 - 60*a*e^6 + 60*(b*d^6*n*x^3 - b*e^6*n)*log((d*x + e*sqrt(x))/x) - 4*(15*b
*d^5*e*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d*e^5*n)*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21075, size = 189, normalized size = 1.39

$$\frac{\left(60\,bd^6nx^3\log\left(d\sqrt{x}+e\right)-60\,bd^6nx^3\log\left(\sqrt{x}\right)-60\,bd^5nx^{\frac{5}{2}}e+30\,bd^4nx^2e^2-20\,bd^3nx^{\frac{3}{2}}e^3+15\,bd^2nxe^4-12\,bdn\sqrt{x}e\right.}{180\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/180*(60*b*d^6*n*x^3*log(d*sqrt(x) + e) - 60*b*d^6*n*x^3*log(sqrt(x)) - 60
*b*d^5*n*x^(5/2)*e + 30*b*d^4*n*x^2*e^2 - 20*b*d^3*n*x^(3/2)*e^3 + 15*b*d^2
*n*x*e^4 - 12*b*d*n*sqrt(x)*e^5 - 60*b*n*e^6*log(d*sqrt(x) + e) + 60*b*n*e^
6*log(sqrt(x)) + 10*b*n*e^6 - 60*b*e^6*log(c) - 60*a*e^6)*e^(-6)/x^3
```

3.429 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=404

$$-\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{3d^6}+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3}-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2be^6n\log\left(1-\right.}{}$$

```
[Out] (-77*b^2*e^5*n^2*Sqrt[x])/(90*d^5) + (47*b^2*e^4*n^2*x)/(180*d^4) - (b^2*e^
3*n^2*x^(3/2))/(10*d^3) + (b^2*e^2*n^2*x^2)/(30*d^2) + (77*b^2*e^6*n^2*Log[
d + e/Sqrt[x]])/(90*d^6) + (2*b*e^5*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*
(d + e/Sqrt[x])^n]))/(3*d^6) - (b*e^4*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))
/(3*d^4) + (2*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*d^3) - (
b*e^2*n*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(6*d^2) + (2*b*e*n*x^(5/2)*(a
 + b*Log[c*(d + e/Sqrt[x])^n]))/(15*d) + (2*b*e^6*n*Log[1 - d/(d + e/Sqrt[x
])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sq
rt[x])^n])^2)/3 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (2*b^2*e^6*n^2*PolyL
og[2, d/(d + e/Sqrt[x])])/(3*d^6)
```

Rubi [A] time = 1.00719, antiderivative size = 428, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{3d^6}+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3}-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}-\frac{e^6\left(a+b\log\left(\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*Sqrt[x])/(90*d^5) + (47*b^2*e^4*n^2*x)/(180*d^4) - (b^2*e^
3*n^2*x^(3/2))/(10*d^3) + (b^2*e^2*n^2*x^2)/(30*d^2) + (77*b^2*e^6*n^2*Log[
d + e/Sqrt[x]])/(90*d^6) + (2*b*e^5*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*
(d + e/Sqrt[x])^n]))/(3*d^6) - (b*e^4*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))
/(3*d^4) + (2*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*d^3) - (
b*e^2*n*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(6*d^2) + (2*b*e*n*x^(5/2)*(a
 + b*Log[c*(d + e/Sqrt[x])^n]))/(15*d) - (e^6*(a + b*Log[c*(d + e/Sqrt[x])^
n])^2)/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/3 + (2*b*e^6*n*(a
 + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/(3*d^6) + (137*b^2*e^
6*n^2*Log[x])/(180*d^6) + (2*b^2*e^6*n^2*PolyLog[2, 1 + e/(d*Sqrt[x])])/(3*
d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=-\left(2\,\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^7}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}dx,x,$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{1}{3}(2bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d+\frac{e}{\sqrt{x}}$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(2bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d}+\frac{(2be$$

$$=\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{(2ben)\,\text{Su}$$

$$=-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}+\frac{1}{3}x^3\left(a\right.$$

$$=-\frac{2b^2e^5n^2\sqrt{x}}{15d^5}+\frac{b^2e^4n^2x}{15d^4}-\frac{2b^2e^3n^2x^{3/2}}{45d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{2b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{15d^6}+\frac{2be$$

$$=-\frac{3b^2e^5n^2\sqrt{x}}{10d^5}+\frac{3b^2e^4n^2x}{20d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{3b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{10d^6}-\frac{be^4$$

$$=-\frac{47b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{47b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}+$$

$$=-\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}+$$

$$=-\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}+$$

Mathematica [A] time = 0.258227, size = 540, normalized size = 1.34

$$-120b^2e^6n^2\text{PolyLog}\left(2,\frac{d\sqrt{x}}{e}+1\right)+60a^2d^6x^3+120abd^6x^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-30abd^4e^2nx^2+40abd^3e^3nx^{3/2}-60abd$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (120*a*b*d*e^5*n*Sqrt[x] - 154*b^2*d*e^5*n^2*Sqrt[x] - 60*a*b*d^2*e^4*n*x +
 47*b^2*d^2*e^4*n^2*x + 40*a*b*d^3*e^3*n*x^(3/2) - 18*b^2*d^3*e^3*n^2*x^(3/
2) - 30*a*b*d^4*e^2*n*x^2 + 6*b^2*d^4*e^2*n^2*x^2 + 24*a*b*d^5*e*n*x^(5/2)
+ 60*a^2*d^6*x^3 + 214*b^2*e^6*n^2*Log[d + e/Sqrt[x]] + 120*b^2*d*e^5*n*Sqr
t[x]*Log[c*(d + e/Sqrt[x])^n] - 60*b^2*d^2*e^4*n*x*Log[c*(d + e/Sqrt[x])^n]
 + 40*b^2*d^3*e^3*n*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] - 30*b^2*d^4*e^2*n*x^2
*Log[c*(d + e/Sqrt[x])^n] + 24*b^2*d^5*e*n*x^(5/2)*Log[c*(d + e/Sqrt[x])^n]
 + 120*a*b*d^6*x^3*Log[c*(d + e/Sqrt[x])^n] + 60*b^2*d^6*x^3*Log[c*(d + e/S
qrt[x])^n]^2 - 120*a*b*e^6*n*Log[e + d*Sqrt[x]] + 60*b^2*e^6*n^2*Log[e + d*
Sqrt[x]] - 120*b^2*e^6*n*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] + 60*b
^2*e^6*n^2*Log[e + d*Sqrt[x]]^2 - 120*b^2*e^6*n^2*Log[e + d*Sqrt[x]]*Log[-(
(d*Sqrt[x])/e)] + 107*b^2*e^6*n^2*Log[x] - 120*b^2*e^6*n^2*PolyLog[2, 1 + (
d*Sqrt[x])/e])/(180*d^6)
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}b^2x^3\log\left(\left(d\sqrt{x}+e\right)^n\right)^2-\int-\frac{3\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^3+3\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x^{\frac{5}{2}}+3\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((d*sqrt(x) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^3 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(5/2) + 3*(b^2*d*x^3 + b^2*e*x^(5/2))*log(x^(1/2*n))^2 - (b^2*d*n*x^3 - 6*
(b^2*d*log(c) + a*b*d)*x^3 - 6*(b^2*e*log(c) + a*b*e)*x^(5/2) + 6*(b^2*d*x^
3 + b^2*e*x^(5/2))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n) - 6*((b^2*d*log(c
) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(5/2))*log(x^(1/2*n)))/(d*x + e*s
qrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+2\,abx^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*sqrt(x))/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x^2, x)
```

3.430 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=288

$$-\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}+\frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}+\frac{be^3n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.}{d^4}$$

```
[Out] (-5*b^2*e^3*n^2*Sqrt[x])/(6*d^3) + (b^2*e^2*n^2*x)/(6*d^2) + (5*b^2*e^4*n^2
*Log[d + e/Sqrt[x]])/(6*d^4) + (b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[
c*(d + e/Sqrt[x])^n]))/d^4 - (b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(
2*d^2) + (b*e*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d) + (b*e^4*n*
Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^4 + (x^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/2 + (11*b^2*e^4*n^2*Log[x])/(12*d^4) - (b^
2*e^4*n^2*PolyLog[2, d/(d + e/Sqrt[x])])/d^4
```

Rubi [A] time = 0.637296, antiderivative size = 311, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^4}-\frac{e^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}+\frac{be^4n\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}+\frac{be^3n\sqrt{x}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (-5*b^2*e^3*n^2*Sqrt[x])/(6*d^3) + (b^2*e^2*n^2*x)/(6*d^2) + (5*b^2*e^4*n^2
*Log[d + e/Sqrt[x]])/(6*d^4) + (b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[
c*(d + e/Sqrt[x])^n]))/d^4 - (b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(
2*d^2) + (b*e*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d) - (e^4*(a +
 b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d^4) + (x^2*(a + b*Log[c*(d + e/Sqrt[x])
^n])^2)/2 + (b*e^4*n*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))]
)/d^4 + (11*b^2*e^4*n^2*Log[x])/(12*d^4) + (b^2*e^4*n^2*PolyLog[2, 1 + e/(d
*Sqrt[x])])/d^4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=-\left(2\,\mathrm{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^5}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(ben)\,\mathrm{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^4(d+ex)}dx,x,\dots\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(bn)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,x,d+\frac{e}{\sqrt{x}}\right)\dots$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(bn)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,x,d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{(be\dots}{\dots}$$

$$=\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{(ben)\,\mathrm{Su}\dots}{\dots}$$

$$=-\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}+\frac{1}{2}x^2\left(a\dots\right.$$

$$=-\frac{b^2e^3n^2\sqrt{x}}{3d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^4}+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\dots\right.\right.}{d^4}$$

$$=-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\dots\right.}{d^4}$$

$$=-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\dots\right.}{d^4}$$

Mathematica [A] time = 0.222446, size = 321, normalized size = 1.11

$$\frac{1}{6}\left(\frac{ben\left(-6be^3n\mathrm{PolyLog}\left(2,\frac{d\sqrt{x}}{e}+1\right)-3ad^2ex+2ad^3x^{3/2}+6ade^2\sqrt{x}-6ae^3\log\left(d\sqrt{x}+e\right)+2bd^3x^{3/2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\dots\right.\right.}{\dots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (3*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(6*a*d*e^2*Sqrt[x] - 5*b
*d*e^2*n*Sqrt[x] - 3*a*d^2*e*x + b*d^2*e*n*x + 2*a*d^3*x^(3/2) + 8*b*e^3*n*
Log[d + e/Sqrt[x]] + 6*b*d*e^2*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 3*b*d^2*e
```

```
*x*Log[c*(d + e/Sqrt[x])^n] + 2*b*d^3*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] - 6*
a*e^3*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]] - 6*b*e^3*Log[c*(d
+ e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]]^2 - 6*b*e
^3*n*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] + 4*b*e^3*n*Log[x] - 6*b*e^3*
n*PolyLog[2, 1 + (d*Sqrt[x])/e]))/d^4)/6
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int x \left(a + b \ln \left(c \left(d + e \frac{1}{\sqrt{x}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} b^2 x^2 \log\left(\left(d\sqrt{x} + e\right)^n\right)^2 - \int -\frac{2\left(b^2 d \log(c)^2 + 2\,abd \log(c) + a^2 d\right)x^2 + 2\left(b^2 dx^2 + b^2 e x^{\frac{3}{2}}\right)\log\left(x^{\frac{1}{2}n}\right)^2 + 2\left(b^2 e \log(c)^2 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((d*sqrt(x) + e)^n)^2 - integrate(-1/2*(2*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 + 2*(b^2*d*x^2 + b^2*e*x^(3/2))*log(x^(1/2*n))^
2 + 2*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(3/2) - (b^2*d*n*x^2 - 4*
(b^2*d*log(c) + a*b*d)*x^2 - 4*(b^2*e*log(c) + a*b*e)*x^(3/2) + 4*(b^2*d*x^
2 + b^2*e*x^(3/2))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n) - 4*((b^2*d*log(c
) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(3/2))*log(x^(1/2*n)))/(d*x + e*s
qrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2 x \log\left(c\left(\frac{dx + e\sqrt{x}}{x}\right)^n\right)^2 + 2\,abx \log\left(c\left(\frac{dx + e\sqrt{x}}{x}\right)^n\right) + a^2 x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*s
qrt(x))/x)^n) + a^2*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right)^2 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x, x)
```

3.431 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=152

$$-\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}+\frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{2ben\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.}{d^2}$$

```
[Out] (2*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 + (2
*b*e^2*n*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 +
 x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b^2*e^2*n^2*Log[x])/d^2 - (2*b^2*e
^2*n^2*PolyLog[2, d/(d + e/Sqrt[x])])/d^2
```

Rubi [A] time = 0.354466, antiderivative size = 174, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^2}-\frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2}+\frac{2be^2n\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{2ben\sqrt{x}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (2*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 - (e
^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d^2 + x*(a + b*Log[c*(d + e/Sqrt[x])
^n])^2 + (2*b*e^2*n*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])
/d^2 + (b^2*e^2*n^2*Log[x])/d^2 + (2*b^2*e^2*n^2*PolyLog[2, 1 + e/(d*Sqrt[x
])])/d^2
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2 dx, x, \sqrt{x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3} dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - (2ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^2(d+ex)} dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - (2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, d+\frac{e}{\sqrt{x}}\right)$$
$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, d+\frac{e}{\sqrt{x}}\right)}{d} + \frac{(2ben)\,\text{Su}}{}$$
$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{(2ben)}{}$$
$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+\right.$$
$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+\right.$$

Mathematica [A] time = 0.122766, size = 170, normalized size = 1.12

$$\frac{ben\left(ben\left(\log\left(d\sqrt{x}+e\right)\left(\log\left(d\sqrt{x}+e\right)-2\log\left(-\frac{d\sqrt{x}}{e}\right)\right)-2\text{PolyLog}\left(2,\frac{d\sqrt{x}}{e}+1\right)\right)-2e\log\left(d\sqrt{x}+e\right)\left(a+b\log\left(c\left(d+\right.\right.\right.\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(2*a*d*Sqrt[x] + 2*b*d*Sqrt[x
]*Log[c*(d + e/Sqrt[x])^n] - 2*e*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d
*Sqrt[x]] + b*e*n*(2*Log[d + e/Sqrt[x]] + Log[x]) + b*e*n*(Log[e + d*Sqrt[x
]]*(Log[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sqr
t[x])/e])))/d^2
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^2}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)ab+\left(x\log\left(\left(d\sqrt{x}+e\right)^n\right)^2-\int-\frac{dx\log\left(c\right)^2+e\sqrt{x}\log\left(c\right)^2+\left(d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] -2*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n)
)*a*b + (x*log((d*sqrt(x) + e)^n)^2 - integrate(-(d*x*log(c)^2 + e*sqrt(x)*
log(c)^2 + (d*x + e*sqrt(x))*log(x^(1/2*n))^2 - (d*n*x - 2*d*x*log(c) - 2*e
*sqrt(x)*log(c) + 2*(d*x + e*sqrt(x))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n
) - 2*(d*x*log(c) + e*sqrt(x)*log(c))*log(x^(1/2*n)))/(d*x + e*sqrt(x)), x)
)*b^2 + a^2*x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqrt(
x))/x)^n) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2, x)
```

3.432 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$-4bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+4b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(\right.\right.\right.$$

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))] - 4*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])] + 4*b^2*n^2*PolyLog
[3, 1 + e/(d*Sqrt[x])]
```

Rubi [A] time = 0.131166, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$-4bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+4b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))] - 4*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])] + 4*b^2*n^2*PolyLog
[3, 1 + e/(d*Sqrt[x])]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = -\left(2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) + (4ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) + (4bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(-\right.}{x}\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) - 4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_2\left(1+\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) - 4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_2\left(1+\right.$$

Mathematica [B] time = 0.346941, size = 386, normalized size = 4.15

$$2bn\left(2\text{PolyLog}\left(2,-\frac{e}{d\sqrt{x}}\right)+\log(x)\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(\frac{e}{d\sqrt{x}}+1\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-bn\log\left(d+\frac{e}{\sqrt{x}}\right)\right)+\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*((Log[d + e/Sqrt[
x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) + (b
^2*n^2*(24*Log[e/d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + 12*Log[d + e/Sqrt[x
]]^2*Log[x] - 12*Log[e/d + Sqrt[x]]^2*Log[x] - 24*Log[d + e/Sqrt[x]]*Log[1
+ (d*Sqrt[x])/e]*Log[x] + 24*Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*Log[
x] + 6*Log[d + e/Sqrt[x]]*Log[x]^2 - 6*Log[1 + (d*Sqrt[x])/e]*Log[x]^2 + Lo
g[x]^3 + 48*Log[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 48*(Log[d +
e/Sqrt[x]] - Log[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 48*PolyLog[
3, 1 + (d*Sqrt[x])/e] - 48*PolyLog[3, -((d*Sqrt[x])/e)]))/12
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2\log\left(\left(d\sqrt{x}+e\right)^n\right)^2\log(x)-\int-\frac{\left(b^2dx+b^2e\sqrt{x}\right)\log\left(x^{\frac{1}{2}n}\right)^2+\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x-\left(b^2dnx\log(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((d*sqrt(x) + e)^n)^2*log(x) - integrate(-((b^2*d*x + b^2*e*sqrt(x))
*log(x^(1/2*n))^2 + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - (b^2*d*n*
x*log(x) - 2*(b^2*d*log(c) + a*b*d)*x + 2*(b^2*d*x + b^2*e*sqrt(x))*log(x^(
1/2*n)) - 2*(b^2*e*log(c) + a*b*e)*sqrt(x))*log((d*sqrt(x) + e)^n) - 2*((b^
2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*sqrt(x))*log(x^(1/2*n)) + (b
^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*sqrt(x))/(d*x^2 + e*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqrt
(x))/x)^n) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x, x)
```

3.433 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=195

$$\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right.\right.}{e^2}$$

```
[Out] -(b^2*n^2*(d + e/Sqrt[x])^2)/(2*e^2) - (4*a*b*d*n)/(e*Sqrt[x]) + (4*b^2*d*n
^2)/(e*Sqrt[x]) - (4*b^2*d*n*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^2
+ (b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^2 + (2*d*(d +
e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2 - ((d + e/Sqrt[x])^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
```

Rubi [A] time = 0.1977, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right.\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -(b^2*n^2*(d + e/Sqrt[x])^2)/(2*e^2) - (4*a*b*d*n)/(e*Sqrt[x]) + (4*b^2*d*n
^2)/(e*Sqrt[x]) - (4*b^2*d*n*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^2
+ (b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^2 + (2*d*(d +
e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2 - ((d + e/Sqrt[x])^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}\right)dx, x \cdots\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\left(c \cdots\right.\right.\right.}{e}$$

$$= -\frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\left(cx^n\right)\right)^2\,dx \cdots\right.}{e^2}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}+\frac{(2b \cdots}{\cdots}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2}+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(\cdots\right.}{\cdots}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{4b^2dn^2}{e\sqrt{x}}-\frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2}+\frac{bn\left(d+\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.29817, size = 298, normalized size = 1.53

$$-\frac{\frac{bn\left(-4bd^2nx\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)+2bd^2nx\left(\log\left(d\sqrt{x}+e\right)\left(\log\left(d\sqrt{x}+e\right)-2\log\left(-\frac{d\sqrt{x}}{e}\right)\right)-2\text{PolyLog}\left(2,\frac{d\sqrt{x}}{e}+1\right)\right)-4d^2x\log\left(d\sqrt{x}+e\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-4d^2x}{e^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -(2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*n*(4*a*d*e*Sqrt[x] - 4*b*d*e*n*
Sqrt[x] + b*n*(e*(e - 2*d*Sqrt[x]) + 2*d^2*x*Log[d + e/Sqrt[x]]) + 4*b*d*(e
 + d*Sqrt[x])*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 2*e^2*(a + b*Log[c*(d + e/
Sqrt[x])^n]) - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d*Sqrt[x]]
- 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 4*b*d^2*
n*x*PolyLog[2, 1 + e/(d*Sqrt[x])] + 2*b*d^2*n*x*(Log[e + d*Sqrt[x]]*(Log[e
+ d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sqrt[x])/e]))
)/e^2)/(2*x)
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

Maxima [A] time = 1.08132, size = 335, normalized size = 1.72

$$aben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)+\frac{1}{4}\left(4\,en\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)\log\left(c\left(d+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] a*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/
(e^2*x)) + 1/4*(4*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d
*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) +
e)^2 + d^2*x*log(x)^2 - 6*d^2*x*log(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*
log(x) - 3*d^2*x)*log(d*sqrt(x) + e))*n^2/(e^2*x))*b^2 - b^2*log(c*(d + e/s
qrt(x))^n)^2/x - 2*a*b*log(c*(d + e/sqrt(x))^n)/x - a^2/x
```

Fricas [A] time = 1.73133, size = 521, normalized size = 2.67

$$-\frac{b^2e^2n^2+2\,b^2e^2\log\left(c\right)^2-2\,abe^2n+2\,a^2e^2-2\left(b^2d^2n^2x-b^2e^2n^2\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^2-2\left(b^2e^2n-2\,abe^2\right)\log\left(c\right)+2\left(2\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*e^2*n^2 + 2*b^2*e^2*log(c)^2 - 2*a*b*e^2*n + 2*a^2*e^2 - 2*(b^2*d
^2*n^2*x - b^2*e^2*n^2)*log((d*x + e*sqrt(x))/x)^2 - 2*(b^2*e^2*n - 2*a*b*e
^2)*log(c) + 2*(2*b^2*d*e*n^2*sqrt(x) - b^2*e^2*n^2 + 2*a*b*e^2*n + (3*b^2*
d^2*n^2 - 2*a*b*d^2*n)*x - 2*(b^2*d^2*n*x - b^2*e^2*n)*log(c))*log((d*x + e
*sqrt(x))/x) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sqrt(x)
)/(e^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x^2, x)
```

3.434 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=341

$$\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}-\frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}+\frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\right.}{e^4}$$

```
[Out] (-3*b^2*d^2*n^2*(d + e/Sqrt[x])^2)/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3
)/(9*e^4) - (b^2*n^2*(d + e/Sqrt[x])^4)/(16*e^4) + (4*b^2*d^3*n^2)/(e^3*Sqr
t[x]) - (b^2*d^4*n^2*Log[d + e/Sqrt[x]]^2)/(2*e^4) - (4*b*d^3*n*(d + e/Sqrt
[x])*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^4 + (3*b*d^2*n*(d + e/Sqrt[x])^2*(
a + b*Log[c*(d + e/Sqrt[x])^n]))/e^4 - (4*b*d*n*(d + e/Sqrt[x])^3*(a + b*Lo
g[c*(d + e/Sqrt[x])^n]))/(3*e^4) + (b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d +
 e/Sqrt[x])^n]))/(4*e^4) + (b*d^4*n*Log[d + e/Sqrt[x]]*(a + b*Log[c*(d + e/
Sqrt[x])^n]))/e^4 - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(2*x^2)
```

Rubi [A] time = 0.366075, antiderivative size = 263, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4}-\frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4}-\frac{12d^4\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^4}+\frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4}-\frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] (-3*b^2*d^2*n^2*(d + e/Sqrt[x])^2)/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3
)/(9*e^4) - (b^2*n^2*(d + e/Sqrt[x])^4)/(16*e^4) + (4*b^2*d^3*n^2)/(e^3*Sqr
t[x]) - (b^2*d^4*n^2*Log[d + e/Sqrt[x]]^2)/(2*e^4) - (b*n*((48*d^3*(d + e/S
qrt[x]))/e^4 - (36*d^2*(d + e/Sqrt[x])^2)/e^4 + (16*d*(d + e/Sqrt[x])^3)/e^
4 - (3*(d + e/Sqrt[x])^4)/e^4 - (12*d^4*Log[d + e/Sqrt[x]])/e^4)*(a + b*Log
[c*(d + e/Sqrt[x])^n]))/12 - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(2*x^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx, x, \frac{1}{\sqrt{}}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx, x, d\right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4} - \frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4} - \frac{12d^4\,\mathrm{l}}{}\right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4} - \frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4} - \frac{12d^4\,\mathrm{l}}{}\right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4} - \frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4} - \frac{12d^4\,\mathrm{l}}{}\right.$$

$$= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4} + \frac{4b^2d^3n^2}{e^3\sqrt{x}} - \frac{1}{12}bn\left(\frac{4}{}\right.$$

$$= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4} + \frac{4b^2d^3n^2}{e^3\sqrt{x}} - \frac{b^2d^4n^2\,\mathrm{l}}{}$$

Mathematica [C] time = 0.35999, size = 473, normalized size = 1.39

$$-\frac{bn\left(-144bd^4nx^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right) - 144bd^4nx^2\text{PolyLog}\left(2,\frac{d\sqrt{x}}{e}+1\right) - 72ad^2e^2x + 144ad^3ex^{3/2} - 144ad^4x^2\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -(72*e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + b*n*(-36*a*e^4 + 9*b*e^4*n +
48*a*d*e^3*Sqrt[x] - 28*b*d*e^3*n*Sqrt[x] - 72*a*d^2*e^2*x + 78*b*d^2*e^2*n
*x + 144*a*d^3*e*x^(3/2) - 300*b*d^3*e*n*x^(3/2) + 156*b*d^4*n*x^2*Log[d +
e/Sqrt[x]] - 36*b*e^4*Log[c*(d + e/Sqrt[x])^n] + 48*b*d*e^3*Sqrt[x]*Log[c*(
d + e/Sqrt[x])^n] - 72*b*d^2*e^2*x*Log[c*(d + e/Sqrt[x])^n] + 144*b*d^3*e*x
^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n] -
144*a*d^4*x^2*Log[e + d*Sqrt[x]] - 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n]*L
og[e + d*Sqrt[x]] + 72*b*d^4*n*x^2*Log[e + d*Sqrt[x]]^2 - 144*a*d^4*x^2*Log
[-(e/(d*Sqrt[x]))] - 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt
[x]))] - 144*b*d^4*n*x^2*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] - 144*b*d
^4*n*x^2*PolyLog[2, 1 + e/(d*Sqrt[x])] - 144*b*d^4*n*x^2*PolyLog[2, 1 + (d*
Sqrt[x])/e]))/(144*e^4*x^2)
```

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

Maxima [A] time = 1.0954, size = 433, normalized size = 1.27

$$\frac{1}{12}aben\left(\frac{12d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6d^4\log(x)}{e^5}-\frac{12d^3x^{\frac{3}{2}}-6d^2ex+4de^2\sqrt{x}-3e^3}{e^4x^2}\right)+\frac{1}{144}\left(12en\left(\frac{12d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6d}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*a*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^
(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/144*(12*e*n*(12
*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*e*
x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*d^4*
x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d
^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(
x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*b^2 - 1/2*b^2*log(c*(d
+ e/sqrt(x))^n)^2/x^2 - a*b*log(c*(d + e/sqrt(x))^n)/x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 1.84748, size = 799, normalized size = 2.34

$$-\frac{9b^2e^4n^2+72b^2e^4\log(c)^2-36abe^4n+72a^2e^4-72\left(b^2d^4n^2x^2-b^2e^4n^2\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^2+6\left(13b^2d^2e^2n^2-12abd^2e^2n\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b^2*e^4*n^2 + 72*b^2*e^4*log(c)^2 - 36*a*b*e^4*n + 72*a^2*e^4 - 7
2*(b^2*d^4*n^2*x^2 - b^2*e^4*n^2)*log((d*x + e*sqrt(x))/x)^2 + 6*(13*b^2*d^
2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 36*(2*b^2*d^2*e^2*n*x + b^2*e^4*n - 4*a*b
*e^4)*log(c) - 12*(6*b^2*d^2*e^2*n^2*x + 3*b^2*e^4*n^2 - 12*a*b*e^4*n - (25
*b^2*d^4*n^2 - 12*a*b*d^4*n)*x^2 + 12*(b^2*d^4*n*x^2 - b^2*e^4*n)*log(c) -
4*(3*b^2*d^3*e*n^2*x + b^2*d*e^3*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 4
*(7*b^2*d*e^3*n^2 - 12*a*b*d*e^3*n + 3*(25*b^2*d^3*e*n^2 - 12*a*b*d^3*e*n)*
x - 12*(3*b^2*d^3*e*n*x + b^2*d*e^3*n)*log(c))*sqrt(x))/(e^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right) + a\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x^3, x)
```

3.435 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=480

$$\frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6}-\frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6}+\frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(\right.\right.}{e^6}$$

```
[Out] (-5*b^2*d^4*n^2*(d + e/Sqrt[x])^2)/(2*e^6) + (40*b^2*d^3*n^2*(d + e/Sqrt[x]
)^3)/(27*e^6) - (5*b^2*d^2*n^2*(d + e/Sqrt[x])^4)/(8*e^6) + (4*b^2*d*n^2*(d
 + e/Sqrt[x])^5)/(25*e^6) - (b^2*n^2*(d + e/Sqrt[x])^6)/(54*e^6) + (4*b^2*d
^5*n^2)/(e^5*Sqrt[x]) - (b^2*d^6*n^2*Log[d + e/Sqrt[x]]^2)/(3*e^6) - (4*b*d
^5*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^6 + (5*b*d^4*n*(d
+ e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^6 - (40*b*d^3*n*(d + e/S
qrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) + (5*b*d^2*n*(d + e/Sqr
t[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) - (4*b*d*n*(d + e/Sqrt[x]
)^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(5*e^6) + (b*n*(d + e/Sqrt[x])^6*(a +
 b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) + (2*b*d^6*n*Log[d + e/Sqrt[x]]*(a +
b*Log[c*(d + e/Sqrt[x])^n]))/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(
3*x^3)
```

Rubi [A] time = 0.470705, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6}-\frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6}+\frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6}-\frac{225d^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^6}-\frac{60d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^6}+\frac{72d\left(d+\frac{e}{\sqrt{x}}\right)^5}{e^6}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-5*b^2*d^4*n^2*(d + e/Sqrt[x])^2)/(2*e^6) + (40*b^2*d^3*n^2*(d + e/Sqrt[x]
)^3)/(27*e^6) - (5*b^2*d^2*n^2*(d + e/Sqrt[x])^4)/(8*e^6) + (4*b^2*d*n^2*(d
 + e/Sqrt[x])^5)/(25*e^6) - (b^2*n^2*(d + e/Sqrt[x])^6)/(54*e^6) + (4*b^2*d
^5*n^2)/(e^5*Sqrt[x]) - (b^2*d^6*n^2*Log[d + e/Sqrt[x]]^2)/(3*e^6) - (b*n*(
(360*d^5*(d + e/Sqrt[x]))/e^6 - (450*d^4*(d + e/Sqrt[x])^2)/e^6 + (400*d^3*
(d + e/Sqrt[x])^3)/e^6 - (225*d^2*(d + e/Sqrt[x])^4)/e^6 + (72*d*(d + e/Sqr
t[x])^5)/e^6 - (10*(d + e/Sqrt[x])^6)/e^6 - (60*d^6*Log[d + e/Sqrt[x]])/e^6
)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/90 - (a + b*Log[c*(d + e/Sqrt[x])^n])^2
/(3*x^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = -\left(2\,\mathrm{Subst}\left(\int x^5\,(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2ben)\,\mathrm{Subst}\left(\int \frac{x^6\,(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx,x,\frac{1}{\sqrt{}}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2bn)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\,(a+b\log\left(cx^n\right))}{x}\,dx,x,d\right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^6} + \right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^6} + \right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^6} + \right.$$

$$= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6}$$

$$= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6}$$

Mathematica [C] time = 0.34083, size = 692, normalized size = 1.44

$$\frac{3600b^2d^6n^2x^3\mathrm{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)+3600b^2d^6n^2x^3\mathrm{PolyLog}\left(2,\frac{d\sqrt{x}}{e}+1\right)-1800a^2e^6-3600abe^6\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+18}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-1800*a^2*e^6 + 600*a*b*e^6*n - 100*b^2*e^6*n^2 - 720*a*b*d*e^5*n*Sqrt[x]
+ 264*b^2*d*e^5*n^2*Sqrt[x] + 900*a*b*d^2*e^4*n*x - 555*b^2*d^2*e^4*n^2*x -
 1200*a*b*d^3*e^3*n*x^(3/2) + 1140*b^2*d^3*e^3*n^2*x^(3/2) + 1800*a*b*d^4*e
^2*n*x^2 - 2610*b^2*d^4*e^2*n^2*x^2 - 3600*a*b*d^5*e*n*x^(5/2) + 8820*b^2*d
^5*e*n^2*x^(5/2) - 5220*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]] - 3600*a*b*e^6*L
og[c*(d + e/Sqrt[x])^n] + 600*b^2*e^6*n*Log[c*(d + e/Sqrt[x])^n] - 720*b^2*
d*e^5*n*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] + 900*b^2*d^2*e^4*n*x*Log[c*(d + e
/Sqrt[x])^n] - 1200*b^2*d^3*e^3*n*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 1800*b
^2*d^4*e^2*n*x^2*Log[c*(d + e/Sqrt[x])^n] - 3600*b^2*d^5*e*n*x^(5/2)*Log[c*
(d + e/Sqrt[x])^n] - 3600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n] - 1800*b^2
*e^6*Log[c*(d + e/Sqrt[x])^n]^2 + 3600*a*b*d^6*n*x^3*Log[e + d*Sqrt[x]] + 3
600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] - 1800*b^2*d^
```

```
6*n^2*x^3*Log[e + d*Sqrt[x]]^2 + 3600*a*b*d^6*n*x^3*Log[-(e/(d*Sqrt[x]))] +
 3600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + 3600*b
^2*d^6*n^2*x^3*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] + 3600*b^2*d^6*n^2*
x^3*PolyLog[2, 1 + e/(d*Sqrt[x])] + 3600*b^2*d^6*n^2*x^3*PolyLog[2, 1 + (d*
Sqrt[x])/e])/(5400*e^6*x^3)
```

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

Maxima [A] time = 1.07684, size = 522, normalized size = 1.09

$$\frac{1}{90}aben\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7}-\frac{30\,d^6\log\left(x\right)}{e^7}-\frac{60\,d^5x^{\frac{5}{2}}-30\,d^4ex^2+20\,d^3e^2x^{\frac{3}{2}}-15\,d^2e^3x+12\,de^4\sqrt{x}-10\,e^5}{e^6x^3}\right)+\frac{1}{540(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/90*a*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x
^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x
) - 10*e^5)/(e^6*x^3)) + 1/5400*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30
*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*
d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(x))^n)
- (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*
log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 555
*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^6*
x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*b^2 - 1/3*b^2*log(c*(d + e/sqrt(x))
^n)^2/x^3 - 2/3*a*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a^2/x^3
```

Fricas [A] time = 1.84532, size = 1094, normalized size = 2.28

$$-\frac{100\,b^2e^6n^2+1800\,b^2e^6\log\left(c\right)^2-600\,abe^6n+1800\,a^2e^6+90\left(29\,b^2d^4e^2n^2-20\,abd^4e^2n\right)x^2-1800\left(b^2d^6n^2x^3-b^2e^6n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/5400*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^2
*e^6 + 90*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x^2 - 1800*(b^2*d^6*n^2*x
^3 - b^2*e^6*n^2)*log((d*x + e*sqrt(x))/x)^2 + 15*(37*b^2*d^2*e^4*n^2 - 60*
```

```
a*b*d^2*e^4*n)*x - 300*(6*b^2*d^4*e^2*n*x^2 + 3*b^2*d^2*e^4*n*x + 2*b^2*e^6
*n - 12*a*b*e^6)*log(c) - 60*(30*b^2*d^4*e^2*n^2*x^2 + 15*b^2*d^2*e^4*n^2*x
 + 10*b^2*e^6*n^2 - 60*a*b*e^6*n - 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^3 +
60*(b^2*d^6*n*x^3 - b^2*e^6*n)*log(c) - 4*(15*b^2*d^5*e*n^2*x^2 + 5*b^2*d^3
*e^3*n^2*x + 3*b^2*d*e^5*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 12*(22*b^
2*d*e^5*n^2 - 60*a*b*d*e^5*n + 15*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e*n)*x^2 +
 5*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(15*b^2*d^5*e*n*x^2 + 5*b
^2*d^3*e^3*n*x + 3*b^2*d*e^5*n)*log(c))*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x^4, x)
```

3.436 $\int x \left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=569

$$-\frac{3b^2e^4n^2\text{PolyLog}\left(2, \frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4} + \frac{5b^3e^4n^3\text{PolyLog}\left(2, \frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{2d^4} - \frac{3b^3e^4n^3\text{PolyLog}\left(2, \frac{e}{d\sqrt{x}} + 1\right)}{d^4}$$

```
[Out] (b^3*e^3*n^3*Sqrt[x])/(2*d^3) - (b^3*e^4*n^3*Log[d + e/Sqrt[x]])/(2*d^4) -
(5*b^2*e^3*n^2*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2
*d^4) + (b^2*e^2*n^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^2) - (5*b^2*e
^4*n^2*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^4)
 + (3*b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(
2*d^4) - (3*b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*d^2) + (b*e*n*
x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d) + (3*b*e^4*n*Log[1 - d/(d
 + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d^4) + (x^2*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^3)/2 - (3*b^2*e^4*n^2*(a + b*Log[c*(d + e/Sqrt[x])^
n])*Log[-(e/(d*Sqrt[x]))])/d^4 - (3*b^3*e^4*n^3*Log[x])/(2*d^4) + (5*b^3*e^
4*n^3*PolyLog[2, d/(d + e/Sqrt[x])])/(2*d^4) - (3*b^2*e^4*n^2*(a + b*Log[c*
(d + e/Sqrt[x])^n])*PolyLog[2, d/(d + e/Sqrt[x])])/d^4 - (3*b^3*e^4*n^3*Pol
yLog[2, 1 + e/(d*Sqrt[x])])/d^4 - (3*b^3*e^4*n^3*PolyLog[3, d/(d + e/Sqrt[x
])])/d^4
```

Rubi [A] time = 1.49154, antiderivative size = 546, normalized size of antiderivative = 0.96, number of steps used = 35, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$\frac{3b^2e^4n^2\text{PolyLog}\left(2, \frac{e}{d\sqrt{x}} + 1\right)\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4} - \frac{11b^3e^4n^3\text{PolyLog}\left(2, \frac{e}{d\sqrt{x}} + 1\right)}{2d^4} - \frac{3b^3e^4n^3\text{PolyLog}\left(3, \frac{e}{d\sqrt{x}}\right.}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e^3*n^3*Sqrt[x])/(2*d^3) - (b^3*e^4*n^3*Log[d + e/Sqrt[x]])/(2*d^4) -
(5*b^2*e^3*n^2*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2
*d^4) + (b^2*e^2*n^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^2) + (5*b*e^4
*n*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*d^4) + (3*b*e^3*n*(d + e/Sqrt[x])
*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d^4) - (3*b*e^2*n*x*(a + b*
Log[c*(d + e/Sqrt[x])^n])^2)/(4*d^2) + (b*e*n*x^(3/2)*(a + b*Log[c*(d + e/S
qrt[x])^n])^2)/(2*d) - (e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*d^4) + (
x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/2 - (11*b^2*e^4*n^2*(a + b*Log[c*(d
 + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/(2*d^4) + (3*b*e^4*n*(a + b*Log[c*
(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))])/(2*d^4) - (3*b^3*e^4*n^3*Log[x
])/(2*d^4) - (11*b^3*e^4*n^3*PolyLog[2, 1 + e/(d*Sqrt[x])])/(2*d^4) + (3*b^
2*e^4*n^2*(a + b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])])/d
^4 - (3*b^3*e^4*n^3*PolyLog[3, 1 + e/(d*Sqrt[x])])/d^4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3dx = -\left(2\,\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^3}{x^5}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{1}{2}(3ben)\,\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^4(d+ex)}dx,x\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{1}{2}(3bn)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,x,d+\frac{e}{\sqrt{}}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{(3bn)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,x,d+\frac{e}{\sqrt{x}}\right)}{2d}+\frac{(3b}{}$$

$$=\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3+\frac{(3ben)\,\text{Sub}}{}$$

$$=-\frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d}+\frac{1}{2}x^2\left(a\right.$$

$$=\frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}$$

$$=-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}+\frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right.\right.}{2d^2}$$

$$=\frac{b^3e^3n^3\sqrt{x}}{2d^3}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}$$

$$=\frac{b^3e^3n^3\sqrt{x}}{2d^3}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}$$

Mathematica [A] time = 0.977875, size = 777, normalized size = 1.37

$$-2b^2n^2\left(-6e^4\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)+3\left(e^4-d^4x^2\right)\log^2\left(d+\frac{e}{\sqrt{x}}\right)-e\log\left(d+\frac{e}{\sqrt{x}}\right)\left(-3d^2ex+2d^3x^{3/2}+6de^2\sqrt{x}+6e^3\log\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (6*b*d*e^3*n*Sqrt[x]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^
n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x
])^n])^2 + 2*b*d^3*e*n*x^(3/2)*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e
/Sqrt[x])^n])^2 + 6*b*d^4*n*x^2*Log[d + e/Sqrt[x]]*(a - b*n*Log[d + e/Sqrt[
x]] + b*Log[c*(d + e/Sqrt[x])^n])^2 + 2*d^4*x^2*(a - b*n*Log[d + e/Sqrt[x]]
 + b*Log[c*(d + e/Sqrt[x])^n])^3 - 6*b*e^4*n*(a - b*n*Log[d + e/Sqrt[x]] +
b*Log[c*(d + e/Sqrt[x])^n])^2*Log[e + d*Sqrt[x]] - 2*b^2*n^2*(a - b*n*Log[d
 + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(3*(e^4 - d^4*x^2)*Log[d + e/Sq
```

```
rt[x]]^2 + e^2*(5*d*e*Sqrt[x] - d^2*x + 11*e^2*Log[-(e/(d*Sqrt[x]))]) - e*L
og[d + e/Sqrt[x]]*(11*e^3 + 6*d*e^2*Sqrt[x] - 3*d^2*e*x + 2*d^3*x^(3/2) + 6
*e^3*Log[-(e/(d*Sqrt[x]))]) - 6*e^4*PolyLog[2, 1 + e/(d*Sqrt[x])]) + b^3*n^
3*(d^2*e^2*x*(2 - 3*Log[d + e/Sqrt[x]])*Log[d + e/Sqrt[x]] + 2*d^3*e*x^(3/2
)*Log[d + e/Sqrt[x]]^2 + 2*d^4*x^2*Log[d + e/Sqrt[x]]^3 + 2*d*e^3*Sqrt[x]*(
1 - 5*Log[d + e/Sqrt[x]] + 3*Log[d + e/Sqrt[x]]^2) + 12*e^4*(-Log[d + e/Sqr
t[x]] + Log[-(e/(d*Sqrt[x]))]) + 11*e^4*(Log[d + e/Sqrt[x]]*(Log[d + e/Sqrt
[x]] - 2*Log[-(e/(d*Sqrt[x]))]) - 2*PolyLog[2, 1 + e/(d*Sqrt[x])]) - 2*e^4*
(Log[d + e/Sqrt[x]]^2*(Log[d + e/Sqrt[x]] - 3*Log[-(e/(d*Sqrt[x]))]) - 6*Lo
g[d + e/Sqrt[x]]*PolyLog[2, 1 + e/(d*Sqrt[x])] + 6*PolyLog[3, 1 + e/(d*Sqrt
[x])])))/(4*d^4)
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b^3x^2\log\left(\left(d\sqrt{x}+e\right)^n\right)^3-\int\frac{4\left(b^3dx^2+b^3ex^{\frac{3}{2}}\right)\log\left(x^{\frac{1}{2}n}\right)^3-4\left(b^3d\log\left(c\right)^3+3\,ab^2d\log\left(c\right)^2+3\,a^2bd\log\left(c\right)+a^3d\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((d*sqrt(x) + e)^n)^3 - integrate(1/4*(4*(b^3*d*x^2 + b^3*e*
x^(3/2))*log(x^(1/2*n))^3 - 4*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*
b*d*log(c) + a^3*d)*x^2 + 3*(b^3*d*n*x^2 - 4*(b^3*d*log(c) + a*b^2*d)*x^2 -
 4*(b^3*e*log(c) + a*b^2*e)*x^(3/2) + 4*(b^3*d*x^2 + b^3*e*x^(3/2))*log(x^(
1/2*n)))*log((d*sqrt(x) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3
*e*log(c) + a*b^2*e)*x^(3/2))*log(x^(1/2*n))^2 - 4*(b^3*e*log(c)^3 + 3*a*b^
2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(3/2) - 12*((b^3*d*log(c)^2 + 2*
a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(3/2))*log(x^(1/2*n))^
2 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2) - 2*((b^3*d*log(c
) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(3/2))*log(x^(1/2*n)))*log((d
*sqrt(x) + e)^n) + 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 +
(b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2))*log(x^(1/2*n)))/(d*x
 + e*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^3+3\,ab^2x\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+3\,a^2bx\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*sqrt(x))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*sqrt(x))/x)^n) + a^3*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right)^3 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3*x, x)
```

3.437 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 \, dx$

Optimal. Leaf size=260

$$-\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}$$

```
[Out] (3*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d^2 +
(3*b*e^2*n*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d
^2 + x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3 - (6*b^2*e^2*n^2*(a + b*Log[c*(d
+ e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/d^2 - (6*b^2*e^2*n^2*(a + b*Log[c*(
d + e/Sqrt[x])^n])*PolyLog[2, d/(d + e/Sqrt[x])])/d^2 - (6*b^3*e^2*n^3*Poly
Log[2, 1 + e/(d*Sqrt[x])])/d^2 - (6*b^3*e^2*n^3*PolyLog[3, d/(d + e/Sqrt[x]
)])/d^2
```

Rubi [A] time = 0.616107, antiderivative size = 281, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391}

$$\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (3*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d^2 -
(e^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/d^2 + x*(a + b*Log[c*(d + e/Sqrt[x
])^n])^3 - (6*b^2*e^2*n^2*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[
x]))])/d^2 + (3*b*e^2*n*(a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[
x]))])/d^2 - (6*b^3*e^2*n^3*PolyLog[2, 1 + e/(d*Sqrt[x])])/d^2 + (6*b^2*e^2
*n^2*(a + b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])])/d^2 -
(6*b^3*e^2*n^3*PolyLog[3, 1 + e/(d*Sqrt[x])])/d^2
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3 dx, x, \sqrt{x}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^3} dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - (3ben)\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(d+ex)} dx, x, \frac{1}{\sqrt{x}}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - (3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, d+\frac{e}{\sqrt{x}}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, d+\frac{e}{\sqrt{x}}\right)}{d} + \frac{(3be}{}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 + \frac{(3}{}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{6b}{}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{d^2} + x$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{d^2} + x$$

Mathematica [A] time = 0.665092, size = 476, normalized size = 1.83

$$3b^2n^2\left(2e^2\text{PolyLog}\left(2, \frac{e}{d\sqrt{x}}+1\right) + \left(d^2x-e^2\right)\log^2\left(d+\frac{e}{\sqrt{x}}\right) - 2e^2\log\left(-\frac{e}{d\sqrt{x}}\right) + 2e\log\left(d+\frac{e}{\sqrt{x}}\right)\left(e\log\left(-\frac{e}{d\sqrt{x}}\right) + d\sqrt{x}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n]
)^2 + 3*b*d^2*n*x*Log[d + e/Sqrt[x]]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*
(d + e/Sqrt[x])^n])^2 + d^2*x*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/
Sqrt[x])^n])^3 - 3*b*e^2*n*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqr
t[x])^n])^2*Log[e + d*Sqrt[x]] + 3*b^2*n^2*(a - b*n*Log[d + e/Sqrt[x]] + b*
Log[c*(d + e/Sqrt[x])^n])*((-e^2 + d^2*x)*Log[d + e/Sqrt[x]]^2 - 2*e^2*Log[
-(e/(d*Sqrt[x]))] + 2*e*Log[d + e/Sqrt[x]]*(e + d*Sqrt[x] + e*Log[-(e/(d*Sq
rt[x]))]) + 2*e^2*PolyLog[2, 1 + e/(d*Sqrt[x])]) + b^3*n^3*(Log[d + e/Sqrt[
x]]*((-e^2 + d^2*x)*Log[d + e/Sqrt[x]]^2 - 6*e^2*Log[-(e/(d*Sqrt[x]))] + 3*
e*Log[d + e/Sqrt[x]]*(e + d*Sqrt[x] + e*Log[-(e/(d*Sqrt[x]))])) + 6*e^2*(-1
 + Log[d + e/Sqrt[x]])*PolyLog[2, 1 + e/(d*Sqrt[x])] - 6*e^2*PolyLog[3, 1 +
 e/(d*Sqrt[x])]))/d^2
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^{3}x\log\left(\left(d\sqrt{x}+e\right)^{n}\right)^{3}-3\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^{2}}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)a^{2}b+a^{3}x-\int\frac{2\left(b^{3}dx+b^{3}e\sqrt{x}\right)\log\left(x^{\frac{1}{2}n}\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log((d*sqrt(x) + e)^n)^3 - 3*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)
/d) - x*log(c*(d + e/sqrt(x))^n))*a^2*b + a^3*x - integrate(1/2*(2*(b^3*d*x
 + b^3*e*sqrt(x))*log(x^(1/2*n))^3 + 3*(b^3*d*n*x - 2*(b^3*d*log(c) + a*b^2
*d)*x + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c) + a*b^
2*e)*sqrt(x))*log((d*sqrt(x) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x + (b
^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))^2 - 2*(b^3*d*log(c)^3 + 3*a*
b^2*d*log(c)^2)*x - 6*((b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n))^2 + (b^3*d*
log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log
(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n)) + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c)
)*sqrt(x))*log((d*sqrt(x) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*
x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*
log(c)^3 + 3*a*b^2*e*log(c)^2)*sqrt(x))/(d*x + e*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^{3}\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{n}\right)^{3}+3\,ab^{2}\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{n}\right)^{2}+3\,a^{2}b\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{n}\right)+a^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*sqr
t(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right) + a\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3, x)
```

3.438 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$12b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-6bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-12b^3$$

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[-(e/(d*Sqrt[x]))] - 6*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])^2*PolyLog[2, 1 + e/(d*Sqrt[x])] + 12*b^2*n^2*(a +
 b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[3, 1 + e/(d*Sqrt[x])] - 12*b^3*n^3*Pol
yLog[4, 1 + e/(d*Sqrt[x])]
```

Rubi [A] time = 0.197535, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$12b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-6bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-12b^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[-(e/(d*Sqrt[x]))] - 6*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])^2*PolyLog[2, 1 + e/(d*Sqrt[x])] + 12*b^2*n^2*(a +
 b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[3, 1 + e/(d*Sqrt[x])] - 12*b^3*n^3*Pol
yLog[4, 1 + e/(d*Sqrt[x])]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = -\left(2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) + (6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\lo}{d+e}\right.$$
$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) + (6bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\,\text{lo}}{x}\right.$$
$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\text{Li}_2\Bigg($$
$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\text{Li}_2\Bigg($$
$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\text{Li}_2\Bigg($$

Mathematica [B] time = 0.262637, size = 532, normalized size = 3.94

$$6b^2n^2\left(-2\text{PolyLog}\left(3,\frac{d\sqrt{x}}{e}+1\right)-2\text{PolyLog}\left(3,-\frac{d\sqrt{x}}{e}\right)+2\log\left(\frac{e}{d}+\sqrt{x}\right)\text{PolyLog}\left(2,\frac{d\sqrt{x}}{e}+1\right)-2\left(\log\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*((Log[d + e/Sqr
t[x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) +
6*b^2*n^2*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(Log[e/
d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + (Log[d + e/Sqrt[x]]^2*Log[x])/2 - (L
```

```
og[e/d + Sqrt[x]]^2*Log[x])/2 - Log[d + e/Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*L
og[x] + Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*Log[x] + (Log[d + e/Sqrt[
x]]*Log[x]^2)/4 - (Log[1 + (d*Sqrt[x])/e]*Log[x]^2)/4 + Log[x]^3/24 + 2*Log
[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 2*(Log[d + e/Sqrt[x]] - Log
[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 2*PolyLog[3, 1 + (d*Sqrt[x]
)/e] - 2*PolyLog[3, -((d*Sqrt[x])/e)]) - 2*b^3*n^3*(Log[d + e/Sqrt[x]]^3*Lo
g[-(e/(d*Sqrt[x]))] + 3*Log[d + e/Sqrt[x]]^2*PolyLog[2, 1 + e/(d*Sqrt[x])]
- 6*Log[d + e/Sqrt[x]]*PolyLog[3, 1 + e/(d*Sqrt[x])] + 6*PolyLog[4, 1 + e/(
d*Sqrt[x])])
```

Maple [F] time = 0.347, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^{3}\log\left(\left(d\sqrt{x}+e\right)^{n}\right)^{3}\log\left(x\right)-\int \frac{2\left(b^{3}dx+b^{3}e\sqrt{x}\right)\log\left(x^{\frac{1}{2}n}\right)^{3}+3\left(b^{3}dnx\log\left(x\right)-2\left(b^{3}d\log\left(c\right)+ab^{2}d\right)x+2\left(b^{3}dx+b^{3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((d*sqrt(x) + e)^n)^3*log(x) - integrate(1/2*(2*(b^3*d*x + b^3*e*sqr
t(x))*log(x^(1/2*n))^3 + 3*(b^3*d*n*x*log(x) - 2*(b^3*d*log(c) + a*b^2*d)*x
 + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c) + a*b^2*e)*
sqrt(x))*log((d*sqrt(x) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*
log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))^2 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d
*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 6*((b^3*d*x + b^3*e*sqrt(x))*log(
x^(1/2*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*d*
log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n)) + (b
^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*sqrt(x))*log((d*sqrt(x) + e)^n)
 + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2
*a*b^2*e*log(c) + a^2*b*e)*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c)^3 + 3*
a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*sqrt(x))/(d*x^2 + e*x^(3/2)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{3}\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{n}\right)^{3}+3\,ab^{2}\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{n}\right)^{2}+3\,a^{2}b\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{n}\right)+a^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*sq
rt(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x, x)
```

3.439 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=285

$$-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2}-\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a\right.}{}$$

```
[Out] (3*b^3*n^3*(d + e/Sqrt[x])^2)/(4*e^2) + (12*a*b^2*d*n^2)/(e*Sqrt[x]) - (12*
b^3*d*n^3)/(e*Sqrt[x]) + (12*b^3*d*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x]
)^n])/e^2 - (3*b^2*n^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/
(2*e^2) - (6*b*d*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
+ (3*b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^2) + (2
*d*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2 - ((d + e/Sqrt[x
])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2
```

Rubi [A] time = 0.272381, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2}-\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```


```
[Out] (3*b^3*n^3*(d + e/Sqrt[x])^2)/(4*e^2) + (12*a*b^2*d*n^2)/(e*Sqrt[x]) - (12*
b^3*d*n^3)/(e*Sqrt[x]) + (12*b^3*d*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x]
)^n])/e^2 - (3*b^2*n^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/
(2*e^2) - (6*b*d*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
+ (3*b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^2) + (2
*d*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2 - ((d + e/Sqrt[x
])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\left(2\,\mathrm{Subst}\left(\int x\,(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\mathrm{Subst}\left(\int\left(-\frac{d\,(a+b\log\left(c(d+ex)^n\right))^3}{e}+\frac{(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^3}{e}\right)dx, x\right.\right.$$

$$= -\frac{2\,\mathrm{Subst}\left(\int(d+ex)\,(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)\,\mathrm{Subst}\left(\int(a+b\log\left(c\right.\right.}{e}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int x\,(a+b\log\left(cx^n\right))^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\,\mathrm{Subst}\left(\int(a+b\log\left(cx^n\right))^3\,dx\right.}{e^2}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}+\frac{(3b}{}$$

$$= -\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right.}{2e^2}$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}-\frac{6bdn\left(d\right.}{}$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}-\frac{12b^3dn^3}{e\sqrt{x}}+\frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2}-\frac{3b^2}{}$$

Mathematica [A] time = 0.614716, size = 558, normalized size = 1.96

$$\frac{-6b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(e\left(2a^2e-2abn\left(e-2d\sqrt{x}\right)+b^2n^2\left(e-6d\sqrt{x}\right)\right)+2bd^2nx(3bn-2a)\log\left(d\sqrt{x}+e\right)+bd^2nx\log(x)(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-4*a^3*e^2 + 6*a^2*b*e^2*n - 6*a*b^2*e^2*n^2 + 3*b^3*e^2*n^3 - 12*a^2*b*d*
e*n*Sqrt[x] + 36*a*b^2*d*e*n^2*Sqrt[x] - 42*b^3*d*e*n^3*Sqrt[x] - 8*b^3*d^2
*n^3*x*Log[d + e/Sqrt[x]]^3 - 4*b^3*e^2*Log[c*(d + e/Sqrt[x])^n]^3 + 12*a^2
*b*d^2*n*x*Log[e + d*Sqrt[x]] - 36*a*b^2*d^2*n^2*x*Log[e + d*Sqrt[x]] + 42*
b^3*d^2*n^3*x*Log[e + d*Sqrt[x]] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]*(-2*a
 + 3*b*n - 2*b*Log[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) -
6*a^2*b*d^2*n*x*Log[x] + 18*a*b^2*d^2*n^2*x*Log[x] - 21*b^3*d^2*n^3*x*Log[x
] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]^2*(2*a - 3*b*n + 2*b*Log[c*(d + e/Sq
rt[x])^n] + 2*b*n*Log[e + d*Sqrt[x]] - b*n*Log[x]) + 6*b^2*Log[c*(d + e/Sqr
t[x])^n]^2*(e*(-2*a*e + b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*x*Log[e + d*Sqrt
[x]] - b*d^2*n*x*Log[x]) - 6*b*Log[c*(d + e/Sqrt[x])^n]*(e*(2*a^2*e + b^2*n
^2*(e - 6*d*Sqrt[x]) - 2*a*b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*(-2*a + 3*b*n
)*x*Log[e + d*Sqrt[x]] + b*d^2*n*(2*a - 3*b*n)*x*Log[x]))/(4*e^2*x)
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

Maxima [B] time = 1.17022, size = 767, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x)
 - e)/(e^2*x)) - b^3*log(c*(d + e/sqrt(x))^n)^3/x + 3/4*(4*e*n*(2*d^2*log(d
*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d +
e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*lo
g(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) +
e))*n^2/(e^2*x))*a*b^2 + 1/8*(12*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*lo
g(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n)^2 + e*n*((8*
d^2*x*log(d*sqrt(x) + e)^3 - d^2*x*log(x)^3 + 9*d^2*x*log(x)^2 - 42*d^2*x*l
og(x) - 12*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) + e)^2 - 84*d*e*sqrt(x) +
 6*e^2 + 6*(d^2*x*log(x)^2 - 6*d^2*x*log(x) + 14*d^2*x)*log(d*sqrt(x) + e))
*n^2/(e^3*x) - 6*(4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*l
og(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) +
 e))*n*log(c*(d + e/sqrt(x))^n)/(e^3*x)))*b^3 - 3*a*b^2*log(c*(d + e/sqrt(x
))^n)^2/x - 3*a^2*b*log(c*(d + e/sqrt(x))^n)/x - a^3/x
```

Fricas [B] time = 1.86846, size = 1162, normalized size = 4.08

$$3\,b^3e^2n^3 - 4\,b^3e^2\log(c)^3 - 6\,ab^2e^2n^2 + 6\,a^2be^2n - 4\,a^3e^2 + 4\left(b^3d^2n^3x - b^3e^2n^3\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^3 + 6\left(b^3e^2n - 2\,ab^2e^2\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^3*e^2*n^3 - 4*b^3*e^2*log(c)^3 - 6*a*b^2*e^2*n^2 + 6*a^2*b*e^2*n -
 4*a^3*e^2 + 4*(b^3*d^2*n^3*x - b^3*e^2*n^3)*log((d*x + e*sqrt(x))/x)^3 + 6
*(b^3*e^2*n - 2*a*b^2*e^2)*log(c)^2 - 6*(2*b^3*d*e*n^3*sqrt(x) - b^3*e^2*n^
3 + 2*a*b^2*e^2*n^2 + (3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2)*x - 2*(b^3*d^2*n^2*
x - b^3*e^2*n^2)*log(c))*log((d*x + e*sqrt(x))/x)^2 - 6*(b^3*e^2*n^2 - 2*a*
b^2*e^2*n + 2*a^2*b*e^2)*log(c) - 6*(b^3*e^2*n^3 - 2*a*b^2*e^2*n^2 + 2*a^2*
b*e^2*n - 2*(b^3*d^2*n*x - b^3*e^2*n)*log(c)^2 - (7*b^3*d^2*n^3 - 6*a*b^2*d
^2*n^2 + 2*a^2*b*d^2*n)*x - 2*(b^3*e^2*n^2 - 2*a*b^2*e^2*n - (3*b^3*d^2*n^2
 - 2*a*b^2*d^2*n)*x)*log(c) - 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log(c) - 2*a
*b^2*d*e*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 6*(7*b^3*d*e*n^3 + 2*b^3*
d*e*n*log(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*n^2 - 2*a*b
^2*d*e*n)*log(c))*sqrt(x))/(e^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x^2, x)
```

3.440 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=595

$$-\frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4}-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\right.}{3e^4}$$

```
[Out] (9*b^3*d^2*n^3*(d + e/Sqrt[x])^2)/(4*e^4) - (4*b^3*d*n^3*(d + e/Sqrt[x])^3)
/(9*e^4) + (3*b^3*n^3*(d + e/Sqrt[x])^4)/(64*e^4) + (12*a*b^2*d^3*n^2)/(e^3
*Sqrt[x]) - (12*b^3*d^3*n^3)/(e^3*Sqrt[x]) + (12*b^3*d^3*n^2*(d + e/Sqrt[x]
)*Log[c*(d + e/Sqrt[x])^n])/e^4 - (9*b^2*d^2*n^2*(d + e/Sqrt[x])^2*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(3*e^4) - (3*b^2*n^2*(d + e/Sqrt[x])^4*(a + b*Log
[c*(d + e/Sqrt[x])^n]))/(16*e^4) - (6*b*d^3*n*(d + e/Sqrt[x])*(a + b*Log[c*
(d + e/Sqrt[x])^n])^2)/e^4 + (9*b*d^2*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d +
 e/Sqrt[x])^n])^2)/(2*e^4) - (2*b*d*n*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e
/Sqrt[x])^n])^2)/e^4 + (3*b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x]
)^n])^2)/(8*e^4) + (2*d^3*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^
3)/e^4 - (3*d^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 +
 (2*d*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 - ((d + e/S
qrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^4)
```

Rubi [A] time = 0.641699, antiderivative size = 595, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4}-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\right.}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```


```
[Out] (9*b^3*d^2*n^3*(d + e/Sqrt[x])^2)/(4*e^4) - (4*b^3*d*n^3*(d + e/Sqrt[x])^3)
/(9*e^4) + (3*b^3*n^3*(d + e/Sqrt[x])^4)/(64*e^4) + (12*a*b^2*d^3*n^2)/(e^3
*Sqrt[x]) - (12*b^3*d^3*n^3)/(e^3*Sqrt[x]) + (12*b^3*d^3*n^2*(d + e/Sqrt[x]
)*Log[c*(d + e/Sqrt[x])^n])/e^4 - (9*b^2*d^2*n^2*(d + e/Sqrt[x])^2*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(3*e^4) - (3*b^2*n^2*(d + e/Sqrt[x])^4*(a + b*Log
[c*(d + e/Sqrt[x])^n]))/(16*e^4) - (6*b*d^3*n*(d + e/Sqrt[x])*(a + b*Log[c*
(d + e/Sqrt[x])^n])^2)/e^4 + (9*b*d^2*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d +
 e/Sqrt[x])^n])^2)/(2*e^4) - (2*b*d*n*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e
/Sqrt[x])^n])^2)/e^4 + (3*b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x]
)^n])^2)/(8*e^4) + (2*d^3*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^
3)/e^4 - (3*d^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 +
 (2*d*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 - ((d + e/S
qrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^4)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}+\frac{3d^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)\right.}{e^3}\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^3}+\frac{(6d)\,\text{Subst}\left(\int(d+ex)\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x^3\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}+\frac{(6d)\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx\right.\right.\right.}{e^4}$$

$$= \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}-\frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right.}{e^4}$$

$$= -\frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4}+\frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{2e^4}$$

$$= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4}-\frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}+\frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4}+\frac{12ab^2d^3n^2}{e^3\sqrt{x}}-\frac{9b^2d^2}{}$$

$$= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4}-\frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}+\frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4}+\frac{12ab^2d^3n^2}{e^3\sqrt{x}}-\frac{12b^3d^3}{e^3\sqrt{}}$$

Mathematica [A] time = 1.00426, size = 766, normalized size = 1.29

$$\frac{-12b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(72a^2e^4-12aben\left(6d^2ex-12d^3x^{3/2}-4de^2\sqrt{x}+3e^3\right)+12bd^4nx^2(25bn-12a)\log\left(d\sqrt{x}+e\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

```
[Out] (-288*a^3*e^4 + 216*a^2*b*e^4*n - 108*a*b^2*e^4*n^2 + 27*b^3*e^4*n^3 - 288*
a^2*b*d*e^3*n*Sqrt[x] + 336*a*b^2*d*e^3*n^2*Sqrt[x] - 148*b^3*d*e^3*n^3*Sqr
t[x] + 432*a^2*b*d^2*e^2*n*x - 936*a*b^2*d^2*e^2*n^2*x + 690*b^3*d^2*e^2*n^
3*x - 864*a^2*b*d^3*e*n*x^(3/2) + 3600*a*b^2*d^3*e*n^2*x^(3/2) - 4980*b^3*d
^3*e*n^3*x^(3/2) - 576*b^3*d^4*n^3*x^2*Log[d + e/Sqrt[x]]^3 - 288*b^3*e^4*L
og[c*(d + e/Sqrt[x])^n]^3 + 864*a^2*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 3600*a
*b^2*d^4*n^2*x^2*Log[e + d*Sqrt[x]] + 4980*b^3*d^4*n^3*x^2*Log[e + d*Sqrt[x
]] + 72*b^2*d^4*n^2*x^2*Log[d + e/Sqrt[x]]*(-12*a + 25*b*n - 12*b*Log[c*(d
+ e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 432*a^2*b*d^4*n*x^2*Log[
x] + 1800*a*b^2*d^4*n^2*x^2*Log[x] - 2490*b^3*d^4*n^3*x^2*Log[x] + 72*b^2*d
^4*n^2*x^2*Log[d + e/Sqrt[x]]^2*(12*a - 25*b*n + 12*b*Log[c*(d + e/Sqrt[x])
^n] + 12*b*n*Log[e + d*Sqrt[x]] - 6*b*n*Log[x]) + 72*b^2*Log[c*(d + e/Sqrt[
x])^n]^2*(e*(-12*a*e^3 + 3*b*e^3*n - 4*b*d*e^2*n*Sqrt[x] + 6*b*d^2*e*n*x -
12*b*d^3*n*x^(3/2)) + 12*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 6*b*d^4*n*x^2*Log
[x]) - 12*b*Log[c*(d + e/Sqrt[x])^n]*(72*a^2*e^4 + b^2*e*n^2*(9*e^3 - 28*d*
e^2*Sqrt[x] + 78*d^2*e*x - 300*d^3*x^(3/2)) - 12*a*b*e*n*(3*e^3 - 4*d*e^2*S
qrt[x] + 6*d^2*e*x - 12*d^3*x^(3/2)) + 12*b*d^4*n*(-12*a + 25*b*n)*x^2*Log[
e + d*Sqrt[x]] + 6*b*d^4*n*(12*a - 25*b*n)*x^2*Log[x]))/(576*e^4*x^2)
```

Maple [F] time = 0.344, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)

[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

Maxima [A] time = 1.1549, size = 988, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="maxima")

[Out] 1/8*a^2*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x
^(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/48*(12*e*n*(12
*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*e*
x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*d^4*
x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d
^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(
x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*a*b^2 + 1/576*(72*e*n*(
12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*
e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n)^2 + e*n*
((288*d^4*x^2*log(d*sqrt(x) + e)^3 - 36*d^4*x^2*log(x)^3 + 450*d^4*x^2*log(
x)^2 - 2490*d^4*x^2*log(x) - 4980*d^3*e*x^(3/2) + 690*d^2*e^2*x - 148*d*e^3
*sqrt(x) + 27*e^4 - 72*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e)^2
 + 12*(18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) + 415*d^4*x^2)*log(d*sqrt(x
) + e))*n^2/(e^5*x^2) - 12*(72*d^4*x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*lo
g(x)^2 - 150*d^4*x^2*log(x) - 300*d^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*s
qrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n*l
og(c*(d + e/sqrt(x))^n)/(e^5*x^2)))*b^3 - 1/2*b^3*log(c*(d + e/sqrt(x))^n)^
3/x^2 - 3/2*a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/s
qrt(x))^n)/x^2 - 1/2*a^3/x^2
```

Fricas [A] time = 1.88304, size = 1894, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="fricas")

[Out] 1/576*(27*b^3*e^4*n^3 - 288*b^3*e^4*log(c)^3 - 108*a*b^2*e^4*n^2 + 216*a^2*
b*e^4*n - 288*a^3*e^4 + 288*(b^3*d^4*n^3*x^2 - b^3*e^4*n^3)*log((d*x + e*sq
rt(x))/x)^3 + 216*(2*b^3*d^2*e^2*n*x + b^3*e^4*n - 4*a*b^2*e^4)*log(c)^2 +
72*(6*b^3*d^2*e^2*n^3*x + 3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 - (25*b^3*d^4*n^
3 - 12*a*b^2*d^4*n^2)*x^2 + 12*(b^3*d^4*n^2*x^2 - b^3*e^4*n^2)*log(c) - 4*(
3*b^3*d^3*e*n^3*x + b^3*d*e^3*n^3)*sqrt(x))*log((d*x + e*sqrt(x))/x)^2 + 6*
(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*x - 36*(
```

```
3*b^3*e^4*n^2 - 12*a*b^2*e^4*n + 24*a^2*b*e^4 + 2*(13*b^3*d^2*e^2*n^2 - 12*
a*b^2*d^2*e^2*n)*x)*log(c) - 12*(9*b^3*e^4*n^3 - 36*a*b^2*e^4*n^2 + 72*a^2*
b*e^4*n - (415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n)*x^2 - 72*(
b^3*d^4*n*x^2 - b^3*e^4*n)*log(c)^2 + 6*(13*b^3*d^2*e^2*n^3 - 12*a*b^2*d^2*
e^2*n^2)*x - 12*(6*b^3*d^2*e^2*n^2*x + 3*b^3*e^4*n^2 - 12*a*b^2*e^4*n - (25
*b^3*d^4*n^2 - 12*a*b^2*d^4*n)*x^2)*log(c) - 4*(7*b^3*d*e^3*n^3 - 12*a*b^2*
d*e^3*n^2 + 3*(25*b^3*d^3*e*n^3 - 12*a*b^2*d^3*e*n^2)*x - 12*(3*b^3*d^3*e*n
^2*x + b^3*d*e^3*n^2)*log(c))*sqrt(x))*log((d*x + e*sqrt(x))/x) - 4*(37*b^3
*d*e^3*n^3 - 84*a*b^2*d*e^3*n^2 + 72*a^2*b*d*e^3*n + 72*(3*b^3*d^3*e*n*x +
b^3*d*e^3*n)*log(c)^2 + 3*(415*b^3*d^3*e*n^3 - 300*a*b^2*d^3*e*n^2 + 72*a^2
*b*d^3*e*n)*x - 12*(7*b^3*d*e^3*n^2 - 12*a*b^2*d*e^3*n + 3*(25*b^3*d^3*e*n^
2 - 12*a*b^2*d^3*e*n)*x)*log(c))*sqrt(x))/(e^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right) + a\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x^3, x)
```

3.441 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=907

result too large to display

```
[Out] (15*b^3*d^4*n^3*(d + e/Sqrt[x])^2)/(4*e^6) - (40*b^3*d^3*n^3*(d + e/Sqrt[x]
)^3)/(27*e^6) + (15*b^3*d^2*n^3*(d + e/Sqrt[x])^4)/(32*e^6) - (12*b^3*d*n^3
*(d + e/Sqrt[x])^5)/(125*e^6) + (b^3*n^3*(d + e/Sqrt[x])^6)/(108*e^6) + (12
*a*b^2*d^5*n^2)/(e^5*Sqrt[x]) - (12*b^3*d^5*n^3)/(e^5*Sqrt[x]) + (12*b^3*d^
5*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^6 - (15*b^2*d^4*n^2*(d +
e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) + (40*b^2*d^3*n^2*(d
 + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) - (15*b^2*d^2*n^2
*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(8*e^6) + (12*b^2*d*n^
2*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(25*e^6) - (b^2*n^2*(
d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(18*e^6) - (6*b*d^5*n*(d
 + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^6 + (15*b*d^4*n*(d + e/
Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^6) - (20*b*d^3*n*(d + e
/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(3*e^6) + (15*b*d^2*n*(d +
e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^6) - (6*b*d*n*(d + e/
Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(5*e^6) + (b*n*(d + e/Sqrt[x
])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(6*e^6) + (2*d^5*(d + e/Sqrt[x])*(
a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^6 - (5*d^4*(d + e/Sqrt[x])^2*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^3)/e^6 + (20*d^3*(d + e/Sqrt[x])^3*(a + b*Log[c*(d
+ e/Sqrt[x])^n])^3)/(3*e^6) - (5*d^2*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/
Sqrt[x])^n])^3)/e^6 + (2*d*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n
])^3)/e^6 - ((d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(3*e^6)
```

Rubi [A] time = 1.01085, antiderivative size = 907, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{3e^6}+\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{6e^6}-\frac{b^2n^2\left(a+b\log\left(c\left(a\cdots\right.\right.\right.}{18\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

```
[Out] (15*b^3*d^4*n^3*(d + e/Sqrt[x])^2)/(4*e^6) - (40*b^3*d^3*n^3*(d + e/Sqrt[x]
)^3)/(27*e^6) + (15*b^3*d^2*n^3*(d + e/Sqrt[x])^4)/(32*e^6) - (12*b^3*d*n^3
*(d + e/Sqrt[x])^5)/(125*e^6) + (b^3*n^3*(d + e/Sqrt[x])^6)/(108*e^6) + (12
*a*b^2*d^5*n^2)/(e^5*Sqrt[x]) - (12*b^3*d^5*n^3)/(e^5*Sqrt[x]) + (12*b^3*d^
5*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^6 - (15*b^2*d^4*n^2*(d +
e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) + (40*b^2*d^3*n^2*(d
 + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) - (15*b^2*d^2*n^2
*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(8*e^6) + (12*b^2*d*n^
2*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(25*e^6) - (b^2*n^2*(
d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(18*e^6) - (6*b*d^5*n*(d
 + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^6 + (15*b*d^4*n*(d + e/
Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^6) - (20*b*d^3*n*(d + e
/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(3*e^6) + (15*b*d^2*n*(d +
e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^6) - (6*b*d*n*(d + e/
```

```
Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(5*e^6) + (b*n*(d + e/Sqrt[x
])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(6*e^6) + (2*d^5*(d + e/Sqrt[x])*(
a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^6 - (5*d^4*(d + e/Sqrt[x])^2*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^3)/e^6 + (20*d^3*(d + e/Sqrt[x])^3*(a + b*Log[c*(d
+ e/Sqrt[x])^n])^3)/(3*e^6) - (5*d^2*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/
Sqrt[x])^n])^3)/e^6 + (2*d*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n
])^3)/e^6 - ((d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(3*e^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = -\left(2\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\mathrm{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}-\right.\right.\right.$$

$$= -\frac{2\,\mathrm{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5}+\frac{(10d)\,\mathrm{Subst}\left(\int(d+ex)^4\,(\right.}{}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6}+\frac{(10d)\,\mathrm{Subst}\left(\int x^4\left(a+b\log\left(cx^n\right)\right)\right.}{e^6}$$

$$= \frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}-\frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}+$$

$$= -\frac{6bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^6}+\frac{15bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.}{2e^6}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6}-\frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}+\frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6}-\frac{12b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right.}{125e^6}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6}-\frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}+\frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6}-\frac{12b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right.}{125e^6}$$

Mathematica [A] time = 1.54843, size = 950, normalized size = 1.05

$$\frac{-72000b^3n^3x^3\log^3\left(d+\frac{e}{\sqrt{x}}\right)d^6+809340b^3n^3x^3\log\left(\sqrt{x}d+e\right)d^6-529200ab^2n^2x^3\log\left(\sqrt{x}d+e\right)d^6+108000a^2bnx^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

```
[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*Sqrt[x] + 15840*a*b^2*d*e^5*n^2*Sqrt[x] - 4368*b^3*d
*e^5*n^3*Sqrt[x] + 27000*a^2*b*d^2*e^4*n*x - 33300*a*b^2*d^2*e^4*n^2*x + 13
785*b^3*d^2*e^4*n^3*x - 36000*a^2*b*d^3*e^3*n*x^(3/2) + 68400*a*b^2*d^3*e^3
*n^2*x^(3/2) - 41180*b^3*d^3*e^3*n^3*x^(3/2) + 54000*a^2*b*d^4*e^2*n*x^2 -
156600*a*b^2*d^4*e^2*n^2*x^2 + 140070*b^3*d^4*e^2*n^3*x^2 - 108000*a^2*b*d^
5*e*n*x^(5/2) + 529200*a*b^2*d^5*e*n^2*x^(5/2) - 809340*b^3*d^5*e*n^3*x^(5/
2) - 72000*b^3*d^6*n^3*x^3*Log[d + e/Sqrt[x]]^3 - 36000*b^3*e^6*Log[c*(d +
e/Sqrt[x])^n]^3 + 108000*a^2*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 529200*a*b^2*
d^6*n^2*x^3*Log[e + d*Sqrt[x]] + 809340*b^3*d^6*n^3*x^3*Log[e + d*Sqrt[x]]
+ 5400*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]*(-20*a + 49*b*n - 20*b*Log[c*(d +
 e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 54000*a^2*b*d^6*n*x^3*Log
[x] + 264600*a*b^2*d^6*n^2*x^3*Log[x] - 404670*b^3*d^6*n^3*x^3*Log[x] + 540
0*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]^2*(20*a - 49*b*n + 20*b*Log[c*(d + e/S
qrt[x])^n] + 20*b*n*Log[e + d*Sqrt[x]] - 10*b*n*Log[x]) + 1800*b^2*Log[c*(d
 + e/Sqrt[x])^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*Sqrt[x] + 15*b
```

```
*d^2*e^3*n*x - 20*b*d^3*e^2*n*x^(3/2) + 30*b*d^4*e*n*x^2 - 60*b*d^5*n*x^(5/
2)) + 60*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 30*b*d^6*n*x^3*Log[x]) - 60*b*Log
[c*(d + e/Sqrt[x])^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 - 264*d*e^4*Sqrt[x
] + 555*d^2*e^3*x - 1140*d^3*e^2*x^(3/2) + 2610*d^4*e*x^2 - 8820*d^5*x^(5/2
)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*Sqrt[x] + 15*d^2*e^3*x - 20*d^3*e^2*x^(3
/2) + 30*d^4*e*x^2 - 60*d^5*x^(5/2)) + 180*b*d^6*n*(-20*a + 49*b*n)*x^3*Log
[e + d*Sqrt[x]] + 90*b*d^6*n*(20*a - 49*b*n)*x^3*Log[x]))/(108000*e^6*x^3)
```

Maple [F] time = 0.391, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)
```

Maxima [A] time = 1.16268, size = 1166, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/60*a^2*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5
*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt
(x) - 10*e^5)/(e^6*x^3)) + 1/1800*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 -
30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 1
5*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(x))^n
) - (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^
3*log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 5
55*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^
6*x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*a*b^2 + 1/108000*(1800*e*n*(60*d^
6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x
^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^
3))*log(c*(d + e/sqrt(x))^n)^2 + e*n*((36000*d^6*x^3*log(d*sqrt(x) + e)^3 -
 4500*d^6*x^3*log(x)^3 + 66150*d^6*x^3*log(x)^2 - 404670*d^6*x^3*log(x) - 8
09340*d^5*e*x^(5/2) + 140070*d^4*e^2*x^2 - 41180*d^3*e^3*x^(3/2) + 13785*d^
2*e^4*x - 4368*d*e^5*sqrt(x) + 1000*e^6 - 5400*(10*d^6*x^3*log(x) - 49*d^6*
x^3)*log(d*sqrt(x) + e)^2 + 60*(450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log(x)
+ 13489*d^6*x^3)*log(d*sqrt(x) + e))*n^2/(e^7*x^3) - 60*(1800*d^6*x^3*log(d
*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log(x) - 8820*d^5*e*x
^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 555*d^2*e^4*x - 264*d*e^
5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^6*x^3)*log(d*sqrt(x) +
e))*n*log(c*(d + e/sqrt(x))^n)/(e^7*x^3)))*b^3 - 1/3*b^3*log(c*(d + e/sqrt(
x))^n)^3/x^3 - a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^3 - a^2*b*log(c*(d + e/sq
rt(x))^n)/x^3 - 1/3*a^3/x^3
```

Fricas [A] time = 1.98959, size = 2684, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/108000*(1000*b^3*e^6*n^3 - 36000*b^3*e^6*log(c)^3 - 6000*a*b^2*e^6*n^2 +
18000*a^2*b*e^6*n - 36000*a^3*e^6 + 36000*(b^3*d^6*n^3*x^3 - b^3*e^6*n^3)*l
og((d*x + e*sqrt(x))/x)^3 + 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n
^2 + 1800*a^2*b*d^4*e^2*n)*x^2 + 9000*(6*b^3*d^4*e^2*n*x^2 + 3*b^3*d^2*e^4*
n*x + 2*b^3*e^6*n - 12*a*b^2*e^6)*log(c)^2 + 1800*(30*b^3*d^4*e^2*n^3*x^2 +
 15*b^3*d^2*e^4*n^3*x + 10*b^3*e^6*n^3 - 60*a*b^2*e^6*n^2 - 3*(49*b^3*d^6*n
^3 - 20*a*b^2*d^6*n^2)*x^3 + 60*(b^3*d^6*n^2*x^3 - b^3*e^6*n^2)*log(c) - 4*
(15*b^3*d^5*e*n^3*x^2 + 5*b^3*d^3*e^3*n^3*x + 3*b^3*d*e^5*n^3)*sqrt(x))*log
((d*x + e*sqrt(x))/x)^2 + 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2
+ 1800*a^2*b*d^2*e^4*n)*x - 300*(20*b^3*e^6*n^2 - 120*a*b^2*e^6*n + 360*a^2
*b*e^6 + 18*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x^2 + 3*(37*b^3*d^2*e
^4*n^2 - 60*a*b^2*d^2*e^4*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 - 600*a*b^2*e^
6*n^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a
^2*b*d^6*n)*x^3 + 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x^2 - 1800
*(b^3*d^6*n*x^3 - b^3*e^6*n)*log(c)^2 + 15*(37*b^3*d^2*e^4*n^3 - 60*a*b^2*d
^2*e^4*n^2)*x - 60*(30*b^3*d^4*e^2*n^2*x^2 + 15*b^3*d^2*e^4*n^2*x + 10*b^3*
e^6*n^2 - 60*a*b^2*e^6*n - 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^3)*log(c)
- 12*(22*b^3*d*e^5*n^3 - 60*a*b^2*d*e^5*n^2 + 15*(49*b^3*d^5*e*n^3 - 20*a*b
^2*d^5*e*n^2)*x^2 + 5*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 20*(1
5*b^3*d^5*e*n^2*x^2 + 5*b^3*d^3*e^3*n^2*x + 3*b^3*d*e^5*n^2)*log(c))*sqrt(x
))*log((d*x + e*sqrt(x))/x) - 4*(1092*b^3*d*e^5*n^3 - 3960*a*b^2*d*e^5*n^2
+ 5400*a^2*b*d*e^5*n + 15*(13489*b^3*d^5*e*n^3 - 8820*a*b^2*d^5*e*n^2 + 180
0*a^2*b*d^5*e*n)*x^2 + 1800*(15*b^3*d^5*e*n*x^2 + 5*b^3*d^3*e^3*n*x + 3*b^3
*d*e^5*n)*log(c)^2 + 5*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 180
0*a^2*b*d^3*e^3*n)*x - 180*(22*b^3*d*e^5*n^2 - 60*a*b^2*d*e^5*n + 15*(49*b^
3*d^5*e*n^2 - 20*a*b^2*d^5*e*n)*x^2 + 5*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*
e^3*n)*x)*log(c))*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x^4, x)
```

3.442 $\int x^3 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right) dx$

Optimal. Leaf size=234

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^{10}nx^{2/3}}{8e^{10}}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}+\frac{bd^5nx^{7/3}}{28e^5}-\frac{bd^4nx^{8/3}}{32e^4}+\frac{bd^3nx^3}{36e^3}-\frac{bd^2nx^{10}}{40e^2}$$

```
[Out] (b*d^11*n*x^(1/3))/(4*e^11) - (b*d^10*n*x^(2/3))/(8*e^10) + (b*d^9*n*x)/(12
*e^9) - (b*d^8*n*x^(4/3))/(16*e^8) + (b*d^7*n*x^(5/3))/(20*e^7) - (b*d^6*n*
x^2)/(24*e^6) + (b*d^5*n*x^(7/3))/(28*e^5) - (b*d^4*n*x^(8/3))/(32*e^4) + (
b*d^3*n*x^3)/(36*e^3) - (b*d^2*n*x^(10/3))/(40*e^2) + (b*d*n*x^(11/3))/(44*
e) - (b*n*x^4)/48 - (b*d^12*n*Log[d + e*x^(1/3)])/(4*e^12) + (x^4*(a + b*Lo
g[c*(d + e*x^(1/3))^n]))/4
```

Rubi [A] time = 0.187993, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^{10}nx^{2/3}}{8e^{10}}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}+\frac{bd^5nx^{7/3}}{28e^5}-\frac{bd^4nx^{8/3}}{32e^4}+\frac{bd^3nx^3}{36e^3}-\frac{bd^2nx^{10}}{40e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```


```
[Out] (b*d^11*n*x^(1/3))/(4*e^11) - (b*d^10*n*x^(2/3))/(8*e^10) + (b*d^9*n*x)/(12
*e^9) - (b*d^8*n*x^(4/3))/(16*e^8) + (b*d^7*n*x^(5/3))/(20*e^7) - (b*d^6*n*
x^2)/(24*e^6) + (b*d^5*n*x^(7/3))/(28*e^5) - (b*d^4*n*x^(8/3))/(32*e^4) + (
b*d^3*n*x^3)/(36*e^3) - (b*d^2*n*x^(10/3))/(40*e^2) + (b*d*n*x^(11/3))/(44*
e) - (b*n*x^4)/48 - (b*d^12*n*Log[d + e*x^(1/3)])/(4*e^12) + (x^4*(a + b*Lo
g[c*(d + e*x^(1/3))^n]))/4
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx = 3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\frac{x^{12}}{d+ex}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\left(-\frac{d^{11}}{e^{12}}+\frac{d^{10}x}{e^{11}}-\frac{d^9x^2}{e^{10}}+\frac{d^8x}{e^9}\right.\right.$$
$$=\frac{bd^{11}n\sqrt[3]{x}}{4e^{11}}-\frac{bd^{10}nx^{2/3}}{8e^{10}}+\frac{bd^9nx}{12e^9}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}+\frac{bd^5nx^{7/3}}{28e^5}-$$

Mathematica [A] time = 0.229194, size = 219, normalized size = 0.94

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{4}ben\left(\frac{d^{10}x^{2/3}}{2e^{11}}+\frac{d^8x^{4/3}}{4e^9}-\frac{d^7x^{5/3}}{5e^8}+\frac{d^6x^2}{6e^7}-\frac{d^5x^{7/3}}{7e^6}+\frac{d^4x^{8/3}}{8e^5}-\frac{d^3x^3}{9e^4}+\frac{d^2x^{10/3}}{10e^3}-\frac{d^{11}\sqrt[3]{}}{e^{12}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^11*x^(1/3))/e^12) + (d^10*x^(2/3))/(2*e^11) - (d^9
*x)/(3*e^10) + (d^8*x^(4/3))/(4*e^9) - (d^7*x^(5/3))/(5*e^8) + (d^6*x^2)/(6
*e^7) - (d^5*x^(7/3))/(7*e^6) + (d^4*x^(8/3))/(8*e^5) - (d^3*x^3)/(9*e^4) +
 (d^2*x^(10/3))/(10*e^3) - (d*x^(11/3))/(11*e^2) + x^4/(12*e) + (d^12*Log[d
 + e*x^(1/3)])/e^13))/4 + (b*x^4*Log[c*(d + e*x^(1/3))^n])/4
```

Maple [F] time = 0.434, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Maxima [A] time = 1.02377, size = 232, normalized size = 0.99

$$\frac{1}{4}bx^4\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{4}ax^4-\frac{1}{110880}ben\left(\frac{27720\,d^{12}\log\left(ex^{\frac{1}{3}}+d\right)}{e^{13}}+\frac{2310\,e^{11}x^4-2520\,de^{10}x^{\frac{11}{3}}+2772\,d^2e^9x^{\frac{10}{3}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*d^12
*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2
*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7
/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240
```

```
*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)
```

Fricas [A] time = 1.81819, size = 513, normalized size = 2.19

$$\frac{27720\, be^{12}x^4 \log(c) + 3080\, bd^3e^9nx^3 - 4620\, bd^6e^6nx^2 + 9240\, bd^9e^3nx - 2310\left(be^{12}n - 12\, ae^{12}\right)x^4 + 27720\left(be^{12}nx^4 - bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/110880*(27720*b*e^12*x^4*log(c) + 3080*b*d^3*e^9*n*x^3 - 4620*b*d^6*e^6*n
*x^2 + 9240*b*d^9*e^3*n*x - 2310*(b*e^12*n - 12*a*e^12)*x^4 + 27720*(b*e^12
*n*x^4 - b*d^12*n)*log(e*x^(1/3) + d) + 63*(40*b*d*e^11*n*x^3 - 55*b*d^4*e^
8*n*x^2 + 88*b*d^7*e^5*n*x - 220*b*d^10*e^2*n)*x^(2/3) - 198*(14*b*d^2*e^10
*n*x^3 - 20*b*d^5*e^7*n*x^2 + 35*b*d^8*e^4*n*x - 140*b*d^11*e*n)*x^(1/3))/e
^12
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2498, size = 714, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/110880*(27720*b*x^4*e*log(c) + 27720*a*x^4*e + (27720*(x^(1/3)*e + d)^12*
e^(-11)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3
)*e + d) + 1829520*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 6098
400*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e
+ d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*e + d)^7*d^5*e^(-
11)*log(x^(1/3)*e + d) + 25613280*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)
*e + d) - 21954240*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 13721
400*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e +
 d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*e + d)^2*d^10*e^(-1
1)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e +
 d) - 2310*(x^(1/3)*e + d)^12*e^(-11) + 30240*(x^(1/3)*e + d)^11*d*e^(-11)
- 182952*(x^(1/3)*e + d)^10*d^2*e^(-11) + 677600*(x^(1/3)*e + d)^9*d^3*e^(-
11) - 1715175*(x^(1/3)*e + d)^8*d^4*e^(-11) + 3136320*(x^(1/3)*e + d)^7*d^5
*e^(-11) - 4268880*(x^(1/3)*e + d)^6*d^6*e^(-11) + 4390848*(x^(1/3)*e + d)^
5*d^7*e^(-11) - 3430350*(x^(1/3)*e + d)^4*d^8*e^(-11) + 2032800*(x^(1/3)*e
+ d)^3*d^9*e^(-11) - 914760*(x^(1/3)*e + d)^2*d^10*e^(-11) + 332640*(x^(1/3
)*e + d)*d^11*e^(-11))*b*n)*e^(-1)
```

3.443 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=185

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+\frac{bd^7nx^{2/3}}{6e^7}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}-\frac{bd^8n\sqrt[3]{x}}{3e^8}-\frac{bd^6nx}{9e^6}+\frac{bd^9n\log}{}$$

```
[Out] -(b*d^8*n*x^(1/3))/(3*e^8) + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^6*n*x)/(9*e^6
) + (b*d^5*n*x^(4/3))/(12*e^5) - (b*d^4*n*x^(5/3))/(15*e^4) + (b*d^3*n*x^2)
/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) + (b*d*n*x^(8/3))/(24*e) - (b*n*x^3)
/27 + (b*d^9*n*Log[d + e*x^(1/3)])/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/
3))^n]))/3
```

Rubi [A] time = 0.134155, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+\frac{bd^7nx^{2/3}}{6e^7}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}-\frac{bd^8n\sqrt[3]{x}}{3e^8}-\frac{bd^6nx}{9e^6}+\frac{bd^9n\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```


```
[Out] -(b*d^8*n*x^(1/3))/(3*e^8) + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^6*n*x)/(9*e^6
) + (b*d^5*n*x^(4/3))/(12*e^5) - (b*d^4*n*x^(5/3))/(15*e^4) + (b*d^3*n*x^2)
/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) + (b*d*n*x^(8/3))/(24*e) - (b*n*x^3)
/27 + (b*d^9*n*Log[d + e*x^(1/3)])/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/
3))^n]))/3
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^n\right)\right)dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\frac{x^9}{d+ex}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\left(\frac{d^8}{e^9}-\frac{d^7x}{e^8}+\frac{d^6x^2}{e^7}-\frac{d^5x^3}{e^6}+\frac{d^4}{e}\right.\right.$$
$$=-\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}+\frac{bdn}{2}$$

Mathematica [A] time = 0.13615, size = 176, normalized size = 0.95

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{3}ben\left(-\frac{d^7x^{2/3}}{2e^8}-\frac{d^5x^{4/3}}{4e^6}+\frac{d^4x^{5/3}}{5e^5}-\frac{d^3x^2}{6e^4}+\frac{d^2x^{7/3}}{7e^3}+\frac{d^8\sqrt[3]{x}}{e^9}+\frac{d^6x}{3e^7}-\frac{d^9\log\left(d+e\sqrt[3]{x}\right)}{e^{10}}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 - (b*e*n*((d^8*x^(1/3))/e^9 - (d^7*x^(2/3))/(2*e^8) + (d^6*x)/(3*
e^7) - (d^5*x^(4/3))/(4*e^6) + (d^4*x^(5/3))/(5*e^5) - (d^3*x^2)/(6*e^4) +
(d^2*x^(7/3))/(7*e^3) - (d*x^(8/3))/(8*e^2) + x^3/(9*e) - (d^9*Log[d + e*x^
(1/3)])/e^10))/3 + (b*x^3*Log[c*(d + e*x^(1/3))^n])/3
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Maxima [A] time = 1.04077, size = 189, normalized size = 1.02

$$\frac{1}{3}bx^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{3}ax^3+\frac{1}{7560}ben\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7x^{\frac{8}{3}}+360\,d^2e^6x^{\frac{7}{3}}-420\,d^3e^5x^2+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a*x^3 + 1/7560*b*e*n*(2520*d^9*log
(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/
3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*
e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)
```

Fricas [A] time = 1.84729, size = 392, normalized size = 2.12

$$\frac{2520\,be^9x^3\log(c) + 420\,bd^3e^6nx^2 - 840\,bd^6e^3nx - 280\left(be^9n - 9\,ae^9\right)x^3 + 2520\left(be^9nx^3 + bd^9n\right)\log\left(ex^{\frac{1}{3}} + d\right) + 63\,(5}{7560\,e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/7560*(2520*b*e^9*x^3*log(c) + 420*b*d^3*e^6*n*x^2 - 840*b*d^6*e^3*n*x - 2
80*(b*e^9*n - 9*a*e^9)*x^3 + 2520*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^(1/3) + d
) + 63*(5*b*d*e^8*n*x^2 - 8*b*d^4*e^5*n*x + 20*b*d^7*e^2*n)*x^(2/3) - 90*(4
*b*d^2*e^7*n*x^2 - 7*b*d^5*e^4*n*x + 28*b*d^8*e*n)*x^(1/3))/e^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34357, size = 540, normalized size = 2.92

$$\frac{1}{7560}\left(2520\,bx^3e\log(c) + 2520\,ax^3e + \left(2520\left(x^{\frac{1}{3}}e + d\right)^9e^{(-8)}\log\left(x^{\frac{1}{3}}e + d\right) - 22680\left(x^{\frac{1}{3}}e + d\right)^8de^{(-8)}\log\left(x^{\frac{1}{3}}e + d\right) + 9\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/7560*(2520*b*x^3*e*log(c) + 2520*a*x^3*e + (2520*(x^(1/3)*e + d)^9*e^(-8)
*log(x^(1/3)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) +
 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e
+ d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*
log(x^(1/3)*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)
 + 211680*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*
e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*l
og(x^(1/3)*e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d
*e^(-8) - 12960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*
e^(-8) - 63504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e
^(-8) - 70560*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^
(-8) - 22680*(x^(1/3)*e + d)*d^8*e^(-8))*b*n)*e^(-1)
```

3.444 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=136

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^4nx^{2/3}}{4e^4}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bd^5n\sqrt[3]{x}}{2e^5}+\frac{bd^3nx}{6e^3}-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2$$

```
[Out] (b*d^5*n*x^(1/3))/(2*e^5) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^3*n*x)/(6*e^3)
 - (b*d^2*n*x^(4/3))/(8*e^2) + (b*d*n*x^(5/3))/(10*e) - (b*n*x^2)/12 - (b*d
^6*n*Log[d + e*x^(1/3)])/(2*e^6) + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/2
```

Rubi [A] time = 0.0949436, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2454, 2395, 43}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^4nx^{2/3}}{4e^4}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bd^5n\sqrt[3]{x}}{2e^5}+\frac{bd^3nx}{6e^3}-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(1/3))/(2*e^5) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^3*n*x)/(6*e^3)
 - (b*d^2*n*x^(4/3))/(8*e^2) + (b*d*n*x^(5/3))/(10*e) - (b*n*x^2)/12 - (b*d
^6*n*Log[d + e*x^(1/3)])/(2*e^6) + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = 3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \sqrt[3]{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{x^6}{d+ex}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - \frac{1}{2}(ben)\,\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}\right.\right.$$
$$= \frac{bd^5n\sqrt[3]{x}}{2e^5} - \frac{bd^4nx^{2/3}}{4e^4} + \frac{bd^3nx}{6e^3} - \frac{bd^2nx^{4/3}}{8e^2} + \frac{bdnx^{5/3}}{10e} - \frac{1}{12}bnx^2 - \frac{bd^6n\log\left(d+e\right.}{2e^6}$$

Mathematica [A] time = 0.0927543, size = 133, normalized size = 0.98

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{2}ben\left(\frac{d^4x^{2/3}}{2e^5}+\frac{d^2x^{4/3}}{4e^3}-\frac{d^5\sqrt[3]{x}}{e^6}-\frac{d^3x}{3e^4}+\frac{d^6\log\left(d+e\sqrt[3]{x}\right)}{e^7}-\frac{dx^{5/3}}{5e^2}+\frac{x^2}{6e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^2)/2 - (b*e*n*(-((d^5*x^(1/3))/e^6) + (d^4*x^(2/3))/(2*e^5) - (d^3*x)/
(3*e^4) + (d^2*x^(4/3))/(4*e^3) - (d*x^(5/3))/(5*e^2) + x^2/(6*e) + (d^6*Lo
g[d + e*x^(1/3)])/e^7))/2 + (b*x^2*Log[c*(d + e*x^(1/3))^n])/2
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Maxima [A] time = 1.00385, size = 143, normalized size = 1.05

$$-\frac{1}{120}\,ben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right)+\frac{1}{2}\,bx^2\log\left(\left(ex^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] -1/120*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3
) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/
e^6) + 1/2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a*x^2
```

Fricas [A] time = 1.8302, size = 292, normalized size = 2.15

$$\frac{60\,be^6x^2\log(c)+20\,bd^3e^3nx-10\left(be^6n-6\,ae^6\right)x^2+60\left(be^6nx^2-bd^6n\right)\log\left(ex^{\frac{1}{3}}+d\right)+6\left(2\,bde^5nx-5\,bd^4e^2n\right)x^{\frac{2}{3}}-15}{120\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/120*(60*b*e^6*x^2*log(c) + 20*b*d^3*e^3*n*x - 10*(b*e^6*n - 6*a*e^6)*x^2
+ 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 6*(2*b*d*e^5*n*x - 5*b*d^
4*e^2*n)*x^(2/3) - 15*(b*d^2*e^4*n*x - 4*b*d^5*e*n)*x^(1/3))/e^6
```

Sympy [A] time = 11.8602, size = 131, normalized size = 0.96

$$\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{3d^6\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{3d^5\sqrt[3]{x}}{e^6}+\frac{3d^4x^{\frac{2}{3}}}{2e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{\frac{4}{3}}}{4e^3}-\frac{3dx^{\frac{5}{3}}}{5e^2}+\frac{x^2}{2e}\right)}{6}+\frac{x^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(3*d**6*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**
(1/3))/e, True))/e**6 - 3*d**5*x**(1/3)/e**6 + 3*d**4*x**(2/3)/(2*e**5) - d
**3*x/e**4 + 3*d**2*x**(4/3)/(4*e**3) - 3*d*x**(5/3)/(5*e**2) + x**2/(2*e))
/6 + x**2*log(c*(d + e*x**(1/3))**n)/2)
```

Giac [B] time = 1.30847, size = 366, normalized size = 2.69

$$\frac{1}{120}\left(60\,bx^2e\log(c)+60\,ax^2e+\left(60\left(x^{\frac{1}{3}}e+d\right)^6e^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)-360\left(x^{\frac{1}{3}}e+d\right)^5de^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)+900\left(x^{\frac{1}{3}}e+d\right)^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/120*(60*b*x^2*e*log(c) + 60*a*x^2*e + (60*(x^(1/3)*e + d)^6*e^(-5)*log(x^
(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(
1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^
(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e +
d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)
^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5
) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 3
60*(x^(1/3)*e + d)*d^5*e^(-5))*b*n)*e^(-1)
```

3.445 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=77

$$ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Rubi [A] time = 0.0531322, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 266, 43}

$$ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx &= ax+b\int \log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)dx\\
&= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{3}(ben)\int \frac{\sqrt[3]{x}}{d+e\sqrt[3]{x}}dx\\
&= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \frac{x^3}{d+ex}dx,x,\sqrt[3]{x}\right)\\
&= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx,x,\sqrt[3]{x}\right)\\
&= -\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)
\end{aligned}
$$

Mathematica [A] time = 0.0417372, size = 77, normalized size = 1.

$$
ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Maple [A] time = 0.088, size = 66, normalized size = 0.9

$$
-\frac{bd^2n}{e^2}\sqrt[3]{x}+\frac{bdn}{2e}x^{\frac{2}{3}}+ax-\frac{bnx}{3}+\frac{bd^3n}{e^3}\ln\left(d+e\sqrt[3]{x}\right)+bx\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e*x^(1/3))^n),x)
```

```
[Out] -b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1/
3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Maxima [A] time = 1.0054, size = 95, normalized size = 1.23

$$
\frac{1}{6}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)\right)b+ax
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="maxima")
```

```
[Out] 1/6*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*b + a*x
```

Fricas [A] time = 1.78471, size = 193, normalized size = 2.51

$$\frac{6\,be^3x\log\left(c\right) + 3\,bde^2nx^{\frac{2}{3}} - 6\,bd^2enx^{\frac{1}{3}} - 2\,\left(be^3n - 3\,ae^3\right)x + 6\,\left(be^3nx + bd^3n\right)\log\left(ex^{\frac{1}{3}} + d\right)}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^3*x*log(c) + 3*b*d*e^2*n*x^(2/3) - 6*b*d^2*e*n*x^(1/3) - 2*(b*e^
3*n - 3*a*e^3)*x + 6*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/e^3
```

Sympy [A] time = 1.54342, size = 82, normalized size = 1.06

$$ax + b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for}\ e = 0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^3} + \frac{3d^2\sqrt[3]{x}}{e^3} - \frac{3dx^{\frac{2}{3}}}{2e^2} + \frac{x}{e}\right)}{3} + x\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(1/3))**n),x)
```

```
[Out] a*x + b*(-e*n*(-3*d**3*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**(1/3
))/e, True))/e**3 + 3*d**2*x**(1/3)/e**3 - 3*d*x**(2/3)/(2*e**2) + x/e)/3 +
 x*log(c*(d + e*x**(1/3))**n))
```

Giac [B] time = 1.32411, size = 182, normalized size = 2.36

$$\frac{1}{6}\left(6\,xe\log\left(c\right) + \left(6\,\left(x^{\frac{1}{3}}e + d\right)^3e^{(-2)}\log\left(x^{\frac{1}{3}}e + d\right) - 18\,\left(x^{\frac{1}{3}}e + d\right)^2de^{(-2)}\log\left(x^{\frac{1}{3}}e + d\right) + 18\,\left(x^{\frac{1}{3}}e + d\right)d^2e^{(-2)}\log\left(x^{\frac{1}{3}}e + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="giac")
```

```
[Out] 1/6*(6*x*e*log(c) + (6*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^
(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*
log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(
-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*n)*b*e^(-1) + a*x
```

3.446 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$3bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] + 3*b*n*PolyLog[2,
 1 + (e*x^(1/3))/d]
```

Rubi [A] time = 0.0505669, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$3bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] + 3*b*n*PolyLog[2,
 1 + (e*x^(1/3))/d]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\sqrt[3]{x}\right)\\ &= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)-(3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,\sqrt[3]{x}\right)\\ &= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+3bn\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)\end{aligned}$$

Mathematica [A] time = 0.0029209, size = 53, normalized size = 1.04

$$3bn\text{PolyLog}\left(2,\frac{d+e\sqrt[3]{x}}{d}\right)+a\log(x)+3b\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

```
[Out] 3*b*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] + a*Log[x] + 3*b*n*PolyL
og[2, (d + e*x^(1/3))/d]
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int\frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
```

Maxima [B] time = 1.59889, size = 224, normalized size = 4.39

$$-3\left(\log\left(\frac{ex^{\frac{1}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-\frac{ex^{\frac{1}{3}}}{d}\right)\right)bn+\frac{4\,bd^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)\log(x)+4\left(bd^2\log(c)+ad^2\right)\log(x)+\frac{2\,be^2nx\log}{}}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b*n + 1/4*(4*b
*d^2*log((e*x^(1/3) + d)^n)*log(x) + 4*(b*d^2*log(c) + a*d^2)*log(x) + (2*b
*e^2*n*x*log(x) - 3*b*e^2*n*x)/x^(1/3) - 4*(b*d*e*n*x*log(x) - 3*b*d*e*n*x)
/x^(2/3))/d^2 + 3/4*(b*e^2*n*x^(2/3) - 4*b*d*e*n*x^(1/3) - 2*(b*e^2*n*x^(2/
3) - 2*b*d*e*n*x^(1/3))*log(x^(1/3)))/d^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(\left(ex^{\frac{1}{3}} + d\right)^n c\right) + a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

3.447 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{ben}{2dx^{2/3}}$$

```
[Out] -(b*e*n)/(2*d*x^(2/3)) + (b*e^2*n)/(d^2*x^(1/3)) - (b*e^3*n*Log[d + e*x^(1/
3)])/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])/x + (b*e^3*n*Log[x])/(3*d^3)
```

Rubi [A] time = 0.0679198, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{ben}{2dx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -(b*e*n)/(2*d*x^(2/3)) + (b*e^2*n)/(d^2*x^(1/3)) - (b*e^3*n*Log[d + e*x^(1/
3)])/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])/x + (b*e^3*n*Log[x])/(3*d^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{e^3}{d^3(d+ex)}\right)dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{ben}{2dx^{2/3}} + \frac{be^2n}{d^2\sqrt[3]{x}} - \frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + \frac{be^3n\log(x)}{3d^3}$$

Mathematica [A] time = 0.0331193, size = 84, normalized size = 0.97

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + ben\left(-\frac{e^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} + \frac{e^2\log(x)}{3d^3} + \frac{e}{d^2\sqrt[3]{x}} - \frac{1}{2dx^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) - (b*Log[c*(d + e*x^(1/3))^n])/x + b*e*n*(-1/(2*d*x^(2/3)) + e/(d^2*
x^(1/3)) - (e^2*Log[d + e*x^(1/3)])/d^3 + (e^2*Log[x])/(3*d^3))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)
```

Maxima [A] time = 1.02714, size = 101, normalized size = 1.16

$$-\frac{1}{6}ben\left(\frac{6\,e^2\log\left(ex^{\frac{1}{3}}+d\right)}{d^3} - \frac{2\,e^2\log(x)}{d^3} - \frac{3\left(2\,ex^{\frac{1}{3}}-d\right)}{d^2x^{\frac{2}{3}}}\right) - \frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*b*e*n*(6*e^2*log(e*x^(1/3) + d)/d^3 - 2*e^2*log(x)/d^3 - 3*(2*e*x^(1/3
) - d)/(d^2*x^(2/3))) - b*log((e*x^(1/3) + d)^n*c)/x - a/x
```

Fricas [A] time = 1.8584, size = 208, normalized size = 2.39

$$\frac{2\,be^3nx\log\left(x^{\frac{1}{3}}\right)+2\,bde^2nx^{\frac{2}{3}}-bd^2enx^{\frac{1}{3}}-2\,bd^3\log\left(c\right)-2\,ad^3-2\left(be^3nx+bd^3n\right)\log\left(ex^{\frac{1}{3}}+d\right)}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*e^3*n*x*log(x^(1/3)) + 2*b*d*e^2*n*x^(2/3) - b*d^2*e*n*x^(1/3) - 2
*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/(d^3*
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3379, size = 378, normalized size = 4.34

$$-\frac{\left(2\left(x^{\frac{1}{3}}e+d\right)^3bne^4\log\left(x^{\frac{1}{3}}e+d\right)-6\left(x^{\frac{1}{3}}e+d\right)^2bdne^4\log\left(x^{\frac{1}{3}}e+d\right)+6\left(x^{\frac{1}{3}}e+d\right)bd^2ne^4\log\left(x^{\frac{1}{3}}e+d\right)-2\left(x^{\frac{1}{3}}e+d\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(x^(1/3)*e + d)^3*b*n*e^4*log(x^(1/3)*e + d) - 6*(x^(1/3)*e + d)^2*
b*d*n*e^4*log(x^(1/3)*e + d) + 6*(x^(1/3)*e + d)*b*d^2*n*e^4*log(x^(1/3)*e
+ d) - 2*(x^(1/3)*e + d)^3*b*n*e^4*log(x^(1/3)*e) + 6*(x^(1/3)*e + d)^2*b*d
*n*e^4*log(x^(1/3)*e) - 6*(x^(1/3)*e + d)*b*d^2*n*e^4*log(x^(1/3)*e) + 2*b*
d^3*n*e^4*log(x^(1/3)*e) - 2*(x^(1/3)*e + d)^2*b*d*n*e^4 + 5*(x^(1/3)*e + d
)*b*d^2*n*e^4 - 3*b*d^3*n*e^4 + 2*b*d^3*e^4*log(c) + 2*a*d^3*e^4)*e^(-1)/((
x^(1/3)*e + d)^3*d^3 - 3*(x^(1/3)*e + d)^2*d^4 + 3*(x^(1/3)*e + d)*d^5 - d^
6)
```

3.448 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=143

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{be^4n}{4d^4x^{2/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}}-\frac{be^3n}{6d^3x}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{ben}{10dx^{5/3}}$$

```
[Out] -(b*e*n)/(10*d*x^(5/3)) + (b*e^2*n)/(8*d^2*x^(4/3)) - (b*e^3*n)/(6*d^3*x) +
 (b*e^4*n)/(4*d^4*x^(2/3)) - (b*e^5*n)/(2*d^5*x^(1/3)) + (b*e^6*n*Log[d + e
*x^(1/3)])/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) - (b*e^6*n*Lo
g[x])/(6*d^6)
```

Rubi [A] time = 0.0927983, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{be^4n}{4d^4x^{2/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}}-\frac{be^3n}{6d^3x}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{ben}{10dx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -(b*e*n)/(10*d*x^(5/3)) + (b*e^2*n)/(8*d^2*x^(4/3)) - (b*e^3*n)/(6*d^3*x) +
 (b*e^4*n)/(4*d^4*x^(2/3)) - (b*e^5*n)/(2*d^5*x^(1/3)) + (b*e^6*n*Log[d + e
*x^(1/3)])/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) - (b*e^6*n*Lo
g[x])/(6*d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}+\frac{e^4}{d^5x^2}\right.\right.$$
$$= -\frac{ben}{10dx^{5/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^3n}{6d^3x}+\frac{be^4n}{4d^4x^{2/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{a+b\log\left(c\right.}{2}$$

Mathematica [A] time = 0.138207, size = 134, normalized size = 0.94

$$-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}ben\left(\frac{e^3}{2d^4x^{2/3}}-\frac{e^4}{d^5\sqrt[3]{x}}-\frac{e^2}{3d^3x}+\frac{e^5\log\left(d+e\sqrt[3]{x}\right)}{d^6}-\frac{e^5\log(x)}{3d^6}+\frac{e}{4d^2x^{4/3}}-\frac{1}{5dx^{5/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) + (b*e*n*(-1/(5*d*x^(5/3)
) + e/(4*d^2*x^(4/3)) - e^2/(3*d^3*x) + e^3/(2*d^4*x^(2/3)) - e^4/(d^5*x^(1
/3)) + (e^5*Log[d + e*x^(1/3)])/d^6 - (e^5*Log[x])/(3*d^6)))/2
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

Maxima [A] time = 1.03044, size = 143, normalized size = 1.

$$\frac{1}{120}ben\left(\frac{60\,e^5\log\left(ex^{\frac{1}{3}}+d\right)}{d^6}-\frac{20\,e^5\log(x)}{d^6}-\frac{60\,e^4x^{\frac{4}{3}}-30\,de^3x+20\,d^2e^2x^{\frac{2}{3}}-15\,d^3ex^{\frac{1}{3}}+12\,d^4}{d^5x^{\frac{5}{3}}}\right)-\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right.}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/120*b*e*n*(60*e^5*log(e*x^(1/3) + d)/d^6 - 20*e^5*log(x)/d^6 - (60*e^4*x^
(4/3) - 30*d*e^3*x + 20*d^2*e^2*x^(2/3) - 15*d^3*e*x^(1/3) + 12*d^4)/(d^5*x
```

^(5/3))) - 1/2*b*log((e*x^(1/3) + d)^n*c)/x^2 - 1/2*a/x^2

Fricas [A] time = 1.93037, size = 312, normalized size = 2.18

$$-\frac{60\,be^{6}nx^{2}\log\left(x^{\frac{1}{3}}\right)+20\,bd^{3}e^{3}nx+60\,bd^{6}\log\left(c\right)+60\,ad^{6}-60\left(be^{6}nx^{2}-bd^{6}n\right)\log\left(ex^{\frac{1}{3}}+d\right)+15\left(4\,bde^{5}nx-bd^{4}e^{2}n\right)\cdots}{120\,d^{6}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/120*(60*b*e^6*n*x^2*log(x^(1/3)) + 20*b*d^3*e^3*n*x + 60*b*d^6*log(c) +
60*a*d^6 - 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 15*(4*b*d*e^5*n*
x - b*d^4*e^2*n)*x^(2/3) - 6*(5*b*d^2*e^4*n*x - 2*b*d^5*e*n)*x^(1/3))/(d^6*
x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30122, size = 732, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/120*(60*(x^(1/3)*e + d)^6*b*n*e^7*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d
)^5*b*d*n*e^7*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*b*d^2*n*e^7*log(x^
(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*b*d^3*n*e^7*log(x^(1/3)*e + d) + 900*
(x^(1/3)*e + d)^2*b*d^4*n*e^7*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*b*d^
5*n*e^7*log(x^(1/3)*e + d) - 60*(x^(1/3)*e + d)^6*b*n*e^7*log(x^(1/3)*e) +
360*(x^(1/3)*e + d)^5*b*d*n*e^7*log(x^(1/3)*e) - 900*(x^(1/3)*e + d)^4*b*d^
2*n*e^7*log(x^(1/3)*e) + 1200*(x^(1/3)*e + d)^3*b*d^3*n*e^7*log(x^(1/3)*e)
- 900*(x^(1/3)*e + d)^2*b*d^4*n*e^7*log(x^(1/3)*e) + 360*(x^(1/3)*e + d)*b*
d^5*n*e^7*log(x^(1/3)*e) - 60*b*d^6*n*e^7*log(x^(1/3)*e) - 60*(x^(1/3)*e +
d)^5*b*d*n*e^7 + 330*(x^(1/3)*e + d)^4*b*d^2*n*e^7 - 740*(x^(1/3)*e + d)^3*
b*d^3*n*e^7 + 855*(x^(1/3)*e + d)^2*b*d^4*n*e^7 - 522*(x^(1/3)*e + d)*b*d^5
*n*e^7 + 137*b*d^6*n*e^7 - 60*b*d^6*e^7*log(c) - 60*a*d^6*e^7)*e^(-1)/((x^(
1/3)*e + d)^6*d^6 - 6*(x^(1/3)*e + d)^5*d^7 + 15*(x^(1/3)*e + d)^4*d^8 - 20
*(x^(1/3)*e + d)^3*d^9 + 15*(x^(1/3)*e + d)^2*d^10 - 6*(x^(1/3)*e + d)*d^11
 + d^12)
```

3.449 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=192

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}-\frac{be^7n}{6d^7x^{2/3}}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^2n}{21d^2x^{7/3}}+\frac{be^8n}{3d^8\sqrt[3]{x}}+\frac{be^6n}{9d^6x}-\frac{be^9n\log\left(d+e\right.}{3d^9}$$

```
[Out] -(b*e*n)/(24*d*x^(8/3)) + (b*e^2*n)/(21*d^2*x^(7/3)) - (b*e^3*n)/(18*d^3*x^
2) + (b*e^4*n)/(15*d^4*x^(5/3)) - (b*e^5*n)/(12*d^5*x^(4/3)) + (b*e^6*n)/(9
*d^6*x) - (b*e^7*n)/(6*d^7*x^(2/3)) + (b*e^8*n)/(3*d^8*x^(1/3)) - (b*e^9*n*
Log[d + e*x^(1/3)])/(3*d^9) - (a + b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b
*e^9*n*Log[x])/(9*d^9)
```

Rubi [A] time = 0.125405, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}-\frac{be^7n}{6d^7x^{2/3}}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^2n}{21d^2x^{7/3}}+\frac{be^8n}{3d^8\sqrt[3]{x}}+\frac{be^6n}{9d^6x}-\frac{be^9n\log\left(d+e\right.}{3d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -(b*e*n)/(24*d*x^(8/3)) + (b*e^2*n)/(21*d^2*x^(7/3)) - (b*e^3*n)/(18*d^3*x^
2) + (b*e^4*n)/(15*d^4*x^(5/3)) - (b*e^5*n)/(12*d^5*x^(4/3)) + (b*e^6*n)/(9
*d^6*x) - (b*e^7*n)/(6*d^7*x^(2/3)) + (b*e^8*n)/(3*d^8*x^(1/3)) - (b*e^9*n*
Log[d + e*x^(1/3)])/(3*d^9) - (a + b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b
*e^9*n*Log[x])/(9*d^9)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{1}{x^9(d+ex)}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^9} - \frac{e}{d^2x^8} + \frac{e^2}{d^3x^7} - \frac{e^3}{d^4x^6} + \frac{e^4}{d^5x^5} - \dots\right.\right.$$
$$= -\frac{ben}{24dx^{8/3}} + \frac{be^2n}{21d^2x^{7/3}} - \frac{be^3n}{18d^3x^2} + \frac{be^4n}{15d^4x^{5/3}} - \frac{be^5n}{12d^5x^{4/3}} + \frac{be^6n}{9d^6x} - \frac{be^7n}{6d^7x^{2/3}} + \frac{be^8n}{3d^8\sqrt[3]{x}} - \dots$$

Mathematica [A] time = 0.191021, size = 177, normalized size = 0.92

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}ben\left(-\frac{e^6}{2d^7x^{2/3}} - \frac{e^4}{4d^5x^{4/3}} + \frac{e^3}{5d^4x^{5/3}} - \frac{e^2}{6d^3x^2} + \frac{e^7}{d^8\sqrt[3]{x}} + \frac{e^5}{3d^6x} - \frac{e^8\log\left(d+e\sqrt[3]{x}\right)}{d^9} + \frac{e^8\log \dots}{3 \dots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b*e*n*(-1/(8*d*x^(8/3)
) + e/(7*d^2*x^(7/3)) - e^2/(6*d^3*x^2) + e^3/(5*d^4*x^(5/3)) - e^4/(4*d^5*
x^(4/3)) + e^5/(3*d^6*x) - e^6/(2*d^7*x^(2/3)) + e^7/(d^8*x^(1/3)) - (e^8*L
og[d + e*x^(1/3)])/d^9 + (e^8*Log[x])/(3*d^9)))/3
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

Maxima [A] time = 1.03738, size = 188, normalized size = 0.98

$$-\frac{1}{2520}ben\left(\frac{840\,e^8\log\left(ex^{\frac{1}{3}}+d\right)}{d^9} - \frac{280\,e^8\log(x)}{d^9} - \frac{840\,e^7x^{\frac{7}{3}} - 420\,de^6x^2 + 280\,d^2e^5x^{\frac{5}{3}} - 210\,d^3e^4x^{\frac{4}{3}} + 168\,d^4e^3x - 140\,d^5 \dots}{d^8x^{\frac{8}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2520*b*e*n*(840*e^8*log(e*x^(1/3) + d)/d^9 - 280*e^8*log(x)/d^9 - (840*e
^7*x^(7/3) - 420*d*e^6*x^2 + 280*d^2*e^5*x^(5/3) - 210*d^3*e^4*x^(4/3) + 16
```

```
8*d^4*e^3*x - 140*d^5*e^2*x^(2/3) + 120*d^6*e*x^(1/3) - 105*d^7)/(d^8*x^(8/
3))) - 1/3*b*log((e*x^(1/3) + d)^n*c)/x^3 - 1/3*a/x^3
```

Fricas [A] time = 1.83991, size = 409, normalized size = 2.13

$$\frac{840\,be^9nx^3\log\left(x^{\frac{1}{3}}\right)+280\,bd^3e^6nx^2-140\,bd^6e^3nx-840\,bd^9\log\left(c\right)-840\,ad^9-840\left(be^9nx^3+bd^9n\right)\log\left(ex^{\frac{1}{3}}+d\right)+}{2520\,d^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/2520*(840*b*e^9*n*x^3*log(x^(1/3)) + 280*b*d^3*e^6*n*x^2 - 140*b*d^6*e^3*
n*x - 840*b*d^9*log(c) - 840*a*d^9 - 840*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^(1
/3) + d) + 30*(28*b*d*e^8*n*x^2 - 7*b*d^4*e^5*n*x + 4*b*d^7*e^2*n)*x^(2/3)
- 21*(20*b*d^2*e^7*n*x^2 - 8*b*d^5*e^4*n*x + 5*b*d^8*e*n)*x^(1/3))/(d^9*x^3
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31043, size = 1091, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/2520*(840*(x^(1/3)*e + d)^9*b*n*e^10*log(x^(1/3)*e + d) - 7560*(x^(1/3)*
e + d)^8*b*d*n*e^10*log(x^(1/3)*e + d) + 30240*(x^(1/3)*e + d)^7*b*d^2*n*e^
10*log(x^(1/3)*e + d) - 70560*(x^(1/3)*e + d)^6*b*d^3*n*e^10*log(x^(1/3)*e
+ d) + 105840*(x^(1/3)*e + d)^5*b*d^4*n*e^10*log(x^(1/3)*e + d) - 105840*(x
^(1/3)*e + d)^4*b*d^5*n*e^10*log(x^(1/3)*e + d) + 70560*(x^(1/3)*e + d)^3*b
*d^6*n*e^10*log(x^(1/3)*e + d) - 30240*(x^(1/3)*e + d)^2*b*d^7*n*e^10*log(x
^(1/3)*e + d) + 7560*(x^(1/3)*e + d)*b*d^8*n*e^10*log(x^(1/3)*e + d) - 840*
(x^(1/3)*e + d)^9*b*n*e^10*log(x^(1/3)*e) + 7560*(x^(1/3)*e + d)^8*b*d*n*e^
10*log(x^(1/3)*e) - 30240*(x^(1/3)*e + d)^7*b*d^2*n*e^10*log(x^(1/3)*e) + 7
0560*(x^(1/3)*e + d)^6*b*d^3*n*e^10*log(x^(1/3)*e) - 105840*(x^(1/3)*e + d)
^5*b*d^4*n*e^10*log(x^(1/3)*e) + 105840*(x^(1/3)*e + d)^4*b*d^5*n*e^10*log(
x^(1/3)*e) - 70560*(x^(1/3)*e + d)^3*b*d^6*n*e^10*log(x^(1/3)*e) + 30240*(x
^(1/3)*e + d)^2*b*d^7*n*e^10*log(x^(1/3)*e) - 7560*(x^(1/3)*e + d)*b*d^8*n*
e^10*log(x^(1/3)*e) + 840*b*d^9*n*e^10*log(x^(1/3)*e) - 840*(x^(1/3)*e + d)
^8*b*d*n*e^10 + 7140*(x^(1/3)*e + d)^7*b*d^2*n*e^10 - 26740*(x^(1/3)*e + d)
```

```
^6*b*d^3*n*e^10 + 57750*(x^(1/3)*e + d)^5*b*d^4*n*e^10 - 78918*(x^(1/3)*e +
 d)^4*b*d^5*n*e^10 + 70252*(x^(1/3)*e + d)^3*b*d^6*n*e^10 - 40188*(x^(1/3)*
e + d)^2*b*d^7*n*e^10 + 13827*(x^(1/3)*e + d)*b*d^8*n*e^10 - 2283*b*d^9*n*e
^10 + 840*b*d^9*e^10*log(c) + 840*a*d^9*e^10)*e^(-1)/((x^(1/3)*e + d)^9*d^9
 - 9*(x^(1/3)*e + d)^8*d^10 + 36*(x^(1/3)*e + d)^7*d^11 - 84*(x^(1/3)*e + d
)^6*d^12 + 126*(x^(1/3)*e + d)^5*d^13 - 126*(x^(1/3)*e + d)^4*d^14 + 84*(x^
(1/3)*e + d)^3*d^15 - 36*(x^(1/3)*e + d)^2*d^16 + 9*(x^(1/3)*e + d)*d^17 -
d^18)
```

3.450 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=680

$$\frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}-\frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}+\frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(\right.}{}$$

```
[Out] (-6*b^2*d^7*n^2*(d + e*x^(1/3))^2)/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3)
/(9*e^9) - (21*b^2*d^5*n^2*(d + e*x^(1/3))^4)/(4*e^9) + (84*b^2*d^4*n^2*(d
+ e*x^(1/3))^5)/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))^6)/(9*e^9) + (24
*b^2*d^2*n^2*(d + e*x^(1/3))^7)/(49*e^9) - (3*b^2*d*n^2*(d + e*x^(1/3))^8)/
(32*e^9) + (2*b^2*n^2*(d + e*x^(1/3))^9)/(243*e^9) + (6*b^2*d^8*n^2*x^(1/3)
)/e^8 - (b^2*d^9*n^2*Log[d + e*x^(1/3)]^2)/(3*e^9) - (6*b*d^8*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^9 + (12*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n]))/e^9 - (56*b*d^6*n*(d + e*x^(1/3))^3*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (21*b*d^5*n*(d + e*x^(1/3))^4*(a + b
*Log[c*(d + e*x^(1/3))^n]))/e^9 - (84*b*d^4*n*(d + e*x^(1/3))^5*(a + b*Log[
c*(d + e*x^(1/3))^n]))/(5*e^9) + (28*b*d^3*n*(d + e*x^(1/3))^6*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(3*e^9) - (24*b*d^2*n*(d + e*x^(1/3))^7*(a + b*Log[c*
(d + e*x^(1/3))^n]))/(7*e^9) + (3*b*d*n*(d + e*x^(1/3))^8*(a + b*Log[c*(d +
 e*x^(1/3))^n]))/(4*e^9) - (2*b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(
1/3))^n]))/(27*e^9) + (2*b*d^9*n*Log[d + e*x^(1/3)]*(a + b*Log[c*(d + e*x^(
1/3))^n]))/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/3
```

Rubi [A] time = 0.697383, antiderivative size = 491, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9}+\frac{63504d^4\left(d+e\sqrt[3]{x}\right)^5}{e^9}-\frac{35280d^3\left(d+e\sqrt[3]{x}\right)^6}{e^9}+\frac{12960d^2\left(d+e\right.}{e^9}\right.}{3780}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-6*b^2*d^7*n^2*(d + e*x^(1/3))^2)/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3)
/(9*e^9) - (21*b^2*d^5*n^2*(d + e*x^(1/3))^4)/(4*e^9) + (84*b^2*d^4*n^2*(d
+ e*x^(1/3))^5)/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))^6)/(9*e^9) + (24
*b^2*d^2*n^2*(d + e*x^(1/3))^7)/(49*e^9) - (3*b^2*d*n^2*(d + e*x^(1/3))^8)/
(32*e^9) + (2*b^2*n^2*(d + e*x^(1/3))^9)/(243*e^9) + (6*b^2*d^8*n^2*x^(1/3)
)/e^8 - (b^2*d^9*n^2*Log[d + e*x^(1/3)]^2)/(3*e^9) - (b*n*((22680*d^8*(d +
e*x^(1/3)))/e^9 - (45360*d^7*(d + e*x^(1/3))^2)/e^9 + (70560*d^6*(d + e*x^(
1/3))^3)/e^9 - (79380*d^5*(d + e*x^(1/3))^4)/e^9 + (63504*d^4*(d + e*x^(1/3
))^5)/e^9 - (35280*d^3*(d + e*x^(1/3))^6)/e^9 + (12960*d^2*(d + e*x^(1/3))^
7)/e^9 - (2835*d*(d + e*x^(1/3))^8)/e^9 + (280*(d + e*x^(1/3))^9)/e^9 - (25
20*d^9*Log[d + e*x^(1/3)])/e^9)*(a + b*Log[c*(d + e*x^(1/3))^n]))/3780 + (x
^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^8 \left(a+b\log\left(c(d+ex)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - \frac{1}{3}(2ben)\,\text{Subst}\left(\int \frac{x^9\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - \frac{1}{3}(2bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^9\left(a+b\log\left(cx^n\right.\right.}{x}\right.$$

$$= -\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9} - \frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9} + \frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9} - \frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9} + \frac{63504}{}\right.}{}$$

$$= -\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9} - \frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9} + \frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9} - \frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9} + \frac{63504}{}\right.}{}$$

$$= -\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9} - \frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9} + \frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9} - \frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9} + \frac{63504}{}\right.}{}$$

$$= -\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9} + \frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9} - \frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9} + \frac{84b^2}{}$$

$$= -\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9} + \frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9} - \frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9} + \frac{84b^2}{}$$

Mathematica [A] time = 0.549727, size = 411, normalized size = 0.6

$$\frac{e\sqrt[3]{x}\left(3175200a^2e^8x^{8/3} - 2520abn\left(840d^6e^2x^{2/3} + 504d^4e^4x^{4/3} - 420d^3e^5x^{5/3} + 360d^2e^6x^2 - 630d^5e^3x - 1260d^7e\sqrt[3]{x} + 2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (e*x^(1/3)*(3175200*a^2*e^8*x^(8/3) - 2520*a*b*n*(2520*d^8 - 1260*d^7*e*x^(
1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x + 504*d^4*e^4*x^(4/3) - 420*d^3*
e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(7/3) + 280*e^8*x^(8/3)) + b^2*
n^2*(17965080*d^8 - 5807340*d^7*e*x^(1/3) + 2813160*d^6*e^2*x^(2/3) - 15806
70*d^5*e^3*x + 947016*d^4*e^4*x^(4/3) - 577500*d^3*e^5*x^(5/3) + 343800*d^2
*e^6*x^2 - 187425*d*e^7*x^(7/3) + 78400*e^8*x^(8/3))) + 2520*b*(2520*a*(d^9
 + e^9*x^3) - b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 8
40*d^6*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2
+ 360*d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3))*Log[c*(d + e*x^(1
/3))^n] + 3175200*b^2*(d^9 + e^9*x^3)*Log[c*(d + e*x^(1/3))^n]^2)/(9525600*
e^9)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Maxima [A] time = 1.04935, size = 572, normalized size = 0.84

$$\frac{1}{3}b^2x^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+\frac{2}{3}abx^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+\frac{1}{3}a^2x^3+\frac{1}{3780}aben\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((e*x^(1/3) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*x^(1/3) + d)^n*
c) + 1/3*a^2*x^3 + 1/3780*a*b*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*
e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d
^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) +
 2520*d^8*x^(1/3))/e^9) + 1/9525600*(2520*e*n*(2520*d^9*log(e*x^(1/3) + d)/
e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5
*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7
*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c) + (78400*e^9*x
^3 - 187425*d*e^8*x^(8/3) + 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3
175200*d^9*log(e*x^(1/3) + d)^2 + 947016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*
x^(4/3) + 2813160*d^6*e^3*x - 17965080*d^9*log(e*x^(1/3) + d) - 5807340*d^7
*e^2*x^(2/3) + 17965080*d^8*e*x^(1/3))*n^2/e^9)*b^2
```

Fricas [A] time = 2.47932, size = 1574, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/9525600*(3175200*b^2*e^9*x^3*log(c)^2 + 39200*(2*b^2*e^9*n^2 - 18*a*b*e^9
*n + 81*a^2*e^9)*x^3 - 2100*(275*b^2*d^3*e^6*n^2 - 504*a*b*d^3*e^6*n)*x^2 +
 3175200*(b^2*e^9*n^2*x^3 + b^2*d^9*n^2)*log(e*x^(1/3) + d)^2 + 840*(3349*b
^2*d^6*e^3*n^2 - 2520*a*b*d^6*e^3*n)*x + 2520*(420*b^2*d^3*e^6*n^2*x^2 - 84
0*b^2*d^6*e^3*n^2*x - 7129*b^2*d^9*n^2 + 2520*a*b*d^9*n - 280*(b^2*e^9*n^2
- 9*a*b*e^9*n)*x^3 + 2520*(b^2*e^9*n*x^3 + b^2*d^9*n)*log(c) + 63*(5*b^2*d*
e^8*n^2*x^2 - 8*b^2*d^4*e^5*n^2*x + 20*b^2*d^7*e^2*n^2)*x^(2/3) - 90*(4*b^2
*d^2*e^7*n^2*x^2 - 7*b^2*d^5*e^4*n^2*x + 28*b^2*d^8*e*n^2)*x^(1/3))*log(e*x
^(1/3) + d) + 352800*(3*b^2*d^3*e^6*n*x^2 - 6*b^2*d^6*e^3*n*x - 2*(b^2*e^9*
n - 9*a*b*e^9)*x^3)*log(c) - 63*(92180*b^2*d^7*e^2*n^2 - 50400*a*b*d^7*e^2*
n + 175*(17*b^2*d*e^8*n^2 - 72*a*b*d*e^8*n)*x^2 - 8*(1879*b^2*d^4*e^5*n^2 -
 2520*a*b*d^4*e^5*n)*x - 2520*(5*b^2*d*e^8*n*x^2 - 8*b^2*d^4*e^5*n*x + 20*b
^2*d^7*e^2*n)*log(c))*x^(2/3) + 90*(199612*b^2*d^8*e*n^2 - 70560*a*b*d^8*e*
n + 20*(191*b^2*d^2*e^7*n^2 - 504*a*b*d^2*e^7*n)*x^2 - 7*(2509*b^2*d^5*e^4*
n^2 - 2520*a*b*d^5*e^4*n)*x - 2520*(4*b^2*d^2*e^7*n*x^2 - 7*b^2*d^5*e^4*n*x
 + 28*b^2*d^8*e*n)*log(c))*x^(1/3))/e^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34386, size = 1926, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/9525600*(3175200*b^2*x^3*e*log(c)^2 + 6350400*a*b*x^3*e*log(c) + 3175200*
a^2*x^3*e + (3175200*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 - 285768
00*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 + 114307200*(x^(1/3)*e +
 d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 - 266716800*(x^(1/3)*e + d)^6*d^3*e^(
-8)*log(x^(1/3)*e + d)^2 + 400075200*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/
3)*e + d)^2 - 400075200*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^2 +
 266716800*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2 - 114307200*(x
^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 + 28576800*(x^(1/3)*e + d)*
d^8*e^(-8)*log(x^(1/3)*e + d)^2 - 705600*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/
3)*e + d) + 7144200*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) - 3265920
0*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) + 88905600*(x^(1/3)*e + d
)^6*d^3*e^(-8)*log(x^(1/3)*e + d) - 160030080*(x^(1/3)*e + d)^5*d^4*e^(-8)*
log(x^(1/3)*e + d) + 200037600*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e +
 d) - 177811200*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) + 114307200
*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) - 57153600*(x^(1/3)*e + d)
*d^8*e^(-8)*log(x^(1/3)*e + d) + 78400*(x^(1/3)*e + d)^9*e^(-8) - 893025*(x
^(1/3)*e + d)^8*d*e^(-8) + 4665600*(x^(1/3)*e + d)^7*d^2*e^(-8) - 14817600*
(x^(1/3)*e + d)^6*d^3*e^(-8) + 32006016*(x^(1/3)*e + d)^5*d^4*e^(-8) - 5000
9400*(x^(1/3)*e + d)^4*d^5*e^(-8) + 59270400*(x^(1/3)*e + d)^3*d^6*e^(-8) -
 57153600*(x^(1/3)*e + d)^2*d^7*e^(-8) + 57153600*(x^(1/3)*e + d)*d^8*e^(-8
))*b^2*n^2 + 2520*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680
*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^
2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/
3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 317520
*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(x^(1/3)*e + d)^3
*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(
1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^
(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e
+ d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e +
 d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e +
d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d
)*d^8*e^(-8))*b^2*n*log(c) + 2520*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3
)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(
1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*
e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)
*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(
x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^
7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*
```

```
e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 1
2960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63
504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 705
60*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 2268
0*(x^(1/3)*e + d)*d^8*e^(-8))*a*b*n)*e^(-1)
```

3.451 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=480

$$-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}+\frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}-\frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(\ldots\right.}{}$$

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(1/3))^2)/(4*e^6) - (20*b^2*d^3*n^2*(d + e*x^(1/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(1/3))^4)/(16*e^6) - (6*b^2*d*n^2*(
d + e*x^(1/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(1/3))^6)/(36*e^6) - (6*b^2*
d^5*n^2*x^(1/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(1/3)]^2)/(2*e^6) + (6*b*d^
5*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^6 - (15*b*d^4*n*(d
+ e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^6) + (20*b*d^3*n*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) - (15*b*d^2*n*(d +
e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) + (6*b*d*n*(d + e*x^
(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(5*e^6) - (b*n*(d + e*x^(1/3))^6
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(6*e^6) - (b*d^6*n*Log[d + e*x^(1/3)]*(a
 + b*Log[c*(d + e*x^(1/3))^n]))/e^6 + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n])
^2)/2
```

Rubi [A] time = 0.461951, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6}-\frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6}+\frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6}-\frac{225d^2\left(d+e\sqrt[3]{x}\right)^4}{e^6}-\frac{60d^6\log\left(d+e\sqrt[3]{x}\right)}{e^6}+\frac{72d\left(\ldots\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(1/3))^2)/(4*e^6) - (20*b^2*d^3*n^2*(d + e*x^(1/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(1/3))^4)/(16*e^6) - (6*b^2*d*n^2*(
d + e*x^(1/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(1/3))^6)/(36*e^6) - (6*b^2*
d^5*n^2*x^(1/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(1/3)]^2)/(2*e^6) + (b*n*((
360*d^5*(d + e*x^(1/3)))/e^6 - (450*d^4*(d + e*x^(1/3))^2)/e^6 + (400*d^3*(
d + e*x^(1/3))^3)/e^6 - (225*d^2*(d + e*x^(1/3))^4)/e^6 + (72*d*(d + e*x^(1
/3))^5)/e^6 - (10*(d + e*x^(1/3))^6)/e^6 - (60*d^6*Log[d + e*x^(1/3)])/e^6)
*(a + b*Log[c*(d + e*x^(1/3))^n]))/60 + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n
])^2)/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (ben)\,\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex} dx\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x} dx\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6} - \frac{225d^2\left(d+\right.}{e^6}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6} - \frac{225d^2\left(d+\right.}{e^6}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6} - \frac{225d^2\left(d+\right.}{e^6}\right.$$

$$= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6} - \frac{6b^2dn^2}{}$$

$$= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6} - \frac{6b^2dn^2}{}$$

Mathematica [A] time = 0.308326, size = 301, normalized size = 0.63

$$\frac{e\sqrt[3]{x}\left(1800a^2e^5x^{5/3} + 60abn\left(20d^3e^2x^{2/3} - 15d^2e^3x - 30d^4e\sqrt[3]{x} + 60d^5 + 12de^4x^{4/3} - 10e^5x^{5/3}\right) + b^2n^2\left(-1140d^3e^2x^{2/3} + \right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (e*x^(1/3)*(1800*a^2*e^5*x^(5/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(1/3) + 20
*d^3*e^2*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5/3)) + b^2*
n^2*(-8820*d^5 + 2610*d^4*e*x^(1/3) - 1140*d^3*e^2*x^(2/3) + 555*d^2*e^3*x
- 264*d*e^4*x^(4/3) + 100*e^5*x^(5/3))) - 60*b*(60*a*(d^6 - e^6*x^2) + b*n*
(-147*d^6 - 60*d^5*e*x^(1/3) + 30*d^4*e^2*x^(2/3) - 20*d^3*e^3*x + 15*d^2*e
^4*x^(4/3) - 12*d*e^5*x^(5/3) + 10*e^6*x^2))*Log[c*(d + e*x^(1/3))^n] - 180
0*b^2*(d^6 - e^6*x^2)*Log[c*(d + e*x^(1/3))^n]^2)/(3600*e^6)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Maxima [A] time = 1.05463, size = 436, normalized size = 0.91

$$\frac{1}{2}b^2x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2-\frac{1}{60}aben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*x^(1/3) + d)^n*c)^2 - 1/60*a*b*e*n*(60*d^6*log(e*x^(1/3)
 + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^
2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/e^6) + a*b*x^2*log((e*x^(1/3) + d)
^n*c) + 1/2*a^2*x^2 - 1/3600*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e
^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^
(2/3) - 60*d^5*x^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2 + 1800
*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1140*
d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d^5*e
*x^(1/3))*n^2/e^6)*b^2
```

Fricas [A] time = 2.18252, size = 1088, normalized size = 2.27

$$\frac{1800\,b^2e^6x^2\log(c)^2+100\left(b^2e^6n^2-6\,abe^6n+18\,a^2e^6\right)x^2+1800\left(b^2e^6n^2x^2-b^2d^6n^2\right)\log\left(ex^{\frac{1}{3}}+d\right)^2-60\left(19\,b^2d^3e^3n^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/3600*(1800*b^2*e^6*x^2*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2
*e^6)*x^2 + 1800*(b^2*e^6*n^2*x^2 - b^2*d^6*n^2)*log(e*x^(1/3) + d)^2 - 60*
(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x + 60*(20*b^2*d^3*e^3*n^2*x + 147*
b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^2 + 60*(b^2*e
^6*n*x^2 - b^2*d^6*n)*log(c) + 6*(2*b^2*d*e^5*n^2*x - 5*b^2*d^4*e^2*n^2)*x^
(2/3) - 15*(b^2*d^2*e^4*n^2*x - 4*b^2*d^5*e*n^2)*x^(1/3))*log(e*x^(1/3) + d
) + 600*(2*b^2*d^3*e^3*n*x - (b^2*e^6*n - 6*a*b*e^6)*x^2)*log(c) + 6*(435*b
^2*d^4*e^2*n^2 - 300*a*b*d^4*e^2*n - 4*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*
x + 60*(2*b^2*d*e^5*n*x - 5*b^2*d^4*e^2*n)*log(c))*x^(2/3) - 15*(588*b^2*d^
5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n)*x + 60*
(b^2*d^2*e^4*n*x - 4*b^2*d^5*e*n)*log(c))*x^(1/3))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33662, size = 1291, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/3600*(1800*b^2*x^2*e*log(c)^2 + 3600*a*b*x^2*e*log(c) + (1800*(x^(1/3)*e
+ d)^6*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)^5*d*e^(-5)*log(x
^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 -
 36000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e
 + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)*d^5*e^(-5)*
log(x^(1/3)*e + d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) + 43
20*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*(x^(1/3)*e + d)^4*
d^2*e^(-5)*log(x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1
/3)*e + d) - 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) + 21600*
(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1/3)*e + d)^6*e^(-5
) - 864*(x^(1/3)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)^4*d^2*e^(-5) - 80
00*(x^(1/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2*d^4*e^(-5) - 2160
0*(x^(1/3)*e + d)*d^5*e^(-5))*b^2*n^2 + 1800*a^2*x^2*e + 60*(60*(x^(1/3)*e
+ d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/
3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200*(x^(
1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^4*e^(
-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d)
- 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3
)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3)*e +
 d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*b^2*n*log(c) + 60*(60*(x
^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*
log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) -
1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^
2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3
)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 22
5*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^
(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*a*b*n)*e^(-1)
```

3.452 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=267

$$\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\right.}{e^3}$$

```
[Out] (-3*b^2*d*n^2*(d + e*x^(1/3))^2)/(2*e^3) + (2*b^2*n^2*(d + e*x^(1/3))^3)/(9
*e^3) + (6*b^2*d^2*n^2*x^(1/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(1/3)]^2)/e^
3 - (6*b*d^2*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 + (3*b
*d*n*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 - (2*b*n*(d +
e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) + (2*b*d^3*n*Log[d +
 e*x^(1/3)]*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 + x*(a + b*Log[c*(d + e*x
^(1/3))^n])^2
```

Rubi [A] time = 0.289783, antiderivative size = 210, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2451, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{6d^3\log\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3}+\frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+x\left(a+b\log\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(1/3))^2)/(2*e^3) + (2*b^2*n^2*(d + e*x^(1/3))^3)/(9
*e^3) + (6*b^2*d^2*n^2*x^(1/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(1/3)]^2)/e^
3 - (b*n*((18*d^2*(d + e*x^(1/3)))/e^3 - (9*d*(d + e*x^(1/3))^2)/e^3 + (2*(
d + e*x^(1/3))^3)/e^3 - (6*d^3*Log[d + e*x^(1/3)])/e^3)*(a + b*Log[c*(d + e
*x^(1/3))^n]))/3 + x*(a + b*Log[c*(d + e*x^(1/3))^n])^2
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^2\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (2ben)\,\text{Subst}\left(\int \frac{x^3(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, \sqrt[3]{x}\right.$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (2bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log(cx^n))}{x}\,dx, x, d-\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3} + \frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3} - \frac{6d^3\log\left(d+e\sqrt[3]{x}\right)}{e^3}\right)\left(a+\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3} + \frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3} - \frac{6d^3\log\left(d+e\sqrt[3]{x}\right)}{e^3}\right)\left(a+\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3} + \frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3} - \frac{6d^3\log\left(d+e\sqrt[3]{x}\right)}{e^3}\right)\left(a+\right.$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{6b^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{9}{}\right.$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{6b^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)}{e^3} -$$

Mathematica [A] time = 0.126614, size = 197, normalized size = 0.74

$$\frac{18a^2\left(d^3+e^3x\right)+6b\left(6a\left(d^3+e^3x\right)-bn\left(6d^2e\sqrt[3]{x}+11d^3-3de^2x^{2/3}+2e^3x\right)\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+6abn\left(-6d^2e\sqrt[3]{x}+7d^3\right.}{18e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (b^2*e*n^2*(66*d^2 - 15*d*e*x^(1/3) + 4*e^2*x^(2/3))*x^(1/3) + 6*a*b*n*(7*d
^3 - 6*d^2*e*x^(1/3) + 3*d*e^2*x^(2/3) - 2*e^3*x) + 18*a^2*(d^3 + e^3*x) +
6*b*(6*a*(d^3 + e^3*x) - b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) +
2*e^3*x))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(d^3 + e^3*x)*Log[c*(d + e*x^(1
/3))^n]^2)/(18*e^3)
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Maxima [A] time = 1.05169, size = 293, normalized size = 1.1

$$\frac{1}{3}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)\right)ab+\frac{1}{18}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a*b + 1/18*(6*e*n*(6*d^3*log(e
*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*x
^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(1/3)
+ d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e*
x^(1/3))*n^2/e^3)*b^2 + a^2*x
```

Fricas [A] time = 1.97478, size = 664, normalized size = 2.49

$$\frac{18\,b^2e^3x\log\left(c\right)^2+18\left(b^2e^3n^2x+b^2d^3n^2\right)\log\left(ex^{\frac{1}{3}}+d\right)^2-12\left(b^2e^3n-3\,abe^3\right)x\log\left(c\right)+2\left(2\,b^2e^3n^2-6\,abe^3n+9\,a^2e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^2*e^3*x*log(c)^2 + 18*(b^2*e^3*n^2*x + b^2*d^3*n^2)*log(e*x^(1/3
) + d)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x*log(c) + 2*(2*b^2*e^3*n^2 - 6*a*b*e
^3*n + 9*a^2*e^3)*x + 6*(3*b^2*d*e^2*n^2*x^(2/3) - 6*b^2*d^2*e*n^2*x^(1/3)
- 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*x + 6*(b^2*e
^3*n*x + b^2*d^3*n)*log(c))*log(e*x^(1/3) + d) - 3*(5*b^2*d*e^2*n^2 - 6*b^2
*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) + 6*(11*b^2*d^2*e*n^2 - 6*b^2*d^2*
e*n*log(c) - 6*a*b*d^2*e*n)*x^(1/3))/e^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Giac [B] time = 1.37276, size = 647, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/18*(18*b^2*x*e*log(c)^2 + (18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)
^2 - 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^(1/3)*e + d
)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)
*e + d) + 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108*(x^(1/3)*e
 + d)*d^2*e^(-2)*log(x^(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2) - 27*(x^(1
/3)*e + d)^2*d*e^(-2) + 108*(x^(1/3)*e + d)*d^2*e^(-2))*b^2*n^2 + 6*(6*(x^(
1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log
(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(
1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^
2*e^(-2))*b^2*n*log(c) + 36*a*b*x*e*log(c) + 6*(6*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 18*
(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e^(-2)
+ 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*a*b*n + 18*
a^2*x*e)*e^(-1)
```

3.453 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$6bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-6b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d\right.\right.\right.$$

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)] + 6*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d] - 6*b^2*n^2*PolyLog[
3, 1 + (e*x^(1/3))/d]
```

Rubi [A] time = 0.13143, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$6bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-6b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```


```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)] + 6*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d] - 6*b^2*n^2*PolyLog[
3, 1 + (e*x^(1/3))/d]
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
```

```
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c\right.$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (6bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(-\right.}{x}\right.$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\text{Li}_2\left(1+\right.$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\text{Li}_2\left(1+\right.$$

Mathematica [B] time = 0.107424, size = 195, normalized size = 2.1

$$2bn\left(\log(x)\left(\log\left(d+e\sqrt[3]{x}\right)-\log\left(\frac{e\sqrt[3]{x}}{d}+1\right)\right)-3\text{PolyLog}\left(2,-\frac{e\sqrt[3]{x}}{d}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-bn\log\left(d+e\sqrt[3]{x}\right)\right)+3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*((Log[d + e*x^(1/
3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) + 3*
b^2*n^2*(Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*
PolyLog[2, 1 + (e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d])
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2 \log\left(\left(ex^{\frac{1}{3}} + d\right)^n\right)^2 \log(x) + \int \frac{3\left(b^2 e \log(c)^2 + 2\,abe \log(c) + a^2 e\right)x - 2\left(b^2 enx \log(x) - 3\left(b^2 e \log(c) + abe\right)x - 3\left(b\right.\right.}{3\left(ex^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((e*x^(1/3) + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*e*log(c)^2 + 2*
a*b*e*log(c) + a^2*e)*x - 2*(b^2*e*n*x*log(x) - 3*(b^2*e*log(c) + a*b*e)*x
- 3*(b^2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*log(c
)^2 + 2*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^2 + d*x^(5/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \log\left(\left(ex^{\frac{1}{3}} + d\right)^n c\right)^2 + 2\,ab \log\left(\left(ex^{\frac{1}{3}} + d\right)^n c\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(\left(ex^{\frac{1}{3}} + d\right)^n c\right) + a\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x, x)
```

3.454 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=231

$$-\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}+\frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}+\frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{d^3\sqrt[3]{x}}$$

```
[Out] -((b^2*e^2*n^2)/(d^2*x^(1/3))) + (b^2*e^3*n^2*Log[d + e*x^(1/3)])/d^3 - (b*
e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d*x^(2/3)) + (2*b*e^2*n*(d + e*x^(1/
3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1/3)) + (2*b*e^3*n*Log[1 - d/
(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/d^3 - (a + b*Log[c*(d +
e*x^(1/3))^n])^2/x - (b^2*e^3*n^2*Log[x])/d^3 - (2*b^2*e^3*n^2*PolyLog[2, d
/(d + e*x^(1/3))])/d^3
```

Rubi [A] time = 0.501207, antiderivative size = 253, normalized size of antiderivative = 1.1, number of steps used = 14, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^3}-\frac{e^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3}+\frac{2be^3n\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}+\frac{2be^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] -((b^2*e^2*n^2)/(d^2*x^(1/3))) + (b^2*e^3*n^2*Log[d + e*x^(1/3)])/d^3 - (b*
e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d*x^(2/3)) + (2*b*e^2*n*(d + e*x^(1/
3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1/3)) - (e^3*(a + b*Log[c*(d
+ e*x^(1/3))^n])^2)/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])^2/x + (2*b*e^3*n
*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 - (b^2*e^3*n^2
*Log[x])/d^3 + (2*b^2*e^3*n^2*PolyLog[2, 1 + (e*x^(1/3))/d])/d^3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + (2ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}dx, x, \sqrt[3]{x}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + (2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+e\sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+e\sqrt[3]{x}\right)}{d} - \frac{(2be}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} - \frac{(2ben)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} -$$

$$= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n\left(d+e\right.}{}$$

$$= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n\left(d+e\right.}{}$$

Mathematica [A] time = 0.222512, size = 274, normalized size = 1.19

$$3\left(\frac{2}{3}ben\left(\frac{be^2n\text{PolyLog}\left(2,\frac{d+e\sqrt[3]{x}}{d}\right)}{d^3} - \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2bd^3n} + \frac{e^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3} + \frac{e\left(a\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] 3*(-(a + b*Log[c*(d + e*x^(1/3))^n])^2/(3*x) + (2*b*e*n*(-(a + b*Log[c*(d +
 e*x^(1/3))^n])/(2*d*x^(2/3)) + (e*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^2*x
^(1/3)) - (e^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*b*d^3*n) + (e^2*(a +
b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 - (b*e^2*n*(-(Log[d
+ e*x^(1/3)]/d) + Log[x]/(3*d)))/d^2 - (b*e*n*(1/(d*x^(1/3)) - (e*Log[d + e
*x^(1/3)])/d^2 + (e*Log[x])/(3*d^2)))/(2*d) + (b*e^2*n*PolyLog[2, (d + e*x^
(1/3))/d])/d^3))/3)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b^2*e^3*n^2/d^
3 - (2*a*b*e^3*n - (3*e^3*n^2 - 2*e^3*n*log(c))*b^2)*log(e*x^(1/3) + d)/d^3
 + 2*(b^2*e^3*n*log(c) + a*b*e^3*n)*log(x^(1/3))/d^3 + integrate((b^2*e^6*n
^2*x - b^2*d^3*e^3*n^2)/x, x)/d^6 - 1/20*(12*b^2*e^8*n^2*x^(5/3) - 15*b^2*d
*e^7*n^2*x^(4/3) + 20*b^2*d^2*e^6*n^2*x - 40*b^2*d^3*e^5*n^2*x^(2/3) + 100*
b^2*d^4*e^4*n^2*x^(1/3) + 20*(b^2*d^3*e^5*n^2*x^(2/3) - 2*b^2*d^4*e^4*n^2*x
^(1/3))*log(x^(1/3)))/d^8 + 1/60*(60*b^2*d^5*e^3*n^2*x^(5/3)*log(e*x^(1/3)
+ d)^2 - 45*b^2*d*e^7*n^2*x^3 - 40*b^2*d^4*e^4*n^2*x^2*log(x) + 300*b^2*d^4
*e^4*n^2*x^2 - 60*b^2*d^8*x^(2/3)*log((e*x^(1/3) + d)^n)^2 - 60*(b^2*d^7*e*
n*log(c) + a*b*d^7*e*n)*x - 20*(6*b^2*d^5*e^3*n*x^(5/3)*log(e*x^(1/3) + d)
- 6*b^2*d^6*e^2*n*x^(4/3) + 3*b^2*d^7*e*n*x - 2*(b^2*d^5*e^3*n*x*log(x) - 3
*b^2*d^8*log(c) - 3*a*b*d^8)*x^(2/3))*log((e*x^(1/3) + d)^n) - 60*(b^2*d^8*
log(c)^2 + 2*a*b*d^8*log(c) + a^2*d^8)*x^(2/3) + 4*(9*b^2*e^8*n^2*x^3 + 5*b
^2*d^3*e^5*n^2*x^2*log(x) - 15*b^2*d^3*e^5*n^2*x^2 + 30*(b^2*d^6*e^2*n*log(
c) + a*b*d^6*e^2*n)*x)*x^(1/3) - 60*(b^2*d^3*e^5*n^2*x^3 + b^2*d^6*e^2*n^2*
x^2)/x^(2/3))/(d^8*x^(5/3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^2, x)
```

3.455 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=405

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6}+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}}-\frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(\right.}{}$$

```
[Out] -(b^2*e^2*n^2)/(20*d^2*x^(4/3)) + (3*b^2*e^3*n^2)/(20*d^3*x) - (47*b^2*e^4*
n^2)/(120*d^4*x^(2/3)) + (77*b^2*e^5*n^2)/(60*d^5*x^(1/3)) - (77*b^2*e^6*n^
2*Log[d + e*x^(1/3)])/(60*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(
5*d*x^(5/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*d^2*x^(4/3)) -
 (b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*d^3*x) + (b*e^4*n*(a + b*Log
[c*(d + e*x^(1/3))^n]))/(2*d^4*x^(2/3)) - (b*e^5*n*(d + e*x^(1/3))*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(d^6*x^(1/3)) - (b*e^6*n*Log[1 - d/(d + e*x^(1/3)
)]*(a + b*Log[c*(d + e*x^(1/3))^n]))/d^6 - (a + b*Log[c*(d + e*x^(1/3))^n])
^2/(2*x^2) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*n^2*PolyLog[2, d
/(d + e*x^(1/3))])/d^6
```

Rubi [A] time = 1.01255, antiderivative size = 430, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^6}+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}}+\frac{e^6\left(a+b\log\left(c\left(d\right.\right.\right.}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -(b^2*e^2*n^2)/(20*d^2*x^(4/3)) + (3*b^2*e^3*n^2)/(20*d^3*x) - (47*b^2*e^4*
n^2)/(120*d^4*x^(2/3)) + (77*b^2*e^5*n^2)/(60*d^5*x^(1/3)) - (77*b^2*e^6*n^
2*Log[d + e*x^(1/3)])/(60*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(
5*d*x^(5/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*d^2*x^(4/3)) -
 (b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*d^3*x) + (b*e^4*n*(a + b*Log
[c*(d + e*x^(1/3))^n]))/(2*d^4*x^(2/3)) - (b*e^5*n*(d + e*x^(1/3))*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(d^6*x^(1/3)) + (e^6*(a + b*Log[c*(d + e*x^(1/3))
^n])^2)/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])^2/(2*x^2) - (b*e^6*n*(a
+ b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^6 + (137*b^2*e^6*n^2
*Log[x])/(180*d^6) - (b^2*e^6*n^2*PolyLog[2, 1 + (e*x^(1/3))/d])/d^6
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
```

```
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right)}{d} - \frac{(ben)\,\text{Su}\ldots}{\ldots}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} - \frac{(ben)\,\text{Subst}\left(\int \frac{a+b\ldots}{(-\ldots}\right.}{\ldots}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} - \frac{\left(a+b\log\left(c\left(\ldots\right.\right.\right.}{2x\ldots}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{b^2e^3n^2}{15d^3x} - \frac{b^2e^4n^2}{10d^4x^{2/3}} + \frac{b^2e^5n^2}{5d^5\sqrt[3]{x}} - \frac{b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{5d^6} - \frac{ben\left(a+b\log\left(\ldots\right.\right.}{5dx\ldots}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{9b^2e^4n^2}{40d^4x^{2/3}} + \frac{9b^2e^5n^2}{20d^5\sqrt[3]{x}} - \frac{9b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{20d^6} - \frac{ben\left(a+b\,\text{l}\ldots\right.}{\ldots}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{47b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{47b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6} - \frac{ben\left(a+\ldots\right.}{\ldots}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6} - \frac{ben\left(a+\ldots\right.}{\ldots}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6} - \frac{ben\left(a+\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.272503, size = 533, normalized size = 1.32

$$-\frac{360b^2e^6n^2x^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)+180a^2d^6+360abd^6\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-360abe^6x^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-90abd}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -(180*a^2*d^6 + 72*a*b*d^5*e*n*x^(1/3) - 90*a*b*d^4*e^2*n*x^(2/3) + 18*b^2*
d^4*e^2*n^2*x^(2/3) + 120*a*b*d^3*e^3*n*x - 54*b^2*d^3*e^3*n^2*x - 180*a*b*
d^2*e^4*n*x^(4/3) + 141*b^2*d^2*e^4*n^2*x^(4/3) + 360*a*b*d*e^5*n*x^(5/3) -
 462*b^2*d*e^5*n^2*x^(5/3) + 822*b^2*e^6*n^2*x^2*Log[d + e*x^(1/3)] + 360*a
*b*d^6*Log[c*(d + e*x^(1/3))^n] + 72*b^2*d^5*e*n*x^(1/3)*Log[c*(d + e*x^(1/
3))^n] - 90*b^2*d^4*e^2*n*x^(2/3)*Log[c*(d + e*x^(1/3))^n] + 120*b^2*d^3*e^
3*n*x*Log[c*(d + e*x^(1/3))^n] - 180*b^2*d^2*e^4*n*x^(4/3)*Log[c*(d + e*x^(
1/3))^n] + 360*b^2*d*e^5*n*x^(5/3)*Log[c*(d + e*x^(1/3))^n] - 360*a*b*e^6*x
^2*Log[c*(d + e*x^(1/3))^n] + 180*b^2*d^6*Log[c*(d + e*x^(1/3))^n]^2 - 180*
b^2*e^6*x^2*Log[c*(d + e*x^(1/3))^n]^2 + 360*a*b*e^6*n*x^2*Log[-((e*x^(1/3)
)/d)] + 360*b^2*e^6*n*x^2*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] -
274*b^2*e^6*n^2*x^2*Log[x] + 360*b^2*e^6*n^2*x^2*PolyLog[2, 1 + (e*x^(1/3))
/d])/(360*d^6*x^2)
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^2}{2\,x^2}+\int\frac{3\left(b^2e\log\left(c\right)^2+2\,abe\log\left(c\right)+a^2e\right)x+\left(b^2enx+6\left(b^2e\log\left(c\right)+abe\right)x+6\left(b^2d\log\left(c\right)+a\right.\right.}{3\left(ex^4+dx^{\frac{11}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((e*x^(1/3) + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(b^
2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^4 + d*x^(11/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^3, x)
```

3.456 $\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=1835

result too large to display

```
[Out] (-99*b^3*d^10*n^3*(d + e*x^(1/3))^2)/(8*e^12) + (110*b^3*d^9*n^3*(d + e*x^(
1/3))^3)/(9*e^12) - (1485*b^3*d^8*n^3*(d + e*x^(1/3))^4)/(128*e^12) + (1188
*b^3*d^7*n^3*(d + e*x^(1/3))^5)/(125*e^12) - (77*b^3*d^6*n^3*(d + e*x^(1/3)
)^6)/(12*e^12) + (1188*b^3*d^5*n^3*(d + e*x^(1/3))^7)/(343*e^12) - (1485*b^
3*d^4*n^3*(d + e*x^(1/3))^8)/(1024*e^12) + (110*b^3*d^3*n^3*(d + e*x^(1/3))
^9)/(243*e^12) - (99*b^3*d^2*n^3*(d + e*x^(1/3))^10)/(1000*e^12) + (18*b^3*
d*n^3*(d + e*x^(1/3))^11)/(1331*e^12) - (b^3*n^3*(d + e*x^(1/3))^12)/(1152*
e^12) - (18*a*b^2*d^11*n^2*x^(1/3))/e^11 + (18*b^3*d^11*n^3*x^(1/3))/e^11 -
 (18*b^3*d^11*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^12 + (99*b^2*
d^10*n^2*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^12) - (11
0*b^2*d^9*n^2*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^12)
 + (1485*b^2*d^8*n^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32
*e^12) - (1188*b^2*d^7*n^2*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n
]))/(25*e^12) + (77*b^2*d^6*n^2*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/
3))^n]))/(2*e^12) - (1188*b^2*d^5*n^2*(d + e*x^(1/3))^7*(a + b*Log[c*(d + e
*x^(1/3))^n]))/(49*e^12) + (1485*b^2*d^4*n^2*(d + e*x^(1/3))^8*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(128*e^12) - (110*b^2*d^3*n^2*(d + e*x^(1/3))^9*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(27*e^12) + (99*b^2*d^2*n^2*(d + e*x^(1/3))^10
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(100*e^12) - (18*b^2*d*n^2*(d + e*x^(1/3
))^11*(a + b*Log[c*(d + e*x^(1/3))^n]))/(121*e^12) + (b^2*n^2*(d + e*x^(1/3
))^12*(a + b*Log[c*(d + e*x^(1/3))^n]))/(96*e^12) + (9*b*d^11*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (99*b*d^10*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^12) + (55*b*d^9*n*(d + e*x^(1/
3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (1485*b*d^8*n*(d + e*x^(1/
3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) + (594*b*d^7*n*(d + e*x
^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^12) - (231*b*d^6*n*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^12) + (594*b*d^5*n*(d
 + e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^12) - (1485*b*d^4*
n*(d + e*x^(1/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(32*e^12) + (55*b*d
^3*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^12) - (99*b
*d^2*n*(d + e*x^(1/3))^10*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(20*e^12) + (
9*b*d*n*(d + e*x^(1/3))^11*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(11*e^12) -
(b*n*(d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) - (3*
d^11*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^10*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^12) - (55*d^9*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^8*(d + e*x^
(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (198*d^7*(d + e*x^(
1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (231*d^6*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 - (198*d^5*(d + e*x^(1/3))^7*(a
 + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^4*(d + e*x^(1/3))^8*(a + b*
Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (55*d^3*(d + e*x^(1/3))^9*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^2*(d + e*x^(1/3))^10*(a + b*Log[c*(
d + e*x^(1/3))^n])^3)/(2*e^12) - (3*d*(d + e*x^(1/3))^11*(a + b*Log[c*(d +
e*x^(1/3))^n])^3)/e^12 + ((d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n
])^3)/(4*e^12)
```

Rubi [A] time = 2.26676, antiderivative size = 1835, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-99*b^3*d^10*n^3*(d + e*x^(1/3))^2)/(8*e^12) + (110*b^3*d^9*n^3*(d + e*x^(
1/3))^3)/(9*e^12) - (1485*b^3*d^8*n^3*(d + e*x^(1/3))^4)/(128*e^12) + (1188
*b^3*d^7*n^3*(d + e*x^(1/3))^5)/(125*e^12) - (77*b^3*d^6*n^3*(d + e*x^(1/3)
)^6)/(12*e^12) + (1188*b^3*d^5*n^3*(d + e*x^(1/3))^7)/(343*e^12) - (1485*b^
3*d^4*n^3*(d + e*x^(1/3))^8)/(1024*e^12) + (110*b^3*d^3*n^3*(d + e*x^(1/3))
^9)/(243*e^12) - (99*b^3*d^2*n^3*(d + e*x^(1/3))^10)/(1000*e^12) + (18*b^3*
d*n^3*(d + e*x^(1/3))^11)/(1331*e^12) - (b^3*n^3*(d + e*x^(1/3))^12)/(1152*
e^12) - (18*a*b^2*d^11*n^2*x^(1/3))/e^11 + (18*b^3*d^11*n^3*x^(1/3))/e^11 -
 (18*b^3*d^11*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^12 + (99*b^2*
d^10*n^2*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^12) - (11
0*b^2*d^9*n^2*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^12)
+ (1485*b^2*d^8*n^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32
*e^12) - (1188*b^2*d^7*n^2*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n
]))/(25*e^12) + (77*b^2*d^6*n^2*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/
3))^n]))/(2*e^12) - (1188*b^2*d^5*n^2*(d + e*x^(1/3))^7*(a + b*Log[c*(d + e
*x^(1/3))^n]))/(49*e^12) + (1485*b^2*d^4*n^2*(d + e*x^(1/3))^8*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(128*e^12) - (110*b^2*d^3*n^2*(d + e*x^(1/3))^9*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(27*e^12) + (99*b^2*d^2*n^2*(d + e*x^(1/3))^10
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(100*e^12) - (18*b^2*d*n^2*(d + e*x^(1/3
))^11*(a + b*Log[c*(d + e*x^(1/3))^n]))/(121*e^12) + (b^2*n^2*(d + e*x^(1/3
))^12*(a + b*Log[c*(d + e*x^(1/3))^n]))/(96*e^12) + (9*b*d^11*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (99*b*d^10*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^12) + (55*b*d^9*n*(d + e*x^(1/
3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (1485*b*d^8*n*(d + e*x^(1/
3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) + (594*b*d^7*n*(d + e*x
^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^12) - (231*b*d^6*n*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^12) + (594*b*d^5*n*(d
 + e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^12) - (1485*b*d^4*
n*(d + e*x^(1/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(32*e^12) + (55*b*d
^3*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^12) - (99*b
*d^2*n*(d + e*x^(1/3))^10*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(20*e^12) + (
9*b*d*n*(d + e*x^(1/3))^11*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(11*e^12) -
(b*n*(d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) - (3*
d^11*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^10*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^12) - (55*d^9*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^8*(d + e*x^
(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (198*d^7*(d + e*x^(
1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (231*d^6*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 - (198*d^5*(d + e*x^(1/3))^7*(a
 + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^4*(d + e*x^(1/3))^8*(a + b*
Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (55*d^3*(d + e*x^(1/3))^9*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^2*(d + e*x^(1/3))^10*(a + b*Log[c*(
d + e*x^(1/3))^n])^3)/(2*e^12) - (3*d*(d + e*x^(1/3))^11*(a + b*Log[c*(d +
e*x^(1/3))^n])^3)/e^12 + ((d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n
])^3)/(4*e^12)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int\left(-\frac{d^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^{11}}+\frac{11d^{10}(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)}{e^{11}}\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int (d+ex)^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(33d)\,\text{Subst}\left(\int (d+ex\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(33d)\,\text{Subst}\left(\int x^{10}\left(a+b\log\right.\right.}{e^{12}}$$
$$= -\frac{3d^{11}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^{12}}+\frac{33d^{10}\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d\right.\right.\right.}{2e^{12}}$$
$$= \frac{9bd^{11}n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^{12}}-\frac{99bd^{10}n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\right.\right.}{4e^{12}}$$
$$= -\frac{99b^3d^{10}n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^{12}}+\frac{110b^3d^9n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^{12}}-\frac{1485b^3d^8n^3\left(d+e\sqrt[3]{x}\right)^4}{128e^{12}}+\ldots$$
$$= -\frac{99b^3d^{10}n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^{12}}+\frac{110b^3d^9n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^{12}}-\frac{1485b^3d^8n^3\left(d+e\sqrt[3]{x}\right)^4}{128e^{12}}+\ldots$$

Mathematica [A] time = 1.25083, size = 1009, normalized size = 0.55

$$\frac{-3550000608000b^3\left(d^{12}-e^{12}x^4\right)\log^3\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-384199200b^2\left(27720a\left(d^{12}-e^{12}x^4\right)+bn\left(-86021d^{12}-27720e\sqrt[3]{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (e*x^(1/3)*(3550000608000*a^3*e^11*x^(11/3) + b^3*n^3*(119225632485960*d^11
 - 26563616859780*d^10*e*x^(1/3) + 10242678720120*d^9*e^2*x^(2/3) - 4836309
598890*d^8*e^3*x + 2516628075192*d^7*e^4*x^(4/3) - 1373077023780*d^6*e^5*x^
(5/3) + 761128152840*d^5*e^6*x^2 - 417533743935*d^4*e^7*x^(7/3) + 220161492
320*d^3*e^8*x^(8/3) - 106944990768*d^2*e^9*x^3 + 44119404000*d*e^10*x^(10/3
) - 12326391000*e^11*x^(11/3)) - 27720*a*b^2*n^2*(2384502120*d^11 - 8080518
60*d^10*e*x^(1/3) + 410634840*d^9*e^2*x^(2/3) - 243942930*d^8*e^3*x + 15673
4424*d^7*e^4*x^(4/3) - 104998740*d^6*e^5*x^(5/3) + 71703720*d^5*e^6*x^2 - 4
9019355*d^4*e^7*x^(7/3) + 32900560*d^3*e^8*x^(8/3) - 21072744*d^2*e^9*x^3 +
 12171600*d*e^10*x^(10/3) - 5336100*e^11*x^(11/3)) + 384199200*a^2*b*n*(277
20*d^11 - 13860*d^10*e*x^(1/3) + 9240*d^9*e^2*x^(2/3) - 6930*d^8*e^3*x + 55
44*d^7*e^4*x^(4/3) - 4620*d^6*e^5*x^(5/3) + 3960*d^5*e^6*x^2 - 3465*d^4*e^7
*x^(7/3) + 3080*d^3*e^8*x^(8/3) - 2772*d^2*e^9*x^3 + 2520*d*e^10*x^(10/3) -
 2310*e^11*x^(11/3))) - 27720*b*(b^2*n^2*(4301068993*d^12 + 2384502120*d^11
*e*x^(1/3) - 808051860*d^10*e^2*x^(2/3) + 410634840*d^9*e^3*x - 243942930*d
^8*e^4*x^(4/3) + 156734424*d^7*e^5*x^(5/3) - 104998740*d^6*e^6*x^2 + 717037
20*d^5*e^7*x^(7/3) - 49019355*d^4*e^8*x^(8/3) + 32900560*d^3*e^9*x^3 - 2107
2744*d^2*e^10*x^(10/3) + 12171600*d*e^11*x^(11/3) - 5336100*e^12*x^4) - 277
20*a*b*n*(86021*d^12 + 27720*d^11*e*x^(1/3) - 13860*d^10*e^2*x^(2/3) + 9240
*d^9*e^3*x - 6930*d^8*e^4*x^(4/3) + 5544*d^7*e^5*x^(5/3) - 4620*d^6*e^6*x^2
 + 3960*d^5*e^7*x^(7/3) - 3465*d^4*e^8*x^(8/3) + 3080*d^3*e^9*x^3 - 2772*d^
2*e^10*x^(10/3) + 2520*d*e^11*x^(11/3) - 2310*e^12*x^4) + 384199200*a^2*(d^
12 - e^12*x^4))*Log[c*(d + e*x^(1/3))^n] - 384199200*b^2*(27720*a*(d^12 - e
^12*x^4) + b*n*(-86021*d^12 - 27720*d^11*e*x^(1/3) + 13860*d^10*e^2*x^(2/3)
 - 9240*d^9*e^3*x + 6930*d^8*e^4*x^(4/3) - 5544*d^7*e^5*x^(5/3) + 4620*d^6*
```

```
e^6*x^2 - 3960*d^5*e^7*x^(7/3) + 3465*d^4*e^8*x^(8/3) - 3080*d^3*e^9*x^3 +
2772*d^2*e^10*x^(10/3) - 2520*d*e^11*x^(11/3) + 2310*e^12*x^4))*Log[c*(d +
e*x^(1/3))^n]^2 - 3550000608000*b^3*(d^12 - e^12*x^4)*Log[c*(d + e*x^(1/3))
^n]^3)/(14200002432000*e^12)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x^3 \left(a + b\ln\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A] time = 1.15272, size = 1436, normalized size = 0.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((e*x^(1/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(1/3) + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a^3*x^4 - 1/36960*a^2
*b*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x
^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) -
 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*
e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1/3))/e
^12) - 1/512265600*(27720*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 + (2310*e
^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 +
 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*
e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*e*x^(2/3)
- 27720*d^11*x^(1/3))/e^12)*log((e*x^(1/3) + d)^n*c) - (5336100*e^12*x^4 -
12171600*d*e^11*x^(11/3) + 21072744*d^2*e^10*x^(10/3) - 32900560*d^3*e^9*x^
3 + 49019355*d^4*e^8*x^(8/3) - 71703720*d^5*e^7*x^(7/3) + 104998740*d^6*e^6
*x^2 + 384199200*d^12*log(e*x^(1/3) + d)^2 - 156734424*d^7*e^5*x^(5/3) + 24
3942930*d^8*e^4*x^(4/3) - 410634840*d^9*e^3*x + 2384502120*d^12*log(e*x^(1/
3) + d) + 808051860*d^10*e^2*x^(2/3) - 2384502120*d^11*e*x^(1/3))*n^2/e^12)
*a*b^2 - 1/14200002432000*(384199200*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^1
3 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^
3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2
- 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10
*e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)*log((e*x^(1/3) + d)^n*c)^2 + e*n*((1
2326391000*e^12*x^4 - 44119404000*d*e^11*x^(11/3) + 106944990768*d^2*e^10*x
^(10/3) - 220161492320*d^3*e^9*x^3 + 3550000608000*d^12*log(e*x^(1/3) + d)^
3 + 417533743935*d^4*e^8*x^(8/3) - 761128152840*d^5*e^7*x^(7/3) + 137307702
3780*d^6*e^6*x^2 + 33049199383200*d^12*log(e*x^(1/3) + d)^2 - 2516628075192
*d^7*e^5*x^(5/3) + 4836309598890*d^8*e^4*x^(4/3) - 10242678720120*d^9*e^3*x
 + 119225632485960*d^12*log(e*x^(1/3) + d) + 26563616859780*d^10*e^2*x^(2/3
) - 119225632485960*d^11*e*x^(1/3))*n^2/e^13 - 27720*(5336100*e^12*x^4 - 12
171600*d*e^11*x^(11/3) + 21072744*d^2*e^10*x^(10/3) - 32900560*d^3*e^9*x^3
+ 49019355*d^4*e^8*x^(8/3) - 71703720*d^5*e^7*x^(7/3) + 104998740*d^6*e^6*x
^2 + 384199200*d^12*log(e*x^(1/3) + d)^2 - 156734424*d^7*e^5*x^(5/3) + 2439
```

```
42930*d^8*e^4*x^(4/3) - 410634840*d^9*e^3*x + 2384502120*d^12*log(e*x^(1/3)
 + d) + 808051860*d^10*e^2*x^(2/3) - 2384502120*d^11*e*x^(1/3))*n*log((e*x^
(1/3) + d)^n*c)/e^13))*b^3
```

Fricas [A] time = 4.33575, size = 5488, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/14200002432000*(3550000608000*b^3*e^12*x^4*log(c)^3 - 12326391000*(b^3*e^
12*n^3 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n - 288*a^3*e^12)*x^4 + 603680*(
364699*b^3*d^3*e^9*n^3 - 1510740*a*b^2*d^3*e^9*n^2 + 1960200*a^2*b*d^3*e^9*
n)*x^3 + 3550000608000*(b^3*e^12*n^3*x^4 - b^3*d^12*n^3)*log(e*x^(1/3) + d)
^3 - 4620*(297202819*b^3*d^6*e^6*n^3 - 629992440*a*b^2*d^6*e^6*n^2 + 384199
200*a^2*b*d^6*e^6*n)*x^2 + 384199200*(3080*b^3*d^3*e^9*n^3*x^3 - 4620*b^3*d
^6*e^6*n^3*x^2 + 9240*b^3*d^9*e^3*n^3*x + 86021*b^3*d^12*n^3 - 27720*a*b^2*
d^12*n^2 - 2310*(b^3*e^12*n^3 - 12*a*b^2*e^12*n^2)*x^4 + 27720*(b^3*e^12*n^
2*x^4 - b^3*d^12*n^2)*log(c) + 63*(40*b^3*d*e^11*n^3*x^3 - 55*b^3*d^4*e^8*n
^3*x^2 + 88*b^3*d^7*e^5*n^3*x - 220*b^3*d^10*e^2*n^3)*x^(2/3) - 198*(14*b^3
*d^2*e^10*n^3*x^3 - 20*b^3*d^5*e^7*n^3*x^2 + 35*b^3*d^8*e^4*n^3*x - 140*b^3
*d^11*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 295833384000*(4*b^3*d^3*e^9*n*
x^3 - 6*b^3*d^6*e^6*n*x^2 + 12*b^3*d^9*e^3*n*x - 3*(b^3*e^12*n - 12*a*b^2*e
^12)*x^4)*log(c)^2 + 9240*(1108515013*b^3*d^9*e^3*n^3 - 1231904520*a*b^2*d^
9*e^3*n^2 + 384199200*a^2*b*d^9*e^3*n)*x - 27720*(4301068993*b^3*d^12*n^3 -
 2384502120*a*b^2*d^12*n^2 + 384199200*a^2*b*d^12*n - 5336100*(b^3*e^12*n^3
 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n)*x^4 + 43120*(763*b^3*d^3*e^9*n^3 -
1980*a*b^2*d^3*e^9*n^2)*x^3 - 4620*(22727*b^3*d^6*e^6*n^3 - 27720*a*b^2*d^6
*e^6*n^2)*x^2 - 384199200*(b^3*e^12*n*x^4 - b^3*d^12*n)*log(c)^2 + 9240*(44
441*b^3*d^9*e^3*n^3 - 27720*a*b^2*d^9*e^3*n^2)*x - 27720*(3080*b^3*d^3*e^9*
n^2*x^3 - 4620*b^3*d^6*e^6*n^2*x^2 + 9240*b^3*d^9*e^3*n^2*x + 86021*b^3*d^1
2*n^2 - 27720*a*b^2*d^12*n - 2310*(b^3*e^12*n^2 - 12*a*b^2*e^12*n)*x^4)*log
(c) - 63*(12826220*b^3*d^10*e^2*n^3 - 6098400*a*b^2*d^10*e^2*n^2 - 8400*(23
*b^3*d*e^11*n^3 - 132*a*b^2*d*e^11*n^2)*x^3 + 385*(2021*b^3*d^4*e^8*n^3 - 3
960*a*b^2*d^4*e^8*n^2)*x^2 - 88*(28271*b^3*d^7*e^5*n^3 - 27720*a*b^2*d^7*e^
5*n^2)*x + 27720*(40*b^3*d*e^11*n^2*x^3 - 55*b^3*d^4*e^8*n^2*x^2 + 88*b^3*d
^7*e^5*n^2*x - 220*b^3*d^10*e^2*n^2)*log(c))*x^(2/3) + 198*(12042940*b^3*d^
11*e*n^3 - 3880800*a*b^2*d^11*e*n^2 - 588*(181*b^3*d^2*e^10*n^3 - 660*a*b^2
*d^2*e^10*n^2)*x^3 + 20*(18107*b^3*d^5*e^7*n^3 - 27720*a*b^2*d^5*e^7*n^2)*x
^2 - 35*(35201*b^3*d^8*e^4*n^3 - 27720*a*b^2*d^8*e^4*n^2)*x + 27720*(14*b^3
*d^2*e^10*n^2*x^3 - 20*b^3*d^5*e^7*n^2*x^2 + 35*b^3*d^8*e^4*n^2*x - 140*b^3
*d^11*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 42688800*(3465*(b^3*e^12
*n^2 - 12*a*b^2*e^12*n + 72*a^2*b*e^12)*x^4 - 28*(763*b^3*d^3*e^9*n^2 - 198
0*a*b^2*d^3*e^9*n)*x^3 + 3*(22727*b^3*d^6*e^6*n^2 - 27720*a*b^2*d^6*e^6*n)*
x^2 - 6*(44441*b^3*d^9*e^3*n^2 - 27720*a*b^2*d^9*e^3*n)*x)*log(c) - 63*(421
644712060*b^3*d^10*e^2*n^3 - 355542818400*a*b^2*d^10*e^2*n^2 + 84523824000*
a^2*b*d^10*e^2*n - 1764000*(397*b^3*d*e^11*n^3 - 3036*a*b^2*d*e^11*n^2 + 87
12*a^2*b*d*e^11*n)*x^3 + 2695*(2459191*b^3*d^4*e^8*n^3 - 8003160*a*b^2*d^4*
e^8*n^2 + 7840800*a^2*b*d^4*e^8*n)*x^2 - 384199200*(40*b^3*d*e^11*n*x^3 - 5
5*b^3*d^4*e^8*n*x^2 + 88*b^3*d^7*e^5*n*x - 220*b^3*d^10*e^2*n)*log(c)^2 - 8
8*(453937243*b^3*d^7*e^5*n^3 - 783672120*a*b^2*d^7*e^5*n^2 + 384199200*a^2*
b*d^7*e^5*n)*x - 27720*(12826220*b^3*d^10*e^2*n^2 - 6098400*a*b^2*d^10*e^2*
n - 8400*(23*b^3*d*e^11*n^2 - 132*a*b^2*d*e^11*n)*x^3 + 385*(2021*b^3*d^4*e
^8*n^2 - 3960*a*b^2*d^4*e^8*n)*x^2 - 88*(28271*b^3*d^7*e^5*n^2 - 27720*a*b^
2*d^7*e^5*n)*x)*log(c))*x^(2/3) + 198*(602149659020*b^3*d^11*e*n^3 - 333830
```

```
296800*a*b^2*d^11*e*n^2 + 53787888000*a^2*b*d^11*e*n - 24696*(21871*b^3*d^2
*e^10*n^3 - 119460*a*b^2*d^2*e^10*n^2 + 217800*a^2*b*d^2*e^10*n)*x^3 + 20*(
192204079*b^3*d^5*e^7*n^3 - 501926040*a*b^2*d^5*e^7*n^2 + 384199200*a^2*b*d
^5*e^7*n)*x^2 - 384199200*(14*b^3*d^2*e^10*n*x^3 - 20*b^3*d^5*e^7*n*x^2 + 3
5*b^3*d^8*e^4*n*x - 140*b^3*d^11*e*n)*log(c)^2 - 35*(697880173*b^3*d^8*e^4*
n^3 - 975771720*a*b^2*d^8*e^4*n^2 + 384199200*a^2*b*d^8*e^4*n)*x - 27720*(1
2042940*b^3*d^11*e*n^2 - 3880800*a*b^2*d^11*e*n - 588*(181*b^3*d^2*e^10*n^2
 - 660*a*b^2*d^2*e^10*n)*x^3 + 20*(18107*b^3*d^5*e^7*n^2 - 27720*a*b^2*d^5*
e^7*n)*x^2 - 35*(35201*b^3*d^8*e^4*n^2 - 27720*a*b^2*d^8*e^4*n)*x)*log(c))*
x^(1/3))/e^12
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.62097, size = 5998, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/14200002432000*(3550000608000*b^3*x^4*e*log(c)^3 + 10650001824000*a*b^2*x
^4*e*log(c)^2 + 10650001824000*a^2*b*x^4*e*log(c) + 3550000608000*a^3*x^4*e
 + (3550000608000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^3 - 4260000
7296000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^3 + 234300040128000
*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^3 - 781000133760000*(x^(
1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^3 + 1757250300960000*(x^(1/3)*
e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^3 - 2811600481536000*(x^(1/3)*e + d
)^7*d^5*e^(-11)*log(x^(1/3)*e + d)^3 + 3280200561792000*(x^(1/3)*e + d)^6*d
^6*e^(-11)*log(x^(1/3)*e + d)^3 - 2811600481536000*(x^(1/3)*e + d)^5*d^7*e^
(-11)*log(x^(1/3)*e + d)^3 + 1757250300960000*(x^(1/3)*e + d)^4*d^8*e^(-11)
*log(x^(1/3)*e + d)^3 - 781000133760000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x
^(1/3)*e + d)^3 + 234300040128000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3
)*e + d)^3 - 42600007296000*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)
^3 - 887500152000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^2 + 1161818
3808000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^2 - 70290012038400*
(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^2 + 260333377920000*(x^(1
/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^2 - 658968862860000*(x^(1/3)*e
+ d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^2 + 1204971634944000*(x^(1/3)*e + d)^
7*d^5*e^(-11)*log(x^(1/3)*e + d)^2 - 1640100280896000*(x^(1/3)*e + d)^6*d^6
*e^(-11)*log(x^(1/3)*e + d)^2 + 1686960288921600*(x^(1/3)*e + d)^5*d^7*e^(-
11)*log(x^(1/3)*e + d)^2 - 1317937725720000*(x^(1/3)*e + d)^4*d^8*e^(-11)*l
og(x^(1/3)*e + d)^2 + 781000133760000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(
1/3)*e + d)^2 - 351450060192000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*
e + d)^2 + 127800021888000*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)^
2 + 147916692000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) - 2112397056
```

000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) + 14058002407680*(x^(1/
3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 57851861760000*(x^(1/3)*e + d
)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 164742215715000*(x^(1/3)*e + d)^8*d^4*
e^(-11)*log(x^(1/3)*e + d) - 344277609984000*(x^(1/3)*e + d)^7*d^5*e^(-11)*
log(x^(1/3)*e + d) + 546700093632000*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1
/3)*e + d) - 674784115568640*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e +
d) + 658968862860000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 520
666755840000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 35145006019
2000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 255600043776000*(x
^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) - 12326391000*(x^(1/3)*e + d)
^12*e^(-11) + 192036096000*(x^(1/3)*e + d)^11*d*e^(-11) - 1405800240768*(x^
(1/3)*e + d)^10*d^2*e^(-11) + 6427984640000*(x^(1/3)*e + d)^9*d^3*e^(-11) -
 20592776964375*(x^(1/3)*e + d)^8*d^4*e^(-11) + 49182515712000*(x^(1/3)*e +
 d)^7*d^5*e^(-11) - 91116682272000*(x^(1/3)*e + d)^6*d^6*e^(-11) + 13495682
3113728*(x^(1/3)*e + d)^5*d^7*e^(-11) - 164742215715000*(x^(1/3)*e + d)^4*d
^8*e^(-11) + 173555585280000*(x^(1/3)*e + d)^3*d^9*e^(-11) - 17572503009600
0*(x^(1/3)*e + d)^2*d^10*e^(-11) + 255600043776000*(x^(1/3)*e + d)*d^11*e^(
-11))*b^3*n^3 + 27720*(384199200*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e +
 d)^2 - 4610390400*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^2 + 2535
7147200*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^2 - 84523824000*(
x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^2 + 190178604000*(x^(1/3)*e
 + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^2 - 304285766400*(x^(1/3)*e + d)^7*d
^5*e^(-11)*log(x^(1/3)*e + d)^2 + 355000060800*(x^(1/3)*e + d)^6*d^6*e^(-11
)*log(x^(1/3)*e + d)^2 - 304285766400*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(
1/3)*e + d)^2 + 190178604000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e +
d)^2 - 84523824000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d)^2 + 253
57147200*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d)^2 - 4610390400*(
x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)^2 - 64033200*(x^(1/3)*e + d)
^12*e^(-11)*log(x^(1/3)*e + d) + 838252800*(x^(1/3)*e + d)^11*d*e^(-11)*log
(x^(1/3)*e + d) - 5071429440*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e +
 d) + 18783072000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) - 475446
51000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) + 86938790400*(x^(1/
3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) - 118333353600*(x^(1/3)*e + d)^6
*d^6*e^(-11)*log(x^(1/3)*e + d) + 121714306560*(x^(1/3)*e + d)^5*d^7*e^(-11
)*log(x^(1/3)*e + d) - 95089302000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3
)*e + d) + 56349216000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) - 2
5357147200*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) + 9220780800*(
x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) + 5336100*(x^(1/3)*e + d)^12
e^(-11) - 76204800(x^(1/3)*e + d)^11*d*e^(-11) + 507142944*(x^(1/3)*e + d
)^10*d^2*e^(-11) - 2087008000*(x^(1/3)*e + d)^9*d^3*e^(-11) + 5943081375*(x
^(1/3)*e + d)^8*d^4*e^(-11) - 12419827200*(x^(1/3)*e + d)^7*d^5*e^(-11) + 1
9722225600*(x^(1/3)*e + d)^6*d^6*e^(-11) - 24342861312*(x^(1/3)*e + d)^5*d^
7*e^(-11) + 23772325500*(x^(1/3)*e + d)^4*d^8*e^(-11) - 18783072000*(x^(1/3
)*e + d)^3*d^9*e^(-11) + 12678573600*(x^(1/3)*e + d)^2*d^10*e^(-11) - 92207
80800*(x^(1/3)*e + d)*d^11*e^(-11))*b^3*n^2*log(c) + 384199200*(27720*(x^(1
/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)^11*d*e^(-
11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)
e + d) - 6098400(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 137214
00*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*e +
 d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) + 25613280*(x^(1/3)*e + d)^6*d^6*e^(-1
1)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*
e + d) + 13721400*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 609840
0*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*e + d
)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)*d^11*e^(-11)*l
og(x^(1/3)*e + d) - 2310*(x^(1/3)*e + d)^12*e^(-11) + 30240*(x^(1/3)*e + d)
^11*d*e^(-11) - 182952*(x^(1/3)*e + d)^10*d^2*e^(-11) + 677600*(x^(1/3)*e +
 d)^9*d^3*e^(-11) - 1715175*(x^(1/3)*e + d)^8*d^4*e^(-11) + 3136320*(x^(1/3
)*e + d)^7*d^5*e^(-11) - 4268880*(x^(1/3)*e + d)^6*d^6*e^(-11) + 4390848*(x
^(1/3)*e + d)^5*d^7*e^(-11) - 3430350*(x^(1/3)*e + d)^4*d^8*e^(-11) + 20328

```
00*(x^(1/3)*e + d)^3*d^9*e^(-11) - 914760*(x^(1/3)*e + d)^2*d^10*e^(-11) +
332640*(x^(1/3)*e + d)*d^11*e^(-11))*b^3*n*log(c)^2 + 27720*(384199200*(x^(
1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^2 - 4610390400*(x^(1/3)*e + d)^11
*d*e^(-11)*log(x^(1/3)*e + d)^2 + 25357147200*(x^(1/3)*e + d)^10*d^2*e^(-11
)*log(x^(1/3)*e + d)^2 - 84523824000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1
/3)*e + d)^2 + 190178604000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d
)^2 - 304285766400*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d)^2 + 355
000060800*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d)^2 - 304285766400
*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d)^2 + 190178604000*(x^(1/3)
*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d)^2 - 84523824000*(x^(1/3)*e + d)^3*
d^9*e^(-11)*log(x^(1/3)*e + d)^2 + 25357147200*(x^(1/3)*e + d)^2*d^10*e^(-1
1)*log(x^(1/3)*e + d)^2 - 4610390400*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/
3)*e + d)^2 - 64033200*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) + 8382
52800*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) - 5071429440*(x^(1/3)
*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) + 18783072000*(x^(1/3)*e + d)^9*d
^3*e^(-11)*log(x^(1/3)*e + d) - 47544651000*(x^(1/3)*e + d)^8*d^4*e^(-11)*l
og(x^(1/3)*e + d) + 86938790400*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e
 + d) - 118333353600*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) + 121
714306560*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) - 95089302000*(x
^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) + 56349216000*(x^(1/3)*e + d
)^3*d^9*e^(-11)*log(x^(1/3)*e + d) - 25357147200*(x^(1/3)*e + d)^2*d^10*e^(
-11)*log(x^(1/3)*e + d) + 9220780800*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/
3)*e + d) + 5336100*(x^(1/3)*e + d)^12*e^(-11) - 76204800*(x^(1/3)*e + d)^1
1*d*e^(-11) + 507142944*(x^(1/3)*e + d)^10*d^2*e^(-11) - 2087008000*(x^(1/3
)*e + d)^9*d^3*e^(-11) + 5943081375*(x^(1/3)*e + d)^8*d^4*e^(-11) - 1241982
7200*(x^(1/3)*e + d)^7*d^5*e^(-11) + 19722225600*(x^(1/3)*e + d)^6*d^6*e^(-
11) - 24342861312*(x^(1/3)*e + d)^5*d^7*e^(-11) + 23772325500*(x^(1/3)*e +
d)^4*d^8*e^(-11) - 18783072000*(x^(1/3)*e + d)^3*d^9*e^(-11) + 12678573600*
(x^(1/3)*e + d)^2*d^10*e^(-11) - 9220780800*(x^(1/3)*e + d)*d^11*e^(-11))*a
*b^2*n^2 + 768398400*(27720*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) -
 332640*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*
e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^9*d^3*e^
(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/
3)*e + d) - 21954240*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) + 256
13280*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*
e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^4*d^8*e^
(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3
)*e + d) + 1829520*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 3326
40*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) - 2310*(x^(1/3)*e + d)^1
2*e^(-11) + 30240*(x^(1/3)*e + d)^11*d*e^(-11) - 182952*(x^(1/3)*e + d)^10*
d^2*e^(-11) + 677600*(x^(1/3)*e + d)^9*d^3*e^(-11) - 1715175*(x^(1/3)*e + d
)^8*d^4*e^(-11) + 3136320*(x^(1/3)*e + d)^7*d^5*e^(-11) - 4268880*(x^(1/3)*
e + d)^6*d^6*e^(-11) + 4390848*(x^(1/3)*e + d)^5*d^7*e^(-11) - 3430350*(x^(
1/3)*e + d)^4*d^8*e^(-11) + 2032800*(x^(1/3)*e + d)^3*d^9*e^(-11) - 914760*
(x^(1/3)*e + d)^2*d^10*e^(-11) + 332640*(x^(1/3)*e + d)*d^11*e^(-11))*a*b^2
*n*log(c) + 384199200*(27720*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)
- 332640*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)
*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^9*d^3*e
^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1
/3)*e + d) - 21954240*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) + 25
613280*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)
*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^4*d^8*e
^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/
3)*e + d) + 1829520*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 332
640*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) - 2310*(x^(1/3)*e + d)^
12*e^(-11) + 30240*(x^(1/3)*e + d)^11*d*e^(-11) - 182952*(x^(1/3)*e + d)^10
*d^2*e^(-11) + 677600*(x^(1/3)*e + d)^9*d^3*e^(-11) - 1715175*(x^(1/3)*e +
d)^8*d^4*e^(-11) + 3136320*(x^(1/3)*e + d)^7*d^5*e^(-11) - 4268880*(x^(1/3)
*e + d)^6*d^6*e^(-11) + 4390848*(x^(1/3)*e + d)^5*d^7*e^(-11) - 3430350*(x^
```

```
(1/3)*e + d)^4*d^8*e^(-11) + 2032800*(x^(1/3)*e + d)^3*d^9*e^(-11) - 914760
*(x^(1/3)*e + d)^2*d^10*e^(-11) + 332640*(x^(1/3)*e + d)*d^11*e^(-11))*a^2*
b*n)*e^(-1)
```

3.457 $\int x^2 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right)^3 \, dx$

Optimal. Leaf size=1357

result too large to display

```
[Out] (9*b^3*d^7*n^3*(d + e*x^(1/3))^2)/e^9 - (56*b^3*d^6*n^3*(d + e*x^(1/3))^3)/
(9*e^9) + (63*b^3*d^5*n^3*(d + e*x^(1/3))^4)/(16*e^9) - (252*b^3*d^4*n^3*(d
 + e*x^(1/3))^5)/(125*e^9) + (7*b^3*d^3*n^3*(d + e*x^(1/3))^6)/(9*e^9) - (7
2*b^3*d^2*n^3*(d + e*x^(1/3))^7)/(343*e^9) + (9*b^3*d*n^3*(d + e*x^(1/3))^8
)/(256*e^9) - (2*b^3*n^3*(d + e*x^(1/3))^9)/(729*e^9) + (18*a*b^2*d^8*n^2*x
^(1/3))/e^8 - (18*b^3*d^8*n^3*x^(1/3))/e^8 + (18*b^3*d^8*n^2*(d + e*x^(1/3)
)*Log[c*(d + e*x^(1/3))^n])/e^9 - (18*b^2*d^7*n^2*(d + e*x^(1/3))^2*(a + b*
Log[c*(d + e*x^(1/3))^n]))/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(3*e^9) - (63*b^2*d^5*n^2*(d + e*x^(1/3))^4*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(4*e^9) + (252*b^2*d^4*n^2*(d + e*x^(1/3))^5*(
a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (72*b^2*d^2*n^2*(d + e*x^(1/
3))^7*(a + b*Log[c*(d + e*x^(1/3))^n]))/(49*e^9) - (9*b^2*d*n^2*(d + e*x^(1
/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32*e^9) + (2*b^2*n^2*(d + e*x^(1/
3))^9*(a + b*Log[c*(d + e*x^(1/3))^n]))/(81*e^9) - (9*b*d^8*n*(d + e*x^(1/3
))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (18*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (28*b*d^6*n*(d + e*x^(1/3))^3*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (63*b*d^5*n*(d + e*x^(1/3))^4*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^9) - (126*b*d^4*n*(d + e*x^(1/3))^5*(a +
 b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^9) + (14*b*d^3*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (36*b*d^2*n*(d + e*x^(1/3))^7*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^9) + (9*b*d*n*(d + e*x^(1/3))^8*(a + b*L
og[c*(d + e*x^(1/3))^n])^2)/(8*e^9) - (b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(
d + e*x^(1/3))^n])^2)/(9*e^9) + (3*d^8*(d + e*x^(1/3))*(a + b*Log[c*(d + e*
x^(1/3))^n])^3)/e^9 - (12*d^7*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3)
)^n])^3)/e^9 + (28*d^6*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3
)/e^9 - (42*d^5*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 +
 (42*d^4*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (28*d^
3*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + (12*d^2*(d +
e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (3*d*(d + e*x^(1/3))
^8*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + ((d + e*x^(1/3))^9*(a + b*Log[
c*(d + e*x^(1/3))^n])^3)/(3*e^9)
```

Rubi [A] time = 1.5733, antiderivative size = 1357, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (9*b^3*d^7*n^3*(d + e*x^(1/3))^2)/e^9 - (56*b^3*d^6*n^3*(d + e*x^(1/3))^3)/
(9*e^9) + (63*b^3*d^5*n^3*(d + e*x^(1/3))^4)/(16*e^9) - (252*b^3*d^4*n^3*(d
 + e*x^(1/3))^5)/(125*e^9) + (7*b^3*d^3*n^3*(d + e*x^(1/3))^6)/(9*e^9) - (7
2*b^3*d^2*n^3*(d + e*x^(1/3))^7)/(343*e^9) + (9*b^3*d*n^3*(d + e*x^(1/3))^8
)/(256*e^9) - (2*b^3*n^3*(d + e*x^(1/3))^9)/(729*e^9) + (18*a*b^2*d^8*n^2*x
^(1/3))/e^8 - (18*b^3*d^8*n^3*x^(1/3))/e^8 + (18*b^3*d^8*n^2*(d + e*x^(1/3)
```

```
)*Log[c*(d + e*x^(1/3))^n])/e^9 - (18*b^2*d^7*n^2*(d + e*x^(1/3))^2*(a + b*
Log[c*(d + e*x^(1/3))^n]))/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(3*e^9) - (63*b^2*d^5*n^2*(d + e*x^(1/3))^4*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(4*e^9) + (252*b^2*d^4*n^2*(d + e*x^(1/3))^5*(
a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (72*b^2*d^2*n^2*(d + e*x^(1/
3))^7*(a + b*Log[c*(d + e*x^(1/3))^n]))/(49*e^9) - (9*b^2*d*n^2*(d + e*x^(1
/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32*e^9) + (2*b^2*n^2*(d + e*x^(1/
3))^9*(a + b*Log[c*(d + e*x^(1/3))^n]))/(81*e^9) - (9*b*d^8*n*(d + e*x^(1/3
))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (18*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (28*b*d^6*n*(d + e*x^(1/3))^3*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (63*b*d^5*n*(d + e*x^(1/3))^4*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^9) - (126*b*d^4*n*(d + e*x^(1/3))^5*(a +
 b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^9) + (14*b*d^3*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (36*b*d^2*n*(d + e*x^(1/3))^7*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^9) + (9*b*d*n*(d + e*x^(1/3))^8*(a + b*L
og[c*(d + e*x^(1/3))^n])^2)/(8*e^9) - (b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(
d + e*x^(1/3))^n])^2)/(9*e^9) + (3*d^8*(d + e*x^(1/3))*(a + b*Log[c*(d + e*
x^(1/3))^n])^3)/e^9 - (12*d^7*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3)
)^n])^3)/e^9 + (28*d^6*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3
)/e^9 - (42*d^5*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 +
 (42*d^4*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (28*d^
3*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + (12*d^2*(d +
e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (3*d*(d + e*x^(1/3))
^8*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + ((d + e*x^(1/3))^9*(a + b*Log[
c*(d + e*x^(1/3))^n])^3)/(3*e^9)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^8}-\frac{8d^7(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^8}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^8\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)}{e^8}-\frac{(24d)\,\text{Subst}\left(\int(d+\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^8\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9}-\frac{(24d)\,\text{Subst}\left(\int x^7\left(a+b\log\right.\right.}{e^9}$$

$$= \frac{3d^8\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9}-\frac{12d^7\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{e^9}$$

$$= -\frac{9bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^9}+\frac{18bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\right.}{e^9}$$

$$= \frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9}-\frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9}+\frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^4}{16e^9}-\frac{252b^3d}{}$$

$$= \frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9}-\frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9}+\frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^4}{16e^9}-\frac{252b^3d}{}$$

Mathematica [A] time = 0.771382, size = 808, normalized size = 0.6

$$\frac{2667168000\left(d^9+e^9x^3\right)a^3-3175200bn\left(7129d^9+2520e\sqrt[3]{x}d^8-1260e^2x^{2/3}d^7+840e^3xd^6-630e^4x^{4/3}d^5+504e^5x^{5/3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*x^(1/3)*(-76356985320*d^8 + 15542491860*d^7*e*x^(1/3) - 54834956
40*d^6*e^2*x^(2/3) + 2340330930*d^5*e^3*x - 1075607064*d^4*e^4*x^(4/3) + 49
8592500*d^3*e^5*x^(5/3) - 219465000*d^2*e^6*x^2 + 83734875*d*e^7*x^(7/3) -
```

```
21952000*e^8*x^(8/3)) - 2520*a*b^2*n^2*(26853209*d^9 - 17965080*d^8*e*x^(1/
3) + 5807340*d^7*e^2*x^(2/3) - 2813160*d^6*e^3*x + 1580670*d^5*e^4*x^(4/3)
- 947016*d^4*e^5*x^(5/3) + 577500*d^3*e^6*x^2 - 343800*d^2*e^7*x^(7/3) + 18
7425*d*e^8*x^(8/3) - 78400*e^9*x^3) + 2667168000*a^3*(d^9 + e^9*x^3) - 3175
200*a^2*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 840*d^6
*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2 + 360*
d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3) + 2520*b*(3175200*a^2*(d
^9 + e^9*x^3) - 2520*a*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^
(2/3) + 840*d^6*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3
*e^6*x^2 + 360*d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3) + b^2*n^2
*(30300391*d^9 + 17965080*d^8*e*x^(1/3) - 5807340*d^7*e^2*x^(2/3) + 2813160
*d^6*e^3*x - 1580670*d^5*e^4*x^(4/3) + 947016*d^4*e^5*x^(5/3) - 577500*d^3*
e^6*x^2 + 343800*d^2*e^7*x^(7/3) - 187425*d*e^8*x^(8/3) + 78400*e^9*x^3))*L
og[c*(d + e*x^(1/3))^n] + 3175200*b^2*(2520*a*(d^9 + e^9*x^3) - b*n*(7129*d
^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 840*d^6*e^3*x - 630*d^5*e^
4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2 + 360*d^2*e^7*x^(7/3) - 3
15*d*e^8*x^(8/3) + 280*e^9*x^3))*Log[c*(d + e*x^(1/3))^n]^2 + 2667168000*b^
3*(d^9 + e^9*x^3)*Log[c*(d + e*x^(1/3))^n]^3)/(8001504000*e^9)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int x^2 \left(a + b\ln\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A] time = 1.12994, size = 1170, normalized size = 0.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log((e*x^(1/3) + d)^n*c)^3 + a*b^2*x^3*log((e*x^(1/3) + d)^n*c)
^2 + a^2*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a^3*x^3 + 1/2520*a^2*b*e*n*(2
520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^
2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3)
 + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9) + 1/3175200*
(2520*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8
/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5
*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*
log((e*x^(1/3) + d)^n*c) + (78400*e^9*x^3 - 187425*d*e^8*x^(8/3) + 343800*d
^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3175200*d^9*log(e*x^(1/3) + d)^2 + 94
7016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*x^(4/3) + 2813160*d^6*e^3*x - 179650
80*d^9*log(e*x^(1/3) + d) - 5807340*d^7*e^2*x^(2/3) + 17965080*d^8*e*x^(1/3
))*n^2/e^9)*a*b^2 + 1/8001504000*(3175200*e*n*(2520*d^9*log(e*x^(1/3) + d)/
e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5
*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7
*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c)^2 - e*n*((2195
2000*e^9*x^3 - 2667168000*d^9*log(e*x^(1/3) + d)^3 - 83734875*d*e^8*x^(8/3)
```

```
 + 219465000*d^2*e^7*x^(7/3) - 498592500*d^3*e^6*x^2 - 22636000800*d^9*log(
e*x^(1/3) + d)^2 + 1075607064*d^4*e^5*x^(5/3) - 2340330930*d^5*e^4*x^(4/3)
+ 5483495640*d^6*e^3*x - 76356985320*d^9*log(e*x^(1/3) + d) - 15542491860*d
^7*e^2*x^(2/3) + 76356985320*d^8*e*x^(1/3))*n^2/e^10 - 2520*(78400*e^9*x^3
- 187425*d*e^8*x^(8/3) + 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3175
200*d^9*log(e*x^(1/3) + d)^2 + 947016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*x^(
4/3) + 2813160*d^6*e^3*x - 17965080*d^9*log(e*x^(1/3) + d) - 5807340*d^7*e^
2*x^(2/3) + 17965080*d^8*e*x^(1/3))*n*log((e*x^(1/3) + d)^n*c)/e^10))*b^3
```

Fricas [A] time = 3.95246, size = 4018, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/8001504000*(2667168000*b^3*e^9*x^3*log(c)^3 - 10976000*(2*b^3*e^9*n^3 - 1
8*a*b^2*e^9*n^2 + 81*a^2*b*e^9*n - 243*a^3*e^9)*x^3 + 2667168000*(b^3*e^9*n
^3*x^3 + b^3*d^9*n^3)*log(e*x^(1/3) + d)^3 + 10500*(47485*b^3*d^3*e^6*n^3 -
 138600*a*b^2*d^3*e^6*n^2 + 127008*a^2*b*d^3*e^6*n)*x^2 + 3175200*(420*b^3*
d^3*e^6*n^3*x^2 - 840*b^3*d^6*e^3*n^3*x - 7129*b^3*d^9*n^3 + 2520*a*b^2*d^9
*n^2 - 280*(b^3*e^9*n^3 - 9*a*b^2*e^9*n^2)*x^3 + 2520*(b^3*e^9*n^2*x^3 + b^
3*d^9*n^2)*log(c) + 63*(5*b^3*d*e^8*n^3*x^2 - 8*b^3*d^4*e^5*n^3*x + 20*b^3*
d^7*e^2*n^3)*x^(2/3) - 90*(4*b^3*d^2*e^7*n^3*x^2 - 7*b^3*d^5*e^4*n^3*x + 28
*b^3*d^8*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 444528000*(3*b^3*d^3*e^6*n*
x^2 - 6*b^3*d^6*e^3*n*x - 2*(b^3*e^9*n - 9*a*b^2*e^9)*x^3)*log(c)^2 - 840*(
6527971*b^3*d^6*e^3*n^3 - 8439480*a*b^2*d^6*e^3*n^2 + 3175200*a^2*b*d^6*e^3
*n)*x + 2520*(30300391*b^3*d^9*n^3 - 17965080*a*b^2*d^9*n^2 + 3175200*a^2*b
*d^9*n + 39200*(2*b^3*e^9*n^3 - 18*a*b^2*e^9*n^2 + 81*a^2*b*e^9*n)*x^3 - 21
00*(275*b^3*d^3*e^6*n^3 - 504*a*b^2*d^3*e^6*n^2)*x^2 + 3175200*(b^3*e^9*n*x
^3 + b^3*d^9*n)*log(c)^2 + 840*(3349*b^3*d^6*e^3*n^3 - 2520*a*b^2*d^6*e^3*n
^2)*x + 2520*(420*b^3*d^3*e^6*n^2*x^2 - 840*b^3*d^6*e^3*n^2*x - 7129*b^3*d^
9*n^2 + 2520*a*b^2*d^9*n - 280*(b^3*e^9*n^2 - 9*a*b^2*e^9*n)*x^3)*log(c) -
63*(92180*b^3*d^7*e^2*n^3 - 50400*a*b^2*d^7*e^2*n^2 + 175*(17*b^3*d*e^8*n^3
 - 72*a*b^2*d*e^8*n^2)*x^2 - 8*(1879*b^3*d^4*e^5*n^3 - 2520*a*b^2*d^4*e^5*n
^2)*x - 2520*(5*b^3*d*e^8*n^2*x^2 - 8*b^3*d^4*e^5*n^2*x + 20*b^3*d^7*e^2*n^
2)*log(c))*x^(2/3) + 90*(199612*b^3*d^8*e*n^3 - 70560*a*b^2*d^8*e*n^2 + 20*
(191*b^3*d^2*e^7*n^3 - 504*a*b^2*d^2*e^7*n^2)*x^2 - 7*(2509*b^3*d^5*e^4*n^3
 - 2520*a*b^2*d^5*e^4*n^2)*x - 2520*(4*b^3*d^2*e^7*n^2*x^2 - 7*b^3*d^5*e^4*
n^2*x + 28*b^3*d^8*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 352800*(280
*(2*b^3*e^9*n^2 - 18*a*b^2*e^9*n + 81*a^2*b*e^9)*x^3 - 15*(275*b^3*d^3*e^6*
n^2 - 504*a*b^2*d^3*e^6*n)*x^2 + 6*(3349*b^3*d^6*e^3*n^2 - 2520*a*b^2*d^6*e
^3*n)*x)*log(c) + 63*(246706220*b^3*d^7*e^2*n^3 - 232293600*a*b^2*d^7*e^2*n
^2 + 63504000*a^2*b*d^7*e^2*n + 6125*(217*b^3*d*e^8*n^3 - 1224*a*b^2*d*e^8*
n^2 + 2592*a^2*b*d*e^8*n)*x^2 + 3175200*(5*b^3*d*e^8*n*x^2 - 8*b^3*d^4*e^5*
n*x + 20*b^3*d^7*e^2*n)*log(c)^2 - 8*(2134141*b^3*d^4*e^5*n^3 - 4735080*a*b
^2*d^4*e^5*n^2 + 3175200*a^2*b*d^4*e^5*n)*x - 2520*(92180*b^3*d^7*e^2*n^2 -
 50400*a*b^2*d^7*e^2*n + 175*(17*b^3*d*e^8*n^2 - 72*a*b^2*d*e^8*n)*x^2 - 8*
(1879*b^3*d^4*e^5*n^2 - 2520*a*b^2*d^4*e^5*n)*x)*log(c))*x^(2/3) - 90*(8484
10948*b^3*d^8*e*n^3 - 503022240*a*b^2*d^8*e*n^2 + 88905600*a^2*b*d^8*e*n +
100*(24385*b^3*d^2*e^7*n^3 - 96264*a*b^2*d^2*e^7*n^2 + 127008*a^2*b*d^2*e^7
*n)*x^2 + 3175200*(4*b^3*d^2*e^7*n*x^2 - 7*b^3*d^5*e^4*n*x + 28*b^3*d^8*e*n
)*log(c)^2 - 7*(3714811*b^3*d^5*e^4*n^3 - 6322680*a*b^2*d^5*e^4*n^2 + 31752
00*a^2*b*d^5*e^4*n)*x - 2520*(199612*b^3*d^8*e*n^2 - 70560*a*b^2*d^8*e*n +
20*(191*b^3*d^2*e^7*n^2 - 504*a*b^2*d^2*e^7*n)*x^2 - 7*(2509*b^3*d^5*e^4*n^
2 - 2520*a*b^2*d^5*e^4*n)*x)*log(c))*x^(1/3))/e^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47896, size = 4500, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/8001504000*(2667168000*b^3*x^3*e*log(c)^3 + 8001504000*a*b^2*x^3*e*log(c)
^2 + 8001504000*a^2*b*x^3*e*log(c) + (2667168000*(x^(1/3)*e + d)^9*e^(-8)*l
og(x^(1/3)*e + d)^3 - 24004512000*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e
+ d)^3 + 96018048000*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^3 - 22
4042112000*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d)^3 + 336063168000
*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^3 - 336063168000*(x^(1/3)*
e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^3 + 224042112000*(x^(1/3)*e + d)^3*d
^6*e^(-8)*log(x^(1/3)*e + d)^3 - 96018048000*(x^(1/3)*e + d)^2*d^7*e^(-8)*l
og(x^(1/3)*e + d)^3 + 24004512000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e
+ d)^3 - 889056000*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 + 90016920
00*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 - 41150592000*(x^(1/3)*e
 + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 + 112021056000*(x^(1/3)*e + d)^6*d^
3*e^(-8)*log(x^(1/3)*e + d)^2 - 201637900800*(x^(1/3)*e + d)^5*d^4*e^(-8)*l
og(x^(1/3)*e + d)^2 + 252047376000*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)
*e + d)^2 - 224042112000*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2
+ 144027072000*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 - 72013536
000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d)^2 + 197568000*(x^(1/3)*e
+ d)^9*e^(-8)*log(x^(1/3)*e + d) - 2250423000*(x^(1/3)*e + d)^8*d*e^(-8)*lo
g(x^(1/3)*e + d) + 11757312000*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e +
 d) - 37340352000*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 8065516
0320*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 126023688000*(x^(1/3
)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 149361408000*(x^(1/3)*e + d)^3*d
^6*e^(-8)*log(x^(1/3)*e + d) - 144027072000*(x^(1/3)*e + d)^2*d^7*e^(-8)*lo
g(x^(1/3)*e + d) + 144027072000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e +
d) - 21952000*(x^(1/3)*e + d)^9*e^(-8) + 281302875*(x^(1/3)*e + d)^8*d*e^(-
8) - 1679616000*(x^(1/3)*e + d)^7*d^2*e^(-8) + 6223392000*(x^(1/3)*e + d)^6
*d^3*e^(-8) - 16131032064*(x^(1/3)*e + d)^5*d^4*e^(-8) + 31505922000*(x^(1/
3)*e + d)^4*d^5*e^(-8) - 49787136000*(x^(1/3)*e + d)^3*d^6*e^(-8) + 7201353
6000*(x^(1/3)*e + d)^2*d^7*e^(-8) - 144027072000*(x^(1/3)*e + d)*d^8*e^(-8)
)*b^3*n^3 + 2667168000*a^3*x^3*e + 2520*(3175200*(x^(1/3)*e + d)^9*e^(-8)*l
og(x^(1/3)*e + d)^2 - 28576800*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d
)^2 + 114307200*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 - 2667168
00*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d)^2 + 400075200*(x^(1/3)*e
 + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^2 - 400075200*(x^(1/3)*e + d)^4*d^5*e
^(-8)*log(x^(1/3)*e + d)^2 + 266716800*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(
1/3)*e + d)^2 - 114307200*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2
 + 28576800*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d)^2 - 705600*(x^(1/
```

3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) + 7144200*(x^(1/3)*e + d)^8*d*e^(-8)*
log(x^(1/3)*e + d) - 32659200*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e +
d) + 88905600*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) - 160030080*(
x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) + 200037600*(x^(1/3)*e + d)^
4*d^5*e^(-8)*log(x^(1/3)*e + d) - 177811200*(x^(1/3)*e + d)^3*d^6*e^(-8)*lo
g(x^(1/3)*e + d) + 114307200*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d
) - 57153600*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) + 78400*(x^(1/3)
*e + d)^9*e^(-8) - 893025*(x^(1/3)*e + d)^8*d*e^(-8) + 4665600*(x^(1/3)*e +
d)^7*d^2*e^(-8) - 14817600*(x^(1/3)*e + d)^6*d^3*e^(-8) + 32006016*(x^(1/3
)*e + d)^5*d^4*e^(-8) - 50009400*(x^(1/3)*e + d)^4*d^5*e^(-8) + 59270400*(x
^(1/3)*e + d)^3*d^6*e^(-8) - 57153600*(x^(1/3)*e + d)^2*d^7*e^(-8) + 571536
00*(x^(1/3)*e + d)*d^8*e^(-8))*b^3*n^2*log(c) + 3175200*(2520*(x^(1/3)*e +
d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/
3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*
(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*
d^4*e^(-8)*log(x^(1/3)*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(
1/3)*e + d) + 211680*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 9072
0*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d
^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)
*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e
+ d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e +
d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e +
d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d)*d^8*e^(-8))*b^3*n*log(c)^2 + 2520*(
3175200*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 - 28576800*(x^(1/3)*e
+ d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 + 114307200*(x^(1/3)*e + d)^7*d^2*e^(
-8)*log(x^(1/3)*e + d)^2 - 266716800*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/
3)*e + d)^2 + 400075200*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^2 -
400075200*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^2 + 266716800*(x
^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2 - 114307200*(x^(1/3)*e + d)
^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 + 28576800*(x^(1/3)*e + d)*d^8*e^(-8)*lo
g(x^(1/3)*e + d)^2 - 705600*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) + 7
144200*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) - 32659200*(x^(1/3)*e
+ d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) + 88905600*(x^(1/3)*e + d)^6*d^3*e^(-8
)*log(x^(1/3)*e + d) - 160030080*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e
+ d) + 200037600*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) - 1778112
00*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) + 114307200*(x^(1/3)*e +
d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) - 57153600*(x^(1/3)*e + d)*d^8*e^(-8)*l
og(x^(1/3)*e + d) + 78400*(x^(1/3)*e + d)^9*e^(-8) - 893025*(x^(1/3)*e + d)
^8*d*e^(-8) + 4665600*(x^(1/3)*e + d)^7*d^2*e^(-8) - 14817600*(x^(1/3)*e +
d)^6*d^3*e^(-8) + 32006016*(x^(1/3)*e + d)^5*d^4*e^(-8) - 50009400*(x^(1/3)
*e + d)^4*d^5*e^(-8) + 59270400*(x^(1/3)*e + d)^3*d^6*e^(-8) - 57153600*(x^
(1/3)*e + d)^2*d^7*e^(-8) + 57153600*(x^(1/3)*e + d)*d^8*e^(-8))*a*b^2*n^2
+ 6350400*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680*(x^(1/3
)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)
*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d
) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 317520*(x^(1/3
)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(x^(1/3)*e + d)^3*d^6*e^(
-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e +
d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^(1/3)*e
+ d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e + d)^7*d
^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e + d)^5*d^
4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e + d)^3*d^6
e^(-8) + 45360(x^(1/3)*e + d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d)*d^8*e^
(-8))*a*b^2*n*log(c) + 3175200*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e
+ d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(1/3
)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*e^(
-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e
+ d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(x^(
1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^7*e

```
^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e +
 d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 1296
0*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63504
*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 70560*
(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 22680*(
x^(1/3)*e + d)*d^8*e^(-8))*a^2*b*n)*e^(-1)
```

3.458 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=907

result too large to display

```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(1/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*x^(1/3
))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(1/3))^4)/(64*e^6) + (18*b^3*d*n^3
*(d + e*x^(1/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(1/3))^6)/(72*e^6) - (18*
a*b^2*d^5*n^2*x^(1/3))/e^5 + (18*b^3*d^5*n^3*x^(1/3))/e^5 - (18*b^3*d^5*n^2
*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(
1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) - (20*b^2*d^3*n^2*(d + e*
x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
 e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(16*e^6) - (18*b^2*d*n^2*(d
 + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^6) + (b^2*n^2*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(12*e^6) + (9*b*d^5*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^4*n*(d + e*x^(1
/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) + (10*b*d^3*n*(d + e*x^(
1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(1/3
))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8*e^6) + (9*b*d*n*(d + e*x^(1/3))
^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^6) - (b*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) - (3*d^5*(d + e*x^(1/3))*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^6 + (15*d^4*(d + e*x^(1/3))^2*(a + b*Log[c*(d
+ e*x^(1/3))^n])^3)/(2*e^6) - (10*d^3*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e
*x^(1/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3
))^n])^3)/(2*e^6) - (3*d*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])
^3)/e^6 + ((d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^6)
```

Rubi [A] time = 0.979217, antiderivative size = 907, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^6}{2e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^6}{4e^6}+\frac{b^2n^2\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```


```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(1/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*x^(1/3
))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(1/3))^4)/(64*e^6) + (18*b^3*d*n^3
*(d + e*x^(1/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(1/3))^6)/(72*e^6) - (18*
a*b^2*d^5*n^2*x^(1/3))/e^5 + (18*b^3*d^5*n^3*x^(1/3))/e^5 - (18*b^3*d^5*n^2
*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(
1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) - (20*b^2*d^3*n^2*(d + e*
x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
 e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(16*e^6) - (18*b^2*d*n^2*(d
 + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^6) + (b^2*n^2*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(12*e^6) + (9*b*d^5*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^4*n*(d + e*x^(1
/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) + (10*b*d^3*n*(d + e*x^(
1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(1/3
))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8*e^6) + (9*b*d*n*(d + e*x^(1/3))
^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^6) - (b*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) - (3*d^5*(d + e*x^(1/3))*(a + b*Lo
```

```
g[c*(d + e*x^(1/3))^n])^3)/e^6 + (15*d^4*(d + e*x^(1/3))^2*(a + b*Log[c*(d
+ e*x^(1/3))^n])^3)/(2*e^6) - (10*d^3*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e
*x^(1/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3
))^n])^3)/(2*e^6) - (3*d*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])
^3)/e^6 + ((d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = 3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)\right.$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)}{e^5}-\frac{(15d)\,\text{Subst}\left(\int(d+e\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}-\frac{(15d)\,\text{Subst}\left(\int x^4\left(a+b\log\right.\right.}{e^6}$$
$$= -\frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}+\frac{15d^4\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d\right.\right.\right.}{2e^6}$$
$$= \frac{9bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6}-\frac{45bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\right.\right.}{4e^6}$$
$$= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}+\frac{18b^3}{}$$
$$= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}+\frac{18b^3}{}$$

Mathematica [A] time = 0.466504, size = 589, normalized size = 0.65

$$\frac{-60b\left(1800a^2\left(d^6-e^6x^2\right)-60abn\left(-30d^4e^2x^{2/3}-15d^2e^4x^{4/3}+20d^3e^3x+60d^5e\sqrt[3]{x}+147d^6+12de^5x^{5/3}-10e^6x^2\right)+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*x^(1/3)*(809340*d^5 - 140070*d^4*e*x^(1/3) + 41180*d^3*e^2*x^(2/
3) - 13785*d^2*e^3*x + 4368*d*e^4*x^(4/3) - 1000*e^5*x^(5/3)) + 1800*a^2*b*
n*(147*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*d^2*
e^4*x^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) - 36000*a^3*(d^6 - e^6*x^2) +
60*a*b^2*n^2*(8111*d^6 - 8820*d^5*e*x^(1/3) + 2610*d^4*e^2*x^(2/3) - 1140*d
^3*e^3*x + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(5/3) + 100*e^6*x^2) - 60*b*(b
^2*n^2*(13489*d^6 + 8820*d^5*e*x^(1/3) - 2610*d^4*e^2*x^(2/3) + 1140*d^3*e^
3*x - 555*d^2*e^4*x^(4/3) + 264*d*e^5*x^(5/3) - 100*e^6*x^2) - 60*a*b*n*(14
7*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*d^2*e^4*x
^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) + 1800*a^2*(d^6 - e^6*x^2))*Log[c*(
d + e*x^(1/3))^n] - 1800*b^2*(60*a*(d^6 - e^6*x^2) + b*n*(-147*d^6 - 60*d^5
*e*x^(1/3) + 30*d^4*e^2*x^(2/3) - 20*d^3*e^3*x + 15*d^2*e^4*x^(4/3) - 12*d*
e^5*x^(5/3) + 10*e^6*x^2))*Log[c*(d + e*x^(1/3))^n]^2 - 36000*b^3*(d^6 - e^
6*x^2)*Log[c*(d + e*x^(1/3))^n]^3)/(72000*e^6)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A] time = 1.09962, size = 902, normalized size = 0.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*x^(1/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(1/3) + d)^
n*c)^2 - 1/40*a^2*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d
*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^
5*x^(1/3))/e^6) + 3/2*a^2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a^3*x^2 - 1/
1200*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3
) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/
e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2 + 1800*d^6*log(e*x^(1/3) + d)^
2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1140*d^3*e^3*x + 8820*d^6*log
(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d^5*e*x^(1/3))*n^2/e^6)*a*b^2
 - 1/72000*(1800*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^
4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x
^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c)^2 + e*n*((36000*d^6*log(e*x^(1/3) + d
)^3 + 1000*e^6*x^2 + 264600*d^6*log(e*x^(1/3) + d)^2 - 4368*d*e^5*x^(5/3) +
 13785*d^2*e^4*x^(4/3) - 41180*d^3*e^3*x + 809340*d^6*log(e*x^(1/3) + d) +
140070*d^4*e^2*x^(2/3) - 809340*d^5*e*x^(1/3))*n^2/e^7 - 60*(100*e^6*x^2 +
1800*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1
140*d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d
^5*e*x^(1/3))*n*log((e*x^(1/3) + d)^n*c)/e^7))*b^3
```

Fricas [A] time = 2.58221, size = 2668, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(36000*b^3*e^6*x^2*log(c)^3 + 36000*(b^3*e^6*n^3*x^2 - b^3*d^6*n^3)
*log(e*x^(1/3) + d)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*
n - 36*a^3*e^6)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x + 147*b^3*d^6*n^3 - 60*a*b
^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^2 + 60*(b^3*e^6*n^2*x^2 -
 b^3*d^6*n^2)*log(c) + 6*(2*b^3*d*e^5*n^3*x - 5*b^3*d^4*e^2*n^3)*x^(2/3) -
15*(b^3*d^2*e^4*n^3*x - 4*b^3*d^5*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 18
000*(2*b^3*d^3*e^3*n*x - (b^3*e^6*n - 6*a*b^2*e^6)*x^2)*log(c)^2 + 20*(2059
*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x - 60*(1
3489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100*(b^3*e^6*n^3
 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^2 - 1800*(b^3*e^6*n*x^2 - b^3*d^6*n)
*log(c)^2 + 60*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 60*(20*b^3*d
^3*e^3*n^2*x + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*n - 10*(b^3*e^6*n^2 - 6*a*b^2
*e^6*n)*x^2)*log(c) - 6*(435*b^3*d^4*e^2*n^3 - 300*a*b^2*d^4*e^2*n^2 - 4*(1
1*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x + 60*(2*b^3*d*e^5*n^2*x - 5*b^3*d^4
*e^2*n^2)*log(c))*x^(2/3) + 15*(588*b^3*d^5*e*n^3 - 240*a*b^2*d^5*e*n^2 - (
37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x + 60*(b^3*d^2*e^4*n^2*x - 4*b^
3*d^5*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 1200*(5*(b^3*e^6*n^2 - 6
*a*b^2*e^6*n + 18*a^2*b*e^6)*x^2 - 3*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3
```

```
*n)*x)*log(c) - 6*(23345*b^3*d^4*e^2*n^3 - 26100*a*b^2*d^4*e^2*n^2 + 9000*a
^2*b*d^4*e^2*n - 1800*(2*b^3*d*e^5*n*x - 5*b^3*d^4*e^2*n)*log(c)^2 - 8*(91*
b^3*d*e^5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)*x - 60*(435*b^3*d^
4*e^2*n^2 - 300*a*b^2*d^4*e^2*n - 4*(11*b^3*d*e^5*n^2 - 30*a*b^2*d*e^5*n)*x
)*log(c))*x^(2/3) + 15*(53956*b^3*d^5*e*n^3 - 35280*a*b^2*d^5*e*n^2 + 7200*
a^2*b*d^5*e*n - 1800*(b^3*d^2*e^4*n*x - 4*b^3*d^5*e*n)*log(c)^2 - (919*b^3*
d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2 + 1800*a^2*b*d^2*e^4*n)*x - 60*(588*b^
3*d^5*e*n^2 - 240*a*b^2*d^5*e*n - (37*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)
*x)*log(c))*x^(1/3))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)

[Out] Timed out
```

Giac [B] time = 1.406, size = 3001, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")

[Out] 1/72000*(36000*b^3*x^2*e*log(c)^3 + 108000*a*b^2*x^2*e*log(c)^2 + (36000*(x
^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d)^3 - 216000*(x^(1/3)*e + d)^5*d*e^
(-5)*log(x^(1/3)*e + d)^3 + 540000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)
*e + d)^3 - 720000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^3 + 5400
00*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^3 - 216000*(x^(1/3)*e +
d)*d^5*e^(-5)*log(x^(1/3)*e + d)^3 - 18000*(x^(1/3)*e + d)^6*e^(-5)*log(x^(
1/3)*e + d)^2 + 129600*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d)^2 - 40
5000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 + 720000*(x^(1/3)*e
+ d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 - 810000*(x^(1/3)*e + d)^2*d^4*e^(-5
)*log(x^(1/3)*e + d)^2 + 648000*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e +
d)^2 + 6000*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 51840*(x^(1/3)*e
+ d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 202500*(x^(1/3)*e + d)^4*d^2*e^(-5)*lo
g(x^(1/3)*e + d) - 480000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) +
 810000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 1296000*(x^(1/3)*
e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 1000*(x^(1/3)*e + d)^6*e^(-5) + 1036
8*(x^(1/3)*e + d)^5*d*e^(-5) - 50625*(x^(1/3)*e + d)^4*d^2*e^(-5) + 160000*
(x^(1/3)*e + d)^3*d^3*e^(-5) - 405000*(x^(1/3)*e + d)^2*d^4*e^(-5) + 129600
0*(x^(1/3)*e + d)*d^5*e^(-5))*b^3*n^3 + 60*(1800*(x^(1/3)*e + d)^6*e^(-5)*l
og(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d)^2
 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 - 36000*(x^(1/3)
*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^2*d^4*e^(
-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e +
 d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) + 4320*(x^(1/3)*e +
 d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(
x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) - 27
000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) + 21600*(x^(1/3)*e + d)
```

```
*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1/3)*e + d)^6*e^(-5) - 864*(x^(1/3
)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)^4*d^2*e^(-5) - 8000*(x^(1/3)*e +
 d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2*d^4*e^(-5) - 21600*(x^(1/3)*e +
d)*d^5*e^(-5))*b^3*n^2*log(c) + 108000*a^2*b*x^2*e*log(c) + 1800*(60*(x^(1/
3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(
x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200
*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^
4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e
+ d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x
^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3
)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*b^3*n*log(c)^2 + 60
*(1800*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d
)^5*d*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(
x^(1/3)*e + d)^2 - 36000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2
+ 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*
e + d)*d^5*e^(-5)*log(x^(1/3)*e + d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x
^(1/3)*e + d) + 4320*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*
(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d
^3*e^(-5)*log(x^(1/3)*e + d) - 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/
3)*e + d) + 21600*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1
/3)*e + d)^6*e^(-5) - 864*(x^(1/3)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)
^4*d^2*e^(-5) - 8000*(x^(1/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2
*d^4*e^(-5) - 21600*(x^(1/3)*e + d)*d^5*e^(-5))*a*b^2*n^2 + 36000*a^3*x^2*e
 + 3600*(60*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e +
d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(
1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x
^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(
-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5
*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^
(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*a
*b^2*n*log(c) + 1800*(60*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*
(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e
^(-5)*log(x^(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e
+ d) + 900*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e
 + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(
1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e
+ d)^3*d^3*e^(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*
d^5*e^(-5))*a^2*b*n)*e^(-1)
```

3.459 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=438

$$\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{9bd^2}{}$$

```
[Out] (9*b^3*d*n^3*(d + e*x^(1/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e*x^(1/3))^3)/(9*
e^3) + (18*a*b^2*d^2*n^2*x^(1/3))/e^2 - (18*b^3*d^2*n^3*x^(1/3))/e^2 + (18*
b^3*d^2*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^3 - (9*b^2*d*n^2*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^3) + (2*b^2*n^2*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (9*b*d*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^3) - (b*n*(d + e*x^(1/3))^3*(a
 + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (3*d^2*(d + e*x^(1/3))*(a + b*Log[c
*(d + e*x^(1/3))^n])^3)/e^3 - (3*d*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^
(1/3))^n])^3)/e^3 + ((d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/
e^3
```

Rubi [A] time = 0.443928, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2451, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{9bd^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```


```
[Out] (9*b^3*d*n^3*(d + e*x^(1/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e*x^(1/3))^3)/(9*
e^3) + (18*a*b^2*d^2*n^2*x^(1/3))/e^2 - (18*b^3*d^2*n^3*x^(1/3))/e^2 + (18*
b^3*d^2*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^3 - (9*b^2*d*n^2*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^3) + (2*b^2*n^2*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (9*b*d*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^3) - (b*n*(d + e*x^(1/3))^3*(a
 + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (3*d^2*(d + e*x^(1/3))*(a + b*Log[c
*(d + e*x^(1/3))^n])^3)/e^3 - (3*d*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^
(1/3))^n])^3)/e^3 + ((d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/
e^3
```


Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} + \right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)}{e^2} - \frac{(6d)\,\text{Subst}\left(\int(d+ex)\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+e\sqrt[3]{x}\right)}{e^3} - \frac{(6d)\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right.\right.\right.}{e^3}$$
$$= \frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} - \frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{e^3}$$
$$= -\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} + \frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(\right.\right.\right.}{2e^3}$$
$$= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a\right.}{2}$$
$$= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{18b^3d^2n^3\sqrt[3]{x}}{e^2} + \frac{18b^3d}{}$$

Mathematica [A] time = 0.224464, size = 362, normalized size = 0.83

$$\frac{6b\left(d+e\sqrt[3]{x}\right)\left(18a^2\left(d^2-de\sqrt[3]{x}+e^2x^{2/3}\right)-6abn\left(11d^2-5de\sqrt[3]{x}+2e^2x^{2/3}\right)+b^2n^2\left(85d^2-19de\sqrt[3]{x}+4e^2x^{2/3}\right)\right)\log\left(c\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*(-510*d^2 + 57*d*e*x^(1/3) - 8*e^2*x^(2/3))*x^(1/3) - 6*a*b^2*n^
2*(23*d^3 - 66*d^2*e*x^(1/3) + 15*d*e^2*x^(2/3) - 4*e^3*x) + 36*a^3*(d^3 +
e^3*x) - 18*a^2*b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*x)
+ 6*b*(18*a^2*(d^2 - d*e*x^(1/3) + e^2*x^(2/3)) - 6*a*b*n*(11*d^2 - 5*d*e*x
^(1/3) + 2*e^2*x^(2/3)) + b^2*n^2*(85*d^2 - 19*d*e*x^(1/3) + 4*e^2*x^(2/3))
)*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(6*a*(d^3 + e^3*x) - b*
n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*x))*Log[c*(d + e*x^(1
/3))^n]^2 + 36*b^3*(d^3 + e^3*x)*Log[c*(d + e*x^(1/3))^n]^3)/(36*e^3)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A] time = 1.08376, size = 614, normalized size = 1.4

$$\frac{1}{2}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)\right)a^2b+\frac{1}{6}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a^2*b + 1/6*(6*e*n*(6*d^3*log(
e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*
x^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(1/3)
 + d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e
*x^(1/3))*n^2/e^3)*a*b^2 + 1/36*(18*e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*
e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*x^(1/3) + d)^n*c)^2 + 36
*x*log((e*x^(1/3) + d)^n*c)^3 + e*n*((36*d^3*log(e*x^(1/3) + d)^3 + 198*d^3
*log(e*x^(1/3) + d)^2 - 8*e^3*x + 510*d^3*log(e*x^(1/3) + d) + 57*d*e^2*x^(
2/3) - 510*d^2*e*x^(1/3))*n^2/e^4 - 6*(18*d^3*log(e*x^(1/3) + d)^2 - 4*e^3*
x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e*x^(1/3))*n*log(
(e*x^(1/3) + d)^n*c)/e^4))*b^3 + a^3*x
```

Fricas [A] time = 2.09555, size = 1544, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/36*(36*b^3*e^3*x*log(c)^3 + 36*(b^3*e^3*n^3*x + b^3*d^3*n^3)*log(e*x^(1/3
) + d)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x*log(c)^2 + 18*(3*b^3*d*e^2*n^3*x^
(2/3) - 6*b^3*d^2*e*n^3*x^(1/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(b^3
*e^3*n^3 - 3*a*b^2*e^3*n^2)*x + 6*(b^3*e^3*n^2*x + b^3*d^3*n^2)*log(c))*log
(e*x^(1/3) + d)^2 + 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x*log(
c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x + 6*
(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 18*(b^3*e^3*n*x + b^3
*d^3*n)*log(c)^2 + 2*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n)*x -
6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n^2 - 3*a*b^2*e^3*n)*x)*log(
c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*d*e^2*n^2)*x^(2/
3) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a*b^2*d^2*e*n^2)*x^(1
/3))*log(e*x^(1/3) + d) + 3*(19*b^3*d*e^2*n^3 + 18*b^3*d*e^2*n*log(c)^2 - 3
0*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n
)*log(c))*x^(2/3) - 6*(85*b^3*d^2*e*n^3 + 18*b^3*d^2*e*n*log(c)^2 - 66*a*b^
2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log
(c))*x^(1/3))/e^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3,x)

[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Giac [B] time = 1.33871, size = 1492, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")

[Out] 1/36*(36*b^3*x*e*log(c)^3 + (36*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)
^3 - 108*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^3 + 108*(x^(1/3)*e +
 d)*d^2*e^(-2)*log(x^(1/3)*e + d)^3 - 36*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/
3)*e + d)^2 + 162*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 - 324*(x^
(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 + 24*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) - 162*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 64
8*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 8*(x^(1/3)*e + d)^3*e^(-2
) + 81*(x^(1/3)*e + d)^2*d*e^(-2) - 648*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n^3
 + 6*(18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)^2 - 54*(x^(1/3)*e + d)
^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3
)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) + 54*(x^(1/3)*e
 + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^
(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2) - 27*(x^(1/3)*e + d)^2*d*e^(-2) +
 108*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n^2*log(c) + 18*(6*(x^(1/3)*e + d)^3*e
^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)
 + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e
^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n
*log(c)^2 + 108*a*b^2*x*e*log(c)^2 + 6*(18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(
1/3)*e + d)^2 - 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^
(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) + 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108
*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2)
 - 27*(x^(1/3)*e + d)^2*d*e^(-2) + 108*(x^(1/3)*e + d)*d^2*e^(-2))*a*b^2*n^
2 + 36*(6*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^
2*d*e^(-2)*log(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e
 + d) - 2*(x^(1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(
1/3)*e + d)*d^2*e^(-2))*a*b^2*n*log(c) + 108*a^2*b*x*e*log(c) + 18*(6*(x^(1
/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(
x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1
/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2
*e^(-2))*a^2*b*n + 36*a^3*x*e)*e^(-1)
```

3.460 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$-18b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+9bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+18b$$

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^3*Log[-((e*x^(1/3))/d)] + 9*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])^2*PolyLog[2, 1 + (e*x^(1/3))/d] - 18*b^2*n^2*(a +
b*Log[c*(d + e*x^(1/3))^n])*PolyLog[3, 1 + (e*x^(1/3))/d] + 18*b^3*n^3*Poly
Log[4, 1 + (e*x^(1/3))/d]
```

Rubi [A] time = 0.19455, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-18b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+9bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+18b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```


```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^3*Log[-((e*x^(1/3))/d)] + 9*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])^2*PolyLog[2, 1 + (e*x^(1/3))/d] - 18*b^2*n^2*(a +
b*Log[c*(d + e*x^(1/3))^n])*PolyLog[3, 1 + (e*x^(1/3))/d] + 18*b^3*n^3*Poly
Log[4, 1 + (e*x^(1/3))/d]
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (9ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log}{d+e}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (9bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log}{x}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\text{Li}_2\left(\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\text{Li}_2\left(\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\text{Li}_2\left(\right.$$

Mathematica [B] time = 0.172827, size = 333, normalized size = 2.47

$$9b^2n^2\left(-2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)+2\log\left(d+e\sqrt[3]{x}\right)\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)+\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\log^2\left(d+e\sqrt[3]{x}\right)\right)\left(a+b\log\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*((Log[d + e*x^(
1/3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) +
9*b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(Log[d
+ e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*PolyLog[2, 1 +
(e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d]) + 3*b^3*n^3*(Log[d + e*x^
(1/3)]^3*Log[-((e*x^(1/3))/d)] + 3*Log[d + e*x^(1/3)]^2*PolyLog[2, 1 + (e*x
^(1/3))/d] - 6*Log[d + e*x^(1/3)]*PolyLog[3, 1 + (e*x^(1/3))/d] + 6*PolyLog
```

```
[4, 1 + (e*x^(1/3))/d])
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^{3}\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}\right)^{3}\log(x)+\int-\frac{\left(b^{3}enx\log(x)-3\left(b^{3}e\log(c)+ab^{2}e\right)x-3\left(b^{3}d\log(c)+ab^{2}d\right)x^{\frac{2}{3}}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}\right)^{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((e*x^(1/3) + d)^n)^3*log(x) + integrate(-((b^3*e*n*x*log(x) - 3*(b^
3*e*log(c) + a*b^2*e)*x - 3*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1/3
) + d)^n)^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3
*e)*x - 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^
2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) - (b^3*d*lo
g(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e*x^2 + d
*x^(5/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{3}\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)^{3}+3\,ab^{2}\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)^{2}+3\,a^{2}b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x, x)
```

3.461 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=439

$$-\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}-\frac{6b^3e^3n^3\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^3}$$

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1
/3)) - (3*b^2*e^3*n^2*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3
))^n]))/d^3 - (3*b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d*x^(2/3)) +
(3*b*e^2*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(d^3*x^(1/3)
) + (3*b*e^3*n*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n])^
2)/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])^3/x - (6*b^2*e^3*n^2*(a + b*Log[c
*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 + (b^3*e^3*n^3*Log[x])/d^3
+ (3*b^3*e^3*n^3*PolyLog[2, d/(d + e*x^(1/3))])/d^3 - (6*b^2*e^3*n^2*(a + b
*Log[c*(d + e*x^(1/3))^n])*PolyLog[2, d/(d + e*x^(1/3))])/d^3 - (6*b^3*e^3*
n^3*PolyLog[2, 1 + (e*x^(1/3))/d])/d^3 - (6*b^3*e^3*n^3*PolyLog[3, d/(d + e
*x^(1/3))])/d^3
```

Rubi [A] time = 1.00594, antiderivative size = 414, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}-\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^3}-\frac{6b^3e^3n^3\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1
/3)) + (3*b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d^3) - (3*b*e*n*(a
 + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d*x^(2/3)) + (3*b*e^2*n*(d + e*x^(1/3)
)*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(d^3*x^(1/3)) - (e^3*(a + b*Log[c*(d
+ e*x^(1/3))^n])^3)/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])^3/x - (9*b^2*e^3
*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 + (3*b*e^3
*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)])/d^3 + (b^3*e^3
*n^3*Log[x])/d^3 - (9*b^3*e^3*n^3*PolyLog[2, 1 + (e*x^(1/3))/d])/d^3 + (6*b
^2*e^3*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d])/
d^3 - (6*b^3*e^3*n^3*PolyLog[3, 1 + (e*x^(1/3))/d])/d^3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + (3ben)\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)}\,dx, x, \ldots\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + (3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + \frac{(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d} - \ldots$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} - \frac{(3ben)\,\text{Subst}\ldots}{\ldots}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} + \frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}}\ldots$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\ldots}{2dx^{2/3}}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} + \frac{3be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\ldots\right.}{2d^3}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} + \frac{3be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\ldots\right.}{2d^3}$$

Mathematica [A] time = 0.708821, size = 733, normalized size = 1.67

$$\frac{-6b^2n^2\left(-2e^3x\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)+\log\left(d+e\sqrt[3]{x}\right)\left(d^2e\sqrt[3]{x}-2de^2x^{2/3}-2e^3x\log\left(-\frac{e\sqrt[3]{x}}{d}\right)-3e^3x\right)+\left(d^3+e^3x\right)\log^2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b*d^2*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))
^n])^2 + 6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x
^(1/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] +
b*Log[c*(d + e*x^(1/3))^n])^2 - 6*b*e^3*n*x*Log[d + e*x^(1/3)]*(a - b*n*Log
[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 2*d^3*(a - b*n*Log[d + e*
x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3 + 2*b*e^3*n*x*(a - b*n*Log[d + e*x
^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log[d +
 e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(d*e^2*x^(2/3) + (d^3 + e^3*x)*Lo
g[d + e*x^(1/3)]^2 + 3*e^3*x*Log[-((e*x^(1/3))/d)] + Log[d + e*x^(1/3)]*(d^
2*e*x^(1/3) - 2*d*e^2*x^(2/3) - 3*e^3*x - 2*e^3*x*Log[-((e*x^(1/3))/d)]) -
2*e^3*x*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*n^3*(-6*d*e^2*x^(2/3)*Log[d +
e*x^(1/3)] - 6*e^3*x*Log[d + e*x^(1/3)] - 3*d^2*e*x^(1/3)*Log[d + e*x^(1/3)
]^2 + 6*d*e^2*x^(2/3)*Log[d + e*x^(1/3)]^2 + 9*e^3*x*Log[d + e*x^(1/3)]^2 -
```

```
2*d^3*Log[d + e*x^(1/3)]^3 - 2*e^3*x*Log[d + e*x^(1/3)]^3 + 6*e^3*x*Log[-(
(e*x^(1/3))/d)] - 18*e^3*x*Log[d + e*x^(1/3)]*Log[-((e*x^(1/3))/d)] + 6*e^3
*x*Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 6*e^3*x*(-3 + 2*Log[d + e*x
^(1/3)])*PolyLog[2, 1 + (e*x^(1/3))/d] - 12*e^3*x*PolyLog[3, 1 + (e*x^(1/3)
)/d]))/(2*d^3*x)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,b^3d^3x^{\frac{2}{3}}\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^3+\left(6\,b^3e^3nx^{\frac{5}{3}}\log\left(ex^{\frac{1}{3}}+d\right)-6\,b^3de^2nx^{\frac{4}{3}}+3\,b^3d^2enx-2\left(b^3e^3nx\log\left(x\right)-3\,b^3d^3\log\left(c\right)-3\right.\right.}{2\,d^3x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*d^3*x^(2/3)*log((e*x^(1/3) + d)^n)^3 + (6*b^3*e^3*n*x^(5/3)*log
(e*x^(1/3) + d) - 6*b^3*d*e^2*n*x^(4/3) + 3*b^3*d^2*e*n*x - 2*(b^3*e^3*n*x*
log(x) - 3*b^3*d^3*log(c) - 3*a*b^2*d^3)*x^(2/3))*log((e*x^(1/3) + d)^n)^2)
/(d^3*x^(5/3)) + integrate(1/3*(3*(b^3*d^3*e*log(c)^3 + 3*a*b^2*d^3*e*log(c
)^2 + 3*a^2*b*d^3*e*log(c) + a^3*d^3*e)*x^(5/3) + 3*(b^3*d^4*log(c)^3 + 3*a
*b^2*d^4*log(c)^2 + 3*a^2*b*d^4*log(c) + a^3*d^4)*x^(4/3) + (6*b^3*e^4*n^2*
x^(8/3)*log(e*x^(1/3) + d) - 6*b^3*d*e^3*n^2*x^(7/3) + 3*b^3*d^2*e^2*n^2*x^
2 + 9*(b^3*d^3*e*log(c)^2 + 2*a*b^2*d^3*e*log(c) + a^2*b*d^3*e)*x^(5/3) + 9
*(b^3*d^4*log(c)^2 + 2*a*b^2*d^4*log(c) + a^2*b*d^4)*x^(4/3) - 2*(b^3*e^4*n
^2*x^2*log(x) - 3*(b^3*d^3*e*n*log(c) + a*b^2*d^3*e*n)*x)*x^(2/3))*log((e*x
^(1/3) + d)^n))/(d^3*e*x^(11/3) + d^4*x^(10/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^3+3\,ab^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2+3\,a^2b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^2, x)
```

3.462 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=765

result too large to display

```
[Out] -(b^3*e^3*n^3)/(20*d^3*x) + (3*b^3*e^4*n^3)/(10*d^4*x^(2/3)) - (71*b^3*e^5*
n^3)/(40*d^5*x^(1/3)) + (71*b^3*e^6*n^3*Log[d + e*x^(1/3)])/(40*d^6) - (3*b
^2*e^2*n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^2*x^(4/3)) + (9*b^2*e^3*
n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^3*x) - (47*b^2*e^4*n^2*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(40*d^4*x^(2/3)) + (77*b^2*e^5*n^2*(d + e*x^(1/3)
)*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6*x^(1/3)) + (77*b^2*e^6*n^2*Log[
1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6) - (3*b*e*
n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(10*d*x^(5/3)) + (3*b*e^2*n*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^2)/(8*d^2*x^(4/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x
^(1/3))^n])^2)/(2*d^3*x) + (3*b*e^4*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(
4*d^4*x^(2/3)) - (3*b*e^5*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]
)^2)/(2*d^6*x^(1/3)) - (3*b*e^6*n*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(
d + e*x^(1/3))^n])^2)/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])^3/(2*x^2)
+ (3*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^
6 - (15*b^3*e^6*n^3*Log[x])/(8*d^6) - (77*b^3*e^6*n^3*PolyLog[2, d/(d + e*x
^(1/3))])/(20*d^6) + (3*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*PolyLo
g[2, d/(d + e*x^(1/3))])/d^6 + (3*b^3*e^6*n^3*PolyLog[2, 1 + (e*x^(1/3))/d]
)/d^6 + (3*b^3*e^6*n^3*PolyLog[3, d/(d + e*x^(1/3))])/d^6
```

Rubi [A] time = 3.05255, antiderivative size = 742, normalized size of antiderivative = 0.97, number of steps used = 73, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$-\frac{3b^2e^6n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6}+\frac{137b^3e^6n^3\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{20d^6}+\frac{3b^3e^6n^3\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}\right.}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -(b^3*e^3*n^3)/(20*d^3*x) + (3*b^3*e^4*n^3)/(10*d^4*x^(2/3)) - (71*b^3*e^5*
n^3)/(40*d^5*x^(1/3)) + (71*b^3*e^6*n^3*Log[d + e*x^(1/3)])/(40*d^6) - (3*b
^2*e^2*n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^2*x^(4/3)) + (9*b^2*e^3*
n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^3*x) - (47*b^2*e^4*n^2*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(40*d^4*x^(2/3)) + (77*b^2*e^5*n^2*(d + e*x^(1/3)
)*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6*x^(1/3)) - (77*b*e^6*n*(a + b*L
og[c*(d + e*x^(1/3))^n])^2)/(40*d^6) - (3*b*e*n*(a + b*Log[c*(d + e*x^(1/3)
)^n])^2)/(10*d*x^(5/3)) + (3*b*e^2*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8
*d^2*x^(4/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d^3*x) + (3
*b*e^4*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*d^4*x^(2/3)) - (3*b*e^5*n*(
d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d^6*x^(1/3)) + (e^6*(
a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n
])^3/(2*x^2) + (137*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x
^(1/3))/d)])/(20*d^6) - (3*b*e^6*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-
((e*x^(1/3))/d)])/(2*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (137*b^3*e^6*
n^3*PolyLog[2, 1 + (e*x^(1/3))/d])/(20*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(
d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d])/d^6 + (3*b^3*e^6*n^3*Poly
```

```
Log[3, 1 + (e*x^(1/3))/d])/d^6
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^6(d+ex)}\,dx, x\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right)}{2d} - \frac{(3}{}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} - \frac{(3ben)\,\text{Subst}}{}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8d^2x^{4/3}} - \frac{(a+b}{}$$

$$= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} + \frac{3be^2n}{}$$

$$= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} + \frac{9b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^3x} - \frac{3be}{}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{40d^4x^{2/3}} - \frac{3b^3e^5n^3}{20d^5\sqrt[3]{x}} + \frac{3b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{20d^6} - \frac{3b^2e^2n^2\left(a+b\log\left(c\right.\right.}{20d^2x^{4/3}}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{3b^3e^5n^3}{5d^5\sqrt[3]{x}} + \frac{3b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{5d^6} - \frac{3b^2e^2n^2\left(a+b\log\left(c\right.\right.}{20d^2x^{4/3}}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}} + \frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6} - \frac{3b^2e^2n^2\left(a+b\log\right.}{20d^2x}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}} + \frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6} - \frac{3b^2e^2n^2\left(a+b\log\right.}{20d^2x}$$

Mathematica [A] time = 1.67626, size = 1074, normalized size = 1.4

$$-\frac{20\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d^6+60bn\log\left(d+e\sqrt[3]{x}\right)\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -(12*b*d^5*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3)
)^n])^2 - 15*b*d^4*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d +
 e*x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*
(d + e*x^(1/3))^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e*x^(1/3)]
+ b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d*e^5*n*x^(5/3)*(a - b*n*Log[d + e*x
^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d^6*n*Log[d + e*x^(1/3)]*(a
- b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 60*b*e^6*n*x^2*L
og[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])
^2 + 20*d^6*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3 + 2
0*b*e^6*n*x^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*L
og[x] + b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(
6*d^4*e^2*x^(2/3) - 18*d^3*e^3*x + 47*d^2*e^4*x^(4/3) - 154*d*e^5*x^(5/3) +
 60*(d^6 - e^6*x^2)*Log[d + e*x^(1/3)]^2 - 274*e^6*x^2*Log[-((e*x^(1/3))/d)
] + 2*Log[d + e*x^(1/3)]*(12*d^5*e*x^(1/3) - 15*d^4*e^2*x^(2/3) + 20*d^3*e^
3*x - 30*d^2*e^4*x^(4/3) + 60*d*e^5*x^(5/3) + 137*e^6*x^2 + 60*e^6*x^2*Log[
-((e*x^(1/3))/d)]) + 120*e^6*x^2*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*n^3*(
3*d^4*e^2*x^(2/3)*(2 - 5*Log[d + e*x^(1/3)])*Log[d + e*x^(1/3)] + 12*d^5*e*
x^(1/3)*Log[d + e*x^(1/3)]^2 + 20*d^6*Log[d + e*x^(1/3)]^3 + 2*d^3*e^3*x*(1
 - 9*Log[d + e*x^(1/3)] + 10*Log[d + e*x^(1/3)]^2) - d^2*e^4*x^(4/3)*(12 -
47*Log[d + e*x^(1/3)] + 30*Log[d + e*x^(1/3)]^2) + d*e^5*x^(5/3)*(71 - 154*
Log[d + e*x^(1/3)] + 60*Log[d + e*x^(1/3)]^2) + 225*e^6*x^2*(-Log[d + e*x^(
1/3)] + Log[-((e*x^(1/3))/d)]) + 137*e^6*x^2*(Log[d + e*x^(1/3)]*(Log[d + e
*x^(1/3)] - 2*Log[-((e*x^(1/3))/d)]) - 2*PolyLog[2, 1 + (e*x^(1/3))/d]) - 2
0*e^6*x^2*(Log[d + e*x^(1/3)]^2*(Log[d + e*x^(1/3)] - 3*Log[-((e*x^(1/3))/d
)]) - 6*Log[d + e*x^(1/3)]*PolyLog[2, 1 + (e*x^(1/3))/d] + 6*PolyLog[3, 1 +
 (e*x^(1/3))/d])))/(40*d^6*x^2)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}\right)^{3}}{2\,x^2}+\int\frac{\left(b^3enx+6\left(b^3e\log\left(c\right)+ab^2e\right)x+6\left(b^3d\log\left(c\right)+ab^2d\right)x^{\frac{2}{3}}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}\right)^{2}+2\left(b^3e\log\left(c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log((e*x^(1/3) + d)^n)^3/x^2 + integrate(1/2*((b^3*e*n*x + 6*(b^3*
e*log(c) + a*b^2*e)*x + 6*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1/3)
+ d)^n)^2 + 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3
*e)*x + 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^
2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 2*(b^3*d*
log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e*x^4 +
```

```
d*x^(11/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^3, x)
```

3.463 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=138

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4$$

```
[Out] (b*d^5*n*x^(2/3))/(4*e^5) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^3*n*x^2)/(12*e
^3) - (b*d^2*n*x^(8/3))/(16*e^2) + (b*d*n*x^(10/3))/(20*e) - (b*n*x^4)/24 -
 (b*d^6*n*Log[d + e*x^(2/3)])/(4*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3))^n
]))/4
```

Rubi [A] time = 0.106875, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(2/3))/(4*e^5) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^3*n*x^2)/(12*e
^3) - (b*d^2*n*x^(8/3))/(16*e^2) + (b*d*n*x^(10/3))/(20*e) - (b*n*x^4)/24 -
 (b*d^6*n*Log[d + e*x^(2/3)])/(4*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3))^n
]))/4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx=\frac{3}{2}\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx,x,x^{2/3}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{4}(ben)\,\mathrm{Subst}\left(\int\frac{x^6}{d+ex}dx,x,x^{2/3}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{4}(ben)\,\mathrm{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}\right.\right.$$
$$=\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4-\frac{bd^6n\log\left(d\right.}{4e^6}$$

Mathematica [A] time = 0.109732, size = 135, normalized size = 0.98

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{1}{4}ben\left(-\frac{d^5x^{2/3}}{e^6}+\frac{d^4x^{4/3}}{2e^5}-\frac{d^3x^2}{3e^4}+\frac{d^2x^{8/3}}{4e^3}+\frac{d^6\log\left(d+ex^{2/3}\right)}{e^7}-\frac{dx^{10/3}}{5e^2}+\frac{x^4}{6e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^5*x^(2/3))/e^6) + (d^4*x^(4/3))/(2*e^5) - (d^3*x^2
)/(3*e^4) + (d^2*x^(8/3))/(4*e^3) - (d*x^(10/3))/(5*e^2) + x^4/(6*e) + (d^6
*Log[d + e*x^(2/3)])/e^7))/4 + (b*x^4*Log[c*(d + e*x^(2/3))^n])/4
```

Maple [F] time = 0.416, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Maxima [A] time = 1.03905, size = 146, normalized size = 1.06

$$\frac{1}{4}bx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{4}ax^4-\frac{1}{240}ben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30\,d^4\cdots}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a*x^4 - 1/240*b*e*n*(60*d^6*log(e*
x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 2
0*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)
```

Fricas [A] time = 1.86777, size = 302, normalized size = 2.19

$$\frac{60\,be^6x^4\log(c)+20\,bd^3e^3nx^2-10\left(be^6n-6\,ae^6\right)x^4+60\left(be^6nx^4-bd^6n\right)\log\left(ex^{\frac{2}{3}}+d\right)-15\left(bd^2e^4nx^2-4\,bd^5en\right)x^{\frac{2}{3}}+6}{240\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(60*b*e^6*x^4*log(c) + 20*b*d^3*e^3*n*x^2 - 10*(b*e^6*n - 6*a*e^6)*x^
4 + 60*(b*e^6*n*x^4 - b*d^6*n)*log(e*x^(2/3) + d) - 15*(b*d^2*e^4*n*x^2 - 4
*b*d^5*e*n)*x^(2/3) + 6*(2*b*d*e^5*n*x^3 - 5*b*d^4*e^2*n*x)*x^(1/3))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.303, size = 359, normalized size = 2.6

$$\frac{1}{4}\,bx^4\log(c)+\frac{1}{4}\,ax^4+\frac{1}{240}\left(60\left(x^{\frac{2}{3}}e+d\right)^6e^{(-5)}\log\left(x^{\frac{2}{3}}e+d\right)-360\left(x^{\frac{2}{3}}e+d\right)^5de^{(-5)}\log\left(x^{\frac{2}{3}}e+d\right)+900\left(x^{\frac{2}{3}}e+d\right)^4d^2e^{(-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/
3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3
)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-5
)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)
- 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) - 10*(x^(2/3)*e + d)^6*
e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3)*e + d)^4*d^2*e^(-5) +
 400*(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e + d)^2*d^4*e^(-5) + 360*
(x^(2/3)*e + d)*d^5*e^(-5))*b*n*e^(-1)
```

3.464 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{2bd^2nx^{5/3}}{15e^2}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) - (2*b*n*x^3)/27 + (2*b*d^(9/2)*n*ArcTa
n[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3
))^n]))/3
```

Rubi [A] time = 0.0780661, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 341, 302, 205}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{2bd^2nx^{5/3}}{15e^2}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) - (2*b*n*x^3)/27 + (2*b*d^(9/2)*n*ArcTa
n[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3
))^n]))/3
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx = \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{9}(2ben)\int\frac{x^{8/3}}{d+ex^{2/3}}dx$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{x^{10}}{d+ex^2}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\left(\frac{d^4}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^4}{e^3}-\frac{dx^6}{e^2}+\right.\right.$$
$$=-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\right.\right.\right.$$
$$=-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}$$

Mathematica [A] time = 0.0976569, size = 135, normalized size = 1.04

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{2bd^2nx^{5/3}}{15e^2}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) + (a*x^3)/3 - (2*b*n*x^3)/27 + (2*b*d^(
9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (b*x^3*Log[c*(d + e
*x^(2/3))^n])/3
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93991, size = 806, normalized size = 6.2

$$\left[\frac{315\,be^4nx^3\log\left(ex^{\frac{2}{3}}+d\right)+315\,be^4x^3\log\left(c\right)-126\,bd^2e^2nx^{\frac{5}{3}}+315\,bd^4n\sqrt{-\frac{d}{e}}\log\left(\frac{e^3x^2-2\,de^2x\sqrt{-\frac{d}{e}}-d^3+2\left(e^3x\sqrt{-\frac{d}{e}}+d^2e\right)x^{\frac{2}{3}}-2}{e^3x^2+d^3}\right.}{945\,e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] [1/945*(315*b*e^4*n*x^3*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*d
^2*e^2*n*x^(5/3) + 315*b*d^4*n*sqrt(-d/e)*log((e^3*x^2 - 2*d*e^2*x*sqrt(-d/
e) - d^3 + 2*(e^3*x*sqrt(-d/e) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^2*e*sqrt(-
d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 210*b*d^3*e*n*x - 35*(2*b*e^4*n - 9*a*e^4
)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4, 1/945*(315*b*e^4*n*x^3
*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*d^2*e^2*n*x^(5/3) + 630*
b*d^4*n*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 210*b*d^3*e*n*x - 35*(2*b
*e^4*n - 9*a*e^4)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33877, size = 140, normalized size = 1.08

$$\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3+\frac{1}{945}\left(315\,x^3\log\left(x^{\frac{2}{3}}e+d\right)+2\left(315\,d^{\frac{9}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}-\left(315\,d^4x^{\frac{1}{3}}e^4-105\,d^3xe^5+63\,d^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/945*(315*x^3*log(x^(2/3)*e + d) + 2*(315*d
^(9/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-11/2) - (315*d^4*x^(1/3)*e^4 - 1
05*d^3*x*e^5 + 63*d^2*x^(5/3)*e^6 - 45*d*x^(7/3)*e^7 + 35*x^3*e^8)*e^(-9))*
e)*b*n
```

3.465 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

```
[Out] -(b*d^2*n*x^(2/3))/(2*e^2) + (b*d*n*x^(4/3))/(4*e) - (b*n*x^2)/6 + (b*d^3*n
*Log[d + e*x^(2/3)])/(2*e^3) + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/2
```

Rubi [A] time = 0.065433, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2454, 2395, 43}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] -(b*d^2*n*x^(2/3))/(2*e^2) + (b*d*n*x^(4/3))/(4*e) - (b*n*x^2)/6 + (b*d^3*n
*Log[d + e*x^(2/3)])/(2*e^3) + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx=\frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))\,dx,x,x^{2/3}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^3}{d+ex}dx,x,x^{2/3}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right.\right.$$
$$=-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

Mathematica [A] time = 0.0272043, size = 94, normalized size = 1.06

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] -(b*d^2*n*x^(2/3))/(2*e^2) + (b*d*n*x^(4/3))/(4*e) + (a*x^2)/2 - (b*n*x^2)/
6 + (b*d^3*n*Log[d + e*x^(2/3)])/(2*e^3) + (b*x^2*Log[c*(d + e*x^(2/3))^n])
/2
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Maxima [A] time = 1.03577, size = 103, normalized size = 1.16

$$\frac{1}{12}ben\left(\frac{6d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2e^2x^2-3dex^{\frac{4}{3}}+6d^2x^{\frac{2}{3}}}{e^3}\right)+\frac{1}{2}bx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/12*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d
^2*x^(2/3))/e^3) + 1/2*b*x^2*log((e*x^(2/3) + d)^n*c) + 1/2*a*x^2
```

Fricas [A] time = 1.82717, size = 203, normalized size = 2.28

$$\frac{6\,be^3x^2\log\left(c\right)+3\,bde^2nx^{\frac{4}{3}}-6\,bd^2enx^{\frac{2}{3}}-2\left(be^3n-3\,ae^3\right)x^2+6\left(be^3nx^2+bd^3n\right)\log\left(ex^{\frac{2}{3}}+d\right)}{12\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*e^3*x^2*log(c) + 3*b*d*e^2*n*x^(4/3) - 6*b*d^2*e*n*x^(2/3) - 2*(b
*e^3*n - 3*a*e^3)*x^2 + 6*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))/e^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2815, size = 111, normalized size = 1.25

$$\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{12}\left(6\,x^2\log\left(x^{\frac{2}{3}}e+d\right)+\left(6\,d^3e^{(-4)}\log\left(\left|x^{\frac{2}{3}}e+d\right|\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x
^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b*n
 + 1/2*a*x^2
```

3.466 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=72

$$ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Rubi [A] time = 0.0530306, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2448, 341, 302, 205}

$$ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx &= ax+b\int \log\left(c\left(d+ex^{2/3}\right)^n\right)dx\\ &= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{1}{3}(2ben)\int \frac{x^{2/3}}{d+ex^{2/3}}\,dx\\ &= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\,\text{Subst}\left(\int \frac{x^4}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)\\ &= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\,\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)dx,x,\sqrt[3]{x}\right)\\ &= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{\left(2bd^2n\right)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{e}\\ &= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)\end{aligned}$$

Mathematica [A] time = 0.0267923, size = 72, normalized size = 1.

$$ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Maple [A] time = 0.092, size = 62, normalized size = 0.9

$$ax+bx\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)-\frac{2\,bnx}{3}+2\,\frac{bdn\sqrt[3]{x}}{e}-2\,\frac{bd^2n}{e\sqrt{de}}\arctan\left(\frac{e\sqrt[3]{x}}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e*x^(2/3))^n),x)
```

```
[Out] a*x+b*x*ln(c*(d+e*x^(2/3))^n)-2/3*b*n*x+2*b*d*n*x^(1/3)/e-2*b/e*n*d^2/(d*e)
^(1/2)*arctan(x^(1/3)*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85239, size = 547, normalized size = 7.6

$$\left[\frac{3\,benx\log\left(ex^{\frac{2}{3}}+d\right)+3\,bdn\sqrt{-\frac{d}{e}}\log\left(\frac{e^3x^2+2\,de^2x\sqrt{-\frac{d}{e}}-d^3-2\left(e^3x\sqrt{-\frac{d}{e}}-d^2e\right)x^{\frac{2}{3}}-2\left(de^2x+d^2e\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{e^3x^2+d^3}\right)+3\,bex\log\left(c\right)+6\,bdnx^{\frac{1}{3}}-}{3\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*e*n*x*log(e*x^(2/3) + d) + 3*b*d*n*sqrt(-d/e)*log((e^3*x^2 + 2*d*
e^2*x*sqrt(-d/e) - d^3 - 2*(e^3*x*sqrt(-d/e) - d^2*e)*x^(2/3) - 2*(d*e^2*x
+ d^2*e*sqrt(-d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 3*b*e*x*log(c) + 6*b*d*n*x^
(1/3) - (2*b*e*n - 3*a*e)*x)/e, 1/3*(3*b*e*n*x*log(e*x^(2/3) + d) - 6*b*d*n
*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 3*b*e*x*log(c) + 6*b*d*n*x^(1/3)
 - (2*b*e*n - 3*a*e)*x)/e]
```

Sympy [A] time = 13.4007, size = 133, normalized size = 1.85

$$ax+b\left(-\frac{2en\left(\begin{cases}-\frac{3id^{\frac{3}{2}}\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+\sqrt[3]{x}\right)}{2e^3\sqrt{\frac{1}{e}}}+\frac{3id^{\frac{3}{2}}\log\left(i\sqrt{d}\sqrt{\frac{1}{e}}+\sqrt[3]{x}\right)}{2e^3\sqrt{\frac{1}{e}}}-\frac{3d\sqrt[3]{x}}{e^2}+\frac{x}{e} & \text{for}\ e\neq 0\\ \frac{3x^{\frac{5}{3}}}{5d} & \text{otherwise}\end{cases}\right)}{3}+x\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(2/3))**n),x)
```

```
[Out] a*x + b*(-2*e*n*Piecewise((-3*I*d**(3/2)*log(-I*sqrt(d)*sqrt(1/e) + x**(1/3
))/(2*e**3*sqrt(1/e)) + 3*I*d**(3/2)*log(I*sqrt(d)*sqrt(1/e) + x**(1/3))/(2
*e**3*sqrt(1/e)) - 3*d*x**(1/3)/e**2 + x/e, Ne(e, 0)), (3*x**(5/3)/(5*d), T
rue))/3 + x*log(c*(d + e*x**(2/3))**n))
```

Giac [A] time = 1.28292, size = 92, normalized size = 1.28

$$-\frac{1}{3}\left(\left(2\left(3\,d^{\frac{3}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\left(3\,dx^{\frac{1}{3}}e-xe^2\right)e^{(-3)}\right)e-3\,x\log\left(x^{\frac{2}{3}}e+d\right)\right)n-3\,x\log\left(c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -1/3*((2*(3*d^(3/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-5/2) - (3*d*x^(1/3)
*e - x*e^2)*e^(-3))*e - 3*x*log(x^(2/3)*e + d))*n - 3*x*log(c))*b + a*x
```

3.467 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=55

$$\frac{3}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/2 + (3*b*n*PolyL
og[2, 1 + (e*x^(2/3))/d])/2
```

Rubi [A] time = 0.0507591, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$\frac{3}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/2 + (3*b*n*PolyL
og[2, 1 + (e*x^(2/3))/d])/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx &= \frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,x^{2/3}\right)\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)-\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^{2/3}\right)\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{3}{2}bn\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.011762, size = 55, normalized size = 1.

$$\frac{3}{2}b\left(n\text{PolyLog}\left(2,\frac{d+ex^{2/3}}{d}\right)+\log\left(-\frac{ex^{2/3}}{d}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

```
[Out] a*Log[x] + (3*b*(Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3))/d)] + n*PolyLog
[2, (d + e*x^(2/3))/d]))/2
```

Maple [F] time = 0.384, size = 0, normalized size = 0.

$$\int\frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)
```

Maxima [B] time = 1.64034, size = 154, normalized size = 2.8

$$-\frac{3}{2}\left(2\log\left(\frac{ex^{\frac{2}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-\frac{ex^{\frac{2}{3}}}{d}\right)\right)bn+\frac{3\left(2benx^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)-benx^{\frac{2}{3}}\right)}{2d}+\frac{2bd\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)\log(x)+2(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3/2*(2*log(e*x^(2/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(2/3)/d))*b*n + 3/2*
(2*b*e*n*x^(2/3)*log(x^(1/3)) - b*e*n*x^(2/3))/d + 1/2*(2*b*d*log((e*x^(2/3
) + d)^n)*log(x) + 2*(b*d*log(c) + a*d)*log(x) - (2*b*e*n*x*log(x) - 3*b*e*
n*x)/x^(1/3))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)
```

3.468 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2ben}{d\sqrt[3]{x}}$$

```
[Out] (-2*b*e*n)/(d*x^(1/3)) - (2*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/
d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])/x
```

Rubi [A] time = 0.0401322, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2ben}{d\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

```
[Out] (-2*b*e*n)/(d*x^(1/3)) - (2*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/
d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])/x
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}+\frac{1}{3}(2ben)\int \frac{1}{\left(d+ex^{2/3}\right)x^{4/3}}\,dx$$
$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}+(2ben)\,\text{Subst}\left(\int \frac{1}{x^2\left(d+ex^2\right)}\,dx,x,\sqrt[3]{x}\right)$$
$$= -\frac{2ben}{d\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d}$$
$$= -\frac{2ben}{d\sqrt[3]{x}}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

Mathematica [C] time = 0.0177444, size = 59, normalized size = 0.87

$$-\frac{a}{x}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{2ben\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^{2/3}}{d}\right)}{d\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

```
[Out] -(a/x) - (2*b*e*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(d*x^(
1/3)) - (b*Log[c*(d + e*x^(2/3))^n])/x
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85809, size = 481, normalized size = 7.07

$$\left[\frac{benx\sqrt{-\frac{e}{d}}\log\left(\frac{e^3x^2+2d^2ex\sqrt{-\frac{e}{d}}-d^3-2\left(de^2x\sqrt{-\frac{e}{d}}-d^2e\right)x^{\frac{2}{3}}-2\left(de^2x+d^3\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{e^3x^2+d^3}\right)-bdn\log\left(ex^{\frac{2}{3}}+d\right)-2\,benx^{\frac{2}{3}}-bd\log\left(c\right)-ad}{dx},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] [(b*e*n*x*sqrt(-e/d)*log((e^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - d^3 - 2*(d*e^2*x
*sqrt(-e/d) - d^2*e)*x^(2/3) - 2*(d*e^2*x + d^3*sqrt(-e/d))*x^(1/3))/(e^3*x
^2 + d^3)) - b*d*n*log(e*x^(2/3) + d) - 2*b*e*n*x^(2/3) - b*d*log(c) - a*d)
/(d*x), -(2*b*e*n*x*sqrt(e/d)*arctan(x^(1/3)*sqrt(e/d)) + b*d*n*log(e*x^(2/
3) + d) + 2*b*e*n*x^(2/3) + b*d*log(c) + a*d)/(d*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24545, size = 82, normalized size = 1.21

$$-\left(2\left(\frac{\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}}+\frac{1}{dx^{\frac{1}{3}}}\right)e+\frac{\log\left(x^{\frac{2}{3}}e+d\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -(2*(arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x^(1/3)))*e + l
og(x^(2/3)*e + d)/x)*b*n - b*log(c)/x - a/x
```

3.469 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=94

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{ben}{4dx^{4/3}}$$

```
[Out] -(b*e*n)/(4*d*x^(4/3)) + (b*e^2*n)/(2*d^2*x^(2/3)) - (b*e^3*n*Log[d + e*x^(
2/3)])/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rubi [A] time = 0.0704927, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{ben}{4dx^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

```
[Out] -(b*e*n)/(4*d*x^(4/3)) + (b*e^2*n)/(2*d^2*x^(2/3)) - (b*e^3*n*Log[d + e*x^(
2/3)])/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = \frac{3}{2}\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx,x,x^{2/3}\right)$$
$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx,x,x^{2/3}\right)$$
$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{e^3}{d^3(d+ex)}\right) d$$
$$= -\frac{ben}{4dx^{4/3}}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^3n\log(x)}{3d^3}$$

Mathematica [A] time = 0.0321834, size = 91, normalized size = 0.97

$$-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{2}ben\left(-\frac{e^2\log\left(d+ex^{2/3}\right)}{d^3}+\frac{2e^2\log(x)}{3d^3}+\frac{e}{d^2x^{2/3}}-\frac{1}{2dx^{4/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e*n*(-1/(2*d*x^(4/3)
) + e/(d^2*x^(2/3)) - (e^2*Log[d + e*x^(2/3)])/d^3 + (2*e^2*Log[x])/(3*d^3)
))/2
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
```

Maxima [A] time = 1.04075, size = 104, normalized size = 1.11

$$-\frac{1}{4}\,ben\left(\frac{2\,e^2\log\left(ex^{\frac{2}{3}}+d\right)}{d^3}-\frac{2\,e^2\log\left(x^{\frac{2}{3}}\right)}{d^3}-\frac{2\,ex^{\frac{2}{3}}-d}{d^2x^{\frac{4}{3}}}\right)-\frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e*n*(2*e^2*log(e*x^(2/3) + d)/d^3 - 2*e^2*log(x^(2/3))/d^3 - (2*e*x^
(2/3) - d)/(d^2*x^(4/3))) - 1/2*b*log((e*x^(2/3) + d)^n*c)/x^2 - 1/2*a/x^2
```

Fricas [A] time = 1.91638, size = 216, normalized size = 2.3

$$\frac{4\,be^3nx^2\log\left(x^{\frac{1}{3}}\right)+2\,bde^2nx^{\frac{4}{3}}-bd^2enx^{\frac{2}{3}}-2\,bd^3\log\left(c\right)-2\,ad^3-2\left(be^3nx^2+bd^3n\right)\log\left(ex^{\frac{2}{3}}+d\right)}{4\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*e^3*n*x^2*log(x^(1/3)) + 2*b*d*e^2*n*x^(4/3) - b*d^2*e*n*x^(2/3) -
 2*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))/(
d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33305, size = 128, normalized size = 1.36

$$\frac{1}{4}\left(\left(\left(\frac{2\,\log\left(x^{\frac{2}{3}}e\right)}{d^3}-\frac{2\,\log\left(\left|x^{\frac{2}{3}}e+d\right|\right)}{d^3}+\frac{\left(2\left(x^{\frac{2}{3}}e+d\right)d-3\,d^2\right)e^{(-2)}}{d^3x^{\frac{4}{3}}}\right)e^4-\frac{2\,e\log\left(x^{\frac{2}{3}}e+d\right)}{x^2}\right)bne^{(-1)}-\frac{b\log\left(c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*((2*log(x^(2/3)*e)/d^3 - 2*log(abs(x^(2/3)*e + d))/d^3 + (2*(x^(2/3)*e
+ d)*d - 3*d^2)*e^(-2)/(d^3*x^(4/3)))*e^4 - 2*e*log(x^(2/3)*e + d)/x^2)*b*n
*e^(-1) - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

3.470 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=123

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{2be^2n}{15d^2x^{5/3}}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{2be^3n}{9d^3x}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-\frac{2ben}{21dx^{7/3}}$$

```
[Out] (-2*b*e*n)/(21*d*x^(7/3)) + (2*b*e^2*n)/(15*d^2*x^(5/3)) - (2*b*e^3*n)/(9*d
^3*x) + (2*b*e^4*n)/(3*d^4*x^(1/3)) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Rubi [A] time = 0.0768091, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{2be^2n}{15d^2x^{5/3}}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{2be^3n}{9d^3x}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-\frac{2ben}{21dx^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

```
[Out] (-2*b*e*n)/(21*d*x^(7/3)) + (2*b*e^2*n)/(15*d^2*x^(5/3)) - (2*b*e^3*n)/(9*d
^3*x) + (2*b*e^4*n)/(3*d^4*x^(1/3)) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}dx = -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{1}{9}(2ben)\int \frac{1}{\left(d+ex^{2/3}\right)x^{10/3}}dx$$

$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{1}{3}(2ben)\,\text{Subst}\left(\int \frac{1}{x^8\left(d+ex^2\right)}dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{2ben}{21dx^{7/3}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}-\frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{1}{x^6(d+ex^2)}dx,x,\sqrt[3]{x}\right)}{3d}$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{\left(2be^3n\right)\text{Subst}\left(\int \frac{1}{x^4(d+ex^2)}dx,x,\sqrt[3]{x}\right)}{3d^2}$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}-\frac{\left(2be^4n\right)\text{Subst}\left(\int \frac{1}{x^2(d+ex^2)}dx,x,\sqrt[3]{x}\right)}{3d^3}$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{\left(2be^5n\right)\text{Subst}\left(\int \frac{1}{d+ex^2}dx,x,\sqrt[3]{x}\right)}{3d^4}$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}+\frac{2be^4n}{3d^4\sqrt[3]{x}}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

Mathematica [C] time = 0.0136037, size = 65, normalized size = 0.53

$$-\frac{a}{3x^3}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}-\frac{2ben\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\frac{ex^{2/3}}{d}\right)}{21dx^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

```
[Out] -a/(3*x^3) - (2*b*e*n*Hypergeometric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(
21*d*x^(7/3)) - (b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93159, size = 755, normalized size = 6.14

$$\left[\frac{105\,be^{4}nx^{3}\sqrt{-\frac{e}{d}}\log\left(\frac{e^{3}x^{2}-2\,d^{2}ex\sqrt{-\frac{e}{d}}-d^{3}+2\left(de^{2}x\sqrt{-\frac{e}{d}}+d^{2}e\right)x^{\frac{2}{3}}-2\left(de^{2}x-d^{3}\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{e^{3}x^{2}+d^{3}}\right)-70\,bde^{3}nx^{2}+42\,bd^{2}e^{2}nx^{\frac{4}{3}}-105\,bd^{4}n\log}{315\,d^{4}x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] [1/315*(105*b*e^4*n*x^3*sqrt(-e/d)*log((e^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - d^
3 + 2*(d*e^2*x*sqrt(-e/d) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^3*sqrt(-e/d))*x
^(1/3))/(e^3*x^2 + d^3)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 105*
b*d^4*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4*n*x
^2 - b*d^3*e*n)*x^(2/3))/(d^4*x^3), 1/315*(210*b*e^4*n*x^3*sqrt(e/d)*arctan
(x^(1/3)*sqrt(e/d)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 105*b*d^4
*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4*n*x^2 -
b*d^3*e*n)*x^(2/3))/(d^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36582, size = 127, normalized size = 1.03

$$\frac{1}{315}\left(2\left(\frac{105\,\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{21\,d^{2}x^{\frac{2}{3}}e-35\,dx^{\frac{4}{3}}e^{2}-15\,d^{3}+105\,x^{2}e^{3}}{d^{4}x^{\frac{7}{3}}}\right)e-\frac{105\,\log\left(x^{\frac{2}{3}}e+d\right)}{x^{3}}\right)bn-\frac{b\log\left(c\right)}{3\,x^{3}}-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/315*(2*(105*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(7/2)/d^(9/2) + (21*d^2*x^(
2/3)*e - 35*d*x^(4/3)*e^2 - 15*d^3 + 105*x^2*e^3)/(d^4*x^(7/3)))*e - 105*lo
g(x^(2/3)*e + d)/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

3.471 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=482

$$-\frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6}+\frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6}-\frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a\right.}{}$$

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(2/3))^2)/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x^(2/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(2/3))^4)/(32*e^6) - (3*b^2*d*n^2*(
d + e*x^(2/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(2/3))^6)/(72*e^6) - (3*b^2*
d^5*n^2*x^(2/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(2/3)]^2)/(4*e^6) + (3*b*d^
5*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^6 - (15*b*d^4*n*(d
+ e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^6) + (10*b*d^3*n*(d +
 e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) - (15*b*d^2*n*(d +
e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) + (3*b*d*n*(d + e*x^
(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*e^6) - (b*n*(d + e*x^(2/3))^6
*(a + b*Log[c*(d + e*x^(2/3))^n]))/(12*e^6) - (b*d^6*n*Log[d + e*x^(2/3)]*(
a + b*Log[c*(d + e*x^(2/3))^n]))/(2*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3)
)^n])^2)/4
```

Rubi [A] time = 0.488, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\frac{225d^2\left(d+ex^{2/3}\right)^4}{e^6}-\frac{60d^6\log\left(d+ex^{2/3}\right)}{e^6}+\frac{72d\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(2/3))^2)/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x^(2/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(2/3))^4)/(32*e^6) - (3*b^2*d*n^2*(
d + e*x^(2/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(2/3))^6)/(72*e^6) - (3*b^2*
d^5*n^2*x^(2/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(2/3)]^2)/(4*e^6) + (b*n*((
360*d^5*(d + e*x^(2/3)))/e^6 - (450*d^4*(d + e*x^(2/3))^2)/e^6 + (400*d^3*(
d + e*x^(2/3))^3)/e^6 - (225*d^2*(d + e*x^(2/3))^4)/e^6 + (72*d*(d + e*x^(2
/3))^5)/e^6 - (10*(d + e*x^(2/3))^6)/e^6 - (60*d^6*Log[d + e*x^(2/3)])/e^6)
*(a + b*Log[c*(d + e*x^(2/3))^n]))/120 + (x^4*(a + b*Log[c*(d + e*x^(2/3))^
n])^2)/4
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2dx=\frac{3}{2}\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2dx,x,x^{2/3}\right)$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}dx\right.$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{2}(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}d\right.$$

$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\frac{225d^2\left(d\right.}{e^6}\right.$$

$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\frac{225d^2\left(d\right.}{e^6}\right.$$

$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\frac{225d^2\left(d\right.}{e^6}\right.$$

$$=\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6}-\frac{3b^2dn^2}{}$$

$$=\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6}-\frac{3b^2dn^2}{}$$

Mathematica [A] time = 0.368755, size = 328, normalized size = 0.68

$$\frac{ex^{2/3}\left(1800a^2e^5x^{10/3}+60abn\left(20d^3e^2x^{4/3}-15d^2e^3x^2-30d^4ex^{2/3}+60d^5+12de^4x^{8/3}-10e^5x^{10/3}\right)+b^2n^2\left(-1140d^3e^2x^{4/3}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (e*x^(2/3)*(1800*a^2*e^5*x^(10/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 2
0*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 + 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3)) +
b^2*n^2*(-8820*d^5 + 2610*d^4*e*x^(2/3) - 1140*d^3*e^2*x^(4/3) + 555*d^2*e^
3*x^2 - 264*d*e^4*x^(8/3) + 100*e^5*x^(10/3))) + 5220*b^2*d^6*n^2*Log[d + e
*x^(2/3)] + 60*b*(b*n*(60*d^6 + 60*d^5*e*x^(2/3) - 30*d^4*e^2*x^(4/3) + 20*
d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^(10/3) - 10*e^6*x^4) - 60*a*(
d^6 - e^6*x^4))*Log[c*(d + e*x^(2/3))^n] - 1800*b^2*(d^6 - e^6*x^4)*Log[c*(
d + e*x^(2/3))^n]^2)/(7200*e^6)
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [A] time = 1.05741, size = 446, normalized size = 0.93

$$\frac{1}{4}b^2x^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+\frac{1}{2}abx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{4}a^2x^4-\frac{1}{120}aben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*log((e*x^(2/3) + d)^n*c)^2 + 1/2*a*b*x^4*log((e*x^(2/3) + d)^n*
c) + 1/4*a^2*x^4 - 1/120*a*b*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x
^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(
4/3) - 60*d^5*x^(2/3))/e^6) - 1/7200*(60*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7
 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 +
30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (100*e^6
*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 + 1800*d
^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(2/3) + d
) - 8820*d^5*e*x^(2/3))*n^2/e^6)*b^2
```

Fricas [A] time = 2.41827, size = 1125, normalized size = 2.33

$$\frac{1800\,b^2e^6x^4\log(c)^2+100\left(b^2e^6n^2-6\,abe^6n+18\,a^2e^6\right)x^4-60\left(19\,b^2d^3e^3n^2-20\,abd^3e^3n\right)x^2+1800\left(b^2e^6n^2x^4-b^2d^6n\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(1800*b^2*e^6*x^4*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2
*e^6)*x^4 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 + 1800*(b^2*e^6*
n^2*x^4 - b^2*d^6*n^2)*log(e*x^(2/3) + d)^2 + 60*(20*b^2*d^3*e^3*n^2*x^2 +
147*b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^4 + 60*(b
^2*e^6*n*x^4 - b^2*d^6*n)*log(c) - 15*(b^2*d^2*e^4*n^2*x^2 - 4*b^2*d^5*e*n^
2)*x^(2/3) + 6*(2*b^2*d*e^5*n^2*x^3 - 5*b^2*d^4*e^2*n^2*x)*x^(1/3))*log(e*x
^(2/3) + d) + 600*(2*b^2*d^3*e^3*n*x^2 - (b^2*e^6*n - 6*a*b*e^6)*x^4)*log(c
) - 15*(588*b^2*d^5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 - 60*a*b*
d^2*e^4*n)*x^2 + 60*(b^2*d^2*e^4*n*x^2 - 4*b^2*d^5*e*n)*log(c))*x^(2/3) - 6
*(4*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x^3 - 15*(29*b^2*d^4*e^2*n^2 - 20*a
*b*d^4*e^2*n)*x - 60*(2*b^2*d*e^5*n*x^3 - 5*b^2*d^4*e^2*n*x)*log(c))*x^(1/3
))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51409, size = 1287, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4*log(c)^2 + 1/2*a*b*x^4*log(c) + 1/4*a^2*x^4 + 1/7200*(1800*(x^(
2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^(-5
)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e +
 d)^2 - 36000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^
(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)*d^5*
e^(-5)*log(x^(2/3)*e + d)^2 - 600*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e +
d) + 4320*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) - 13500*(x^(2/3)*e
+ d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) + 24000*(x^(2/3)*e + d)^3*d^3*e^(-5)*l
og(x^(2/3)*e + d) - 27000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) +
 21600*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) + 100*(x^(2/3)*e + d)^
6*e^(-5) - 864*(x^(2/3)*e + d)^5*d*e^(-5) + 3375*(x^(2/3)*e + d)^4*d^2*e^(-
5) - 8000*(x^(2/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(2/3)*e + d)^2*d^4*e^(-5)
 - 21600*(x^(2/3)*e + d)*d^5*e^(-5))*b^2*n^2*e^(-1) + 1/120*(60*(x^(2/3)*e
+ d)^6*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/
3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(
2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(
-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d)
- 10*(x^(2/3)*e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3
)*e + d)^4*d^2*e^(-5) + 400*(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e +
 d)^2*d^4*e^(-5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*b^2*n*e^(-1)*log(c) + 1/
120*(60*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5
*d*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)
*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/
3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*
log(x^(2/3)*e + d) - 10*(x^(2/3)*e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e
^(-5) - 225*(x^(2/3)*e + d)^4*d^2*e^(-5) + 400*(x^(2/3)*e + d)^3*d^3*e^(-5)
 - 450*(x^(2/3)*e + d)^2*d^4*e^(-5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*a*b*n
*e^(-1)
```

3.472 $\int x \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=275

$$\frac{bd^3 n \log \left(d+ex^{2/3}\right) \left(a+b \log \left(c \left(d+ex^{2/3}\right)^n\right)\right)}{e^3}-\frac{3bd^2 n \left(d+ex^{2/3}\right) \left(a+b \log \left(c \left(d+ex^{2/3}\right)^n\right)\right)}{e^3}+\frac{3bdn \left(d+ex^{2/3}\right)^2 \left(a\right.}{}$$

```
[Out] (-3*b^2*d*n^2*(d + e*x^(2/3))^2)/(4*e^3) + (b^2*n^2*(d + e*x^(2/3))^3)/(9*e
^3) + (3*b^2*d^2*n^2*x^(2/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(2/3)]^2)/(2*e
^3) - (3*b*d^2*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^3 + (3
*b*d*n*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*e^3) - (b*n*(
d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) + (b*d^3*n*Log[d
 + e*x^(2/3)]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^3 + (x^2*(a + b*Log[c*(d
+ e*x^(2/3))^n])^2)/2
```

Rubi [A] time = 0.305088, antiderivative size = 217, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{6d^3\log\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{1}{2}x^2\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(2/3))^2)/(4*e^3) + (b^2*n^2*(d + e*x^(2/3))^3)/(9*e
^3) + (3*b^2*d^2*n^2*x^(2/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(2/3)]^2)/(2*e
^3) - (b*n*((18*d^2*(d + e*x^(2/3)))/e^3 - (9*d*(d + e*x^(2/3))^2)/e^3 + (2
*(d + e*x^(2/3))^3)/e^3 - (6*d^3*Log[d + e*x^(2/3)])/e^3)*(a + b*Log[c*(d +
 e*x^(2/3))^n]))/6 + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2dx=\frac{3}{2}\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2dx,x,x^{2/3}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int\frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}d$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log\left(cx^n\right)\right)}{x}d$$
$$=-\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}-\frac{6d^3\log\left(d+ex^{2/3}\right)}{e^3}\right.$$
$$=-\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}-\frac{6d^3\log\left(d+ex^{2/3}\right)}{e^3}\right.$$
$$=-\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}-\frac{6d^3\log\left(d+ex^{2/3}\right)}{e^3}\right.$$
$$=-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}\right.$$
$$=-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{b^2d^3n^2\log^2\left(d+ex^{2/3}\right)}{2e^3}$$

Mathematica [A] time = 0.156801, size = 239, normalized size = 0.87

$$\frac{18a^2d^3+18a^2e^3x^2+6b\left(6a\left(d^3+e^3x^2\right)-bn\left(6d^2ex^{2/3}+6d^3-3de^2x^{4/3}+2e^3x^2\right)\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)-36abd^2enx^{2/3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (18*a^2*d^3 - 36*a*b*d^2*e*n*x^(2/3) + 66*b^2*d^2*e*n^2*x^(2/3) + 18*a*b*d*
e^2*n*x^(4/3) - 15*b^2*d*e^2*n^2*x^(4/3) + 18*a^2*e^3*x^2 - 12*a*b*e^3*n*x^
2 + 4*b^2*e^3*n^2*x^2 - 30*b^2*d^3*n^2*Log[d + e*x^(2/3)] + 6*b*(6*a*(d^3 +
 e^3*x^2) - b*n*(6*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2))*Lo
g[c*(d + e*x^(2/3))^n] + 18*b^2*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3))^n]^2)
/(36*e^3)
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [A] time = 1.04515, size = 312, normalized size = 1.13

$$\frac{1}{2}b^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+\frac{1}{6}aben\left(\frac{6d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2e^2x^2-3dex^{\frac{4}{3}}+6d^2x^{\frac{2}{3}}}{e^3}\right)+abx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{2}a^2x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 1/6*a*b*e*n*(6*d^3*log(e*x^(2/3) +
 d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3) + a*b*x^2*log((e
*x^(2/3) + d)^n*c) + 1/2*a^2*x^2 + 1/36*(6*e*n*(6*d^3*log(e*x^(2/3) + d)/e^
4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*
c) + (4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*d^3*l
og(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n^2/e^3)*b^2
```

Fricas [A] time = 2.10064, size = 689, normalized size = 2.51

$$\frac{18b^2e^3x^2\log(c)^2-12\left(b^2e^3n-3abe^3\right)x^2\log(c)+2\left(2b^2e^3n^2-6abe^3n+9a^2e^3\right)x^2+18\left(b^2e^3n^2x^2+b^2d^3n^2\right)\log\left(ex^{\frac{2}{3}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/36*(18*b^2*e^3*x^2*log(c)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x^2*log(c) + 2*(
2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x^2 + 18*(b^2*e^3*n^2*x^2 + b^2*d^
3*n^2)*log(e*x^(2/3) + d)^2 + 6*(3*b^2*d*e^2*n^2*x^(4/3) - 6*b^2*d^2*e*n^2*
x^(2/3) - 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*x^2
+ 6*(b^2*e^3*n*x^2 + b^2*d^3*n)*log(c))*log(e*x^(2/3) + d) + 6*(11*b^2*d^2*
e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) + 3*(6*b^2*d*e^2*n*x*
log(c) - (5*b^2*d*e^2*n^2 - 6*a*b*d*e^2*n)*x)*x^(1/3))/e^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.70717, size = 427, normalized size = 1.55

$$\frac{1}{2}b^2x^2\log(c)^2+\frac{1}{36}\left(18x^2\log\left(x^{\frac{2}{3}}e+d\right)^2+\left(18d^3\log\left(x^{\frac{2}{3}}e+d\right)^2-12\left(x^{\frac{2}{3}}e+d\right)^3\log\left(x^{\frac{2}{3}}e+d\right)+54\left(x^{\frac{2}{3}}e+d\right)^2d\log\left(x^{\frac{2}{3}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2*log(c)^2 + 1/36*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2
/3)*e + d)^2 - 12*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)
^2*d*log(x^(2/3)*e + d) - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x
^(2/3)*e + d)^3 - 27*(x^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))
*b^2*n^2 + 1/6*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e
+ d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b^2*n*log(c)
 + a*b*x^2*log(c) + 1/6*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x
^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a*b
*n + 1/2*a^2*x^2
```

3.473 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=95

$$3bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-3b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.$$

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d)])/2 + 3*b*n*(a +
 b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, 1 + (e*x^(2/3))/d] - 3*b^2*n^2*Poly
Log[3, 1 + (e*x^(2/3))/d]
```

Rubi [A] time = 0.131004, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$3bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-3b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d)])/2 + 3*b*n*(a +
 b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, 1 + (e*x^(2/3))/d] - 3*b^2*n^2*Poly
Log[3, 1 + (e*x^(2/3))/d]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
```

```
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, x^{2/3}\right)$$
$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) - (3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\,l}{d+}\right.$$
$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) - (3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\,l}{x}\right.$$
$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\text{Li}_2$$
$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\text{Li}_2$$

Mathematica [B] time = 0.114874, size = 199, normalized size = 2.09

$$2bn\left(\log(x)\left(\log\left(d+ex^{2/3}\right)-\log\left(\frac{ex^{2/3}}{d}+1\right)\right)-\frac{3}{2}\text{PolyLog}\left(2,-\frac{ex^{2/3}}{d}\right)\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)-bn\log\left(d+ex^{2/3}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((Log[d + e*x^(2/
3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2)
+ (3*b^2*n^2*(Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2
/3)]*PolyLog[2, 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2
```

Maple [F] time = 0.344, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)^2\log(x)+\int\frac{3\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x-2\left(2\,b^2enx\log(x)-3\left(b^2e\log(c)+abe\right)x-3\left(b^2\right.\right.}{3\left(ex^2+dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((e*x^(2/3) + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*e*log(c)^2 + 2*
a*b*e*log(c) + a^2*e)*x - 2*(2*b^2*e*n*x*log(x) - 3*(b^2*e*log(c) + a*b*e)*
x - 3*(b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log
(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^2 + d*x^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x, x)
```

3.474 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=238

$$-\frac{b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{d^3}+\frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}+\frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+e\right.\right.\right.}{d^3x^{2/3}}$$

```
[Out] -(b^2*e^2*n^2)/(2*d^2*x^(2/3)) + (b^2*e^3*n^2*Log[d + e*x^(2/3)])/(2*d^3) -
 (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d*x^(4/3)) + (b*e^2*n*(d + e*x
^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^3*x^(2/3)) + (b*e^3*n*Log[1 -
d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^3 - (a + b*Log[c*(d
+ e*x^(2/3))^n])^2/(2*x^2) - (b^2*e^3*n^2*Log[x])/d^3 - (b^2*e^3*n^2*PolyLo
g[2, d/(d + e*x^(2/3))])/d^3
```

Rubi [A] time = 0.500007, antiderivative size = 261, normalized size of antiderivative = 1.1, number of steps used = 14, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^3n^2\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)}{d^3}-\frac{e^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3}+\frac{be^3n\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}+\frac{be^2n}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -(b^2*e^2*n^2)/(2*d^2*x^(2/3)) + (b^2*e^3*n^2*Log[d + e*x^(2/3)])/(2*d^3) -
 (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d*x^(4/3)) + (b*e^2*n*(d + e*x
^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^3*x^(2/3)) - (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n])^2)/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(2*x^2
) + (b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/d^3 -
(b^2*e^3*n^2*Log[x])/d^3 + (b^2*e^3*n^2*PolyLog[2, 1 + (e*x^(2/3))/d])/d^3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \frac{3}{2}\operatorname{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx,x,x^{2/3}\right)$$
$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (ben)\operatorname{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}\,dx,x,x^{2/3}\right.$$
$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (bn)\operatorname{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+ex^{2/3}\right)$$
$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\operatorname{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+ex^{2/3}\right)}{d} - \frac{(ben}{}$$
$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} - \frac{(ben)\operatorname{Subst}\left(\right.}{}$$
$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}} -$$
$$= -\frac{b^2e^2n^2}{2d^2x^{2/3}} + \frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{be^2n\left(d+\right.}{}$$
$$= -\frac{b^2e^2n^2}{2d^2x^{2/3}} + \frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{be^2n\left(d+\right.}{}$$

Mathematica [A] time = 0.303298, size = 264, normalized size = 1.11

$$-\frac{\frac{ex^{2/3}\left(-6be^2nx^{4/3}\left(bn\operatorname{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\right)+3bd^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+3e^2x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-6bdenx^2\right.}{d^3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -(3*(a + b*Log[c*(d + e*x^(2/3))^n])^2 + (e*x^(2/3)*(3*b*d^2*n*(a + b*Log[c
*(d + e*x^(2/3))^n]) - 6*b*d*e*n*x^(2/3)*(a + b*Log[c*(d + e*x^(2/3))^n]) +
 3*e^2*x^(4/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*b^2*e^2*n^2*x^(4/3)*(
3*Log[d + e*x^(2/3)] - 2*Log[x]) + b^2*e*n^2*x^(2/3)*(3*d - 3*e*x^(2/3)*Log
[d + e*x^(2/3)] + 2*e*x^(2/3)*Log[x]) - 6*b*e^2*n*x^(4/3)*((a + b*Log[c*(d
+ e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)] + b*n*PolyLog[2, 1 + (e*x^(2/3))/d])
))/d^3)/(6*x^2)
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)^2}{2\,x^2}+\int\frac{3\left(b^2e\log\left(c\right)^2+2\,abe\log\left(c\right)+a^2e\right)x+2\left(b^2enx+3\left(b^2e\log\left(c\right)+abe\right)x+3\left(b^2d\log\left(c\right)+ab\right.\right.}{3\left(ex^4+dx^{\frac{10}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((e*x^(2/3) + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + 2*(b^2*e*n*x + 3*(b^2*e*log(c) + a*b*e)*x + 3*(
b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^4 + d*x^(10/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^3, x)
```

3.475 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=412

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^6}-\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6}-\frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right.\right.}{2d^6x^{2/3}}$$

```
[Out] -(b^2*e^2*n^2)/(40*d^2*x^(8/3)) + (3*b^2*e^3*n^2)/(40*d^3*x^2) - (47*b^2*e^
4*n^2)/(240*d^4*x^(4/3)) + (77*b^2*e^5*n^2)/(120*d^5*x^(2/3)) - (77*b^2*e^6
*n^2*Log[d + e*x^(2/3)])/(120*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]
))/(10*d*x^(10/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*d^2*x^(8
/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(6*d^3*x^2) + (b*e^4*n*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(4*d^4*x^(4/3)) - (b*e^5*n*(d + e*x^(2/3))*
(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6*x^(2/3)) - (b*e^6*n*Log[1 - d/(d +
 e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6) - (a + b*Log[c*(d +
e*x^(2/3))^n])^2/(4*x^4) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*n^
2*PolyLog[2, d/(d + e*x^(2/3))])/(2*d^6)
```

Rubi [A] time = 1.01578, antiderivative size = 436, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)}{2d^6}+\frac{e^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4d^6}-\frac{be^6n\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6}-\frac{be^5n\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -(b^2*e^2*n^2)/(40*d^2*x^(8/3)) + (3*b^2*e^3*n^2)/(40*d^3*x^2) - (47*b^2*e^
4*n^2)/(240*d^4*x^(4/3)) + (77*b^2*e^5*n^2)/(120*d^5*x^(2/3)) - (77*b^2*e^6
*n^2*Log[d + e*x^(2/3)])/(120*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]
))/(10*d*x^(10/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*d^2*x^(8
/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(6*d^3*x^2) + (b*e^4*n*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(4*d^4*x^(4/3)) - (b*e^5*n*(d + e*x^(2/3))*
(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6*x^(2/3)) + (e^6*(a + b*Log[c*(d +
e*x^(2/3))^n])^2)/(4*d^6) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(4*x^4) - (b
*e^6*n*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/(2*d^6) + (1
37*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^2*PolyLog[2, 1 + (e*x^(2/3))/
d])/(2*d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
```

```
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
```

```
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+ex^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+ex^{2/3}\right)}{2d} - \frac{(ben)\,\text{S}\cdots}{\cdots}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} - \frac{(ben)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} - \frac{\left(a+b\log\left(c\cdots\right.\right.}{4\cdots}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{b^2e^3n^2}{30d^3x^2} - \frac{b^2e^4n^2}{20d^4x^{4/3}} + \frac{b^2e^5n^2}{10d^5x^{2/3}} - \frac{b^2e^6n^2\log\left(d+ex^{2/3}\right)}{10d^6} - \frac{ben\left(a+b\,\text{lo}\cdots\right.}{1\cdots}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{9b^2e^4n^2}{80d^4x^{4/3}} + \frac{9b^2e^5n^2}{40d^5x^{2/3}} - \frac{9b^2e^6n^2\log\left(d+ex^{2/3}\right)}{40d^6} - \frac{ben\left(a+b\cdots\right.}{\cdots}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{47b^2e^5n^2}{120d^5x^{2/3}} - \frac{47b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6} - \frac{ben\left(a\cdots\right.}{\cdots}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{77b^2e^5n^2}{120d^5x^{2/3}} - \frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6} - \frac{ben\left(a\cdots\right.}{\cdots}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{77b^2e^5n^2}{120d^5x^{2/3}} - \frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6} - \frac{ben\left(a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.275875, size = 539, normalized size = 1.31

$$-\frac{360b^2e^6n^2x^4\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+180a^2d^6+360abd^6\log\left(c\left(d+ex^{2/3}\right)^n\right)-360abe^6x^4\log\left(c\left(d+ex^{2/3}\right)^n\right)-90ab}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -(180*a^2*d^6 + 72*a*b*d^5*e*n*x^(2/3) - 90*a*b*d^4*e^2*n*x^(4/3) + 18*b^2*
d^4*e^2*n^2*x^(4/3) + 120*a*b*d^3*e^3*n*x^2 - 54*b^2*d^3*e^3*n^2*x^2 - 180*
a*b*d^2*e^4*n*x^(8/3) + 141*b^2*d^2*e^4*n^2*x^(8/3) + 360*a*b*d*e^5*n*x^(10
/3) - 462*b^2*d*e^5*n^2*x^(10/3) + 822*b^2*e^6*n^2*x^4*Log[d + e*x^(2/3)] +
 360*a*b*d^6*Log[c*(d + e*x^(2/3))^n] + 72*b^2*d^5*e*n*x^(2/3)*Log[c*(d + e
*x^(2/3))^n] - 90*b^2*d^4*e^2*n*x^(4/3)*Log[c*(d + e*x^(2/3))^n] + 120*b^2*
d^3*e^3*n*x^2*Log[c*(d + e*x^(2/3))^n] - 180*b^2*d^2*e^4*n*x^(8/3)*Log[c*(d
 + e*x^(2/3))^n] + 360*b^2*d*e^5*n*x^(10/3)*Log[c*(d + e*x^(2/3))^n] - 360*
a*b*e^6*x^4*Log[c*(d + e*x^(2/3))^n] + 180*b^2*d^6*Log[c*(d + e*x^(2/3))^n]
^2 - 180*b^2*e^6*x^4*Log[c*(d + e*x^(2/3))^n]^2 + 360*a*b*e^6*n*x^4*Log[-((
e*x^(2/3))/d)] + 360*b^2*e^6*n*x^4*Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3
))/d)] - 548*b^2*e^6*n^2*x^4*Log[x] + 360*b^2*e^6*n^2*x^4*PolyLog[2, 1 + (e
*x^(2/3))/d])/(720*d^6*x^4)
```

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)^2}{4x^4}+\int \frac{3\left(b^2e\log(c)^2+2abe\log(c)+a^2e\right)x+\left(b^2enx+6\left(b^2e\log(c)+abe\right)x+6\left(b^2d\log(c)+a\right.\right.}{3\left(ex^6+dx^{\frac{16}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log((e*x^(2/3) + d)^n)^2/x^4 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(b^
2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^6 + d*x^(16/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^5, x)
```

3.476 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=547

$$\frac{4ib^2d^{9/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+$$

```
[Out] (-4*a*b*d^4*n*x^(1/3))/(3*e^4) + (4504*b^2*d^4*n^2*x^(1/3))/(945*e^4) - (19
84*b^2*d^3*n^2*x)/(2835*e^3) + (1144*b^2*d^2*n^2*x^(5/3))/(4725*e^2) - (128
*b^2*d*n^2*x^(7/3))/(1323*e) + (8*b^2*n^2*x^3)/243 - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(945*e^(9/2)) + (((4*I)/3)*b^2*d^(9/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(9/2) + (8*b^2*d^(9/2)*n^2*ArcTan
[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])
/(3*e^(9/2)) - (4*b^2*d^4*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/(3*e^4) + (4*
b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e^3) - (4*b*d^2*n*x^(5/3)*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(15*e^2) + (4*b*d*n*x^(7/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(21*e) - (4*b*n*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/27
 + (4*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(
2/3))^n]))/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/3 + (((4*
I)/3)*b^2*d^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/
3))])/e^(9/2)
```

Rubi [A] time = 0.769075, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2458, 2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ib^2d^{9/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```


```
[Out] (-4*a*b*d^4*n*x^(1/3))/(3*e^4) + (4504*b^2*d^4*n^2*x^(1/3))/(945*e^4) - (19
84*b^2*d^3*n^2*x)/(2835*e^3) + (1144*b^2*d^2*n^2*x^(5/3))/(4725*e^2) - (128
*b^2*d*n^2*x^(7/3))/(1323*e) + (8*b^2*n^2*x^3)/243 - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(945*e^(9/2)) + (((4*I)/3)*b^2*d^(9/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(9/2) + (8*b^2*d^(9/2)*n^2*ArcTan
[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])
/(3*e^(9/2)) - (4*b^2*d^4*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/(3*e^4) + (4*
b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e^3) - (4*b*d^2*n*x^(5/3)*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(15*e^2) + (4*b*d*n*x^(7/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(21*e) - (4*b*n*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/27
 + (4*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(
2/3))^n]))/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/3 + (((4*
I)/3)*b^2*d^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/
3))])/e^(9/2)
```


Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
```

```
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2dx=3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{3}(4ben)\,\text{Subst}\left(\int\frac{x^{10}\left(a+b\log\left(c\left(d+ex^2\right)^n\right.\right.}{d+ex^2}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{3}(4ben)\,\text{Subst}\left(\int\left(\frac{d^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right.\right.}{e^5}\right.\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{3}(4bn)\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\right.$$

$$=-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+\right.\right.\right.}{15e^2}$$

$$=-\frac{4abd^4n\sqrt[3]{x}}{3e^4}-\frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right.\right.}{9e^3}$$

$$=-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}$$

$$=-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}$$

$$=-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}$$

$$=-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}$$

Mathematica [A] time = 0.485456, size = 438, normalized size = 0.8

$$\frac{396900ib^2d^{9/2}n^2\text{PolyLog}\left(2,\frac{\sqrt{e}\sqrt[3]{x}+i\sqrt{d}}{\sqrt{e}\sqrt[3]{x}-i\sqrt{d}}\right)+\sqrt{e}\sqrt[3]{x}\left(99225a^2e^4x^{8/3}-630b\left(2bn\left(63d^2e^2x^{4/3}-105d^3ex^{2/3}+315d^4-45de^3x^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((396900*I)*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 + 1260*b*d^
(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(315*a - 1126*b*n + 630*b*n*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 315*b*Log[c*(d + e*x^(2/3))^n])
 + Sqrt[e]*x^(1/3)*(99225*a^2*e^4*x^(8/3) - 1260*a*b*n*(315*d^4 - 105*d^3*e
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)) + 8*b^2*n^2*
(177345*d^4 - 26040*d^3*e*x^(2/3) + 9009*d^2*e^2*x^(4/3) - 3600*d*e^3*x^2 +
```

```
 1225*e^4*x^(8/3)) - 630*b*(-315*a*e^4*x^(8/3) + 2*b*n*(315*d^4 - 105*d^3*e
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)))*Log[c*(d +
e*x^(2/3))^n] + 99225*b^2*e^4*x^(8/3)*Log[c*(d + e*x^(2/3))^n]^2) + (396900
*I)*b^2*d^(9/2)*n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d]
+ Sqrt[e]*x^(1/3))])/(297675*e^(9/2))
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \ln \left(c \left(d + e x^{\frac{2}{3}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^{2} x^{2} \log \left(\left(e x^{\frac{2}{3}} + d\right)^{n} c\right)^{2} + 2 \, a b x^{2} \log \left(\left(e x^{\frac{2}{3}} + d\right)^{n} c\right) + a^{2} x^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*x^2*log((e*x^(2/3) + d)
^n*c) + a^2*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2*x^2, x)
```

3.477 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=364

$$-\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+\right.\right.\right.$$

```
[Out] (4*a*b*d*n*x^(1/3))/e - (32*b^2*d*n^2*x^(1/3))/(3*e) + (8*b^2*n^2*x)/9 + (3
2*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) - ((4*I)*b
^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) - (8*b^2*d^(3/2
)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e
]*x^(1/3))])/e^(3/2) + (4*b^2*d*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/e - (4*
b*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 - (4*b*d^(3/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^(3/2) + x*(a + b*Log[
c*(d + e*x^(2/3))^n])^2 - ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])
/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/e^(3/2)
```

Rubi [A] time = 0.446778, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {2451, 2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```


```
[Out] (4*a*b*d*n*x^(1/3))/e - (32*b^2*d*n^2*x^(1/3))/(3*e) + (8*b^2*n^2*x)/9 + (3
2*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) - ((4*I)*b
^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) - (8*b^2*d^(3/2
)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e
]*x^(1/3))])/e^(3/2) + (4*b^2*d*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/e - (4*
b*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 - (4*b*d^(3/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^(3/2) + x*(a + b*Log[
c*(d + e*x^(2/3))^n])^2 - ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])
/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/e^(3/2)
```


Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d+ex^2} dx, x,\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4ben)\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{e^2} + \right.\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4bn)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right) dx, x,\right.$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\right.\right.}{e^{3/2}}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} + \frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} - \frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{4bd}{}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}} + \frac{4b^2dn\sqrt[3]{x}\log\left(c\left(\right.\right.}{e}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\right.}{e^{3/2}}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\right.}{e^{3/2}}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\right.}{e^{3/2}}$$

Mathematica [A] time = 0.216445, size = 319, normalized size = 0.88

$$\frac{-36ib^2d^{3/2}n^2\text{PolyLog}\left(2,\frac{\sqrt{e}\sqrt[3]{x}+i\sqrt{d}}{\sqrt{e}\sqrt[3]{x}-i\sqrt{d}}\right) + \sqrt{e}\sqrt[3]{x}\left(9a^2ex^{2/3} + 6b\left(3aex^{2/3} + 6bdn - 2benx^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right) + 12abn\left(3d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((-36*I)*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 12*b*d^(3/2)
*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(3*a - 8*b*n + 6*b*n*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 3*b*Log[c*(d + e*x^(2/3))^n]) + Sqrt[e]*x^(
1/3)*(12*a*b*n*(3*d - e*x^(2/3)) + 8*b^2*n^2*(-12*d + e*x^(2/3)) + 9*a^2*e*
x^(2/3) + 6*b*(6*b*d*n + 3*a*e*x^(2/3) - 2*b*e*n*x^(2/3))*Log[c*(d + e*x^(2
/3))^n] + 9*b^2*e*x^(2/3)*Log[c*(d + e*x^(2/3))^n]^2) - (36*I)*b^2*d^(3/2)*
```

```
n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3
))])/(9*e^(3/2))
```

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int \left(a+b\ln \left(c\left(d+e x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^{2}\log\left(\left(e x^{\frac{2}{3}}+d\right)^{n}c\right)^{2}+2\,ab\log\left(\left(e x^{\frac{2}{3}}+d\right)^{n}c\right)+a^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\log \left(c\left(d+e x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2, x)
```

3.478 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=298

$$-\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{4ben\left(a+b\log\left(c\left(d+e\right.\right.\right.}{d\sqrt[3]{x}}$$

```
[Out] (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(3/2) - ((4*I)*b^2*
e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) - (8*b^2*e^(3/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x
^(1/3))])/d^(3/2) - (4*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d*x^(1/3))
- (4*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2
/3))^n]))/d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/x - ((4*I)*b^2*e^(3/
2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(3/2)
```

Rubi [A] time = 0.405929, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2458, 2457, 2476, 2455, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{4ben\left(a+b\log\left(c\left(d+e\right.\right.\right.}{d\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```


```
[Out] (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(3/2) - ((4*I)*b^2*
e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) - (8*b^2*e^(3/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x
^(1/3))])/d^(3/2) - (4*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d*x^(1/3))
- (4*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2
/3))^n]))/d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/x - ((4*I)*b^2*e^(3/
2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(3/2)
```


Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
```

```
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + (4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2\left(d+ex^2\right)}\,dx, x,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + (4ben)\,\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^2} - \frac{e\left(\right.}{}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + \frac{(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx, x, \sqrt[3]{x}\right)}{d} - \frac{(4}{}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.}{d^{3/2}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{}}{\sqrt{d}}\right.}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{d\sqrt[3]{x}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\text{lo}}{d^{3/2}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\text{lo}}{d^{3/2}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\text{lo}}{d^{3/2}}$$

Mathematica [A] time = 0.187435, size = 247, normalized size = 0.83

$$\frac{-4ib^2e^{3/2}n^2x\text{PolyLog}\left(2, \frac{\sqrt{e}\sqrt[3]{x}+i\sqrt{d}}{\sqrt{e}\sqrt[3]{x}-i\sqrt{d}}\right) - 4be^{3/2}nx\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right) + 2bn\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right) - 2bn\right)}{d^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

```
[Out] ((-4*I)*b^2*e^(3/2)*n^2*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 4*b*e^(3/2)
*n*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - 2*b*n + 2*b*n*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + b*Log[c*(d + e*x^(2/3))^n]) - Sqrt[d]*(a +
```

```
b*Log[c*(d + e*x^(2/3))^n])*(a*d + 4*b*e*n*x^(2/3) + b*d*Log[c*(d + e*x^(2/
3))^n]) - (4*I)*b^2*e^(3/2)*n^2*x*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/
((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))])/(d^(3/2)*x)
```

Maple [F] time = 0.347, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^2, x)
```

3.479 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=476

$$\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}}-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x}+\frac{4be^2n\left(\ldots\right.}{}$$

```
[Out] (-8*b^2*e^2*n^2)/(105*d^2*x^(5/3)) + (32*b^2*e^3*n^2)/(105*d^3*x) - (568*b^
2*e^4*n^2)/(315*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(315*d^(9/2)) + (((4*I)/3)*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(
1/3))/Sqrt[d]]^2)/d^(9/2) + (8*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(3*d^(9/2)) - (4*b*e*
n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(21*d*x^(7/3)) + (4*b*e^2*n*(a + b*Log[
c*(d + e*x^(2/3))^n]))/(15*d^2*x^(5/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(9*d^3*x) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^4
*x^(1/3)) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(
d + e*x^(2/3))^n]))/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(3*x^3
) + (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt
[e]*x^(1/3))])/d^(9/2)
```

Rubi [A] time = 0.620084, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2458, 2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}}-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x}+\frac{4be^2n\left(\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```


```
[Out] (-8*b^2*e^2*n^2)/(105*d^2*x^(5/3)) + (32*b^2*e^3*n^2)/(105*d^3*x) - (568*b^
2*e^4*n^2)/(315*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(315*d^(9/2)) + (((4*I)/3)*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(
1/3))/Sqrt[d]]^2)/d^(9/2) + (8*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(3*d^(9/2)) - (4*b*e*
n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(21*d*x^(7/3)) + (4*b*e^2*n*(a + b*Log[
c*(d + e*x^(2/3))^n]))/(15*d^2*x^(5/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(9*d^3*x) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^4
*x^(1/3)) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(
d + e*x^(2/3))^n]))/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(3*x^3
) + (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt
[e]*x^(1/3))])/d^(9/2)
```


Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^8\left(d+ex^2\right)}\,dx, x, \sqrt[3]{\ldots}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^8} - \frac{e(a\ldots}{\ldots}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^8}\,dx, x, \sqrt[3]{x}\right)}{3d} - \frac{(4be^{\ldots}}{\ldots}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4be^3n\left(a+\ldots\right.}{\ldots}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{8b^2e^3n^2}{45d^3x} - \frac{8b^2e^4n^2}{9d^4\sqrt[3]{x}} - \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}} - \frac{4ben\left(a+b\log\left(c\left(d\ldots\right.\right.\right.}{21dx^{7/3}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{64b^2e^4n^2}{45d^4\sqrt[3]{x}} - \frac{32b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{9d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan^{-1}\ldots}{3d^{9/2}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{184b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{45d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan\ldots}{3d^{9/2}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan\ldots}{3d^{9\ldots}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan\ldots}{3d^{9\ldots}}$$

Mathematica [C] time = 0.531639, size = 473, normalized size = 0.99

$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3}+\frac{4}{3}ben\left(\frac{ibe^{7/2}n\left(\text{PolyLog}\left(2,\frac{\sqrt{e}\sqrt[3]{x}+i\sqrt{d}}{\sqrt{e}\sqrt[3]{x}-i\sqrt{d}}\right)+\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)-2i\log\left(\frac{2\sqrt{}}{\sqrt{d}+i\sqrt{}}\right.\right.\right.}{d^{9/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

```
[Out] -(a + b*Log[c*(d + e*x^(2/3))^n])^2/(3*x^3) + (4*b*e*n*((-2*b*e^(7/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(9/2) - (2*b*e*n*Hypergeometric2F1[-5/2,
1, -3/2, -((e*x^(2/3))/d)])/(35*d^2*x^(5/3)) + (2*b*e^2*n*Hypergeometric2F1
[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^3*x) - (2*b*e^3*n*Hypergeometric2F
1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^4*x^(1/3)) - (a + b*Log[c*(d + e*x^
(2/3))^n])/(7*d*x^(7/3)) + (e*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^2*x^(5
/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^3*x) + (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(d^4*x^(1/3)) + (e^(7/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(9/2) + (I*b*e^(7/2)*n*(ArcTan[(S
qrt[e]*x^(1/3))/Sqrt[d]]*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*
x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))]))/d^(9/2)))/3
```

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int\frac{1}{x^4}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(\left(ex^{\frac{2}{3}} + d\right)^{n} c\right) + a\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^4, x)
```

3.480 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=640

$$-\frac{4ib^2e^{15/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}}+\frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x}-\frac{4b}{}$$

```
[Out] (-8*b^2*e^2*n^2)/(715*d^2*x^(11/3)) + (64*b^2*e^3*n^2)/(2145*d^3*x^3) - (28
72*b^2*e^4*n^2)/(45045*d^4*x^(7/3)) + (1216*b^2*e^5*n^2)/(9009*d^5*x^(5/3))
 - (224072*b^2*e^6*n^2)/(675675*d^6*x) + (344192*b^2*e^7*n^2)/(225225*d^7*x
^(1/3)) + (704552*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(2252
25*d^(15/2)) - (((4*I)/5)*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]
]^2)/d^(15/2) - (8*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(5*d^(15/2)) - (4*b*e*n*(a + b*L
og[c*(d + e*x^(2/3))^n]))/(65*d*x^(13/3)) + (4*b*e^2*n*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(55*d^2*x^(11/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n
]))/(45*d^3*x^3) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(35*d^4*x^(
7/3)) - (4*b*e^5*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*d^5*x^(5/3)) + (4*
b*e^6*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(15*d^6*x) - (4*b*e^7*n*(a + b*Lo
g[c*(d + e*x^(2/3))^n]))/(5*d^7*x^(1/3)) - (4*b*e^(15/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^(15/2)) - (a + b*L
og[c*(d + e*x^(2/3))^n])^2/(5*x^5) - (((4*I)/5)*b^2*e^(15/2)*n^2*PolyLog[2,
 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(15/2)
```

Rubi [A] time = 0.944001, antiderivative size = 640, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2458, 2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^2e^{15/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}}+\frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x}-\frac{4b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```



```
[Out] (-8*b^2*e^2*n^2)/(715*d^2*x^(11/3)) + (64*b^2*e^3*n^2)/(2145*d^3*x^3) - (28
72*b^2*e^4*n^2)/(45045*d^4*x^(7/3)) + (1216*b^2*e^5*n^2)/(9009*d^5*x^(5/3))
 - (224072*b^2*e^6*n^2)/(675675*d^6*x) + (344192*b^2*e^7*n^2)/(225225*d^7*x
^(1/3)) + (704552*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(2252
25*d^(15/2)) - (((4*I)/5)*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]
]^2)/d^(15/2) - (8*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(5*d^(15/2)) - (4*b*e*n*(a + b*L
og[c*(d + e*x^(2/3))^n]))/(65*d*x^(13/3)) + (4*b*e^2*n*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(55*d^2*x^(11/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n
]))/(45*d^3*x^3) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(35*d^4*x^(
7/3)) - (4*b*e^5*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*d^5*x^(5/3)) + (4*
b*e^6*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(15*d^6*x) - (4*b*e^7*n*(a + b*Lo
g[c*(d + e*x^(2/3))^n]))/(5*d^7*x^(1/3)) - (4*b*e^(15/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^(15/2)) - (a + b*L
og[c*(d + e*x^(2/3))^n])^2/(5*x^5) - (((4*I)/5)*b^2*e^(15/2)*n^2*PolyLog[2,
 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(15/2)
```


Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{14}\left(d+ex^2\right)}\,dx, x, \sqrt[3]{\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^{14}} - \frac{e\left(a\right.}{}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{14}}\,dx, x, \sqrt[3]{x}\right)}{5d} - \frac{\left(4be^{\cdots}\right.}{}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+\right.}{}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{8b^2e^3n^2}{495d^3x^3} - \frac{8b^2e^4n^2}{315d^4x^{7/3}} + \frac{8b^2e^5n^2}{175d^5x^{5/3}} - \frac{8b^2e^6n^2}{75d^6x} + \frac{8b^2e^7n^2}{15d^7\sqrt[3]{x}} + \frac{8b^2e^{15/2}n}{}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{32b^2e^4n^2}{693d^4x^{7/3}} + \frac{128b^2e^5n^2}{1575d^5x^{5/3}} - \frac{32b^2e^6n^2}{175d^6x} + \frac{64b^2e^7n^2}{75d^7\sqrt[3]{x}} + \frac{32b^{\cdots}}{}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1912b^2e^5n^2}{17325d^5x^{5/3}} - \frac{1144b^2e^6n^2}{4725d^6x} + \frac{568b^2e^7n^2}{525d^7\sqrt[3]{x}}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{15104b^2e^6n^2}{51975d^6x} + \frac{1984b^2e^7n^{\cdots}}{1575d^7\sqrt[3]{\cdots}}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} + \frac{24344b^2\cdots}{17325d^{\cdots}}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} + \frac{344192b\cdots}{225225\cdots}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} + \frac{344192b\cdots}{225225\cdots}$$

Mathematica [C] time = 1.03363, size = 678, normalized size = 1.06

$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{4}{5}ben\left(-\frac{ibe^{13/2}n\left(\text{PolyLog}\left(2, \frac{\sqrt{e}\sqrt[3]{x}+i\sqrt{d}}{\sqrt{e}\sqrt[3]{x}-i\sqrt{d}}\right) + \tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right) - 2i\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\cdots}\right.\right.\right.}{d^{15/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

```
[Out] -(a + b*Log[c*(d + e*x^(2/3))^n])^2/(5*x^5) + (4*b*e*n*((2*b*e^(13/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(15/2) - (2*b*e*n*Hypergeometric2F1[-11/2
, 1, -9/2, -((e*x^(2/3))/d)])/(143*d^2*x^(11/3)) + (2*b*e^2*n*Hypergeometri
c2F1[-9/2, 1, -7/2, -((e*x^(2/3))/d)])/(99*d^3*x^3) - (2*b*e^3*n*Hypergeome
tric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(63*d^4*x^(7/3)) + (2*b*e^4*n*Hyp
ergeometric2F1[-5/2, 1, -3/2, -((e*x^(2/3))/d)])/(35*d^5*x^(5/3)) - (2*b*e^
5*n*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^6*x) + (2*b*e
^6*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^7*x^(1/3)) - (
a + b*Log[c*(d + e*x^(2/3))^n])/(13*d*x^(13/3)) + (e*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(11*d^2*x^(11/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*d
^3*x^3) + (e^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(7*d^4*x^(7/3)) - (e^4*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(5*d^5*x^(5/3)) + (e^5*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(3*d^6*x) - (e^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^7*x^(1/3
)) - (e^(13/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3
))^n]))/d^(15/2) - (I*b*e^(13/2)*n*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(ArcT
an[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[
e]*x^(1/3))]))/d^(15/2)))/5
```

Maple [F] time = 0.351, size = 0, normalized size = 0.

$$\int \frac{1}{x^{6}} \left(a + b \ln \left(c \left(d + e x^{\frac{2}{3}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{2} \log \left(\left(e x^{\frac{2}{3}} + d\right)^{n} c\right)^{2} + 2 \, a b \log \left(\left(e x^{\frac{2}{3}} + d\right)^{n} c\right) + a^{2}}{x^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(\left(ex^{\frac{2}{3}} + d\right)^{n} c\right) + a\right)^{2}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^6, x)
```

3.481 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=913

result too large to display

```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(2/3))^2)/(16*e^6) + (10*b^3*d^3*n^3*(d + e*x^(2/
3))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(2/3))^4)/(128*e^6) + (9*b^3*d*n^
3*(d + e*x^(2/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(2/3))^6)/(144*e^6) - (9
*a*b^2*d^5*n^2*x^(2/3))/e^5 + (9*b^3*d^5*n^3*x^(2/3))/e^5 - (9*b^3*d^5*n^2*
(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(2
/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x
^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(32*e^6) - (9*b^2*d*n^2*(d +
 e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*e^6) + (b^2*n^2*(d + e*
x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(24*e^6) + (9*b*d^5*n*(d + e*x
^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^6) - (45*b*d^4*n*(d + e*x^
(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) + (5*b*d^3*n*(d + e*x^
(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(2/
3))^4*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(16*e^6) + (9*b*d*n*(d + e*x^(2/3
))^5*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(10*e^6) - (b*n*(d + e*x^(2/3))^6*
(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) - (3*d^5*(d + e*x^(2/3))*(a + b
*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) + (15*d^4*(d + e*x^(2/3))^2*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^3)/(4*e^6) - (5*d^3*(d + e*x^(2/3))^3*(a + b*Log[c*
(d + e*x^(2/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*
x^(2/3))^n])^3)/(4*e^6) - (3*d*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3
))^n])^3)/(2*e^6) + ((d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/
(4*e^6)
```

Rubi [A] time = 1.03238, antiderivative size = 913, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6}+\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^6}{4e^6}-\frac{bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^6}{8e^6}+\frac{b^2n^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```


```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(2/3))^2)/(16*e^6) + (10*b^3*d^3*n^3*(d + e*x^(2/
3))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(2/3))^4)/(128*e^6) + (9*b^3*d*n^
3*(d + e*x^(2/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(2/3))^6)/(144*e^6) - (9
*a*b^2*d^5*n^2*x^(2/3))/e^5 + (9*b^3*d^5*n^3*x^(2/3))/e^5 - (9*b^3*d^5*n^2*
(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(2
/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x
^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(32*e^6) - (9*b^2*d*n^2*(d +
 e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*e^6) + (b^2*n^2*(d + e*
x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(24*e^6) + (9*b*d^5*n*(d + e*x
^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^6) - (45*b*d^4*n*(d + e*x^
(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) + (5*b*d^3*n*(d + e*x^
(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(2/
3))^4*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(16*e^6) + (9*b*d*n*(d + e*x^(2/3
))^5*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(10*e^6) - (b*n*(d + e*x^(2/3))^6*
```

```
(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) - (3*d^5*(d + e*x^(2/3))*(a + b
*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) + (15*d^4*(d + e*x^(2/3))^2*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^3)/(4*e^6) - (5*d^3*(d + e*x^(2/3))^3*(a + b*Log[c*
(d + e*x^(2/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*
x^(2/3))^n])^3)/(4*e^6) - (3*d*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3
))^n])^3)/(2*e^6) + ((d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/
(4*e^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx = \frac{3}{2}\text{Subst}\left(\int x^5 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right.}{e^5}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, x^{2/3}\right)}{2e^5} - \frac{(15d)\,\text{Subst}\left(\int (d+\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+ex^{2/3}\right)}{2e^6} - \frac{(15d)\,\text{Subst}\left(\int x^4 (a+b\log\right.}{2e^6}$$

$$= -\frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6}+\frac{15d^4\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\right.\right.}{4e^6}$$

$$= \frac{9bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^6} - \frac{45bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\right.}{8e^6}$$

$$= -\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+ex^{2/3}\right)^4}{128e^6}+\frac{9b}{}$$

$$= -\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+ex^{2/3}\right)^4}{128e^6}+\frac{9b}{}$$

Mathematica [A] time = 1.01787, size = 598, normalized size = 0.65

$$\frac{-60b\left(1800a^2\left(d^6-e^6x^4\right)-60abn\left(-30d^4e^2x^{4/3}+20d^3e^3x^2-15d^2e^4x^{8/3}+60d^5ex^{2/3}+147d^6+12de^5x^{10/3}-10e^6x^4\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (e*x^(2/3)*(36000*a^3*e^5*x^(10/3) + b^3*n^3*(809340*d^5 - 140070*d^4*e*x^(
2/3) + 41180*d^3*e^2*x^(4/3) - 13785*d^2*e^3*x^2 + 4368*d*e^4*x^(8/3) - 100
0*e^5*x^(10/3)) - 60*a*b^2*n^2*(8820*d^5 - 2610*d^4*e*x^(2/3) + 1140*d^3*e^
2*x^(4/3) - 555*d^2*e^3*x^2 + 264*d*e^4*x^(8/3) - 100*e^5*x^(10/3)) + 1800*
a^2*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 20*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 +
12*d*e^4*x^(8/3) - 10*e^5*x^(10/3))) - 280140*b^3*d^6*n^3*Log[d + e*x^(2/3)
] - 60*b*(b^2*n^2*(8820*d^6 + 8820*d^5*e*x^(2/3) - 2610*d^4*e^2*x^(4/3) + 1
140*d^3*e^3*x^2 - 555*d^2*e^4*x^(8/3) + 264*d*e^5*x^(10/3) - 100*e^6*x^4) -
 60*a*b*n*(147*d^6 + 60*d^5*e*x^(2/3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2
 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^(10/3) - 10*e^6*x^4) + 1800*a^2*(d^6 - e
^6*x^4))*Log[c*(d + e*x^(2/3))^n] + 1800*b^2*(b*n*(147*d^6 + 60*d^5*e*x^(2/
3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^
(10/3) - 10*e^6*x^4) - 60*a*(d^6 - e^6*x^4))*Log[c*(d + e*x^(2/3))^n]^2 - 3
6000*b^3*(d^6 - e^6*x^4)*Log[c*(d + e*x^(2/3))^n]^3)/(144000*e^6)
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [A] time = 1.12021, size = 918, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((e*x^(2/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(2/3) + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a^3*x^4 - 1/80*a^2*b*
e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d
^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6) -
 1/2400*(60*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(
10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(
2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (100*e^6*x^4 - 264*d*e^5*x^(10/3) + 5
55*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 + 1800*d^6*log(e*x^(2/3) + d)^2 + 261
0*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(2/3) + d) - 8820*d^5*e*x^(2/3))*n^2/e
^6)*a*b^2 - 1/144000*(1800*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4
 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/
3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c)^2 + e*n*((1000*e^6*x^4 -
 4368*d*e^5*x^(10/3) + 36000*d^6*log(e*x^(2/3) + d)^3 + 13785*d^2*e^4*x^(8/
3) - 41180*d^3*e^3*x^2 + 264600*d^6*log(e*x^(2/3) + d)^2 + 140070*d^4*e^2*x
^(4/3) + 809340*d^6*log(e*x^(2/3) + d) - 809340*d^5*e*x^(2/3))*n^2/e^7 - 60
*(100*e^6*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2
 + 1800*d^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^
(2/3) + d) - 8820*d^5*e*x^(2/3))*n*log((e*x^(2/3) + d)^n*c)/e^7))*b^3
```

Fricas [A] time = 3.21431, size = 2745, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/144000*(36000*b^3*e^6*x^4*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
+ 18*a^2*b*e^6*n - 36*a^3*e^6)*x^4 + 36000*(b^3*e^6*n^3*x^4 - b^3*d^6*n^3)*
log(e*x^(2/3) + d)^3 + 20*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 +
1800*a^2*b*d^3*e^3*n)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x^2 + 147*b^3*d^6*n^3
- 60*a*b^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^4 + 60*(b^3*e^6*n
^2*x^4 - b^3*d^6*n^2)*log(c) - 15*(b^3*d^2*e^4*n^3*x^2 - 4*b^3*d^5*e*n^3)*x
^(2/3) + 6*(2*b^3*d*e^5*n^3*x^3 - 5*b^3*d^4*e^2*n^3*x)*x^(1/3))*log(e*x^(2/
3) + d)^2 + 18000*(2*b^3*d^3*e^3*n*x^2 - (b^3*e^6*n - 6*a*b^2*e^6)*x^4)*log
(c)^2 - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100
*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^4 + 60*(19*b^3*d^3*e^3*
n^3 - 20*a*b^2*d^3*e^3*n^2)*x^2 - 1800*(b^3*e^6*n*x^4 - b^3*d^6*n)*log(c)^2
 - 60*(20*b^3*d^3*e^3*n^2*x^2 + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*n - 10*(b^3*
e^6*n^2 - 6*a*b^2*e^6*n)*x^4)*log(c) + 15*(588*b^3*d^5*e*n^3 - 240*a*b^2*d^
```

```
5*e*n^2 - (37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 + 60*(b^3*d^2*e^4
*n^2*x^2 - 4*b^3*d^5*e*n^2)*log(c))*x^(2/3) + 6*(4*(11*b^3*d*e^5*n^3 - 30*a
*b^2*d*e^5*n^2)*x^3 - 15*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x - 60
*(2*b^3*d*e^5*n^2*x^3 - 5*b^3*d^4*e^2*n^2*x)*log(c))*x^(1/3))*log(e*x^(2/3)
 + d) + 1200*(5*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^4 - 3*(19*b^
3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x^2)*log(c) + 15*(53956*b^3*d^5*e*n^3 -
 35280*a*b^2*d^5*e*n^2 + 7200*a^2*b*d^5*e*n - (919*b^3*d^2*e^4*n^3 - 2220*a
*b^2*d^2*e^4*n^2 + 1800*a^2*b*d^2*e^4*n)*x^2 - 1800*(b^3*d^2*e^4*n*x^2 - 4*
b^3*d^5*e*n)*log(c)^2 - 60*(588*b^3*d^5*e*n^2 - 240*a*b^2*d^5*e*n - (37*b^3
*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)*x^2)*log(c))*x^(2/3) + 6*(8*(91*b^3*d*e^
5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)*x^3 + 1800*(2*b^3*d*e^5*n*
x^3 - 5*b^3*d^4*e^2*n*x)*log(c)^2 - 5*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^
4*e^2*n^2 + 1800*a^2*b*d^4*e^2*n)*x - 60*(4*(11*b^3*d*e^5*n^2 - 30*a*b^2*d*
e^5*n)*x^3 - 15*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x)*log(c))*x^(1/3
))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)

[Out] Timed out
```

Giac [B] time = 2.15474, size = 3002, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")

[Out] 1/4*b^3*x^4*log(c)^3 + 3/4*a*b^2*x^4*log(c)^2 + 1/144000*(36000*(x^(2/3)*e
+ d)^6*e^(-5)*log(x^(2/3)*e + d)^3 - 216000*(x^(2/3)*e + d)^5*d*e^(-5)*log(
x^(2/3)*e + d)^3 + 540000*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d)^3
 - 720000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d)^3 + 540000*(x^(2/
3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)^3 - 216000*(x^(2/3)*e + d)*d^5*e^
(-5)*log(x^(2/3)*e + d)^3 - 18000*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e +
d)^2 + 129600*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d)^2 - 405000*(x^(
2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d)^2 + 720000*(x^(2/3)*e + d)^3*d^
3*e^(-5)*log(x^(2/3)*e + d)^2 - 810000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(
2/3)*e + d)^2 + 648000*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d)^2 + 60
00*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) - 51840*(x^(2/3)*e + d)^5*d*
e^(-5)*log(x^(2/3)*e + d) + 202500*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)
*e + d) - 480000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 810000*(
x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) - 1296000*(x^(2/3)*e + d)*d^
5*e^(-5)*log(x^(2/3)*e + d) - 1000*(x^(2/3)*e + d)^6*e^(-5) + 10368*(x^(2/3
)*e + d)^5*d*e^(-5) - 50625*(x^(2/3)*e + d)^4*d^2*e^(-5) + 160000*(x^(2/3)*
e + d)^3*d^3*e^(-5) - 405000*(x^(2/3)*e + d)^2*d^4*e^(-5) + 1296000*(x^(2/3
)*e + d)*d^5*e^(-5))*b^3*n^3*e^(-1) + 3/4*a^2*b*x^4*log(c) + 1/2400*(1800*(
x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^
(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*
```

e + d)^2 - 36000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*
(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)*d
^5*e^(-5)*log(x^(2/3)*e + d)^2 - 600*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e
 + d) + 4320*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) - 13500*(x^(2/3)
*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) + 24000*(x^(2/3)*e + d)^3*d^3*e^(-5
)*log(x^(2/3)*e + d) - 27000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d
) + 21600*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) + 100*(x^(2/3)*e +
d)^6*e^(-5) - 864*(x^(2/3)*e + d)^5*d*e^(-5) + 3375*(x^(2/3)*e + d)^4*d^2*e
^(-5) - 8000*(x^(2/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(2/3)*e + d)^2*d^4*e^(
-5) - 21600*(x^(2/3)*e + d)*d^5*e^(-5))*b^3*n^2*e^(-1)*log(c) + 1/80*(60*(x
^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*
log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) -
1200*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^
2*d^4*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3
)*e + d) - 10*(x^(2/3)*e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 22
5*(x^(2/3)*e + d)^4*d^2*e^(-5) + 400*(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^
(2/3)*e + d)^2*d^4*e^(-5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*b^3*n*e^(-1)*lo
g(c)^2 + 1/4*a^3*x^4 + 1/2400*(1800*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e
+ d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^(
2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d)^2 - 36000*(x^(2/3)*e + d)^3*d^3
*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/
3)*e + d)^2 - 10800*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d)^2 - 600*(
x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) + 4320*(x^(2/3)*e + d)^5*d*e^(-5
)*log(x^(2/3)*e + d) - 13500*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d
) + 24000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) - 27000*(x^(2/3)*
e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) + 21600*(x^(2/3)*e + d)*d^5*e^(-5)*l
og(x^(2/3)*e + d) + 100*(x^(2/3)*e + d)^6*e^(-5) - 864*(x^(2/3)*e + d)^5*d*
e^(-5) + 3375*(x^(2/3)*e + d)^4*d^2*e^(-5) - 8000*(x^(2/3)*e + d)^3*d^3*e^(
-5) + 13500*(x^(2/3)*e + d)^2*d^4*e^(-5) - 21600*(x^(2/3)*e + d)*d^5*e^(-5)
)*a*b^2*n^2*e^(-1) + 1/40*(60*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) -
 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*
d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/
3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2
/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) - 10*(x^(2/3)*e + d)^6*e^(-5) + 72
*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3)*e + d)^4*d^2*e^(-5) + 400*(x^(2/
3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e + d)^2*d^4*e^(-5) + 360*(x^(2/3)*e
+ d)*d^5*e^(-5))*a*b^2*n*e^(-1)*log(c) + 1/80*(60*(x^(2/3)*e + d)^6*e^(-5)*
log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) + 90
0*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*
d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3
)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) - 10*(x^(2/3)*
e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3)*e + d)^4*d^2
e^(-5) + 400(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e + d)^2*d^4*e^(-
5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*a^2*b*n*e^(-1)

3.482 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=449

$$\frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9bd^2}{}$$

```
[Out] (9*b^3*d*n^3*(d + e*x^(2/3))^2)/(8*e^3) - (b^3*n^3*(d + e*x^(2/3))^3)/(9*e^
3) + (9*a*b^2*d^2*n^2*x^(2/3))/e^2 - (9*b^3*d^2*n^3*x^(2/3))/e^2 + (9*b^3*d
^2*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^3 - (9*b^2*d*n^2*(d + e*
x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^3) + (b^2*n^2*(d + e*x^(2
/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e*x^(2/3
))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (9*b*d*n*(d + e*x^(2/3))^2
*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*e^3) - (b*n*(d + e*x^(2/3))^3*(a +
b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (3*d^2*(d + e*x^(2/3))*(a + b*Log[
c*(d + e*x^(2/3))^n])^3)/(2*e^3) - (3*d*(d + e*x^(2/3))^2*(a + b*Log[c*(d +
 e*x^(2/3))^n])^3)/(2*e^3) + ((d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3)
)^n])^3)/(2*e^3)
```

Rubi [A] time = 0.458871, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9bd^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```


```
[Out] (9*b^3*d*n^3*(d + e*x^(2/3))^2)/(8*e^3) - (b^3*n^3*(d + e*x^(2/3))^3)/(9*e^
3) + (9*a*b^2*d^2*n^2*x^(2/3))/e^2 - (9*b^3*d^2*n^3*x^(2/3))/e^2 + (9*b^3*d
^2*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^3 - (9*b^2*d*n^2*(d + e*
x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^3) + (b^2*n^2*(d + e*x^(2
/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e*x^(2/3
))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (9*b*d*n*(d + e*x^(2/3))^2
*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*e^3) - (b*n*(d + e*x^(2/3))^3*(a +
b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (3*d^2*(d + e*x^(2/3))*(a + b*Log[
c*(d + e*x^(2/3))^n])^3)/(2*e^3) - (3*d*(d + e*x^(2/3))^2*(a + b*Log[c*(d +
 e*x^(2/3))^n])^3)/(2*e^3) + ((d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3)
)^n])^3)/(2*e^3)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{3}{2}\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,x^{2/3}\right)$$

$$=\frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}-\frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}\right.\right.$$

$$=\frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,x^{2/3}\right)}{2e^2}-\frac{(3d)\,\text{Subst}\left(\int(d+e\right.}{}$$

$$=\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+ex^{2/3}\right)}{2e^3}-\frac{(3d)\,\text{Subst}\left(\int x\left(a+b\log\left(c\right.\right.\right.}{e^3}$$

$$=\frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}-\frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{2e^3}$$

$$=-\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}+\frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\right.}{4e^3}$$

$$=\frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(\right.}{}$$

$$=\frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9b^3d^2n^3x^{2/3}}{e^2}+\frac{9b^3d^2}{}$$

Mathematica [A] time = 0.407071, size = 428, normalized size = 0.95

$$\frac{6b\left(18a^2\left(d^3+e^3x^2\right)-6abn\left(6d^2ex^{2/3}+11d^3-3de^2x^{4/3}+2e^3x^2\right)+b^2n^2\left(66d^2ex^{2/3}+66d^3-15de^2x^{4/3}+4e^3x^2\right)\right)\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (36*a^3*d^3 - 198*a^2*b*d^3*n - 108*a^2*b*d^2*e*n*x^(2/3) + 396*a*b^2*d^2*e
*n^2*x^(2/3) - 510*b^3*d^2*e*n^3*x^(2/3) + 54*a^2*b*d*e^2*n*x^(4/3) - 90*a*
b^2*d*e^2*n^2*x^(4/3) + 57*b^3*d*e^2*n^3*x^(4/3) + 36*a^3*e^3*x^2 - 36*a^2*
b*e^3*n*x^2 + 24*a*b^2*e^3*n^2*x^2 - 8*b^3*e^3*n^3*x^2 + 114*b^3*d^3*n^3*Lo
g[d + e*x^(2/3)] + 6*b*(18*a^2*(d^3 + e^3*x^2) - 6*a*b*n*(11*d^3 + 6*d^2*e*
x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2) + b^2*n^2*(66*d^3 + 66*d^2*e*x^(2/3)
 - 15*d*e^2*x^(4/3) + 4*e^3*x^2))*Log[c*(d + e*x^(2/3))^n] + 18*b^2*(6*a*(d
^3 + e^3*x^2) - b*n*(11*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2
))*Log[c*(d + e*x^(2/3))^n]^2 + 36*b^3*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3)
)^n]^3)/(72*e^3)
```

Maple [F] time = 0.347, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [A] time = 1.11061, size = 653, normalized size = 1.45

$$\frac{1}{2}b^3x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+\frac{3}{2}ab^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+\frac{1}{4}a^2ben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)+\frac{3}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(2/3) + d)^
n*c)^2 + 1/4*a^2*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x
^(4/3) + 6*d^2*x^(2/3))/e^3) + 3/2*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) + 1/2
*a^3*x^2 + 1/12*(6*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x
^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*c) + (4*e^3*x^2 - 18*d^3
*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*d^3*log(e*x^(2/3) + d) + 66*d
^2*e*x^(2/3))*n^2/e^3)*a*b^2 + 1/72*(18*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 -
 (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*c)^
2 + e*n*((36*d^3*log(e*x^(2/3) + d)^3 - 8*e^3*x^2 + 198*d^3*log(e*x^(2/3) +
 d)^2 + 57*d*e^2*x^(4/3) + 510*d^3*log(e*x^(2/3) + d) - 510*d^2*e*x^(2/3))*
n^2/e^4 + 6*(4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 6
6*d^3*log(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n*log((e*x^(2/3) + d)^n*c)/e^4
))*b^3
```

Fricas [A] time = 2.3055, size = 1590, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72*(36*b^3*e^3*x^2*log(c)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x^2*log(c)^2 +
 36*(b^3*e^3*n^3*x^2 + b^3*d^3*n^3)*log(e*x^(2/3) + d)^3 + 12*(2*b^3*e^3*n^
2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x^2*log(c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2*e^
3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x^2 + 18*(3*b^3*d*e^2*n^3*x^(4/3) - 6*b^
3*d^2*e*n^3*x^(2/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(b^3*e^3*n^3 - 3
*a*b^2*e^3*n^2)*x^2 + 6*(b^3*e^3*n^2*x^2 + b^3*d^3*n^2)*log(c))*log(e*x^(2/
3) + d)^2 + 6*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 2*(2*b^
3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n)*x^2 + 18*(b^3*e^3*n*x^2 + b^3*
d^3*n)*log(c)^2 - 6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n^2 - 3*a*
b^2*e^3*n)*x^2)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a
*b^2*d^2*e*n^2)*x^(2/3) + 3*(6*b^3*d*e^2*n^2*x*log(c) - (5*b^3*d*e^2*n^3 -
6*a*b^2*d*e^2*n^2)*x)*x^(1/3))*log(e*x^(2/3) + d) - 6*(85*b^3*d^2*e*n^3 + 1
8*b^3*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*
d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) + 3*(18*b^3*d*e^2*n*x*log(c)^2
 - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n)*x*log(c) + (19*b^3*d*e^2*n^3 - 30*
a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n)*x)*x^(1/3))/e^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.30135, size = 1050, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*log(c)^3 + 1/72*(36*x^2*log(x^(2/3)*e + d)^3 + (36*d^3*log(x^(2
/3)*e + d)^3 - 36*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d)^2 + 162*(x^(2/3)*e +
 d)^2*d*log(x^(2/3)*e + d)^2 - 324*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d)^2
 + 24*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) - 162*(x^(2/3)*e + d)^2*d*log(x^
(2/3)*e + d) + 648*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) - 8*(x^(2/3)*e +
d)^3 + 81*(x^(2/3)*e + d)^2*d - 648*(x^(2/3)*e + d)*d^2)*e^(-3))*b^3*n^3 +
1/12*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2/3)*e + d)^2 - 12*(x^(2
/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)^2*d*log(x^(2/3)*e + d)
 - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x^(2/3)*e + d)^3 - 27*(x
^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))*b^3*n^2*log(c) + 1/4*(
6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e + d)) + (3*d*x^(
4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b^3*n*log(c)^2 + 3/2*a*b^2*x
^2*log(c)^2 + 1/12*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2/3)*e + d
)^2 - 12*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)^2*d*log(
x^(2/3)*e + d) - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x^(2/3)*e
+ d)^3 - 27*(x^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))*a*b^2*n^
2 + 1/2*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e + d)) +
 (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a*b^2*n*log(c) + 3/
2*a^2*b*x^2*log(c) + 1/4*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(
x^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a^
2*b*n + 1/2*a^3*x^2
```

3.483 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=139

$$-9b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{9}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+9b$$

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^3*Log[-((e*x^(2/3))/d)])/2 + (9*b*n*(a
+ b*Log[c*(d + e*x^(2/3))^n])^2*PolyLog[2, 1 + (e*x^(2/3))/d])/2 - 9*b^2*n^
2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[3, 1 + (e*x^(2/3))/d] + 9*b^3*n^
3*PolyLog[4, 1 + (e*x^(2/3))/d]
```

Rubi [A] time = 0.201417, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-9b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{9}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+9b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^3*Log[-((e*x^(2/3))/d)])/2 + (9*b*n*(a
+ b*Log[c*(d + e*x^(2/3))^n])^2*PolyLog[2, 1 + (e*x^(2/3))/d])/2 - 9*b^2*n^
2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[3, 1 + (e*x^(2/3))/d] + 9*b^3*n^
3*PolyLog[4, 1 + (e*x^(2/3))/d]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) - \frac{1}{2}(9ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b}{d}\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) - \frac{1}{2}(9bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))}{}\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\text{L}$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\text{L}$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\text{L}$$

Mathematica [B] time = 0.192884, size = 339, normalized size = 2.44

$$\frac{9}{2}b^2n^2\left(-2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)+2\log\left(d+ex^{2/3}\right)\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+\log\left(-\frac{ex^{2/3}}{d}\right)\log^2\left(d+ex^{2/3}\right)\right)\left(a+b\,\text{lo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*((Log[d + e*x^(
2/3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2
) + (9*b^2*n^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*(L
og[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2/3)]*PolyLog[2,
 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2 + (3*b^3*n^3*(Log
[d + e*x^(2/3)]^3*Log[-((e*x^(2/3))/d)] + 3*Log[d + e*x^(2/3)]^2*PolyLog[2,
 1 + (e*x^(2/3))/d] - 6*Log[d + e*x^(2/3)]*PolyLog[3, 1 + (e*x^(2/3))/d] +
```

```
6*PolyLog[4, 1 + (e*x^(2/3))/d]))/2
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)^3\log(x)+\int -\frac{\left(2\,b^3enx\log(x)-3\left(b^3e\log(c)+ab^2e\right)x-3\left(b^3d\log(c)+ab^2d\right)x^{\frac{1}{3}}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((e*x^(2/3) + d)^n)^3*log(x) + integrate(-((2*b^3*e*n*x*log(x) - 3*(
b^3*e*log(c) + a*b^2*e)*x - 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*log((e*x^(2
/3) + d)^n)^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a
^3*e)*x - 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c
)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) - (b^3*d*
log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e*x^2 +
 d*x^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x, x)
```

3.484 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=451

$$-\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right.}{d^3}$$

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^3*x^
(2/3)) - (3*b^2*e^3*n^2*Log[1 - d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2
/3))^n]))/(2*d^3) - (3*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*d*x^(4/
3)) + (3*b*e^2*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*d^3
*x^(2/3)) + (3*b*e^3*n*Log[1 - d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/
3))^n])^2)/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/(2*x^2) - (3*b^2*e^
3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/d^3 + (b^3*e^
3*n^3*Log[x])/d^3 + (3*b^3*e^3*n^3*PolyLog[2, d/(d + e*x^(2/3))])/(2*d^3) -
 (3*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, d/(d + e*x^(2/3
))])/d^3 - (3*b^3*e^3*n^3*PolyLog[2, 1 + (e*x^(2/3))/d])/d^3 - (3*b^3*e^3*n
^3*PolyLog[3, d/(d + e*x^(2/3))])/d^3
```

Rubi [A] time = 1.00763, antiderivative size = 428, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}-\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)}{2d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^3*x^
(2/3)) + (3*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*d^3) - (3*b*e*n*
(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*d*x^(4/3)) + (3*b*e^2*n*(d + e*x^(2/
3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*d^3*x^(2/3)) - (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n])^3)/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/(2*x^2
) - (9*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/
(2*d^3) + (3*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d
)])/(2*d^3) + (b^3*e^3*n^3*Log[x])/d^3 - (9*b^3*e^3*n^3*PolyLog[2, 1 + (e*x
^(2/3))/d])/(2*d^3) + (3*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyL
og[2, 1 + (e*x^(2/3))/d])/d^3 - (3*b^3*e^3*n^3*PolyLog[3, 1 + (e*x^(2/3))/d
])/d^3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \frac{3}{2}\operatorname{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\operatorname{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)}\,dx, x\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\operatorname{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\operatorname{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)}{2d} - $$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} - \frac{(3ben)\operatorname{Subs}}{}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} + \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right.\right.}{2d^3x^{2/3}}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} - \frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right.\right.}{4dx^{4/3}}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} + \frac{3be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.}{4d^3}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} + \frac{3be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.}{4d^3}$$

Mathematica [A] time = 0.781538, size = 764, normalized size = 1.69

$$\frac{-6b^2n^2\left(-2e^3x^2\operatorname{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+\left(d^3+e^3x^2\right)\log^2\left(d+ex^{2/3}\right)+\log\left(d+ex^{2/3}\right)\left(d^2ex^{2/3}-2de^2x^{4/3}-2e^3x^2\log\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))
^n])^2 + 6*b*d*e^2*n*x^(4/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x
^(2/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] +
b*Log[c*(d + e*x^(2/3))^n])^2 - 6*b*e^3*n*x^2*Log[d + e*x^(2/3)]*(a - b*n*L
og[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*d^3*(a - b*n*Log[d +
e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3 + 4*b*e^3*n*x^2*(a - b*n*Log[d +
 e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log
[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((d^3 + e^3*x^2)*Log[d + e*x^
(2/3)]^2 + e^2*x^(4/3)*(d + 3*e*x^(2/3)*Log[-((e*x^(2/3))/d)]) + Log[d + e*
x^(2/3)]*(d^2*e*x^(2/3) - 2*d*e^2*x^(4/3) - 3*e^3*x^2 - 2*e^3*x^2*Log[-((e*
x^(2/3))/d)]) - 2*e^3*x^2*PolyLog[2, 1 + (e*x^(2/3))/d]) + b^3*n^3*(-6*d*e^
2*x^(4/3)*Log[d + e*x^(2/3)] - 6*e^3*x^2*Log[d + e*x^(2/3)] - 3*d^2*e*x^(2/
3)*Log[d + e*x^(2/3)]^2 + 6*d*e^2*x^(4/3)*Log[d + e*x^(2/3)]^2 + 9*e^3*x^2*
Log[d + e*x^(2/3)]^2 - 2*d^3*Log[d + e*x^(2/3)]^3 - 2*e^3*x^2*Log[d + e*x^(
```

```
2/3)]^3 + 6*e^3*x^2*Log[-((e*x^(2/3))/d)] - 18*e^3*x^2*Log[d + e*x^(2/3)]*L
og[-((e*x^(2/3))/d)] + 6*e^3*x^2*Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)]
 + 6*e^3*x^2*(-3 + 2*Log[d + e*x^(2/3)])*PolyLog[2, 1 + (e*x^(2/3))/d] - 12
*e^3*x^2*PolyLog[3, 1 + (e*x^(2/3))/d]))/(4*d^3*x^2)
```

Maple [F] time = 0.399, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)^3}{2x^2}+\int \frac{\left(b^3enx+3\left(b^3e\log(c)+ab^2e\right)x+3\left(b^3d\log(c)+ab^2d\right)x^{\frac{1}{3}}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)^2+\left(b^3e\log(c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log((e*x^(2/3) + d)^n)^3/x^2 + integrate(((b^3*e*n*x + 3*(b^3*e*lo
g(c) + a*b^2*e)*x + 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*log((e*x^(2/3) + d)
^n)^2 + (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x
+ 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^2 + 2*
a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + (b^3*d*log(c)^3
 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e*x^4 + d*x^(10
/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^3, x)
```

3.485 $\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^3 \, dx$

Optimal. Leaf size=793

result too large to display

```
[Out] (4504*a*b^2*d^4*n^2*x^(1/3))/(315*e^4) - (3475504*b^3*d^4*n^3*x^(1/3))/(992
25*e^4) + (637984*b^3*d^3*n^3*x)/(297675*e^3) - (221344*b^3*d^2*n^3*x^(5/3)
)/(496125*e^2) + (3088*b^3*d*n^3*x^(7/3))/(27783*e) - (16*b^3*n^3*x^3)/729
+ (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(99225*e^(9/2
)) - (((4504*I)/315)*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e
^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqr
t[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(315*e^(9/2)) + (4504*b^3*d^4*n^2*x^(
1/3)*Log[c*(d + e*x^(2/3))^n])/(315*e^4) - (1984*b^2*d^3*n^2*x*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(945*e^3) + (1144*b^2*d^2*n^2*x^(5/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(1575*e^2) - (128*b^2*d*n^2*x^(7/3)*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(441*e) + (8*b^2*n^2*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/81
 - (4504*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d
+ e*x^(2/3))^n]))/(315*e^(9/2)) - (2*b*d^4*n*x^(1/3)*(a + b*Log[c*(d + e*x^
(2/3))^n])^2)/e^4 + (2*b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*e^3
) - (2*b*d^2*n*x^(5/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(5*e^2) + (2*b*d
*n*x^(7/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(7*e) - (2*b*n*x^3*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^2)/9 + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/3 -
 (((4504*I)/315)*b^3*d^(9/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x^(1/3))])/e^(9/2) + (2*b*d^5*n*Unintegrable[(a + b*Log[c*(d + e*x^(2
/3))^n])^2/((d + e*x^(2/3))*x^(2/3)), x])/(3*e^4)
```

Rubi [A] time = 3.00639, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```


```
[Out] (4504*a*b^2*d^4*n^2*x^(1/3))/(315*e^4) - (3475504*b^3*d^4*n^3*x^(1/3))/(992
25*e^4) + (637984*b^3*d^3*n^3*x)/(297675*e^3) - (221344*b^3*d^2*n^3*x^(5/3)
)/(496125*e^2) + (3088*b^3*d*n^3*x^(7/3))/(27783*e) - (16*b^3*n^3*x^3)/729
+ (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(99225*e^(9/2
)) - (((4504*I)/315)*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e
^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqr
t[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(315*e^(9/2)) + (4504*b^3*d^4*n^2*x^(
1/3)*Log[c*(d + e*x^(2/3))^n])/(315*e^4) - (1984*b^2*d^3*n^2*x*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(945*e^3) + (1144*b^2*d^2*n^2*x^(5/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(1575*e^2) - (128*b^2*d*n^2*x^(7/3)*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(441*e) + (8*b^2*n^2*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/81
 - (4504*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d
+ e*x^(2/3))^n]))/(315*e^(9/2)) - (2*b*d^4*n*x^(1/3)*(a + b*Log[c*(d + e*x^
(2/3))^n])^2)/e^4 + (2*b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*e^3
) - (2*b*d^2*n*x^(5/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(5*e^2) + (2*b*d
*n*x^(7/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(7*e) - (2*b*n*x^3*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^2)/9 + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/3 -
 (((4504*I)/315)*b^3*d^(9/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
```

```
rt[e]*x^(1/3))])/e^(9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*
(d + e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2ben)\,\text{Subst}\left(\int \frac{x^{10}\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right.}{d+ex^2}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2ben)\,\text{Subst}\left(\int\left(\frac{d^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right.}{e^5}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2bn)\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2\right.$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3e^3} -$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3e^3} -$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3e^3} -$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{1984b^2d^3n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{945e^3} + \frac{1144b^2d^2n^2x^{5/3}\left(a\right.}{1}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} + \frac{4504b^3d^4n^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{315e^4} - \frac{1984b^2d^3n^2x\left(a+b\log\right.}{945e^3}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3d^2n^3x^{5/3}}{496125e^2} +$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3d^2n^3x^{5/3}}{496125e^2} +$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3d^2n^3x^{5/3}}{496125e^2} +$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3d^2n^3x^{5/3}}{496125e^2} +$$

Mathematica [A] time = 9.0109, size = 3146, normalized size = 3.97

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*x^(1/3)*(32*d^4 - 32*d^4*Sqrt[1 - (d + e*x^(2/3))/d] + 128*d^3*Sqr
t[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) - 192*d^2*Sqrt[1 - (d + e*x^(2/3))
/d]*(d + e*x^(2/3))^2 + 128*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3
 - 32*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4 + 1584*d^3*(d + e*x^(2/
3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 4536
*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x
^(2/3))/d] + 3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2
, 2, 2}, (d + e*x^(2/3))/d] - 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7
/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] + 3024*d^3*(d + e*x^(2/3)
)*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] -
3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2}
, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1,
 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] - 240*d^4*Log[d + e*x^(2/3)] +
240*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 672*d^3*Sqrt[1 - (
d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3)] + 576*d^2*Sqrt[1 - (d
+ e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2/3)] - 96*d*Sqrt[1 - (d + e
*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2/3)] - 48*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3)] - 3780*d^3*(d + e*x^(2/3))*Hyp
ergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]
+ 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d
+ e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 3024*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 37
80*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e
*x^(2/3))/d]*Log[d + e*x^(2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPF
Q[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 284*d
^4*Log[d + e*x^(2/3)]^2 - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^2 + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^2 - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
^(2/3)]^2 + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^2 - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^2 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 + 945*d^3*(d + e*x^(2/3))*Hypergeome
tricPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 - 70*
d^4*Log[d + e*x^(2/3)]^3 + 70*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^3 - 280*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^3 + 420*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
^(2/3)]^3 - 280*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^3 + 70*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^3 + 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*(1 + 3*Log[d + e*x^(2/3)] + Log[d + e*x^(2/3)]^2) - 144*d
^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/
d]*(6 + 11*Log[d + e*x^(2/3)] + 3*Log[d + e*x^(2/3)]^2)))/(210*e^4*Sqrt[1 -
 (d + e*x^(2/3))/d]) + (b^2*n^2*x^(1/3)*(-120*d^4 + 120*d^4*Sqrt[1 - (d + e
*x^(2/3))/d] - 336*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) + 288*d^
2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2 - 48*d*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^3 - 24*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^
4 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e
*x^(2/3))/d] + 432*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {
2, 2, 2}, (d + e*x^(2/3))/d] - 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{
-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*Hy
pergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 945*d^3*(d
```

```
 + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))
/d] + 284*d^4*Log[d + e*x^(2/3)] - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[
d + e*x^(2/3)] + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d
+ e*x^(2/3)] - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d
+ e*x^(2/3)] + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d +
e*x^(2/3)] - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(
2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d
 + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 945*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 105*d^4*
Log[d + e*x^(2/3)]^2 + 105*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3
)]^2 - 420*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3
)]^2 + 630*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2
/3)]^2 - 420*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2
/3)]^2 + 105*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3
)]^2 + 756*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (d +
 e*x^(2/3))/d]*(3 + 2*Log[d + e*x^(2/3)]) - 72*d^3*(d + e*x^(2/3))*Hypergeo
metricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*(11 + 6*Log[d + e*x^(2/3
)]))*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n])))/(105*e^4
*Sqrt[1 - (d + e*x^(2/3))/d]) - (2*b*d^4*n*x^(1/3)*(a + b*(-(n*Log[d + e*x^
(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/e^4 + (2*b*d^3*n*x*(a + b*(-(n*Log[
d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/(3*e^3) - (2*b*d^2*n*x^(5/3
)*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/(5*e^2) +
 (2*b*d*n*x^(7/3)*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n
]))^2)/(7*e) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*(-(n
*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/e^(9/2) + b*n*x^3*Log[
d + e*x^(2/3)]*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))
^2 + (x^3*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2*(3
*a - 2*b*n + 3*b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n])))/9
```

Maple [A] time = 0.341, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \ln \left(c \left(d + e x^{\frac{2}{3}}\right)^{n}\right)\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^{3} x^{2} \log\left(\left(e x^{\frac{2}{3}} + d\right)^{n} c\right)^{3} + 3 \, a b^{2} x^{2} \log\left(\left(e x^{\frac{2}{3}} + d\right)^{n} c\right)^{2} + 3 \, a^{2} b x^{2} \log\left(\left(e x^{\frac{2}{3}} + d\right)^{n} c\right) + a^{3} x^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*x^2*log((e*x^(2/3) +
d)^n*c)^2 + 3*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) + a^3*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3*x^2, x)
```

3.486 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=485

$$-\frac{2bd^2n\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}\left(d+ex^{2/3}\right)},x\right)}{e}+\frac{32ib^3d^{3/2}n^3\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+\right.}{e^{3/2}}$$

```
[Out] (-32*a*b^2*d*n^2*x^(1/3))/e + (208*b^3*d*n^3*x^(1/3))/(3*e) - (16*b^3*n^3*x
)/9 - (208*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) +
 ((32*I)*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) + (64
*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x^(1/3))])/e^(3/2) - (32*b^3*d*n^2*x^(1/3)*Log[c*(d + e*x^(2/3
))^n])/e + (8*b^2*n^2*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 + (32*b^2*d^(3/
2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/
e^(3/2) + (6*b*d*n*x^(1/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e - 2*b*n*x*
(a + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a + b*Log[c*(d + e*x^(2/3))^n])^3 +
 ((32*I)*b^3*d^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^
(1/3))])/e^(3/2) - (2*b*d^2*n*Unintegrable[(a + b*Log[c*(d + e*x^(2/3))^n])
^2/((d + e*x^(2/3))*x^(2/3)), x])/e
```

Rubi [A] time = 1.07922, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (-32*a*b^2*d*n^2*x^(1/3))/e + (208*b^3*d*n^3*x^(1/3))/(3*e) - (16*b^3*n^3*x
)/9 - (208*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) +
 ((32*I)*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) + (64
*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x^(1/3))])/e^(3/2) - (32*b^3*d*n^2*x^(1/3)*Log[c*(d + e*x^(2/3
))^n])/e + (8*b^2*n^2*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 + (32*b^2*d^(3/
2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/
e^(3/2) + (6*b*d*n*x^(1/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e - 2*b*n*x*
(a + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a + b*Log[c*(d + e*x^(2/3))^n])^3 +
 ((32*I)*b^3*d^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^
(1/3))])/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x
^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\, d\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6ben)\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{e^2}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6bn)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2 dx\right.$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a\right.$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a\right.$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a\right.$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} - \frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} + \frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^2\right.\right.\right.$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x + \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}} - \frac{32b^3d}{}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{32ib^3d}{}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{32ib^3d}{}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{32ib^3d}{}$$

Mathematica [A] time = 1.24265, size = 598, normalized size = 1.23

$$\frac{3b^2n^2x\left(-a-b\log\left(c\left(d+ex^{2/3}\right)^n\right)+bn\log\left(d+ex^{2/3}\right)\right)\left(3\left(d+ex^{2/3}\right){}_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;\frac{x^{2/3}e}{d}+1\right)+\log\left(d+ex^{2/3}\right)\left(\right.\right.}{d\left(-\frac{ex^{2/3}}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] -(b^3*n^3*x*(-18*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2,
2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(18*(d + e*x^(2/3))*Hyper
geometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(
2/3)]*(-9*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (e*x^
(2/3))/d] + 2*(d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))))/(2*d*
(-((e*x^(2/3))/d))^(3/2)) + (3*b^2*n^2*x*(3*(d + e*x^(2/3))*HypergeometricP
FQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(-3*
(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (e*x^(2/3))/d]
+ (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d + e
*x^(2/3)] - b*Log[c*(d + e*x^(2/3))^n]))/(d*(-((e*x^(2/3))/d))^(3/2)) + (6*
b*d*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/
e - (6*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x^(
2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/e^(3/2) + 3*b*n*x*Log[d + e*x^(2/3)]
*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a - b*n*L
og[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*(a - 2*b*n - b*n*Log[d +
e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])
```

Maple [A] time = 0.334, size = 0, normalized size = 0.

$$\int\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)^
2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(\left(ex^{\frac{2}{3}} + d\right)^n c\right) + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3, x)
```

3.487 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=318

$$-\frac{2be^2n\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}\left(d+ex^{2/3}\right)},x\right)}{d}+\frac{24ib^3e^{3/2}n^3\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}+\frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+\right.}{d^{3/2}}$$

```
[Out] ((24*I)*b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) + (48*
b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/d^(3/2) + (24*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(3/2) - (6*b*e*n*(a + b*Log
[c*(d + e*x^(2/3))^n])^2)/(d*x^(1/3)) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/
x + ((24*I)*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]
*x^(1/3))])/d^(3/2) - (2*b*e^2*n*Unintegrable[(a + b*Log[c*(d + e*x^(2/3))^
n])^2/((d + e*x^(2/3))*x^(2/3)), x])/d
```

Rubi [A] time = 0.486985, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] ((24*I)*b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) + (48*
b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/d^(3/2) + (24*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(3/2) - (6*b*e*n*(a + b*Log
[c*(d + e*x^(2/3))^n])^2)/(d*x^(1/3)) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/
x + ((24*I)*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]
*x^(1/3))])/d^(3/2) - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d +
e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}dx = 3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^4}dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + (6ben)\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^2\left(d+ex^2\right)}dx,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + (6ben)\,\mathrm{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{dx^2} - \right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + \frac{(6ben)\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^2}dx, x, \sqrt[3]{x}\right)}{d} -$$

$$= -\frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} - \frac{\left(6be^2n\right)\mathrm{Sul}}{}$$

$$= \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{d\sqrt[3]{x}}$$

$$= \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{d\sqrt[3]{x}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right.}{d^{3/2}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{24b^2e^{3/}}{}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{24b^2e^{3/}}{}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{24b^2e^{3/}}{}$$

Mathematica [A] time = 7.16765, size = 1028, normalized size = 3.23

$$\frac{b^3\left(\frac{d^{5/2}\log^3\left(d+ex^{2/3}\right)}{\sqrt{-d}} + 6\sqrt{-d}\left(d+ex^{2/3}\right)^{3/2}\left(\frac{ex^{2/3}}{d+ex^{2/3}}\right)^{3/2}\sin^{-1}\left(\frac{\sqrt{d}}{\sqrt{d+ex^{2/3}}}\right)\log^2\left(d+ex^{2/3}\right) - 6\sqrt{-d^2}ex^{2/3}\log^2\left(d+ex^{2/3}\right) - 2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] (-3*b^2*n^2*(-3*d*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*HypergeometricPF
Q[{1, 1, 1, 5/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] - d*Log[d + e*x^(2/3)]*(-4*
e*(-1 + Sqrt[-((e*x^(2/3))/d)])*x^(2/3) + 4*d*(-((e*x^(2/3))/d))^(3/2)*Log[
(1 + Sqrt[-((e*x^(2/3))/d)])/2] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d +
e*x^(2/3)]))*(-a + b*n*Log[d + e*x^(2/3)] - b*Log[c*(d + e*x^(2/3))^n]))/(d
^2*x) - (6*b*e*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^
2)/(d*x^(1/3)) - (6*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*
Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/d^(3/2) - (3*b*n*Log[d
+ e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/x
 - (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3/x + (b^3*n^3
*(48*Sqrt[-d^2]*e*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*HypergeometricP
FQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^(2/3))] - 12*d*Sqrt[-d
^2]*(-((e*x^(2/3))/d))^(3/2)*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 24*Sqr
t[d]*(e*x^(2/3))^(3/2)*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[d + e*x^(2/3)]
 + 24*Sqrt[-d^2]*e*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*Hypergeometric
PFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^(2/3))]*Log[d + e*x^(2/3)] - 6*
Sqrt[-d^2]*e*x^(2/3)*Log[d + e*x^(2/3)]^2 + 6*Sqrt[-d]*(d + e*x^(2/3))^(3/2
)*((e*x^(2/3))/(d + e*x^(2/3)))^(3/2)*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]]*L
og[d + e*x^(2/3)]^2 + (d^(5/2)*Log[d + e*x^(2/3)]^3)/Sqrt[-d] + 24*Sqrt[d]*
(e*x^(2/3))^(3/2)*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[1 + (e*x^(2/3))/d]
+ 24*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*Log[(1 + Sqrt[-((e*x^(2/3))/d)])
/2]*Log[1 + (e*x^(2/3))/d] - 6*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*Log[1
+ (e*x^(2/3))/d]^2 + 24*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*PolyLog[2, 1/
2 - Sqrt[-((e*x^(2/3))/d)]/2]))/(Sqrt[-d]*d^(3/2)*x)
```

Maple [A] time = 0.341, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3 + 3\,ab^2 \log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2 + 3\,a^2 b \log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right) + a^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^2, x)
```

3.488 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=631

$$\frac{2be^5n\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}\left(d+ex^{2/3}\right)},x\right)}{3d^4}-\frac{1408ib^3e^{9/2}n^3\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}-\frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{105d^4\sqrt[3]{x}}$$

```
[Out] (-16*b^3*e^3*n^3)/(105*d^3*x) + (16*b^3*e^4*n^3)/(7*d^4*x^(1/3)) + (1376*b^
3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(105*d^(9/2)) - (((1408*I)
/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(9/2) - (2816*
b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/(105*d^(9/2)) - (8*b^2*e^2*n^2*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(35*d^2*x^(5/3)) + (32*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3)
)^n]))/(35*d^3*x) - (568*b^2*e^4*n^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(105
*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(105*d^(9/2)) - (2*b*e*n*(a + b*Log[c*(d + e
*x^(2/3))^n])^2)/(7*d*x^(7/3)) + (2*b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]
)^2)/(5*d^2*x^(5/3)) - (2*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*d^
3*x) + (2*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(d^4*x^(1/3)) - (a +
b*Log[c*(d + e*x^(2/3))^n])^3/(3*x^3) - (((1408*I)/105)*b^3*e^(9/2)*n^3*Pol
yLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(9/2) + (2*b*e^5*
n*Unintegrable[(a + b*Log[c*(d + e*x^(2/3))^n])^2/((d + e*x^(2/3))*x^(2/3))
, x])/(3*d^4)
```

Rubi [A] time = 1.87028, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (-16*b^3*e^3*n^3)/(105*d^3*x) + (16*b^3*e^4*n^3)/(7*d^4*x^(1/3)) + (1376*b^
3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(105*d^(9/2)) - (((1408*I)
/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(9/2) - (2816*
b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/(105*d^(9/2)) - (8*b^2*e^2*n^2*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(35*d^2*x^(5/3)) + (32*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3)
)^n]))/(35*d^3*x) - (568*b^2*e^4*n^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(105
*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(105*d^(9/2)) - (2*b*e*n*(a + b*Log[c*(d + e
*x^(2/3))^n])^2)/(7*d*x^(7/3)) + (2*b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]
)^2)/(5*d^2*x^(5/3)) - (2*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*d^
3*x) + (2*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(d^4*x^(1/3)) - (a +
b*Log[c*(d + e*x^(2/3))^n])^3/(3*x^3) - (((1408*I)/105)*b^3*e^(9/2)*n^3*Pol
yLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(9/2) + (2*b*e^5*
n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^n])^2/(d + e*x^2), x], x
, x^(1/3)])/d^4
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + (2ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^8\left(d+ex^2\right)}\,dx, x,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + (2ben)\,\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{dx^8} - \frac{e\left(\right.}{}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{(2ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^8}\,dx, x, \sqrt[3]{x}\right)}{d} - \frac{(2}{}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}} - \frac{2be^3n\left(a\right.}{}$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}} - \frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right.\right.}{7dx^{7/3}}$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}} - \frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right.\right.}{7dx^{7/3}}$$

$$= -\frac{8ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{9/2}} - \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3n^2\left(a+b\right.}{3}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{64b^3e^4n^3}{35d^4\sqrt[3]{x}} + \frac{1136b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{8ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{9/2}} -$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1328b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$

Mathematica [A] time = 2.8917, size = 803, normalized size = 1.27

$$-70\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3d^5-210bn\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+e\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (35*b^3*n^3*(54*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*Hypergeo
metricPFQ[{1, 1, 1, 1, 11/2}, {2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*
x^(2/3)]*(54*d*e^3*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*x^2*Hypergeomet
ricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*
(27*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*HypergeometricPFQ[{1
, 1, 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] - 2*d*(d^4 + d*e^3*(-((e*x^(2/3))/d)
)^(3/2)*x^2)*Log[d + e*x^(2/3)]))) + (210*b^2*n^2*(-9*e^5*(d + e*x^(2/3))*x
^(10/3)*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] +
Log[d + e*x^(2/3)]*(9*e^5*(d + e*x^(2/3))*x^(10/3)*HypergeometricPFQ[{1, 1,
 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] + d*(d^5*Sqrt[-((e*x^(2/3))/d)] + e^5*x^
(10/3))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d + e*x^(2/3)] - b*Log[c*(d + e*
x^(2/3))^n]))/(d*Sqrt[-((e*x^(2/3))/d)]) - 60*b*d^4*e*n*x^(2/3)*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 84*b*d^3*e^2*n*x^(4/3)*(
a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 140*b*d^2*e^3*
n*x^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 420*b*d
*e^4*n*x^(8/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2
+ 420*b*Sqrt[d]*e^(9/2)*n*x^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 210*b*d^5*n*Log[d + e*x^
(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 70*d^5
*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3)/(210*d^5*x^3)
```

Maple [A] time = 0.364, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^4, x)
```

3.489 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=239

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^{10}nx^{2/3}}{8d^{10}}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}-\frac{be^4nx^{8/3}}{32d^4}+\frac{be^3nx^3}{36d^3}-\frac{be^2nx}{40d}$$

```
[Out] (b*e^11*n*x^(1/3))/(4*d^11) - (b*e^10*n*x^(2/3))/(8*d^10) + (b*e^9*n*x)/(12
*d^9) - (b*e^8*n*x^(4/3))/(16*d^8) + (b*e^7*n*x^(5/3))/(20*d^7) - (b*e^6*n*
x^2)/(24*d^6) + (b*e^5*n*x^(7/3))/(28*d^5) - (b*e^4*n*x^(8/3))/(32*d^4) + (
b*e^3*n*x^3)/(36*d^3) - (b*e^2*n*x^(10/3))/(40*d^2) + (b*e*n*x^(11/3))/(44*
d) - (b*e^12*n*Log[d + e/x^(1/3)])/(4*d^12) + (x^4*(a + b*Log[c*(d + e/x^(1
/3))^n]))/4 - (b*e^12*n*Log[x])/(12*d^12)
```

Rubi [A] time = 0.170841, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^{10}nx^{2/3}}{8d^{10}}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}-\frac{be^4nx^{8/3}}{32d^4}+\frac{be^3nx^3}{36d^3}-\frac{be^2nx}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^11*n*x^(1/3))/(4*d^11) - (b*e^10*n*x^(2/3))/(8*d^10) + (b*e^9*n*x)/(12
*d^9) - (b*e^8*n*x^(4/3))/(16*d^8) + (b*e^7*n*x^(5/3))/(20*d^7) - (b*e^6*n*
x^2)/(24*d^6) + (b*e^5*n*x^(7/3))/(28*d^5) - (b*e^4*n*x^(8/3))/(32*d^4) + (
b*e^3*n*x^3)/(36*d^3) - (b*e^2*n*x^(10/3))/(40*d^2) + (b*e*n*x^(11/3))/(44*
d) - (b*e^12*n*Log[d + e/x^(1/3)])/(4*d^12) + (x^4*(a + b*Log[c*(d + e/x^(1
/3))^n]))/4 - (b*e^12*n*Log[x])/(12*d^12)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=-\left(3\,\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^{13}}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\frac{1}{x^{12}(d+ex)}dx,x,\frac{1}{\sqrt[3]{x}}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^{12}}-\frac{e}{d^2x^{11}}+\frac{e^2}{d^3x^{10}}-\frac{e^3}{d^4x^9}\right.\right.$$
$$=\frac{be^{11}n\sqrt[3]{x}}{4d^{11}}-\frac{be^{10}nx^{2/3}}{8d^{10}}+\frac{be^9nx}{12d^9}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}-\frac{be^4n}{32d}$$

Mathematica [A] time = 0.226955, size = 218, normalized size = 0.91

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{4}ben\left(\frac{e^9x^{2/3}}{2d^{10}}+\frac{e^7x^{4/3}}{4d^8}-\frac{e^6x^{5/3}}{5d^7}+\frac{e^5x^2}{6d^6}-\frac{e^4x^{7/3}}{7d^5}+\frac{e^3x^{8/3}}{8d^4}-\frac{e^2x^3}{9d^3}-\frac{e^{10}\sqrt[3]{x}}{d^{11}}-\frac{e^8x}{3d^9}+\frac{e^{11}\log}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(1/3))^n])/4 - (b*e*n*(-((e^10*x^(1/3))/d
^11) + (e^9*x^(2/3))/(2*d^10) - (e^8*x)/(3*d^9) + (e^7*x^(4/3))/(4*d^8) - (
e^6*x^(5/3))/(5*d^7) + (e^5*x^2)/(6*d^6) - (e^4*x^(7/3))/(7*d^5) + (e^3*x^(
8/3))/(8*d^4) - (e^2*x^3)/(9*d^3) + (e*x^(10/3))/(10*d^2) - x^(11/3)/(11*d)
 + (e^11*Log[d + e/x^(1/3)])/d^12 + (e^11*Log[x])/(3*d^12)))/4
```

Maple [F] time = 0.542, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Maxima [A] time = 1.04676, size = 219, normalized size = 0.92

$$\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{4}ax^4-\frac{1}{110880}ben\left(\frac{27720\,e^{11}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{12}}-\frac{2520\,d^{10}x^{\frac{11}{3}}-2772\,d^9ex^{\frac{10}{3}}+3080\,d^8e^2x^3-346}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/x^(1/3))^n) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*e^11
*log(d*x^(1/3) + e)/d^12 - (2520*d^10*x^(11/3) - 2772*d^9*e*x^(10/3) + 3080
*d^8*e^2*x^3 - 3465*d^7*e^3*x^(8/3) + 3960*d^6*e^4*x^(7/3) - 4620*d^5*e^5*x
^2 + 5544*d^4*e^6*x^(5/3) - 6930*d^3*e^7*x^(4/3) + 9240*d^2*e^8*x - 13860*d
*e^9*x^(2/3) + 27720*e^10*x^(1/3))/d^11)
```

Fricas [A] time = 2.18807, size = 605, normalized size = 2.53

$$\frac{27720\,bd^{12}x^4\log(c) + 3080\,bd^9e^3nx^3 + 27720\,ad^{12}x^4 - 4620\,bd^6e^6nx^2 + 9240\,bd^3e^9nx - 27720\,bd^{12}n\log\left(x^{\frac{1}{3}}\right) + 277\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/110880*(27720*b*d^12*x^4*log(c) + 3080*b*d^9*e^3*n*x^3 + 27720*a*d^12*x^4
 - 4620*b*d^6*e^6*n*x^2 + 9240*b*d^3*e^9*n*x - 27720*b*d^12*n*log(x^(1/3))
+ 27720*(b*d^12 - b*e^12)*n*log(d*x^(1/3) + e) + 27720*(b*d^12*n*x^4 - b*d^
12*n)*log((d*x + e*x^(2/3))/x) + 63*(40*b*d^11*e*n*x^3 - 55*b*d^8*e^4*n*x^2
 + 88*b*d^5*e^7*n*x - 220*b*d^2*e^10*n)*x^(2/3) - 198*(14*b*d^10*e^2*n*x^3
- 20*b*d^7*e^5*n*x^2 + 35*b*d^4*e^8*n*x - 140*b*d*e^11*n)*x^(1/3))/d^12
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3343, size = 217, normalized size = 0.91

$$\frac{1}{4}\,bx^4\log(c) + \frac{1}{4}\,ax^4 + \frac{1}{110880}\left(27720\,x^4\log\left(d + \frac{e}{x^{\frac{1}{3}}}\right) + \left(\frac{2520\,d^{10}x^{\frac{11}{3}} - 2772\,d^9x^{\frac{10}{3}}e + 3080\,d^8x^3e^2 - 3465\,d^7x^{\frac{8}{3}}e^3 + \ldots}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/110880*(27720*x^4*log(d + e/x^(1/3)) + ((2
520*d^10*x^(11/3) - 2772*d^9*x^(10/3)*e + 3080*d^8*x^3*e^2 - 3465*d^7*x^(8/
3)*e^3 + 3960*d^6*x^(7/3)*e^4 - 4620*d^5*x^2*e^5 + 5544*d^4*x^(5/3)*e^6 - 6
930*d^3*x^(4/3)*e^7 + 9240*d^2*x*e^8 - 13860*d*x^(2/3)*e^9 + 27720*x^(1/3)*
e^10)/d^11 - 27720*e^11*log(abs(d*x^(1/3) + e))/d^12)*e)*b*n
```

3.490 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=190

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+\frac{be^7nx^{2/3}}{6d^7}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}-\frac{be^8n\sqrt[3]{x}}{3d^8}-\frac{be^6nx}{9d^6}+\frac{be^9n\log\left(d\right.}{3d^9}$$

```
[Out] -(b*e^8*n*x^(1/3))/(3*d^8) + (b*e^7*n*x^(2/3))/(6*d^7) - (b*e^6*n*x)/(9*d^6
) + (b*e^5*n*x^(4/3))/(12*d^5) - (b*e^4*n*x^(5/3))/(15*d^4) + (b*e^3*n*x^2)
/(18*d^3) - (b*e^2*n*x^(7/3))/(21*d^2) + (b*e*n*x^(8/3))/(24*d) + (b*e^9*n*
Log[d + e/x^(1/3)])/(3*d^9) + (x^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/3 + (b
*e^9*n*Log[x])/(9*d^9)
```

Rubi [A] time = 0.130131, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+\frac{be^7nx^{2/3}}{6d^7}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}-\frac{be^8n\sqrt[3]{x}}{3d^8}-\frac{be^6nx}{9d^6}+\frac{be^9n\log\left(d\right.}{3d^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] -(b*e^8*n*x^(1/3))/(3*d^8) + (b*e^7*n*x^(2/3))/(6*d^7) - (b*e^6*n*x)/(9*d^6
) + (b*e^5*n*x^(4/3))/(12*d^5) - (b*e^4*n*x^(5/3))/(15*d^4) + (b*e^3*n*x^2)
/(18*d^3) - (b*e^2*n*x^(7/3))/(21*d^2) + (b*e*n*x^(8/3))/(24*d) + (b*e^9*n*
Log[d + e/x^(1/3)])/(3*d^9) + (x^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/3 + (b
*e^9*n*Log[x])/(9*d^9)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx = -\left(3\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\frac{1}{x^9(d+ex)}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^9}-\frac{e}{d^2x^8}+\frac{e^2}{d^3x^7}-\frac{e^3}{d^4x^6}\right.\right.$$
$$=-\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}+\frac{be}{}$$

Mathematica [A] time = 0.132517, size = 175, normalized size = 0.92

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{3}ben\left(-\frac{e^6x^{2/3}}{2d^7}-\frac{e^4x^{4/3}}{4d^5}+\frac{e^3x^{5/3}}{5d^4}-\frac{e^2x^2}{6d^3}+\frac{e^7\sqrt[3]{x}}{d^8}+\frac{e^5x}{3d^6}-\frac{e^8\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^9}-\frac{e^8\log(x}{3d^9}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/x^(1/3))^n])/3 - (b*e*n*((e^7*x^(1/3))/d^8
- (e^6*x^(2/3))/(2*d^7) + (e^5*x)/(3*d^6) - (e^4*x^(4/3))/(4*d^5) + (e^3*x^
(5/3))/(5*d^4) - (e^2*x^2)/(6*d^3) + (e*x^(7/3))/(7*d^2) - x^(8/3)/(8*d) -
(e^8*Log[d + e/x^(1/3)])/d^9 - (e^8*Log[x])/(3*d^9)))/3
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Maxima [A] time = 1.0329, size = 173, normalized size = 0.91

$$\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{3}ax^3+\frac{1}{2520}ben\left(\frac{840\,e^8\log\left(dx^{\frac{1}{3}}+e\right)}{d^9}+\frac{105\,d^7x^{\frac{8}{3}}-120\,d^6ex^{\frac{7}{3}}+140\,d^5e^2x^2-168\,d^4e^3x^{\frac{5}{3}}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/x^(1/3))^n) + 1/3*a*x^3 + 1/2520*b*e*n*(840*e^8*log(
d*x^(1/3) + e)/d^9 + (105*d^7*x^(8/3) - 120*d^6*e*x^(7/3) + 140*d^5*e^2*x^2
 - 168*d^4*e^3*x^(5/3) + 210*d^3*e^4*x^(4/3) - 280*d^2*e^5*x + 420*d*e^6*x^
```

```
(2/3) - 840*e^7*x^(1/3))/d^8)
```

Fricas [A] time = 2.19577, size = 473, normalized size = 2.49

$$\frac{840\,bd^9x^3\log(c) + 140\,bd^6e^3nx^2 + 840\,ad^9x^3 - 280\,bd^3e^6nx - 840\,bd^9n\log\left(x^{\frac{1}{3}}\right) + 840\left(bd^9 + be^9\right)n\log\left(dx^{\frac{1}{3}} + e\right) + 840}{2520}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/2520*(840*b*d^9*x^3*log(c) + 140*b*d^6*e^3*n*x^2 + 840*a*d^9*x^3 - 280*b*
d^3*e^6*n*x - 840*b*d^9*n*log(x^(1/3)) + 840*(b*d^9 + b*e^9)*n*log(d*x^(1/3
) + e) + 840*(b*d^9*n*x^3 - b*d^9*n)*log((d*x + e*x^(2/3))/x) + 21*(5*b*d^8
*e*n*x^2 - 8*b*d^5*e^4*n*x + 20*b*d^2*e^7*n)*x^(2/3) - 30*(4*b*d^7*e^2*n*x^
2 - 7*b*d^4*e^5*n*x + 28*b*d*e^8*n)*x^(1/3))/d^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39846, size = 177, normalized size = 0.93

$$\frac{1}{3}\,bx^3\log(c) + \frac{1}{3}\,ax^3 + \frac{1}{2520}\left(840\,x^3\log\left(d + \frac{e}{x^{\frac{1}{3}}}\right) + \left(\frac{105\,d^7x^{\frac{8}{3}} - 120\,d^6x^{\frac{7}{3}}e + 140\,d^5x^2e^2 - 168\,d^4x^{\frac{5}{3}}e^3 + 210\,d^3x^{\frac{4}{3}}e^4 - 28}{d^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/2520*(840*x^3*log(d + e/x^(1/3)) + ((105*d
^7*x^(8/3) - 120*d^6*x^(7/3)*e + 140*d^5*x^2*e^2 - 168*d^4*x^(5/3)*e^3 + 21
0*d^3*x^(4/3)*e^4 - 280*d^2*x*e^5 + 420*d*x^(2/3)*e^6 - 840*x^(1/3)*e^7)/d^
8 + 840*e^8*log(abs(d*x^(1/3) + e))/d^9)*e)*b*n
```

3.491 $\int x \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) dx$

Optimal. Leaf size=141

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^4nx^{2/3}}{4d^4}-\frac{be^2nx^{4/3}}{8d^2}+\frac{be^5n\sqrt[3]{x}}{2d^5}+\frac{be^3nx}{6d^3}-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{benx^{5/3}}{10d}$$

```
[Out] (b*e^5*n*x^(1/3))/(2*d^5) - (b*e^4*n*x^(2/3))/(4*d^4) + (b*e^3*n*x)/(6*d^3)
 - (b*e^2*n*x^(4/3))/(8*d^2) + (b*e*n*x^(5/3))/(10*d) - (b*e^6*n*Log[d + e/
x^(1/3)])/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/2 - (b*e^6*n*Log
[x])/(6*d^6)
```

Rubi [A] time = 0.0916034, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2454, 2395, 44}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^4nx^{2/3}}{4d^4}-\frac{be^2nx^{4/3}}{8d^2}+\frac{be^5n\sqrt[3]{x}}{2d^5}+\frac{be^3nx}{6d^3}-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{benx^{5/3}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^5*n*x^(1/3))/(2*d^5) - (b*e^4*n*x^(2/3))/(4*d^4) + (b*e^3*n*x)/(6*d^3)
 - (b*e^2*n*x^(4/3))/(8*d^2) + (b*e*n*x^(5/3))/(10*d) - (b*e^6*n*Log[d + e/
x^(1/3)])/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/2 - (b*e^6*n*Log
[x])/(6*d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=-\left(3\,\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^7}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\frac{1}{x^6(d+ex)}dx,x,\frac{1}{\sqrt[3]{x}}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}+\right.\right.$$
$$=\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}+\frac{1}{2}x^2\left(a+\right.$$

Mathematica [A] time = 0.0861445, size = 132, normalized size = 0.94

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{2}ben\left(\frac{e^3x^{2/3}}{2d^4}-\frac{e^4\sqrt[3]{x}}{d^5}-\frac{e^2x}{3d^3}+\frac{e^5\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}+\frac{e^5\log(x)}{3d^6}+\frac{ex^{4/3}}{4d^2}-\frac{x^{5/3}}{5d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(1/3))^n])/2 - (b*e*n*(-((e^4*x^(1/3))/d^
5) + (e^3*x^(2/3))/(2*d^4) - (e^2*x)/(3*d^3) + (e*x^(4/3))/(4*d^2) - x^(5/3
)/(5*d) + (e^5*Log[d + e/x^(1/3)])/d^6 + (e^5*Log[x])/(3*d^6)))/2
```

Maple [F] time = 0.34, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Maxima [A] time = 1.01447, size = 130, normalized size = 0.92

$$-\frac{1}{120}ben\left(\frac{60\,e^5\log\left(dx^{\frac{1}{3}}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3ex^{\frac{4}{3}}+20\,d^2e^2x-30\,de^3x^{\frac{2}{3}}+60\,e^4x^{\frac{1}{3}}}{d^5}\right)+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] -1/120*b*e*n*(60*e^5*log(d*x^(1/3) + e)/d^6 - (12*d^4*x^(5/3) - 15*d^3*e*x^
(4/3) + 20*d^2*e^2*x - 30*d*e^3*x^(2/3) + 60*e^4*x^(1/3))/d^5) + 1/2*b*x^2*
log(c*(d + e/x^(1/3))^n) + 1/2*a*x^2
```

Fricas [A] time = 2.14783, size = 373, normalized size = 2.65

$$\frac{60\,bd^6x^2\log(c) + 20\,bd^3e^3nx + 60\,ad^6x^2 - 60\,bd^6n\log\left(x^{\frac{1}{3}}\right) + 60\left(bd^6 - be^6\right)n\log\left(dx^{\frac{1}{3}} + e\right) + 60\left(bd^6nx^2 - bd^6n\right)\log}{120\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/120*(60*b*d^6*x^2*log(c) + 20*b*d^3*e^3*n*x + 60*a*d^6*x^2 - 60*b*d^6*n*l
og(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(1/3) + e) + 60*(b*d^6*n*x^2 - b
*d^6*n)*log((d*x + e*x^(2/3))/x) + 6*(2*b*d^5*e*n*x - 5*b*d^2*e^4*n)*x^(2/3
) - 15*(b*d^4*e^2*n*x - 4*b*d*e^5*n)*x^(1/3))/d^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44522, size = 136, normalized size = 0.96

$$\frac{1}{2}\,bx^2\log(c) + \frac{1}{120}\left(60\,x^2\log\left(d + \frac{e}{x^{\frac{1}{3}}}\right) + \left(\frac{12\,d^4x^{\frac{5}{3}} - 15\,d^3x^{\frac{4}{3}}e + 20\,d^2xe^2 - 30\,dx^{\frac{2}{3}}e^3 + 60\,x^{\frac{1}{3}}e^4}{d^5} - \frac{60\,e^5\log\left(\left|dx^{\frac{1}{3}} + e\right|\right.}{d^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/120*(60*x^2*log(d + e/x^(1/3)) + ((12*d^4*x^(5/3) - 15
*d^3*x^(4/3)*e + 20*d^2*x*e^2 - 30*d*x^(2/3)*e^3 + 60*x^(1/3)*e^4)/d^5 - 60
*e^5*log(abs(d*x^(1/3) + e))/d^6)*e)*b*n + 1/2*a*x^2
```

3.492 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=70

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{be^3n\log\left(d\sqrt[3]{x}+e\right)}{d^3}+\frac{benx^{2/3}}{2d}$$

```
[Out] -((b*e^2*n*x^(1/3))/d^2) + (b*e*n*x^(2/3))/(2*d) + a*x + b*x*Log[c*(d + e/x
^(1/3))^n] + (b*e^3*n*Log[e + d*x^(1/3)])/d^3
```

Rubi [A] time = 0.0501666, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2448, 263, 190, 43}

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{be^3n\log\left(d\sqrt[3]{x}+e\right)}{d^3}+\frac{benx^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

```
[Out] -((b*e^2*n*x^(1/3))/d^2) + (b*e*n*x^(2/3))/(2*d) + a*x + b*x*Log[c*(d + e/x
^(1/3))^n] + (b*e^3*n*Log[e + d*x^(1/3)])/d^3
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx = ax+b\int \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}(ben)\int \frac{1}{e+d\sqrt[3]{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+(ben)\,\text{Subst}\left(\int \frac{x^2}{e+dx}dx,x,\sqrt[3]{x}\right)$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+(ben)\,\text{Subst}\left(\int \left(-\frac{e}{d^2}+\frac{x}{d}+\frac{e^2}{d^2(e+dx)}\right)dx,x,\sqrt[3]{x}\right)$$
$$= -\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(e+d\sqrt[3]{x}\right)}{d^3}$$

Mathematica [A] time = 0.0470079, size = 79, normalized size = 1.13

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-ben\left(-\frac{e^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{e^2\log(x)}{3d^3}+\frac{e\sqrt[3]{x}}{d^2}-\frac{x^{2/3}}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

```
[Out] a*x + b*x*Log[c*(d + e/x^(1/3))^n] - b*e*n*((e*x^(1/3))/d^2 - x^(2/3)/(2*d)
 - (e^2*Log[d + e/x^(1/3)])/d^3 - (e^2*Log[x])/(3*d^3))
```

Maple [A] time = 0.102, size = 115, normalized size = 1.6

$$ax+xb\ln\left(c\left(\left(e+d\sqrt[3]{x}\right)\frac{1}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\ln\left(d^3x+e^3\right)}{3\,d^3}+\frac{enb}{2\,d}x^{\frac{2}{3}}-\frac{be^3n}{3\,d^3}\ln\left(d^2x^{\frac{2}{3}}-ed\sqrt[3]{x}+e^2\right)+\frac{2\,be^3n}{3\,d^3}\ln\left(e+d\sqrt[3]{x}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/x^(1/3))^n),x)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(1/3))/x^(1/3))^n)+1/3*b*e^3*n*ln(d^3*x+e^3)/d^3+1/2*b
*e*n*x^(2/3)/d-1/3*b*e^3*n/d^3*ln(d^2*x^(2/3)-e*d*x^(1/3)+e^2)+2/3*b*e^3*n*
ln(e+d*x^(1/3))/d^3-b*e^2*n*x^(1/3)/d^2
```

Maxima [A] time = 1.03024, size = 80, normalized size = 1.14

$$\frac{1}{2}\left(en\left(\frac{2\,e^2\log\left(dx^{\frac{1}{3}}+e\right)}{d^3}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^2}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="maxima")
```

```
[Out] 1/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2
*x*log(c*(d + e/x^(1/3))^n))*b + a*x
```

Fricas [A] time = 2.15552, size = 269, normalized size = 3.84

$$\frac{2\,bd^3x\log\left(c\right) - 2\,bd^3n\log\left(x^{\frac{1}{3}}\right) + bd^2enx^{\frac{2}{3}} - 2\,bde^2nx^{\frac{1}{3}} + 2\,ad^3x + 2\left(bd^3 + be^3\right)n\log\left(dx^{\frac{1}{3}} + e\right) + 2\left(bd^3nx - bd^3n\right)\log\left(\right.}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d^3*x*log(c) - 2*b*d^3*n*log(x^(1/3)) + b*d^2*e*n*x^(2/3) - 2*b*d*
e^2*n*x^(1/3) + 2*a*d^3*x + 2*(b*d^3 + b*e^3)*n*log(d*x^(1/3) + e) + 2*(b*d
^3*n*x - b*d^3*n)*log((d*x + e*x^(2/3))/x))/d^3
```

Sympy [A] time = 15.0529, size = 92, normalized size = 1.31

$$ax + b\left(\frac{en\left(\frac{3x^{\frac{2}{3}}}{2d} - \frac{3e\sqrt[3]{x}}{d^2} + \frac{3e^3\left(\begin{cases}\frac{1}{d\sqrt[3]{x}} & \text{for}\ e = 0\\ \frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{3e^2\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^3}\right)}{3} + x\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(1/3))**n),x)
```

```
[Out] a*x + b*(e*n*(3*x**(2/3)/(2*d) - 3*e*x**(1/3)/d**2 + 3*e**3*Piecewise((1/(d
*x**(1/3)), Eq(e, 0)), (log(d + e/x**(1/3))/e, True))/d**3 - 3*e**2*log(x**
(-1/3))/d**3)/3 + x*log(c*(d + e/x**(1/3))**n))
```

Giac [A] time = 1.38545, size = 89, normalized size = 1.27

$$\frac{1}{2}\left(\left(\left(\left(\frac{dx^{\frac{2}{3}} - 2\,x^{\frac{1}{3}}e}{d^2} + \frac{2\,e^2\log\left(\left|dx^{\frac{1}{3}} + e\right|\right)}{d^3}\right)e + 2\,x\log\left(d + \frac{e}{x^{\frac{1}{3}}}\right)\right)n + 2\,x\log\left(c\right)\right)b + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="giac")
```

```
[Out] 1/2*((((d*x^(2/3) - 2*x^(1/3)*e)/d^2 + 2*e^2*log(abs(d*x^(1/3) + e))/d^3)*e
 + 2*x*log(d + e/x^(1/3)))*n + 2*x*log(c))*b + a*x
```

3.493 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$-3bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] - 3*b*n*PolyLog[2
, 1 + e/(d*x^(1/3))]
```

Rubi [A] time = 0.0499843, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$-3bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] - 3*b*n*PolyLog[2
, 1 + e/(d*x^(1/3))]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx = -\left(3\,\mathrm{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (3ben)\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$
$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 3bn\mathrm{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)$$

Mathematica [A] time = 0.0029341, size = 53, normalized size = 1.04

$$-3bn\mathrm{PolyLog}\left(2, \frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right) + a\log(x) - 3b\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*b*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)))] + a*Log[x] - 3*b*n*Poly
Log[2, (d + e/x^(1/3))/d]
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)
```

Maxima [B] time = 2.31771, size = 250, normalized size = 4.9

$$-3\left(\log\left(\frac{dx^{\frac{1}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right) + \mathrm{Li}_2\left(-\frac{dx^{\frac{1}{3}}}{e}\right)\right)bn + \frac{2\,be^2n\log(x)^2 + 12\,be^2\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)\log(x) - 12\,be^2\log(x)\log\left(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3*(log(d*x^(1/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(1/3)/e))*b*n + 1/12*(2*
b*e^2*n*log(x)^2 + 12*b*e^2*log((d*x^(1/3) + e)^n)*log(x) - 12*b*e^2*log(x)
*log(x^(1/3*n)) + 9*b*d^2*n*x^(2/3) - 36*b*d*e*n*x^(1/3) - 6*(b*d^2*n*x^(2/
3) - 2*b*d*e*n*x^(1/3))*log(x) + 12*(b*e^2*log(c) + a*e^2)*log(x) + 3*(2*b*
d^2*n*x*log(x) - 3*b*d^2*n*x)/x^(1/3) - 12*(b*d*e*n*x*log(x) - 3*b*d*e*n*x)
/x^(2/3))/e^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + e*x^(2/3))/x)^n) + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)/x, x)
```

3.494 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{bdn}{2ex^{2/3}}+\frac{bn}{3x}$$

```
[Out] (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b*d^3*n*Lo
g[d + e/x^(1/3)])/e^3 - (a + b*Log[c*(d + e/x^(1/3))^n])/x
```

Rubi [A] time = 0.0682031, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{bdn}{2ex^{2/3}}+\frac{bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```


```
[Out] (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b*d^3*n*Lo
g[d + e/x^(1/3)])/e^3 - (a + b*Log[c*(d + e/x^(1/3))^n])/x
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx = -\left(3\,\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\,\mathrm{Subst}\left(\int \frac{x^3}{d+ex}dx, x, \frac{1}{\sqrt[3]{x}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\,\mathrm{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)$$
$$= \frac{bn}{3x} - \frac{bdn}{2ex^{2/3}} + \frac{bd^2n}{e^2\sqrt[3]{x}} - \frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

Mathematica [A] time = 0.0325174, size = 85, normalized size = 1.04

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + \frac{bd^2n}{e^2\sqrt[3]{x}} - \frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{bdn}{2ex^{2/3}} + \frac{bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b
*d^3*n*Log[d + e/x^(1/3)])/e^3 - (b*Log[c*(d + e/x^(1/3))^n])/x
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

Maxima [A] time = 1.025, size = 116, normalized size = 1.41

$$-\frac{1}{6}\,ben\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4} - \frac{2\,d^3\log(x)}{e^4} - \frac{6\,d^2x^{\frac{2}{3}} - 3\,dex^{\frac{1}{3}} + 2\,e^2}{e^3x}\right) - \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3
) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b*log(c*(d + e/x^(1/3))^n)/x - a/x
```

Fricas [A] time = 1.76232, size = 250, normalized size = 3.05

$$\frac{6\,bd^2enx^{\frac{2}{3}} - 3\,bde^2nx^{\frac{1}{3}} + 2\,be^3n - 6\,ae^3 - 2\left(be^3n - 3\,ae^3\right)x + 6\left(be^3x - be^3\right)\log\left(c\right) - 6\left(bd^3nx + be^3n\right)\log\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)}{6\,e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*d^2*e*n*x^(2/3) - 3*b*d*e^2*n*x^(1/3) + 2*b*e^3*n - 6*a*e^3 - 2*(b
*e^3*n - 3*a*e^3)*x + 6*(b*e^3*x - b*e^3)*log(c) - 6*(b*d^3*n*x + b*e^3*n)*
log((d*x + e*x^(2/3))/x))/(e^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34764, size = 128, normalized size = 1.56

$$-\frac{1}{6}\left(\left(6\,d^3e^{(-4)}\log\left(\left|dx^{\frac{1}{3}} + e\right|\right) - 2\,d^3e^{(-4)}\log\left(|x|\right) - \frac{\left(6\,d^2x^{\frac{2}{3}}e - 3\,dx^{\frac{1}{3}}e^2 + 2\,e^3\right)e^{(-4)}}{x}\right)e + \frac{6\,\log\left(d + \frac{e}{x^{\frac{1}{3}}}\right)}{x}\right)bn - \frac{b\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/6*((6*d^3*e^(-4)*log(abs(d*x^(1/3) + e)) - 2*d^3*e^(-4)*log(abs(x)) - (6
*d^2*x^(2/3)*e - 3*d*x^(1/3)*e^2 + 2*e^3)*e^(-4)/x)*e + 6*log(d + e/x^(1/3)
)/x)*b*n - b*log(c)/x - a/x
```

3.495 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{bd^4n}{4e^4x^{2/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}-\frac{bd^3n}{6e^3x}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{bdn}{10ex^{5/3}}+\frac{bn}{12x^2}$$

```
[Out] (b*n)/(12*x^2) - (b*d*n)/(10*e*x^(5/3)) + (b*d^2*n)/(8*e^2*x^(4/3)) - (b*d^
3*n)/(6*e^3*x) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^5*x^(1/3)) + (b
*d^6*n*Log[d + e/x^(1/3)])/(2*e^6) - (a + b*Log[c*(d + e/x^(1/3))^n])/(2*x^
2)
```

Rubi [A] time = 0.097263, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{bd^4n}{4e^4x^{2/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}-\frac{bd^3n}{6e^3x}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{bdn}{10ex^{5/3}}+\frac{bn}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```


```
[Out] (b*n)/(12*x^2) - (b*d*n)/(10*e*x^(5/3)) + (b*d^2*n)/(8*e^2*x^(4/3)) - (b*d^
3*n)/(6*e^3*x) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^5*x^(1/3)) + (b
*d^6*n*Log[d + e/x^(1/3)])/(2*e^6) - (a + b*Log[c*(d + e/x^(1/3))^n])/(2*x^
2)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\,(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{x^6}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \left(-\frac{d^5}{e^6} + \frac{d^4x}{e^5} - \frac{d^3x^2}{e^4} + \frac{d^2x^3}{e^3} - \frac{dx^4}{e^2} + \frac{x}{e}\right.\right.$$

$$= \frac{bn}{12x^2} - \frac{bdn}{10ex^{5/3}} + \frac{bd^2n}{8e^2x^{4/3}} - \frac{bd^3n}{6e^3x} + \frac{bd^4n}{4e^4x^{2/3}} - \frac{bd^5n}{2e^5\sqrt[3]{x}} + \frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} - \frac{a+b\log}{}$$

Mathematica [A] time = 0.0894655, size = 135, normalized size = 0.98

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2} + \frac{1}{2}ben\left(\frac{d^4}{2e^5x^{2/3}} + \frac{d^2}{4e^3x^{4/3}} - \frac{d^5}{e^6\sqrt[3]{x}} - \frac{d^3}{3e^4x} + \frac{d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^7} - \frac{d}{5e^2x^{5/3}} + \frac{1}{6ex^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*e*n*(1/(6*e*x^2) - d/(5*e^2*x^(5/3)) + d^2/(4*e^3*x^(4/3))
- d^3/(3*e^4*x) + d^4/(2*e^5*x^(2/3)) - d^5/(e^6*x^(1/3)) + (d^6*Log[d + e/
x^(1/3)])/e^7))/2 - (b*Log[c*(d + e/x^(1/3))^n])/(2*x^2)
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
```

Maxima [A] time = 1.03237, size = 158, normalized size = 1.14

$$\frac{1}{120}ben\left(\frac{60\,d^6\log\left(dx^{\frac{1}{3}}+e\right)}{e^7} - \frac{20\,d^6\log(x)}{e^7} - \frac{60\,d^5x^{\frac{5}{3}} - 30\,d^4ex^{\frac{4}{3}} + 20\,d^3e^2x - 15\,d^2e^3x^{\frac{2}{3}} + 12\,de^4x^{\frac{1}{3}} - 10\,e^5}{e^6x^2}\right) - \frac{b\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/120*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^
(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(
1/3) - 10*e^5)/(e^6*x^2)) - 1/2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a/x^2
```

Fricas [A] time = 1.81379, size = 377, normalized size = 2.73

$$-\frac{20\,bd^3e^3nx - 10\,be^6n + 60\,ae^6 - 10\left(6\,ae^6 + \left(2\,bd^3e^3 - be^6\right)n\right)x^2 - 60\left(be^6x^2 - be^6\right)\log\left(c\right) - 60\left(bd^6nx^2 - be^6n\right)\log\left(\frac{dx+\cdots}{x}\right.}{120\,e^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b*d^3*e^3*n*x - 10*b*e^6*n + 60*a*e^6 - 10*(6*a*e^6 + (2*b*d^3*e
^3 - b*e^6)*n)*x^2 - 60*(b*e^6*x^2 - b*e^6)*log(c) - 60*(b*d^6*n*x^2 - b*e^
6*n)*log((d*x + e*x^(2/3))/x) + 15*(4*b*d^5*e*n*x - b*d^2*e^4*n)*x^(2/3) -
6*(5*b*d^4*e^2*n*x - 2*b*d*e^5*n)*x^(1/3))/(e^6*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33372, size = 166, normalized size = 1.2

$$\frac{1}{120}\left(\left(60\,d^6e^{(-7)}\log\left(\left|dx^{\frac{1}{3}} + e\right|\right) - 20\,d^6e^{(-7)}\log\left(|x|\right) - \frac{\left(60\,d^5x^{\frac{5}{3}}e - 30\,d^4x^{\frac{4}{3}}e^2 + 20\,d^3xe^3 - 15\,d^2x^{\frac{2}{3}}e^4 + 12\,dx^{\frac{1}{3}}e^5 - 10\,e^6\right)e\cdots}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/120*((60*d^6*e^(-7)*log(abs(d*x^(1/3) + e)) - 20*d^6*e^(-7)*log(abs(x)) -
 (60*d^5*x^(5/3)*e - 30*d^4*x^(4/3)*e^2 + 20*d^3*x*e^3 - 15*d^2*x^(2/3)*e^4
 + 12*d*x^(1/3)*e^5 - 10*e^6)*e^(-7)/x^2)*e - 60*log(d + e/x^(1/3))/x^2)*b*
n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

3.496 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=187

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}-\frac{bd^7n}{6e^7x^{2/3}}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^2n}{21e^2x^{7/3}}+\frac{bd^8n}{3e^8\sqrt[3]{x}}+\frac{bd^6n}{9e^6x}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}$$

```
[Out] (b*n)/(27*x^3) - (b*d*n)/(24*e*x^(8/3)) + (b*d^2*n)/(21*e^2*x^(7/3)) - (b*d
^3*n)/(18*e^3*x^2) + (b*d^4*n)/(15*e^4*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)
) + (b*d^6*n)/(9*e^6*x) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8*n)/(3*e^8*x^(1
/3)) - (b*d^9*n*Log[d + e/x^(1/3)])/(3*e^9) - (a + b*Log[c*(d + e/x^(1/3))^
n])/(3*x^3)
```

Rubi [A] time = 0.133426, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}-\frac{bd^7n}{6e^7x^{2/3}}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^2n}{21e^2x^{7/3}}+\frac{bd^8n}{3e^8\sqrt[3]{x}}+\frac{bd^6n}{9e^6x}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

```
[Out] (b*n)/(27*x^3) - (b*d*n)/(24*e*x^(8/3)) + (b*d^2*n)/(21*e^2*x^(7/3)) - (b*d
^3*n)/(18*e^3*x^2) + (b*d^4*n)/(15*e^4*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)
) + (b*d^6*n)/(9*e^6*x) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8*n)/(3*e^8*x^(1
/3)) - (b*d^9*n*Log[d + e/x^(1/3)])/(3*e^9) - (a + b*Log[c*(d + e/x^(1/3))^
n])/(3*x^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = -\left(3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{x^9}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int\left(\frac{d^8}{e^9} - \frac{d^7x}{e^8} + \frac{d^6x^2}{e^7} - \frac{d^5x^3}{e^6} + \frac{d^4x^4}{e^5} - \frac{d^3x^5}{e^4}\right.\right.$$

$$= \frac{bn}{27x^3} - \frac{bdn}{24ex^{8/3}} + \frac{bd^2n}{21e^2x^{7/3}} - \frac{bd^3n}{18e^3x^2} + \frac{bd^4n}{15e^4x^{5/3}} - \frac{bd^5n}{12e^5x^{4/3}} + \frac{bd^6n}{9e^6x} - \frac{bd^7n}{6e^7x^{2/3}} + \frac{bd^8n}{3e^8\sqrt[3]{x}}$$

Mathematica [A] time = 0.150398, size = 178, normalized size = 0.95

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} + \frac{1}{3}ben\left(-\frac{d^7}{2e^8x^{2/3}} - \frac{d^5}{4e^6x^{4/3}} + \frac{d^4}{5e^5x^{5/3}} - \frac{d^3}{6e^4x^2} + \frac{d^2}{7e^3x^{7/3}} + \frac{d^8}{e^9\sqrt[3]{x}} + \frac{d^6}{3e^7x} - \frac{d^9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^{10}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

```
[Out] -a/(3*x^3) + (b*e*n*(1/(9*e*x^3) - d/(8*e^2*x^(8/3)) + d^2/(7*e^3*x^(7/3))
- d^3/(6*e^4*x^2) + d^4/(5*e^5*x^(5/3)) - d^5/(4*e^6*x^(4/3)) + d^6/(3*e^7*
x) - d^7/(2*e^8*x^(2/3)) + d^8/(e^9*x^(1/3)) - (d^9*Log[d + e/x^(1/3)])/e^1
0))/3 - (b*Log[c*(d + e/x^(1/3))^n])/(3*x^3)
```

Maple [F] time = 0.38, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)
```

Maxima [A] time = 1.04253, size = 203, normalized size = 1.09

$$-\frac{1}{7560}ben\left(\frac{2520\,d^9\log\left(dx^{\frac{1}{3}}+e\right)}{e^{10}} - \frac{840\,d^9\log(x)}{e^{10}} - \frac{2520\,d^8x^{\frac{8}{3}} - 1260\,d^7ex^{\frac{7}{3}} + 840\,d^6e^2x^2 - 630\,d^5e^3x^{\frac{5}{3}} + 504\,d^4e^4x^{\frac{4}{3}} -}{e^9x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/7560*b*e*n*(2520*d^9*log(d*x^(1/3) + e)/e^10 - 840*d^9*log(x)/e^10 - (25
20*d^8*x^(8/3) - 1260*d^7*e*x^(7/3) + 840*d^6*e^2*x^2 - 630*d^5*e^3*x^(5/3)
 + 504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x + 360*d^2*e^6*x^(2/3) - 315*d*e^7*x^
(1/3) + 280*e^8)/(e^9*x^3)) - 1/3*b*log(c*(d + e/x^(1/3))^n)/x^3 - 1/3*a/x^
3
```

Fricas [A] time = 1.81207, size = 502, normalized size = 2.68

$$\frac{840\,bd^6e^3nx^2 - 420\,bd^3e^6nx + 280\,be^9n - 2520\,ae^9 + 140\left(18\,ae^9 - \left(6\,bd^6e^3 - 3\,bd^3e^6 + 2\,be^9\right)n\right)x^3 + 2520\left(be^9x^3 - be^9 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/7560*(840*b*d^6*e^3*n*x^2 - 420*b*d^3*e^6*n*x + 280*b*e^9*n - 2520*a*e^9
+ 140*(18*a*e^9 - (6*b*d^6*e^3 - 3*b*d^3*e^6 + 2*b*e^9)*n)*x^3 + 2520*(b*e^
9*x^3 - b*e^9)*log(c) - 2520*(b*d^9*n*x^3 + b*e^9*n)*log((d*x + e*x^(2/3))/
x) + 90*(28*b*d^8*e*n*x^2 - 7*b*d^5*e^4*n*x + 4*b*d^2*e^7*n)*x^(2/3) - 63*(
20*b*d^7*e^2*n*x^2 - 8*b*d^4*e^5*n*x + 5*b*d*e^8*n)*x^(1/3))/(e^9*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38991, size = 207, normalized size = 1.11

$$-\frac{1}{7560}\left(\left(2520\,d^9e^{(-10)}\log\left(\left|dx^{\frac{1}{3}} + e\right|\right) - 840\,d^9e^{(-10)}\log\left(|x|\right) - \frac{\left(2520\,d^8x^{\frac{8}{3}}e - 1260\,d^7x^{\frac{7}{3}}e^2 + 840\,d^6x^2e^3 - 630\,d^5x^{\frac{5}{3}}e^4 \cdots\right.}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/7560*((2520*d^9*e^(-10)*log(abs(d*x^(1/3) + e)) - 840*d^9*e^(-10)*log(ab
s(x)) - (2520*d^8*x^(8/3)*e - 1260*d^7*x^(7/3)*e^2 + 840*d^6*x^2*e^3 - 630*
d^5*x^(5/3)*e^4 + 504*d^4*x^(4/3)*e^5 - 420*d^3*x*e^6 + 360*d^2*x^(2/3)*e^7
 - 315*d*x^(1/3)*e^8 + 280*e^9)*e^(-10)/x^3)*e + 2520*log(d + e/x^(1/3))/x^
3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

3.497 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=572

$$\frac{2b^2e^9n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{3d^9}+\frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7}+\frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5}-\frac{2be^4nx^{5/3}\left(a+b\right.}{}$$

```
[Out] (481*b^2*e^8*n^2*x^(1/3))/(420*d^8) - (341*b^2*e^7*n^2*x^(2/3))/(840*d^7) +
 (743*b^2*e^6*n^2*x)/(3780*d^6) - (533*b^2*e^5*n^2*x^(4/3))/(5040*d^5) + (7
3*b^2*e^4*n^2*x^(5/3))/(1260*d^4) - (5*b^2*e^3*n^2*x^2)/(168*d^3) + (b^2*e^
2*n^2*x^(7/3))/(84*d^2) - (481*b^2*e^9*n^2*Log[d + e/x^(1/3)])/(420*d^9) -
(2*b*e^8*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9
) + (b*e^7*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^7) - (2*b*e^6*n
*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^6) + (b*e^5*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n]))/(6*d^5) - (2*b*e^4*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
1/3))^n]))/(15*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^3
) - (2*b*e^2*n*x^(7/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(21*d^2) + (b*e*n*
x^(8/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*d) - (2*b*e^9*n*Log[1 - d/(d
+ e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9) + (x^3*(a + b*Log[c
*(d + e/x^(1/3))^n])^2)/3 - (761*b^2*e^9*n^2*Log[x])/(1260*d^9) + (2*b^2*e^
9*n^2*PolyLog[2, d/(d + e/x^(1/3))])/(3*d^9)
```

Rubi [A] time = 1.71509, antiderivative size = 596, normalized size of antiderivative = 1.04, number of steps used = 38, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{2b^2e^9n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{3d^9}+\frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7}+\frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5}-\frac{2be^4nx^{5/3}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (481*b^2*e^8*n^2*x^(1/3))/(420*d^8) - (341*b^2*e^7*n^2*x^(2/3))/(840*d^7) +
 (743*b^2*e^6*n^2*x)/(3780*d^6) - (533*b^2*e^5*n^2*x^(4/3))/(5040*d^5) + (7
3*b^2*e^4*n^2*x^(5/3))/(1260*d^4) - (5*b^2*e^3*n^2*x^2)/(168*d^3) + (b^2*e^
2*n^2*x^(7/3))/(84*d^2) - (481*b^2*e^9*n^2*Log[d + e/x^(1/3)])/(420*d^9) -
(2*b*e^8*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9
) + (b*e^7*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^7) - (2*b*e^6*n
*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^6) + (b*e^5*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n]))/(6*d^5) - (2*b*e^4*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
1/3))^n]))/(15*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^3
) - (2*b*e^2*n*x^(7/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(21*d^2) + (b*e*n*
x^(8/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*d) + (e^9*(a + b*Log[c*(d + e
/x^(1/3))^n])^2)/(3*d^9) + (x^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/3 - (2*
b*e^9*n*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/(3*d^9) - (
761*b^2*e^9*n^2*Log[x])/(1260*d^9) - (2*b^2*e^9*n^2*PolyLog[2, 1 + e/(d*x^(
1/3))])/(3*d^9)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx &= -\left(3\,\mathrm{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^{10}}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2ben)\,\mathrm{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^9(d+ex)}\,dx, \cdots\right.\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2bn)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^9}\,dx, x, d+\cdots\right.\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(2bn)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^9}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d} + \cdots\\
&= \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{(2ben)\cdots}{\cdots}\\
&= -\frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}\cdots\\
&= \frac{b^2e^8n^2\sqrt[3]{x}}{12d^8} - \frac{b^2e^7n^2x^{2/3}}{24d^7} + \frac{b^2e^6n^2x}{36d^6} - \frac{b^2e^5n^2x^{4/3}}{48d^5} + \frac{b^2e^4n^2x^{5/3}}{60d^4} - \frac{b^2e^3n^2x^2}{72d^3} + \frac{b^2\cdots}{\cdots}\\
&= \frac{5b^2e^8n^2\sqrt[3]{x}}{28d^8} - \frac{5b^2e^7n^2x^{2/3}}{56d^7} + \frac{5b^2e^6n^2x}{84d^6} - \frac{5b^2e^5n^2x^{4/3}}{112d^5} + \frac{b^2e^4n^2x^{5/3}}{28d^4} - \frac{5b^2e^3n^2x\cdots}{168d^3}\cdots\\
&= \frac{73b^2e^8n^2\sqrt[3]{x}}{252d^8} - \frac{73b^2e^7n^2x^{2/3}}{504d^7} + \frac{73b^2e^6n^2x}{756d^6} - \frac{73b^2e^5n^2x^{4/3}}{1008d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4} - \cdots\\
&= \frac{533b^2e^8n^2\sqrt[3]{x}}{1260d^8} - \frac{533b^2e^7n^2x^{2/3}}{2520d^7} + \frac{533b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4}\cdots\\
&= \frac{743b^2e^8n^2\sqrt[3]{x}}{1260d^8} - \frac{743b^2e^7n^2x^{2/3}}{2520d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4}\cdots\\
&= \frac{341b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4}\cdots\\
&= \frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4}\cdots\\
&= \frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4}\cdots
\end{aligned}
$$

Mathematica [A] time = 0.417994, size = 738, normalized size = 1.29

$$\frac{10080b^2e^9n^2\text{PolyLog}\left(2,\frac{d\sqrt[3]{x}}{e}+1\right)+5040a^2d^9x^3+10080abd^9x^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-1440abd^7e^2nx^{7/3}+1680abd^6e^3nx^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (-10080*a*b*d*e^8*n*x^(1/3) + 17316*b^2*d*e^8*n^2*x^(1/3) + 5040*a*b*d^2*e^
7*n*x^(2/3) - 6138*b^2*d^2*e^7*n^2*x^(2/3) - 3360*a*b*d^3*e^6*n*x + 2972*b^
2*d^3*e^6*n^2*x + 2520*a*b*d^4*e^5*n*x^(4/3) - 1599*b^2*d^4*e^5*n^2*x^(4/3)
 - 2016*a*b*d^5*e^4*n*x^(5/3) + 876*b^2*d^5*e^4*n^2*x^(5/3) + 1680*a*b*d^6*
e^3*n*x^2 - 450*b^2*d^6*e^3*n^2*x^2 - 1440*a*b*d^7*e^2*n*x^(7/3) + 180*b^2*
d^7*e^2*n^2*x^(7/3) + 1260*a*b*d^8*e*n*x^(8/3) + 5040*a^2*d^9*x^3 - 22356*b
^2*e^9*n^2*Log[d + e/x^(1/3)] - 10080*b^2*d*e^8*n*x^(1/3)*Log[c*(d + e/x^(1
/3))^n] + 5040*b^2*d^2*e^7*n*x^(2/3)*Log[c*(d + e/x^(1/3))^n] - 3360*b^2*d^
3*e^6*n*x*Log[c*(d + e/x^(1/3))^n] + 2520*b^2*d^4*e^5*n*x^(4/3)*Log[c*(d +
e/x^(1/3))^n] - 2016*b^2*d^5*e^4*n*x^(5/3)*Log[c*(d + e/x^(1/3))^n] + 1680*
b^2*d^6*e^3*n*x^2*Log[c*(d + e/x^(1/3))^n] - 1440*b^2*d^7*e^2*n*x^(7/3)*Log
[c*(d + e/x^(1/3))^n] + 1260*b^2*d^8*e*n*x^(8/3)*Log[c*(d + e/x^(1/3))^n] +
 10080*a*b*d^9*x^3*Log[c*(d + e/x^(1/3))^n] + 5040*b^2*d^9*x^3*Log[c*(d + e
/x^(1/3))^n]^2 + 10080*a*b*e^9*n*Log[e + d*x^(1/3)] - 5040*b^2*e^9*n^2*Log[
e + d*x^(1/3)] + 10080*b^2*e^9*n*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)
] - 5040*b^2*e^9*n^2*Log[e + d*x^(1/3)]^2 + 10080*b^2*e^9*n^2*Log[e + d*x^(
1/3)]*Log[-((d*x^(1/3))/e)] - 7452*b^2*e^9*n^2*Log[x] + 10080*b^2*e^9*n^2*P
olyLog[2, 1 + (d*x^(1/3))/e])/(15120*d^9)
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}b^2x^3\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)^2-\int-\frac{9\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^3+9\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x^{\frac{8}{3}}+9\left(b^2dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((d*x^(1/3) + e)^n)^2 - integrate(-1/9*(9*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^3 + 9*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(8/3) + 9*(b^2*d*x^3 + b^2*e*x^(8/3))*log(x^(1/3*n))^2 - 2*(b^2*d*n*x^3 -
```

```
9*(b^2*d*log(c) + a*b*d)*x^3 - 9*(b^2*e*log(c) + a*b*e)*x^(8/3) + 9*(b^2*d*
x^3 + b^2*e*x^(8/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 18*((b^2*d*lo
g(c) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(8/3))*log(x^(1/3*n)))/(d*x +
e*x^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2x^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2+2\,abx^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*x^(2/3))/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x^2, x)
```

3.498 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=400

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^6}-\frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4}-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}+\frac{be^6n\log\left(1-\frac{d}{d+\dots}\right.}{\dots}$$

```
[Out] (-77*b^2*e^5*n^2*x^(1/3))/(60*d^5) + (47*b^2*e^4*n^2*x^(2/3))/(120*d^4) - (
3*b^2*e^3*n^2*x)/(20*d^3) + (b^2*e^2*n^2*x^(4/3))/(20*d^2) + (77*b^2*e^6*n^
2*Log[d + e/x^(1/3)])/(60*d^6) + (b*e^5*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Lo
g[c*(d + e/x^(1/3))^n]))/d^6 - (b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3)
)^n]))/(2*d^4) + (b*e^3*n*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^3) - (b*
e^2*n*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*d^2) + (b*e*n*x^(5/3)*(a
 + b*Log[c*(d + e/x^(1/3))^n]))/(5*d) + (b*e^6*n*Log[1 - d/(d + e/x^(1/3))]
*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^6 + (x^2*(a + b*Log[c*(d + e/x^(1/3))^
n])^2)/2 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^2*PolyLog[2, d/(
d + e/x^(1/3))])/d^6
```

Rubi [A] time = 1.02412, antiderivative size = 423, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^6}-\frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4}-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}-\frac{e^6\left(a+b\log\left(\dots\right.\right.}{2\dots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(1/3))/(60*d^5) + (47*b^2*e^4*n^2*x^(2/3))/(120*d^4) - (
3*b^2*e^3*n^2*x)/(20*d^3) + (b^2*e^2*n^2*x^(4/3))/(20*d^2) + (77*b^2*e^6*n^
2*Log[d + e/x^(1/3)])/(60*d^6) + (b*e^5*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Lo
g[c*(d + e/x^(1/3))^n]))/d^6 - (b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3)
)^n]))/(2*d^4) + (b*e^3*n*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^3) - (b*
e^2*n*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*d^2) + (b*e*n*x^(5/3)*(a
 + b*Log[c*(d + e/x^(1/3))^n]))/(5*d) - (e^6*(a + b*Log[c*(d + e/x^(1/3))^n
])^2)/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/2 + (b*e^6*n*(a +
b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^6 + (137*b^2*e^6*n^2*L
og[x])/(180*d^6) + (b^2*e^6*n^2*PolyLog[2, 1 + e/(d*x^(1/3))])/d^6
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx=-\left(3\,\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^7}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(ben)\,\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}dx,x,\frac{1}{\sqrt[3]{x}}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d+\frac{e}{\sqrt[3]{x}}\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{(bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{(ben)}{}$$

$$=\frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{(ben)\,\text{Subst}}{}$$

$$=-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}+\frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d}+\frac{1}{2}x^2\left(a\right.$$

$$=-\frac{b^2e^5n^2\sqrt[3]{x}}{5d^5}+\frac{b^2e^4n^2x^{2/3}}{10d^4}-\frac{b^2e^3n^2x}{15d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{5d^6}+\frac{be^3nx}{}$$

$$=-\frac{9b^2e^5n^2\sqrt[3]{x}}{20d^5}+\frac{9b^2e^4n^2x^{2/3}}{40d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{9b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{20d^6}-\frac{be}{}$$

$$=-\frac{47b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{47b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}$$

$$=-\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}$$

$$=-\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}$$

Mathematica [A] time = 0.238926, size = 546, normalized size = 1.36

$$-360b^2e^6n^2\text{PolyLog}\left(2,\frac{d\sqrt[3]{x}}{e}+1\right)+180a^2d^6x^2+360abd^6x^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-90abd^4e^2nx^{4/3}-180abd^2e^4nx^{2/3}+1$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (360*a*b*d*e^5*n*x^(1/3) - 462*b^2*d*e^5*n^2*x^(1/3) - 180*a*b*d^2*e^4*n*x^
(2/3) + 141*b^2*d^2*e^4*n^2*x^(2/3) + 120*a*b*d^3*e^3*n*x - 54*b^2*d^3*e^3*
n^2*x - 90*a*b*d^4*e^2*n*x^(4/3) + 18*b^2*d^4*e^2*n^2*x^(4/3) + 72*a*b*d^5*
e*n*x^(5/3) + 180*a^2*d^6*x^2 + 642*b^2*e^6*n^2*Log[d + e/x^(1/3)] + 360*b^
2*d*e^5*n*x^(1/3)*Log[c*(d + e/x^(1/3))^n] - 180*b^2*d^2*e^4*n*x^(2/3)*Log[
c*(d + e/x^(1/3))^n] + 120*b^2*d^3*e^3*n*x*Log[c*(d + e/x^(1/3))^n] - 90*b^
2*d^4*e^2*n*x^(4/3)*Log[c*(d + e/x^(1/3))^n] + 72*b^2*d^5*e*n*x^(5/3)*Log[c
*(d + e/x^(1/3))^n] + 360*a*b*d^6*x^2*Log[c*(d + e/x^(1/3))^n] + 180*b^2*d^
6*x^2*Log[c*(d + e/x^(1/3))^n]^2 - 360*a*b*e^6*n*Log[e + d*x^(1/3)] + 180*b
^2*e^6*n^2*Log[e + d*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(1/3))^n]*Log[
e + d*x^(1/3)] + 180*b^2*e^6*n^2*Log[e + d*x^(1/3)]^2 - 360*b^2*e^6*n^2*Log
[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] + 214*b^2*e^6*n^2*Log[x] - 360*b^2*e^
6*n^2*PolyLog[2, 1 + (d*x^(1/3))/e])/(360*d^6)
```

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b^2x^2\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)^2-\int-\frac{3\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^2+3\left(b^2dx^2+b^2ex^{\frac{5}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^2+3\left(b^2e\log(c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 + 3*(b^2*d*x^2 + b^2*e*x^(5/3))*log(x^(1/3*n))^
2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(5/3) - (b^2*d*n*x^2 - 6*
(b^2*d*log(c) + a*b*d)*x^2 - 6*(b^2*e*log(c) + a*b*e)*x^(5/3) + 6*(b^2*d*x^
2 + b^2*e*x^(5/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*((b^2*d*log(c
) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(5/3))*log(x^(1/3*n)))/(d*x + e*x
^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2+2\,abx\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a^2x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*x
^(2/3))/x)^n) + a^2*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x, x)
```

3.499 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=227

$$\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}-\frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{2be^2n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^3}$$

```
[Out] (b^2*e^2*n^2*x^(1/3))/d^2 - (b^2*e^3*n^2*Log[d + e/x^(1/3)])/d^3 - (2*b*e^2
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + (b*e*n*x
^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d - (2*b*e^3*n*Log[1 - d/(d + e/x^
(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + x*(a + b*Log[c*(d + e/x^(1/
3))^n])^2 - (b^2*e^3*n^2*Log[x])/d^3 + (2*b^2*e^3*n^2*PolyLog[2, d/(d + e/x
^(1/3))])/d^3
```

Rubi [A] time = 0.53316, antiderivative size = 248, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^3}+\frac{e^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}-\frac{2be^3n\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{2be^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(1/3))/d^2 - (b^2*e^3*n^2*Log[d + e/x^(1/3)])/d^3 - (2*b*e^2
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + (b*e*n*x
^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d + (e^3*(a + b*Log[c*(d + e/x^(1/
3))^n])^2)/d^3 + x*(a + b*Log[c*(d + e/x^(1/3))^n])^2 - (2*b*e^3*n*(a + b*L
og[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^3 - (b^2*e^3*n^2*Log[x])/
d^3 - (2*b^2*e^3*n^2*PolyLog[2, 1 + e/(d*x^(1/3))])/d^3
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= -\left(3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4} dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (2ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)} dx, x, \frac{1}{\sqrt[3]{x}}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx, x, d+\frac{e}{\sqrt[3]{x}}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d} + \frac{(2ben}{}$$

$$= \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{(2ben)\,\text{Subst}}{}$$

$$= -\frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right.\right.}{d}$$

$$= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$

$$= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$

Mathematica [A] time = 0.18659, size = 237, normalized size = 1.04

$$x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{ben\left(3be^2n\left(\log\left(d\sqrt[3]{x}+e\right)\left(\log\left(d\sqrt[3]{x}+e\right)-2\log\left(-\frac{d\sqrt[3]{x}}{e}\right)\right)-2\text{PolyLog}\left(2,\frac{d\sqrt[3]{x}}{e}+1\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/x^(1/3))^n])^2 - (b*e*n*(6*a*d*e*x^(1/3) + 6*b*d*e*x^
(1/3)*Log[c*(d + e/x^(1/3))^n] - 3*d^2*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))
^n]) - 6*e^2*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[e + d*x^(1/3)] + 3*b*e*n*
(-(d*x^(1/3)) + e*Log[e + d*x^(1/3)]) + 2*b*e^2*n*(3*Log[d + e/x^(1/3)] + L
og[x]) + 3*b*e^2*n*(Log[e + d*x^(1/3)]*(Log[e + d*x^(1/3)] - 2*Log[-((d*x^(
1/3))/e)]) - 2*PolyLog[2, 1 + (d*x^(1/3))/e])))/(3*d^3)
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(en\left(\frac{2\,e^{2}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{3}}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^{2}}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)ab+\left(x\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)^{2}-\int-\frac{3\,dx\log\left(c\right)^{2}+3\,ex^{\frac{2}{3}}\log(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] (e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*l
og(c*(d + e/x^(1/3))^n))*a*b + (x*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3
*(3*d*x*log(c)^2 + 3*e*x^(2/3)*log(c)^2 + 3*(d*x + e*x^(2/3))*log(x^(1/3*n)
)^2 - 2*(d*n*x - 3*d*x*log(c) - 3*e*x^(2/3)*log(c) + 3*(d*x + e*x^(2/3))*lo
g(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*(d*x*log(c) + e*x^(2/3)*log(c))*lo
g(x^(1/3*n)))/(d*x + e*x^(2/3)), x))*b^2 + a^2*x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^{2}\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)^{2}+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)+a^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(2/
3))/x)^n) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)^{n}\right) + a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2, x)
```

3.500 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$-6bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+6b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.$$

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))] - 6*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))] + 6*b^2*n^2*PolyLog
[3, 1 + e/(d*x^(1/3))]
```

Rubi [A] time = 0.130883, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$-6bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+6b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))] - 6*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))] + 6*b^2*n^2*PolyLog
[3, 1 + e/(d*x^(1/3))]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = -\left(3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log\ldots}{d+e\ldots}\right.$$
$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (6bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\ldots}{x}\right.$$
$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\text{Li}_2\left(\ldots\right.$$
$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\text{Li}_2\left(\ldots\right.$$

Mathematica [B] time = 0.209201, size = 389, normalized size = 4.18

$$2bn\left(3\text{PolyLog}\left(2,-\frac{e}{d\sqrt[3]{x}}\right)+\log(x)\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(\frac{e}{d\sqrt[3]{x}}+1\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((Log[d + e/x^(1/
3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) + 3*
b^2*n^2*(2*Log[e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d + e/
x^(1/3)] - Log[e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[e/d +
 x^(1/3)]^2*Log[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] - 27*Log
[e/d + x^(1/3)]^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log
[x] + 54*Log[e/d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d + e/x^(
1/3)]*Log[x]^2 - 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 162*PolyLog
[3, 1 + (d*x^(1/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81)
```

Maple [F] time = 0.351, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)^2\log(x) - \int -\frac{3\left(b^2dx + b^2ex^{\frac{2}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^2 + 3\left(b^2d\log(c)^2 + 2\,abd\log(c) + a^2d\right)x - 2\left(b^2dnx\log(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((d*x^(1/3) + e)^n)^2*log(x) - integrate(-1/3*(3*(b^2*d*x + b^2*e*x^
(2/3))*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - 2
*(b^2*d*n*x*log(x) - 3*(b^2*d*log(c) + a*b*d)*x + 3*(b^2*d*x + b^2*e*x^(2/3
))*log(x^(1/3*n)) - 3*(b^2*e*log(c) + a*b*e)*x^(2/3))*log((d*x^(1/3) + e)^n
) - 6*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(2/3))*log(x^(1/
3*n)) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(2/3))/(d*x^2 + e*x^(
5/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2 + 2\,ab\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(2
/3))/x)^n) + a^2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2/x, x)
```

3.501 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=269

$$-\frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}+\frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}-\frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\right.}{e^3}$$

```
[Out] (3*b^2*d*n^2*(d + e/x^(1/3))^2)/(2*e^3) - (2*b^2*n^2*(d + e/x^(1/3))^3)/(9*
e^3) - (6*b^2*d^2*n^2)/(e^2*x^(1/3)) + (b^2*d^3*n^2*Log[d + e/x^(1/3)]^2)/e
^3 + (6*b*d^2*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 - (3*
b*d*n*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 + (2*b*n*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) - (2*b*d^3*n*Log[d
+ e/x^(1/3)]*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 - (a + b*Log[c*(d + e/x^
(1/3))^n])^2/x
```

Rubi [A] time = 0.311076, antiderivative size = 212, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3}+\frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{\left(a+b\log\left(c\left(d+\right.\right.\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(1/3))^2)/(2*e^3) - (2*b^2*n^2*(d + e/x^(1/3))^3)/(9*
e^3) - (6*b^2*d^2*n^2)/(e^2*x^(1/3)) + (b^2*d^3*n^2*Log[d + e/x^(1/3)]^2)/e
^3 + (b*n*((18*d^2*(d + e/x^(1/3)))/e^3 - (9*d*(d + e/x^(1/3))^2)/e^3 + (2*
(d + e/x^(1/3))^3)/e^3 - (6*d^3*Log[d + e/x^(1/3)])/e^3)*(a + b*Log[c*(d +
e/x^(1/3))^n]))/3 - (a + b*Log[c*(d + e/x^(1/3))^n])^2/x
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\left(3\,\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2ben)\,\mathrm{Subst}\left(\int \frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx,x,\frac{1}{\sqrt[3]{x}}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2bn)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx,x,d+\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}\right)\left(a+b\log\left(\right.\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}\right)\left(a+b\log\left(\right.\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}\right)\left(a+b\log\left(\right.\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right.}{e^3}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{1}{3}bn\left(\frac{18\ldots}{\ldots}\right.$$

Mathematica [C] time = 0.360091, size = 374, normalized size = 1.39

$$\frac{bn\left(-36bd^3nx\,\mathrm{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)-36bd^3nx\,\mathrm{PolyLog}\left(2,\frac{d\sqrt[3]{x}}{e}+1\right)-36d^3x\log\left(d\sqrt[3]{x}+e\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-36d^3x\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-18de^2\sqrt[3]{x}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (-18*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*n*(-2*b*e*n*(2*e^2 - 3*d*e*x^(
1/3) + 6*d^2*x^(2/3)) + 9*b*d*e*n*(e - 2*d*x^(1/3))*x^(1/3) + 36*a*d^2*e*x^
(2/3) - 36*b*d^2*e*n*x^(2/3) + 30*b*d^3*n*x*Log[d + e/x^(1/3)] + 36*b*d^2*(
e + d*x^(1/3))*x^(2/3)*Log[c*(d + e/x^(1/3))^n] + 12*e^3*(a + b*Log[c*(d +
e/x^(1/3))^n]) - 18*d*e^2*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]) - 36*d^3
*x*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[e + d*x^(1/3)] - 36*d^3*x*(a + b*Lo
g[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] + 18*b*d^3*n*x*Log[e + d*x^(1
/3)]*(Log[e + d*x^(1/3)] - 2*Log[-((d*x^(1/3))/e)]) - 36*b*d^3*n*x*PolyLog[
2, 1 + e/(d*x^(1/3))] - 36*b*d^3*n*x*PolyLog[2, 1 + (d*x^(1/3))/e]))/e^3)/(
18*x)
```

Maple [F] time = 0.576, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

Maxima [A] time = 1.06979, size = 383, normalized size = 1.42

$$-\frac{1}{3}aben\left(\frac{6d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2d^3\log(x)}{e^4}-\frac{6d^2x^{\frac{2}{3}}-3dex^{\frac{1}{3}}+2e^2}{e^3x}\right)-\frac{1}{18}\left(6en\left(\frac{6d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2d^3\log(x)}{e^4}-6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2
/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - 1/18*(6*e*n*(6*d^3*log(d*x^(1/3) +
e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)
)*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x
)^2 - 22*d^3*x*log(x) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*
d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e))*n^2/(e^3*x))*b^2 - b^2*log(c*(
d + e/x^(1/3))^n)^2/x - 2*a*b*log(c*(d + e/x^(1/3))^n)/x - a^2/x
```

Fricas [A] time = 1.90434, size = 798, normalized size = 2.97

$$-\frac{4b^2e^3n^2-12abe^3n+18a^2e^3-18\left(b^2e^3x-b^2e^3\right)\log(c)^2+18\left(b^2d^3n^2x+b^2e^3n^2\right)\log\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^2-2\left(2b^2e^3n^2-6abe\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/18*(4*b^2*e^3*n^2 - 12*a*b*e^3*n + 18*a^2*e^3 - 18*(b^2*e^3*x - b^2*e^3)
*log(c)^2 + 18*(b^2*d^3*n^2*x + b^2*e^3*n^2)*log((d*x + e*x^(2/3))/x)^2 - 2
*(2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x - 12*(b^2*e^3*n - 3*a*b*e^3 -
(b^2*e^3*n - 3*a*b*e^3)*x)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(2/3) - 3*b^2*d*e^
2*n^2*x^(1/3) + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*d^3*n
)*x - 6*(b^2*d^3*n*x + b^2*e^3*n)*log(c))*log((d*x + e*x^(2/3))/x) + 6*(11*
b^2*d^2*e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) - 3*(5*b^2*d*
e^2*n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(1/3))/(e^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2/x^2, x)
```

3.502 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=479

$$\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}-\frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}+\frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\right.}{2e^6}$$

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(1/3))^2)/(4*e^6) + (20*b^2*d^3*n^2*(d + e/x^(1/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(1/3))^4)/(16*e^6) + (6*b^2*d*n^2*
(d + e/x^(1/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(1/3))^6)/(36*e^6) + (6*b^2
*d^5*n^2)/(e^5*x^(1/3)) - (b^2*d^6*n^2*Log[d + e/x^(1/3)]^2)/(2*e^6) - (6*b
*d^5*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^6 + (15*b*d^4*n*
(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^6) - (20*b*d^3*n*(
d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) + (15*b*d^2*n*(d
 + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) - (6*b*d*n*(d + e
/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(5*e^6) + (b*n*(d + e/x^(1/3)
)^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(6*e^6) + (b*d^6*n*Log[d + e/x^(1/3)]
*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^6 - (a + b*Log[c*(d + e/x^(1/3))^n])^2
/(2*x^2)
```

Rubi [A] time = 0.479669, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}-\frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6}+\frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6}-\frac{225d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{e^6}-\frac{60d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}+\frac{72d\left(d+\right.}{e^6}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(1/3))^2)/(4*e^6) + (20*b^2*d^3*n^2*(d + e/x^(1/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(1/3))^4)/(16*e^6) + (6*b^2*d*n^2*
(d + e/x^(1/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(1/3))^6)/(36*e^6) + (6*b^2
*d^5*n^2)/(e^5*x^(1/3)) - (b^2*d^6*n^2*Log[d + e/x^(1/3)]^2)/(2*e^6) - (b*n
*((360*d^5*(d + e/x^(1/3)))/e^6 - (450*d^4*(d + e/x^(1/3))^2)/e^6 + (400*d^
3*(d + e/x^(1/3))^3)/e^6 - (225*d^2*(d + e/x^(1/3))^4)/e^6 + (72*d*(d + e/x
^(1/3))^5)/e^6 - (10*(d + e/x^(1/3))^6)/e^6 - (60*d^6*Log[d + e/x^(1/3)])/e
^6)*(a + b*Log[c*(d + e/x^(1/3))^n]))/60 - (a + b*Log[c*(d + e/x^(1/3))^n])
^2/(2*x^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{}}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}\,dx, x, d\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{\sqrt[3]{x}}\right.}{e^6}\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{\sqrt[3]{x}}\right.}{e^6}\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{\sqrt[3]{x}}\right.}{e^6}\right.$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6} + \frac{6b^2dn^2\left(d\right.}{25e^6}$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6} + \frac{6b^2dn^2\left(d\right.}{25e^6}$$

Mathematica [C] time = 0.339805, size = 698, normalized size = 1.46

$$\frac{3600b^2d^6n^2x^2\text{PolyLog}\left(2, \frac{e}{d\sqrt[3]{x}}+1\right) + 3600b^2d^6n^2x^2\text{PolyLog}\left(2, \frac{d\sqrt[3]{x}}{e}+1\right) - 1800a^2e^6 - 3600abe^6\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

```
[Out] (-1800*a^2*e^6 + 600*a*b*e^6*n - 100*b^2*e^6*n^2 - 720*a*b*d*e^5*n*x^(1/3)
+ 264*b^2*d*e^5*n^2*x^(1/3) + 900*a*b*d^2*e^4*n*x^(2/3) - 555*b^2*d^2*e^4*n
^2*x^(2/3) - 1200*a*b*d^3*e^3*n*x + 1140*b^2*d^3*e^3*n^2*x + 1800*a*b*d^4*e
^2*n*x^(4/3) - 2610*b^2*d^4*e^2*n^2*x^(4/3) - 3600*a*b*d^5*e*n*x^(5/3) + 88
20*b^2*d^5*e*n^2*x^(5/3) - 5220*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)] - 3600*a
*b*e^6*Log[c*(d + e/x^(1/3))^n] + 600*b^2*e^6*n*Log[c*(d + e/x^(1/3))^n] -
720*b^2*d*e^5*n*x^(1/3)*Log[c*(d + e/x^(1/3))^n] + 900*b^2*d^2*e^4*n*x^(2/3
)*Log[c*(d + e/x^(1/3))^n] - 1200*b^2*d^3*e^3*n*x*Log[c*(d + e/x^(1/3))^n]
+ 1800*b^2*d^4*e^2*n*x^(4/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b^2*d^5*e*n*x^
(5/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n
] - 1800*b^2*e^6*Log[c*(d + e/x^(1/3))^n]^2 + 3600*a*b*d^6*n*x^2*Log[e + d*
x^(1/3)] + 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] -
```

```
1800*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)]^2 + 3600*a*b*d^6*n*x^2*Log[-(e/(d*
x^(1/3)))] + 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)
))] + 3600*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] + 3600*
b^2*d^6*n^2*x^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 3600*b^2*d^6*n^2*x^2*PolyLo
g[2, 1 + (d*x^(1/3))/e])/(3600*e^6*x^2)
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)
```

Maxima [A] time = 1.08619, size = 522, normalized size = 1.09

$$\frac{1}{60}aben\left(\frac{60\,d^6\log\left(dx^{\frac{1}{3}}+e\right)}{e^7}-\frac{20\,d^6\log(x)}{e^7}-\frac{60\,d^5x^{\frac{5}{3}}-30\,d^4ex^{\frac{4}{3}}+20\,d^3e^2x-15\,d^2e^3x^{\frac{2}{3}}+12\,de^4x^{\frac{1}{3}}-10\,e^5}{e^6x^2}\right)+\frac{1}{3600}\left(6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/60*a*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x
^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^
(1/3) - 10*e^5)/(e^6*x^2)) + 1/3600*(60*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7
- 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 1
5*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e/x^(1
/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 2940*
d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3*x
 + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log(x
) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*b^2 - 1/2*b^2*log(c*(d
+ e/x^(1/3))^n)^2/x^2 - a*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 2.04804, size = 1310, normalized size = 2.73

$$-\frac{100\,b^2e^6n^2-600\,abe^6n+1800\,a^2e^6-20\left(90\,a^2e^6-\left(57\,b^2d^3e^3-5\,b^2e^6\right)n^2+30\left(2\,abd^3e^3-abe^6\right)n\right)x^2-1800\left(b^2e^6x^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/3600*(100*b^2*e^6*n^2 - 600*a*b*e^6*n + 1800*a^2*e^6 - 20*(90*a^2*e^6 -
(57*b^2*d^3*e^3 - 5*b^2*e^6)*n^2 + 30*(2*a*b*d^3*e^3 - a*b*e^6)*n)*x^2 - 18
```

```
00*(b^2*e^6*x^2 - b^2*e^6)*log(c)^2 - 1800*(b^2*d^6*n^2*x^2 - b^2*e^6*n^2)*
log((d*x + e*x^(2/3))/x)^2 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x +
 600*(2*b^2*d^3*e^3*n*x - b^2*e^6*n + 6*a*b*e^6 - (6*a*b*e^6 + (2*b^2*d^3*e
^3 - b^2*e^6)*n)*x^2)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x - 10*b^2*e^6*n^2 +
60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^2 - 60*(b^2*d^6*n*x^2 -
b^2*e^6*n)*log(c) + 15*(4*b^2*d^5*e*n^2*x - b^2*d^2*e^4*n^2)*x^(2/3) - 6*(5
*b^2*d^4*e^2*n^2*x - 2*b^2*d*e^5*n^2)*x^(1/3))*log((d*x + e*x^(2/3))/x) + 1
5*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n - 12*(49*b^2*d^5*e*n^2 - 20*a*b*d^
5*e*n)*x + 60*(4*b^2*d^5*e*n*x - b^2*d^2*e^4*n)*log(c))*x^(2/3) - 6*(44*b^2
*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x
 + 60*(5*b^2*d^4*e^2*n*x - 2*b^2*d*e^5*n)*log(c))*x^(1/3))/(e^6*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2/x^3, x)
```

3.503 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=759

result too large to display

```
[Out] (71*b^3*e^5*n^3*x^(1/3))/(40*d^5) - (3*b^3*e^4*n^3*x^(2/3))/(10*d^4) + (b^3
*e^3*n^3*x)/(20*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(1/3)])/(40*d^6) - (77*b
^2*e^5*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^
6) + (47*b^2*e^4*n^2*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(40*d^4) - (
9*b^2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^3) + (3*b^2*e^2*n^2
*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^2) - (77*b^2*e^6*n^2*Log[1
 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^6) + (3*b*e^5
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^6) - (3
*b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*d^4) + (b*e^3*n*x*(
a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^3) - (3*b*e^2*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n])^2)/(8*d^2) + (3*b*e*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
1/3))^n])^2)/(10*d) + (3*b*e^6*n*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d
 + e/x^(1/3))^n])^2)/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/2 -
 (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^6
 - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (77*b^3*e^6*n^3*PolyLog[2, d/(d + e/x^
(1/3))])/(20*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*PolyLog
[2, d/(d + e/x^(1/3))])/d^6 - (3*b^3*e^6*n^3*PolyLog[2, 1 + e/(d*x^(1/3))])
/d^6 - (3*b^3*e^6*n^3*PolyLog[3, d/(d + e/x^(1/3))])/d^6
```

Rubi [A] time = 2.97455, antiderivative size = 736, normalized size of antiderivative = 0.97, number of steps used = 73, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$\frac{3b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}-\frac{137b^3e^6n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{20d^6}-\frac{3b^3e^6n^3\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+\right.}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (71*b^3*e^5*n^3*x^(1/3))/(40*d^5) - (3*b^3*e^4*n^3*x^(2/3))/(10*d^4) + (b^3
*e^3*n^3*x)/(20*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(1/3)])/(40*d^6) - (77*b
^2*e^5*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^
6) + (47*b^2*e^4*n^2*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(40*d^4) - (
9*b^2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^3) + (3*b^2*e^2*n^2
*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^2) + (77*b*e^6*n*(a + b*Lo
g[c*(d + e/x^(1/3))^n])^2)/(40*d^6) + (3*b*e^5*n*(d + e/x^(1/3))*x^(1/3)*(a
 + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^6) - (3*b*e^4*n*x^(2/3)*(a + b*Log[c
*(d + e/x^(1/3))^n])^2)/(4*d^4) + (b*e^3*n*x*(a + b*Log[c*(d + e/x^(1/3))^n
])^2)/(2*d^3) - (3*b*e^2*n*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*d
^2) + (3*b*e*n*x^(5/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(10*d) - (e^6*(a
 + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3
))^n])^3)/2 - (137*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*
x^(1/3)))])/(20*d^6) + (3*b*e^6*n*(a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(
e/(d*x^(1/3)))])/(2*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) - (137*b^3*e^6*n
^3*PolyLog[2, 1 + e/(d*x^(1/3))])/(20*d^6) + (3*b^2*e^6*n^2*(a + b*Log[c*(d
 + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))])/d^6 - (3*b^3*e^6*n^3*PolyL
```

```
og[3, 1 + e/(d*x^(1/3))])/d^6
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = -\left(3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^7}dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^6(d+ex)}d$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{1}{2}(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx, x, d+$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d} +$$

$$= \frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 + \frac{(3ben)}{}$$

$$= -\frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2} + \frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d} +$$

$$= \frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2} + \frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^3} - \frac{3b}{}$$

$$= -\frac{9b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^3} + \frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2} -$$

$$= \frac{3b^3e^5n^3\sqrt[3]{x}}{20d^5} - \frac{3b^3e^4n^3x^{2/3}}{40d^4} + \frac{b^3e^3n^3x}{20d^3} - \frac{3b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{20d^6} + \frac{47b^2e^4n^2x^{2/3}\left(a\right.}{}$$

$$= \frac{3b^3e^5n^3\sqrt[3]{x}}{5d^5} - \frac{3b^3e^4n^3x^{2/3}}{10d^4} + \frac{b^3e^3n^3x}{20d^3} - \frac{3b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{5d^6} - \frac{77b^2e^5n^2\left(d+\right.}{}$$

$$= \frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5} - \frac{3b^3e^4n^3x^{2/3}}{10d^4} + \frac{b^3e^3n^3x}{20d^3} - \frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{40d^6} - \frac{77b^2e^5n^2\left(d\right.}{}$$

$$= \frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5} - \frac{3b^3e^4n^3x^{2/3}}{10d^4} + \frac{b^3e^3n^3x}{20d^3} - \frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{40d^6} - \frac{77b^2e^5n^2\left(d\right.}{}$$

Mathematica [A] time = 1.58747, size = 1006, normalized size = 1.33

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (60*b*d*e^5*n*x^(1/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))
^n])^2 - 30*b*d^2*e^4*n*x^(2/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d +
e/x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(
d + e/x^(1/3))^n])^2 - 15*b*d^4*e^2*n*x^(4/3)*(a - b*n*Log[d + e/x^(1/3)] +
 b*Log[c*(d + e/x^(1/3))^n])^2 + 12*b*d^5*e*n*x^(5/3)*(a - b*n*Log[d + e/x^
(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2 + 60*b*d^6*n*x^2*Log[d + e/x^(1/3)]*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2 + 20*d^6*x^2*(a
 - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^3 - 60*b*e^6*n*(a -
 b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[e + d*x^(1/3)]
+ b^2*n^2*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*(d*e^2*
x^(1/3)*(-154*e^3 + 47*d*e^2*x^(1/3) - 18*d^2*e*x^(2/3) + 6*d^3*x) - 60*(e^
6 - d^6*x^2)*Log[d + e/x^(1/3)]^2 - 274*e^6*Log[-(e/(d*x^(1/3)))] + 2*e*Log
[d + e/x^(1/3)]*(137*e^5 + 60*d*e^4*x^(1/3) - 30*d^2*e^3*x^(2/3) + 20*d^3*e
^2*x - 15*d^4*e*x^(4/3) + 12*d^5*x^(5/3) + 60*e^5*Log[-(e/(d*x^(1/3)))]) +
120*e^6*PolyLog[2, 1 + e/(d*x^(1/3))]) + b^3*n^3*(3*d^4*e^2*x^(4/3)*(2 - 5*
Log[d + e/x^(1/3)])*Log[d + e/x^(1/3)] + 12*d^5*e*x^(5/3)*Log[d + e/x^(1/3)
]^2 + 20*d^6*x^2*Log[d + e/x^(1/3)]^3 + 2*d^3*e^3*x*(1 - 9*Log[d + e/x^(1/3
)] + 10*Log[d + e/x^(1/3)]^2) - d^2*e^4*x^(2/3)*(12 - 47*Log[d + e/x^(1/3)]
 + 30*Log[d + e/x^(1/3)]^2) + d*e^5*x^(1/3)*(71 - 154*Log[d + e/x^(1/3)] +
60*Log[d + e/x^(1/3)]^2) + 225*e^6*(-Log[d + e/x^(1/3)] + Log[-(e/(d*x^(1/3
)))]) + 137*e^6*(Log[d + e/x^(1/3)]*(Log[d + e/x^(1/3)] - 2*Log[-(e/(d*x^(1
/3)))]) - 2*PolyLog[2, 1 + e/(d*x^(1/3))]) - 20*e^6*(Log[d + e/x^(1/3)]^2*(
Log[d + e/x^(1/3)] - 3*Log[-(e/(d*x^(1/3)))]) - 6*Log[d + e/x^(1/3)]*PolyLo
g[2, 1 + e/(d*x^(1/3))] + 6*PolyLog[3, 1 + e/(d*x^(1/3))])))/(40*d^6)
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,b^{3}x^{2}\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)^{3}-\int\frac{2\left(b^{3}dx^{2}+b^{3}ex^{\frac{5}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^{3}-2\left(b^{3}d\log\left(c\right)^{3}+3\,ab^{2}d\log\left(c\right)^{2}+3\,a^{2}bd\log\left(c\right)+a^{3}d\right)x^{2}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((d*x^(1/3) + e)^n)^3 - integrate(1/2*(2*(b^3*d*x^2 + b^3*e*
x^(5/3))*log(x^(1/3*n))^3 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*
b*d*log(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 6*(b^3*d*log(c) + a*b^2*d)*x^2 - 6
*(b^3*e*log(c) + a*b^2*e)*x^(5/3) + 6*(b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/
3*n)))*log((d*x^(1/3) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*
log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n))^2 - 2*(b^3*e*log(c)^3 + 3*a*b^2*e
*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(5/3) - 6*((b^3*d*log(c)^2 + 2*a*b^
```

```
2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/3*n))^2 +
(b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3) - 2*((b^3*d*log(c) +
a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n)))*log((d*x^(
1/3) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3))*log(x^(1/3*n)))/(d*x + e*
x^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^3+3\,ab^2x\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2+3\,a^2bx\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*x^(2/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(2/3))/x)^n) + a^3*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3*x, x)
```

3.504 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=436

$$\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}+\frac{6b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^3}+\cdots$$

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^
3 + (3*b^2*e^3*n^2*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^
n]))/d^3 - (3*b*e^2*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^
n])^2)/d^3 + (3*b*e*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d) - (
3*b*e^3*n*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/d^
3 + x*(a + b*Log[c*(d + e/x^(1/3))^n])^3 + (6*b^2*e^3*n^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^3 + (b^3*e^3*n^3*Log[x])/d^3 - (3*
b^3*e^3*n^3*PolyLog[2, d/(d + e/x^(1/3))])/d^3 + (6*b^2*e^3*n^2*(a + b*Log[
c*(d + e/x^(1/3))^n])*PolyLog[2, d/(d + e/x^(1/3))])/d^3 + (6*b^3*e^3*n^3*P
olyLog[2, 1 + e/(d*x^(1/3))])/d^3 + (6*b^3*e^3*n^3*PolyLog[3, d/(d + e/x^(1
/3))])/d^3
```

Rubi [A] time = 1.04215, antiderivative size = 410, normalized size of antiderivative = 0.94, number of steps used = 23, number of rules used = 17, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.85, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$-\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^3}+\frac{6b^3e^3n^3\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+\cdots\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^
3 - (3*b*e^3*n*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^3) - (3*b*e^2*n*(d
+ e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/d^3 + (3*b*e*n*x^(
2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d) + (e^3*(a + b*Log[c*(d + e/x
^(1/3))^n])^3)/d^3 + x*(a + b*Log[c*(d + e/x^(1/3))^n])^3 + (9*b^2*e^3*n^2*
(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^3 - (3*b*e^3*n*(a
 + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))])/d^3 + (b^3*e^3*n^3*
Log[x])/d^3 + (9*b^3*e^3*n^3*PolyLog[2, 1 + e/(d*x^(1/3))])/d^3 - (6*b^2*e^
3*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))])/d^3 +
 (6*b^3*e^3*n^3*PolyLog[3, 1 + e/(d*x^(1/3))])/d^3
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log \left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= -\left(3\,\text{Subst}\left(\int \frac{(a+b\log (c(d+ex)^n))^3}{x^4} dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= x\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - (3ben)\,\text{Subst}\left(\int \frac{(a+b\log (c(d+ex)^n))^2}{x^3(d+ex)} dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= x\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - (3bn)\,\text{Subst}\left(\int \frac{(a+b\log (cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx, x, d+\frac{e}{\sqrt[3]{x}}\right)$$

$$= x\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{(3bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d} + \ldots$$

$$= \frac{3benx^{2/3}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d} + x\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 + \frac{(3ben)\,\text{Sub}\ldots}{\ldots}$$

$$= -\frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + \frac{3benx^{2/3}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\ldots\right.\right.\right.}{2d}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log \ldots\right.}{d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^3n\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\ldots\right.}{2d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^3n\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\ldots\right.}{2d^3}$$

Mathematica [A] time = 0.742077, size = 675, normalized size = 1.55

$$\frac{6b^2n^2\left(-2e^3\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)+e\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(d^2x^{2/3}-2e^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-2de\sqrt[3]{x}-3e^2\right)+\left(d^3x+e^3\right)\log^2\left(d+\frac{e}{\sqrt[3]{x}}\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (-6*b*d*e^2*n*x^(1/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))
^n])^2 + 3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x
^(1/3))^n])^2 + 6*b*d^3*n*x*Log[d + e/x^(1/3)]*(a - b*n*Log[d + e/x^(1/3)]
+ b*Log[c*(d + e/x^(1/3))^n])^2 + 2*d^3*x*(a - b*n*Log[d + e/x^(1/3)] + b*L
og[c*(d + e/x^(1/3))^n])^3 + 6*b*e^3*n*(a - b*n*Log[d + e/x^(1/3)] + b*Log[
c*(d + e/x^(1/3))^n])^2*Log[e + d*x^(1/3)] + 6*b^2*n^2*(a - b*n*Log[d + e/x
^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((e^3 + d^3*x)*Log[d + e/x^(1/3)]^2 +
 e^2*(d*x^(1/3) + 3*e*Log[-(e/(d*x^(1/3)))]) + e*Log[d + e/x^(1/3)]*(-3*e^2
 - 2*d*e*x^(1/3) + d^2*x^(2/3) - 2*e^2*Log[-(e/(d*x^(1/3)))]]) - 2*e^3*PolyL
og[2, 1 + e/(d*x^(1/3))]) - b^3*n^3*(-6*e^3*Log[d + e/x^(1/3)] - 6*d*e^2*x^
```

```
(1/3)*Log[d + e/x^(1/3)] + 9*e^3*Log[d + e/x^(1/3)]^2 + 6*d*e^2*x^(1/3)*Log
[d + e/x^(1/3)]^2 - 3*d^2*e*x^(2/3)*Log[d + e/x^(1/3)]^2 - 2*e^3*Log[d + e/
x^(1/3)]^3 - 2*d^3*x*Log[d + e/x^(1/3)]^3 + 6*e^3*Log[-(e/(d*x^(1/3)))] - 1
8*e^3*Log[d + e/x^(1/3)]*Log[-(e/(d*x^(1/3)))] + 6*e^3*Log[d + e/x^(1/3)]^2
*Log[-(e/(d*x^(1/3)))] + 6*e^3*(-3 + 2*Log[d + e/x^(1/3)])*PolyLog[2, 1 + e
/(d*x^(1/3))] - 12*e^3*PolyLog[3, 1 + e/(d*x^(1/3))]))/(2*d^3)
```

Maple [F] time = 0.534, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)

[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^3x\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)^{3}+\frac{3}{2}\left(en\left(\frac{2\,e^2\log\left(dx^{\frac{1}{3}}+e\right)}{d^3}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^2}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)a^2b+a^3x-\int\frac{\left(b^3dx+b^3ex^{\frac{2}{3}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")

[Out] b^3*x*log((d*x^(1/3) + e)^n)^3 + 3/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (
d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*log(c*(d + e/x^(1/3))^n))*a^2*b + a^3*x
 - integrate(((b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))^3 + (b^3*d*n*x - 3*(
b^3*d*log(c) + a*b^2*d)*x + 3*(b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n)) - 3*
(b^3*e*log(c) + a*b^2*e)*x^(2/3))*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d*log(
c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2 - (b^3
*d*log(c)^3 + 3*a*b^2*d*log(c)^2)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*log(x^(1
/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c) + a*b^2
*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3*e*log(c)^2
+ 2*a*b^2*e*log(c))*x^(2/3))*log((d*x^(1/3) + e)^n) + 3*((b^3*d*log(c)^2 +
2*a*b^2*d*log(c))*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*x^(2/3))*log(x^(1
/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*x^(2/3))/(d*x + e*x^(2/3)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)^{3}+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)^{2}+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^(
2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{1}{3}}}\right)^n\right) + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3, x)
```

3.505 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$18b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-9bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-18b^3$$

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^3*Log[-(e/(d*x^(1/3)))] - 9*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 18*b^2*n^2*(a +
 b*Log[c*(d + e/x^(1/3))^n])*PolyLog[3, 1 + e/(d*x^(1/3))] - 18*b^3*n^3*Pol
yLog[4, 1 + e/(d*x^(1/3))]
```

Rubi [A] time = 0.196575, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$18b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-9bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-18b^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```


```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^3*Log[-(e/(d*x^(1/3)))] - 9*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 18*b^2*n^2*(a +
 b*Log[c*(d + e/x^(1/3))^n])*PolyLog[3, 1 + e/(d*x^(1/3))] - 18*b^3*n^3*Pol
yLog[4, 1 + e/(d*x^(1/3))]
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx = -\left(3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (9ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\,\text{lo}}{d+}\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (9bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\,\text{l}}{x}\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\text{Li}_2$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\text{Li}_2$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\text{Li}_2$$

Mathematica [B] time = 0.319472, size = 527, normalized size = 3.9

$$9b^2n^2\left(\frac{1}{81}\left(-162\text{PolyLog}\left(3,\frac{d\sqrt[3]{x}}{e}+1\right)-162\text{PolyLog}\left(3,-\frac{d\sqrt[3]{x}}{e}\right)+9\log^2(x)\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-9\log^2(x)\log\left(\frac{d\sqrt[3]{x}}{e}+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*((Log[d + e/x^(
1/3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) +
9*b^2*n^2*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*(2*Log[
e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d + e/x^(1/3)] - Log[
```

```
e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[e/d + x^(1/3)]^2*Log
[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] - 27*Log[e/d + x^(1/3)]
^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 54*Log[e/
d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d + e/x^(1/3)]*Log[x]^2
- 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 162*PolyLog[3, 1 + (d*x^(1
/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81) - 3*b^3*n^3*(Log[d + e/x^(1
/3)]^3*Log[-(e/(d*x^(1/3)))] + 3*Log[d + e/x^(1/3)]^2*PolyLog[2, 1 + e/(d*x
^(1/3))] - 6*Log[d + e/x^(1/3)]*PolyLog[3, 1 + e/(d*x^(1/3))] + 6*PolyLog[4
, 1 + e/(d*x^(1/3))])
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^3\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)^3\log(x)-\int\frac{\left(b^3dx+b^3ex^{\frac{2}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^3+\left(b^3dnx\log(x)-3\left(b^3d\log(c)+ab^2d\right)x+3\left(b^3dx+b^3ex^{\frac{2}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((d*x^(1/3) + e)^n)^3*log(x) - integrate(((b^3*d*x + b^3*e*x^(2/3))*
log(x^(1/3*n))^3 + (b^3*d*n*x*log(x) - 3*(b^3*d*log(c) + a*b^2*d)*x + 3*(b^
3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(2/3))
*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) +
 a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2
+ 3*a^2*b*d*log(c) + a^3*d)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))
^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*d*log(c) + a
*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3*e*log(c
)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(2/3))*log((d*x^(1/3) + e)^n) + 3*((b^3
*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*b^2*e*l
og(c) + a^2*b*e)*x^(2/3))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(
c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(2/3))/(d*x^2 + e*x^(5/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3/x, x)
```

3.506 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=438

$$-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{9bd^2n\left(d+\right.}{}$$

```
[Out] (-9*b^3*d*n^3*(d + e/x^(1/3))^2)/(4*e^3) + (2*b^3*n^3*(d + e/x^(1/3))^3)/(9
*e^3) - (18*a*b^2*d^2*n^2)/(e^2*x^(1/3)) + (18*b^3*d^2*n^3)/(e^2*x^(1/3)) -
 (18*b^3*d^2*n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^3 + (9*b^2*d*n
^2*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^3) - (2*b^2*n^2
*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) + (9*b*d^2*n*(
d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (9*b*d*n*(d + e/x^
(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*e^3) + (b*n*(d + e/x^(1/3))
^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (3*d^2*(d + e/x^(1/3))*(a + b*
Log[c*(d + e/x^(1/3))^n])^3)/e^3 + (3*d*(d + e/x^(1/3))^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])^3)/e^3 - ((d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]
)^3)/e^3
```

Rubi [A] time = 0.452248, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{9bd^2n\left(d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```


```
[Out] (-9*b^3*d*n^3*(d + e/x^(1/3))^2)/(4*e^3) + (2*b^3*n^3*(d + e/x^(1/3))^3)/(9
*e^3) - (18*a*b^2*d^2*n^2)/(e^2*x^(1/3)) + (18*b^3*d^2*n^3)/(e^2*x^(1/3)) -
 (18*b^3*d^2*n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^3 + (9*b^2*d*n
^2*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^3) - (2*b^2*n^2
*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) + (9*b*d^2*n*(
d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (9*b*d*n*(d + e/x^
(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*e^3) + (b*n*(d + e/x^(1/3))
^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (3*d^2*(d + e/x^(1/3))*(a + b*
Log[c*(d + e/x^(1/3))^n])^3)/e^3 + (3*d*(d + e/x^(1/3))^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])^3)/e^3 - ((d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]
)^3)/e^3
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} + \left(\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\,\text{Subst}\left(\int (d+ex)\,(a\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3\right.}{e^3}$$

$$= -\frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3} + \frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$

$$= \frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3} - \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right.\right.}{2e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3} + \frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\right.}{2e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3} + \frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{18b^3d^2n^3}{e^2\sqrt[3]{x}} - \frac{18b^3d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right.}{}$$

Mathematica [A] time = 0.794028, size = 666, normalized size = 1.52

$$\frac{-6b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\left(18a^2e^3 - 6aben\left(6d^2x^{2/3} - 3de\sqrt[3]{x} + 2e^2\right) + 6bd^3nx(6a-11bn)\log\left(d\sqrt[3]{x}+e\right) + 2bd^3nx\log(x)(11b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(1/3) + 90*a*b^2*d*e^2*n^2*x^(1/3) - 57*b^3*d*e^2*n^3*x^(1/3) +
108*a^2*b*d^2*e*n*x^(2/3) - 396*a*b^2*d^2*e*n^2*x^(2/3) + 510*b^3*d^2*e*n^3
*x^(2/3) + 72*b^3*d^3*n^3*x*Log[d + e/x^(1/3)]^3 - 36*b^3*e^3*Log[c*(d + e/
x^(1/3))^n]^3 - 108*a^2*b*d^3*n*x*Log[e + d*x^(1/3)] + 396*a*b^2*d^3*n^2*x*
Log[e + d*x^(1/3)] - 510*b^3*d^3*n^3*x*Log[e + d*x^(1/3)] + 12*b^2*d^3*n^2*
x*Log[d + e/x^(1/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e
 + d*x^(1/3)] - Log[x]) + 36*a^2*b*d^3*n*x*Log[x] - 132*a*b^2*d^3*n^2*x*Log
[x] + 170*b^3*d^3*n^3*x*Log[x] - 18*b^2*d^3*n^2*x*Log[d + e/x^(1/3)]^2*(6*a
 - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n] + 6*b*n*Log[e + d*x^(1/3)] - 2*b*n
*Log[x]) + 18*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-6*a*e^2 + 2*b*e^2*n - 3*b
*d*e*n*x^(1/3) + 6*b*d^2*n*x^(2/3)) - 6*b*d^3*n*x*Log[e + d*x^(1/3)] + 2*b*
d^3*n*x*Log[x]) - 6*b*Log[c*(d + e/x^(1/3))^n]*(18*a^2*e^3 - 6*a*b*e*n*(2*e
^2 - 3*d*e*x^(1/3) + 6*d^2*x^(2/3)) + b^2*e*n^2*(4*e^2 - 15*d*e*x^(1/3) + 6
6*d^2*x^(2/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x*Log[e + d*x^(1/3)] + 2*b*d^3*n*
(-6*a + 11*b*n)*x*Log[x]))/(36*e^3*x)
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

Maxima [A] time = 1.16536, size = 864, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^
(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b^3*log(c*(d + e/x^(1/3))^n)^3/x
- 1/6*(6*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2
/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*
log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x) - 66*d^2*e*x^(2/3
) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3)
+ e))*n^2/(e^3*x))*a*b^2 - 1/108*(54*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*
d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*log(c*(d
+ e/x^(1/3))^n)^2 + e*n*((108*d^3*x*log(d*x^(1/3) + e)^3 - 4*d^3*x*log(x)^3
 + 66*d^3*x*log(x)^2 - 510*d^3*x*log(x) - 1530*d^2*e*x^(2/3) + 171*d*e^2*x^
(1/3) - 24*e^3 - 54*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e)^2 + 18*(
2*d^3*x*log(x)^2 - 22*d^3*x*log(x) + 85*d^3*x)*log(d*x^(1/3) + e))*n^2/(e^4
*x) - 18*(18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x
) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*log(x) - 11*d^
3*x)*log(d*x^(1/3) + e))*n*log(c*(d + e/x^(1/3))^n)/(e^4*x)))*b^3 - 3*a*b^2
*log(c*(d + e/x^(1/3))^n)^2/x - 3*a^2*b*log(c*(d + e/x^(1/3))^n)/x - a^3/x
```

Fricas [B] time = 2.22042, size = 1781, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/36*(8*b^3*e^3*n^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^3*n - 36*a^3*e^3 + 36*(
b^3*e^3*x - b^3*e^3)*log(c)^3 - 36*(b^3*d^3*n^3*x + b^3*e^3*n^3)*log((d*x +
 e*x^(2/3))/x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3 - (b^3*e^3*n - 3*a*b^2*e^3)*
x)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(2/3) - 3*b^3*d*e^2*n^3*x^(1/3) + 2*b^3
*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d^3*n^3 - 6*a*b^2*d^3*n^2)*x - 6*(b^3*
d^3*n^2*x + b^3*e^3*n^2)*log(c))*log((d*x + e*x^(2/3))/x)^2 - 4*(2*b^3*e^3*
n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x - 12*(2*b^3*e^3*n^2 -
6*a*b^2*e^3*n + 9*a^2*b*e^3 - (2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)
*x)*log(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n + 18*(b^3
*d^3*n*x + b^3*e^3*n)*log(c)^2 + (85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^
```

```
2*b*d^3*n)*x - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^2 - 6*a*b^2
*d^3*n)*x)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a*b^2*
d^2*e*n^2)*x^(2/3) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*
d*e^2*n^2)*x^(1/3))*log((d*x + e*x^(2/3))/x) + 6*(85*b^3*d^2*e*n^3 + 18*b^3
*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*d^2*e
*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) - 3*(19*b^3*d*e^2*n^3 + 18*b^3*d*e^
2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*d*e^2*n^2 -
 6*a*b^2*d*e^2*n)*log(c))*x^(1/3))/(e^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3/x^2, x)
```

3.507 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=907

result too large to display

```
[Out] (45*b^3*d^4*n^3*(d + e/x^(1/3))^2)/(8*e^6) - (20*b^3*d^3*n^3*(d + e/x^(1/3)
)^3)/(9*e^6) + (45*b^3*d^2*n^3*(d + e/x^(1/3))^4)/(64*e^6) - (18*b^3*d*n^3*
(d + e/x^(1/3))^5)/(125*e^6) + (b^3*n^3*(d + e/x^(1/3))^6)/(72*e^6) + (18*a
*b^2*d^5*n^2)/(e^5*x^(1/3)) - (18*b^3*d^5*n^3)/(e^5*x^(1/3)) + (18*b^3*d^5*
n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^6 - (45*b^2*d^4*n^2*(d + e/
x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) + (20*b^2*d^3*n^2*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) - (45*b^2*d^2*n^2*(
d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(16*e^6) + (18*b^2*d*n^2
*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(25*e^6) - (b^2*n^2*(d
 + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*e^6) - (9*b*d^5*n*(d
+ e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^4*n*(d + e/x
^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) - (10*b*d^3*n*(d + e/
x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^2*n*(d + e/x^(
1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*e^6) - (9*b*d*n*(d + e/x^(1/
3))^5*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(5*e^6) + (b*n*(d + e/x^(1/3))^6*
(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) + (3*d^5*(d + e/x^(1/3))*(a + b
*Log[c*(d + e/x^(1/3))^n])^3)/e^6 - (15*d^4*(d + e/x^(1/3))^2*(a + b*Log[c*
(d + e/x^(1/3))^n])^3)/(2*e^6) + (10*d^3*(d + e/x^(1/3))^3*(a + b*Log[c*(d
+ e/x^(1/3))^n])^3)/e^6 - (15*d^2*(d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(
1/3))^n])^3)/(2*e^6) + (3*d*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^
n])^3)/e^6 - ((d + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*e^6)
```

Rubi [A] time = 1.0013, antiderivative size = 907, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{72e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{2e^6}+\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{4e^6}-\frac{b^2n^2\left(a+b\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (45*b^3*d^4*n^3*(d + e/x^(1/3))^2)/(8*e^6) - (20*b^3*d^3*n^3*(d + e/x^(1/3)
)^3)/(9*e^6) + (45*b^3*d^2*n^3*(d + e/x^(1/3))^4)/(64*e^6) - (18*b^3*d*n^3*
(d + e/x^(1/3))^5)/(125*e^6) + (b^3*n^3*(d + e/x^(1/3))^6)/(72*e^6) + (18*a
*b^2*d^5*n^2)/(e^5*x^(1/3)) - (18*b^3*d^5*n^3)/(e^5*x^(1/3)) + (18*b^3*d^5*
n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^6 - (45*b^2*d^4*n^2*(d + e/
x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) + (20*b^2*d^3*n^2*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) - (45*b^2*d^2*n^2*(
d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(16*e^6) + (18*b^2*d*n^2
*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(25*e^6) - (b^2*n^2*(d
 + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*e^6) - (9*b*d^5*n*(d
+ e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^4*n*(d + e/x
^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) - (10*b*d^3*n*(d + e/
x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^2*n*(d + e/x^(
1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*e^6) - (9*b*d*n*(d + e/x^(1/
```

```
3))^5*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(5*e^6) + (b*n*(d + e/x^(1/3))^6*
(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) + (3*d^5*(d + e/x^(1/3))*(a + b
*Log[c*(d + e/x^(1/3))^n])^3)/e^6 - (15*d^4*(d + e/x^(1/3))^2*(a + b*Log[c*
(d + e/x^(1/3))^n])^3)/(2*e^6) + (10*d^3*(d + e/x^(1/3))^3*(a + b*Log[c*(d
+ e/x^(1/3))^n])^3)/e^6 - (15*d^2*(d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(
1/3))^n])^3)/(2*e^6) + (3*d*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^
n])^3)/e^6 - ((d + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*e^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\,(a+b\log(c(d+ex)^n))^3\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(-\frac{d^5\,(a+b\log(c(d+ex)^n))^3}{e^5}+\frac{5d^4(d+ex)\,(a+b\log(c(d+ex)^n)}{e^5}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^5\,(a+b\log(c(d+ex)^n))^3\,dx,x,\frac{1}{\sqrt[3]{x}}\right)}{e^5}+\frac{(15d)\,\text{Subst}\left(\int(d+e\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^5\,(a+b\log(cx^n))^3\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}+\frac{(15d)\,\text{Subst}\left(\int x^4\,(a+b\log(c\right.}{e^6}$$

$$= \frac{3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6}-\frac{15d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right.\right.}{2e^6}$$

$$= -\frac{9bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6}+\frac{45bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d\right.\right.\right.}{4e^6}$$

$$= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6}-\frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}+\frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6}-\frac{18b^3dn^3\left(d\right.}{125e^6}$$

$$= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6}-\frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}+\frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6}-\frac{18b^3dn^3\left(d\right.}{125e^6}$$

Mathematica [A] time = 1.63296, size = 962, normalized size = 1.06

$$\frac{-72000b^3n^3x^2\log^3\left(d+\frac{e}{\sqrt[3]{x}}\right)d^6+809340b^3n^3x^2\log\left(\sqrt[3]{x}d+e\right)d^6-529200ab^2n^2x^2\log\left(\sqrt[3]{x}d+e\right)d^6+108000a^2bnx^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*x^(1/3) + 15840*a*b^2*d*e^5*n^2*x^(1/3) - 4368*b^3*d
*e^5*n^3*x^(1/3) + 27000*a^2*b*d^2*e^4*n*x^(2/3) - 33300*a*b^2*d^2*e^4*n^2*
x^(2/3) + 13785*b^3*d^2*e^4*n^3*x^(2/3) - 36000*a^2*b*d^3*e^3*n*x + 68400*a
*b^2*d^3*e^3*n^2*x - 41180*b^3*d^3*e^3*n^3*x + 54000*a^2*b*d^4*e^2*n*x^(4/3
) - 156600*a*b^2*d^4*e^2*n^2*x^(4/3) + 140070*b^3*d^4*e^2*n^3*x^(4/3) - 108
000*a^2*b*d^5*e*n*x^(5/3) + 529200*a*b^2*d^5*e*n^2*x^(5/3) - 809340*b^3*d^5
*e*n^3*x^(5/3) - 72000*b^3*d^6*n^3*x^2*Log[d + e/x^(1/3)]^3 - 36000*b^3*e^6
*Log[c*(d + e/x^(1/3))^n]^3 + 108000*a^2*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 5
29200*a*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)] + 809340*b^3*d^6*n^3*x^2*Log[e +
 d*x^(1/3)] + 3600*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]*(-20*a + 49*b*n - 20*
b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e + d*x^(1/3)] - Log[x]) - 36000*a^2*b*d
^6*n*x^2*Log[x] + 176400*a*b^2*d^6*n^2*x^2*Log[x] - 269780*b^3*d^6*n^3*x^2*
Log[x] + 1800*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]^2*(60*a - 147*b*n + 60*b*L
og[c*(d + e/x^(1/3))^n] + 60*b*n*Log[e + d*x^(1/3)] - 20*b*n*Log[x]) + 1800
*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*x
```

```
^(1/3) + 15*b*d^2*e^3*n*x^(2/3) - 20*b*d^3*e^2*n*x + 30*b*d^4*e*n*x^(4/3) -
 60*b*d^5*n*x^(5/3)) + 60*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 20*b*d^6*n*x^2*L
og[x]) - 60*b*Log[c*(d + e/x^(1/3))^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 -
 264*d*e^4*x^(1/3) + 555*d^2*e^3*x^(2/3) - 1140*d^3*e^2*x + 2610*d^4*e*x^(4
/3) - 8820*d^5*x^(5/3)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*x^(1/3) + 15*d^2*e^
3*x^(2/3) - 20*d^3*e^2*x + 30*d^4*e*x^(4/3) - 60*d^5*x^(5/3)) + 180*b*d^6*n
*(-20*a + 49*b*n)*x^2*Log[e + d*x^(1/3)] + 60*b*d^6*n*(20*a - 49*b*n)*x^2*L
og[x]))/(72000*e^6*x^2)
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

Maxima [A] time = 1.17795, size = 1166, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/40*a^2*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5
*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*
x^(1/3) - 10*e^5)/(e^6*x^2)) + 1/1200*(60*e*n*(60*d^6*log(d*x^(1/3) + e)/e^
7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x -
 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e/x^
(1/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 294
0*d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3
*x + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log
(x) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*a*b^2 + 1/216000*(540
0*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3)
- 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) -
 10*e^5)/(e^6*x^2))*log(c*(d + e/x^(1/3))^n)^2 + e*n*((108000*d^6*x^2*log(d
*x^(1/3) + e)^3 - 4000*d^6*x^2*log(x)^3 + 88200*d^6*x^2*log(x)^2 - 809340*d
^6*x^2*log(x) - 2428020*d^5*e*x^(5/3) + 420210*d^4*e^2*x^(4/3) - 123540*d^3
*e^3*x + 41355*d^2*e^4*x^(2/3) - 13104*d*e^5*x^(1/3) + 3000*e^6 - 5400*(20*
d^6*x^2*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e)^2 + 180*(200*d^6*x^2*log(x
)^2 - 2940*d^6*x^2*log(x) + 13489*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^7*x^2
) - 180*(1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 2940*d^
6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3*x +
 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log(x)
- 147*d^6*x^2)*log(d*x^(1/3) + e))*n*log(c*(d + e/x^(1/3))^n)/(e^7*x^2)))*b
^3 - 1/2*b^3*log(c*(d + e/x^(1/3))^n)^3/x^2 - 3/2*a*b^2*log(c*(d + e/x^(1/3
))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a^3/x^2
```

Fricas [A] time = 2.42146, size = 3106, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(1000*b^3*e^6*n^3 - 6000*a*b^2*e^6*n^2 + 18000*a^2*b*e^6*n - 36000*
a^3*e^6 + 36000*(b^3*e^6*x^2 - b^3*e^6)*log(c)^3 + 36000*(b^3*d^6*n^3*x^2 -
 b^3*e^6*n^3)*log((d*x + e*x^(2/3))/x)^3 + 20*(1800*a^3*e^6 + (2059*b^3*d^3
*e^3 - 50*b^3*e^6)*n^3 - 60*(57*a*b^2*d^3*e^3 - 5*a*b^2*e^6)*n^2 + 900*(2*a
^2*b*d^3*e^3 - a^2*b*e^6)*n)*x^2 - 18000*(2*b^3*d^3*e^3*n*x - b^3*e^6*n + 6
*a*b^2*e^6 - (6*a*b^2*e^6 + (2*b^3*d^3*e^3 - b^3*e^6)*n)*x^2)*log(c)^2 - 18
00*(20*b^3*d^3*e^3*n^3*x - 10*b^3*e^6*n^3 + 60*a*b^2*e^6*n^2 + 3*(49*b^3*d^
6*n^3 - 20*a*b^2*d^6*n^2)*x^2 - 60*(b^3*d^6*n^2*x^2 - b^3*e^6*n^2)*log(c) +
 15*(4*b^3*d^5*e*n^3*x - b^3*d^2*e^4*n^3)*x^(2/3) - 6*(5*b^3*d^4*e^2*n^3*x
- 2*b^3*d*e^5*n^3)*x^(1/3))*log((d*x + e*x^(2/3))/x)^2 - 20*(2059*b^3*d^3*e
^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x - 1200*(5*b^3*e^6
*n^2 - 30*a*b^2*e^6*n + 90*a^2*b*e^6 - (90*a^2*b*e^6 - (57*b^3*d^3*e^3 - 5*
b^3*e^6)*n^2 + 30*(2*a*b^2*d^3*e^3 - a*b^2*e^6)*n)*x^2 - 3*(19*b^3*d^3*e^3*
n^2 - 20*a*b^2*d^3*e^3*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 - 600*a*b^2*e^6*n
^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*
b*d^6*n)*x^2 - 1800*(b^3*d^6*n*x^2 - b^3*e^6*n)*log(c)^2 - 60*(19*b^3*d^3*e
^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x + 60*(20*b^3*d^3*e^3*n^2*x - 10*b^3*e^6*n^
2 + 60*a*b^2*e^6*n + 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^2)*log(c) + 15*(
37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2 - 12*(49*b^3*d^5*e*n^3 - 20*a*b^2
*d^5*e*n^2)*x + 60*(4*b^3*d^5*e*n^2*x - b^3*d^2*e^4*n^2)*log(c))*x^(2/3) -
6*(44*b^3*d*e^5*n^3 - 120*a*b^2*d*e^5*n^2 - 15*(29*b^3*d^4*e^2*n^3 - 20*a*b
^2*d^4*e^2*n^2)*x + 60*(5*b^3*d^4*e^2*n^2*x - 2*b^3*d*e^5*n^2)*log(c))*x^(1
/3))*log((d*x + e*x^(2/3))/x) + 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^
4*n^2 + 1800*a^2*b*d^2*e^4*n - 1800*(4*b^3*d^5*e*n*x - b^3*d^2*e^4*n)*log(c
)^2 - 4*(13489*b^3*d^5*e*n^3 - 8820*a*b^2*d^5*e*n^2 + 1800*a^2*b*d^5*e*n)*x
 - 60*(37*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n - 12*(49*b^3*d^5*e*n^2 - 20*
a*b^2*d^5*e*n)*x)*log(c))*x^(2/3) - 6*(728*b^3*d*e^5*n^3 - 2640*a*b^2*d*e^5
*n^2 + 3600*a^2*b*d*e^5*n - 1800*(5*b^3*d^4*e^2*n*x - 2*b^3*d*e^5*n)*log(c)
^2 - 5*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n^2 + 1800*a^2*b*d^4*e^2*
n)*x - 60*(44*b^3*d*e^5*n^2 - 120*a*b^2*d*e^5*n - 15*(29*b^3*d^4*e^2*n^2 -
20*a*b^2*d^4*e^2*n)*x)*log(c))*x^(1/3))/(e^6*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3/x^3, x)
```

3.508 $\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx$

Optimal. Leaf size=143

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}-\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{ben}{2}$$

```
[Out] (b*e^5*n*x^(2/3))/(4*d^5) - (b*e^4*n*x^(4/3))/(8*d^4) + (b*e^3*n*x^2)/(12*d
^3) - (b*e^2*n*x^(8/3))/(16*d^2) + (b*e*n*x^(10/3))/(20*d) - (b*e^6*n*Log[d
 + e/x^(2/3)])/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/4 - (b*e^6*
n*Log[x])/(6*d^6)
```

Rubi [A] time = 0.105398, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}-\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{ben}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```


```
[Out] (b*e^5*n*x^(2/3))/(4*d^5) - (b*e^4*n*x^(4/3))/(8*d^4) + (b*e^3*n*x^2)/(12*d
^3) - (b*e^2*n*x^(8/3))/(16*d^2) + (b*e*n*x^(10/3))/(20*d) - (b*e^6*n*Log[d
 + e/x^(2/3)])/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/4 - (b*e^6*
n*Log[x])/(6*d^6)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^7}dx,x,\frac{1}{x^{2/3}}\right)\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\frac{1}{x^6(d+ex)}dx,x,\frac{1}{x^{2/3}}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}+\right.\right.$$
$$=\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}+\frac{benx^{10/3}}{20d}-\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}+\frac{1}{4}x^4\left(\right.$$

Mathematica [A] time = 0.108891, size = 134, normalized size = 0.94

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{1}{4}ben\left(-\frac{e^4x^{2/3}}{d^5}+\frac{e^3x^{4/3}}{2d^4}-\frac{e^2x^2}{3d^3}+\frac{e^5\log\left(d+\frac{e}{x^{2/3}}\right)}{d^6}+\frac{2e^5\log(x)}{3d^6}+\frac{ex^{8/3}}{4d^2}-\frac{x^{10/3}}{5d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(2/3))^n])/4 - (b*e*n*(-((e^4*x^(2/3))/d^
5) + (e^3*x^(4/3))/(2*d^4) - (e^2*x^2)/(3*d^3) + (e*x^(8/3))/(4*d^2) - x^(1
0/3)/(5*d) + (e^5*Log[d + e/x^(2/3)])/d^6 + (2*e^5*Log[x])/(3*d^6)))/4
```

Maple [F] time = 0.6, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Maxima [A] time = 1.03153, size = 132, normalized size = 0.92

$$\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{4}ax^4-\frac{1}{240}ben\left(\frac{60e^5\log\left(dx^{\frac{2}{3}}+e\right)}{d^6}-\frac{12d^4x^{\frac{10}{3}}-15d^3ex^{\frac{8}{3}}+20d^2e^2x^2-30de^3x^{\frac{4}{3}}+60e^4x^{\frac{2}{3}}}{d^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/x^(2/3))^n) + 1/4*a*x^4 - 1/240*b*e*n*(60*e^5*log(d*
x^(2/3) + e)/d^6 - (12*d^4*x^(10/3) - 15*d^3*e*x^(8/3) + 20*d^2*e^2*x^2 - 3
0*d*e^3*x^(4/3) + 60*e^4*x^(2/3))/d^5)
```

Fricas [A] time = 1.95869, size = 385, normalized size = 2.69

$$\frac{60\,bd^6x^4\log\left(c\right)+60\,ad^6x^4+20\,bd^3e^3nx^2-120\,bd^6n\log\left(x^{\frac{1}{3}}\right)+60\,\left(bd^6-be^6\right)n\log\left(dx^{\frac{2}{3}}+e\right)+60\,\left(bd^6nx^4-bd^6n\right)\mathrm{l}}{240\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(60*b*d^6*x^4*log(c) + 60*a*d^6*x^4 + 20*b*d^3*e^3*n*x^2 - 120*b*d^6*
n*log(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(2/3) + e) + 60*(b*d^6*n*x^4
- b*d^6*n)*log((d*x + e*x^(1/3))/x) - 15*(b*d^4*e^2*n*x^2 - 4*b*d*e^5*n)*x^
(2/3) + 6*(2*b*d^5*e*n*x^3 - 5*b*d^2*e^4*n*x)*x^(1/3))/d^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35651, size = 139, normalized size = 0.97

$$\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4+\frac{1}{240}\left(60\,x^4\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+\left(\frac{12\,d^4x^{\frac{10}{3}}-15\,d^3x^{\frac{8}{3}}e+20\,d^2x^2e^2-30\,dx^{\frac{4}{3}}e^3+60\,x^{\frac{2}{3}}e^4}{d^5}-\frac{60\,e^5\log}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*x^4*log(d + e/x^(2/3)) + ((12*d^4*
x^(10/3) - 15*d^3*x^(8/3)*e + 20*d^2*x^2*e^2 - 30*d*x^(4/3)*e^3 + 60*x^(2/3
)*e^4)/d^5 - 60*e^5*log(abs(d*x^(2/3) + e))/d^6)*e)*b*n
```

3.509 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=121

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{2be^2nx^{5/3}}{15d^2}-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+\frac{2benx^{7/3}}{21d}$$

```
[Out] (-2*b*e^4*n*x^(1/3))/(3*d^4) + (2*b*e^3*n*x)/(9*d^3) - (2*b*e^2*n*x^(5/3))/
(15*d^2) + (2*b*e*n*x^(7/3))/(21*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(3*d^(9/2)) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/3
```

Rubi [A] time = 0.0718126, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2455, 263, 341, 302, 205}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{2be^2nx^{5/3}}{15d^2}-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+\frac{2benx^{7/3}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (-2*b*e^4*n*x^(1/3))/(3*d^4) + (2*b*e^3*n*x)/(9*d^3) - (2*b*e^2*n*x^(5/3))/
(15*d^2) + (2*b*e*n*x^(7/3))/(21*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(3*d^(9/2)) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/3
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^{4/3}}{d+\frac{e}{x^{2/3}}}dx$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^2}{e+dx^{2/3}}dx$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{x^8}{e+dx^2}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{3}(2ben)\,\text{Subst}\left(\int\left(-\frac{e^3}{d^4}+\frac{e^2x^2}{d^3}-\frac{ex^4}{d^2}+\frac{x^6}{d}\right.\right.$$
$$=-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$
$$=-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+\frac{1}{3}x^3\Big($$

Mathematica [C] time = 0.0153182, size = 65, normalized size = 0.54

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2benx^{7/3}\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\frac{e}{dx^{2/3}}\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^3)/3 + (2*b*e*n*x^(7/3)*Hypergeometric2F1[-7/2, 1, -5/2, -(e/(d*x^(2/3
)))])/(21*d) + (b*x^3*Log[c*(d + e/x^(2/3))^n])/3
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07102, size = 971, normalized size = 8.02

$$\left[\frac{105\,bd^4x^3\log\left(c\right)+105\,ad^4x^3-42\,bd^2e^2nx^{\frac{5}{3}}+105\,be^4n\sqrt{-\frac{e}{d}}\log\left(\frac{d^3x^2-2\,d^2ex\sqrt{-\frac{e}{d}}-e^3+2\left(d^3x\sqrt{-\frac{e}{d}}+de^2\right)x^{\frac{2}{3}}-2\left(d^2ex-de^2\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)+\cdots}{315\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] [1/315*(105*b*d^4*x^3*log(c) + 105*a*d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 105
*b*e^4*n*sqrt(-e/d)*log((d^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - e^3 + 2*(d^3*x*sq
rt(-e/d) + d*e^2)*x^(2/3) - 2*(d^2*e*x - d*e^2*sqrt(-e/d))*x^(1/3))/(d^3*x^
2 + e^3)) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n*l
og(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/x) + 30*(b*
d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4, 1/315*(105*b*d^4*x^3*log(c) + 105*a*
d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 210*b*e^4*n*sqrt(e/d)*arctan(d*x^(1/3)*s
qrt(e/d)/e) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n
*log(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/x) + 30*(
b*d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36602, size = 131, normalized size = 1.08

$$\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3+\frac{1}{315}\left(105\,x^3\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+2\left(\frac{105\,\arctan\left(\sqrt{d}x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{15\,d^6x^{\frac{7}{3}}-21\,d^5x^{\frac{5}{3}}e+35\,d^4xe^2-10\cdots}{d^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/315*(105*x^3*log(d + e/x^(2/3)) + 2*(105*a
rctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(7/2)/d^(9/2) + (15*d^6*x^(7/3) - 21*d^5*
x^(5/3)*e + 35*d^4*x*e^2 - 105*d^3*x^(1/3)*e^3)/d^7)*e)*b*n
```

3.510 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$

Optimal. Leaf size=94

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{be^2nx^{2/3}}{2d^2}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{benx^{4/3}}{4d}$$

```
[Out] -(b*e^2*n*x^(2/3))/(2*d^2) + (b*e*n*x^(4/3))/(4*d) + (b*e^3*n*Log[d + e/x^(
2/3)])/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/2 + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rubi [A] time = 0.0624697, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2454, 2395, 44}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{be^2nx^{2/3}}{2d^2}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{benx^{4/3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] -(b*e^2*n*x^(2/3))/(2*d^2) + (b*e*n*x^(4/3))/(4*d) + (b*e^3*n*Log[d + e/x^(
2/3)])/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/2 + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^4}dx,x,\frac{1}{x^{2/3}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\frac{1}{x^3(d+ex)}dx,x,\frac{1}{x^{2/3}}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{e^3}{d^3(d+ex}\right.\right.$$
$$=-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n}{}$$

Mathematica [A] time = 0.0229221, size = 91, normalized size = 0.97

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{1}{2}ben\left(-\frac{e^2\log\left(d+\frac{e}{x^{2/3}}\right)}{d^3}-\frac{2e^2\log(x)}{3d^3}+\frac{ex^{2/3}}{d^2}-\frac{x^{4/3}}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(2/3))^n])/2 - (b*e*n*((e*x^(2/3))/d^2 -
x^(4/3)/(2*d) - (e^2*Log[d + e/x^(2/3)])/d^3 - (2*e^2*Log[x])/(3*d^3)))/2
```

Maple [F] time = 0.358, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Maxima [A] time = 1.04099, size = 85, normalized size = 0.9

$$\frac{1}{4}ben\left(\frac{2e^2\log\left(dx^{\frac{2}{3}}+e\right)}{d^3}+\frac{dx^{\frac{4}{3}}-2ex^{\frac{2}{3}}}{d^2}\right)+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*e*n*(2*e^2*log(d*x^(2/3) + e)/d^3 + (d*x^(4/3) - 2*e*x^(2/3))/d^2) +
1/2*b*x^2*log(c*(d + e/x^(2/3))^n) + 1/2*a*x^2
```

Fricas [A] time = 1.8801, size = 277, normalized size = 2.95

$$\frac{2\,bd^3x^2\log\left(c\right) + bd^2enx^{\frac{4}{3}} + 2\,ad^3x^2 - 4\,bd^3n\log\left(x^{\frac{1}{3}}\right) - 2\,bde^2nx^{\frac{2}{3}} + 2\,\left(bd^3 + be^3\right)n\log\left(dx^{\frac{2}{3}} + e\right) + 2\,\left(bd^3nx^2 - bd^3n\right.}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d^3*x^2*log(c) + b*d^2*e*n*x^(4/3) + 2*a*d^3*x^2 - 4*b*d^3*n*log(x
^(1/3)) - 2*b*d*e^2*n*x^(2/3) + 2*(b*d^3 + b*e^3)*n*log(d*x^(2/3) + e) + 2*
(b*d^3*n*x^2 - b*d^3*n)*log((d*x + e*x^(1/3))/x))/d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26951, size = 97, normalized size = 1.03

$$\frac{1}{2}\,bx^2\log\left(c\right) + \frac{1}{4}\left(2\,x^2\log\left(d + \frac{e}{x^{\frac{2}{3}}}\right) + \left(\frac{dx^{\frac{4}{3}} - 2\,x^{\frac{2}{3}}e}{d^2} + \frac{2\,e^2\log\left(\left|dx^{\frac{2}{3}} + e\right|\right)}{d^3}\right)e\right)bn + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/4*(2*x^2*log(d + e/x^(2/3)) + ((d*x^(4/3) - 2*x^(2/3)*
e)/d^2 + 2*e^2*log(abs(d*x^(2/3) + e))/d^3)*e)*b*n + 1/2*a*x^2
```

3.511 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$

Optimal. Leaf size=65

$$ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+\frac{2ben\sqrt[3]{x}}{d}$$

```
[Out] (2*b*e*n*x^(1/3))/d + a*x - (2*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + b*x*Log[c*(d + e/x^(2/3))^n]
```

Rubi [A] time = 0.0417612, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2448, 263, 243, 321, 205}

$$ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+\frac{2ben\sqrt[3]{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

```
[Out] (2*b*e*n*x^(1/3))/d + a*x - (2*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + b*x*Log[c*(d + e/x^(2/3))^n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx &= ax+b\int \log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)dx \\
&= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}}\,dx \\
&= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}(2ben)\int \frac{1}{e+dx^{2/3}}\,dx \\
&= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+(2ben)\,\text{Subst}\left(\int \frac{x^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right) \\
&= \frac{2ben\sqrt[3]{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{d} \\
&= \frac{2ben\sqrt[3]{x}}{d}+ax-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)
\end{aligned}$$

Mathematica [C] time = 0.0133864, size = 53, normalized size = 0.82

$$ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2ben\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{e}{dx^{2/3}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

```
[Out] a*x + (2*b*e*n*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/d
 + b*x*Log[c*(d + e/x^(2/3))^n]
```

Maple [B] time = 0.204, size = 168, normalized size = 2.6

$$ax+xb\ln\left(c\left(\left(e+dx^{\frac{2}{3}}\right)x^{-\frac{2}{3}}\right)^n\right)+\frac{2\,be^2n}{3\,d}\arctan\left(\frac{xd^2}{e}\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+2\,\frac{enb\sqrt[3]{x}}{d}-\frac{2\,be^2n}{3\,d}\arctan\left(\left(2\,d\sqrt[3]{x}+\sqrt{3}\sqrt{d}\sqrt{e}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/x^(2/3))^n),x)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(2/3))/x^(2/3))^n)+2/3*b*e^2*n/d/(d*e)^(1/2)*arctan(x*
d^2/e/(d*e)^(1/2))+2*b*e*n*x^(1/3)/d-2/3*b*e^2*n/d/(d*e)^(1/2)*arctan((2*d*
x^(1/3)+3^(1/2)*d^(1/2)*e^(1/2))/(d*e)^(1/2))+2/3*b*e^2*n/d/(d*e)^(1/2)*arc
tan((3^(1/2)*d^(1/2)*e^(1/2)-2*d*x^(1/3))/(d*e)^(1/2))-4/3*b*e^2*n/d/(d*e)^
(1/2)*arctan(d*x^(1/3)/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99349, size = 664, normalized size = 10.22

$$\left[\frac{ben\sqrt{-\frac{e}{d}}\log\left(\frac{d^3x^2+2\,d^2ex\sqrt{-\frac{e}{d}}-e^3-2\left(d^3x\sqrt{-\frac{e}{d}}-de^2\right)x^{\frac{2}{3}}-2\left(d^2ex+de^2\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)+bdn\log\left(dx^{\frac{2}{3}}+e\right)+bdx\log\left(c\right)-2\,bdn\log\left(x^{\frac{1}{3}}\right)+2}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="fricas")
```

```
[Out] [(b*e*n*sqrt(-e/d)*log((d^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - e^3 - 2*(d^3*x*sqr
t(-e/d) - d*e^2)*x^(2/3) - 2*(d^2*e*x + d*e^2*sqrt(-e/d))*x^(1/3))/(d^3*x^2
 + e^3)) + b*d*n*log(d*x^(2/3) + e) + b*d*x*log(c) - 2*b*d*n*log(x^(1/3)) +
 2*b*e*n*x^(1/3) + a*d*x + (b*d*n*x - b*d*n)*log((d*x + e*x^(1/3))/x))/d, -
(2*b*e*n*sqrt(e/d)*arctan(d*x^(1/3)*sqrt(e/d)/e) - b*d*n*log(d*x^(2/3) + e)
 - b*d*x*log(c) + 2*b*d*n*log(x^(1/3)) - 2*b*e*n*x^(1/3) - a*d*x - (b*d*n*x
 - b*d*n)*log((d*x + e*x^(1/3))/x))/d]
```

Sympy [A] time = 161.897, size = 61, normalized size = 0.94

$$ax+b\left(\frac{2en\left(\frac{3\sqrt[3]{x}}{d}-\frac{3e\operatorname{atan}\left(\frac{\sqrt[3]{x}}{\sqrt{\frac{e}{d}}}\right)}{d^2\sqrt{\frac{e}{d}}}\right)}{3}+x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(2/3))**n),x)
```

```
[Out] a*x + b*(2*e*n*(3*x**(1/3)/d - 3*e*atan(x**(1/3)/sqrt(e/d))/(d**2*sqrt(e/d)
))/3 + x*log(c*(d + e/x**(2/3))**n))
```

Giac [A] time = 1.32507, size = 77, normalized size = 1.18

$$-\left(\left(2\left(\frac{\arctan\left(\sqrt{d}x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}}-\frac{x^{\frac{1}{3}}}{d}\right)e-x\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)n-x\log\left(c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -((2*(arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(1/2)/d^(3/2) - x^(1/3)/d)*e - x*l
og(d + e/x^(2/3)))*n - x*log(c))*b + a*x
```

3.512 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=55

$$-\frac{3}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/2 - (3*b*n*Poly
Log[2, 1 + e/(d*x^(2/3))])/2
```

Rubi [A] time = 0.0514524, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$-\frac{3}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/2 - (3*b*n*Poly
Log[2, 1 + e/(d*x^(2/3))])/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx &= -\left(\frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\frac{1}{x^{2/3}}\right)\right)\\
&= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,\frac{1}{x^{2/3}}\right)\\
&= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{3}{2}bn\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0114116, size = 55, normalized size = 1.

$$a\log(x)-\frac{3}{2}b\left(n\text{PolyLog}\left(2,\frac{d+\frac{e}{x^{2/3}}}{d}\right)+\log\left(-\frac{e}{dx^{2/3}}\right)\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

```
[Out] a*Log[x] - (3*b*(Log[c*(d + e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + n*PolyLog
[2, (d + e/x^(2/3))/d]))/2
```

Maple [F] time = 0.417, size = 0, normalized size = 0.

$$\int\frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)
```

Maxima [B] time = 2.41227, size = 171, normalized size = 3.11

$$-\frac{3}{2}\left(2\log\left(\frac{dx^{\frac{2}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-\frac{dx^{\frac{2}{3}}}{e}\right)\right)bn+\frac{2\,ben\log(x)^2+6\,bdnx^{\frac{2}{3}}\log(x)+6\,be\log\left(\left(dx^{\frac{2}{3}}+e\right)^n\right)\log(x)-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3/2*(2*log(d*x^(2/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(2/3)/e))*b*n + 1/6*
(2*b*e*n*log(x)^2 + 6*b*d*n*x^(2/3)*log(x) + 6*b*e*log((d*x^(2/3) + e)^n)*l
og(x) - 12*b*e*log(x)*log(x^(1/3*n)) - 9*b*d*n*x^(2/3) + 6*(b*e*log(c) + a*
e)*log(x) - 3*(2*b*d*n*x*log(x) - 3*b*d*n*x)/x^(1/3))/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + e*x^(1/3))/x)^n) + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)/x, x)
```

3.513 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=77

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bn}{3x}$$

```
[Out] (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) - (2*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1
/3))/Sqrt[e]])/e^(3/2) - (a + b*Log[c*(d + e/x^(2/3))^n])/x
```

Rubi [A] time = 0.0509336, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2455, 263, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

```
[Out] (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) - (2*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1
/3))/Sqrt[e]])/e^(3/2) - (a + b*Log[c*(d + e/x^(2/3))^n])/x
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx &= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{1}{3}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{8/3}}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{1}{3}(2ben)\int \frac{1}{\left(e+dx^{2/3}\right)x^2}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-(2ben)\,\text{Subst}\left(\int \frac{1}{x^4\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bn}{3x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}+(2bdn)\,\text{Subst}\left(\int \frac{1}{x^2\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{\left(2bd^2n\right)\text{Subst}\left(\int \frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e}\\
&= \frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0489569, size = 80, normalized size = 1.04

$$-\frac{a}{x}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}+\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

```
[Out] -(a/x) + (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) + (2*b*d^(3/2)*n*ArcTan[Sqrt
[e]/(Sqrt[d]*x^(1/3))])/e^(3/2) - (b*Log[c*(d + e/x^(2/3))^n])/x
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87208, size = 554, normalized size = 7.19

$$\left[\frac{3\,bdnx\sqrt{-\frac{d}{e}}\log\left(\frac{d^3x^2+2\,de^2x\sqrt{-\frac{d}{e}}-e^3-2\left(d^2ex\sqrt{-\frac{d}{e}}-de^2\right)x^{\frac{2}{3}}-2\left(d^2ex+e^3\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)-3\,ben\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)-6\,bdnx^{\frac{2}{3}}+2\,ben-3\,be\log}{3\,ex}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*d*n*x*sqrt(-d/e)*log((d^3*x^2 + 2*d*e^2*x*sqrt(-d/e) - e^3 - 2*(d
^2*e*x*sqrt(-d/e) - d*e^2)*x^(2/3) - 2*(d^2*e*x + e^3*sqrt(-d/e))*x^(1/3))/
(d^3*x^2 + e^3)) - 3*b*e*n*log((d*x + e*x^(1/3))/x) - 6*b*d*n*x^(2/3) + 2*b
*e*n - 3*b*e*log(c) - 3*a*e)/(e*x), -1/3*(6*b*d*n*x*sqrt(d/e)*arctan(x^(1/3
)*sqrt(d/e)) + 3*b*e*n*log((d*x + e*x^(1/3))/x) + 6*b*d*n*x^(2/3) - 2*b*e*n
 + 3*b*e*log(c) + 3*a*e)/(e*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34372, size = 99, normalized size = 1.29

$$-\frac{1}{3}\left(2\left(3\,d^{\frac{3}{2}}\arctan\left(\sqrt{d}x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}+\frac{\left(3\,dx^{\frac{2}{3}}-e\right)e^{(-2)}}{x}\right)e+\frac{3\,\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*(3*d^(3/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-5/2) + (3*d*x^(2/3)
 - e)*e^(-2)/x)*e + 3*log(d + e/x^(2/3))/x)*b*n - b*log(c)/x - a/x
```

3.514 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=89

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{bdn}{4ex^{4/3}}+\frac{bn}{6x^2}$$

```
[Out] (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/3)) - (b*d^3*
n*Log[d + e/x^(2/3)])/(2*e^3) - (a + b*Log[c*(d + e/x^(2/3))^n])/(2*x^2)
```

Rubi [A] time = 0.0680862, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{bdn}{4ex^{4/3}}+\frac{bn}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/3)) - (b*d^3*
n*Log[d + e/x^(2/3)])/(2*e^3) - (a + b*Log[c*(d + e/x^(2/3))^n])/(2*x^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = -\left(\frac{3}{2}\operatorname{Subst}\left(\int x^2\,(a+b\log(c(d+ex)^n))\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\operatorname{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, \frac{1}{x^{2/3}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\operatorname{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx, x, \frac{1}{x^{2/3}}\right)$$
$$= \frac{bn}{6x^2} - \frac{bdn}{4ex^{4/3}} + \frac{bd^2n}{2e^2x^{2/3}} - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}$$

Mathematica [A] time = 0.0325246, size = 94, normalized size = 1.06

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2} + \frac{bd^2n}{2e^2x^{2/3}} - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3} - \frac{bdn}{4ex^{4/3}} + \frac{bn}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/
3)) - (b*d^3*n*Log[d + e/x^(2/3)])/(2*e^3) - (b*Log[c*(d + e/x^(2/3))^n])/(
2*x^2)
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
```

Maxima [A] time = 1.0359, size = 119, normalized size = 1.34

$$-\frac{1}{12}ben\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4} - \frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4} - \frac{6\,d^2x^{\frac{4}{3}} - 3\,dex^{\frac{2}{3}} + 2\,e^2}{e^3x^2}\right) - \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/12*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2
*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/2*b*log(c*(d + e/x^(2/3))^
n)/x^2 - 1/2*a/x^2
```

Fricas [A] time = 1.78914, size = 205, normalized size = 2.3

$$\frac{6\,bd^2enx^{\frac{4}{3}} - 3\,bde^2nx^{\frac{2}{3}} + 2\,be^3n - 6\,be^3\log\left(c\right) - 6\,ae^3 - 6\left(bd^3nx^2 + be^3n\right)\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)}{12\,e^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*d^2*e*n*x^(4/3) - 3*b*d*e^2*n*x^(2/3) + 2*b*e^3*n - 6*b*e^3*log(c
) - 6*a*e^3 - 6*(b*d^3*n*x^2 + b*e^3*n)*log((d*x + e*x^(1/3))/x))/(e^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45338, size = 140, normalized size = 1.57

$$\frac{1}{12}\left(\left(12\,d^3e^{(-4)}\log\left(x^{\frac{1}{3}}\right) - 6\,d^3e^{(-4)}\log\left(\left|dx^{\frac{2}{3}} + e\right|\right) - \frac{\left(11\,d^3x^2 - 6\,d^2x^{\frac{4}{3}}e + 3\,dx^{\frac{2}{3}}e^2 - 2\,e^3\right)e^{(-4)}}{x^2}\right)e - \frac{6\,\log\left(d + \frac{e}{x^{\frac{2}{3}}}\right)}{x^2}\right)bn - \frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/12*((12*d^3*e^(-4)*log(x^(1/3)) - 6*d^3*e^(-4)*log(abs(d*x^(2/3) + e)) -
(11*d^3*x^2 - 6*d^2*x^(4/3)*e + 3*d*x^(2/3)*e^2 - 2*e^3)*e^(-4)/x^2)*e - 6*
log(d + e/x^(2/3))/x^2)*b*n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

3.515 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=132

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{2bd^2n}{15e^2x^{5/3}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

```
[Out] (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*e^2*x^(5/3))
- (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(9/2)) - (a + b*Log[c*(d + e/x^(2/3))^n
])/(3*x^3)
```

Rubi [A] time = 0.088075, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2455, 263, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{2bd^2n}{15e^2x^{5/3}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*e^2*x^(5/3))
- (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(9/2)) - (a + b*Log[c*(d + e/x^(2/3))^n
])/(3*x^3)
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{9}(2ben)\int\frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{14/3}}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{9}(2ben)\int\frac{1}{\left(e+dx^{2/3}\right)x^4}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{3}(2ben)\,\mathrm{Subst}\left(\int\frac{1}{x^{10}\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{2bn}{27x^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{1}{3}(2bdn)\,\mathrm{Subst}\left(\int\frac{1}{x^8\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{\left(2bd^2n\right)\mathrm{Subst}\left(\int\frac{1}{x^6\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)}{3e}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{\left(2bd^3n\right)\mathrm{Subst}\left(\int\frac{1}{x^4\left(e+dx^2\right)}\,dx,x\ldots\right.}{3e^2}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{\left(2bd^4n\right)\mathrm{Subst}\left(\int\frac{1}{x^2(e\ldots}\right.}{3e^3}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{\left(2bd^5n\right)\mathrm{Sub}\ldots}{}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{a+b\log\left(c\left(d\ldots\right.\right.}{3x^3}$$

Mathematica [A] time = 0.0643829, size = 137, normalized size = 1.04

$$-\frac{a}{3x^3}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{2bd^2n}{15e^2x^{5/3}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}-\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{3e^{9/2}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] -a/(3*x^3) + (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*
e^2*x^(5/3)) - (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) - (2*b*d
^(9/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/(3*e^(9/2)) - (b*Log[c*(d + e/x
^(2/3))^n])/(3*x^3)
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int\frac{1}{x^4}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89939, size = 811, normalized size = 6.14

$$\left[\frac{315\,bd^4nx^3\sqrt{-\frac{d}{e}}\log\left(\frac{d^3x^2-2\,de^2x\sqrt{-\frac{d}{e}}-e^3+2\left(d^2ex\sqrt{-\frac{d}{e}}+de^2\right)x^{\frac{2}{3}}-2\left(d^2ex-e^3\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)-210\,bd^3enx^2+126\,bd^2e^2nx^{\frac{4}{3}}-315\,be^4n\,\mathrm{lo}}{945\,e^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] [1/945*(315*b*d^4*n*x^3*sqrt(-d/e)*log((d^3*x^2 - 2*d*e^2*x*sqrt(-d/e) - e^
3 + 2*(d^2*e*x*sqrt(-d/e) + d*e^2)*x^(2/3) - 2*(d^2*e*x - e^3*sqrt(-d/e))*x
^(1/3))/(d^3*x^2 + e^3)) - 210*b*d^3*e*n*x^2 + 126*b*d^2*e^2*n*x^(4/3) - 31
5*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 315*b*e^4*log(c) - 315*a*
e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*x^3), 1/945*(630*b*d^4*n
*x^3*sqrt(d/e)*arctan(x^(1/3)*sqrt(d/e)) - 210*b*d^3*e*n*x^2 + 126*b*d^2*e^
2*n*x^(4/3) - 315*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 315*b*e^4
*log(c) - 315*a*e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28727, size = 139, normalized size = 1.05

$$\frac{1}{945}\left(2\left(315\,d^{\frac{9}{2}}\arctan\left(\sqrt{d}x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{11}{2}\right)}+\frac{\left(315\,d^4x^{\frac{8}{3}}-105\,d^3x^2e+63\,d^2x^{\frac{4}{3}}e^2-45\,dx^{\frac{2}{3}}e^3+35\,e^4\right)e^{(-5)}}{x^3}\right)e-\frac{315\log\left(d\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/945*(2*(315*d^(9/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-11/2) + (315*d^4
*x^(8/3) - 105*d^3*x^2*e + 63*d^2*x^(4/3)*e^2 - 45*d*x^(2/3)*e^3 + 35*e^4)*
e^(-5)/x^3)*e - 315*log(d + e/x^(2/3))/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/
x^3
```

3.516 $\int x^3 \left(a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=412

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^6}+\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}+\frac{be^5nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{2d^6}$$

```
[Out] (-77*b^2*e^5*n^2*x^(2/3))/(120*d^5) + (47*b^2*e^4*n^2*x^(4/3))/(240*d^4) -
(3*b^2*e^3*n^2*x^2)/(40*d^3) + (b^2*e^2*n^2*x^(8/3))/(40*d^2) + (77*b^2*e^6
*n^2*Log[d + e/x^(2/3)])/(120*d^6) + (b*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a +
b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) - (b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e
/x^(2/3))^n]))/(4*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(6*
d^3) - (b*e^2*n*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*d^2) + (b*e*n*
x^(10/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(10*d) + (b*e^6*n*Log[1 - d/(d +
 e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) + (x^4*(a + b*Log[c*
(d + e/x^(2/3))^n])^2)/4 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^
2*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^6)
```

Rubi [A] time = 1.01667, antiderivative size = 436, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{2d^6}-\frac{e^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^6}+\frac{be^6n\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}+\frac{be^5nx^{2/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(2/3))/(120*d^5) + (47*b^2*e^4*n^2*x^(4/3))/(240*d^4) -
(3*b^2*e^3*n^2*x^2)/(40*d^3) + (b^2*e^2*n^2*x^(8/3))/(40*d^2) + (77*b^2*e^6
*n^2*Log[d + e/x^(2/3)])/(120*d^6) + (b*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a +
b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) - (b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e
/x^(2/3))^n]))/(4*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(6*
d^3) - (b*e^2*n*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*d^2) + (b*e*n*
x^(10/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(10*d) - (e^6*(a + b*Log[c*(d +
e/x^(2/3))^n])^2)/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/4 + (b
*e^6*n*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/(2*d^6) + (1
37*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*n^2*PolyLog[2, 1 + e/(d*x^(2/3)
)])/(2*d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, \cdots\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{1}{2}(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\cdots\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2d} + \cdots$$

$$= \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(ben)\cdots}{\cdots}$$

$$= -\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2} + \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{1}{4}\cdots$$

$$= -\frac{b^2e^5n^2x^{2/3}}{10d^5} + \frac{b^2e^4n^2x^{4/3}}{20d^4} - \frac{b^2e^3n^2x^2}{30d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{10d^6} + \cdots$$

$$= -\frac{9b^2e^5n^2x^{2/3}}{40d^5} + \frac{9b^2e^4n^2x^{4/3}}{80d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{9b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{40d^6}\cdots$$

$$= -\frac{47b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{47b^2e^6n^2\log\left(d+\cdots\right.}{120d^6}$$

$$= -\frac{77b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{77b^2e^6n^2\log\left(d+\cdots\right.}{120d^6}$$

$$= -\frac{77b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{77b^2e^6n^2\log\left(d+\cdots\right.}{120d^6}$$

Mathematica [B] time = 0.434584, size = 968, normalized size = 2.35

$$\frac{180a^2x^4d^6 + 180b^2x^4\log^2\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)d^6 + 360abx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)d^6 + 72abenx^{10/3}d^5 + 72b^2enx^{10/3}\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (360*a*b*d*e^5*n*x^(2/3) - 462*b^2*d*e^5*n^2*x^(2/3) - 180*a*b*d^2*e^4*n*x^
(4/3) + 141*b^2*d^2*e^4*n^2*x^(4/3) + 120*a*b*d^3*e^3*n*x^2 - 54*b^2*d^3*e^
3*n^2*x^2 - 90*a*b*d^4*e^2*n*x^(8/3) + 18*b^2*d^4*e^2*n^2*x^(8/3) + 72*a*b*
d^5*e*n*x^(10/3) + 180*a^2*d^6*x^4 + 822*b^2*e^6*n^2*Log[d + e/x^(2/3)] + 3
60*b^2*d*e^5*n*x^(2/3)*Log[c*(d + e/x^(2/3))^n] - 180*b^2*d^2*e^4*n*x^(4/3)
*Log[c*(d + e/x^(2/3))^n] + 120*b^2*d^3*e^3*n*x^2*Log[c*(d + e/x^(2/3))^n]
- 90*b^2*d^4*e^2*n*x^(8/3)*Log[c*(d + e/x^(2/3))^n] + 72*b^2*d^5*e*n*x^(10/
3)*Log[c*(d + e/x^(2/3))^n] + 360*a*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n] + 18
0*b^2*d^6*x^4*Log[c*(d + e/x^(2/3))^n]^2 - 360*a*b*e^6*n*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[e] - Sqrt[-
d]*x^(1/3)] + 180*b^2*e^6*n^2*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 - 360*a*b*e
^6*n*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(2/3))^
n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 180*b^2*e^6*n^2*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2 + 360*b^2*e^6*n^2*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sq
rt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b^2*e^6*n^2*Log[Sqrt[e] - Sqrt[-d]*x^(1/
3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b^2*e^6*n^2*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] - 720*b^2*e^6*n^2*Log
[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 548*b^2*e^6*
n^2*Log[x] - 720*b^2*e^6*n^2*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 3
60*b^2*e^6*n^2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b^2*e
^6*n^2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b^2*e^6*n^2*Pol
yLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(720*d^6)
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b^2x^4\log\left(\left(dx^{\frac{2}{3}}+e\right)^n\right)^2 - \int -\frac{3\left(b^2d\log\left(c\right)^2+2\,abd\log\left(c\right)+a^2d\right)x^4+3\left(b^2e\log\left(c\right)^2+2\,abe\log\left(c\right)+a^2e\right)x^{\frac{10}{3}}+12\left(b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*log((d*x^(2/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^4 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(10/3) + 12*(b^2*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n))^2 - (b^2*d*n*x^4 -
 6*(b^2*d*log(c) + a*b*d)*x^4 - 6*(b^2*e*log(c) + a*b*e)*x^(10/3) + 12*(b^2
*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n)))*log((d*x^(2/3) + e)^n) - 12*((b^2*
d*log(c) + a*b*d)*x^4 + (b^2*e*log(c) + a*b*e)*x^(10/3))*log(x^(1/3*n)))/(d
*x + e*x^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+2\,abx^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^2x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^3*log(c*((d*x +
 e*x^(1/3))/x)^n) + a^2*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^3, x)
```

3.517 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=239

$$\frac{b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{d^3}-\frac{be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}-\frac{be^2nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}$$

```
[Out] (b^2*e^2*n^2*x^(2/3))/(2*d^2) - (b^2*e^3*n^2*Log[d + e/x^(2/3)])/(2*d^3) -
(b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (b
*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d) - (b*e^3*n*Log[1 - d/(
d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (x^2*(a + b*Log[c*(
d + e/x^(2/3))^n])^2)/2 - (b^2*e^3*n^2*Log[x])/d^3 + (b^2*e^3*n^2*PolyLog[2
, d/(d + e/x^(2/3))])/d^3
```

Rubi [A] time = 0.484285, antiderivative size = 264, normalized size of antiderivative = 1.1, number of steps used = 14, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2e^3n^2\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{d^3}+\frac{e^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}-\frac{be^3n\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}-\frac{be^2nx^{2/3}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(2/3))/(2*d^2) - (b^2*e^3*n^2*Log[d + e/x^(2/3)])/(2*d^3) -
(b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (b
*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d) + (e^3*(a + b*Log[c*(d
 + e/x^(2/3))^n])^2)/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/2 -
 (b*e^3*n*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/d^3 - (b^
2*e^3*n^2*Log[x])/d^3 - (b^2*e^3*n^2*PolyLog[2, 1 + e/(d*x^(2/3))])/d^3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx=-\left(\frac{3}{2}\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^4}dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}dx,x,\frac{1}{x^{2/3}}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx,x,d+\frac{e}{x^{2/3}}\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx,x,d+\frac{e}{x^{2/3}}\right)}{d}+\frac{(ben}{}$$

$$=\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(ben)\text{Subs}}{}$$

$$=-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}$$

$$=\frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}$$

$$=\frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}$$

Mathematica [B] time = 0.44262, size = 542, normalized size = 2.27

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{ben\left(3be^2n\left(-4\text{PolyLog}\left(2,1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)+2\text{PolyLog}\left(2,\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)+\log\left(\sqrt{e}-\sqrt{-d}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/2 - (b*e*n*(6*d*e*x^(2/3)*(a + b*L
og[c*(d + e/x^(2/3))^n]) - 3*d^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]) -
 6*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 6
*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*b
*e^2*n*(3*Log[d + e/x^(2/3)] + 2*Log[x]) + b*e*n*(-3*d*x^(2/3) + 3*e*Log[d
+ e/x^(2/3)] + 2*e*Log[x]) + 3*b*e^2*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Lo
g[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] -
 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/S
qrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 3*b*e^2*n*(
Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/
2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])
```

```
 + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqr
t[-d]*x^(1/3))/Sqrt[e]])))/(6*d^3)
```

Maple [F] time = 0.35, size = 0, normalized size = 0.

$$\int x \left(a + b\ln\left(c\left(d + e x^{-\frac{2}{3}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)

[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} b^2 x^2 \log\left(\left(d x^{\frac{2}{3}} + e\right)^{n}\right)^2 - \int -\frac{3\left(b^2 d \log\left(c\right)^2 + 2\,abd\log\left(c\right) + a^2 d\right)x^2 + 12\left(b^2 d x^2 + b^2 e x^{\frac{4}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^2 + 3\left(b^2 e \log\left(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")

[Out] 1/2*b^2*x^2*log((d*x^(2/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 + 12*(b^2*d*x^2 + b^2*e*x^(4/3))*log(x^(1/3*n))
^2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(4/3) - 2*(b^2*d*n*x^2 -
 3*(b^2*d*log(c) + a*b*d)*x^2 - 3*(b^2*e*log(c) + a*b*e)*x^(4/3) + 6*(b^2*d
*x^2 + b^2*e*x^(4/3))*log(x^(1/3*n)))*log((d*x^(2/3) + e)^n) - 12*((b^2*d*l
og(c) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(4/3))*log(x^(1/3*n)))/(d*x +
 e*x^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2 x \log\left(c\left(\frac{dx + e x^{\frac{1}{3}}}{x}\right)^{n}\right)^2 + 2\,abx\log\left(c\left(\frac{dx + e x^{\frac{1}{3}}}{x}\right)^{n}\right) + a^2 x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")

[Out] integral(b^2*x*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*x
^(1/3))/x)^n) + a^2*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x, x)
```

3.518 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=95

$$-3bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+3b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(\right.\right.$$

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))])/2 - 3*b*n*(a
+ b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, 1 + e/(d*x^(2/3))] + 3*b^2*n^2*Pol
yLog[3, 1 + e/(d*x^(2/3))]
```

Rubi [A] time = 0.131049, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$-3bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+3b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```


```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))])/2 - 3*b*n*(a
+ b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, 1 + e/(d*x^(2/3))] + 3*b^2*n^2*Pol
yLog[3, 1 + e/(d*x^(2/3))]
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = -\left(\frac{3}{2}\operatorname{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$
$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) + (3ben)\operatorname{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log\right.}{d+ex}$$
$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) + (3bn)\operatorname{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(\right.}{x}\right.$$
$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) - 3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\operatorname{Li}_2\left(1\right.$$
$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) - 3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\operatorname{Li}_2\left(1\right.$$

Mathematica [B] time = 0.142723, size = 199, normalized size = 2.09

$$2bn\left(\frac{3}{2}\operatorname{PolyLog}\left(2,-\frac{e}{dx^{2/3}}\right)+\log(x)\left(\log\left(d+\frac{e}{x^{2/3}}\right)-\log\left(\frac{e}{dx^{2/3}}+1\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-bn\log\left(d+\frac{e}{x^{2/3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((Log[d + e/x^(2/
3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/2)
- (3*b^2*n^2*(Log[d + e/x^(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 2*Log[d + e/x^(2
/3)]*PolyLog[2, 1 + e/(d*x^(2/3))] - 2*PolyLog[3, 1 + e/(d*x^(2/3))]))/2
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2 \log\left(\left(dx^{\frac{2}{3}} + e\right)^n\right)^2 \log(x) - \int -\frac{12\left(b^2dx + b^2ex^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^2 + 3\left(b^2d\log(c)^2 + 2\,abd\log(c) + a^2d\right)x - 2\left(2\,b^2dnx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((d*x^(2/3) + e)^n)^2*log(x) - integrate(-1/3*(12*(b^2*d*x + b^2*e*x
^(1/3))*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x -
2*(2*b^2*d*n*x*log(x) - 3*(b^2*d*log(c) + a*b*d)*x + 6*(b^2*d*x + b^2*e*x^(
1/3))*log(x^(1/3*n)) - 3*(b^2*e*log(c) + a*b*e)*x^(1/3))*log((d*x^(2/3) + e
)^n) - 12*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(1/3))*log(x
^(1/3*n)) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(1/3))/(d*x^2 + e
*x^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \log\left(c\left(\frac{dx + ex^{\frac{1}{3}}}{x}\right)^n\right)^2 + 2\,ab\log\left(c\left(\frac{dx + ex^{\frac{1}{3}}}{x}\right)^n\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1
/3))/x)^n) + a^2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x, x)
```

3.519 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=276

$$-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}+\frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}-\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\right.}{2e^3}$$

```
[Out] (3*b^2*d*n^2*(d + e/x^(2/3))^2)/(4*e^3) - (b^2*n^2*(d + e/x^(2/3))^3)/(9*e^
3) - (3*b^2*d^2*n^2)/(e^2*x^(2/3)) + (b^2*d^3*n^2*Log[d + e/x^(2/3)]^2)/(2*
e^3) + (3*b*d^2*n*(d + e/x^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^3 - (
3*b*d*n*(d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*e^3) + (b*n*
(d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) - (b*d^3*n*Log[
d + e/x^(2/3)]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^3 - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/(2*x^2)
```

Rubi [A] time = 0.300981, antiderivative size = 217, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3}-\frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}-\frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3}+\frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{\left(a+b\log\left(c\left(\right.\right.\right.}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(2/3))^2)/(4*e^3) - (b^2*n^2*(d + e/x^(2/3))^3)/(9*e^
3) - (3*b^2*d^2*n^2)/(e^2*x^(2/3)) + (b^2*d^3*n^2*Log[d + e/x^(2/3)]^2)/(2*
e^3) + (b*n*((18*d^2*(d + e/x^(2/3)))/e^3 - (9*d*(d + e/x^(2/3))^2)/e^3 + (
2*(d + e/x^(2/3))^3)/e^3 - (6*d^3*Log[d + e/x^(2/3)])/e^3)*(a + b*Log[c*(d
+ e/x^(2/3))^n]))/6 - (a + b*Log[c*(d + e/x^(2/3))^n])^2/(2*x^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(\frac{3}{2}\,\text{Subst}\left(\int x^2\,(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{x^3\,(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx, x, \frac{1}{x^{2/3}}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log(cx^n))}{x}\,dx, x, d\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}\right)\left(a+b\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}\right)\left(a+b\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}\right)\left(a+b\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} + \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\right.}{e}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3} + \frac{1}{6}bn\left(\right.$$

Mathematica [C] time = 0.528384, size = 691, normalized size = 2.5

$$\frac{bn\left(-36d^3x^2\left(bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)+\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\right)+18bd^3nx^2\left(-4\text{PolyLog}\left(2,1-\frac{\sqrt{-d}}{\sqrt{e}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (-18*e^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*n*(9*b*d*n*x^(2/3)*(e*(e -
2*d*x^(2/3)) + 2*d^2*x^(4/3)*Log[d + e/x^(2/3)]) - 2*b*n*(e*(2*e^2 - 3*d*e*
x^(2/3) + 6*d^2*x^(4/3)) - 6*d^3*x^2*Log[d + e/x^(2/3)]) + 12*e^3*(a + b*Lo
g[c*(d + e/x^(2/3))^n]) - 18*d*e^2*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])
 + 36*d^2*x^(4/3)*(e*(a - b*n) + b*(e + d*x^(2/3))*Log[c*(d + e/x^(2/3))^n]
) - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)] - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x
^(1/3)] - 36*d^3*x^2*((a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))
] + b*n*PolyLog[2, 1 + e/(d*x^(2/3))]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/
3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sq
rt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e]
)]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-
d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] -
 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/(36*e^3*x^2)
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

Maxima [A] time = 1.0588, size = 402, normalized size = 1.46

$$-\frac{1}{6}\,aben\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)-\frac{1}{36}\left(6\,en\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*a*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^
2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/36*(6*e*n*(6*d^3*log(d*x^
(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3) +
2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3) + e)
^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2*x
^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n^2/
(e^3*x^2))*b^2 - 1/2*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - a*b*log(c*(d + e/
x^(2/3))^n)/x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 1.86134, size = 695, normalized size = 2.52

$$-\frac{4\,b^2e^3n^2+18\,b^2e^3\log\left(c\right)^2-12\,abe^3n+18\,a^2e^3+18\left(b^2d^3n^2x^2+b^2e^3n^2\right)\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^2-12\left(b^2e^3n-3\,abe^3\right)\log\left(c\right)-6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(4*b^2*e^3*n^2 + 18*b^2*e^3*log(c)^2 - 12*a*b*e^3*n + 18*a^2*e^3 + 18
*(b^2*d^3*n^2*x^2 + b^2*e^3*n^2)*log((d*x + e*x^(1/3))/x)^2 - 12*(b^2*e^3*n
 - 3*a*b*e^3)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(4/3) - 3*b^2*d*e^2*n^2*x^(2/3)
 + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*d^3*n)*x^2 - 6*(b^
2*d^3*n*x^2 + b^2*e^3*n)*log(c))*log((d*x + e*x^(1/3))/x) - 3*(5*b^2*d*e^2*
n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) - 6*(6*b^2*d^2*e*n*x*lo
g(c) - (11*b^2*d^2*e*n^2 - 6*a*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^3, x)
```

3.520 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=482

$$\frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6}-\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6}+\frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\right.}{4e^6}$$

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(2/3))^2)/(8*e^6) + (10*b^2*d^3*n^2*(d + e/x^(2/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(2/3))^4)/(32*e^6) + (3*b^2*d*n^2*
(d + e/x^(2/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(2/3))^6)/(72*e^6) + (3*b^2
*d^5*n^2)/(e^5*x^(2/3)) - (b^2*d^6*n^2*Log[d + e/x^(2/3)]^2)/(4*e^6) - (3*b
*d^5*n*(d + e/x^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^6 + (15*b*d^4*n*
(d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^6) - (10*b*d^3*n*(
d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^6) + (15*b*d^2*n*(d
 + e/x^(2/3))^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*e^6) - (3*b*d*n*(d + e
/x^(2/3))^5*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*e^6) + (b*n*(d + e/x^(2/3)
)^6*(a + b*Log[c*(d + e/x^(2/3))^n]))/(12*e^6) + (b*d^6*n*Log[d + e/x^(2/3)
]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*e^6) - (a + b*Log[c*(d + e/x^(2/3))^
n])^2/(4*x^4)
```

Rubi [A] time = 0.476294, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6}-\frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6}+\frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6}-\frac{225d^2\left(d+\frac{e}{x^{2/3}}\right)^4}{e^6}-\frac{60d^6\log\left(d+\frac{e}{x^{2/3}}\right)}{e^6}+\frac{72d\left(d+\frac{e}{x}\right.}{e^6}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(2/3))^2)/(8*e^6) + (10*b^2*d^3*n^2*(d + e/x^(2/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(2/3))^4)/(32*e^6) + (3*b^2*d*n^2*
(d + e/x^(2/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(2/3))^6)/(72*e^6) + (3*b^2
*d^5*n^2)/(e^5*x^(2/3)) - (b^2*d^6*n^2*Log[d + e/x^(2/3)]^2)/(4*e^6) - (b*n
*((360*d^5*(d + e/x^(2/3)))/e^6 - (450*d^4*(d + e/x^(2/3))^2)/e^6 + (400*d^
3*(d + e/x^(2/3))^3)/e^6 - (225*d^2*(d + e/x^(2/3))^4)/e^6 + (72*d*(d + e/x
^(2/3))^5)/e^6 - (10*(d + e/x^(2/3))^6)/e^6 - (60*d^6*Log[d + e/x^(2/3)])/e
^6)*(a + b*Log[c*(d + e/x^(2/3))^n]))/120 - (a + b*Log[c*(d + e/x^(2/3))^n]
)^2/(4*x^4)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = -\left(\frac{3}{2}\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(ben)\,\mathrm{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx,x,\frac{1}{x^{2/3}}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(bn)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}\,dx,x,d+\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^6}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^6}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{225d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^6}\right.$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} + \frac{3b^2dn^2\left(d+\right.}{25e^6}$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} + \frac{3b^2dn^2\left(d+\right.}{25e^6}$$

Mathematica [C] time = 0.837764, size = 1021, normalized size = 2.12

$$bn\left(-1800bnx^4\log^2\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)d^6-1800bnx^4\log^2\left(\sqrt[3]{x}\sqrt{-d}+\sqrt{e}\right)d^6-5220bnx^4\log\left(d+\frac{e}{x^{2/3}}\right)d^6-3600bx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)d^6+3600ax^4\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)d^6+3600\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] (-1800*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(600*a*e^6 - 100*b*e^6*n -
 720*a*d*e^5*x^(2/3) + 264*b*d*e^5*n*x^(2/3) + 900*a*d^2*e^4*x^(4/3) - 555*
b*d^2*e^4*n*x^(4/3) - 1200*a*d^3*e^3*x^2 + 1140*b*d^3*e^3*n*x^2 + 1800*a*d^
4*e^2*x^(8/3) - 2610*b*d^4*e^2*n*x^(8/3) - 3600*a*d^5*e*x^(10/3) + 8820*b*d
^5*e*n*x^(10/3) - 5220*b*d^6*n*x^4*Log[d + e/x^(2/3)] + 600*b*e^6*Log[c*(d
+ e/x^(2/3))^n] - 720*b*d*e^5*x^(2/3)*Log[c*(d + e/x^(2/3))^n] + 900*b*d^2*
e^4*x^(4/3)*Log[c*(d + e/x^(2/3))^n] - 1200*b*d^3*e^3*x^2*Log[c*(d + e/x^(2
/3))^n] + 1800*b*d^4*e^2*x^(8/3)*Log[c*(d + e/x^(2/3))^n] - 3600*b*d^5*e*x^
(10/3)*Log[c*(d + e/x^(2/3))^n] - 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n] +
 3600*a*d^6*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 3600*b*d^6*x^4*Log[c*(d +
 e/x^(2/3))^n]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 1800*b*d^6*n*x^4*Log[Sqrt[
e] - Sqrt[-d]*x^(1/3)]^2 + 3600*a*d^6*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] +
 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] -
1800*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 - 3600*b*d^6*n*x^4*Log[S
qrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 3600
*b*d^6*n*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2] + 3600*a*d^6*x^4*Log[-(e/(d*x^(2/3)))] + 3600*b*d^6*x^4*Log[c*(d
```

```
+ e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + 7200*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt
[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] + 7200*b*d^6*n*x^4*Log[Sqr
t[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 3600*b*d^6*n*x^4
*PolyLog[2, 1 + e/(d*x^(2/3))] + 7200*b*d^6*n*x^4*PolyLog[2, 1 - (Sqrt[-d]*
x^(1/3))/Sqrt[e]] - 3600*b*d^6*n*x^4*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2
*Sqrt[e])] - 3600*b*d^6*n*x^4*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2
] + 7200*b*d^6*n*x^4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/e^6)/(720
0*x^4)
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

Maxima [A] time = 1.09712, size = 536, normalized size = 1.11

$$\frac{1}{120}aben\left(\frac{60\,d^6\log\left(dx^{\frac{2}{3}}+e\right)}{e^7}-\frac{60\,d^6\log\left(x^{\frac{2}{3}}\right)}{e^7}-\frac{60\,d^5x^{\frac{10}{3}}-30\,d^4ex^{\frac{8}{3}}+20\,d^3e^2x^2-15\,d^2e^3x^{\frac{4}{3}}+12\,de^4x^{\frac{2}{3}}-10\,e^5}{e^6x^4}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/120*a*b*e*n*(60*d^6*log(d*x^(2/3) + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (6
0*d^5*x^(10/3) - 30*d^4*e*x^(8/3) + 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) + 1
2*d*e^4*x^(2/3) - 10*e^5)/(e^6*x^4)) + 1/7200*(60*e*n*(60*d^6*log(d*x^(2/3)
 + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (60*d^5*x^(10/3) - 30*d^4*e*x^(8/3) +
 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) + 12*d*e^4*x^(2/3) - 10*e^5)/(e^6*x^4)
)*log(c*(d + e/x^(2/3))^n) - (1800*d^6*x^4*log(d*x^(2/3) + e)^2 + 800*d^6*x
^4*log(x)^2 - 5880*d^6*x^4*log(x) - 8820*d^5*e*x^(10/3) + 2610*d^4*e^2*x^(8
/3) - 1140*d^3*e^3*x^2 + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(2/3) + 100*e^6
- 60*(40*d^6*x^4*log(x) - 147*d^6*x^4)*log(d*x^(2/3) + e))*n^2/(e^6*x^4))*b
^2 - 1/4*b^2*log(c*(d + e/x^(2/3))^n)^2/x^4 - 1/2*a*b*log(c*(d + e/x^(2/3))
^n)/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 1.87401, size = 1137, normalized size = 2.36

$$-\frac{100\,b^2e^6n^2+1800\,b^2e^6\log(c)^2-600\,abe^6n+1800\,a^2e^6-60\left(19\,b^2d^3e^3n^2-20\,abd^3e^3n\right)x^2-1800\left(b^2d^6n^2x^4-b^2e^6n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/7200*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^2
*e^6 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 - 1800*(b^2*d^6*n^2*x
^4 - b^2*e^6*n^2)*log((d*x + e*x^(1/3))/x)^2 + 600*(2*b^2*d^3*e^3*n*x^2 - b
^2*e^6*n + 6*a*b*e^6)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x^2 - 10*b^2*e^6*n^2
+ 60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^4 - 60*(b^2*d^6*n*x^4
- b^2*e^6*n)*log(c) - 6*(5*b^2*d^4*e^2*n^2*x^2 - 2*b^2*d*e^5*n^2)*x^(2/3) +
 15*(4*b^2*d^5*e*n^2*x^3 - b^2*d^2*e^4*n^2*x)*x^(1/3))*log((d*x + e*x^(1/3)
)/x) - 6*(44*b^2*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n^2 - 20*
a*b*d^4*e^2*n)*x^2 + 60*(5*b^2*d^4*e^2*n*x^2 - 2*b^2*d*e^5*n)*log(c))*x^(2/
3) - 15*(12*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e*n)*x^3 - (37*b^2*d^2*e^4*n^2 -
 60*a*b*d^2*e^4*n)*x - 60*(4*b^2*d^5*e*n*x^3 - b^2*d^2*e^4*n*x)*log(c))*x^(
1/3))/(e^6*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) + a\right)^{2}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^5, x)
```

3.521 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=490

$$-\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2}+$$

```
[Out] (-4*a*b*e^4*n*x^(1/3))/(3*d^4) + (568*b^2*e^4*n^2*x^(1/3))/(315*d^4) - (32*
b^2*e^3*n^2*x)/(105*d^3) + (8*b^2*e^2*n^2*x^(5/3))/(105*d^2) - (1408*b^2*e^
(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(315*d^(9/2)) - (((4*I)/3)*b^2
*e^(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^(9/2) + (8*b^2*e^(9/2)*
n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt
[d]*x^(1/3))])/(3*d^(9/2)) - (4*b^2*e^4*n*x^(1/3)*Log[c*(d + e/x^(2/3))^n])
/(3*d^4) + (4*b*e^3*n*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(9*d^3) - (4*b*e^
2*n*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(15*d^2) + (4*b*e*n*x^(7/3)*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(21*d) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^
(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*d^(9/2)) + (x^3*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/3 - (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, -1
+ (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(9/2)
```

Rubi [A] time = 0.806912, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2458, 2457, 2476, 2448, 263, 205, 2455, 193, 321, 302, 2470, 12, 260, 6688, 4924, 4868, 2447}

$$-\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```


```
[Out] (-4*a*b*e^4*n*x^(1/3))/(3*d^4) + (568*b^2*e^4*n^2*x^(1/3))/(315*d^4) - (32*
b^2*e^3*n^2*x)/(105*d^3) + (8*b^2*e^2*n^2*x^(5/3))/(105*d^2) - (1408*b^2*e^
(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(315*d^(9/2)) - (((4*I)/3)*b^2
*e^(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^(9/2) + (8*b^2*e^(9/2)*
n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt
[d]*x^(1/3))])/(3*d^(9/2)) - (4*b^2*e^4*n*x^(1/3)*Log[c*(d + e/x^(2/3))^n])
/(3*d^4) + (4*b*e^3*n*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(9*d^3) - (4*b*e^
2*n*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(15*d^2) + (4*b*e*n*x^(7/3)*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(21*d) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^
(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*d^(9/2)) + (x^3*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/3 - (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, -1
+ (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(9/2)
```


Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2dx=3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{1}{3}(4ben)\,\text{Subst}\left(\int\frac{x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{d+\frac{e}{x^2}}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{1}{3}(4ben)\,\text{Subst}\left(\int\left(-\frac{e^3\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right.\right.}{d^4}\right.\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(4ben)\,\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)d\right.}{3d}$$

$$=-\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{15d^2}$$

$$=-\frac{4abe^4n\sqrt[3]{x}}{3d^4}-\frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-$$

$$=-\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{8b^2e^4n^2\sqrt[3]{x}}{9d^4}-\frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4}+\frac{4be^3nx\left(a+b\log\left(c\right.\right.}{9d^3}$$

$$=-\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4}-\frac{32b^2e^3n^2x}{105d^3}+\frac{8b^2e^2n^2x^{5/3}}{105d^2}-\frac{32b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{}}\right.}{9d^{9/2}}$$

$$=-\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4}-\frac{32b^2e^3n^2x}{105d^3}+\frac{8b^2e^2n^2x^{5/3}}{105d^2}-\frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\right.}{315d^{9/2}}$$

$$=-\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4}-\frac{32b^2e^3n^2x}{105d^3}+\frac{8b^2e^2n^2x^{5/3}}{105d^2}-\frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\right.}{315d^{9/2}}$$

Mathematica [C] time = 2.22864, size = 735, normalized size = 1.5

$$\frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-4ben\left(-\frac{be^{7/2}n\left(-4\text{PolyLog}\left(2,1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)+2\text{PolyLog}\left(2,\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)+\log\left(\sqrt{e}-\right.\right.}{4(-d)^{9}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 - 4*b*e*n*((a*e^3*x^(1/3))/d^4 - (2
*b*e^(7/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(9/2) - (2*b*e*n*x^(5/3)*
Hypergeometric2F1[-5/2, 1, -3/2, -(e/(d*x^(2/3)))])/(35*d^2) + (2*b*e^2*n*x
```

```
*Hypergeometric2F1[-3/2, 1, -1/2, -(e/(d*x^(2/3)))]])/(15*d^3) - (2*b*e^3*n*
x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))]])/(3*d^4) + (b*e^3
*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/d^4 - (e^2*x*(a + b*Log[c*(d + e/x^(2/3)
)^n]))/(3*d^3) + (e*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*d^2) - (x^
(7/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(7*d) + (e^(7/2)*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(2*(-d)^(9/2)) - (e^(7/2)*(
a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(2*(-d)^(9
/2)) - (b*e^(7/2)*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d
]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^
(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog
[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]))/(4*(-d)^(9/2)) + (b*e^(7/2)*n*(
Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/
2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])
 + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqr
t[-d]*x^(1/3))/Sqrt[e]]))/(4*(-d)^(9/2))))/3
```

Maple [F] time = 0.347, size = 0, normalized size = 0.

$$\int x^{2} \left(a +b \ln \left(c \left(d +e x^{-\frac{2}{3}}\right)^{n}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^{2} x^{2} \log\left(c \left(\frac{d x + e x^{\frac{1}{3}}}{x}\right)^{n}\right)^{2} + 2\, a b x^{2} \log\left(c \left(\frac{d x + e x^{\frac{1}{3}}}{x}\right)^{n}\right) + a^{2} x^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*x^(1/3))/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^2, x)
```

3.522 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=309

$$\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

```
[Out] (4*a*b*e*n*x^(1/3))/d + (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + ((4*I)*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^
(3/2) - (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqr
t[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b^2*e*n*x^(1/3)*Log[c*(d
 + e/x^(2/3))^n])/d - (4*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a +
 b*Log[c*(d + e/x^(2/3))^n]))/d^(3/2) + x*(a + b*Log[c*(d + e/x^(2/3))^n])^
2 + ((4*I)*b^2*e^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]
*x^(1/3))])/d^(3/2)
```

Rubi [A] time = 0.444566, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {2451, 2457, 2471, 2448, 263, 205, 2470, 12, 260, 6688, 4924, 4868, 2447}

$$\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (4*a*b*e*n*x^(1/3))/d + (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + ((4*I)*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^
(3/2) - (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqr
t[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b^2*e*n*x^(1/3)*Log[c*(d
 + e/x^(2/3))^n])/d - (4*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a +
 b*Log[c*(d + e/x^(2/3))^n]))/d^(3/2) + x*(a + b*Log[c*(d + e/x^(2/3))^n])^
2 + ((4*I)*b^2*e^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]
*x^(1/3))])/d^(3/2)
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + (4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{d+\frac{e}{x^2}} dx, x, \sqrt[3]{}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + (4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{d} - \frac{e\left(a\right.}{}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(4ben)\,\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)dx, x,\right.}{d}$$

$$= \frac{4aben\sqrt[3]{x}}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}} + x\left(a+b\log\left(c\left(d\right.\right.\right.$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{d^{3/2}}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{d^{3/2}}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}} + \frac{4b^2en\sqrt[3]{x}\log}{}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan}{}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan}{}$$

Mathematica [A] time = 1.13613, size = 523, normalized size = 1.69

$$ben\left(\frac{b\sqrt{e}n\left(-4\text{PolyLog}\left(2,1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)+2\text{PolyLog}\left(2,\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)+\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)+2\log\left(\frac{1}{2}\right.\right.\right.}{(-d)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*e*n*((4*a*x^(1/3))/d - (8*b*Sqrt[e
]*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b*x^(1/3)*Log[c*(d + e/
x^(2/3))^n])/d - (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] -
Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))^n])
*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (b*Sqrt[e]*n*(Log[Sqrt[e] -
Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x
^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 -
 (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqr
t[e])]))/(-d)^(3/2) + (b*d*Sqrt[e]*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[
Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] -
 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3
))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(-d)^(5/2))
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^{2}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{2}+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)+a^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1/
3))/x)^n) + a^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) + a\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2, x)
```

3.523 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=361

$$\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}-\frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right.\right.}{e\sqrt[3]{x}}$$

```
[Out] (-8*b^2*n^2)/(9*x) + (32*b^2*d*n^2)/(3*e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) + ((4*I)*b^2*d^(3/2)*n^2*ArcTan
[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) - (8*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(3
/2) + (4*b*n*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) - (4*b*d*n*(a + b*Log[
c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) - (4*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3)
)/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/x + ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt
[e] - I*Sqrt[d]*x^(1/3))])/e^(3/2)
```

Rubi [A] time = 0.594036, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2458, 2457, 2476, 2455, 263, 325, 205, 2470, 12, 260, 6688, 4924, 4868, 2447}

$$\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}-\frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right.\right.}{e\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```


```
[Out] (-8*b^2*n^2)/(9*x) + (32*b^2*d*n^2)/(3*e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) + ((4*I)*b^2*d^(3/2)*n^2*ArcTan
[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) - (8*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(3
/2) + (4*b*n*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) - (4*b*d*n*(a + b*Log[
c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) - (4*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3)
)/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/x + ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt
[e] - I*Sqrt[d]*x^(1/3))])/e^(3/2)
```


Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^6}\,dx, x, \sqrt[3]{x}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{ex^4} - \frac{d\left(a+\right.}{}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4bn)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}}{}$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}}{}$$

$$= -\frac{8b^2n^2}{9x} + \frac{8b^2dn^2}{e\sqrt[3]{x}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{e\sqrt[3]{x}}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} + \frac{4bn}{}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} - \frac{8b}{}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} - \frac{8b}{}$$

Mathematica [A] time = 1.2665, size = 598, normalized size = 1.66

$$\frac{bn\left(9b(-d)^{3/2}nx\left(2\text{PolyLog}\left(2,\frac{1}{2}\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}+1\right)\right)-4\text{PolyLog}\left(2,\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}+1\right)+\log\left(\sqrt{-d}\sqrt[3]{x}+\sqrt{e}\right)\left(\log\left(\sqrt{-d}\sqrt[3]{x}+\sqrt{e}\right)+2\log\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)-4\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)\right)+9bd\sqrt{-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

```
[Out] (-9*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(72*b*d*Sqrt[e]*n*x^(2/3) - 7
2*b*d^(3/2)*n*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))] - 8*b*n*(Sqrt[e]*(e - 3*d
```

```
*x^(2/3)) + 3*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))]) + 12*e^(3/2)*(a
+ b*Log[c*(d + e/x^(2/3))^n]) - 36*d*Sqrt[e]*x^(2/3)*(a + b*Log[c*(d + e/x^
(2/3))^n]) + 18*(-d)^(3/2)*x*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] -
 Sqrt[-d]*x^(1/3)] + 18*Sqrt[-d]*d*x*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[S
qrt[e] + Sqrt[-d]*x^(1/3)] + 9*b*Sqrt[-d]*d*n*x*(Log[Sqrt[e] - Sqrt[-d]*x^(
1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt
[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x
^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 9*
b*(-d)^(3/2)*n*x*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x
^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x
^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*P
olyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/e^(3/2))/(9*x)
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1
/3))/x)^n) + a^2)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^2, x)
```

3.524 $\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 \, dx$

Optimal. Leaf size=773

result too large to display

```
[Out] (71*b^3*e^5*n^3*x^(2/3))/(80*d^5) - (3*b^3*e^4*n^3*x^(4/3))/(20*d^4) + (b^3
*e^3*n^3*x^2)/(40*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(2/3)])/(80*d^6) - (77
*b^2*e^5*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*
d^6) + (47*b^2*e^4*n^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(80*d^4) -
 (9*b^2*e^3*n^2*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^3) + (3*b^2*e^2
*n^2*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^2) - (77*b^2*e^6*n^2*L
og[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^6) + (3*b
*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6)
- (3*b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(8*d^4) + (b*e^3*n
*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^3) - (3*b*e^2*n*x^(8/3)*(a +
b*Log[c*(d + e/x^(2/3))^n])^2)/(16*d^2) + (3*b*e*n*x^(10/3)*(a + b*Log[c*(d
 + e/x^(2/3))^n])^2)/(20*d) + (3*b*e^6*n*Log[1 - d/(d + e/x^(2/3))]*(a + b*
Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]
)^3)/4 - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)
))])/(2*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (77*b^3*e^6*n^3*PolyLog[2,
 d/(d + e/x^(2/3))])/(40*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))
^n])*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^6) - (3*b^3*e^6*n^3*PolyLog[2, 1 +
 e/(d*x^(2/3))])/(2*d^6) - (3*b^3*e^6*n^3*PolyLog[3, d/(d + e/x^(2/3))])/(2
*d^6)
```

Rubi [A] time = 3.01947, antiderivative size = 746, normalized size of antiderivative = 0.97, number of steps used = 73, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$\frac{3b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{137b^3e^6n^3\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{40d^6}-\frac{3b^3e^6n^3\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}\right.}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (71*b^3*e^5*n^3*x^(2/3))/(80*d^5) - (3*b^3*e^4*n^3*x^(4/3))/(20*d^4) + (b^3
*e^3*n^3*x^2)/(40*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(2/3)])/(80*d^6) - (77
*b^2*e^5*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*
d^6) + (47*b^2*e^4*n^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(80*d^4) -
 (9*b^2*e^3*n^2*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^3) + (3*b^2*e^2
*n^2*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^2) + (77*b*e^6*n*(a +
b*Log[c*(d + e/x^(2/3))^n])^2)/(80*d^6) + (3*b*e^5*n*(d + e/x^(2/3))*x^(2/3
)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6) - (3*b*e^4*n*x^(4/3)*(a + b*L
og[c*(d + e/x^(2/3))^n])^2)/(8*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(2
/3))^n])^2)/(4*d^3) - (3*b*e^2*n*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2
)/(16*d^2) + (3*b*e*n*x^(10/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(20*d) -
 (e^6*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/(4*d^6) + (x^4*(a + b*Log[c*(d +
e/x^(2/3))^n])^3)/4 - (137*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log
[-(e/(d*x^(2/3)))])/(40*d^6) + (3*b*e^6*n*(a + b*Log[c*(d + e/x^(2/3))^n])^
2*Log[-(e/(d*x^(2/3)))])/(4*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) - (137*b
^3*e^6*n^3*PolyLog[2, 1 + e/(d*x^(2/3))])/(40*d^6) + (3*b^2*e^6*n^2*(a + b*
Log[c*(d + e/x^(2/3))^n])*PolyLog[2, 1 + e/(d*x^(2/3))])/(2*d^6) - (3*b^3*e
```

```
^6*n^3*PolyLog[3, 1 + e/(d*x^(2/3))])/(2*d^6)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3dx=-\left(\frac{3}{2}\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^3}{x^7}dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{1}{4}(3ben)\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^6(d+ex)}\right.$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{1}{4}(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d\right.$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d+\frac{e}{x^{2/3}}\right)}{4d}$$

$$=\frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{(3b}{\ }$$

$$=-\frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2}+\frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d}$$

$$=\frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2}+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^3}-$$

$$=-\frac{9b^2e^3n^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^3}+\frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2}$$

$$=\frac{3b^3e^5n^3x^{2/3}}{40d^5}-\frac{3b^3e^4n^3x^{4/3}}{80d^4}+\frac{b^3e^3n^3x^2}{40d^3}-\frac{3b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{40d^6}+\frac{47b^2e^4n^2x^{4/}}{\ }$$

$$=\frac{3b^3e^5n^3x^{2/3}}{10d^5}-\frac{3b^3e^4n^3x^{4/3}}{20d^4}+\frac{b^3e^3n^3x^2}{40d^3}-\frac{3b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{10d^6}-\frac{77b^2e^5n^2\left(d\right.}{\ }$$

$$=\frac{71b^3e^5n^3x^{2/3}}{80d^5}-\frac{3b^3e^4n^3x^{4/3}}{20d^4}+\frac{b^3e^3n^3x^2}{40d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6}-\frac{77b^2e^5n^2}{\ }$$

$$=\frac{71b^3e^5n^3x^{2/3}}{80d^5}-\frac{3b^3e^4n^3x^{4/3}}{20d^4}+\frac{b^3e^3n^3x^2}{40d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6}-\frac{77b^2e^5n^2}{\ }$$

Mathematica [A] time = 2.23248, size = 1014, normalized size = 1.31

$$\frac{20x^4\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3d^6+60bnx^4\log\left(d+\frac{e}{x^{2/3}}\right)\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (60*b*d*e^5*n*x^(2/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))
^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d +
e/x^(2/3))^n])^2 + 20*b*d^3*e^3*n*x^2*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c
*(d + e/x^(2/3))^n])^2 - 15*b*d^4*e^2*n*x^(8/3)*(a - b*n*Log[d + e/x^(2/3)]
 + b*Log[c*(d + e/x^(2/3))^n])^2 + 12*b*d^5*e*n*x^(10/3)*(a - b*n*Log[d + e
/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + 60*b*d^6*n*x^4*Log[d + e/x^(2/3
)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + 20*d^6*x^4
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3 - 60*b*e^6*n*(
a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[e + d*x^(2/3
)] + b^2*n^2*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*(d*e
^2*x^(2/3)*(-154*e^3 + 47*d*e^2*x^(2/3) - 18*d^2*e*x^(4/3) + 6*d^3*x^2) - 6
0*(e^6 - d^6*x^4)*Log[d + e/x^(2/3)]^2 - 274*e^6*Log[-(e/(d*x^(2/3)))] + 2*
e*Log[d + e/x^(2/3)]*(137*e^5 + 60*d*e^4*x^(2/3) - 30*d^2*e^3*x^(4/3) + 20*
d^3*e^2*x^2 - 15*d^4*e*x^(8/3) + 12*d^5*x^(10/3) + 60*e^5*Log[-(e/(d*x^(2/3
)))]) + 120*e^6*PolyLog[2, 1 + e/(d*x^(2/3))]) + b^3*n^3*(3*d^4*e^2*x^(8/3)
*(2 - 5*Log[d + e/x^(2/3)])*Log[d + e/x^(2/3)] + 12*d^5*e*x^(10/3)*Log[d +
e/x^(2/3)]^2 + 20*d^6*x^4*Log[d + e/x^(2/3)]^3 + 2*d^3*e^3*x^2*(1 - 9*Log[d
 + e/x^(2/3)] + 10*Log[d + e/x^(2/3)]^2) - d^2*e^4*x^(4/3)*(12 - 47*Log[d +
 e/x^(2/3)] + 30*Log[d + e/x^(2/3)]^2) + d*e^5*x^(2/3)*(71 - 154*Log[d + e/
x^(2/3)] + 60*Log[d + e/x^(2/3)]^2) + 225*e^6*(-Log[d + e/x^(2/3)] + Log[-(
e/(d*x^(2/3)))]) + 137*e^6*(Log[d + e/x^(2/3)]*(Log[d + e/x^(2/3)] - 2*Log[
-(e/(d*x^(2/3)))]) - 2*PolyLog[2, 1 + e/(d*x^(2/3))]) - 20*e^6*(Log[d + e/x
^(2/3)]^2*(Log[d + e/x^(2/3)] - 3*Log[-(e/(d*x^(2/3)))]) - 6*Log[d + e/x^(2
/3)]*PolyLog[2, 1 + e/(d*x^(2/3))] + 6*PolyLog[3, 1 + e/(d*x^(2/3))]))))/(80
*d^6)
```

Maple [F] time = 0.351, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,b^3x^4\log\left(\left(dx^{\frac{2}{3}}+e\right)^n\right)^3-\int-\frac{2\left(b^3d\log\left(c\right)^3+3\,ab^2d\log\left(c\right)^2+3\,a^2bd\log\left(c\right)+a^3d\right)x^4+2\left(b^3e\log\left(c\right)^3+3\,ab^2e\log\left(c\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((d*x^(2/3) + e)^n)^3 - integrate(-1/2*(2*(b^3*d*log(c)^3 +
3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^4 + 2*(b^3*e*log(c)^3 + 3*
a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(10/3) - 16*(b^3*d*x^4 + b^3
*e*x^(10/3))*log(x^(1/3*n))^3 - (b^3*d*n*x^4 - 6*(b^3*d*log(c) + a*b^2*d)*x
^4 - 6*(b^3*e*log(c) + a*b^2*e)*x^(10/3) + 12*(b^3*d*x^4 + b^3*e*x^(10/3))*
log(x^(1/3*n)))*log((d*x^(2/3) + e)^n)^2 + 24*((b^3*d*log(c) + a*b^2*d)*x^4
```

```
 + (b^3*e*log(c) + a*b^2*e)*x^(10/3))*log(x^(1/3*n))^2 + 6*((b^3*d*log(c)^2
 + 2*a*b^2*d*log(c) + a^2*b*d)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a
^2*b*e)*x^(10/3) + 4*(b^3*d*x^4 + b^3*e*x^(10/3))*log(x^(1/3*n))^2 - 4*((b^
3*d*log(c) + a*b^2*d)*x^4 + (b^3*e*log(c) + a*b^2*e)*x^(10/3))*log(x^(1/3*n
)))*log((d*x^(2/3) + e)^n) - 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b
*d)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(10/3))*log(x^(1/
3*n)))/(d*x + e*x^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2x^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2bx^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^3*log(c*((d*x
 + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^3*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x^
3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^3, x)
```

3.525 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=451

$$\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{d^3}+$$

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2
*d^3) + (3*b^2*e^3*n^2*Log[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/
3))^n]))/(2*d^3) - (3*b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(2*d^3) + (3*b*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^
2)/(4*d) - (3*b*e^3*n*Log[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3
))^n])^2)/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/2 + (3*b^2*e^3
*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/d^3 + (b^3*e^3
*n^3*Log[x])/d^3 - (3*b^3*e^3*n^3*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^3) +
(3*b^2*e^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, d/(d + e/x^(2/3)
)])/d^3 + (3*b^3*e^3*n^3*PolyLog[2, 1 + e/(d*x^(2/3))])/d^3 + (3*b^3*e^3*n^
3*PolyLog[3, d/(d + e/x^(2/3))])/d^3
```

Rubi [A] time = 1.00322, antiderivative size = 428, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$-\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{2d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2
*d^3) - (3*b*e^3*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^3) - (3*b*e^2*n
*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*d^3) + (3*b
*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d) + (e^3*(a + b*Log[c*
(d + e/x^(2/3))^n])^3)/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/2
 + (9*b^2*e^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/(
2*d^3) - (3*b*e^3*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))
])/(2*d^3) + (b^3*e^3*n^3*Log[x])/d^3 + (9*b^3*e^3*n^3*PolyLog[2, 1 + e/(d*
x^(2/3))])/(2*d^3) - (3*b^2*e^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLo
g[2, 1 + e/(d*x^(2/3))])/d^3 + (3*b^3*e^3*n^3*PolyLog[3, 1 + e/(d*x^(2/3))]
)/d^3
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^3}{x^4}dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{2}(3ben)\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)} \cdots\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{2}(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d \cdots\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\frac{e}{x^{2/3}}\right)}{2d} \cdots$$

$$= \frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(3ben \cdots}{\cdots}$$

$$= -\frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3} + \frac{3benx^{4/3}\left(a+b\log\left(c\left(d+ \cdots\right.\right.\right.}{4d}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\,\text{l}\cdots\right.}{2d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^3n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\cdots\right.\right.\right.}{4d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^3n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\cdots\right.\right.\right.}{4d^3}$$

Mathematica [A] time = 1.35184, size = 683, normalized size = 1.51

$$\frac{6b^2n^2\left(-2e^3\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)+\left(d^3x^2+e^3\right)\log^2\left(d+\frac{e}{x^{2/3}}\right)+e\log\left(d+\frac{e}{x^{2/3}}\right)\left(d^2x^{4/3}-2e^2\log\left(-\frac{e}{dx^{2/3}}\right)-2dex^{2/3}-\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (-6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))
^n])^2 + 3*b*d^2*e*n*x^(4/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x
^(2/3))^n])^2 + 6*b*d^3*n*x^2*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)
] + b*Log[c*(d + e/x^(2/3))^n])^2 + 2*d^3*x^2*(a - b*n*Log[d + e/x^(2/3)] +
 b*Log[c*(d + e/x^(2/3))^n])^3 + 6*b*e^3*n*(a - b*n*Log[d + e/x^(2/3)] + b*
Log[c*(d + e/x^(2/3))^n])^2*Log[e + d*x^(2/3)] + 6*b^2*n^2*(a - b*n*Log[d +
 e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((e^3 + d^3*x^2)*Log[d + e/x^(2/3
)]^2 + e^2*(d*x^(2/3) + 3*e*Log[-(e/(d*x^(2/3)))]) + e*Log[d + e/x^(2/3)]*(
-3*e^2 - 2*d*e*x^(2/3) + d^2*x^(4/3) - 2*e^2*Log[-(e/(d*x^(2/3)))]) - 2*e^3
*PolyLog[2, 1 + e/(d*x^(2/3))]) - b^3*n^3*(-6*e^3*Log[d + e/x^(2/3)] - 6*d*
e^2*x^(2/3)*Log[d + e/x^(2/3)] + 9*e^3*Log[d + e/x^(2/3)]^2 + 6*d*e^2*x^(2/
3)*Log[d + e/x^(2/3)]^2 - 3*d^2*e*x^(4/3)*Log[d + e/x^(2/3)]^2 - 2*e^3*Log[
d + e/x^(2/3)]^3 - 2*d^3*x^2*Log[d + e/x^(2/3)]^3 + 6*e^3*Log[-(e/(d*x^(2/3
)))] - 18*e^3*Log[d + e/x^(2/3)]*Log[-(e/(d*x^(2/3)))] + 6*e^3*Log[d + e/x^
```

```
(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 6*e^3*(-3 + 2*Log[d + e/x^(2/3)])*PolyLog[
2, 1 + e/(d*x^(2/3))] - 12*e^3*PolyLog[3, 1 + e/(d*x^(2/3))]))/(4*d^3)
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b^{3}x^{2}\log\left(\left(dx^{\frac{2}{3}}+e\right)^{n}\right)^{3}-\int\frac{8\left(b^{3}dx^{2}+b^{3}ex^{\frac{4}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^{3}-\left(b^{3}d\log\left(c\right)^{3}+3\,ab^{2}d\log\left(c\right)^{2}+3\,a^{2}bd\log\left(c\right)+a^{3}d\right)x^{2}+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((d*x^(2/3) + e)^n)^3 - integrate((8*(b^3*d*x^2 + b^3*e*x^(4
/3))*log(x^(1/3*n))^3 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*lo
g(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 3*(b^3*d*log(c) + a*b^2*d)*x^2 - 3*(b^3*
e*log(c) + a*b^2*e)*x^(4/3) + 6*(b^3*d*x^2 + b^3*e*x^(4/3))*log(x^(1/3*n)))
*log((d*x^(2/3) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*log(c
) + a*b^2*e)*x^(4/3))*log(x^(1/3*n))^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)
^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(4/3) - 3*((b^3*d*log(c)^2 + 2*a*b^2*d*log
(c) + a^2*b*d)*x^2 + 4*(b^3*d*x^2 + b^3*e*x^(4/3))*log(x^(1/3*n))^2 + (b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3) - 4*((b^3*d*log(c) + a*b^2
*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(4/3))*log(x^(1/3*n)))*log((d*x^(2/3)
+ e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*e*log
(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3))*log(x^(1/3*n)))/(d*x + e*x^(1/
3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^{3}x\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{3}+3\,ab^{2}x\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{2}+3\,a^{2}bx\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)+a^{3}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*x^(1/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x, x)
```

3.526 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=139

$$9b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{9}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-9b^3$$

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^3*Log[-(e/(d*x^(2/3)))])/2 - (9*b*n*(a
 + b*Log[c*(d + e/x^(2/3))^n])^2*PolyLog[2, 1 + e/(d*x^(2/3))])/2 + 9*b^2*n
^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[3, 1 + e/(d*x^(2/3))] - 9*b^3*n
^3*PolyLog[4, 1 + e/(d*x^(2/3))]
```

Rubi [A] time = 0.201533, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$9b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{9}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-9b^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```


```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^3*Log[-(e/(d*x^(2/3)))])/2 - (9*b*n*(a
 + b*Log[c*(d + e/x^(2/3))^n])^2*PolyLog[2, 1 + e/(d*x^(2/3))])/2 + 9*b^2*n
^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[3, 1 + e/(d*x^(2/3))] - 9*b^3*n
^3*PolyLog[4, 1 + e/(d*x^(2/3))]
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx = -\left(\frac{3}{2}\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(9ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+}{d}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(9bn)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{L}$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{L}$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{L}$$

Mathematica [B] time = 0.25123, size = 341, normalized size = 2.45

$$\frac{9}{2}b^2n^2\left(-2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)+2\log\left(d+\frac{e}{x^{2/3}}\right)\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)+\log\left(-\frac{e}{dx^{2/3}}\right)\log^2\left(d+\frac{e}{x^{2/3}}\right)\right)\left(-a-b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*((Log[d + e/x^(
2/3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/2
) + (9*b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*(
Log[d + e/x^(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 2*Log[d + e/x^(2/3)]*PolyLog[2
, 1 + e/(d*x^(2/3))] - 2*PolyLog[3, 1 + e/(d*x^(2/3))]))/2 - (3*b^3*n^3*(Lo
```

```
g[d + e/x^(2/3)]^3*Log[-(e/(d*x^(2/3)))] + 3*Log[d + e/x^(2/3)]^2*PolyLog[2
, 1 + e/(d*x^(2/3))] - 6*Log[d + e/x^(2/3)]*PolyLog[3, 1 + e/(d*x^(2/3))] +
 6*PolyLog[4, 1 + e/(d*x^(2/3))]))/2
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^{3}\log\left(\left(dx^{\frac{2}{3}}+e\right)^{n}\right)^{3}\log\left(x\right)-\int\frac{8\left(b^{3}dx+b^{3}ex^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^{3}+\left(2\,b^{3}dnx\log\left(x\right)-3\left(b^{3}d\log\left(c\right)+ab^{2}d\right)x+6\left(b^{3}dx+b^{3}e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((d*x^(2/3) + e)^n)^3*log(x) - integrate((8*(b^3*d*x + b^3*e*x^(1/3)
)*log(x^(1/3*n))^3 + (2*b^3*d*n*x*log(x) - 3*(b^3*d*log(c) + a*b^2*d)*x + 6
*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(1
/3))*log((d*x^(2/3) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log
(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(
c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 3*(4*(b^3*d*x + b^3*e*x^(1/3))*log(x^(
1/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^3*d*log
(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n)) + (b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*log((d*x^(2/3) + e)^n) +
6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*
b^2*e*log(c) + a^2*b*e)*x^(1/3))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2
*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(1/3))/(d*x^2 + e*x^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{3}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{3}+3\,ab^{2}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{2}+3\,a^{2}b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)+a^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x, x)
```

3.527 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=449

$$-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3}-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right.}{}$$

```
[Out] (-9*b^3*d*n^3*(d + e/x^(2/3))^2)/(8*e^3) + (b^3*n^3*(d + e/x^(2/3))^3)/(9*e
^3) - (9*a*b^2*d^2*n^2)/(e^2*x^(2/3)) + (9*b^3*d^2*n^3)/(e^2*x^(2/3)) - (9*
b^3*d^2*n^2*(d + e/x^(2/3))*Log[c*(d + e/x^(2/3))^n])/e^3 + (9*b^2*d*n^2*(d
 + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^3) - (b^2*n^2*(d + e
/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) + (9*b*d^2*n*(d + e/x
^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (9*b*d*n*(d + e/x^(2/
3))^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*e^3) + (b*n*(d + e/x^(2/3))^3*
(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (3*d^2*(d + e/x^(2/3))*(a + b
*Log[c*(d + e/x^(2/3))^n])^3)/(2*e^3) + (3*d*(d + e/x^(2/3))^2*(a + b*Log[c
*(d + e/x^(2/3))^n])^3)/(2*e^3) - ((d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^
(2/3))^n])^3)/(2*e^3)
```

Rubi [A] time = 0.462811, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3}-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```


```
[Out] (-9*b^3*d*n^3*(d + e/x^(2/3))^2)/(8*e^3) + (b^3*n^3*(d + e/x^(2/3))^3)/(9*e
^3) - (9*a*b^2*d^2*n^2)/(e^2*x^(2/3)) + (9*b^3*d^2*n^3)/(e^2*x^(2/3)) - (9*
b^3*d^2*n^2*(d + e/x^(2/3))*Log[c*(d + e/x^(2/3))^n])/e^3 + (9*b^2*d*n^2*(d
 + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^3) - (b^2*n^2*(d + e
/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) + (9*b*d^2*n*(d + e/x
^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (9*b*d*n*(d + e/x^(2/
3))^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*e^3) + (b*n*(d + e/x^(2/3))^3*
(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (3*d^2*(d + e/x^(2/3))*(a + b
*Log[c*(d + e/x^(2/3))^n])^3)/(2*e^3) + (3*d*(d + e/x^(2/3))^2*(a + b*Log[c
*(d + e/x^(2/3))^n])^3)/(2*e^3) - ((d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^
(2/3))^n])^3)/(2*e^3)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(\frac{3}{2}\,\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$= -\left(\frac{3}{2}\,\mathrm{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}-\frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2}+\right.\right.\right.$$

$$= -\frac{3\,\mathrm{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\frac{1}{x^{2/3}}\right)}{2e^2}+\frac{(3d)\,\mathrm{Subst}\left(\int(d+ex)\,(a\right.}{}$$

$$= -\frac{3\,\mathrm{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+\frac{e}{x^{2/3}}\right)}{2e^3}+\frac{(3d)\,\mathrm{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3\right.}{e^3}$$

$$= -\frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3}+\frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3}$$

$$= \frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3}-\frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right.\right.}{4e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3}+\frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\right.\right.}{4e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3}+\frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9b^3d^2n^3}{e^2x^{2/3}}-\frac{9b^3d^2n^2\left(d+\frac{e}{x^{2/3}}\right)\mathrm{l}}{e^3}$$

Mathematica [A] time = 1.43404, size = 692, normalized size = 1.54

$$\frac{-6b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\left(18a^2e^3-6aben\left(6d^2x^{4/3}-3dex^{2/3}+2e^2\right)+6bd^3nx^2(6a-11bn)\log\left(dx^{2/3}+e\right)+4bd^3nx^2\log(x)(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(2/3) + 90*a*b^2*d*e^2*n^2*x^(2/3) - 57*b^3*d*e^2*n^3*x^(2/3) +
108*a^2*b*d^2*e*n*x^(4/3) - 396*a*b^2*d^2*e*n^2*x^(4/3) + 510*b^3*d^2*e*n^3
*x^(4/3) + 72*b^3*d^3*n^3*x^2*Log[d + e/x^(2/3)]^3 - 36*b^3*e^3*Log[c*(d +
e/x^(2/3))^n]^3 - 108*a^2*b*d^3*n*x^2*Log[e + d*x^(2/3)] + 396*a*b^2*d^3*n^
2*x^2*Log[e + d*x^(2/3)] - 510*b^3*d^3*n^3*x^2*Log[e + d*x^(2/3)] + 12*b^2*
d^3*n^2*x^2*Log[d + e/x^(2/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n]
)*(3*Log[e + d*x^(2/3)] - 2*Log[x]) + 72*a^2*b*d^3*n*x^2*Log[x] - 264*a*b^2
*d^3*n^2*x^2*Log[x] + 340*b^3*d^3*n^3*x^2*Log[x] - 18*b^2*d^3*n^2*x^2*Log[d
 + e/x^(2/3)]^2*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n] + 6*b*n*Log[e
+ d*x^(2/3)] - 4*b*n*Log[x]) + 18*b^2*Log[c*(d + e/x^(2/3))^n]^2*(e*(-6*a*e
^2 + 2*b*e^2*n - 3*b*d*e*n*x^(2/3) + 6*b*d^2*n*x^(4/3)) - 6*b*d^3*n*x^2*Log
[e + d*x^(2/3)] + 4*b*d^3*n*x^2*Log[x]) - 6*b*Log[c*(d + e/x^(2/3))^n]*(18*
a^2*e^3 - 6*a*b*e*n*(2*e^2 - 3*d*e*x^(2/3) + 6*d^2*x^(4/3)) + b^2*e*n^2*(4*
e^2 - 15*d*e*x^(2/3) + 66*d^2*x^(4/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x^2*Log[e
 + d*x^(2/3)] + 4*b*d^3*n*(-6*a + 11*b*n)*x^2*Log[x]))/(72*e^3*x^2)
```

Maple [F] time = 0.358, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

Maxima [A] time = 1.15112, size = 923, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*
d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/12*(6*e*n*(6*d^3*log(d*
x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3)
+ 2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3) +
e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2
*x^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n^
2/(e^3*x^2))*a*b^2 - 1/216*(54*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*lo
g(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2))*log(c*(
d + e/x^(2/3))^n)^2 + e*n*((108*d^3*x^2*log(d*x^(2/3) + e)^3 - 32*d^3*x^2*l
og(x)^3 + 264*d^3*x^2*log(x)^2 - 1020*d^3*x^2*log(x) - 1530*d^2*e*x^(4/3) +
 171*d*e^2*x^(2/3) - 24*e^3 - 54*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2
/3) + e)^2 + 18*(8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) + 85*d^3*x^2)*log(d
*x^(2/3) + e))*n^2/(e^4*x^2) - 18*(18*d^3*x^2*log(d*x^(2/3) + e)^2 + 8*d^3*
x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2*x^(2/3) - 4*
e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n*log(c*(d + e/
x^(2/3))^n)/(e^4*x^2)))*b^3 - 1/2*b^3*log(c*(d + e/x^(2/3))^n)^3/x^2 - 3/2*
a*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/x^(2/3))^n)/x
^2 - 1/2*a^3/x^2
```

Fricas [A] time = 1.96166, size = 1600, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/72*(8*b^3*e^3*n^3 - 36*b^3*e^3*log(c)^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^3
*n - 36*a^3*e^3 - 36*(b^3*d^3*n^3*x^2 + b^3*e^3*n^3)*log((d*x + e*x^(1/3))/
x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(4/3)
- 3*b^3*d*e^2*n^3*x^(2/3) + 2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d^3*n
^3 - 6*a*b^2*d^3*n^2)*x^2 - 6*(b^3*d^3*n^2*x^2 + b^3*e^3*n^2)*log(c))*log((
d*x + e*x^(1/3))/x)^2 - 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*lo
g(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n + (85*b^3*d^3*n
^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n)*x^2 + 18*(b^3*d^3*n*x^2 + b^3*e^3*n
)*log(c)^2 - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^2 - 6*a*b^2*d
^3*n)*x^2)*log(c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*d
*e^2*n^2)*x^(2/3) - 6*(6*b^3*d^2*e*n^2*x*log(c) - (11*b^3*d^2*e*n^3 - 6*a*b
^2*d^2*e*n^2)*x)*x^(1/3))*log((d*x + e*x^(1/3))/x) - 3*(19*b^3*d*e^2*n^3 +
18*b^3*d*e^2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*
```

```
d*e^2*n^2 - 6*a*b^2*d*e^2*n)*log(c))*x^(2/3) + 6*(18*b^3*d^2*e*n*x*log(c)^2
 - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*x*log(c) + (85*b^3*d^2*e*n^3 - 66
*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^3, x)
```

3.528 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=1277

result too large to display

```
[Out] (568*a*b^2*e^4*n^2*x^(1/3))/(105*d^4) - (16*b^3*e^4*n^3*x^(1/3))/(7*d^4) +
(16*b^3*e^3*n^3*x)/(105*d^3) + (1376*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(105*d^(9/2)) + (((568*I)/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]^2)/d^(9/2) - (1136*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/(105*d^(9/
2)) + (568*b^3*e^4*n^2*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/(105*d^4) - (32*b^
2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^3) + (8*b^2*e^2*n^2*x^(
5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^2) - (568*b^2*e^(9/2)*n^2*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(105*d^(9/2
)) - (2*b*e^4*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3*
n*x*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) - (2*b*e^2*n*x^(5/3)*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a + b*Log[c*(d +
e/x^(2/3))^n])^2)/(7*d) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/3 + (4*b
^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)])/(-d)^(9/2) - (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2)/(-
d)^(9/2) - (4*b^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e]
+ Sqrt[-d]*x^(1/3)])/(-d)^(9/2) + (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2)/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)
]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-d)^(9/2) - (4*b^3*e^(9/2)*n^
3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2])/
(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt
[-d]*x^(1/3))/Sqrt[e])])/(-d)^(9/2) + (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]])/(-d)^(9/2) + (((568*I)/105)*
b^3*e^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])
/d^(9/2) + (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(
-d)^(9/2) - (4*b^3*e^(9/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[
e])])/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2])/(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3)
)/Sqrt[e]])/(-d)^(9/2) + (2*b*e^5*n*Unintegrable[(a + b*Log[c*(d + e/x^(2/3
))^n])^2/((e + d*x^(2/3))*x^(2/3)), x])/(3*d^4)
```

Rubi [A] time = 3.16869, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```


```
[Out] (568*a*b^2*e^4*n^2*x^(1/3))/(105*d^4) - (16*b^3*e^4*n^3*x^(1/3))/(7*d^4) +
(16*b^3*e^3*n^3*x)/(105*d^3) + (1376*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(105*d^(9/2)) + (((568*I)/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]^2)/d^(9/2) - (1136*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/(105*d^(9/
2)) + (568*b^3*e^4*n^2*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/(105*d^4) - (32*b^
2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^3) + (8*b^2*e^2*n^2*x^(
```

```
5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^2) - (568*b^2*e^(9/2)*n^2*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(105*d^(9/2
)) - (2*b*e^4*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3*
n*x*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) - (2*b*e^2*n*x^(5/3)*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a + b*Log[c*(d +
e/x^(2/3))^n])^2)/(7*d) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/3 + (4*b
^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)])/(-d)^(9/2) - (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2)/(-
d)^(9/2) - (4*b^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e]
+ Sqrt[-d]*x^(1/3)])/(-d)^(9/2) + (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2)/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)
]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-d)^(9/2) - (4*b^3*e^(9/2)*n^
3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2])/
(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt
[-d]*x^(1/3))/Sqrt[e])])/(-d)^(9/2) + (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]])/(-d)^(9/2) + (((568*I)/105)*
b^3*e^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])
/d^(9/2) + (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(
-d)^(9/2) - (4*b^3*e^(9/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[
e])])/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2])/(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3)
)/Sqrt[e]])/(-d)^(9/2) + (2*b*e^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d
 + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d^4
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3dx=3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+(2ben)\,\text{Subst}\left(\int\frac{x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right.\right.}{d+\frac{e}{x^2}}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+(2ben)\,\text{Subst}\left(\int\left(-\frac{e^3\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right.\right.\right.}{d^4}\right.\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{(2ben)\,\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right.\right.}{d}$$

$$=-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3}-$$

$$=-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3}-$$

$$=-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3}-$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{32b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{35d^3}+\frac{8b^2e^2n^2x^{5/3}\left(a+b\log\right.}{35d^2}$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}+\frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{105d^4}-\frac{32b^2e^3n^2x\left(a+b\log\left(c\left(\right.\right.\right.}{35d^3}$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{64b^3e^4n^3\sqrt[3]{x}}{35d^4}+\frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{105d^4}-\frac{32b^2e^3n^2}{}$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4}+\frac{16b^3e^3n^3x}{105d^3}+\frac{1328b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{105d^{9/2}}+$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4}+\frac{16b^3e^3n^3x}{105d^3}+\frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{105d^{9/2}}+$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4}+\frac{16b^3e^3n^3x}{105d^3}+\frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{105d^{9/2}}+$$

$$\frac{568ab^2e^4n^2\sqrt[3]{x}}{}\quad\frac{16b^3e^4n^3\sqrt[3]{x}}{}\quad\frac{16b^3e^3n^3x}{}\quad\frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{}$$

Mathematica [A] time = 4.8286, size = 764, normalized size = 0.6

$$-\frac{b^2n^2\left(-a-b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+bn\log\left(d+\frac{e}{x^{2/3}}\right)\right)\left(\log\left(d+\frac{e}{x^{2/3}}\right)\left(9e^5\left(dx^{2/3}+e\right){}_3F_2\left(1,1,\frac{11}{2};2,2;\frac{e}{dx^{2/3}}+1\right)+dx^{2/3}\left(d\right.\right.\right.}{d^6x\sqrt{-\frac{e}{dx^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*(54*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 1, 11/2}, {2,
2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)]*(-54*e^5*(e + d*x^(2/3))*
HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d +
e/x^(2/3)]*(27*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 11/2}, {2, 2},
1 + e/(d*x^(2/3))] + 2*d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2/3)))]*x^(10/3)
)*Log[d + e/x^(2/3)]))))/(6*d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (b^2*n^2*(-9*e^
5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^
(2/3))] + Log[d + e/x^(2/3)]*(9*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1
, 11/2}, {2, 2}, 1 + e/(d*x^(2/3))] + d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2
/3)))]*x^(10/3))*Log[d + e/x^(2/3)]))*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[
c*(d + e/x^(2/3))^n]))/(d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (2*b*e^4*n*x^(1/3)*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3
*n*x*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) -
 (2*b*e^2*n*x^(5/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n
])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d +
 e/x^(2/3))^n])^2)/(7*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^(9/2) + b*n
*x^3*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3
))^n])^2 + (x^3*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3
)/3
```

Maple [A] time = 0.355, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2x^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2bx^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^2*log(c*((d*x
 + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^2*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x^
2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^2, x)
```

3.529 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=737

$$-\frac{2be^2n\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}\left(dx^{2/3}+e\right)},x\right)}{d}+\frac{24b^3e^{3/2}n^3\text{PolyLog}\left(2,1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}-\frac{12b^3e^{3/2}n^3\text{PolyLog}\left(2,\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right.}{(-d)^{3/2}}$$

```
[Out] (6*b*e*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d + x*(a + b*Log[c*(d
+ e/x^(2/3))^n])^3 + (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*L
og[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(-d)^(3/2) - (6*b^3*e^(3/2)*n^3*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) - (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (6*b^3*e^(3/2)
*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-
d)^(3/2) - (12*b^3*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sq
rt[-d]*x^(1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])/(-d)^(3/2) + (24*b^3
*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]
])/(-d)^(3/2) + (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[
e]])/(-d)^(3/2) - (12*b^3*e^(3/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(
2*Sqrt[e])])/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(
1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 + (Sqrt[-d
]*x^(1/3))/Sqrt[e]])/(-d)^(3/2) - (2*b*e^2*n*Unintegrable[(a + b*Log[c*(d +
 e/x^(2/3))^n])^2/((e + d*x^(2/3))*x^(2/3)), x])/d
```

Rubi [A] time = 1.29714, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```


```
[Out] (6*b*e*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d + x*(a + b*Log[c*(d
+ e/x^(2/3))^n])^3 + (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*L
og[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(-d)^(3/2) - (6*b^3*e^(3/2)*n^3*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) - (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (6*b^3*e^(3/2)
*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-
d)^(3/2) - (12*b^3*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sq
rt[-d]*x^(1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])/(-d)^(3/2) + (24*b^3
*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]
])/(-d)^(3/2) + (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[
e]])/(-d)^(3/2) - (12*b^3*e^(3/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(
2*Sqrt[e])])/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(
1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 + (Sqrt[-d
]*x^(1/3))/Sqrt[e]])/(-d)^(3/2)
```

```
 - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b
```

```
*Log[c*(d + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (6ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d+\frac{e}{x^2}} dx, x, \sqrt[3]{\ldots}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (6ben)\,\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d} - e\left(a \ldots\right.\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(6ben)\,\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\ldots\right.}{d}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{\left(6be^2n\right)\text{Subs}\ldots}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{\left(6be^2n\right)\text{Subs}\ldots}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{\left(6be^2n\right)\text{Subs}\ldots}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12b^2e^{3/2}n^2\left(a \ldots\right.}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12b^2e^{3/2}n^2\left(a \ldots\right.}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12b^2e^{3/2}n^2\left(a \ldots\right.}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12b^2e^{3/2}n^2\left(a \ldots\right.}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12b^2e^{3/2}n^2\left(a \ldots\right.}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12b^2e^{3/2}n^2\left(a \ldots\right.}{\ldots}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12b^2e^{3/2}n^2\left(a \ldots\right.}{\ldots}$$

Mathematica [A] time = 5.68845, size = 824, normalized size = 1.12

$$b^3\sqrt[3]{x}\left(\sqrt{d}\left(6e+dx^{2/3}\log\left(d+\frac{e}{x^{2/3}}\right)\right)\log^2\left(d+\frac{e}{x^{2/3}}\right)-6e\sqrt{\frac{e}{x^{2/3}d+e}}\left(8\sqrt{d}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{d}{d+\frac{e}{x^{2/3}}}\right)+\log\left(d+\frac{e}{x^{2/3}}\right)\left(4\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (3*b^2*n^2*(-3*e^2*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 5/2}, {2, 2,
 2}, 1 + e/(d*x^(2/3))] - d*x^(2/3)*Log[d + e/x^(2/3)]*(4*e*(e - e/Sqrt[-(e
/(d*x^(2/3)))]) + 4*e^2*Log[(1 + Sqrt[-(e/(d*x^(2/3)))])/2] + (-e^2 + d^2*S
qrt[-(e/(d*x^(2/3)))]*x^(4/3))*Log[d + e/x^(2/3)]))*(-a + b*n*Log[d + e/x^(
2/3)] - b*Log[c*(d + e/x^(2/3))^n]))/(d^3*Sqrt[-(e/(d*x^(2/3)))]*x) + (6*b*
e*n*x^(1/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d
- (6*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/
3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^(3/2) + 3*b*n*x*Log[d + e/x^(2/3)]*(
a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + x*(a - b*n*Log
[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3 + (b^3*n^3*x^(1/3)*(Sqrt[d]
*Log[d + e/x^(2/3)]^2*(6*e + d*x^(2/3)*Log[d + e/x^(2/3)]) - 6*e*Sqrt[e/(e
+ d*x^(2/3))]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2,
 3/2}, d/(d + e/x^(2/3))] + Log[d + e/x^(2/3)]*(4*Sqrt[d]*HypergeometricPFQ
[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e/x^(2/3))] + Sqrt[d + e/x^(2/3)]*ArcS
in[Sqrt[d]/Sqrt[d + e/x^(2/3)]]*Log[d + e/x^(2/3)])) + 6*Sqrt[d]*e*((4*Sqrt
[e/x^(2/3)]*ArcTanh[Sqrt[e/x^(2/3)]/Sqrt[-d]]*(Log[d + e/x^(2/3)] - Log[1 +
 e/(d*x^(2/3))]))/Sqrt[-d] - Sqrt[-(e/(d*x^(2/3)))]*(2*Log[(1 + Sqrt[-(e/(d
*x^(2/3)))])/2]^2 - 4*Log[(1 + Sqrt[-(e/(d*x^(2/3)))])/2]*Log[1 + e/(d*x^(2
/3))] + Log[1 + e/(d*x^(2/3))]^2 - 4*PolyLog[2, 1/2 - Sqrt[-(e/(d*x^(2/3)))
]/2]))))/d^(3/2)
```

Maple [A] time = 0.345, size = 0, normalized size = 0.

$$\int\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^(
1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3, x)
```

3.530 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=482

$$-\frac{2bd^2n\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}\left(dx^{2/3}+e\right)},x\right)}{e}-\frac{32ib^3d^{3/2}n^3\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e}$$

```
[Out] (16*b^3*n^3)/(9*x) - (208*b^3*d*n^3)/(3*e*x^(1/3)) - (208*b^3*d^(3/2)*n^3*A
rcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) - ((32*I)*b^3*d^(3/2)*n^3*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) + (64*b^3*d^(3/2)*n^3*ArcTan[(Sqr
t[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))]])/
e^(3/2) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) + (32*b^2*d*n^
2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*ArcTa
n[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) + (2
*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/x - (6*b*d*n*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(e*x^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/x - ((32*I)
*b^3*d^(3/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))]
)/e^(3/2) - (2*b*d^2*n*Unintegrable[(a + b*Log[c*(d + e/x^(2/3))^n])^2/((e
+ d*x^(2/3))*x^(2/3)), x])/e
```

Rubi [A] time = 1.38325, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (16*b^3*n^3)/(9*x) - (208*b^3*d*n^3)/(3*e*x^(1/3)) - (208*b^3*d^(3/2)*n^3*A
rcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) - ((32*I)*b^3*d^(3/2)*n^3*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) + (64*b^3*d^(3/2)*n^3*ArcTan[(Sqr
t[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))]])/
e^(3/2) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) + (32*b^2*d*n^
2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*ArcTa
n[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) + (2
*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/x - (6*b*d*n*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(e*x^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/x - ((32*I)
*b^3*d^(3/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))]
)/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])
^2/(e + d*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^2}\right)x^6}\,dx, x, \sqrt[3]{x}\right.\cdots$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6ben)\,\text{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{ex^4} - \frac{d\left(a+\cdots\right.}{\cdots}\right.\right.\cdots$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)\cdots$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d\cdots\right.\right.\right.}{x}\cdots$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d\cdots\right.\right.\right.}{x}\cdots$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d\cdots\right.\right.\right.}{x}\cdots$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\cdots}{\cdots}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\cdots}{\cdots}$$

$$= \frac{16b^3n^3}{9x} - \frac{64b^3dn^3}{e\sqrt[3]{x}} - \frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.}{e\sqrt[3]{x}}\cdots$$

$$= \frac{16b^3n^3}{9x} - \frac{208b^3dn^3}{3e\sqrt[3]{x}} - \frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} - \frac{8b^2n\cdots}{\cdots}$$

$$= \frac{16b^3n^3}{9x} - \frac{208b^3dn^3}{3e\sqrt[3]{x}} - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} + \frac{64\cdots}{\cdots}$$

$$= \frac{16b^3n^3}{\cdots} - \frac{208b^3dn^3}{\cdots} - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\cdots}\right)}{\cdots} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\cdots}\right)^2}{\cdots} + 64\cdots$$

Mathematica [A] time = 2.23848, size = 1097, normalized size = 2.28

$$\frac{b^3\left(18\left(x^{2/3}d+e\right){}_5F_4\left(-\frac{1}{2},1,1,1,1;2,2,2,2;\frac{e}{dx^{2/3}}+1\right)-\log\left(d+\frac{e}{x^{2/3}}\right)\left(18\left(x^{2/3}d+e\right){}_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;\frac{e}{dx^{2/3}}+1\right)\right.\right.}{2e\sqrt{-\frac{e}{dx^{2/3}}}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (b^3*n^3*(18*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2
, 2}, 1 + e/(d*x^(2/3))] - Log[d + e/x^(2/3)]*(18*(e + d*x^(2/3))*Hypergeom
etricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)
]*(-9*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2
/3))] + 2*(e*Sqrt[-(e/(d*x^(2/3)))] + d*x^(2/3))*Log[d + e/x^(2/3)]))))/(2*
e*Sqrt[-(e/(d*x^(2/3)))]*x) - (6*b*d*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[
c*(d + e/x^(2/3))^n])^2)/(e*x^(1/3)) - (6*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/e
^(3/2) - (3*b*n*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d
 + e/x^(2/3))^n])^2)/x - ((a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2
/3))^n])^2*(a - 2*b*n - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n]
))/x + (b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*
(8*e^(3/2) - 96*d*Sqrt[e]*x^(2/3) + 96*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^
(1/3))] - 12*e^(3/2)*Log[d + e/x^(2/3)] + 36*d*Sqrt[e]*x^(2/3)*Log[d + e/x^
(2/3)] + 9*e^(3/2)*Log[d + e/x^(2/3)]^2 + 18*Sqrt[-d]*d*x*Log[d + e/x^(2/3)
]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 9*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x
^(1/3)]^2 + 18*(-d)^(3/2)*x*Log[d + e/x^(2/3)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/
3)] + 9*Sqrt[-d]*d*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 + 18*Sqrt[-d]*d*x*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 1
8*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/
Sqrt[e])/2] + 36*(-d)^(3/2)*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])] + 36*Sqrt[-d]*d*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log
[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 36*Sqrt[-d]*d*x*PolyLog[2, 1 - (Sqrt[-d]*x^(
1/3))/Sqrt[e]] + 18*(-d)^(3/2)*x*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqr
t[e])] + 18*Sqrt[-d]*d*x*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] + 3
6*(-d)^(3/2)*x*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(3*e^(3/2)*x)
```

Maple [A] time = 0.341, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^2, x)
```

3.531 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=783

result too large to display

```
[Out] (16*b^3*n^3)/(729*x^3) - (3088*b^3*d*n^3)/(27783*e*x^(7/3)) + (221344*b^3*d
^2*n^3)/(496125*e^2*x^(5/3)) - (637984*b^3*d^3*n^3)/(297675*e^3*x) + (34755
04*b^3*d^4*n^3)/(99225*e^4*x^(1/3)) + (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt
[d]*x^(1/3))/Sqrt[e]])/(99225*e^(9/2)) + (((4504*I)/315)*b^3*d^(9/2)*n^3*Ar
cTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(
Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))
])/(315*e^(9/2)) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(81*x^3) +
(128*b^2*d*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(441*e*x^(7/3)) - (1144*b^
2*d^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(1575*e^2*x^(5/3)) + (1984*b^2*
d^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(945*e^3*x) - (4504*b^2*d^4*n^2*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^4*x^(1/3)) - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^
(9/2)) + (2*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(9*x^3) - (2*b*d*n*(a +
 b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a + b*Log[c*(d
+ e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2*b*d^3*n*(a + b*Log[c*(d + e/x^(2/3
))^n])^2)/(3*e^3*x) + (2*b*d^4*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(e^4*x
^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/(3*x^3) + (((4504*I)/315)*b^3*
d^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(
9/2) + (2*b*d^5*n*Unintegrable[(a + b*Log[c*(d + e/x^(2/3))^n])^2/((e + d*x
^(2/3))*x^(2/3)), x])/(3*e^4)
```

Rubi [A] time = 3.62489, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```


```
[Out] (16*b^3*n^3)/(729*x^3) - (3088*b^3*d*n^3)/(27783*e*x^(7/3)) + (221344*b^3*d
^2*n^3)/(496125*e^2*x^(5/3)) - (637984*b^3*d^3*n^3)/(297675*e^3*x) + (34755
04*b^3*d^4*n^3)/(99225*e^4*x^(1/3)) + (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt
[d]*x^(1/3))/Sqrt[e]])/(99225*e^(9/2)) + (((4504*I)/315)*b^3*d^(9/2)*n^3*Ar
cTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(
Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))
])/(315*e^(9/2)) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(81*x^3) +
(128*b^2*d*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(441*e*x^(7/3)) - (1144*b^
2*d^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(1575*e^2*x^(5/3)) + (1984*b^2*
d^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(945*e^3*x) - (4504*b^2*d^4*n^2*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^4*x^(1/3)) - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^
(9/2)) + (2*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(9*x^3) - (2*b*d*n*(a +
 b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a + b*Log[c*(d
+ e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2*b*d^3*n*(a + b*Log[c*(d + e/x^(2/3
))^n])^2)/(3*e^3*x) + (2*b*d^4*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(e^4*x
```

```
^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/(3*x^3) + (((4504*I)/315)*b^3*
d^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(
9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])^2/(e
 + d*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^2}\right)x^{12}}\,dx, x, \cdots\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\,\text{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{ex^{10}} - \frac{d\left(\cdots\right.}{\cdots}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^{10}}\,dx, x, \sqrt[3]{\cdots}\right.$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2n\left(a+b\cdots\right.}{\cdots}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2n\left(a+b\cdots\right.}{\cdots}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2n\left(a+b\cdots\right.}{\cdots}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{81x^3} + \frac{128b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{441ex^{7/3}} - \frac{1144b^2\cdots}{\cdots}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{81x^3} + \frac{128b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{441ex^{7/3}} - \frac{1144b^2\cdots}{\cdots}$$

$$= \frac{16b^3n^3}{729x^3} - \frac{256b^3dn^3}{3087ex^{7/3}} + \frac{2288b^3d^2n^3}{7875e^2x^{5/3}} - \frac{3968b^3d^3n^3}{2835e^3x} + \frac{9008b^3d^4n^3}{315e^4\sqrt[3]{x}} - \frac{8b^2n^2\left(a+b\,\mathrm{l}\cdots\right.}{\cdots}$$

$$= \frac{16b^3n^3}{729x^3} - \frac{3088b^3dn^3}{27783ex^{7/3}} + \frac{7472b^3d^2n^3}{18375e^2x^{5/3}} - \frac{26704b^3d^3n^3}{14175e^3x} + \frac{30992b^3d^4n^3}{945e^4\sqrt[3]{x}} + \frac{9008b^3d^{9/\cdots}}{\cdots}$$

$$= \frac{16b^3n^3}{729x^3} - \frac{3088b^3dn^3}{27783ex^{7/3}} + \frac{221344b^3d^2n^3}{496125e^2x^{5/3}} - \frac{206128b^3d^3n^3}{99225e^3x} + \frac{161824b^3d^4n^3}{4725e^4\sqrt[3]{x}} + \frac{3099\cdots}{\cdots}$$

$$\frac{16b^3n^3}{\cdots} \quad \frac{3088b^3dn^3}{\cdots} \quad \frac{221344b^3d^2n^3}{\cdots} \quad \frac{637984b^3d^3n^3}{\cdots} \quad \frac{1151968b^3d^4n^3}{\cdots} \quad \frac{161\cdots}{\cdots}$$

Mathematica [A] time = 9.0079, size = 2858, normalized size = 3.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

```
[Out] -(b^3*n^3*(32*d^4 - 32*d^4*Sqrt[1 - (d + e/x^(2/3))/d] + 128*d^3*Sqrt[1 - (
d + e/x^(2/3))/d]*(d + e/x^(2/3)) - 192*d^2*Sqrt[1 - (d + e/x^(2/3))/d]*(d
+ e/x^(2/3))^2 + 128*d*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^3 - 32*S
qrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^4 + 1584*d^3*(d + e/x^(2/3))*Hyp
ergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d + e/x^(2/3))/d] - 4536*d^3*(d
 + e/x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e/x^(2/3))
/d] + 3780*d^3*(d + e/x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}
, (d + e/x^(2/3))/d] - 864*d^3*(d + e/x^(2/3))*HypergeometricPFQ[{-7/2, 1,
1, 1, 1}, {2, 2, 2, 2}, (d + e/x^(2/3))/d] + 3024*d^3*(d + e/x^(2/3))*Hyper
geometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e/x^(2/3))/d] - 3780*d^
3*(d + e/x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d +
e/x^(2/3))/d] + 1890*d^3*(d + e/x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1,
1}, {2, 2, 2, 2}, (d + e/x^(2/3))/d] - 240*d^4*Log[d + e/x^(2/3)] + 240*d^4
*Sqrt[1 - (d + e/x^(2/3))/d]*Log[d + e/x^(2/3)] - 672*d^3*Sqrt[1 - (d + e/x
^(2/3))/d]*(d + e/x^(2/3))*Log[d + e/x^(2/3)] + 576*d^2*Sqrt[1 - (d + e/x^(
2/3))/d]*(d + e/x^(2/3))^2*Log[d + e/x^(2/3)] - 96*d*Sqrt[1 - (d + e/x^(2/3
))/d]*(d + e/x^(2/3))^3*Log[d + e/x^(2/3)] - 48*Sqrt[1 - (d + e/x^(2/3))/d]
*(d + e/x^(2/3))^4*Log[d + e/x^(2/3)] - 3780*d^3*(d + e/x^(2/3))*Hypergeome
tricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e/x^(2/3))/d]*Log[d + e/x^(2/3)] + 864*d
^3*(d + e/x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d + e/x^(
2/3))/d]*Log[d + e/x^(2/3)] - 3024*d^3*(d + e/x^(2/3))*HypergeometricPFQ[{-
5/2, 1, 1, 1}, {2, 2, 2}, (d + e/x^(2/3))/d]*Log[d + e/x^(2/3)] + 3780*d^3*
(d + e/x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e/x^(2/3
))/d]*Log[d + e/x^(2/3)] - 1890*d^3*(d + e/x^(2/3))*HypergeometricPFQ[{-1/2
, 1, 1, 1}, {2, 2, 2}, (d + e/x^(2/3))/d]*Log[d + e/x^(2/3)] + 284*d^4*Log[
d + e/x^(2/3)]^2 - 284*d^4*Sqrt[1 - (d + e/x^(2/3))/d]*Log[d + e/x^(2/3)]^2
 + 668*d^3*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))*Log[d + e/x^(2/3)]^2
 - 552*d^2*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^2*Log[d + e/x^(2/3)]
^2 + 236*d*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^3*Log[d + e/x^(2/3)]
^2 - 68*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^4*Log[d + e/x^(2/3)]^2
- 1890*d^3*(d + e/x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e/x
^(2/3))/d]*Log[d + e/x^(2/3)]^2 + 945*d^3*(d + e/x^(2/3))*HypergeometricPFQ
[{-1/2, 1, 1}, {2, 2}, (d + e/x^(2/3))/d]*Log[d + e/x^(2/3)]^2 - 70*d^4*Log
[d + e/x^(2/3)]^3 + 70*d^4*Sqrt[1 - (d + e/x^(2/3))/d]*Log[d + e/x^(2/3)]^3
 - 280*d^3*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))*Log[d + e/x^(2/3)]^3
 + 420*d^2*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^2*Log[d + e/x^(2/3)]
^3 - 280*d*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^3*Log[d + e/x^(2/3)]
^3 + 70*Sqrt[1 - (d + e/x^(2/3))/d]*(d + e/x^(2/3))^4*Log[d + e/x^(2/3)]^3
+ 1512*d^3*(d + e/x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (d + e/x
^(2/3))/d]*(1 + 3*Log[d + e/x^(2/3)] + Log[d + e/x^(2/3)]^2) - 144*d^3*(d +
 e/x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e/x^(2/3))/d]*(6 +
 11*Log[d + e/x^(2/3)] + 3*Log[d + e/x^(2/3)]^2)))/(210*e^4*Sqrt[1 - (d + e
/x^(2/3))/d]*x^(1/3)) - (2*b*d*n*(a + b*(-(n*Log[d + e/x^(2/3)]) + Log[c*(d
 + e/x^(2/3))^n]))^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a + b*(-(n*Log[d + e/x^(2
/3)]) + Log[c*(d + e/x^(2/3))^n]))^2)/(5*e^2*x^(5/3)) - (2*b*d^3*n*(a + b*(
-(n*Log[d + e/x^(2/3)]) + Log[c*(d + e/x^(2/3))^n]))^2)/(3*e^3*x) + (2*b*d^
4*n*(a + b*(-(n*Log[d + e/x^(2/3)]) + Log[c*(d + e/x^(2/3))^n]))^2)/(e^4*x^
(1/3)) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*(-(n*Log[d
 + e/x^(2/3)]) + Log[c*(d + e/x^(2/3))^n]))^2)/e^(9/2) - (b*n*Log[d + e/x^(
2/3)]*(a + b*(-(n*Log[d + e/x^(2/3)]) + Log[c*(d + e/x^(2/3))^n]))^2)/x^3 -
 ((a + b*(-(n*Log[d + e/x^(2/3)]) + Log[c*(d + e/x^(2/3))^n]))^2*(3*a - 2*b
```

```
*n + 3*b*(-(n*Log[d + e/x^(2/3)]) + Log[c*(d + e/x^(2/3))^n])))/(9*x^3) + 9
*b^2*n^2*(a + b*(-(n*Log[d + e/x^(2/3)]) + Log[c*(d + e/x^(2/3))^n]))*(-Log
[(e + d*x^(2/3))/x^(2/3)]^2/(9*x^3) + (-9800*e^(9/2) + 28800*d*e^(7/2)*x^(2
/3) - 72072*d^2*e^(5/2)*x^(4/3) + 208320*d^3*e^(3/2)*x^2 - 1418760*d^4*Sqrt
[e]*x^(8/3) + 1418760*d^(9/2)*x^3*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))] + 44100
*e^(9/2)*Log[d + e/x^(2/3)] - 56700*d*e^(7/2)*x^(2/3)*Log[d + e/x^(2/3)] +
79380*d^2*e^(5/2)*x^(4/3)*Log[d + e/x^(2/3)] - 132300*d^3*e^(3/2)*x^2*Log[d
 + e/x^(2/3)] + 396900*d^4*Sqrt[e]*x^(8/3)*Log[d + e/x^(2/3)] + 198450*(-d)
^(9/2)*x^3*Log[d + e/x^(2/3)]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 99225*(-d)^
(9/2)*x^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 - 198450*(-d)^(9/2)*x^3*Log[d +
 e/x^(2/3)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 99225*(-d)^(9/2)*x^3*Log[Sqrt
[e] + Sqrt[-d]*x^(1/3)]^2 + 198450*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqrt[-d]*x^
(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 198450*(-d)^(9/2)*x^3*Lo
g[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 396
900*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))
/Sqrt[e])] + 396900*(-d)^(9/2)*x^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqr
t[-d]*x^(1/3))/Sqrt[e]] + 396900*(-d)^(9/2)*x^3*PolyLog[2, 1 - (Sqrt[-d]*x^
(1/3))/Sqrt[e]] - 198450*(-d)^(9/2)*x^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))
/(2*Sqrt[e])] + 198450*(-d)^(9/2)*x^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2] - 396900*(-d)^(9/2)*x^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]
])/(893025*e^(9/2)*x^3))
```

Maple [A] time = 0.351, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e x^{-\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{3}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{3}+3\,ab^{2}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{2}+3\,a^{2}b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)+a^{3}}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) + a\right)^{3}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^4, x)
```

3.532 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx$

Optimal. Leaf size=730

result too large to display

```
[Out] (2^(-2 - 3*p)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^8*e^8*E^((8*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(7^p*c^7*e^8*E^((7*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (7*d^2*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(6^p*c^6*e^8*E^((6*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^3*Gamma[1 + p, (-5*(a + b*Log[c*(d
+ e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^8*E^((5*a)/
b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (35*2^(-1 - 2*p)*d^4*Gamma[1
+ p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^
p)/(c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^5*
Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqr
t[x])])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p
) + (7*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c
*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[
x])])/b))^p) - (2*d^7*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a
+ b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p)
```

Rubi [A] time = 1.3364, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{7d^2 6^{-p} e^{-\frac{6a}{b}} \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^6e^8} - \frac{14d^3 5^{-p} e^{-\frac{5a}{b}} \left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```


```
[Out] (2^(-2 - 3*p)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^8*e^8*E^((8*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(7^p*c^7*e^8*E^((7*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (7*d^2*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(6^p*c^6*e^8*E^((6*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^3*Gamma[1 + p, (-5*(a + b*Log[c*(d
+ e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^8*E^((5*a)/
b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (35*2^(-1 - 2*p)*d^4*Gamma[1
+ p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^
p)/(c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^5*
Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqr
t[x])])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p
) + (7*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c
*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[
x])])/b))^p) - (2*d^7*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a
+ b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=2\,\text{Subst}\left(\int x^7(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)$$
$$=2\,\text{Subst}\left(\int\left(-\frac{d^7(a+b\log(c(d+ex)))^p}{e^7}+\frac{7d^6(d+ex)(a+b\log(c(d+ex)))^p}{e^7}-\right.\right.$$
$$=\frac{2\,\text{Subst}\left(\int(d+ex)^7(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7}-\frac{(14d)\,\text{Subst}\left(\int(d+ex)^6\right.}{}$$
$$=\frac{2\,\text{Subst}\left(\int x^7(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8}-\frac{(14d)\,\text{Subst}\left(\int x^6(a+b\log(cx)\right.}{e^8}$$
$$=\frac{2\,\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^8e^8}-\frac{(14d)\,\text{Subst}\left(\int e^{7x}(a+bx)^p\right.}{c^7e^8}$$
$$=\frac{2^{-2-3p}e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{c^8e^8}$$

Mathematica [A] time = 0.986439, size = 435, normalized size = 0.6

$$\frac{2^{-3p-2}105^{-p}e^{-\frac{8a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(c^7d^7\left(-8^{p+1}\right)105^pe^{\frac{7a}{b}}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right.\right.}{b}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-2 - 3*p)*(105^p*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b] -
8^(1 + p)*15^p*c*d*E^(a/b)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))
/b] + 5^p*28^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-6*(a + b*Log[c*(d +
 e*Sqrt[x])]))/b] - 3^p*56^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1 + p, (-5*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b] + 3^p*70^(1 + p)*c^4*d^4*E^((4*a)/b)*Gamma[
1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 5^p*56^(1 + p)*c^5*d^5*E^((
5*a)/b)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b] + 15^p*28^(1 +
p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]
- 8^(1 + p)*105^p*c^7*d^7*E^((7*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sq
rt[x])])/b)])*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(105^p*c^8*e^8*E^((8*a)/b)*
(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Maple [F] time = 0.489, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^px^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^{p}x^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

3.533 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx$

Optimal. Leaf size=551

$$\frac{5d^2 4^{-p} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^6} - \frac{20d^3 3^{-p-1} e^{-\frac{3a}{b}} \left(a+\right.}{}$$

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[
c*(d + e*Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])])/b))^p) - (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*Sqrt[x])])/b))^p) + (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x
])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p) - (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^3*e^6*
E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (5*d^4*Gamma[1 + p,
(-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2
^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^5*Ga
mma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x]
)])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rubi [A] time = 0.863902, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{5d^2 4^{-p} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^6} - \frac{20d^3 3^{-p-1} e^{-\frac{3a}{b}} \left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```


```
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[
c*(d + e*Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])])/b))^p) - (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*Sqrt[x])])/b))^p) + (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x
])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p) - (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^3*e^6*
E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (5*d^4*Gamma[1 + p,
(-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2
^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^5*Ga
mma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x]
)])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}-\frac{10d\right.\right.$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^5}-\frac{(10d)\,\text{Subst}\left(\int(d+ex)^4(a\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^6}-\frac{(10d)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\,d\right.}{e^6}$$
$$= \frac{2\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^6e^6}-\frac{(10d)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,\right.}{c^5e^6}$$
$$= \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{c^6e^6}$$

Mathematica [A] time = 0.893107, size = 325, normalized size = 0.59

$$3^{-p-1}20^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(10^p\text{Gamma}\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{2p+}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (3^(-1 - p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - c*d
*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x
])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e*Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a +
 b*Log[c*(d + e*Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a +
b*Log[c*(d + e*Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(20^p*
c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

3.534 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=360

$$\frac{3d^2 2^{-p} e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^4}+\frac{2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\right.}{}$$

```
[Out] (2^(-1 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^4*e^4*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^3*e^4*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (3*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^4*E^((2*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^3*Gamma[1 + p, -((a + b*Log[c*(d + e
*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^4*E^(a/b)*(-((a + b*
Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rubi [A] time = 0.543432, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3d^2 2^{-p} e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^4}+\frac{2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^4*e^4*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^3*e^4*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (3*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^4*E^((2*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^3*Gamma[1 + p, -((a + b*Log[c*(d + e
*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^4*E^(a/b)*(-((a + b*
Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx &= 2\,\mathrm{Subst}\left(\int x^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)\\
&= 2\,\mathrm{Subst}\left(\int\left(-\frac{d^3(a+b\log(c(d+ex)))^p}{e^3}+\frac{3d^2(d+ex)(a+b\log(c(d+ex)))^p}{e^3}-\frac{3d(d\right.\right.\\
&= \frac{2\,\mathrm{Subst}\left(\int(d+ex)^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^3}-\frac{(6d)\,\mathrm{Subst}\left(\int(d+ex)^2(a+\right.}{e}\\
&= \frac{2\,\mathrm{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^4}-\frac{(6d)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,\right.}{e^4}\\
&= \frac{2\,\mathrm{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^4e^4}-\frac{(6d)\,\mathrm{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\right.}{c^3e^4}\\
&= \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right.}{c^4e^4}
\end{aligned}$$

Mathematica [A] time = 0.421726, size = 229, normalized size = 0.64

$$\frac{2^{-2p-1}3^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(3^p\mathrm{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-cd2^{p+1}e^{a/b}\left(2^{p+1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*(3^p*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^
(1 + p)*c*d*E^(a/b)*(2^(1 + p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x]
)]))/b] + 3^p*c*d*E^(a/b)*(-3*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])]
)/b)])))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^4*e^4*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^{p}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

3.535 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=174

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^2}-\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d\right.\right.\right.}{}$$

```
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sq
rt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^
p) - (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d
 + e*Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rubi [A] time = 0.221995, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2451, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^2}-\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sq
rt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^
p) - (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d
 + e*Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx &= 2\,\mathrm{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)\\
&= 2\,\mathrm{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)))^p}{e}+\frac{(d+ex)(a+b\log(c(d+ex)))^p}{e}\right)dx,x,\sqrt{x}\right)\\
&= \frac{2\,\mathrm{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e}-\frac{(2d)\,\mathrm{Subst}\left(\int(a+b\log(c(d+\right.}{e}\\
&= \frac{2\,\mathrm{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^2}-\frac{(2d)\,\mathrm{Subst}\left(\int(a+b\log(cx))^p\,dx,x,d+\right.}{e^2}\\
&= \frac{2\,\mathrm{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^2e^2}-\frac{(2d)\,\mathrm{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\right.}{ce^2}\\
&= \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^2}
\end{aligned}$$

Mathematica [A] time = 0.123858, size = 130, normalized size = 0.75

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(\mathrm{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-cd2^{p+1}e^{a/b}\mathrm{Gamma}\left(p+\right.\right.}{c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] ((Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^(1 + p)*c*d*E^(a/
b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)])*(a + b*Log[c*(d + e*S
qrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))
^p)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

3.536 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*Sqrt[x])])^p/x, x]
```

Rubi [A] time = 0.0528729, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.264424, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x, x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

3.537 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2, x]
```

Rubi [A] time = 0.0533969, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]

[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.317068, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]

[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2, x]
```

Maple [A] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)

[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

3.538 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=907

result too large to display

```
[Out] (Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*
Sqrt[x])^2])^p)/(2^(2*(1 + p))*c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(7^p
*e^8*E^((7*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(7/2)*(-((a + b*Log[c*(d + e*Sqr
t[x])^2])/b))^p) + (7*d^2*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)*d^3*(d + e*Sqrt[x])^5*Gamm
a[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*S
qrt[x])^2])^p)/(5^p*e^8*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a +
 b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (35*2^(-1 - p)*d^4*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^
2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)
*d^5*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(
2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^8*E^((3*a)/(2*b))*(c*(d +
e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (7*d^6*Gam
ma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x
])^2])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1
 + p)*d^7*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/(2
*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^8*E^(a/(2*b))*Sqrt[c*(d + e*Sqr
t[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rubi [A] time = 1.40442, antiderivative size = 907, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{2^{-2(p+1)}e^{-\frac{4a}{b}}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}-\frac{2^{p+1}7^{-p}de^{-\frac{7a}{2b}}\left(d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```


```
[Out] (Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*
Sqrt[x])^2])^p)/(2^(2*(1 + p))*c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(7^p
*e^8*E^((7*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(7/2)*(-((a + b*Log[c*(d + e*Sqr
t[x])^2])/b))^p) + (7*d^2*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)*d^3*(d + e*Sqrt[x])^5*Gamm
a[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*S
qrt[x])^2])^p)/(5^p*e^8*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a +
 b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (35*2^(-1 - p)*d^4*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^
2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)
*d^5*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(
```

```
2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^8*E^((3*a)/(2*b))*(c*(d +
e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (7*d^6*Gam
ma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x
])^2])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1
 + p)*d^7*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/(2
*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^8*E^(a/(2*b))*Sqrt[c*(d + e*Sqr
t[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx = 2\,\text{Subst}\left(\int x^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}+\frac{7d^6(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}\right.\right.$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)}{e^7} - \frac{(14d)\,\text{Subst}\left(\int(d+ex\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x^7\left(a+b\log\left(cx^2\right)\right)^p dx, x, d+e\sqrt{x}\right)}{e^8} - \frac{(14d)\,\text{Subst}\left(\int x^6\left(a+b\log\left(c\right.\right.\right.}{e^8}$$
$$= \frac{\text{Subst}\left(\int e^{4x}(a+bx)^p dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^4e^8} + \frac{\left(21d^2\right)\text{Subst}\left(\int e^{3x}(a+bx)^p d\right.}{c^3e^8}$$
$$= \frac{4^{-1-p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{c^4e^8}$$

Mathematica [F] time = 0.473294, size = 0, normalized size = 0.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F] time = 0.571, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^px^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

3.539 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=677

result too large to display

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(5^p*e^
6*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b
]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e*Sqrt[x])^2])/b))^p) - (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e*Sqrt[x]
)^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*
(d + e*Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-(
(a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (5*d^4*Gamma[1 + p, -((a + b*Log[
c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^6*E^(a/b
)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1 + p)*d^5*(d + e*Sqrt[x
])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d +
e*Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[
c*(d + e*Sqrt[x])^2])/b))^p)
```

Rubi [A] time = 0.981997, antiderivative size = 677, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{5d^2 2^{-p} e^{-\frac{2a}{b}} \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2 e^6} + \frac{3^{-p-1} e^{-\frac{3a}{b}} \left(a+b\log \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```


```
[Out] (3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(5^p*e^
6*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b
]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e*Sqrt[x])^2])/b))^p) - (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e*Sqrt[x]
)^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*
(d + e*Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-(
(a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (5*d^4*Gamma[1 + p, -((a + b*Log[
c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^6*E^(a/b
)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1 + p)*d^5*(d + e*Sqrt[x
])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d +
e*Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[
c*(d + e*Sqrt[x])^2])/b))^p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx = 2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5} + \frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right.\right.$$

$$= \frac{2\,\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)}{e^5} - \frac{(10d)\,\text{Subst}\left(\int (d+ex\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(10d)\,\text{Subst}\left(\int x^4\left(a+b\log\left(c\right.\right.\right.}{e^6}$$

$$= \frac{\text{Subst}\left(\int e^{3x}(a+bx)^p dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^3e^6} + \frac{\left(10d^2\right)\text{Subst}\left(\int e^{2x}(a+bx)^p d\right.}{c^2e^6}$$

$$= \frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{c^3e^6}$$

Mathematica [F] time = 0.326332, size = 0, normalized size = 0.

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^2 \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)^2 c\right)+a\right)^p x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^p x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

3.540 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=445

$$\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^4}+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\right.\right.}{}$$

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^2*e^4*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^
4*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*
(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^4*E^(a/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d^3*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Lo
g[c*(d + e*Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^4*E^(
a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^
p)
```

Rubi [A] time = 0.626889, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^4}+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```


```
[Out] (2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^2*e^4*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^
4*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*
(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^4*E^(a/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d^3*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Lo
g[c*(d + e*Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^4*E^(
a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^
p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^3}+\frac{3d^2(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^3}\right.\right.$$

$$= \frac{2\,\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^3}-\frac{(6d)\,\text{Subst}\left(\int(d+ex)^2\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x^3\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt{x}\right)}{e^4}-\frac{(6d)\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right.\right.\right.}{e^4}$$

$$= \frac{\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^2e^4}+\frac{\left(3d^2\right)\text{Subst}\left(\int e^x(a+bx)^p\,dx,\right.}{ce^4}$$

$$= \frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right.\right.}{b}\right.}{c^2e^4}$$

Mathematica [F] time = 0.238173, size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^px,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

3.541 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=213

$$\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^2}-\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\log\right.}{}$$

```
[Out] (Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sq
rt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (
2^(1 + p)*d*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/
(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d + e*S
qrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rubi [A] time = 0.261162, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2451, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^2}-\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sq
rt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (
2^(1 + p)*d*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/
(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d + e*S
qrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}\right)dx \cdots\right.$$

$$= \frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e}-\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\cdots\right.\right.}{\cdots}$$

$$= \frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt{x}\right)}{e^2}-\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\left(cx^2\right)\right)\cdots\right.}{e^2}$$

$$= \frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{ce^2}-\frac{\left(d\left(d+e\sqrt{x}\right)\right)\text{Subst}\left(\int e^{x/2}(a\cdots\right.}{e^2\sqrt{c\left(d\cdots\right.}}$$

$$= \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\cdots}{ce^2}$$

Mathematica [F] time = 0.116794, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)^{2}c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^{2}x+2\,cde\sqrt{x}+cd^{2}\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)^{2}c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

3.542 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x, x]
```

Rubi [A] time = 0.0540136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.116114, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x, x]
```

Maple [A] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2 c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2 c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

3.543 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2, x]
```

Rubi [A] time = 0.0526444, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.12041, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2, x]
```

Maple [A] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

3.544 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x*(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Rubi [A] time = 0.0449333, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx=2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.64589, size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Maple [A] time = 0.483, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

3.545 $\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Rubi [A] time = 0.021141, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx = 2 \text{Subst}\left(\int x \left(a+b \log \left(c \left(d+\frac{e}{x}\right)\right)\right)^p \, dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.174579, size = 0, normalized size = 0.

$$\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Maple [A] time = 0.351, size = 0, normalized size = 0.

$$\int \left(a+b \ln \left(c \left(d+e\frac{1}{\sqrt{x}}\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

3.546 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/Sqrt[x])])^p/x, x]
```

Rubi [A] time = 0.0548585, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.303821, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x, x]
```

Maple [A] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

3.547 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=175

$$\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{ce^2}-\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{}$$

```
[Out] -((Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)) + (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c
*(d + e/Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^
p)
```

Rubi [A] time = 0.246925, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{ce^2}-\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)) + (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c
*(d + e/Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^
p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)))^p}{e}+\frac{(d+ex)(a+b\log(c(d+ex)))^p}{e}\right)dx, x,\right.\right.$$
$$= -\frac{2\,\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)\,\text{Subst}\left(\int(a+b\log(c(\right.}{e}$$
$$= -\frac{2\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\,\text{Subst}\left(\int(a+b\log(cx))^p\,dx, x,\right.}{e^2}$$
$$= -\frac{2\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^2e^2}+\frac{(2d)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x\right.}{ce^2}$$
$$= -\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-}}{c^2e^2}$$

Mathematica [A] time = 0.171015, size = 131, normalized size = 0.75

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\left(cd2^{p+1}e^{a/b}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)-\text{Gamma}\left(p+1\right.\right.}{c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

```
[Out] ((-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)])*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)
```

Maple [F] time = 0.361, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

3.548 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=552

$$-\frac{5d^2 4^{-p} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^4 e^6}+\frac{20d^3 3^{-p-1} e^{-\frac{3a}{b}} \left(a+b\right.}{}$$

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e/Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]
*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/Sqrt[x])])/b))^p) - (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqr
t[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*
(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (5*d^4*Gamma[1 +
p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)
/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2*d^5
*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/Sqrt
[x])])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rubi [A] time = 0.848473, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{5d^2 4^{-p} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^4 e^6}+\frac{20d^3 3^{-p-1} e^{-\frac{3a}{b}} \left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```


```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e/Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]
*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/Sqrt[x])])/b))^p) - (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqr
t[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*
(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (5*d^4*Gamma[1 +
p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)
/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2*d^5
*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/Sqrt
[x])])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = -\left(2\,\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}-\frac{10d$$
$$= -\frac{2\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}+\frac{(10d)\,\text{Subst}\left(\int (d+ex)^4(a+\right.}{e^5}$$
$$= -\frac{2\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6}+\frac{(10d)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx,\right.}{e^6}$$
$$= -\frac{2\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^6e^6}+\frac{(10d)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\right.}{c^5e^6}$$
$$= -\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right.}{c^6e^6}$$

Mathematica [A] time = 0.777938, size = 325, normalized size = 0.59

$$\frac{3^{-p-1}20^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\left(cde^{a/b}\left(2^{2p+1}3^{p+1}\text{Gamma}\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cd^5\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

```
[Out] (3^(-1 - p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]) +
c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqr
t[x])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(
d + e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b
*Log[c*(d + e/Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a
 + b*Log[c*(d + e/Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(20
^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

3.549 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$

Optimal. Leaf size=926

result too large to display

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*L
og[c*(d + e/Sqrt[x])])^p)/(2^p*c^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/Sqrt[x])])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(9^p*c^9*e^10*E^((9*a)/b)*(-((a + b*
Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^2*Gamma[1 + p, (-8*(a + b*Log[c*(d +
e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(8^p*c^8*e^10*E^((8*a)/b
)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (24*d^3*Gamma[1 + p, (-7*(a +
b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(7^p*c^7*e^
10*E^((7*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (7*6^(1 - p)*d^4*
Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqr
t[x])])^p)/(c^6*e^10*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) +
 (252*5^(-1 - p)*d^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e/Sqrt[x])])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p) - (21*2^(1 - 2*p)*d^6*Gamma[1 + p, (-4*(a + b*Log[c*(d
+ e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^4*e^10*E^((4*a)/b)*
(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (8*3^(1 - p)*d^7*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3
*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^8*Gamma[1
 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])
^p)/(2^p*c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2
*d^9*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(c*e^10*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rubi [A] time = 1.5566, antiderivative size = 926, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*L
og[c*(d + e/Sqrt[x])])^p)/(2^p*c^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/Sqrt[x])])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(9^p*c^9*e^10*E^((9*a)/b)*(-((a + b*
Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^2*Gamma[1 + p, (-8*(a + b*Log[c*(d +
e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(8^p*c^8*e^10*E^((8*a)/b
)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (24*d^3*Gamma[1 + p, (-7*(a +
b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(7^p*c^7*e^
10*E^((7*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (7*6^(1 - p)*d^4*
Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqr
t[x])])^p)/(c^6*e^10*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) +
 (252*5^(-1 - p)*d^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e/Sqrt[x])])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p) - (21*2^(1 - 2*p)*d^6*Gamma[1 + p, (-4*(a + b*Log[c*(d
+ e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^4*e^10*E^((4*a)/b)*
(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (8*3^(1 - p)*d^7*Gamma[1 + p, (-
```

```
3*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3
*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^8*Gamma[1
 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])
^p)/(2^p*c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2
*d^9*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(c*e^10*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = -\left(2\,\text{Subst}\left(\int x^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int \left(-\frac{d^9(a+b\log(c(d+ex)))^p}{e^9} + \frac{9d^8(d+ex)(a+b\log(c(d+ex)))^p}{e^9} - \frac{36d\right.$$

$$= -\frac{2\,\text{Subst}\left(\int (d+ex)^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(18d)\,\text{Subst}\left(\int (d+ex)^8(a+\right.}{e^9}$$

$$= -\frac{2\,\text{Subst}\left(\int x^9(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^{10}} + \frac{(18d)\,\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx,\right.}{e^{10}}$$

$$= -\frac{2\,\text{Subst}\left(\int e^{10x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^{10}e^{10}} + \frac{(18d)\,\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx, x,\right.}{c^9e^{10}}$$

$$= -\frac{2^{-p}5^{-1-p}e^{-\frac{10a}{b}}\Gamma\left(1+p, -\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.}{b}\right.}{c^{10}e^{10}}$$

Mathematica [A] time = 3.25503, size = 525, normalized size = 0.57

$$\frac{5^{-p-1}504^{-p}e^{-\frac{10a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\left(cde^{a/b}\left(2^{3p+1}5^{p+1}7^p\text{Gamma}\left(p+1, -\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] (5^(-1 - p)*(-(252^p*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b])
+ c*d*E^(a/b)*(2^(1 + 3*p)*5^(1 + p)*7^p*Gamma[1 + p, (-9*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + c*d*E^(a/b)*(-(7^p*45^(1 + p)*Gamma[1 + p, (-8*(a + b*L
og[c*(d + e/Sqrt[x])]))/b]) + 2^p*c*d*E^(a/b)*(2^(3 + 2*p)*3^(1 + 2*p)*5^(1
 + p)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 7^p*c*d*E^(a/b)
*(-7*30^(1 + p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + c*d*E
^(a/b)*(7*36^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b] +
3^p*5^(1 + p)*c*d*E^(a/b)*(-14*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(3*2^(3 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e/Sqrt[x])]))/b] + 3^p*c*d*E^(a/b)*(-9*Gamma[1 + p, (-2*(a + b*Log
[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[
c*(d + e/Sqrt[x])])/b)])))))))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(504^p*c
^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

3.550 $\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Rubi [A] time = 0.0468434, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p \, dx = 2 \, \text{Subst}\left(\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x}\right)^2\right)\right)^p \, dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.225437, size = 0, normalized size = 0.

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Maple [A] time = 0.607, size = 0, normalized size = 0.

$$\int x \left(a+b \ln \left(c \left(d+e\frac{1}{\sqrt{x}}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^2\right) + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log \left(\frac{cd^2x + 2\, cde\sqrt{x} + ce^2}{x}\right) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^2\right) + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

3.551 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Rubi [A] time = 0.0225073, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=2\,\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.104124, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Maple [A] time = 0.327, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

3.552 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x, x]
```

Rubi [A] time = 0.0598604, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]

[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.164973, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]

[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x, x]
```

Maple [A] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

3.553 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=213

$$\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}-\frac{e^{-\frac{a}{b}}\left(a+b\log\left(\right.\right.}{}$$

```
[Out] -((Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p))
+ (2^(1 + p)*d*(d + e/Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2
])/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d +
e/Sqrt[x])^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rubi [A] time = 0.287876, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}-\frac{e^{-\frac{a}{b}}\left(a+b\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p))
+ (2^(1 + p)*d*(d + e/Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2
])/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d +
e/Sqrt[x])^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}\right)\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\right.\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\left(cx^2\right)\right)^p\right.}{e^2}$$

$$= -\frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{ce^2}+\frac{\left(d\left(d+\frac{e}{\sqrt{x}}\right)\right)\text{Subst}\left(\int e^{x/2}(a+\right.}{e^2\sqrt{c\left(d\right.}}$$

$$= -\frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^2}$$

Mathematica [F] time = 0.139233, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2, x]
```

Maple [F] time = 0.334, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

3.554 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=676

result too large to display

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b
*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(5^p
*e^6*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqr
t[x])^2])/b))^p) - (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b
*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e/Sqrt
[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log
[c*(d + e/Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*
(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (5*d^4*Gamma[1 + p, -((a + b*L
og[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^6*E^(
a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^5*(d + e/Sqr
t[x])*Gamma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d
 + e/Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*L
og[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rubi [A] time = 0.995055, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$-\frac{5d^2 2^{-p} e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)}{c^2e^6}-\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\,\mathrm{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
```


```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b
*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(5^p
*e^6*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqr
t[x])^2])/b))^p) - (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b
*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e/Sqrt
[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log
[c*(d + e/Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*
(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (5*d^4*Gamma[1 + p, -((a + b*L
og[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^6*E^(
a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^5*(d + e/Sqr
t[x])*Gamma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d
 + e/Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*L
og[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx = -\left(2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5} + \frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(10d)\,\text{Subst}\left(\int (d+ex)^4\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(10d)\,\text{Subst}\left(\int x^4\left(a+b\log\left(cx^2\right)\right.\right.}{e^6}$$

$$= -\frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^3e^6} - \frac{\left(10d^2\right)\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x\right.}{c^2e^6}$$

$$= -\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right.}{b}\right.}{c^3e^6}$$

Mathematica [F] time = 0.143448, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4, x]
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

3.555 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$

Optimal. Leaf size=1141

result too large to display

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^9*Gamma[1 + p, (-9*(a +
b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(9^
p*e^10*E^((9*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(9/2)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p) - (9*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqrt[x])^2
]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(4^p*c^4*e^10*E^((4*a)/b)*(-((a
+ b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (3*2^(3 + p)*d^3*(d + e/Sqrt[x])^7*G
amma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d +
e/Sqrt[x])^2])^p)/(7^p*e^10*E^((7*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(7/2)*(-(
(a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (14*3^(1 - p)*d^4*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/
(c^3*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (63*2^(2
 + p)*5^(-1 - p)*d^5*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e
/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^10*E^((5*a)/(2
*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
 - (21*2^(1 - p)*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*
(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(
d + e/Sqrt[x])^2])/b))^p) + (2^(3 + p)*3^(1 - p)*d^7*(d + e/Sqrt[x])^3*Gamm
a[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/S
qrt[x])^2])^p)/(e^10*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*(-((a + b*
Log[c*(d + e/Sqrt[x])^2])/b))^p) - (9*d^8*Gamma[1 + p, -((a + b*Log[c*(d +
e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^10*E^(a/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^9*(d + e/Sqrt[x])*Gam
ma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d + e/Sqrt
[x])^2])^p)/(e^10*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*Log[c*(d
+ e/Sqrt[x])^2])/b))^p)
```

Rubi [A] time = 1.73843, antiderivative size = 1141, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
```

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^9*Gamma[1 + p, (-9*(a +
b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(9^
p*e^10*E^((9*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(9/2)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p) - (9*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqrt[x])^2
]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(4^p*c^4*e^10*E^((4*a)/b)*(-((a
+ b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (3*2^(3 + p)*d^3*(d + e/Sqrt[x])^7*G
amma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d +
e/Sqrt[x])^2])^p)/(7^p*e^10*E^((7*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(7/2)*(-(
```

```
(a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (14*3^(1 - p)*d^4*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/
(c^3*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (63*2^(2
 + p)*5^(-1 - p)*d^5*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e
/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^10*E^((5*a)/(2
*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
 - (21*2^(1 - p)*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*
(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(
d + e/Sqrt[x])^2])/b))^p) + (2^(3 + p)*3^(1 - p)*d^7*(d + e/Sqrt[x])^3*Gamm
a[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/S
qrt[x])^2])^p)/(e^10*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*(-((a + b*
Log[c*(d + e/Sqrt[x])^2])/b))^p) - (9*d^8*Gamma[1 + p, -((a + b*Log[c*(d +
e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^10*E^(a/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^9*(d + e/Sqrt[x])*Gam
ma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d + e/Sqrt
[x])^2])^p)/(e^10*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*Log[c*(d
+ e/Sqrt[x])^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = -\left(2\,\text{Subst}\left(\int x^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^9} + \frac{9d^8(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^9}\right.\right.\right.$$
$$= -\frac{2\,\text{Subst}\left(\int(d+ex)^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(18d)\,\text{Subst}\left(\int(d+ex)^8\right.}{}$$
$$= -\frac{2\,\text{Subst}\left(\int x^9\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^{10}} + \frac{(18d)\,\text{Subst}\left(\int x^8\left(a+b\log\left(cx^2\right)\right.\right.}{e^{10}}$$
$$= -\frac{\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^5e^{10}} - \frac{\left(36d^2\right)\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x\right.}{c^4e^{10}}$$
$$= -\frac{5^{-1-p}e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right.}{c^5e^{10}}$$

Mathematica [F] time = 0.132578, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6, x]
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

3.556 $\int x^3 \left(a + b \log \left(c \left(d + e\sqrt[3]{x}\right)\right)\right)^p \, dx$

Optimal. Leaf size=1121

result too large to display

```
[Out] (4^(-1 - p)*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(3^p*c^12*e^12*E^((12*a)/b)*(-((a + b*Log[c*(d + e*
x^(1/3))])/b))^p) - (3*d*Gamma[1 + p, (-11*(a + b*Log[c*(d + e*x^(1/3))]))/
b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(11^p*c^11*e^12*E^((11*a)/b)*(-((a + b
*Log[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^2*Gamma[1 + p, (-10*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^10*
e^12*E^((10*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (55*d^3*Gamma[
1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))]
)^p)/(9^p*c^9*e^12*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (
495*2^(-2 - 3*p)*d^4*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(c^8*e^12*E^((8*a)/b)*(-((a + b*Log[c*(d +
e*x^(1/3))])/b))^p) - (198*d^5*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^12*E^((7*a)/b)*(-((a +
 b*Log[c*(d + e*x^(1/3))])/b))^p) + (77*3^(1 - p)*d^6*Gamma[1 + p, (-6*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(2^p*c^6*e
^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (198*d^7*Gamma[1
 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])
^p)/(5^p*c^5*e^12*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (4
95*4^(-1 - p)*d^8*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x
^(1/3))])/b))^p) - (55*d^9*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))
/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^12*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^10*Gamma[1 + p, (-2*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^12*E^
((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (3*d^11*Gamma[1 + p, -
((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^
12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rubi [A] time = 1.86769, antiderivative size = 1121, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```


```
[Out] (4^(-1 - p)*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(3^p*c^12*e^12*E^((12*a)/b)*(-((a + b*Log[c*(d + e*
x^(1/3))])/b))^p) - (3*d*Gamma[1 + p, (-11*(a + b*Log[c*(d + e*x^(1/3))]))/
b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(11^p*c^11*e^12*E^((11*a)/b)*(-((a + b
*Log[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^2*Gamma[1 + p, (-10*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^10*
e^12*E^((10*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (55*d^3*Gamma[
1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))]
)^p)/(9^p*c^9*e^12*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (
495*2^(-2 - 3*p)*d^4*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(c^8*e^12*E^((8*a)/b)*(-((a + b*Log[c*(d +
e*x^(1/3))])/b))^p) - (198*d^5*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
```

```
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^12*E^((7*a)/b)*(-((a +
 b*Log[c*(d + e*x^(1/3))])/b))^p) + (77*3^(1 - p)*d^6*Gamma[1 + p, (-6*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(2^p*c^6*e
^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (198*d^7*Gamma[1
 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])
^p)/(5^p*c^5*e^12*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (4
95*4^(-1 - p)*d^8*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x
^(1/3))])/b))^p) - (55*d^9*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))
/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^12*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^10*Gamma[1 + p, (-2*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^12*E^
((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (3*d^11*Gamma[1 + p, -
((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^
12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(-\frac{d^{11}(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{11d^{10}(d+ex)(a+b\log(c(d+ex)))^p}{e^{11}}-\cdots\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^{11}(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}}-\frac{(33d)\,\text{Subst}\left(\int(d+ex)^{10}(\cdots\right.}{\cdots}$$

$$= \frac{3\,\text{Subst}\left(\int x^{11}(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^{12}}-\frac{(33d)\,\text{Subst}\left(\int x^{10}(a+b\log(cx))^{\cdots}\right.}{e^{12}}$$

$$= \frac{3\,\text{Subst}\left(\int e^{12x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{12}e^{12}}-\frac{(33d)\,\text{Subst}\left(\int e^{11x}(a+bx)^p\,d\cdots\right.}{c^{11}e^{12}}$$

$$= \frac{3^{-p}4^{-1-p}e^{-\frac{12a}{b}}\Gamma\left(1+p,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(d\cdots}{b}\right.}{c^{12}e^{12}}$$

Mathematica [A] time = 2.66171, size = 670, normalized size = 0.6

$$-\frac{2^{-3p-2}3465^{-p}e^{-\frac{12a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(c^2d^2e^{\frac{2a}{b}}\left(c^2d^2e^{\frac{2a}{b}}\left(c^7d^7 2^{3p+2}3^{2p+1}385^p e^{\frac{7a}{b}}\text{Gamma}\left(\cdots\right.\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] -((2^(-2 - 3*p)*(-(2310^p*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))
/b]) + 2^(2 + 3*p)*3^(1 + 2*p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-11*(a + b*Lo
g[c*(d + e*x^(1/3))]))/b] + c^2*d^2*E^((2*a)/b)*(-(6^(1 + 2*p)*7^p*11^(1 +
p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e*x^(1/3))]))/b]) + 2^(2 + 3*p)*7^p*
55^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]
+ c^2*d^2*E^((2*a)/b)*(-(7^p*495^(1 + p)*Gamma[1 + p, (-8*(a + b*Log[c*(d +
 e*x^(1/3))]))/b]) + 5^p*792^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))]))/b] - 5^p*924^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p
, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 7^p*792^(1 + p)*c^3*d^3*E^((3*a)
/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b] - 14^p*495^(1 + p)*
c^4*d^4*E^((4*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2
^(2 + 3*p)*21^p*55^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-3*(a + b*Log[
c*(d + e*x^(1/3))]))/b] - 6^(1 + 2*p)*11^(1 + p)*35^p*c^6*d^6*E^((6*a)/b)*G
amma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(2 + 3*p)*3^(1 + 2*p
)*385^p*c^7*d^7*E^((7*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b
)]))))*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3465^p*c^12*e^12*E^((12*a)/b)*(-((
a + b*Log[c*(d + e*x^(1/3))])/b))^p))
```

Maple [F] time = 0.484, size = 0, normalized size = 0.

$$\int x^3 \left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^{p}x^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

3.557 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=831

result too large to display

```
[Out] (3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) - (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(d +
 e*x^(1/3))])/b))^p) + (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a +
b*Log[c*(d + e*x^(1/3))])/b))^p) - (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^9
*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (42*d^4*Gamma[1 + p
, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/
(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (21*2^(
1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p) + (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) - (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*(d
+ e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (3*d^8*Gamma[1 + p, -((a + b*Log[
c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^9*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rubi [A] time = 1.35217, antiderivative size = 831, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3^{-2p-1}e^{-\frac{9a}{b}}\text{Gamma}\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^9}-\frac{3\,8^{-p}de^{-\frac{8a}{b}}\text{Gamma}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) - (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(d +
 e*x^(1/3))])/b))^p) + (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a +
b*Log[c*(d + e*x^(1/3))])/b))^p) - (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^9
*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (42*d^4*Gamma[1 + p
, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/
(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (21*2^(
1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p) + (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) - (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*(d
+ e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b)*(
```

```
-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (3*d^8*Gamma[1 + p, -((a + b*Log[
c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^9*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)$$
$$=3\,\text{Subst}\left(\int\left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8}-\frac{8d^7(d+ex)(a+b\log(c(d+ex)))^p}{e^8}+\frac{28d^6}{}\right.\right.$$
$$=\frac{3\,\text{Subst}\left(\int(d+ex)^8(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^8}-\frac{(24d)\,\text{Subst}\left(\int(d+ex)^7(a\right.}{}$$
$$=\frac{3\,\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^9}-\frac{(24d)\,\text{Subst}\left(\int x^7(a+b\log(cx))^p\,d\right.}{e^9}$$
$$=\frac{3\,\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^9e^9}-\frac{(24d)\,\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx,\right.}{c^8e^9}$$
$$=\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right.}{c^9e^9}$$

Mathematica [A] time = 0.972082, size = 501, normalized size = 0.6

$$\frac{3^{-2p-1}280^{-p}e^{-\frac{9a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(c^8d^89^{p+1}280^pe^{\frac{8a}{b}}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)-c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b] -
9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))
/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1 +
 p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*c^4*d^4
*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b] - 5^p*126^
(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))])
)/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a)/b)*Gamma[
1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 9^(1 + p)*280^p*c^8*d^8*E^(
(8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)])*(a + b*Log[c*(d
 + e*x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^{p}x^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

3.558 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=553

$$\frac{15d^2 2^{-2p-1} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^4e^6} - \frac{10d^3 3^{-p} e^{-\frac{3a}{b}} \left(a+\right.}{}$$

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[
c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) + (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^6*E^((4*a)/
b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (10*d^3*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (15*2^(-1 - p)*d^
4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x
^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
- (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d
+ e*x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rubi [A] time = 0.845662, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{15d^2 2^{-2p-1} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^4e^6} - \frac{10d^3 3^{-p} e^{-\frac{3a}{b}} \left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[
c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) + (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^6*E^((4*a)/
b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (10*d^3*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (15*2^(-1 - p)*d^
4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x
^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
- (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d
+ e*x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx &= 3\,\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\,\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}-1\ldots\right.\right.\\
&= \frac{3\,\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5}-\frac{(15d)\,\text{Subst}\left(\int(d+ex)^4(\ldots\right.}{\ldots}\\
&= \frac{3\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}-\frac{(15d)\,\text{Subst}\left(\int x^4(a+b\log(cx))\ldots\right.}{e^6}\\
&= \frac{3\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^6e^6}-\frac{(15d)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,d\ldots\right.}{c^5e^6}\\
&= \frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(\ldots}{b}\right.}{c^6e^6}
\end{aligned}
$$

Mathematica [A] time = 0.910723, size = 325, normalized size = 0.59

$$2^{-2p-1}15^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(10^p\text{Gamma}\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{2p+1}3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (2^(-1 - 2*p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] - c
*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1
/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d
 + e*x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*
Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a
 + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a
+ b*Log[c*(d + e*x^(1/3))])/b)])))))*(a + b*Log[c*(d + e*x^(1/3))])^p)/(15^
p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(\left(ex^{\frac{1}{3}} + d\right)c\right) + a\right)^{p} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

3.559 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=266

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^3}-\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(\cdots\right.\right.\right.}{\cdots}$$

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^
(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^
p) - (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*
(d + e*x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p)
```

Rubi [A] time = 0.393357, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2451, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^3}-\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^
(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^
p) - (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*
(d + e*x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p)
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = 3\,\mathrm{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\mathrm{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2} - \frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2} + \frac{(d+$$
$$= \frac{3\,\mathrm{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^2} - \frac{(6d)\,\mathrm{Subst}\left(\int (d+ex)(a+\right.}{}$$
$$= \frac{3\,\mathrm{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^3} - \frac{(6d)\,\mathrm{Subst}\left(\int x(a+b\log(cx))^p\,dx\right.}{e^3}$$
$$= \frac{3\,\mathrm{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^3e^3} - \frac{(6d)\,\mathrm{Subst}\left(\int e^{2x}(a+bx)^p\,dx,\right.}{c^2e^3}$$
$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right.}{c^3e^3}$$

Mathematica [A] time = 0.194662, size = 174, normalized size = 0.65

$$\frac{6^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(2^p\mathrm{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+cd3^{p+1}e^{a/b}\left(cd2^pe^{a/}\right.\right.}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] ((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*E
^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^p*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]))*(a + b*Log[c*(d + e
```

```
*x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b
))^p)
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \left(a+b\ln \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log \left(cex^{\frac{1}{3}}+cd\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

3.560 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(1/3))])^p/x, x]
```

Rubi [A] time = 0.0520717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.248104, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x, x]
```

Maple [A] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^{p}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

3.561 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2, x]
```

Rubi [A] time = 0.0520194, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.359465, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2, x]
```

Maple [A] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

3.562 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=1363

result too large to display

```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))^2])^p)/(3^p*c^6*e^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) - (3*(2/11)^p*d*(d + e*x^(1/3))^11*Gamma[1 + p, (-11*(
a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)
/(e^12*E^((11*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(11/2)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) + (33*d^2*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3)
)^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*5^p*c^5*e^12*E^((5*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/9)^p*d^3*(d + e*x^(1/3))
^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(
d + e*x^(1/3))^2])^p)/(e^12*E^((9*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(9/2)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (495*d^4*Gamma[1 + p, (-4*(a + b*
Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2^(2*(1
+ p))*c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (99
*2^(1 + p)*d^5*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/
3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^12*E^((7*a)/(2*b
))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) +
 (77*3^(1 - p)*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^3*e^12*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e*x^(1/3))^2])/b))^p) - (99*2^(1 + p)*d^7*(d + e*x^(1/3))^5*Gamma[1 + p,
(-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2
])^p)/(5^p*e^12*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c
*(d + e*x^(1/3))^2])/b))^p) + (495*2^(-2 - p)*d^8*Gamma[1 + p, (-2*(a + b*L
og[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^12*
E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/3)^p*d^9*(d
 + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))
^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (33*d^10*Gamma[1 + p
, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p
)/(2*c*e^12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^1
1*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(a
+ b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2
]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rubi [A] time = 2.13277, antiderivative size = 1363, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```


```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))^2])^p)/(3^p*c^6*e^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) - (3*(2/11)^p*d*(d + e*x^(1/3))^11*Gamma[1 + p, (-11*(
a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)
/(e^12*E^((11*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(11/2)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) + (33*d^2*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3)
)^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*5^p*c^5*e^12*E^((5*a)/b)*(
```

```
-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/9)^p*d^3*(d + e*x^(1/3))
^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(
d + e*x^(1/3))^2])^p)/(e^12*E^((9*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(9/2)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (495*d^4*Gamma[1 + p, (-4*(a + b*
Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2^(2*(1
+ p))*c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (99
*2^(1 + p)*d^5*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/
3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^12*E^((7*a)/(2*b
))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) +
 (77*3^(1 - p)*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^3*e^12*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e*x^(1/3))^2])/b))^p) - (99*2^(1 + p)*d^7*(d + e*x^(1/3))^5*Gamma[1 + p,
(-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2
])^p)/(5^p*e^12*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c
*(d + e*x^(1/3))^2])/b))^p) + (495*2^(-2 - p)*d^8*Gamma[1 + p, (-2*(a + b*L
og[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^12*
E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/3)^p*d^9*(d
 + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))
^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (33*d^10*Gamma[1 + p
, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p
)/(2*c*e^12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^1
1*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(a
+ b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2
]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
```

```
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(-\frac{d^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^{11}}+\frac{11d^{10}(d+ex)\left(a+b\log\left(c(d+ex)^2\right.\right.}{e^{11}}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}}-\frac{(33d)\,\text{Subst}\left(\int(d+e\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^{12}}-\frac{(33d)\,\text{Subst}\left(\int x^{10}\left(a+b\log\right.\right.}{e^{12}}$$

$$= \frac{3\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^6e^{12}}+\frac{\left(165d^2\right)\text{Subst}\left(\int e^{5x}(a+bx\right.}{2c^5}$$

$$= \frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log(c}{}\right.}{c^6e^{12}}$$

Mathematica [F] time = 0.649893, size = 0, normalized size = 0.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F] time = 0.576, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(\left(ex^{\frac{1}{3}} + d\right)^{2} c\right) + a\right)^{p} x^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \log \left(ce^{2} x^{\frac{2}{3}} + 2 \, cdex^{\frac{1}{3}} + cd^{2}\right) + a\right)^{p} x^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(\left(ex^{\frac{1}{3}} + d\right)^{2} c\right) + a\right)^{p} x^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

3.563 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=1035

result too large to display

```
[Out] (2^p*3^(-1 - 2*p)*(d + e*x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^
(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b))
*(c*(d + e*x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (
3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d +
 e*x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))
^2])/b))^p) + (3*2^(2 + p)*d^2*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^
9*E^((7*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3
))^2])/b))^p) - (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/
b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (21*2^(1 + p)*d^4*(d + e*x^(1/3))^5*Gamma
[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^
(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a +
b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*(a
+ b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*
e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (7*2^(2 + p)*d
^6*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*
b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d + e*
x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (12*d^7*Gamm
a[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p
*d^8*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*
(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))
^2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rubi [A] time = 1.56564, antiderivative size = 1035, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```


```
[Out] (2^p*3^(-1 - 2*p)*(d + e*x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^
(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b))
*(c*(d + e*x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (
3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d +
 e*x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))
^2])/b))^p) + (3*2^(2 + p)*d^2*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^
9*E^((7*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3
))^2])/b))^p) - (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/
b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (21*2^(1 + p)*d^4*(d + e*x^(1/3))^5*Gamma
[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^
(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a +
b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*(a
+ b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*
```

```
e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (7*2^(2 + p)*d
^6*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*
b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d + e*
x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (12*d^7*Gamm
a[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p
*d^8*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*
(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))
^2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx = 3\,\text{Subst}\left(\int x^8 \left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{d^8 \left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8} - \frac{8d^7(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8} + \cdots\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int (d+ex)^8 \left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d)\,\text{Subst}\left(\int (d+ex \cdots\right.}{\cdots}$$

$$= \frac{3\,\text{Subst}\left(\int x^8 \left(a+b\log\left(cx^2\right)\right)^p dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(24d)\,\text{Subst}\left(\int x^7 \left(a+b\log\left(\cdots\right.\right.\right.}{e^9}$$

$$= -\frac{(12d)\,\text{Subst}\left(\int e^{4x}(a+bx)^p dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^4 e^9} - \frac{\left(84d^3\right)\text{Subst}\left(\int e^{3x}(a \cdots\right.}{\cdots}$$

$$= \frac{2^p 3^{-1-2p} e^{-\frac{9a}{2b}} \left(d+e\sqrt[3]{x}\right)^9 \Gamma\left(1+p, -\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)\cdots}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}$$

Mathematica [F] time = 0.440129, size = 0, normalized size = 0.

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x^2 \left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2 c\right)+a\right)^p x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{2}{3}} + 2\,cdex^{\frac{1}{3}} + cd^2\right) + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}} + d\right)^2 c\right) + a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

3.564 $\int x \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=673

result too large to display

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^
2])/b))^p) - (3*(2/5)^p*d*(d + e*x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(
d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^6*E^((5*a
)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b)
)^p) + (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))^2])
)/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log
[c*(d + e*x^(1/3))^2])/b))^p) - (5*2^(1 + p)*d^3*(d + e*x^(1/3))^3*Gamma[1
+ p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/
3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(3/2)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^5*(d + e*x^(1/3))*Gamma[
1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2]*(-((a + b*Log[c*(d + e*
x^(1/3))^2])/b))^p)
```

Rubi [A] time = 0.971375, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{15 d^2 2^{-p-1} e^{-\frac{2a}{b}} \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)}{c^2 e^6} + \frac{3^{-p} e^{-\frac{3a}{b}} \left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```


```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^
2])/b))^p) - (3*(2/5)^p*d*(d + e*x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(
d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^6*E^((5*a
)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b)
)^p) + (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))^2])
)/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log
[c*(d + e*x^(1/3))^2])/b))^p) - (5*2^(1 + p)*d^3*(d + e*x^(1/3))^3*Gamma[1
+ p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/
3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(3/2)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^5*(d + e*x^(1/3))*Gamma[
1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2]*(-((a + b*Log[c*(d + e*
x^(1/3))^2])/b))^p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\mathrm{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right.\right.$$
$$= \frac{3\,\mathrm{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^5} - \frac{(15d)\,\mathrm{Subst}\left(\int (d+ex)^4\right.}{}$$
$$= \frac{3\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} - \frac{(15d)\,\mathrm{Subst}\left(\int x^4\left(a+b\log\left(cx\right.\right.\right.}{e^6}$$
$$= \frac{3\,\mathrm{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^3e^6} + \frac{(15d^2)\,\mathrm{Subst}\left(\int e^{2x}(a+bx)^p\right.}{c^2e^6}$$
$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{2c^3e^6}$$

Mathematica [F] time = 0.302593, size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{2}{3}} + 2\,cdex^{\frac{1}{3}} + cd^2\right) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}} + d\right)^2 c\right) + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)
```

3.565 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=338

$$\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}+\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(\right.}{}$$

```
[Out] ((2/3)^p*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]
))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d +
e*x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*d*Gamma
[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))
^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p*
d^2*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^
2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rubi [A] time = 0.459609, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2451, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}+\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```



```
[Out] ((2/3)^p*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]
))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d +
e*x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*d*Gamma
[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))
^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p*
d^2*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^
2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```


Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} + \right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^2} - \frac{(6d)\,\text{Subst}\left(\int (d+ex)\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^3} - \frac{(6d)\,\text{Subst}\left(\int x\left(a+b\log\left(cx\right.\right.\right.}{e^3}$$

$$= -\frac{(3d)\,\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{ce^3} + \frac{\left(3\left(d+e\sqrt[3]{x}\right)^3\right)\text{Subst}\left(\int\right.}{2e^3}$$

$$= \frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(\right.}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

Mathematica [F] time = 0.126521, size = 0, normalized size = 0.

$$\int \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \left(a + b \ln \left(c \left(d + e \sqrt[3]{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(\left(ex^{\frac{1}{3}} + d\right)^2 c\right) + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log \left(ce^2 x^{\frac{2}{3}} + 2\, cdex^{\frac{1}{3}} + cd^2\right) + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

3.566 $\int \frac{\left(a+b \log\left(c\left(d+e \sqrt[3]{x}\right)^2\right)\right)^p}{x} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x, x]
```

Rubi [A] time = 0.0540624, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.11715, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x, x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(ce^{2}x^{\frac{2}{3}}+2\,cdex^{\frac{1}{3}}+cd^{2}\right)+a\right)^{p}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

3.567 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2, x]
```

Rubi [A] time = 0.0537709, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.115935, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(ce^{2}x^{\frac{2}{3}}+2\,cdex^{\frac{1}{3}}+cd^{2}\right)+a\right)^{p}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

3.568 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=557

$$\frac{15d^2 2^{-2(p+1)}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^4e^6}-\frac{5d^3 3^{-p}e^{-\frac{3a}{b}}\left(a\right.}{}$$

```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[
c*(d + e*x^(2/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a
 + b*Log[c*(d + e*x^(2/3))])^p)/(2*5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e*x^(2/3))])/b))^p) + (15*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(
2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2^(2*(1 + p))*c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) - (5*d^3*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(3^p*c^3
*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) + (15*2^(-2 - p)*
d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e
*x^(2/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p
) - (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]*(a + b*Log[c*(
d + e*x^(2/3))])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^
p)
```

Rubi [A] time = 0.868733, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{15d^2 2^{-2(p+1)}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^4e^6}-\frac{5d^3 3^{-p}e^{-\frac{3a}{b}}\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```


```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[
c*(d + e*x^(2/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a
 + b*Log[c*(d + e*x^(2/3))])^p)/(2*5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e*x^(2/3))])/b))^p) + (15*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(
2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2^(2*(1 + p))*c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) - (5*d^3*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(3^p*c^3
*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) + (15*2^(-2 - p)*
d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e
*x^(2/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p
) - (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]*(a + b*Log[c*(
d + e*x^(2/3))])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^
p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = \frac{3}{2}\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)$$
$$= \frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}-10\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{2e^5}-\frac{(15d)\,\text{Subst}\left(\int(d+ex)^4(a\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{2e^6}-\frac{(15d)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\right.}{2e^6}$$
$$= \frac{3\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^6e^6}-\frac{(15d)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx\right.}{2c^5e^6}$$
$$= \frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log(c(d\right.}{b}}{c^6e^6}$$

Mathematica [A] time = 0.919903, size = 325, normalized size = 0.58

$$\frac{4^{-p-1}15^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\left(10^p\text{Gamma}\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{2p+1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (4^(-1 - p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b] - c*d
*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3
))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e*x^(2/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*x^(2/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a +
 b*Log[c*(d + e*x^(2/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a +
b*Log[c*(d + e*x^(2/3))])/b)])))))*(a + b*Log[c*(d + e*x^(2/3))])^p)/(15^p*
c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p)
```

Maple [F] time = 0.475, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(cex^{\frac{2}{3}} + cd\right) + a\right)^{p} x^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^{p} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

3.569 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=273

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{2c^3e^3}-\frac{3d2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(\right.\right.}{}$$

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^
(2/3))])^p)/(2*3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b)
)^p) - (3*2^(-1 - p)*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*
(a + b*Log[c*(d + e*x^(2/3))])^p)/(c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d +
 e*x^(2/3))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])
/b)]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d
+ e*x^(2/3))])/b))^p)
```

Rubi [A] time = 0.38078, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{2c^3e^3}-\frac{3d2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^
(2/3))])^p)/(2*3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b)
)^p) - (3*2^(-1 - p)*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*
(a + b*Log[c*(d + e*x^(2/3))])^p)/(c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d +
 e*x^(2/3))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])
/b)]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d
+ e*x^(2/3))])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\frac{3}{2}\operatorname{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)$$
$$=\frac{3}{2}\operatorname{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2}+\frac{(d$$
$$=\frac{3\operatorname{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{2e^2}-\frac{(3d)\operatorname{Subst}\left(\int(d+ex)(a\right.}{}$$
$$=\frac{3\operatorname{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{2e^3}-\frac{(3d)\operatorname{Subst}\left(\int x(a+b\log(cx))^p\right.}{e^3}$$
$$=\frac{3\operatorname{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^3e^3}-\frac{(3d)\operatorname{Subst}\left(\int e^{2x}(a+bx)^p\,d\right.}{c^2e^3}$$
$$=\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/\right.\right.}{b}\right.}{2c^3e^3}$$

Mathematica [A] time = 0.236596, size = 181, normalized size = 0.66

$$\frac{2^{-p-1}3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\left(2^p\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+cd3^{p+1}e^{a/b}\left(\right.\right.}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (2^(-1 - p)*(2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b] + 3^(1
 + p)*c*d*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b] + 2
^p*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]))*(a + b*L
```

```
og[c*(d + e*x^(2/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x
^(2/3))])/b))^p)
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^{p}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

3.570 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))])^p/x, x]
```

Rubi [A] time = 0.0527489, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.284578, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x, x]
```

Maple [A] time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

3.571 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3, x]
```

Rubi [A] time = 0.0530978, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^7, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.386046, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3, x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^{p}}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

3.572 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Rubi [A] time = 0.0547608, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.719293, size = 0, normalized size = 0.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int x^2\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^{p}x^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

3.573 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Rubi [A] time = 0.0319482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.169608, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [A] time = 0.09, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

3.574 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2, x]
```

Rubi [A] time = 0.0533673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.466748, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

3.575 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx$

Optimal. Leaf size=675

$$\frac{15d^2 2^{-p-2} e^{-\frac{2a}{b}} \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{c^2e^6} + \frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\right.}{}$$

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(4*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))^
2])/b))^p) - (3*2^(-1 + p)*d*(d + e*x^(2/3))^5*Gamma[1 + p, (-5*(a + b*Log[
c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(5^p*e^6*
E^((5*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(2/3))
^2])/b))^p) + (15*2^(-2 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/
3))^2]))/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a
+ b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (5*(2/3)^p*d^3*(d + e*x^(2/3))^3*Gam
ma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*
Log[c*(d + e*x^(2/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
 e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(4*c*e^6*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*2^(-1 + p)*d^5*(d + e*x^(2/3)
)*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(2/3))^2])/(2*b)]*(a + b*Log[c*(d + e
*x^(2/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(2/3))^2]*(-((a + b*Log[c
*(d + e*x^(2/3))^2])/b))^p)
```

Rubi [A] time = 0.970561, antiderivative size = 675, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{15d^2 2^{-p-2} e^{-\frac{2a}{b}} \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{c^2e^6} + \frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```


```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(4*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))^
2])/b))^p) - (3*2^(-1 + p)*d*(d + e*x^(2/3))^5*Gamma[1 + p, (-5*(a + b*Log[
c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(5^p*e^6*
E^((5*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(2/3))
^2])/b))^p) + (15*2^(-2 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/
3))^2]))/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a
+ b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (5*(2/3)^p*d^3*(d + e*x^(2/3))^3*Gam
ma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*
Log[c*(d + e*x^(2/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
 e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(4*c*e^6*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*2^(-1 + p)*d^5*(d + e*x^(2/3)
)*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(2/3))^2])/(2*b)]*(a + b*Log[c*(d + e
*x^(2/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(2/3))^2]*(-((a + b*Log[c
```

```
*(d + e*x^(2/3))^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx = \frac{3}{2}\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, x^{2/3}\right)}{2e^5} - \frac{(15d)\,\text{Subst}\left(\int(d+e\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p dx, x, d+ex^{2/3}\right)}{2e^6} - \frac{(15d)\,\text{Subst}\left(\int x^4\left(a+b\log\left(\right.\right.\right.}{2e^6}$$

$$= \frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4c^3e^6} + \frac{\left(15d^2\right)\text{Subst}\left(\int e^{2x}(a+bx)\right.}{2c^2}$$

$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{4c^3e^6}$$

Mathematica [F] time = 0.492355, size = 0, normalized size = 0.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [F] time = 0.594, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{4}{3}} + 2\,cdex^{\frac{2}{3}} + cd^2\right) + a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

3.576 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=347

$$\frac{3d^2 2^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}+\frac{2^{p-1}3^{-p}\cdots}{\cdots}$$

```
[Out] (2^(-1 + p)*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))
^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(3^p*e^3*E^((3*a)/(2*b))*(
c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*
d*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x
^(2/3))^2])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
+ (3*2^(-1 + p)*d^2*(d + e*x^(2/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(2/
3))^2])/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*
(d + e*x^(2/3))^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
```

Rubi [A] time = 0.453242, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{3d^2 2^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}+\frac{2^{p-1}3^{-p}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```


```
[Out] (2^(-1 + p)*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))
^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(3^p*e^3*E^((3*a)/(2*b))*(
c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*
d*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x
^(2/3))^2])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
+ (3*2^(-1 + p)*d^2*(d + e*x^(2/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(2/
3))^2])/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*
(d + e*x^(2/3))^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\frac{3}{2}\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)$$

$$=\frac{3}{2}\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}-\frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}\right.\right.$$

$$=\frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)}{2e^2}-\frac{(3d)\,\text{Subst}\left(\int(d+\right.}{}$$

$$=\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+ex^{2/3}\right)}{2e^3}-\frac{(3d)\,\text{Subst}\left(\int x\left(a+b\log\left(\right.\right.\right.}{e^3}$$

$$=-\frac{(3d)\,\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2ce^3}+\frac{\left(3\left(d+ex^{2/3}\right)^3\right)\text{Subst}}{}$$

$$=\frac{2^{-1+p}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$

Mathematica [F] time = 0.284323, size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)
```

3.577 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x, x]
```

Rubi [A] time = 0.0552981, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.153891, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x, x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(ce^{2}x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^{2}\right)+a\right)^{p}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

3.578 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3, x]
```

Rubi [A] time = 0.0551799, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^7, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^7}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.155079, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3, x]
```

Maple [A] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(\left(ex^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b \log\left(ce^2x^{\frac{4}{3}} + 2\,cdex^{\frac{2}{3}} + cd^2\right) + a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left(\left(ex^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

3.579 $\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Rubi [A] time = 0.0581975, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx = 3 \,\text{Subst}\left(\int x^8 \left(a+b \log \left(c \left(d+e x^2\right)^2\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.172214, size = 0, normalized size = 0.

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int x^2 \left(a+b \ln \left(c \left(d+e x^{\frac{2}{3}}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(ce^{2}x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^{2}\right)+a\right)^{p}x^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

3.580 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Rubi [A] time = 0.033729, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.0904817, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

3.581 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2, x]
```

Rubi [A] time = 0.0561046, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^4, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.123803, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2, x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(\left(ex^{\frac{2}{3}} + d\right)^{2} c\right) + a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b \log \left(ce^{2}x^{\frac{4}{3}} + 2\,cdex^{\frac{2}{3}} + cd^{2}\right) + a\right)^{p}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log \left(\left(ex^{\frac{2}{3}} + d\right)^{2} c\right) + a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

3.582 $\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x*(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Rubi [A] time = 0.0471712, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx=3 \,\text{Subst}\left(\int x^5 \left(a+b \log \left(c \left(d+\frac{e}{x}\right)\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.814806, size = 0, normalized size = 0.

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Maple [A] time = 0.501, size = 0, normalized size = 0.

$$\int x \left(a+b\ln \left(c \left(d+e{\frac{1}{\sqrt[3]{x}}} \right) \right) \right) ^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^{p} x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

3.583 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Rubi [A] time = 0.0333854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.183279, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Maple [A] time = 0.332, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

3.584 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(1/3))])^p/x, x]
```

Rubi [A] time = 0.0545988, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.309246, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x, x]
```

Maple [A] time = 0.335, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

3.585 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=267

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^3}+\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(\right.\right.\right.}{}$$

```
[Out] -((Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/
x^(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b)
)^p)) + (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log
[c*(d + e/x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(
1/3))])/b))^p) - (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)
```

Rubi [A] time = 0.392388, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^3}+\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/
x^(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b)
)^p)) + (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log
[c*(d + e/x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(
1/3))])/b))^p) - (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2} - \frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2} + \frac{(d$$

$$= -\frac{3\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\,\text{Subst}\left(\int (d+ex)(a\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx\right.}{e^3}$$

$$= -\frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^3e^3} + \frac{(6d)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,\right.}{c^2e^3}$$

$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^3e^3}$$

Mathematica [A] time = 0.23436, size = 175, normalized size = 0.66

$$-\frac{6^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\left(2^p\text{Gamma}\left(p+1, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+cd3^{p+1}e^{a/b}\left(cd2^pe^{a/}\right.\right.}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -(((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d
*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^
(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]))*(a + b*Log[c*(d +
 e/x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])
/b))^p))
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

3.586 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=554

$$-\frac{15d^2 2^{-2p-1} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^4e^6}+\frac{10d^3 3^{-p} e^{-\frac{3a}{b}} \left(a+\right.}{}$$

```
[Out] -((2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e/x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^
(1/3))])/b))^p)) + (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Lo
g[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (10*d^3*Gamma[1 + p, (-3*(
a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^
3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - p)
*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d +
e/x^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^
p) + (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p
)
```

Rubi [A] time = 0.833211, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{15d^2 2^{-2p-1} e^{-\frac{4a}{b}} \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^4e^6}+\frac{10d^3 3^{-p} e^{-\frac{3a}{b}} \left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```


```
[Out] -((2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e/x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^
(1/3))])/b))^p)) + (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Lo
g[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (10*d^3*Gamma[1 + p, (-3*(
a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^
3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - p)
*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d +
e/x^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^
p) + (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p
)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5} + \frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5} - \frac{10d}{\ }\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)\,\text{Subst}\left(\int (d+ex)^4(a+\right.}{e}$$

$$= -\frac{3\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx,\right.}{e^6}$$

$$= -\frac{3\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^6e^6} + \frac{(15d)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x,\right.}{c^5e^6}$$

$$= -\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p, -\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^6e^6}$$

Mathematica [A] time = 0.770917, size = 325, normalized size = 0.59

$$\frac{2^{-2p-1}15^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\left(cde^{a/b}\left(2^{2p+1}3^{p+1}\text{Gamma}\left(p+1, -\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+c\right.\right.}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

```
[Out] (2^(-1 - 2*p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b])
+ c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x
^(1/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2
*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -(
(a + b*Log[c*(d + e/x^(1/3))])/b)])))))*(a + b*Log[c*(d + e/x^(1/3))])^p)/(
15^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

3.587 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=832

result too large to display

```
[Out] -((3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)) + (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(
d + e/x^(1/3))])/b))^p) - (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/x^(1
/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a
 + b*Log[c*(d + e/x^(1/3))])/b))^p) + (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a
+ b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^6*
e^9*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (42*d^4*Gamma[1
+ p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^
p)/(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (21*
2^(1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p) - (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) + (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*
(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b
)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (3*d^8*Gamma[1 + p, -((a + b*L
og[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^9*E^(a/b)
*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Rubi [A] time = 1.3026, antiderivative size = 832, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{3^{-2p-1}e^{-\frac{9a}{b}}\text{Gamma}\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^9e^9}+\frac{3\,8^{-p}de^{-\frac{8a}{b}}\text{Gamma}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```


```
[Out] -((3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)) + (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(
d + e/x^(1/3))])/b))^p) - (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/x^(1
/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a
 + b*Log[c*(d + e/x^(1/3))])/b))^p) + (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a
+ b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^6*
e^9*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (42*d^4*Gamma[1
+ p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^
p)/(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (21*
2^(1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p) - (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c
```

```
*(d + e/x^(1/3))])/b))^p) + (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*
(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b
)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (3*d^8*Gamma[1 + p, -((a + b*L
og[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^9*E^(a/b)
*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx = -\left(3\,\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8} - \frac{8d^7(d+ex)(a+b\log(c(d+ex)))^p}{e^8} + \frac{28d^6}{}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d)\,\text{Subst}\left(\int(d+ex)^7(a+\right.}{e}$$

$$= -\frac{3\,\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} + \frac{(24d)\,\text{Subst}\left(\int x^7(a+b\log(cx))^p\,dx,\right.}{e^9}$$

$$= -\frac{3\,\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^9e^9} + \frac{(24d)\,\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx, x,\right.}{c^8e^9}$$

$$= -\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p, -\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^9e^9}$$

Mathematica [A] time = 0.830324, size = 502, normalized size = 0.6

$$-\frac{3^{-2p-1}280^{-p}e^{-\frac{9a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\left(c^8d^89^{p+1}280^pe^{\frac{8a}{b}}\,\text{Gamma}\left(p+1, -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right) - c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

```
[Out] -((3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]
- 9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]
))/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1
 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*c^4*d
^4*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*12
6^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))
]))/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e/x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a)/b)*Gamm
a[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 9^(1 + p)*280^p*c^8*d^8*E
^((8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)])*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p))
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

3.588 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Rubi [A] time = 0.0469373, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.228788, size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Maple [A] time = 0.574, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

3.589 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Rubi [A] time = 0.0350846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.10971, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Maple [A] time = 0.335, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{1}{3}}}\right)^2\right) + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log \left(\frac{cd^2x + 2\,cdex^{\frac{2}{3}} + ce^2x^{\frac{1}{3}}}{x}\right) + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{1}{3}}}\right)^2\right) + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

3.590 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x, x]
```

Rubi [A] time = 0.0569865, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.164801, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x, x]
```

Maple [A] time = 0.329, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cd^{2}x+2\,cdex^{\frac{2}{3}}+ce^{2}x^{\frac{1}{3}}}{x}\right)+a\right)^{p}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

3.591 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=339

$$-\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}-\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\cdots}{\cdots}$$

```
[Out] -(((2/3)^p*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^
2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d
+ e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)) + (3*d*Ga
mma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/
3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*2
^p*d^2*(d + e/x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b)
]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3
))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rubi [A] time = 0.478085, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$-\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}-\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
```


```
[Out] -(((2/3)^p*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^
2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d
+ e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)) + (3*d*Ga
mma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/
3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*2
^p*d^2*(d + e/x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b)
]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3
))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\,\text{Subst}\left(\int(d+ex\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\,\text{Subst}\left(\int x\left(a+b\log\left(cx\right.\right.\right.}{e^3}$$

$$= \frac{(3d)\,\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{ce^3} - \frac{\left(3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\right)\text{Subst}\left(\int e^{3x/}\right.}{2e^3\left(c\right.}$$

$$= -\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\right.}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

Mathematica [F] time = 0.139524, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2, x]
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^2,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

3.592 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=673

result too large to display

```
[Out] -(Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))
^2])/b))^p) + (3*(2/5)^p*d*(d + e/x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*
(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^6*E^((5*
a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b
))^p) - (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e/x^(1/3))^2])/b))^p) + (5*2^(1 + p)*d^3*(d + e/x^(1/3))^3*Gamma[1
 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(3/2)*(-((a + b*
Log[c*(d + e/x^(1/3))^2])/b))^p) - (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
 e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-
((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (3*2^p*d^5*(d + e/x^(1/3))*Gamma
[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b)]*(a + b*Log[c*(d + e/x^(1/3
))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3))^2]*(-((a + b*Log[c*(d + e
/x^(1/3))^2])/b))^p)
```

Rubi [A] time = 0.972384, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$-\frac{15 d^2 2^{-p-1} e^{-\frac{2a}{b}} \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p} \text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)}{c^2 e^6} - \frac{3^{-p} e^{-\frac{3a}{b}} \left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```


```
[Out] -(Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))
^2])/b))^p) + (3*(2/5)^p*d*(d + e/x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*
(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^6*E^((5*
a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b
))^p) - (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e/x^(1/3))^2])/b))^p) + (5*2^(1 + p)*d^3*(d + e/x^(1/3))^3*Gamma[1
 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(3/2)*(-((a + b*
Log[c*(d + e/x^(1/3))^2])/b))^p) - (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
 e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-
((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (3*2^p*d^5*(d + e/x^(1/3))*Gamma
[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b)]*(a + b*Log[c*(d + e/x^(1/3
))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3))^2]*(-((a + b*Log[c*(d + e
/x^(1/3))^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5} + \frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right.\right.}{e^5}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)\,\text{Subst}\left(\int(d+e\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\,\text{Subst}\left(\int x^4\left(a+b\log\left(\right.\right.\right.}{e^6}$$

$$= -\frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2c^3e^6} - \frac{\left(15d^2\right)\text{Subst}\left(\int e^{2x}(a+bx)^p\right.}{c^2e^6}$$

$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.}{b}\right.}{2c^3e^6}$$

Mathematica [F] time = 0.134207, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3, x]
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^3,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

3.593 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=1036

result too large to display

```
[Out] -((2^p*3^(-1 - 2*p)*(d + e/x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/
x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b
))*(c*(d + e/x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p))
+ (3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(
d + e/x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1/
3))^2])/b))^p) - (3*2^(2 + p)*d^2*(d + e/x^(1/3))^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(7^p
*e^9*E^((7*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e/x^(
1/3))^2])/b))^p) + (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a +
b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (21*2^(1 + p)*d^4*(d + e/x^(1/3))^5*Ga
mma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a
 + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*
(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c
^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (7*2^(2 + p
)*d^6*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/
(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d +
 e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (12*d^7*G
amma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*
2^p*d^8*(d + e/x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b
)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/
3))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rubi [A] time = 1.51902, antiderivative size = 1036, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
```

```
[Out] -((2^p*3^(-1 - 2*p)*(d + e/x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/
x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b
))*(c*(d + e/x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p))
+ (3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(
d + e/x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1/
3))^2])/b))^p) - (3*2^(2 + p)*d^2*(d + e/x^(1/3))^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(7^p
*e^9*E^((7*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e/x^(
1/3))^2])/b))^p) + (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a +
b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (21*2^(1 + p)*d^4*(d + e/x^(1/3))^5*Ga
mma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a
```

```
 + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*
(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c
^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (7*2^(2 + p
)*d^6*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/
(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d +
 e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (12*d^7*G
amma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*
2^p*d^8*(d + e/x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b
)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/
3))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = -\left(3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}-\frac{8d^7(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)}{e^8}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt[3]{x}}\right)}{e^8}+\frac{(24d)\,\text{Subst}\left(\int(d+e\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^8\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}+\frac{(24d)\,\text{Subst}\left(\int x^7\left(a+b\log\left(\right.\right.\right.}{e^9}$$

$$= \frac{(12d)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{c^4e^9}+\frac{\left(84d^3\right)\text{Subst}\left(\int e^{3x}(a+b\right.}{c}$$

$$= -\frac{2^p3^{-1-2p}e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}}$$

Mathematica [F] time = 0.132336, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4, x]
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\ln\left(c\left(d+e\frac{1}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^4,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

3.594 $\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [A] time = 0.0561933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)
]
```

Rubi steps

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=3 \text{Subst}\left(\int x^{11} \left(a+b \log \left(c \left(d+\frac{e}{x^2}\right)\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.850858, size = 0, normalized size = 0.

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A] time = 0.505, size = 0, normalized size = 0.

$$\int x^3 \left(a+b \ln \left(c \left(d+e x^{-\frac{2}{3}}\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

3.595 $\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [A] time = 0.0563921, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx = 3 \operatorname{Subst}\left(\int x^8 \left(a+b \log \left(c \left(d+\frac{e}{x^2}\right)\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.721864, size = 0, normalized size = 0.

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A] time = 0.372, size = 0, normalized size = 0.

$$\int x^2 \left(a+b \ln \left(c \left(d+e x^{-\frac{2}{3}}\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^{p}x^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)
```

3.596 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [A] time = 0.0446302, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.554859, size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A] time = 0.319, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log \left(\frac{cdx + cex^{\frac{1}{3}}}{x}\right) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

3.597 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [A] time = 0.0332621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.229584, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A] time = 0.309, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

3.598 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(2/3))])^p/x, x]
```

Rubi [A] time = 0.0543736, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.455853, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x, x]
```

Maple [A] time = 0.309, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

3.599 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2, x]
```

Rubi [A] time = 0.0538691, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.396273, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2, x]
```

Maple [A] time = 0.309, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="maxima")

[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^{p}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="fricas")

[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)

3.600 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [A] time = 0.0606486, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/
3)]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.290518, size = 0, normalized size = 0.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A] time = 0.573, size = 0, normalized size = 0.

$$\int x^3\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right) + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")

[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log \left(\frac{cd^2x^2 + 2\,cdex^{\frac{4}{3}} + ce^2x^{\frac{2}{3}}}{x^2}\right) + a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")

[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right) + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)

3.601 $\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [A] time = 0.0590958, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = 3 \text{Subst}\left(\int x^8 \left(a+b \log \left(c \left(d+\frac{e}{x^2}\right)^2\right)\right)^p dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.204486, size = 0, normalized size = 0.

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A] time = 0.306, size = 0, normalized size = 0.

$$\int x^2 \left(a+b \ln \left(c \left(d+e x^{-\frac{2}{3}}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{2}\right) + a\right)^{p} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \log \left(\frac{cd^{2} x^{2} + 2\, cdex^{\frac{4}{3}} + ce^{2} x^{\frac{2}{3}}}{x^{2}}\right) + a\right)^{p} x^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{2}\right) + a\right)^{p} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

3.602 $\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [A] time = 0.0458282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx=3 \, \text{Subst}\left(\int x^5 \left(a+b \log \left(c \left(d+\frac{e}{x^2}\right)^2\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.241849, size = 0, normalized size = 0.

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A] time = 0.308, size = 0, normalized size = 0.

$$\int x \left(a+b\ln \left(c \left(d+e{x}^{-{\frac{2}{3}}} \right)^{2} \right) \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right) + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log \left(\frac{cd^2x^2 + 2\,cdex^{\frac{4}{3}} + ce^2x^{\frac{2}{3}}}{x^2}\right) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right) + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

3.603 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [A] time = 0.0344933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.113606, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A] time = 0.314, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\log\left(\frac{cd^2x^2+2\,cdex^{\frac{4}{3}}+ce^2x^{\frac{2}{3}}}{x^2}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

3.604 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x, x]
```

Rubi [A] time = 0.0569371, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x} \, dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x} \, dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.164814, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x, x]
```

Maple [A] time = 0.311, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^{2}x^{2}+2\,cdex^{\frac{4}{3}}+ce^{2}x^{\frac{2}{3}}}{x^{2}}\right)+a\right)^{p}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/x
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

3.605 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2, x]
```

Rubi [A] time = 0.0553495, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x^4, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.144324, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2, x]
```

Maple [A] time = 0.312, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\ln\left(c\left(d+ex^{-\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cd^2x^2+2\,cdex^{\frac{4}{3}}+ce^2x^{\frac{2}{3}}}{x^2}\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/x
^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

3.606 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal. Leaf size=631

$$\frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2}+\frac{2af\sqrt{hx}}{h}+\frac{2bf\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h}+\frac{\sqrt{2}bd^{3/4}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{}\right.}{3e^{3/4}\sqrt{h}}$$

```
[Out] (2*a*f*Sqrt[h*x])/h - (8*b*f*p*Sqrt[h*x])/h - (8*b*g*p*(h*x)^(3/2))/(9*h^2)
 - (2*Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt[2]*b
*d^(1/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^
(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*b*f*Sqrt[h*x]*Log[c*(d +
 e*x^2)^p])/h + (2*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2) - (S
qrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (Sqrt[2]*b*d^(3/4)*g*p*Log[Sqrt
[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^
(3/4)*Sqrt[h]) + (Sqrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[
h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (Sqrt[2]*b*d
^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(3*e^(3/4)*Sqrt[h])
```

Rubi [A] time = 0.895228, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2467, 2471, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$\frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2}+\frac{2af\sqrt{hx}}{h}+\frac{2bf\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h}+\frac{\sqrt{2}bd^{3/4}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{}\right.}{3e^{3/4}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```


```
[Out] (2*a*f*Sqrt[h*x])/h - (8*b*f*p*Sqrt[h*x])/h - (8*b*g*p*(h*x)^(3/2))/(9*h^2)
 - (2*Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt[2]*b
*d^(1/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^
(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*b*f*Sqrt[h*x]*Log[c*(d +
 e*x^2)^p])/h + (2*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2) - (S
qrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (Sqrt[2]*b*d^(3/4)*g*p*Log[Sqrt
[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^
(3/4)*Sqrt[h]) + (Sqrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[
h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (Sqrt[2]*b*d
^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(3*e^(3/4)*Sqrt[h])
```


Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
```

```
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2\,\text{Subst}\left(\int\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int\left(f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{gx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int\left(a+\right.\right.}{}$$

$$= \frac{2af\sqrt{hx}}{h} + \frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2} + \frac{(2bf)\,\text{Subst}\left(\int\log\left(c\left(d+\right.\right.\right.}{h}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h} + \frac{2g(hx)^{3/2}\left(a+b\log\left(c\right.\right.}{3h^2}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h} + \frac{2g(hx)^{3/2}}{}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h} + \frac{2g(hx)^{3/2}}{}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h} + \frac{2g(hx)^{3/2}}{}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} - \frac{2\sqrt{2}bd^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} + \frac{2\sqrt{}}{}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} - \frac{2\sqrt{2}b\sqrt[4]{d}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}} - \frac{2\sqrt{}}{}$$

Mathematica [A] time = 0.45806, size = 344, normalized size = 0.55

$$\frac{2\sqrt{x}\left(\frac{1}{3}gx^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+af\sqrt{x}+bf\sqrt{x}\log\left(c\left(d+ex^2\right)^p\right)-\frac{2bgp\left(2\sqrt[4]{-d}e^{3/4}x^{3/2}-3d\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+3d\tanh^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)\right)}{9\sqrt[4]{-d}e^{3/4}}\right.}{\sqrt{hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*Sqrt[x]*(a*f*Sqrt[x] - (2*b*g*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*d*ArcT
an[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(1/4)
]))/(9*(-d)^(1/4)*e^(3/4)) - (b*f*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1
+ (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt
```

```
[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) + b*f*Sqrt[x]*Log[c*
(d + e*x^2)^p] + (g*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3))/Sqrt[h*x]
```

Maple [F] time = 1.281, size = 0, normalized size = 0.

$$\int (gx+f)\left(a+b\ln\left(c\left(ex^2+d\right)^p\right)\right)\frac{1}{\sqrt{hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93234, size = 2461, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fricas
")
```

```
[Out] -2/9*(3*h*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2
*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 -
b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4
*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d*e*f
*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*
d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b^2*d*f
*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*
p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3
 - 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g
^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*
d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^
4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4
*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log
(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g*h^2*sqrt(-(81
*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) - 3*(9*
b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*
b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h)))
 + 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*
```

```
f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3
*d^2*g^4)*sqrt(h*x)*p^3 - 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^
2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) - 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^
2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2
*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) + (36*b*f*p - 9*a*f + (4*
b*g*p - 3*a*g)*x - 3*(b*g*p*x + 3*b*f*p)*log(e*x^2 + d) - 3*(b*g*x + 3*b*f)
*log(c))*sqrt(h*x))/h
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41609, size = 694, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*(6*sqrt(h*x)*b*g*x*log(c) + 9*((2*sqrt(2)*(d*h^2)^(1/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e
^(-5/4) + 2*sqrt(2)*(d*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4
)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) + sqrt(2)*(d*h^2)
^(1/4)*e^(-5/4)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d
*h^2)*e^(-1/2)) - sqrt(2)*(d*h^2)^(1/4)*e^(-5/4)*log(-sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 8*sqrt(h*x)*e^(-1))*e +
 2*sqrt(h*x)*log(x^2*e + d))*b*f*p + 6*sqrt(h*x)*a*g*x + 18*sqrt(h*x)*b*f*l
og(c) + (6*sqrt(h*x)*h*x*log(x^2*e + d) - (8*sqrt(h*x)*h*x*e^(-1) - 6*sqrt(
2)*(d*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-7/4) - 6*sqrt(2)*(d*h^2)^(3/4)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(
1/4))*e^(-7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*e^(-7/4)*log(sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 3*sqrt(2)*(d*h^2)^(3/4)
*e^(-7/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)
*e^(-1/2)))*e)*b*g*p/h + 18*sqrt(h*x)*a*f)/h
```

3.607 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal. Leaf size=603

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}}+\frac{2ag\sqrt{hx}}{h^2}+\frac{2bg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2}+\frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\right.}{\sqrt[4]{d}h^{3/2}}$$

```
[Out] (2*a*g*Sqrt[h*x])/h^2 - (8*b*g*p*Sqrt[h*x])/h^2 - (2*Sqrt[2]*b*e^(1/4)*f*p*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)
) - (2*Sqrt[2]*b*d^(1/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f*p*ArcTan[1 + (Sqrt[
2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*
d^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(
1/4)*h^(3/2)) + (2*b*g*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^2 - (2*f*(a + b*Lo
g[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]) + (Sqrt[2]*b*e^(1/4)*f*p*Log[Sqrt[d]*Sqr
t[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(
3/2)) - (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*f*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(d^(1/4)*h^(3/2)) + (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[
e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2))
```

Rubi [A] time = 0.794842, antiderivative size = 603, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}}+\frac{2ag\sqrt{hx}}{h^2}+\frac{2bg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2}+\frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\right.}{\sqrt[4]{d}h^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*a*g*Sqrt[h*x])/h^2 - (8*b*g*p*Sqrt[h*x])/h^2 - (2*Sqrt[2]*b*e^(1/4)*f*p*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)
) - (2*Sqrt[2]*b*d^(1/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f*p*ArcTan[1 + (Sqrt[
2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*
d^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(
1/4)*h^(3/2)) + (2*b*g*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^2 - (2*f*(a + b*Lo
g[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]) + (Sqrt[2]*b*e^(1/4)*f*p*Log[Sqrt[d]*Sqr
t[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(
3/2)) - (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*f*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(d^(1/4)*h^(3/2)) + (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[
e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2))
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
```

```
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h} + \frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log(}{}\right.}{h}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}} + \frac{(2bg)\,\text{Subst}\left(\int \log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right.}{h^2}$$

$$= \frac{2ag\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}} - \frac{(8beg}{}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right.\right.\right.}{h\sqrt{hx}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right.\right.\right.}{h\sqrt{hx}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}(}{\sqrt[4]{d}h^{3}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}(}{\sqrt[4]{d}h^{3}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} - \frac{2\sqrt{2}b\sqrt[4]{d}gp\tan^{-1}(}{\sqrt[4]{e}h^{3/}}$$

Mathematica [A] time = 0.477263, size = 316, normalized size = 0.52

$$\frac{2x^{3/2}\left(-\frac{f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} + ag\sqrt{x} + bg\sqrt{x}\log\left(c\left(d+ex^2\right)^p\right) + \frac{2b\sqrt[4]{e}fp\left(\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\tanh^{-1}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{bgp\left(\sqrt{2}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{\right.\right.}{}\right.}{(hx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(a*g*Sqrt[x] + (2*b*e^(1/4)*f*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(
1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*g*p*(8*e^
(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1
```

```
/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sq
rt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] -
Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])
)/(2*e^(1/4)) + b*g*Sqrt[x]*Log[c*(d + e*x^2)^p] - (f*(a + b*Log[c*(d + e*x
^2)^p]))/Sqrt[x]))/(h*x)^(3/2)
```

Maple [F] time = 1.149, size = 0, normalized size = 0.

$$\int (gx+f)\left(a+b\ln\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98322, size = 2245, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fricas
")
```

```
[Out] 2*(h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2
+ b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqr
t(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2
*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*
f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*
e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^
4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^
3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2
*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)
*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^
2*g^4)*p^4/(d*e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*
e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^
3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 -
 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) + (b^2*d*e*f^2*g - b^2*d^2
*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^
```

```
2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) + h^2*x*sqrt(-(2*b^2*f*g*p^2 - h
^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^
3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(d*e*f*h^5*sqrt(-
(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) + (b^2*d*e*f
^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4
- 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) - (a*f + (4*b*g*p
- a*g)*x - (b*g*p*x - b*f*p)*log(e*x^2 + d) - (b*g*x - b*f)*log(c))*sqrt(h*
x))/(h^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.58716, size = 576, normalized size = 0.96

$$\frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}} bdghpe^{\frac{7}{4}} + \sqrt{2}\left(dh^2\right)^{\frac{3}{4}} bfpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-2)}}{dh^3} + \frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}} bdghpe^{\frac{7}{4}} + \sqrt{2}\left(dh^2\right)^{\frac{3}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) + sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^
(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^
(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h^3) + 2*(sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^
(7/4) + sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(
d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h^3)
+ (sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^
(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h
^2)*e^(-1/2))/(d*h^3) - (sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - sqrt(2)*
(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^
(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^3) + 2*(b*g*h*p*x*log(h^2*x^2*e +
 d*h^2) - b*g*h*p*x*log(h^2) - 4*b*g*h*p*x - b*f*h*p*log(h^2*x^2*e + d*h^2)
 + b*f*h*p*log(h^2) + b*g*h*x*log(c) + a*g*h*x - b*f*h*log(c) - a*f*h)/(sqr
t(h*x)*h^2)
```

3.608 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal. Leaf size=588

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}}-\frac{\sqrt{2}be^{3/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}}+$$

```
[Out] (-2*Sqrt[2]*b*e^(3/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) - (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b*e
^(3/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^
(3/4)*h^(5/2)) + (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(3*h*(h*x)^(3/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(h^2*Sqrt[h*x]
) - (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) + (Sqrt[2]*b*e^(1/4)*g*p*
Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]
])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]
*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) - (Sqr
t[2]*b*e^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4
)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2))
```

Rubi [A] time = 0.74087, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2467, 2476, 2455, 211, 1165, 628, 1162, 617, 204, 297}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}}-\frac{\sqrt{2}be^{3/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```


```
[Out] (-2*Sqrt[2]*b*e^(3/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) - (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b*e
^(3/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^
(3/4)*h^(5/2)) + (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(3*h*(h*x)^(3/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(h^2*Sqrt[h*x]
) - (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) + (Sqrt[2]*b*e^(1/4)*g*p*
Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]
])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]
*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) - (Sqr
t[2]*b*e^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4
)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2))
```


Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
```

```
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^2}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)\right.}{x^4}\right.}{h}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(8begp)\,\text{S}}{}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(4befp)\,\text{S}}{}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} - \frac{\left(\sqrt{2}be^{3/4}f\right.}{}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} - \frac{\sqrt{2}be^{3/4}f}{}$$

$$= -\frac{2\sqrt{2}be^{3/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2}{}$$

Mathematica [A] time = 0.433418, size = 271, normalized size = 0.46

$$\frac{2x^{5/2}\left(-\frac{f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}} - \frac{g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} - \frac{be^{3/4}fp\left(\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)\right.}{3\sqrt{2}d^{3/4}}}{(hx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*((2*b*e^(1/4)*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTan
h[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/4)*f*p*(2*ArcTan[1
 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
```

```
x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] -
 Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(3*Sqrt[2]*d^
(3/4)) - (f*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g*(a + b*Log[c*(d
+ e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

Maple [F] time = 1.139, size = 0, normalized size = 0.

$$\int \left(gx+f\right)\left(a+b\ln\left(c\left(ex^{2}+d\right)^{p}\right)\right)(hx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95346, size = 2557, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fricas
")
```

```
[Out] -2/3*(h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*
e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*
f^4 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4
- 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*e^2*f^3
 - 9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^
3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5)))
 - h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2
*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4
 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 - 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 1
8*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*e^2*f^3 -
9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f
^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) -
h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^
2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4 -
81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 18*b
^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) - (b^2*d*e^2*f^3 - 9*b
```

```
^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4
- 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) + h^3
*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^2*g
^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4 - 81*
b^3*d^2*e*g^4)*sqrt(h*x)*p^3 - 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 18*b^4*
d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) - (b^2*d*e^2*f^3 - 9*b^2*
d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 1
8*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) + (3*a*g
*x + a*f + (3*b*g*p*x + b*f*p)*log(e*x^2 + d) + (3*b*g*x + b*f)*log(c))*sqr
t(h*x))/(h^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.65191, size = 599, normalized size = 1.02

$$\frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-2)}}{dh^2}+\frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e\right.}{2\left(d\right.}\right.}{dh^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*
g*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*
x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h^2) + 2*(sqrt(2)*(d*h^2)^(1/4)*b*f*h*
p*e^(11/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sq
rt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(
d*h^2) + (sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 3*sqrt(2)*(d*h^2)^(3/4)*
b*g*p*e^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x +
sqrt(d*h^2)*e^(-1/2))/(d*h^2) - (sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 3
*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2))/h^2 - 2/3*(3*b*g*h^2
*p*x*log(h^2*x^2*e + d*h^2) - 3*b*g*h^2*p*x*log(h^2) + b*f*h^2*p*log(h^2*x^
2*e + d*h^2) - b*f*h^2*p*log(h^2) + 3*b*g*h^2*x*log(c) + 3*a*g*h^2*x + b*f*
h^2*log(c) + a*f*h^2)/(sqrt(h*x)*h^4*x)
```

3.609 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal. Leaf size=620

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}-\frac{\sqrt{2}be^{5/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{7/2}}+\ldots$$

```
[Out] (-8*b*e*f*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 - (Sqr
t[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (2*Sqrt[2
]*b*e^(3/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/
(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*
Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(3/4)*g
*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^
(7/2)) - (2*f*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (2*g*(a + b
*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (Sqrt[2]*b*e^(5/4)*f*p*Log[Sq
rt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*
d^(5/4)*h^(7/2)) - (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqr
t[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]
*b*e^(5/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^
(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2)) + (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4
)*h^(7/2))
```

Rubi [A] time = 0.799025, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2467, 2476, 2455, 325, 297, 1162, 617, 204, 1165, 628, 211}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}-\frac{\sqrt{2}be^{5/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{7/2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (-8*b*e*f*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 - (Sqr
t[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (2*Sqrt[2
]*b*e^(3/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/
(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*
Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(3/4)*g
*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^
(7/2)) - (2*f*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (2*g*(a + b
*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (Sqrt[2]*b*e^(5/4)*f*p*Log[Sq
rt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*
d^(5/4)*h^(7/2)) - (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqr
t[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]
*b*e^(5/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^
(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2)) + (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4
)*h^(7/2))
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
```

```
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4}\right) dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\right.}{h}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} + \frac{(8begp)\,\text{Subs}}{}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} -$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} +$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} -$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2\sqrt{2}be^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} + \frac{2\sqrt{2}be^{3/4}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}}{\sqrt[4]{d}}\right.}{3d^{3/4}h^{7/2}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} + \frac{2\sqrt{2}be^{5/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{2\sqrt{2}be^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}}{\sqrt[4]{d}}\right.}{3d^{3/4}h^{7/2}}$$

Mathematica [C] time = 0.182868, size = 309, normalized size = 0.5

$$\frac{2x^{7/2}\left(-\frac{f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5x^{5/2}}-\frac{g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}}-\frac{1}{6}bgp\left(\frac{\frac{\sqrt{2}e^{3/4}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)}{\sqrt[4]{d}}-\frac{\sqrt{2}e^{3/4}\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)}{\sqrt[4]{d}}}{\sqrt{d}}+\frac{2\left(\frac{\sqrt{2}e^{3/4}\tan\right.}{}}{}\right.\right.}{(hx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (2*x^(7/2)*((-4*b*e*f*p*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d
*Sqrt[x]) - (b*g*p*((2*((Sqrt[2]*e^(3/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x
])/d^(1/4)])/d^(1/4) - (Sqrt[2]*e^(3/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x]
)/d^(1/4)])/d^(1/4)))/Sqrt[d] + ((Sqrt[2]*e^(3/4)*Log[Sqrt[d] - Sqrt[2]*d^(
1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/d^(1/4) - (Sqrt[2]*e^(3/4)*Log[Sqrt[d] +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/d^(1/4))/Sqrt[d]))/6 - (f*(a
 + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (g*(a + b*Log[c*(d + e*x^2)^p]))/
(3*x^(3/2))))/(h*x)^(7/2)
```

Maple [F] time = 1.239, size = 0, normalized size = 0.

$$\int(gx+f)\left(a+b\ln\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08976, size = 2877, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2
 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*lo
g(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 + 32*(3*d^4*f*h^
11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4
/(d^5*h^14)) - 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt((d^
2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*
p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7
*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(
d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^4 - 625*b
^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 - 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^5*f^4 - 45
0*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 5*(9*b^2*d^2*e
^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 -
 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*
f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450
*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*
p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^
3 + 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^
4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)
*h^4*p^2)*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 62
5*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) + d*h^
4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^
4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81
*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 - 32*(3*d^4*f*h^11*sqrt(-
(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^1
4)) + 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5
*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - (12*b*e*f*p*x^2 + 5*a*d*g*x + 3
*a*d*f + (5*b*d*g*p*x + 3*b*d*f*p)*log(e*x^2 + d) + (5*b*d*g*x + 3*b*d*f)*l
og(c))*sqrt(h*x))/(d*h^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.62067, size = 647, normalized size = 1.04

$$\frac{2\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^2h^2}+\frac{2\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{(\cdots}\right.}{2\left(dh^{\cdots}\right.}\right.}{d^2h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*(5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - 3*sqrt(2)*(d*h^2)^(3/4
)*b*f*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h^2) + 2*(5*sqrt(2)*(d*h^2)^(1/4
)*b*d*g*h*p*e^(7/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*arctan(-1/2*sq
rt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))
*e^(-1)/(d^2*h^2) + (5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) + 3*sqrt(2)*
(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(
-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2) - (5*sqrt(2)*(d*h^2)^(1/4)*b*
d*g*h*p*e^(7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-1)*log(-sqrt(2
)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2))
/h^3 - 2/15*(12*b*f*h^3*p*x^2*e + 5*b*d*g*h^3*p*x*log(h^2*x^2*e + d*h^2) -
5*b*d*g*h^3*p*x*log(h^2) + 3*b*d*f*h^3*p*log(h^2*x^2*e + d*h^2) - 3*b*d*f*h
^3*p*log(h^2) + 5*b*d*g*h^3*x*log(c) + 5*a*d*g*h^3*x + 3*b*d*f*h^3*log(c) +
 3*a*d*f*h^3)/(sqrt(h*x)*d*h^6*x^2)
```

3.610 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal. Leaf size=641

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}}+\frac{\sqrt{2}be^{7/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}-\sqrt{}$$

```
[Out] (-8*b*e*f*p)/(21*d*h^3*(h*x)^(3/2)) - (8*b*e*g*p)/(5*d*h^4*Sqrt[h*x]) + (2*
Sqrt[2]*b*e^(7/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 - (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(
7/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7
/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*
x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(7*h*(h*x)^(7/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2*(h*x)^(
5/2)) + (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (Sqrt[2]*b*e^(5/4)*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2))
+ (Sqrt[2]*b*e^(5/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(9/2))
```

Rubi [A] time = 0.826927, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2467, 2476, 2455, 325, 211, 1165, 628, 1162, 617, 204, 297}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}}+\frac{\sqrt{2}be^{7/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}-\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-8*b*e*f*p)/(21*d*h^3*(h*x)^(3/2)) - (8*b*e*g*p)/(5*d*h^4*Sqrt[h*x]) + (2*
Sqrt[2]*b*e^(7/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 - (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(
7/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7
/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*
x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(7*h*(h*x)^(7/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2*(h*x)^(
5/2)) + (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (Sqrt[2]*b*e^(5/4)*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2))
+ (Sqrt[2]*b*e^(5/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(9/2))
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
```

```
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^8}\,d\right.}{h}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}} + \frac{(8begp)\,\text{Subs}}{}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c\left(\right.\right.\right.}{5h^2(hx)}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c\left(\right.\right.\right.}{5h^2(hx)}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c\left(\right.\right.\right.}{5h^2(hx)}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c\left(\right.\right.\right.}{5h^2(hx)}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}be^{7/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} + \frac{2\sqrt{2}be^{5/4}gp}{}$$

Mathematica [C] time = 0.0687117, size = 100, normalized size = 0.16

$$-\frac{2\sqrt{hx}\left(3d(5f+7gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+20befpx^2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{ex^2}{d}\right)+84begpx^3\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{ex^2}{d}\right)\right)}{105dh^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[h*x]*(20*b*e*f*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)]
 + 84*b*e*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] + 3*d*(5*f
+ 7*g*x)*(a + b*Log[c*(d + e*x^2)^p])))/(105*d*h^5*x^4)
```

Maple [F] time = 1.201, size = 0, normalized size = 0.

$$\int \left(gx+f\right)\left(a+b\ln\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12623, size = 2996, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(3*d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f
^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h
^9))*log(-32*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32*(7
*d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*
e^5*g^4)*p^4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^
5*p^2)*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 240
1*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - 3*d*
```

```
h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 24
01*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-3
2*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 - 32*(7*d^6*g*h^14
*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^
4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt
(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e
^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - 3*d*h^5*x^4*sqr
t((d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e
^5*g^4)*p^4/(d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-32*(625*b^3*e
^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32*(7*d^6*g*h^14*sqrt(-(625*
b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)
) - 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt((d^3*h^9*sq
rt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(
d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) + 3*d*h^5*x^4*sqrt((d^3*h^9*sq
rt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(
d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-32*(625*b^3*e^6*f^4 - 2401
*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 - 32*(7*d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 -
 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) - 5*(25*b^2
*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt((d^3*h^9*sqrt(-(625*b^4*
e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) -
70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - (84*b*e*g*p*x^3 + 20*b*e*f*p*x^2 + 21*a*d
*g*x + 15*a*d*f + 3*(7*b*d*g*p*x + 5*b*d*f*p)*log(e*x^2 + d) + 3*(7*b*d*g*x
 + 5*b*d*f)*log(c))*sqrt(h*x))/(d*h^5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61487, size = 659, normalized size = 1.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(2*(5*sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) + 7*sqrt(2)*(d*h^2)^(3/4
)*b*g*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h^2) + 2*(5*sqrt(2)*(d*h^2)^(1/4
)*b*f*h*p*e^(11/4) + 7*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*arctan(-1/2*sqr
t(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*
e^(-1)/(d^2*h^2) + (5*sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 7*sqrt(2)*(d
*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1
/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2) - (5*sqrt(2)*(d*h^2)^(1/4)*b*f*
h*p*e^(11/4) - 7*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(
d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2))/h^
4 - 2/105*(84*b*g*h^4*p*x^3*e + 20*b*f*h^4*p*x^2*e + 21*b*d*g*h^4*p*x*log(h
^2*x^2*e + d*h^2) - 21*b*d*g*h^4*p*x*log(h^2) + 15*b*d*f*h^4*p*log(h^2*x^2*
```

```
e + d*h^2) - 15*b*d*f*h^4*p*log(h^2) + 21*b*d*g*h^4*x*log(c) + 21*a*d*g*h^4
*x + 15*b*d*f*h^4*log(c) + 15*a*d*f*h^4)/(sqrt(h*x)*d*h^8*x^3)
```

3.611 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal. Leaf size=1002

result too large to display

```
[Out] (2*a*f^2*Sqrt[h*x])/h - (8*b*f^2*p*Sqrt[h*x])/h + (8*b*d*g^2*p*Sqrt[h*x])/(
5*e*h) - (16*b*f*g*p*(h*x)^(3/2))/(9*h^2) - (8*b*g^2*p*(h*x)^(5/2))/(25*h^3
) - (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1
/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (4*Sqrt[2]*b*d^(3/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt
[2]*b*d^(5/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(5*e^(5/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) + (4*Sqrt[2]*b*d^(3/
4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3
/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(5/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(5*e^(5/4)*Sqrt[h]) + (2*b*f^2*Sqrt[h*x]*Log[c*(d
 + e*x^2)^p])/h + (4*f*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2)
+ (2*g^2*(h*x)^(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^3) - (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) + (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h]) + (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) - (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h])
```

Rubi [A] time = 1.30943, antiderivative size = 1002, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2467, 2471, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297, 302}

$$-\frac{8bg^2p(hx)^{5/2}}{25h^3}+\frac{2g^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{5/2}}{5h^3}-\frac{16bfgp(hx)^{3/2}}{9h^2}+\frac{4fg\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{3/2}}{3h^2}-\frac{8bf^2p}{h}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```


```
[Out] (2*a*f^2*Sqrt[h*x])/h - (8*b*f^2*p*Sqrt[h*x])/h + (8*b*d*g^2*p*Sqrt[h*x])/(
5*e*h) - (16*b*f*g*p*(h*x)^(3/2))/(9*h^2) - (8*b*g^2*p*(h*x)^(5/2))/(25*h^3
) - (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1
/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (4*Sqrt[2]*b*d^(3/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt
[2]*b*d^(5/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(5*e^(5/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) + (4*Sqrt[2]*b*d^(3/
4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3
/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(5/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(5*e^(5/4)*Sqrt[h]) + (2*b*f^2*Sqrt[h*x]*Log[c*(d
 + e*x^2)^p])/h + (4*f*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2)
```

```
+ (2*g^2*(h*x)^(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^3) - (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) + (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h]) + (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) - (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h])
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\mathrm{Subst}\left(\int\left(f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{2fgx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}+\frac{g^2x^4\left(\right.}{}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\mathrm{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^3}+\frac{(4fg)\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{2af^2\sqrt{hx}}{h}+\frac{4fg(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2}+\frac{2g^2(hx)^{5/2}\left(a+b\log\right.}{5h^3}$$

$$= \frac{2af^2\sqrt{hx}}{h}-\frac{16bfgp(hx)^{3/2}}{9h^2}+\frac{2bf^2\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h}+\frac{4fg(hx)^{3/2}\left(\right.}{}$$

$$= \frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}-\frac{8bg^2p(hx)^{5/2}}{25h^3}+$$

$$= \frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}-\frac{8bg^2p(hx)^{5/2}}{25h^3}+$$

$$= \frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}-\frac{8bg^2p(hx)^{5/2}}{25h^3}+$$

$$= \frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}-\frac{8bg^2p(hx)^{5/2}}{25h^3}-$$

$$= \frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}-\frac{8bg^2p(hx)^{5/2}}{25h^3}-$$

Mathematica [A] time = 1.33385, size = 588, normalized size = 0.59

$$2\sqrt{x}\left(\frac{2}{3}fgx^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+\frac{1}{5}g^2x^{5/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+af^2\sqrt{x}+bf^2\sqrt{x}\log\left(c\left(d+ex^2\right)^p\right)-\frac{bg^2}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*Sqrt[x]*(a*f^2*Sqrt[x] - (4*b*f*g*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*d*
ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(
1/4)]))/(9*(-d)^(1/4)*e^(3/4)) - (b*f^2*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^
(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*Arc
Tan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] -
```

```
Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d]
+ Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) - (b*g^2*p*(-4
0*d*e^(1/4)*Sqrt[x] + 8*e^(5/4)*x^(5/2) - 10*Sqrt[2]*d^(5/4)*ArcTan[1 - (Sq
rt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + 10*Sqrt[2]*d^(5/4)*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[x])/d^(1/4)] - 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e
^(1/4)*Sqrt[x] + Sqrt[e]*x] + 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/
4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(50*e^(5/4)) + b*f^2*Sqrt[x]*Log[c*(d + e
*x^2)^p] + (2*f*g*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3 + (g^2*x^(5/2)*(a
 + b*Log[c*(d + e*x^2)^p]))/5))/Sqrt[h*x]
```

Maple [F] time = 1.281, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^{2}\left(a+b\ln\left(c\left(ex^{2}+d\right)^{p}\right)\right)\frac{1}{\sqrt{hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36029, size = 4782, normalized size = 4.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/225*(15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f
^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^
8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(
16*(50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4
- 1620*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(10*e^4*f*g*h
^2*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e
^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) + 3*(1
125*b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*
d^3*e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e
^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^
5*g^8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h)))
```

```
 - 15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g
^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p
^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(
50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 16
20*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(10*e^4*f*g*h^2*s
qrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f
^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) + 3*(1125*
b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*
e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f
^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^
8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) - 1
5*e*h*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 +
40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e
^5*h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(50625
*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 1620*b^
3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(10*e^4*f*g*h^2*sqrt(-
(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^
4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) - 3*(1125*b^2*e
^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*e*g^6
)*h*p^2)*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2
 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4
/(e^5*h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) + 15*e*h*
sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*
b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2
)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(50625*b^3*e
^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 1620*b^3*d^3*
e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(10*e^4*f*g*h^2*sqrt(-(50625
*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 34
20*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) - 3*(1125*b^2*e^4*f^6
 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*e*g^6)*h*p^
2)*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 401
50*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*
h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) + (225*a*e*f^2
- 9*(4*b*e*g^2*p - 5*a*e*g^2)*x^2 - 180*(5*b*e*f^2 - b*d*g^2)*p - 50*(4*b*e
*f*g*p - 3*a*e*f*g)*x + 15*(3*b*e*g^2*p*x^2 + 10*b*e*f*g*p*x + 15*b*e*f^2*p
)*log(e*x^2 + d) + 15*(3*b*e*g^2*x^2 + 10*b*e*f*g*x + 15*b*e*f^2)*log(c))*s
qrt(h*x))/(e*h)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38448, size = 1107, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/225*(90*sqrt(h*x)*b*g^2*x^2*log(c) + 90*sqrt(h*x)*a*g^2*x^2 + 300*sqrt(h*
x)*b*f*g*x*log(c) + 225*((2*sqrt(2)*(d*h^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(
2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) +
2*sqrt(2)*(d*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4)
 - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) + sqrt(2)*(d*h^2)^(1/4)*e^(
-5/4)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-
1/2)) - sqrt(2)*(d*h^2)^(1/4)*e^(-5/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 8*sqrt(h*x)*e^(-1))*e + 2*sqrt(h*
x)*log(x^2*e + d))*b*f^2*p + 9*(10*sqrt(h*x)*x^2*log(x^2*e + d) - (10*sqrt(
2)*(d*h^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*s
qrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-9/4) + 10*sqrt(2)*(d*h^2)^(1/4)*d*arct
an(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h
^2)^(1/4))*e^(-9/4) + 5*sqrt(2)*(d*h^2)^(1/4)*d*e^(-9/4)*log(sqrt(2)*(d*h^2
)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 5*sqrt(2)*(d*h^2
)^(1/4)*d*e^(-9/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sq
rt(d*h^2)*e^(-1/2)) + 8*(sqrt(h*x)*h^10*x^2*e^4 - 5*sqrt(h*x)*d*h^10*e^3)*e
^(-5)/h^10)*e)*b*g^2*p + 300*sqrt(h*x)*a*f*g*x + 450*sqrt(h*x)*b*f^2*log(c)
 + 50*(6*sqrt(h*x)*h*x*log(x^2*e + d) - (8*sqrt(h*x)*h*x*e^(-1) - 6*sqrt(2)
*(d*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(
h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-7/4) - 6*sqrt(2)*(d*h^2)^(3/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/
4))*e^(-7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*e^(-7/4)*log(sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 3*sqrt(2)*(d*h^2)^(3/4)*e
^(-7/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e
^(-1/2)))*e)*b*f*g*p/h + 450*sqrt(h*x)*a*f^2)/h
```

3.612 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal. Leaf size=949

result too large to display

```
[Out] (4*a*f*g*Sqrt[h*x])/h^2 - (16*b*f*g*p*Sqrt[h*x])/h^2 - (8*b*g^2*p*(h*x)^(3/
2))/(9*h^3) - (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h
*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) - (4*Sqrt[2]*b*d^(1/4)*f*g*p*Arc
Tan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(3/2)) -
 (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 + (Sq
rt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (4*Sqrt[2]
*b*d^(1/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])
/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)
*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (4*b*f*g*Sqrt[h*x]*Lo
g[c*(d + e*x^2)^p])/h^2 - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]
) + (2*g^2*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3) + (Sqrt[2]*b*e
^(1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) - (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*
h^(3/2)) + (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2)) - (Sqrt[2]*b*e^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^
(3/2)) - (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2))
```

Rubi [A] time = 1.26132, antiderivative size = 949, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$-\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{\sqrt[4]{d}h^{3/2}}+\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{\sqrt[4]{d}h^{3/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{h\sqrt{hx}}+\frac{\sqrt{2}b\sqrt[4]{e}p}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (4*a*f*g*Sqrt[h*x])/h^2 - (16*b*f*g*p*Sqrt[h*x])/h^2 - (8*b*g^2*p*(h*x)^(3/
2))/(9*h^3) - (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h
*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) - (4*Sqrt[2]*b*d^(1/4)*f*g*p*Arc
Tan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(3/2)) -
 (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 + (Sq
rt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (4*Sqrt[2]
*b*d^(1/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])
/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)
*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (4*b*f*g*Sqrt[h*x]*Lo
g[c*(d + e*x^2)^p])/h^2 - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]
) + (2*g^2*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3) + (Sqrt[2]*b*e
^(1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) - (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*
```

```
Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*
h^(3/2)) + (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2)) - (Sqrt[2]*b*e^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^
(3/2)) - (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2))
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\mathrm{Subst}\left(\int\left(\frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h} + \frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2} + \frac{g^2x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)\right.\right.}{h^2}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\,\mathrm{Subst}\left(\int\left(\right.\right.}{}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right.\right.\right.}{3h^3}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f^2\left(a+b\log\left(c\right.\right.}{h\sqrt{hx}}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f}{}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f}{}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$

Mathematica [A] time = 0.829528, size = 436, normalized size = 0.46

$$2x^{3/2}\left(-\frac{f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} + \frac{1}{3}g^2x^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right) + 2afg\sqrt{x} + 2bfg\sqrt{x}\log\left(c\left(d+ex^2\right)^p\right) - \frac{2bg^2p\left(2\sqrt[4]{-d}e^{3/4}x^{3/2}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(2*a*f*g*Sqrt[x] - (2*b*g^2*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*
d*ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)
```

```
^(1/4)]))/(9*(-d)^(1/4)*e^(3/4)) + (2*b*e^(1/4)*f^2*p*(ArcTan[(e^(1/4)*Sqrt
[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (
b*f*g*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*
Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/
d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x]
+ Sqrt[e]*x]))/e^(1/4) + 2*b*f*g*Sqrt[x]*Log[c*(d + e*x^2)^p] - (f^2*(a + b
*Log[c*(d + e*x^2)^p]))/Sqrt[x] + (g^2*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p])
)/3))/(h*x)^(3/2)
```

Maple [F] time = 1.332, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^{2}\left(a+b\ln\left(c\left(ex^{2}+d\right)^{p}\right)\right)(hx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)

[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxi
ma")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42991, size = 4340, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fric
as")

[Out] -2/9*(3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 +
918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^
6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*b^3*e^4*f^8
 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3*e*f^2*g^6
+ b^3*d^4*g^8)*sqrt(h*x)*p^3 + 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h^5*sqrt(-(8
1*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^
3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b^2*d*e^3*f^5*g - 30*b^2
*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4
*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g
^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/
(e*h^3))) - 3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*
```

```
g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*
e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*b^3*e
^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3*e*f^
2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 - 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h^5*sq
rt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*
b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b^2*d*e^3*f^5*g -
30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-(e*h^3*sqrt(-(81*b
^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e
*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)
*p^2)/(e*h^3))) - 3*h^2*x*sqrt((e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3
*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^
4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*
b^3*e^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3
*e*f^2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 + 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h
^5*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4
- 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 6*(9*b^2*d*e^3*f^5
*g - 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt((e*h^3*sqrt(-(
81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d
^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*
g^3)*p^2)/(e*h^3))) + 3*h^2*x*sqrt((e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d
*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8
)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*
(81*b^3*e^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3
*d^3*e*f^2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 - 32*((3*d*e^3*f^2 + d^2*e^2*g^
2)*h^5*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*
g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 6*(9*b^2*d*e^3
*f^5*g - 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt((e*h^3*sqr
t(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b
^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*
d*f*g^3)*p^2)/(e*h^3))) + (9*a*f^2 + (4*b*g^2*p - 3*a*g^2)*x^2 + 18*(4*b*f*
g*p - a*f*g)*x - 3*(b*g^2*p*x^2 + 6*b*f*g*p*x - 3*b*f^2*p)*log(e*x^2 + d) -
 3*(b*g^2*x^2 + 6*b*f*g*x - 3*b*f^2)*log(c))*sqrt(h*x))/(h^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)

[Out] Timed out
```

Giac [A] time = 1.7384, size = 869, normalized size = 0.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac
")

[Out] 2/3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + sqrt(2)*(d*h^2)^(3/4)*b
*d*g^2*p*e^(9/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*arctan(1/2*sqr
```

```
t(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*
e^(-3)/(d*h^3) + 2/3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + sqrt(2
)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4
))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/
4)/(d*h^2)^(1/4))*e^(-3)/(d*h^3) + 1/3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p
*e^(11/4) - sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/
4)*b*f^2*p*e^(13/4))*e^(-3)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) +
h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^3) - 1/3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*
h*p*e^(11/4) - sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^
(3/4)*b*f^2*p*e^(13/4))*e^(-3)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4
) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^3) + 2/9*(3*b*g^2*h^2*p*x^2*log(h^2*x^
2*e + d*h^2) - 3*b*g^2*h^2*p*x^2*log(h^2) - 4*b*g^2*h^2*p*x^2 + 18*b*f*g*h^
2*p*x*log(h^2*x^2*e + d*h^2) - 18*b*f*g*h^2*p*x*log(h^2) + 3*b*g^2*h^2*x^2*
log(c) - 72*b*f*g*h^2*p*x + 3*a*g^2*h^2*x^2 - 9*b*f^2*h^2*p*log(h^2*x^2*e +
 d*h^2) + 9*b*f^2*h^2*p*log(h^2) + 18*b*f*g*h^2*x*log(c) + 18*a*f*g*h^2*x -
 9*b*f^2*h^2*log(c) - 9*a*f^2*h^2)/(sqrt(h*x)*h^3)
```

3.613 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal. Leaf size=932

result too large to display

[Out] (2*a*g^2*Sqrt[h*x])/h^3 - (8*b*g^2*p*Sqrt[h*x])/h^3 - (2*Sqrt[2]*b*e^(3/4)*
f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)
*h^(5/2)) - (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x
])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*Sqrt[2]*b*d^(1/4)*g^2*p*ArcTa
n[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(5/2)) + (
2*Sqrt[2]*b*e^(3/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) + (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 + (Sqrt
[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b
*d^(1/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(
e^(1/4)*h^(5/2)) + (2*b*g^2*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^3 - (2*f^2*(a
 + b*Log[c*(d + e*x^2)^p]))/(3*h*(h*x)^(3/2)) - (4*f*g*(a + b*Log[c*(d + e*
x^2)^p]))/(h^2*Sqrt[h*x]) - (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] +
Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2))
 + (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqr
t[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) - (Sqrt[2]*b*d^(1/4)*g^2
*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h
*x]])/(e^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sq
rt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) -
 (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[
2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*d^(1/4)*g^2*p
*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x
]])/(e^(1/4)*h^(5/2))

Rubi [A] time = 1.21505, antiderivative size = 932, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$-\frac{2\sqrt{2}be^{3/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{3d^{3/4}h^{5/2}}+\frac{2\sqrt{2}be^{3/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{3d^{3/4}h^{5/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{3h(hx)^{3/2}}-\frac{\sqrt{2}be^{3/4}p\log}{}$$

Antiderivative was successfully verified.

[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]

[Out] (2*a*g^2*Sqrt[h*x])/h^3 - (8*b*g^2*p*Sqrt[h*x])/h^3 - (2*Sqrt[2]*b*e^(3/4)*
f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)
*h^(5/2)) - (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x
])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*Sqrt[2]*b*d^(1/4)*g^2*p*ArcTa
n[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(5/2)) + (
2*Sqrt[2]*b*e^(3/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) + (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 + (Sqrt
[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b
*d^(1/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(
e^(1/4)*h^(5/2)) + (2*b*g^2*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^3 - (2*f^2*(a
 + b*Log[c*(d + e*x^2)^p]))/(3*h*(h*x)^(3/2)) - (4*f*g*(a + b*Log[c*(d + e*
x^2)^p]))/(h^2*Sqrt[h*x]) - (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] +
Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2))
 + (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqr

```
t[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) - (Sqrt[2]*b*d^(1/4)*g^2
*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h
*x]])/(e^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sq
rt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) -
 (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[
2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*d^(1/4)*g^2*p
*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x
]])/(e^(1/4)*h^(5/2))
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2}+\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}+\frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^2}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^3}+\frac{(4fg)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3}-\frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}}-\frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right.}{h^2\sqrt{hx}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3}+\frac{2bg^2\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^3}-\frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3}-\frac{8bg^2p\sqrt{hx}}{h^3}+\frac{2bg^2\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^3}-\frac{2f^2\left(a+b\log\left(c\right.\right.}{3h(hx)}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3}-\frac{8bg^2p\sqrt{hx}}{h^3}+\frac{2bg^2\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^3}-\frac{2f^2\left(a+b\log\left(c\right.\right.}{3h(hx)}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3}-\frac{8bg^2p\sqrt{hx}}{h^3}-\frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}-\frac{4\sqrt{2}b\sqrt[4]{e}fg}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3}-\frac{8bg^2p\sqrt{hx}}{h^3}-\frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}-\frac{4\sqrt{2}b\sqrt[4]{e}fg}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3}-\frac{8bg^2p\sqrt{hx}}{h^3}-\frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}-\frac{4\sqrt{2}b\sqrt[4]{e}fg}{}$$

Mathematica [A] time = 0.959661, size = 503, normalized size = 0.54

$$\frac{2x^{5/2}\left(-\frac{f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}}-\frac{2fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}}+ag^2\sqrt{x}+bg^2\sqrt{x}\log\left(c\left(d+ex^2\right)^p\right)-\frac{be^{3/4}f^2p\left(\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*(a*g^2*Sqrt[x] + (4*b*e^(1/4)*f*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-
d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/
4)*f^2*p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (S
```

```
qrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sq
rt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e
]*x]))/(3*Sqrt[2]*d^(3/4)) - (b*g^2*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4
)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[
1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) + b*g^2*Sqrt[x]*Lo
g[c*(d + e*x^2)^p] - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (2*f*
g*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

Maple [F] time = 1.299, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^{2} \left(a+b\ln\left(c\left(ex^{2}+d\right)^{p}\right)\right) (hx)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46429, size = 4329, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fric
as")
```

```
[Out] 2/3*(h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b
^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^1
0)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(16*(b^3*e^4*f^8 +
12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*d^3*e*f^2*g^6 + 8
1*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(6*d^3*e*f*g*h^8*sqrt(-(b^4*e^4*f^8 - 60*
b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^
4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 27*b^2*d^2*e^2*f^4*g^2 - 81
*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f
^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6
 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2
)/(d*h^5))) - h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^
```

```
2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(
d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(16*(b^3*e
^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*d^3*e*f^
2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(6*d^3*e*f*g*h^8*sqrt(-(b^4*e^4*
f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^
6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 27*b^2*d^2*e^2*f^4
*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(-(d*h^5*sqrt(-(
b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*
e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f
*g^3)*p^2)/(d*h^5))) - h^3*x^2*sqrt((d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^
3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^
8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(1
6*(b^3*e^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*
d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(6*d^3*e*f*g*h^8*sqrt(-(
b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*
e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - (b^2*d*e^3*f^6 - 27*b^2*d^2
*e^2*f^4*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt((d*h^5*
sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b
^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*
b^2*d*f*g^3)*p^2)/(d*h^5))) + h^3*x^2*sqrt((d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b
^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4
*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5)
)*log(16*(b^3*e^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 1
08*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(6*d^3*e*f*g*h^8*
sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b
^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - (b^2*d*e^3*f^6 - 27*
b^2*d^2*e^2*f^4*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(
(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4
- 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3
*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))) - (6*a*f*g*x + a*f^2 + 3*(4*b*g^2*p - a*
g^2)*x^2 - (3*b*g^2*p*x^2 - 6*b*f*g*p*x - b*f^2*p)*log(e*x^2 + d) - (3*b*g^
2*x^2 - 6*b*f*g*x - b*f^2)*log(c))*sqrt(h*x))/(h^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)

[Out] Timed out
```

Giac [A] time = 1.64771, size = 863, normalized size = 0.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac
")

[Out] 1/3*(2*(3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sqrt(2)*(d*h^2)^(1/4)
*b*f^2*h*p*e^(11/4) + 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*arctan(1/2*s
```

```
qrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4)
)*e^(-2)/(d*h^2) + 2*(3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sqrt(2)
*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4)
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4
)/(d*h^2)^(1/4))*e^(-2)/(d*h^2) + (3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7
/4) + sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 6*sqrt(2)*(d*h^2)^(3/4)*b*
f*g*p*e^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x +
sqrt(d*h^2)*e^(-1/2))/(d*h^2) - (3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4
) + sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 6*sqrt(2)*(d*h^2)^(3/4)*b*f*
g*p*e^(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + s
qrt(d*h^2)*e^(-1/2))/(d*h^2))/h^2 + 2/3*(3*b*g^2*h^2*p*x^2*log(h^2*x^2*e +
d*h^2) - 3*b*g^2*h^2*p*x^2*log(h^2) - 12*b*g^2*h^2*p*x^2 - 6*b*f*g*h^2*p*x*
log(h^2*x^2*e + d*h^2) + 6*b*f*g*h^2*p*x*log(h^2) + 3*b*g^2*h^2*x^2*log(c)
+ 3*a*g^2*h^2*x^2 - b*f^2*h^2*p*log(h^2*x^2*e + d*h^2) + b*f^2*h^2*p*log(h^
2) - 6*b*f*g*h^2*x*log(c) - 6*a*f*g*h^2*x - b*f^2*h^2*log(c) - a*f^2*h^2)/(
sqrt(h*x)*h^4*x)
```

3.614 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal. Leaf size=935

result too large to display

```
[Out] (-8*b*e*f^2*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f^2*p*ArcTan[1 -
(Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (4*Sq
rt[2]*b*e^(3/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(1/4)*g^2*p*ArcTan[1 - (Sqrt[2]*
e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(
5/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(7/2)) + (4*Sqrt[2]*b*e^(3/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(1/4)*g^2*
p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/
2)) - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (4*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (2*g^2*(a + b*Log[c*(d + e*x
^2)^p]))/(h^3*Sqrt[h*x]) - (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
- (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2)) + (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
+ (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) - (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2))
```

Rubi [A] time = 1.18265, antiderivative size = 935, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2467, 2476, 2455, 325, 297, 1162, 617, 204, 1165, 628, 211}

$$\frac{2\sqrt{2}be^{5/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}be^{5/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{5d^{5/4}h^{7/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{5h(hx)^{5/2}}-\frac{\sqrt{2}be^{5/4}p\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```


```
[Out] (-8*b*e*f^2*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f^2*p*ArcTan[1 -
(Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (4*Sq
rt[2]*b*e^(3/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(1/4)*g^2*p*ArcTan[1 - (Sqrt[2]*
e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(
5/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(7/2)) + (4*Sqrt[2]*b*e^(3/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(1/4)*g^2*
p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/
2)) - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (4*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (2*g^2*(a + b*Log[c*(d + e*x
^2)^p]))/(h^3*Sqrt[h*x]) - (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
- (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt
```

```
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2)) + (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
+ (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) - (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2))
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4} + \frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2x^2}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\right.}{h^2}$$

$$= -\frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{2g^2\left(a+b\right.}{}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{4\sqrt{2}be^{3/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\tan^{-1}\left(1-\right.}{\sqrt[4]{d}h^{7/2}}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} + \frac{2\sqrt{2}be^{5/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{4\sqrt{2}be^{3/4}fgp\tan^{-1}\left(1-\right.}{3d^{3/4}h^{7/2}}$$

Mathematica [C] time = 0.952199, size = 340, normalized size = 0.36

$$\frac{2x^{7/2}\left(-\frac{f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5x^{5/2}} - \frac{2fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}} - \frac{g^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} - \frac{\sqrt{2}be^{3/4}fgp\left(\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\right.\right.}{3d^{3/4}}\right.}{(hx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (2*x^(7/2)*((2*b*e^(1/4)*g^2*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcT
anh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (4*b*e*f^2*p*Hypergeomet
ric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d*Sqrt[x]) - (Sqrt[2]*b*e^(3/4)*f*g*
p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*
e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
 Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/
(3*d^(3/4)) - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (2*f*g*(a +
```

```
b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g^2*(a + b*Log[c*(d + e*x^2)^p]))/S
qrt[x]))/(h*x)^(7/2)
```

Maple [F] time = 1.289, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^{2}\left(a+b\ln\left(c\left(ex^{2}+d\right)^{p}\right)\right)\left(hx\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45978, size = 4817, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fric
as")
```

```
[Out] -2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g
^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*
e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7
))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g^2 + 2150*b^3*d^2*e^3*f^4*g
^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e*g^8)*sqrt(h*x)*p^3 + 32*(3
*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^
2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e
*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f^5*g - 190*b^2*d^3*e^2*f^3*g^3 +
 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b
^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 +
50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)
*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^
4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 5
0625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*
p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g^2 + 2150*b^3*
d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e*g^8)*sqrt(h*x
)*p^3 - 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4
*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50
```

```
625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f^5*g - 190*b^2*d^3*
e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5
*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e
^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b
^2*d*e*f*g^3)*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5
*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e
^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*e^2*f^3*g - 5*b
^2*d*e*f*g^3)*p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g
^2 + 2150*b^3*d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e
*g^8)*sqrt(h*x)*p^3 + 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*
f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^
2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 10*(9*b^2*d^2*e^3*f^5*g
- 190*b^2*d^3*e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 -
85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*
e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))) + d*h^4*x^3*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 -
85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*
e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*
b^3*d*e^4*f^6*g^2 + 2150*b^3*d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 +
50625*b^3*d^4*e*g^8)*sqrt(h*x)*p^3 - 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqr
t(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 8
5500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 10*(9*b^2
*d^2*e^3*f^5*g - 190*b^2*d^3*e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sq
rt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2
*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^
14)) - 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))) + (10*a*d*f*g*
x + 3*a*d*f^2 + 3*(4*b*e*f^2*p + 5*a*d*g^2)*x^2 + (15*b*d*g^2*p*x^2 + 10*b*
d*f*g*p*x + 3*b*d*f^2*p)*log(e*x^2 + d) + (15*b*d*g^2*x^2 + 10*b*d*f*g*x +
3*b*d*f^2)*log(c))*sqrt(h*x))/(d*h^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6748, size = 892, normalized size = 0.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac
")
```

```
[Out] 1/15*(2*(10*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + 15*sqrt(2)*(d*h^2)
^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*arctan
(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)
^(1/4))*e^(-2)/(d^2*h^2) + 2*(10*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4)
```

```
 + 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f
^2*p*e^(13/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt
(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d^2*h^2) + (10*sqrt(2)*(d*h^2)^(1/4)*
b*d*f*g*h*p*e^(11/4) - 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(
2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*
x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2) - (10*sqrt(2)*(d*h^2)^(
1/4)*b*d*f*g*h*p*e^(11/4) - 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*
sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2))/h^3 - 2/15*(15*b
*d*g^2*h^3*p*x^2*log(h^2*x^2*e + d*h^2) - 15*b*d*g^2*h^3*p*x^2*log(h^2) + 1
2*b*f^2*h^3*p*x^2*e + 10*b*d*f*g*h^3*p*x*log(h^2*x^2*e + d*h^2) - 10*b*d*f*
g*h^3*p*x*log(h^2) + 15*b*d*g^2*h^3*x^2*log(c) + 15*a*d*g^2*h^3*x^2 + 3*b*d
*f^2*h^3*p*log(h^2*x^2*e + d*h^2) - 3*b*d*f^2*h^3*p*log(h^2) + 10*b*d*f*g*h
^3*x*log(c) + 10*a*d*f*g*h^3*x + 3*b*d*f^2*h^3*log(c) + 3*a*d*f^2*h^3)/(sqr
t(h*x)*d*h^6*x^2)
```

3.615 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal. Leaf size=968

result too large to display

```
[Out] (-8*b*e*f^2*p)/(21*d*h^3*(h*x)^(3/2)) - (16*b*e*f*g*p)/(5*d*h^4*Sqrt[h*x])
+ (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) + (4*Sqrt[2]*b*e^(5/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt
[2]*b*e^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(3*d^(3/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) - (4*Sqrt[2]*b*e^(
5/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(9/2)) - (2*f^2*(a + b*Log[c*(d +
e*x^2)^p]))/(7*h*(h*x)^(7/2)) - (4*f*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2
*(h*x)^(5/2)) - (2*g^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3*(h*x)^(3/2)) +
(Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*f*g*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2))
- (Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*f*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) + (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] +
 Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2)
)
```

Rubi [A] time = 1.23186, antiderivative size = 968, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2467, 2476, 2455, 325, 211, 1165, 628, 1162, 617, 204, 297}

$$\frac{2\sqrt{2}be^{7/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{7d^{7/4}h^{9/2}}-\frac{2\sqrt{2}be^{7/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{7d^{7/4}h^{9/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{7h(hx)^{7/2}}+\frac{\sqrt{2}be^{7/4}p\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```


```
[Out] (-8*b*e*f^2*p)/(21*d*h^3*(h*x)^(3/2)) - (16*b*e*f*g*p)/(5*d*h^4*Sqrt[h*x])
+ (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) + (4*Sqrt[2]*b*e^(5/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt
[2]*b*e^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(3*d^(3/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) - (4*Sqrt[2]*b*e^(
5/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(9/2)) - (2*f^2*(a + b*Log[c*(d +
e*x^2)^p]))/(7*h*(h*x)^(7/2)) - (4*f*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2
*(h*x)^(5/2)) - (2*g^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3*(h*x)^(3/2)) +
(Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*
```

```
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*f*g*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2))
- (Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*f*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) + (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] +
 Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2)
)
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6} + \frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right.\right.\right.}{h^2x^4}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\right.\right.}{x^6}\right.}{h^2}$$

$$= -\frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{2g^2\left(a\right.}{}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\right.}{5}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\right.}{5}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\right.}{5}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2\sqrt{2}be^{3/4}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} + \frac{2\sqrt{2}be}{}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}be^{7/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} + \frac{4\sqrt{2}be}{}$$

Mathematica [C] time = 0.262752, size = 294, normalized size = 0.3

$$\frac{x\left(-30df^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right) - 84dfgx\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right) - 70dg^2x^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right) - 35\sqrt{2}b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (x*(-40*b*e*f^2*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)] - 336*b
*e*f*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] - 35*Sqrt[2]*b*d
^(1/4)*e^(3/4)*g^2*p*x^(7/2)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4
)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[x] + Sqrt[e]*x]) - 30*d*f^2*(a + b*Log[c*(d + e*x^2)^p]) - 84*d*f*
g*x*(a + b*Log[c*(d + e*x^2)^p]) - 70*d*g^2*x^2*(a + b*Log[c*(d + e*x^2)^p]
)))/(105*d*(h*x)^(9/2))
```

Maple [F] time = 1.354, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left(a + b\ln\left(c\left(ex^2 + d\right)^p\right)\right)(hx)^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56233, size = 5276, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fric
as")
```

```
[Out] -2/105*(d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e
^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 15
00625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2
*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^
2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3
*d^4*e^2*g^8)*sqrt(h*x)*p^3 + 16*(42*d^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8
- 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3
*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 5*(675*b^2*d^2*e^
4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^
5*e*g^6)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e
^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 15
00625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2
*f*g^3)*p^2)/(d^3*h^9))) - d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^
8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d
^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*
f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^3*e^6*f^8 - 4725
00*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^
2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 - 16*(42*d^6*f*g*h^14*sqrt(-
(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^
4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
+ 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^
2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^
```

```
8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d
^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*
f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))) - d*h^5*x^4*sqrt((d^3*h^9*sqrt(
-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g
^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
 - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*
b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572
500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 + 16*(42*d
^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*
b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8
)*p^4/(d^7*h^18)) - 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23
373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqrt((d^3*h^9*sqrt(-
(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^
4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
- 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))) + d*h^5*x^4*sq
rt((d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*
b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8
)*p^4/(d^7*h^18)) - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9
))*log(16*(50625*b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e
^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*
x)*p^3 - 16*(42*d^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f
^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 150062
5*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) - 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3
*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqr
t((d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b
^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)
*p^4/(d^7*h^18)) - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9)
)) + (168*b*e*f*g*p*x^3 + 42*a*d*f*g*x + 15*a*d*f^2 + 5*(4*b*e*f^2*p + 7*a*
d*g^2)*x^2 + (35*b*d*g^2*p*x^2 + 42*b*d*f*g*p*x + 15*b*d*f^2*p)*log(e*x^2 +
 d) + (35*b*d*g^2*x^2 + 42*b*d*f*g*x + 15*b*d*f^2)*log(c))*sqrt(h*x))/(d*h^
5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.58614, size = 911, normalized size = 0.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac
")
```

```
[Out] 1/105*(2*(35*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)
^(1/4)*b*f^2*h*p*e^(11/4) - 42*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2
```

```
)^(1/4))*e^(-1)/(d^2*h^2) + 2*(35*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4)
 - 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 42*sqrt(2)*(d*h^2)^(3/4)*b
*f*g*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h^2) + (35*sqrt(2)*(d*h^2)^(1/4)
*b*d*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 42*sqr
t(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2) - (35*sqrt(2)*(d*h^2)^
(1/4)*b*d*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 4
2*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(d*h^2)^(1/4)*
sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h^2))/h^4 - 2/105*(16
8*b*f*g*h^4*p*x^3*e + 35*b*d*g^2*h^4*p*x^2*log(h^2*x^2*e + d*h^2) - 35*b*d*
g^2*h^4*p*x^2*log(h^2) + 20*b*f^2*h^4*p*x^2*e + 42*b*d*f*g*h^4*p*x*log(h^2*
x^2*e + d*h^2) - 42*b*d*f*g*h^4*p*x*log(h^2) + 35*b*d*g^2*h^4*x^2*log(c) +
35*a*d*g^2*h^4*x^2 + 15*b*d*f^2*h^4*p*log(h^2*x^2*e + d*h^2) - 15*b*d*f^2*h
^4*p*log(h^2) + 42*b*d*f*g*h^4*x*log(c) + 42*a*d*f*g*h^4*x + 15*b*d*f^2*h^4
*log(c) + 15*a*d*f^2*h^4)/(sqrt(h*x)*d*h^8*x^3)
```

3.616 $\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}\,dx$

Optimal. Leaf size=1680

result too large to display

```
[Out] (2*a*Sqrt[h*x])/g - (8*b*p*Sqrt[h*x])/g - (2*Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*g) + (2*S
qrt[2]*b*d^(1/4)*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*
Sqrt[h])])/(e^(1/4)*g) + (2*b*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/g - (2*Sqrt[f
]*Sqrt[h]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e
*x^2)^p]))/g^(3/2) - (Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqr
t[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) + (Sqrt[2]
*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) - (8*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g
]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] -
I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqr
t[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[
-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]
*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]
*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sq
rt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^
(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sq
rt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f
]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*S
qrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*S
qrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[
h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[
g]*Sqrt[h*x]))])/g^(3/2) + ((4*I)*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqr
t[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) - (I*b*Sqrt
[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h
] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*S
qrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*S
qrt[h]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sq
rt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sq
rt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt
[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)
*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]
*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)
^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2)
```

Rubi [A] time = 3.08461, antiderivative size = 1680, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 20, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.645, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 205, 2470, 12, 260, 6725, 4928, 4856, 2402, 2315, 2447}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```


```
[Out] (2*a*Sqrt[h*x])/g - (8*b*p*Sqrt[h*x])/g - (2*Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*g) + (2*S
```

```
qrt[2]*b*d^(1/4)*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*
Sqrt[h])]])/(e^(1/4)*g) + (2*b*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/g - (2*Sqrt[f
]*Sqrt[h]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e
*x^2)^p]))/g^(3/2) - (Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqr
t[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) + (Sqrt[2]
*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) - (8*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g
]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] -
I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqr
t[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[
-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]
*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]
*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sq
rt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^
(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sq
rt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f
]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*S
qrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*S
qrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[
h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[
g]*Sqrt[h*x]))])/g^(3/2) + ((4*I)*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqr
t[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) - (I*b*Sqrt
[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h
] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*S
qrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*S
qrt[h]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sq
rt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sq
rt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt
[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)
*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]
*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)
^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2)
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f+\frac{gx^2}{h}}\,dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int\left(\frac{h\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g}-\frac{fh\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g\left(f+\frac{gx^2}{h}\right)}\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{g} - \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\right.\right.}{f+\frac{gx^2}{h}}\right.}{g}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{g^{3/2}} + \frac{(2b)\,\text{Subst}\left(\int \text{lo}\right.}{}$$

$$= \frac{2a\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\right.}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\right.}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\right.}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\right.}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} - \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g} + \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\tan^{-1}}{\sqrt[4]{e}g}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} - \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g} + \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\tan^{-1}}{\sqrt[4]{e}g}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} - \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g} + \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\tan^{-1}}{\sqrt[4]{e}g}$$

Mathematica [A] time = 1.34706, size = 1471, normalized size = 0.88

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

```
[Out] (Sqrt[h*x]*(2*a*Sqrt[g]*Sqrt[x] - (b*Sqrt[g]*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[
2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4
)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[
d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqr
t[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/e^(1/4) + 2*b*Sqrt[g]
*Sqrt[x]*Log[c*(d + e*x^2)^p] + Sqrt[-f]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a
 + b*Log[c*(d + e*x^2)^p]) - Sqrt[-f]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a +
b*Log[c*(d + e*x^2)^p]) - b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*
Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g
]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt
[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*
(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])
]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt
[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[
x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] -
 (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(
e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] -
 Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2,
(e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[
g])]) + b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)
*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqr
t[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqr
t[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/
4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + S
qrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*S
qrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + PolyLog[2
, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqr
t[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f
] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[
-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])])))/(g^(3/2
)*Sqrt[x])
```

Maple [F] time = 1.358, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c\left(ex^2+d\right)^p\right)}{gx+f}\sqrt{hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)
```

```
[Out] int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{hx}b\log\left(\left(ex^2+d\right)^p c\right)+\sqrt{hx}a}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**(1/2)*(a+b*ln(c*(e*x**2+d)**p))/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{hx}\left(b\log\left(\left(ex^2+d\right)^p c\right)+a\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(sqrt(h*x)*(b*log((e*x^2 + d)^p*c) + a)/(g*x + f), x)
```

3.617 $\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}(f+gx)}\,dx$

Optimal. Leaf size=1361

result too large to display

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p
]))/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (8*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*
Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])
/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqr
t[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h
*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*S
qrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[
(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*
ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h
]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h]
+ I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sq
rt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h]
)]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(
1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))
])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - ((4*I)*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h]
)/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*
b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4
)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(S
qrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p
*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x])
)/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sq
rt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[
g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[
h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[
g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/
4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqr
t[h])
```

Rubi [A] time = 1.83058, antiderivative size = 1361, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2467, 205, 2470, 12, 260, 6725, 4928, 4856, 2402, 2315, 2447}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}+\frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{2bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\cdots}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-\cdots\right.}\right.}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```


```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p
]))/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (8*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*
Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])
/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqr
t[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h
```

```
*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*S
qrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[
(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*
ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h
]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h]
+ I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sq
rt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h]
)]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(
1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))
])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - ((4*I)*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h]
)/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*
b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4
)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(S
qrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p
*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x])
)/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sq
rt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[
g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[
h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[
g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/
4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqr
t[h])
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
```

```
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{f+\frac{gx^2}{h}}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\,\text{Subst}\left(\int \frac{\sqrt{h}x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{\ldots}\right)}{h^3}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\,\text{Subst}\left(\int \frac{x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{h\ldots}\right)}{\sqrt{f}\sqrt{g}h^{5/2}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\,\text{Subst}\left(\int \left(\frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2\left(-\sqrt{-d}\sqrt{e}h+ex^2\right)} + \frac{h^2x\tan\ldots}{2\left(\sqrt{-\ldots}\right.}\ldots\right)\right)}{\sqrt{f}\sqrt{g}h^{5/2}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\,\text{Subst}\left(\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{-\sqrt{-d}\sqrt{e}h+ex^2}\,dx, x, \sqrt{hx}\ldots\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\,\text{Subst}\left(\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x\right)} + \ldots\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{\left(2b\sqrt[4]{e}p\right)\,\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x}\,dx, x, \sqrt{\ldots}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \ldots$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \ldots$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \ldots$$

Mathematica [A] time = 0.387796, size = 1297, normalized size = 0.95

$$\sqrt{x}\left(a\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right) - bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}-\sqrt[4]{e}\sqrt{x}\right)}{\sqrt[4]{-d}\sqrt{g}-\sqrt[4]{e}\sqrt{-f}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right) - bp\log\left(\frac{\sqrt{g}\left(i\sqrt[4]{e}\sqrt{x}+\sqrt[4]{-d}\right)}{i\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right) - \ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

```
[Out] (Sqrt[x]*(a*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4)
- e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f
] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(
I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] -
 b*p*Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(
-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d
)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sq
rt[-f] - Sqrt[g]*Sqrt[x]] - a*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sq
rt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g
])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1
/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqr
t[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^
(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p
*Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(
1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*Log[Sqrt[-f] - Sqrt[g]*S
qrt[x]]*Log[c*(d + e*x^2)^p] - b*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*Log[c*(d +
 e*x^2)^p] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)
*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt
[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (
e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt
[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt
[-f] + (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*S
qrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)
*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] +
 b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] +
I*(-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]))/(Sqrt[-f]*Sqrt[g]*Sqrt[h*x])
```

Maple [F] time = 1.306, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c\left(e x^{2}+d\right)^{p}\right)}{g x+f}\frac{1}{\sqrt{h x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)
```

```
[Out] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{hx}b\log\left(\left(ex^2+d\right)^p c\right)+\sqrt{hx}a}{ghx^2+fhx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h*x^2 + f*h*x)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(\left(ex^2+d\right)^p c\right)+a}{\left(gx+f\right)\sqrt{hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*sqrt(h*x)), x)
```

3.618 $\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{(hx)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=1659

result too large to display

```
[Out] (-2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqr
t[h])])/(d^(1/4)*f*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*f*h^(3/2)) - (2*(a + b*Log[c*
(d + e*x^2)^p]))/(f*h*Sqrt[h*x]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p]))/(f^(3/2)*h^(3/2)) + (Sqrt[2]
*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*f*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt
[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*f*h^
(3/2)) - (8*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(
2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/
2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*S
qrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1
/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt
[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h
*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/
4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] -
 I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]
*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*S
qrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqr
t[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
+ (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[
f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] +
(-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(
3/2)) + ((4*I)*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt
[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2,
1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(
((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h]
- I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 +
 (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*S
qrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^
(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*(
(-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I
*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/
2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
```

Rubi [A] time = 2.52854, antiderivative size = 1659, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 19, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.613, Rules used = {2467, 2476, 2455, 297, 1162, 617, 204, 1165, 628, 205, 2470, 12, 260, 6725, 4928, 4856, 2402, 2315, 2447}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```


```
[Out] (-2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqr
t[h])])/(d^(1/4)*f*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 + (Sqrt[2]*e^
```

```
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*f*h^(3/2)) - (2*(a + b*Log[c*
(d + e*x^2)^p]))/(f*h*Sqrt[h*x]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p]))/(f^(3/2)*h^(3/2)) + (Sqrt[2]
*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*f*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt
[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*f*h^
(3/2)) - (8*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(
2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/
2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*S
qrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1
/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt
[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h
*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/
4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] -
 I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]
*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*S
qrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqr
t[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
+ (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[
f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] +
(-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(
3/2)) + ((4*I)*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt
[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2,
1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(
((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h]
- I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 +
 (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*S
qrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^
(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*(
(-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I
*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/
2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2\left(f+\frac{gx^2}{h}\right)}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fx^2} - \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f\left(fh+gx^2\right)}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}dx, x, \sqrt{hx}\right)}{fh} - \frac{(2g)\,\mathrm{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fh+gx^2}dx, x, \sqrt{hx}\right)}{fh}$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \frac{(8bep)\,\mathrm{S}}{}$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \frac{\left(8be\sqrt{g}\right.}{}$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \frac{\left(8be\sqrt{g}\right.}{}$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \frac{\sqrt{2}b\sqrt[4]{e}p}{}$$

$$= -\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{2\left(a+b\log\left(c\left(d+\right.\right.\right.}{fh\sqrt{hx}}$$

$$= -\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{2\left(a+b\log\left(c\left(d+\right.\right.\right.}{fh\sqrt{hx}}$$

$$= -\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{2\left(a+b\log\left(c\left(d+\right.\right.\right.}{fh\sqrt{hx}}$$

$$= -\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{2\left(a+b\log\left(c\left(d+\right.\right.\right.}{fh\sqrt{hx}}$$

$$= -\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{2\left(a+b\log\left(c\left(d+\right.\right.\right.}{fh\sqrt{hx}}$$

Mathematica [A] time = 1.63769, size = 1336, normalized size = 0.81

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (x^(3/2)*((4*b*e^(1/4)*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d
*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (2*(a + b*Log[c*(d + e*x^2)^p]
))/Sqrt[x] + (f*Sqrt[g]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a + b*Log[c*(d + e
*x^2)^p]))/(-f)^(3/2) + (Sqrt[g]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a + b*Log
[c*(d + e*x^2)^p]))/Sqrt[-f] + (b*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(
1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - S
qrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)
*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqr
t[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqr
t[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)
*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*
Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[
-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[
-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLo
g[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*
Sqrt[g])]))/Sqrt[-f] + (b*f*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*S
qrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sq
rt[x]] + Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f]
 + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d
)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]
*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[
x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqr
t[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f]
 - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))
/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f]
 + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2
, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqr
t[g])]))/(-f)^(3/2)))/(f*(h*x)^(3/2))
```

Maple [F] time = 1.321, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c\left(ex^2+d\right)^p\right)}{gx+f}(hx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)
```

```
[Out] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{hx}b\log\left(\left(ex^2+d\right)^pc\right)+\sqrt{hx}a}{gh^2x^3+fh^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h^2*x^3 + f*h^
2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(\left(ex^2+d\right)^pc\right)+a}{\left(gx+f\right)\left(hx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*(h*x)^(3/2)), x)
```

3.619 $\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx$

Optimal. Leaf size=33

$$\frac{p\text{PolyLog}(3,-ex^m)}{m^2}-\frac{\log(fx^p)\,\text{PolyLog}(2,-ex^m)}{m}$$

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Rubi [A] time = 0.0260934, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2374, 6589}

$$\frac{p\text{PolyLog}(3,-ex^m)}{m^2}-\frac{\log(fx^p)\,\text{PolyLog}(2,-ex^m)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx &= -\frac{\log(fx^p)\,\text{Li}_2(-ex^m)}{m}+\frac{p\int \frac{\text{Li}_2(-ex^m)}{x}\,dx}{m}\\ &= -\frac{\log(fx^p)\,\text{Li}_2(-ex^m)}{m}+\frac{p\text{Li}_3(-ex^m)}{m^2}\end{aligned}$$

Mathematica [A] time = 0.0115781, size = 33, normalized size = 1.

$$\frac{p\text{PolyLog}(3,-ex^m)}{m^2}-\frac{\log(fx^p)\,\text{PolyLog}(2,-ex^m)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Maple [C] time = 2.384, size = 191, normalized size = 5.8

$$-\frac{p\ln(x)\,\mathrm{polylog}(2,-ex^m)}{m}+\frac{p\mathrm{polylog}(3,-ex^m)}{m^2}-\frac{\left(\ln(x^p)-p\ln(x)\right)\mathrm{dilog}(1+ex^m)}{m}+\frac{\frac{i}{2}\mathrm{dilog}(1+ex^m)\,\pi\,\mathrm{csgn}\left(if\right)\,\mathrm{cs}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)*ln(1+e*x^m)/x,x)
```

```
[Out] -p/m*ln(x)*polylog(2,-e*x^m)+p*polylog(3,-e*x^m)/m^2-(ln(x^p)-p*ln(x))/m*di
log(1+e*x^m)+1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-
1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f)*csgn(I*f*x^p)^2-1/2*I/m*dilog(1+e*x^m)*
Pi*csgn(I*x^p)*csgn(I*f*x^p)^2+1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f*x^p)^3-1/
m*dilog(1+e*x^m)*ln(f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(p\log(x)^2-2\,\log\left(f\right)\log(x)-2\,\log(x)\log(x^p)\right)\log(ex^m+1)-\int\frac{2\,emx^m\log(x)\log(x^p)-\left(emp\log(x)^2-2\,em\,\mathrm{lo}\right.}{2\,(exx^m+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(p*log(x)^2 - 2*log(f)*log(x) - 2*log(x)*log(x^p))*log(e*x^m + 1) - in
tegrate(1/2*(2*e*m*x^m*log(x)*log(x^p) - (e*m*p*log(x)^2 - 2*e*m*log(f)*log
(x))*x^m)/(e*x*x^m + x), x)
```

Fricas [C] time = 1.56477, size = 93, normalized size = 2.82

$$-\frac{\left(mp\log(x)+m\log\left(f\right)\right)\mathrm{Li}_2(-ex^m)-p\mathrm{polylog}(3,-ex^m)}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="fricas")
```

```
[Out] -((m*p*log(x) + m*log(f))*dilog(-e*x^m) - p*polylog(3, -e*x^m))/m^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)*ln(1+e*x**m)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ex^m+1)\log\left(fx^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^m + 1)*log(f*x^p)/x, x)
```

3.620 $\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx$

Optimal. Leaf size=75

$$\frac{2p\log(fx^p)\,\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}-\frac{2p^2\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}+\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}$$

```
[Out] (Log[f*x^p]^2*Log[1 + (e*x^m)/d])/(e*m) + (2*p*Log[f*x^p]*PolyLog[2, -((e*x
^m)/d)])/(e*m^2) - (2*p^2*PolyLog[3, -((e*x^m)/d)])/(e*m^3)
```

Rubi [A] time = 0.120039, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2337, 2374, 6589}

$$\frac{2p\log(fx^p)\,\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}-\frac{2p^2\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}+\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (Log[f*x^p]^2*Log[1 + (e*x^m)/d])/(e*m) + (2*p*Log[f*x^p]*PolyLog[2, -((e*x
^m)/d)])/(e*m^2) - (2*p^2*PolyLog[3, -((e*x^m)/d)])/(e*m^3)
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} - \frac{(2p)\int \frac{\log(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx}{em}$$
$$= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2p\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} - \frac{(2p^2)\int \frac{\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{em^2}$$
$$= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2p\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} - \frac{2p^2\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{em^3}$$

Mathematica [B] time = 0.129107, size = 210, normalized size = 2.8

$$\frac{6mp\left(p\log(x)-\log\left(fx^p\right)\right)\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)-6p^2\text{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)-6mp^2\log(x)\text{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)+3m^2 \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (m^3*p^2*Log[x]^3 + 3*m^2*p^2*Log[x]^2*Log[1 + d/(e*x^m)] - 3*m^2*p^2*Log[x
]^2*Log[d + e*x^m] + 6*m*p^2*Log[x]*Log[-((e*x^m)/d)]*Log[d + e*x^m] - 6*m*
p*Log[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m] + 3*m^2*Log[f*x^p]^2*Log[d +
e*x^m] - 6*m*p^2*Log[x]*PolyLog[2, -(d/(e*x^m))] + 6*m*p*(p*Log[x] - Log[f*
x^p])*PolyLog[2, 1 + (e*x^m)/d] - 6*p^2*PolyLog[3, -(d/(e*x^m))])/(3*e*m^3)
```

Maple [C] time = 0.345, size = 1373, normalized size = 18.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*ln(f*x^p)^2/(d+e*x^m),x)
```

```
[Out] 1/2/m*ln(d+e*x^m)/e*Pi^2*csgn(I*x^p)*csgn(I*f*x^p)^5-1/4/m*ln(d+e*x^m)/e*Pi
^2*csgn(I*f)^2*csgn(I*f*x^p)^4+I/m*ln(d+e*x^m)/e*p*ln(x)*Pi*csgn(I*f*x^p)^3
+I/m*p*ln(x)*ln((d+e*x^m)/d)/e*Pi*csgn(I*f)*csgn(I*f*x^p)^2-I/m*ln(d+e*x^m)
/e*ln(x^p)*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-I/m*ln(d+e*x^m)/e*ln(f)*P
i*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)+1/2/m*ln(d+e*x^m)/e*Pi^2*csgn(I*f)^2*
csgn(I*x^p)*csgn(I*f*x^p)^3+1/m*(ln(x^p)-p*ln(x))^2*ln(d+e*x^m)/e+1/m*ln(d+
e*x^m)/e*ln(f)^2+1/2/m*ln(d+e*x^m)/e*Pi^2*csgn(I*f)*csgn(I*f*x^p)^5-1/4/m*l
n(d+e*x^m)/e*Pi^2*csgn(I*x^p)^2*csgn(I*f*x^p)^4+I/m*ln(d+e*x^m)/e*p*ln(x)*P
i*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-I/m*p*ln(x)*ln((d+e*x^m)/d)/e*Pi*csgn
(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-I/m*ln(d+e*x^m)/e*ln(x^p)*Pi*csgn(I*f*x^p)^
3-I/m^2*p*dilog((d+e*x^m)/d)/e*Pi*csgn(I*f*x^p)^3+1/m*p^2/e*ln(x)^2*ln(1+e*
x^m/d)-I/m*p*ln(x)*ln((d+e*x^m)/d)/e*Pi*csgn(I*f*x^p)^3+I/m^2*p*dilog((d+e*
x^m)/d)/e*Pi*csgn(I*f)*csgn(I*f*x^p)^2-I/m*ln(d+e*x^m)/e*p*ln(x)*Pi*csgn(I*
x^p)*csgn(I*f*x^p)^2-2*p^2*polylog(3,-e*x^m/d)/e/m^3+2/m*p*ln(x)*ln((d+e*x^
m)/d)/e*ln(f)+I/m*ln(d+e*x^m)/e*ln(f)*Pi*csgn(I*f)*csgn(I*f*x^p)^2+I/m*ln(d
+e*x^m)/e*ln(x^p)*Pi*csgn(I*f)*csgn(I*f*x^p)^2+I/m*p*ln(x)*ln((d+e*x^m)/d)/
e*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2+I/m^2*p*dilog((d+e*x^m)/d)/e*Pi*csgn(I*x^p
)*csgn(I*f*x^p)^2+I/m*ln(d+e*x^m)/e*ln(x^p)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2+
I/m*ln(d+e*x^m)/e*ln(f)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-2/m*ln(d+e*x^m)/e*p*
ln(x)*ln(f)+2/m*p*(ln(x^p)-p*ln(x))*ln(x)*ln((d+e*x^m)/d)/e-1/4/m*ln(d+e*x^
```

```
m)/e*Pi^2*csgn(I*f)^2*csgn(I*x^p)^2*csgn(I*f*x^p)^2+1/2/m*ln(d+e*x^m)/e*Pi^
2*csgn(I*f)*csgn(I*x^p)^2*csgn(I*f*x^p)^3-I/m*ln(d+e*x^m)/e*ln(f)*Pi*csgn(I
*f*x^p)^3-I/m^2*p*dilog((d+e*x^m)/d)/e*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^
p)-I/m*ln(d+e*x^m)/e*p*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^p)^2-1/m*ln(d+e*x^m)/e
*Pi^2*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)^4+2/m^2*p^2/e*ln(x)*polylog(2,-e*
x^m/d)+2/m^2*p*(ln(x^p)-p*ln(x))*dilog((d+e*x^m)/d)/e-1/4/m*ln(d+e*x^m)/e*P
i^2*csgn(I*f*x^p)^6+2/m*ln(d+e*x^m)/e*ln(x^p)*ln(f)+2/m^2*p*dilog((d+e*x^m)
/d)/e*ln(f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}\log\left(fx^p\right)^2}{ex^m+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

Fricas [C] time = 1.58564, size = 257, normalized size = 3.43

$$\frac{m^2\log\left(ex^m+d\right)\log\left(f\right)^2-2\,p^2\mathrm{polylog}\left(3,-\frac{ex^m}{d}\right)+2\left(mp^2\log\left(x\right)+mp\log\left(f\right)\right)\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+\left(m^2p^2\log\left(x\right)^2+2\,m\right.}{em^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="fricas")
```

```
[Out] (m^2*log(e*x^m + d)*log(f)^2 - 2*p^2*polylog(3, -e*x^m/d) + 2*(m*p^2*log(x)
 + m*p*log(f))*dilog(-(e*x^m + d)/d + 1) + (m^2*p^2*log(x)^2 + 2*m^2*p*log(
f)*log(x))*log((e*x^m + d)/d))/(e*m^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*ln(f*x**p)**2/(d+e*x**m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}\log\left(fx^p\right)^2}{ex^m+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

3.621 $\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=161

$$-\frac{6bnp^2\log(fx^p)\,\text{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{3bnp\log^2(fx^p)\,\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^3(fx^p)\,\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{6bnp^3}{}$$

```
[Out] (Log[f*x^p]^4*(a + b*Log[c*(d + e*x^m)^n]))/(4*p) - (b*n*Log[f*x^p]^4*Log[1
 + (e*x^m)/d])/(4*p) - (b*n*Log[f*x^p]^3*PolyLog[2, -((e*x^m)/d)])/m + (3*b
*n*p*Log[f*x^p]^2*PolyLog[3, -((e*x^m)/d)])/m^2 - (6*b*n*p^2*Log[f*x^p]*Pol
yLog[4, -((e*x^m)/d)])/m^3 + (6*b*n*p^3*PolyLog[5, -((e*x^m)/d)])/m^4
```

Rubi [A] time = 0.224429, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2481, 2337, 2374, 2383, 6589}

$$-\frac{6bnp^2\log(fx^p)\,\text{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{3bnp\log^2(fx^p)\,\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^3(fx^p)\,\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{6bnp^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```


```
[Out] (Log[f*x^p]^4*(a + b*Log[c*(d + e*x^m)^n]))/(4*p) - (b*n*Log[f*x^p]^4*Log[1
 + (e*x^m)/d])/(4*p) - (b*n*Log[f*x^p]^3*PolyLog[2, -((e*x^m)/d)])/m + (3*b
*n*p*Log[f*x^p]^2*PolyLog[3, -((e*x^m)/d)])/m^2 - (6*b*n*p^2*Log[f*x^p]*Pol
yLog[4, -((e*x^m)/d)])/m^3 + (6*b*n*p^3*PolyLog[5, -((e*x^m)/d)])/m^4
```


Rule 2481

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^q]^(m + 1)*(a + b*Log[c*(
d + e*x^n)^p]))/(q*(m + 1)), x] - Dist[(b*e*n*p)/(q*(m + 1)), Int[(x^(n - 1
)*Log[f*x^q]^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{x}\,dx = \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{(bemn)\int \frac{x^{-1+m}\log^4\left(fx^p\right)}{d+ex^m}\,dx}{4p}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{4p} + (b$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{4p} - \frac{bn}{}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{4p} - \frac{bn}{}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{4p} - \frac{bn}{}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{4p} - \frac{bn}{}$$

Mathematica [B] time = 0.255645, size = 659, normalized size = 4.09

$$\frac{6bnp^2\log\left(fx^p\right)\text{PolyLog}\left(4,-\frac{dx^{-m}}{e}\right)}{m^3} + \frac{3bnp\log^2\left(fx^p\right)\text{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2} + \frac{bnp\log(x)\left(3\log^2\left(fx^p\right)-3p\log(x)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (-3*b*m*n*p^3*Log[x]^5)/10 + (3*b*m*n*p^2*Log[x]^4*Log[f*x^p])/4 - (b*m*n*p
*Log[x]^3*Log[f*x^p]^2)/2 + (a*Log[f*x^p]^4)/(4*p) - (3*b*n*p^3*Log[x]^4*Lo
g[1 + d/(e*x^m)])/4 + 2*b*n*p^2*Log[x]^3*Log[f*x^p]*Log[1 + d/(e*x^m)] - (3
*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[1 + d/(e*x^m)])/2 + b*n*p^3*Log[x]^4*Log[d
 + e*x^m] - (b*n*p^3*Log[x]^3*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m - 3*b*n*p
^2*Log[x]^3*Log[f*x^p]*Log[d + e*x^m] + (3*b*n*p^2*Log[x]^2*Log[-((e*x^m)/d
)]*Log[f*x^p]*Log[d + e*x^m])/m + 3*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[d + e*x
^m] - (3*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]^2*Log[d + e*x^m])/m - b*
n*Log[x]*Log[f*x^p]^3*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^3*
Log[d + e*x^m])/m - (b*p^3*Log[x]^4*Log[c*(d + e*x^m)^n])/4 + b*p^2*Log[x]^
3*Log[f*x^p]*Log[c*(d + e*x^m)^n] - (3*b*p*Log[x]^2*Log[f*x^p]^2*Log[c*(d +
 e*x^m)^n])/2 + b*Log[x]*Log[f*x^p]^3*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*
(p^2*Log[x]^2 - 3*p*Log[x]*Log[f*x^p] + 3*Log[f*x^p]^2)*PolyLog[2, -(d/(e*x
^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])^3*PolyLog[2, 1 + (e*x^m)/d])/m + (3
```

```
*b*n*p*Log[f*x^p]^2*PolyLog[3, -(d/(e*x^m))])/m^2 + (6*b*n*p^2*Log[f*x^p]*P
olyLog[4, -(d/(e*x^m))])/m^3 + (6*b*n*p^3*PolyLog[5, -(d/(e*x^m))])/m^4
```

Maple [F] time = 0.51, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(fx^{p}\right)\right)^{3}\left(a+b\ln\left(c\left(d+ex^{m}\right)^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

```
[Out] int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\left(bp^{3}\log\left(x\right)^{4}-4\,bp^{2}\log\left(f\right)\log\left(x\right)^{3}+6\,bp\log\left(f\right)^{2}\log\left(x\right)^{2}-4\,b\log\left(f\right)^{3}\log\left(x\right)-4\,b\log\left(x\right)\log\left(x^{p}\right)^{3}+6\left(bp\log\left(x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b*p^3*log(x)^4 - 4*b*p^2*log(f)*log(x)^3 + 6*b*p*log(f)^2*log(x)^2 -
4*b*log(f)^3*log(x) - 4*b*log(x)*log(x^p)^3 + 6*(b*p*log(x)^2 - 2*b*log(f)*
log(x))*log(x^p)^2 - 4*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)
^2*log(x))*log(x^p))*log((e*x^m + d)^n) - integrate(-1/4*(4*b*d*log(c)*log(
f)^3 + 4*a*d*log(f)^3 + 4*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c)
- a*e)*x^m)*log(x^p)^3 + 6*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p
*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x
^m)*log(x^p)^2 + (b*e*m*n*p^3*log(x)^4 - 4*b*e*m*n*p^2*log(f)*log(x)^3 + 6*
b*e*m*n*p*log(f)^2*log(x)^2 - 4*b*e*m*n*log(f)^3*log(x) + 4*b*e*log(c)*log(
f)^3 + 4*a*e*log(f)^3)*x^m + 4*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2 - (b
*e*m*n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*log(f)^2*log(
x) - 3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m)*log(x^p))/(e*x*x^m + d*x)
, x)
```

Fricas [C] time = 1.7741, size = 1072, normalized size = 6.66

$$\frac{24\,bnp^{3}\mathrm{polylog}\left(5,-\frac{ex^{m}}{d}\right)+4\left(bm^{4}\log\left(c\right)+am^{4}\right)\log\left(f\right)^{3}\log\left(x\right)+6\left(bm^{4}p\log\left(c\right)+am^{4}p\right)\log\left(f\right)^{2}\log\left(x\right)^{2}+4\left(bm^{4}p^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(24*b*n*p^3*polylog(5, -e*x^m/d) + 4*(b*m^4*log(c) + a*m^4)*log(f)^3*lo
g(x) + 6*(b*m^4*p*log(c) + a*m^4*p)*log(f)^2*log(x)^2 + 4*(b*m^4*p^2*log(c)
 + a*m^4*p^2)*log(f)*log(x)^3 + (b*m^4*p^3*log(c) + a*m^4*p^3)*log(x)^4 - 4
*(b*m^3*n*p^3*log(x)^3 + 3*b*m^3*n*p^2*log(f)*log(x)^2 + 3*b*m^3*n*p*log(f)
^2*log(x) + b*m^3*n*log(f)^3)*dilog(-(e*x^m + d)/d + 1) + (b*m^4*n*p^3*log(
```

```
x)^4 + 4*b*m^4*n*p^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*
m^4*n*log(f)^3*log(x))*log(e*x^m + d) - (b*m^4*n*p^3*log(x)^4 + 4*b*m^4*n*p
^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*m^4*n*log(f)^3*log
(x))*log((e*x^m + d)/d) - 24*(b*m*n*p^3*log(x) + b*m*n*p^2*log(f))*polylog(
4, -e*x^m/d) + 12*(b*m^2*n*p^3*log(x)^2 + 2*b*m^2*n*p^2*log(f)*log(x) + b*m
^2*n*p*log(f)^2)*polylog(3, -e*x^m/d))/m^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)**3*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \log\left((ex^m + d)^n c\right) + a\right) \log\left(fx^p\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^3/x, x)
```

3.622 $\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=132

$$\frac{2bnp\log\left(fx^p\right)\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^2\left(fx^p\right)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}-\frac{2bnp^2\text{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{\log^3\left(fx^p\right)\left(a+b\log\right.}{3p}$$

```
[Out] (Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/(3*p) - (b*n*Log[f*x^p]^3*Log[1
 + (e*x^m)/d])/(3*p) - (b*n*Log[f*x^p]^2*PolyLog[2, -((e*x^m)/d)])/m + (2*b
*n*p*Log[f*x^p]*PolyLog[3, -((e*x^m)/d)])/m^2 - (2*b*n*p^2*PolyLog[4, -((e*
x^m)/d)])/m^3
```

Rubi [A] time = 0.187696, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2481, 2337, 2374, 2383, 6589}

$$\frac{2bnp\log\left(fx^p\right)\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^2\left(fx^p\right)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}-\frac{2bnp^2\text{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{\log^3\left(fx^p\right)\left(a+b\log\right.}{3p}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```


```
[Out] (Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/(3*p) - (b*n*Log[f*x^p]^3*Log[1
 + (e*x^m)/d])/(3*p) - (b*n*Log[f*x^p]^2*PolyLog[2, -((e*x^m)/d)])/m + (2*b
*n*p*Log[f*x^p]*PolyLog[3, -((e*x^m)/d)])/m^2 - (2*b*n*p^2*PolyLog[4, -((e*
x^m)/d)])/m^3
```


Rule 2481

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^q]^(m + 1)*(a + b*Log[c*(
d + e*x^n)^p]))/(q*(m + 1)), x] - Dist[(b*e*n*p)/(q*(m + 1)), Int[(x^(n - 1
)*Log[f*x^q]^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{x}\,dx = \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{(bemn)\int\frac{x^{-1+m}\log^3\left(fx^p\right)}{d+ex^m}\,dx}{3p}$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{3p} + (b$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{3p} - \frac{bn}{}$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{3p} - \frac{bn}{}$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{3p} - \frac{bn}{}$$

Mathematica [B] time = 0.22785, size = 456, normalized size = 3.45

$$\frac{2bnp\log\left(fx^p\right)\text{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2} - \frac{bnp\log(x)\left(p\log(x)-2\log\left(fx^p\right)\right)\text{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} + \frac{bn\left(\log\left(fx^p\right)-p\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (b*m*n*p^2*Log[x]^4)/4 - (b*m*n*p*Log[x]^3*Log[f*x^p])/3 + (a*Log[f*x^p]^3)
/(3*p) + (2*b*n*p^2*Log[x]^3*Log[1 + d/(e*x^m)])/3 - b*n*p*Log[x]^2*Log[f*x
^p]*Log[1 + d/(e*x^m)] - b*n*p^2*Log[x]^3*Log[d + e*x^m] + (b*n*p^2*Log[x]^
2*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m + 2*b*n*p*Log[x]^2*Log[f*x^p]*Log[d +
 e*x^m] - (2*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m -
b*n*Log[x]*Log[f*x^p]^2*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^
2*Log[d + e*x^m])/m + (b*p^2*Log[x]^3*Log[c*(d + e*x^m)^n])/3 - b*p*Log[x]^
2*Log[f*x^p]*Log[c*(d + e*x^m)^n] + b*Log[x]*Log[f*x^p]^2*Log[c*(d + e*x^m)
^n] - (b*n*p*Log[x]*(p*Log[x] - 2*Log[f*x^p])*PolyLog[2, -(d/(e*x^m))])/m +
 (b*n*(-(p*Log[x]) + Log[f*x^p])^2*PolyLog[2, 1 + (e*x^m)/d])/m + (2*b*n*p*
Log[f*x^p]*PolyLog[3, -(d/(e*x^m))])/m^2 + (2*b*n*p^2*PolyLog[4, -(d/(e*x^m
))])/m^3
```

Maple [F] time = 0.458, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(fx^p\right)\right)^2\left(a+b\ln\left(c\left(d+ex^m\right)^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

```
[Out] int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(bp^2\log(x)^3 - 3\,bp\log\left(f\right)\log(x)^2 + 3\,b\log\left(f\right)^2\log(x) + 3\,b\log(x)\log(x^p)^2 - 3\left(bp\log(x)^2 - 2\,b\log\left(f\right)\log(x)\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)^2*log(x) + 3*b*log
(x)*log(x^p)^2 - 3*(b*p*log(x)^2 - 2*b*log(f)*log(x))*log(x^p))*log((e*x^m
+ d)^n) - integrate(-1/3*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2 + 3*(b*d*l
og(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p)^2 - (b*e*m*
n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*log(f)^2*log(x) -
3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m + 3*(2*b*d*log(c)*log(f) + 2*a*
d*log(f) + (b*e*m*n*p*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log
(f) + 2*a*e*log(f))*x^m)*log(x^p))/(e*x*x^m + d*x), x)
```

Fricas [C] time = 1.75331, size = 724, normalized size = 5.48

$$-\frac{6\,bnp^2\mathrm{polylog}\left(4,-\frac{ex^m}{d}\right) - 3\left(bm^3\log(c) + am^3\right)\log\left(f\right)^2\log(x) - 3\left(bm^3p\log(c) + am^3p\right)\log\left(f\right)\log(x)^2 - \left(bm^3p^2\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*b*n*p^2*polylog(4, -e*x^m/d) - 3*(b*m^3*log(c) + a*m^3)*log(f)^2*lo
g(x) - 3*(b*m^3*p*log(c) + a*m^3*p)*log(f)*log(x)^2 - (b*m^3*p^2*log(c) + a
*m^3*p^2)*log(x)^3 + 3*(b*m^2*n*p^2*log(x)^2 + 2*b*m^2*n*p*log(f)*log(x) +
b*m^2*n*log(f)^2)*dilog(-(e*x^m + d)/d + 1) - (b*m^3*n*p^2*log(x)^3 + 3*b*m
^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*log(e*x^m + d) + (b*m^3
*n*p^2*log(x)^3 + 3*b*m^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*
log((e*x^m + d)/d) - 6*(b*m*n*p^2*log(x) + b*m*n*p*log(f))*polylog(3, -e*x^
m/d))/m^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)**2*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left((ex^m+d)^n c\right)+a\right)\log\left(fx^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^2/x, x)
```

3.623 $\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=102

$$-\frac{bn\log\left(fx^p\right)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{bnp\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}+\frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p}-\frac{bn\log^2\left(fx^p\right)\log\left(\right.}{2p}$$

```
[Out] (Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/(2*p) - (b*n*Log[f*x^p]^2*Log[1
 + (e*x^m)/d])/(2*p) - (b*n*Log[f*x^p]*PolyLog[2, -((e*x^m)/d)])/m + (b*n*p
*PolyLog[3, -((e*x^m)/d)])/m^2
```

Rubi [A] time = 0.144893, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2481, 2337, 2374, 6589}

$$-\frac{bn\log\left(fx^p\right)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{bnp\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}+\frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p}-\frac{bn\log^2\left(fx^p\right)\log\left(\right.}{2p}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/(2*p) - (b*n*Log[f*x^p]^2*Log[1
 + (e*x^m)/d])/(2*p) - (b*n*Log[f*x^p]*PolyLog[2, -((e*x^m)/d)])/m + (b*n*p
*PolyLog[3, -((e*x^m)/d)])/m^2
```

Rule 2481

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^q]^(m + 1)*(a + b*Log[c*(
d + e*x^n)^p]))/(q*(m + 1)), x] - Dist[(b*e*n*p)/(q*(m + 1)), Int[(x^(n - 1
)*Log[f*x^q]^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{x}\,dx = \frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p} - \frac{(bemn)\int \frac{x^{-1+m}\log^2\left(fx^p\right)}{d+ex^m}\,dx}{2p}$$

$$= \frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p} - \frac{bn\log^2\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{2p} + (bn$$

$$= \frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p} - \frac{bn\log^2\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{2p} - bn\,l$$

$$= \frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p} - \frac{bn\log^2\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{2p} - bn\,l$$

Mathematica [B] time = 0.179673, size = 265, normalized size = 2.6

$$-\frac{bn\left(p\log(x)-\log\left(fx^p\right)\right)\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)}{m} + \frac{bnp\text{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2} + \frac{bnp\log(x)\text{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} + a\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] -(b*m*n*p*Log[x]^3)/6 + (a*Log[f*x^p]^2)/(2*p) - (b*n*p*Log[x]^2*Log[1 + d/
(e*x^m)])/2 + b*n*p*Log[x]^2*Log[d + e*x^m] - (b*n*p*Log[x]*Log[-((e*x^m)/d
)]*Log[d + e*x^m])/m - b*n*Log[x]*Log[f*x^p]*Log[d + e*x^m] + (b*n*Log[-((e
*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m - (b*p*Log[x]^2*Log[c*(d + e*x^m)^n]
)/2 + b*Log[x]*Log[f*x^p]*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*PolyLog[2, -
(d/(e*x^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])*PolyLog[2, 1 + (e*x^m)/d])/m
 + (b*n*p*PolyLog[3, -(d/(e*x^m))])/m^2
```

Maple [F] time = 0.46, size = 0, normalized size = 0.

$$\int \frac{\ln\left(fx^p\right)\left(a+b\ln\left(c\left(d+ex^m\right)^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

```
[Out] int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(bp\log(x)^2-2b\log\left(f\right)\log(x)-2b\log(x)\log\left(x^p\right)\right)\log\left(\left(ex^m+d\right)^n\right)-\int -\frac{2bd\log(c)\log\left(f\right)+2ad\log\left(f\right)+(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*p*log(x)^2 - 2*b*log(f)*log(x) - 2*b*log(x)*log(x^p))*log((e*x^m +
d)^n) - integrate(-1/2*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p*log
(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x^m +
 2*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p))/(
e*x*x^m + d*x), x)
```

Fricas [C] time = 1.67641, size = 423, normalized size = 4.15

$$2\,bnp\mathrm{polylog}\left(3,-\frac{ex^m}{d}\right)+2\left(bm^2\log(c)+am^2\right)\log(f)\log(x)+\left(bm^2p\log(c)+am^2p\right)\log(x)^2-2\left(bmnp\log(x)+bmn\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*n*p*polylog(3, -e*x^m/d) + 2*(b*m^2*log(c) + a*m^2)*log(f)*log(x)
+ (b*m^2*p*log(c) + a*m^2*p)*log(x)^2 - 2*(b*m*n*p*log(x) + b*m*n*log(f))*d
ilog(-(e*x^m + d)/d + 1) + (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*l
og(e*x^m + d) - (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*log((e*x^m +
 d)/d))/m^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log\left((ex^m+d)^n c\right)+a\right)\log\left(fx^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)/x, x)
```

3.624 $\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx$

Optimal. Leaf size=49

$$\frac{bn\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)}{m}+\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}$$

```
[Out] (Log[-((e*x^m)/d)]*(a + b*Log[c*(d + e*x^m)^n]))/m + (b*n*PolyLog[2, 1 + (e
*x^m)/d])/m
```

Rubi [A] time = 0.0528, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2454, 2394, 2315}

$$\frac{bn\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)}{m}+\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

```
[Out] (Log[-((e*x^m)/d)]*(a + b*Log[c*(d + e*x^m)^n]))/m + (b*n*PolyLog[2, 1 + (e
*x^m)/d])/m
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x}\,dx&=\frac{\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx,x,x^m\right)}{m}\\&=\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}-\frac{(ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^m\right)}{m}\\&=\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}+\frac{bn\text{Li}_2\left(1+\frac{ex^m}{d}\right)}{m}\end{aligned}$$

Mathematica [A] time = 0.0147763, size = 49, normalized size = 1.

$$\frac{b\left(n\text{PolyLog}\left(2,\frac{d+ex^m}{d}\right)+\log\left(-\frac{ex^m}{d}\right)\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

```
[Out] a*Log[x] + (b*(Log[-((e*x^m)/d)]*Log[c*(d + e*x^m)^n] + n*PolyLog[2, (d + e
*x^m)/d]))/m
```

Maple [C] time = 3.542, size = 189, normalized size = 3.9

$$b\ln(x)\ln\left(\left(d+ex^m\right)^n\right)-\frac{i}{2}\ln(x)b\pi\,\text{csgn}(ic)\,\text{csgn}\left(i\left(d+ex^m\right)^n\right)\text{csgn}\left(ic\left(d+ex^m\right)^n\right)+\frac{i}{2}\ln(x)b\pi\,\text{csgn}(ic)\left(\text{csgn}\left(ic\left(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x,x)
```

```
[Out] b*ln(x)*ln((d+e*x^m)^n)-1/2*I*ln(x)*b*Pi*csgn(I*c)*csgn(I*(d+e*x^m)^n)*csgn
(I*c*(d+e*x^m)^n)+1/2*I*ln(x)*b*Pi*csgn(I*c)*csgn(I*c*(d+e*x^m)^n)^2+1/2*I*
ln(x)*b*Pi*csgn(I*(d+e*x^m)^n)*csgn(I*c*(d+e*x^m)^n)^2-1/2*I*ln(x)*b*Pi*csg
n(I*c*(d+e*x^m)^n)^3+ln(c)*ln(x)*b+ln(x)*a-b/m*n*dilog((d+e*x^m)/d)-b*n*ln(
x)*ln((d+e*x^m)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(2\,dmn\int\frac{\log(x)}{exx^m+dx}\,dx-mn\log(x)^2+2\,\log\left(\left(ex^m+d\right)^n\right)\log(x)+2\,\log(c)\log(x)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*m*n*integrate(log(x)/(e*x*x^m + d*x), x) - m*n*log(x)^2 + 2*log((e
*x^m + d)^n)*log(x) + 2*log(c)*log(x))*b + a*log(x)
```

Fricas [A] time = 1.66569, size = 171, normalized size = 3.49

$$\frac{bmn\log\left(ex^m+d\right)\log(x)-bmn\log(x)\log\left(\frac{ex^m+d}{d}\right)-bn\text{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+\left(bm\log(c)+am\right)\log(x)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] (b*m*n*log(e*x^m + d)*log(x) - b*m*n*log(x)*log((e*x^m + d)/d) - b*n*dilog(
-(e*x^m + d)/d + 1) + (b*m*log(c) + a*m)*log(x))/m
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\log\left(c\left(d + ex^m\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**m)**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(\left(ex^m + d\right)^n c\right) + a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/x, x)
```

3.625 $\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx$

Optimal. Leaf size=41

$$b\text{Unintegrable}\left(\frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)},x\right)+\frac{a\log\left(\log\left(fx^p\right)\right)}{p}$$

```
[Out] (a*Log[Log[f*x^p]])/p + b*Unintegrable[Log[c*(d + e*x^m)^n]/(x*Log[f*x^p]),
 x]
```

Rubi [A] time = 0.292362, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

```
[Out] (a*Log[Log[f*x^p]])/p + b*Defer[Int][Log[c*(d + e*x^m)^n]/(x*Log[f*x^p]), x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx &= \int \left(\frac{a}{x\log\left(fx^p\right)}+\frac{b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\right)dx\\
&= a\int \frac{1}{x\log\left(fx^p\right)}\,dx+b\int \frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx\\
&= b\int \frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx+\frac{a\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,\log\left(fx^p\right)\right)}{p}\\
&= \frac{a\log\left(\log\left(fx^p\right)\right)}{p}+b\int \frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx
\end{aligned}$$

Mathematica [A] time = 0.54297, size = 0, normalized size = 0.

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]), x]
```

Maple [A] time = 0.474, size = 0, normalized size = 0.

$$\int \frac{a + b\ln\left(c\left(d + e x^{m}\right)^{n}\right)}{x\ln\left(f x^{p}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
```

```
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\log\left(\left(e x^{m} + d\right)^{n}\right) + \log\left(c\right)}{x\log\left(f\right) + x\log\left(x^{p}\right)}\,dx + \frac{a\log\left(\log\left(f x^{p}\right)\right)}{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="maxima")
```

```
[Out] b*integrate((log((e*x^m + d)^n) + log(c))/(x*log(f) + x*log(x^p)), x) + a*l
og(log(f*x^p))/p
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(\left(e x^{m} + d\right)^{n} c\right) + a}{x\log\left(f x^{p}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(\left(e x^{m} + d\right)^{n} c\right) + a}{x\log\left(f x^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

3.626 $\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^2\left(fx^p\right)}\,dx$

Optimal. Leaf size=63

$$\frac{bemn\text{Unintegrable}\left(\frac{x^{m-1}}{(d+ex^m)\log(fx^p)},x\right)}{p}-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{p\log\left(fx^p\right)}$$

```
[Out] -((a + b*Log[c*(d + e*x^m)^n])/(p*Log[f*x^p])) + (b*e*m*n*Unintegrable[x^(-
1 + m)/((d + e*x^m)*Log[f*x^p]), x])/p
```

Rubi [A] time = 0.119513, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]

[Out] -((a + b*Log[c*(d + e*x^m)^n])/(p*Log[f*x^p])) + (b*e*m*n*Defer[Int][x^(-1
+ m)/((d + e*x^m)*Log[f*x^p]), x])/p
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^2\left(fx^p\right)}\,dx=-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{p\log\left(fx^p\right)}+\frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log(fx^p)}\,dx}{p}$$

Mathematica [A] time = 2.00025, size = 0, normalized size = 0.

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]

[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2), x]
```

Maple [A] time = 0.48, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c\left(d+ex^m\right)^n\right)}{x\left(\ln\left(fx^p\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\left(emn\int \frac{x^m}{epxx^m\log\left(f\right)+dpx\log\left(f\right)+\left(epxx^m+dpx\right)\log\left(x^p\right)}\,dx-\frac{\log\left(\left(ex^m+d\right)^n\right)+\log\left(c\right)}{p\log\left(f\right)+p\log\left(x^p\right)}\right)b-\frac{a}{p\log\left(fx^p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="maxima")
```

```
[Out] (e*m*n*integrate(x^m/(e*p*x*x^m*log(f) + d*p*x*log(f) + (e*p*x*x^m + d*p*x)
*log(x^p)), x) - (log((e*x^m + d)^n) + log(c))/(p*log(f) + p*log(x^p)))*b -
 a/(p*log(f*x^p))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\log\left(\left(ex^m+d\right)^nc\right)+a}{x\log\left(fx^p\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(\left(ex^m+d\right)^nc\right)+a}{x\log\left(fx^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

3.627 $\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx$

Optimal. Leaf size=68

$$\frac{bemn\text{Unintegrable}\left(\frac{x^{m-1}}{(d+ex^m)\log^2(fx^p)},x\right)}{2p}-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{2p\log^2\left(fx^p\right)}$$

```
[Out] -(a + b*Log[c*(d + e*x^m)^n])/(2*p*Log[f*x^p]^2) + (b*e*m*n*Unintegrable[x^
(-1 + m)/((d + e*x^m)*Log[f*x^p]^2), x])/(2*p)
```

Rubi [A] time = 0.116714, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^3\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

```
[Out] -(a + b*Log[c*(d + e*x^m)^n])/(2*p*Log[f*x^p]^2) + (b*e*m*n*Defer[Int][x^(-
1 + m)/((d + e*x^m)*Log[f*x^p]^2), x])/(2*p)
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^3\left(fx^p\right)}\,dx=-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{2p\log^2\left(fx^p\right)}+\frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log^2(fx^p)}\,dx}{2p}$$

Mathematica [A] time = 10.6062, size = 0, normalized size = 0.

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^3\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3), x]
```

Maple [A] time = 0.468, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c\left(d+ex^m\right)^n\right)}{x\left(\ln\left(fx^p\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(2\,dem^2n\int\frac{x^m}{2\left(e^2p^2xx^{2\,m}\log\left(f\right)+2\,dep^2xx^m\log\left(f\right)+d^2p^2x\log\left(f\right)+\left(e^2p^2xx^{2\,m}+2\,dep^2xx^m+d^2p^2x\right)\log\left(x^p\right)\right)}\,dx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*e*m^2*n*integrate(1/2*x^m/(e^2*p^2*x*x^(2*m)*log(f) + 2*d*e*p^2*x*
x^m*log(f) + d^2*p^2*x*log(f) + (e^2*p^2*x*x^(2*m) + 2*d*e*p^2*x*x^m + d^2*
p^2*x)*log(x^p)), x) - (e*m*n*x^m*log(x^p) + d*p*log(c) + (e*m*n*log(f) + e
*p*log(c))*x^m + (e*p*x^m + d*p)*log((e*x^m + d)^n))/(e*p^2*x^m*log(f)^2 +
d*p^2*log(f)^2 + (e*p^2*x^m + d*p^2)*log(x^p)^2 + 2*(e*p^2*x^m*log(f) + d*p
^2*log(f))*log(x^p)))*b - 1/2*a/(p*log(f*x^p)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\log\left(\left(ex^m+d\right)^nc\right)+a}{x\log\left(fx^p\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\log\left(\left(ex^m+d\right)^nc\right)+a}{x\log\left(fx^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

3.628 $\int \log \left(c \left(d+e(f+gx)^p\right)^q\right) \, dx$

Optimal. Leaf size=76

$$\frac{(f+gx)\log \left(c \left(d+e(f+gx)^p\right)^q\right)}{g}-\frac{epq(f+gx)^{p+1}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(p+1)}$$

```
[Out] -((e*p*q*(f + g*x)^(1 + p)*Hypergeometric2F1[1, 1 + p^(-1), 2 + p^(-1), -((
e*(f + g*x)^p)/d)])/(d*g*(1 + p))) + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q
])/g
```

Rubi [A] time = 0.0443628, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2483, 2448, 364}

$$\frac{(f+gx)\log \left(c \left(d+e(f+gx)^p\right)^q\right)}{g}-\frac{epq(f+gx)^{p+1}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^p)^q],x]
```

```
[Out] -((e*p*q*(f + g*x)^(1 + p)*Hypergeometric2F1[1, 1 + p^(-1), 2 + p^(-1), -((
e*(f + g*x)^p)/d)])/(d*g*(1 + p))) + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q
])/g
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \log\left(c\left(d+e(f+gx)^p\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c(d+ex^p)^q\right)dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+e(f+gx)^p\right)^q\right)}{g} - \frac{(epq)\,\text{Subst}\left(\int \frac{x^p}{d+ex^p}dx, x, f+gx\right)}{g}$$
$$= -\frac{epq(f+gx)^{1+p}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(1+p)} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^p\right)^q\right)}{g}$$

Mathematica [A] time = 0.024157, size = 65, normalized size = 0.86

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^p\right)^q\right)}{g} + \frac{pq(f+gx)\,{}_2F_1\left(1,\frac{1}{p};1+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{g} - pqx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^p)^q],x]
```

```
[Out] -(p*q*x) + (p*q*(f + g*x)*Hypergeometric2F1[1, p^(-1), 1 + p^(-1), -((e*(f
+ g*x)^p)/d)])/g + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q])/g
```

Maple [F] time = 2.597, size = 0, normalized size = 0.

$$\int \ln\left(c\left(d+e\left(gx+f\right)^p\right)^q\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

```
[Out] int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$dgpq\int \frac{x}{dgx+\left(egx+ef\right)\left(gx+f\right)^p+df}dx + \frac{fpq\log\left(gx+f\right)+gx\log\left(\left(\left(gx+f\right)^p e+d\right)^q\right)-\left(gpq-g\log\left(c\right)\right)x}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="maxima")
```

```
[Out] d*g*p*q*integrate(x/(d*g*x + (e*g*x + e*f)*(g*x + f)^p + d*f), x) + (f*p*q*
log(g*x + f) + g*x*log(((g*x + f)^p*e + d)^q) - (g*p*q - g*log(c))*x)/g
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\left(\left(gx+f\right)^p e+d\right)^q c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="fricas")
```

```
[Out] integral(log(((g*x + f)^p*e + d)^q*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(c\left(d+e\left(f+gx\right)^{p}\right)^{q}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**p)**q),x)
```

```
[Out] Integral(log(c*(d + e*(f + g*x)**p)**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(\left(\left(gx+f\right)^{p}e+d\right)^{q}c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="giac")
```

```
[Out] integrate(log(((g*x + f)^p*e + d)^q*c), x)
```

3.629 $\int \log \left(c \left(d+e(f+gx)^3\right)^q\right) dx$

Optimal. Leaf size=169

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}-\frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g}-$$

```
[Out] -3*q*x - (Sqrt[3]*d^(1/3)*q*ArcTan[(d^(1/3) - 2*e^(1/3)*(f + g*x))/(Sqrt[3]
*d^(1/3))])/(e^(1/3)*g) + (d^(1/3)*q*Log[d^(1/3) + e^(1/3)*(f + g*x)])/(e^(
1/3)*g) - (d^(1/3)*q*Log[d^(2/3) - d^(1/3)*e^(1/3)*(f + g*x) + e^(2/3)*(f +
 g*x)^2])/(2*e^(1/3)*g) + ((f + g*x)*Log[c*(d + e*(f + g*x)^3)^q])/g
```

Rubi [A] time = 0.205736, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2483, 2448, 321, 200, 31, 634, 617, 204, 628}

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}-\frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^3)^q],x]
```


```
[Out] -3*q*x - (Sqrt[3]*d^(1/3)*q*ArcTan[(d^(1/3) - 2*e^(1/3)*(f + g*x))/(Sqrt[3]
*d^(1/3))])/(e^(1/3)*g) + (d^(1/3)*q*Log[d^(1/3) + e^(1/3)*(f + g*x)])/(e^(
1/3)*g) - (d^(1/3)*q*Log[d^(2/3) - d^(1/3)*e^(1/3)*(f + g*x) + e^(2/3)*(f +
 g*x)^2])/(2*e^(1/3)*g) + ((f + g*x)*Log[c*(d + e*(f + g*x)^3)^q])/g
```


Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \log\left(c\left(d+e(f+gx)^3\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+ex^3\right)^q\right)dx,x,f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} - \frac{(3eq)\,\text{Subst}\left(\int \frac{x^3}{d+ex^3}dx,x,f+gx\right)}{g}$$
$$= -3qx + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} + \frac{(3dq)\,\text{Subst}\left(\int \frac{1}{d+ex^3}dx,x,f+gx\right)}{g}$$
$$= -3qx + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} + \frac{\left(\sqrt[3]{d}q\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}dx,x,f+gx\right)}{g}$$
$$= -3qx + \frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} + \frac{\left(3d^{2/3}q\right)}{}$$
$$= -3qx + \frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g} - \frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)\right.}{2\sqrt[3]{e}g}$$
$$= -3qx - \frac{\sqrt{3}\sqrt[3]{d}q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}g} + \frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g} - \frac{\sqrt[3]{d}q\log\left(d^{2/3}-\right.}{}$$

Mathematica [A] time = 0.0991185, size = 147, normalized size = 0.87

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} + \frac{\sqrt[3]{d}q\left(-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)+2\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)+2\right.}{2\sqrt[3]{e}g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^3)^q],x]
```

```
[Out] -3*q*x + (d^(1/3)*q*(2*Sqrt[3]*ArcTan[(-d^(1/3) + 2*e^(1/3)*(f + g*x))/(Sqr
t[3]*d^(1/3))] + 2*Log[d^(1/3) + e^(1/3)*(f + g*x)] - Log[d^(2/3) - d^(1/3)
*e^(1/3)*(f + g*x) + e^(2/3)*(f + g*x)^2]))/(2*e^(1/3)*g) + ((f + g*x)*Log[
c*(d + e*(f + g*x)^3)^q])/g
```

Maple [C] time = 0.569, size = 145, normalized size = 0.9

$$\ln\left(c\left(eg^3x^3+3efg^2x^2+3ef^2gx+ef^3+d\right)^q\right)x - 3qx - \frac{q}{eg}\sum_{\_R=\text{RootOf}\left(eg^3\_Z^3+3efg^2\_Z^2+3ef^2g\_Z+ef^3+d\right)} \frac{\left(-\_R^2efg^2-2\_\right.}{g^2\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^3)^q),x)
```

```
[Out] ln(c*(e*g^3*x^3+3*e*f*g^2*x^2+3*e*f^2*g*x+e*f^3+d)^q)*x-3*q*x-1/g/e*q*sum((
-_R^2*e*f*g^2-2*_R*e*f^2*g-e*f^3-d)/(_R^2*g^2+2*_R*f*g+f^2)*ln(x-_R),_R=Roo
tOf(_Z^3*e*g^3+3*_Z^2*e*f*g^2+3*_Z*e*f^2*g+e*f^3+d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-(3q-\log(c))x+3q\int\frac{efg^2x^2+2ef^2gx+ef^3+d}{eg^3x^3+3efg^2x^2+3ef^2gx+ef^3+d}\,dx+x\log\left(\left(eg^3x^3+3efg^2x^2+3ef^2gx+ef^3+d\right)^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="maxima")
```

```
[Out] -(3*q - log(c))*x + 3*q*integrate((e*f*g^2*x^2 + 2*e*f^2*g*x + e*f^3 + d)/(
e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d), x) + x*log((e*g^3*x^3
 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d)^q)
```

Fricas [C] time = 9.78525, size = 3071, normalized size = 18.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d) - 12*
g*q*x - 4*sqrt(3)*g*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f
^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*
f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I
*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2)*arctan(-1/24*(2*sqrt(3)*sq
rt(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) +
1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 1
2*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) +
1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g))*(((-1/2*
f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I
*sqrt(3) + 1) - 2*f*q/g)*e*g^2 + 2*e*f*g*q)*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*
d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*
f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 +
d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2) -
sqrt(3)*(8*e*f*g^2*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*
f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*e*g^3 + 12*e*f
^2*g*q^2 + 4*(e*g^3*q*x + 2*e*f*g^2*q)*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^
3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g))*sqr
t((((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3)
)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3
/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g
)*f*g*q + 4*f^2*q^2)/g^2))/(d*q^3)) - 2*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g
^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*l
og(q*x - 1/2*(-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3
)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/
g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*f*q/g)*g + 6*f*q)*log(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q
^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt
(3) + 1) - 2*f*q/g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q
^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt
(3) + 1) - 2*f*q/g)))/g
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**3)**q),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42366, size = 402, normalized size = 2.38

$$qx\log\left(g^3x^3e+3\,fg^2x^2e+3\,f^2gxe+f^3e+d\right)+\sqrt{3}\left(\frac{dq^3}{g^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{g^4xe^2+fg^3e^2+d^{\frac{1}{3}}g^3e^{\frac{5}{3}}}{\sqrt{3}g^4xe^2+\sqrt{3}fg^3e^2-\sqrt{3}d^{\frac{1}{3}}g^3e^{\frac{5}{3}}}\right)e^{\left(-\frac{1}{3}\right)}-\frac{1}{2}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="giac")
```

```
[Out] q*x*log(g^3*x^3*e + 3*f*g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d) + sqrt(3)*(d*q
^3/g^3)^(1/3)*arctan(-(g^4*x*e^2 + f*g^3*e^2 + d^(1/3)*g^3*e^(5/3))/(sqrt(3
)*g^4*x*e^2 + sqrt(3)*f*g^3*e^2 - sqrt(3)*d^(1/3)*g^3*e^(5/3)))*e^(-1/3) -
1/2*(d*q^3/g^3)^(1/3)*e^(-1/3)*log(9*(sqrt(3)*g^4*x*e^2 + sqrt(3)*f*g^3*e^2
 - sqrt(3)*d^(1/3)*g^3*e^(5/3))^2 + 9*(g^4*x*e^2 + f*g^3*e^2 + d^(1/3)*g^3*
e^(5/3))^2) + (d*q^3/g^3)^(1/3)*e^(-1/3)*log(abs(3*g^4*x*e^2 + 3*f*g^3*e^2
+ 3*d^(1/3)*g^3*e^(5/3))) - 3*q*x + x*log(c) + f*q*log(abs(g^3*x^3*e + 3*f*
g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d))/g
```

3.630 $\int \log\left(c\left(d+e(f+gx)^2\right)^q\right)\,dx$

Optimal. Leaf size=63

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}+\frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}-2qx$$

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Rubi [A] time = 0.047829, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2483, 2448, 321, 205}

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}+\frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}-2qx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^2)^q],x]
```

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \log\left(c\left(d+e(f+gx)^2\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+ex^2\right)^q\right)dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} - \frac{(2eq)\,\text{Subst}\left(\int \frac{x^2}{d+ex^2}dx, x, f+gx\right)}{g}$$
$$= -2qx + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} + \frac{(2dq)\,\text{Subst}\left(\int \frac{1}{d+ex^2}dx, x, f+gx\right)}{g}$$
$$= -2qx + \frac{2\sqrt{d}q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

Mathematica [A] time = 0.0351679, size = 63, normalized size = 1.

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} + \frac{2\sqrt{d}q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g} - 2qx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^2)^q],x]
```

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Maple [A] time = 0.171, size = 98, normalized size = 1.6

$$\ln\left(c\left(eg^2x^2+2\,efgx+ef^2+d\right)^q\right)x - 2\,qx + \frac{qf\ln\left(eg^2x^2+2\,efgx+ef^2+d\right)}{g} + 2\,\frac{qd}{g\sqrt{de}}\arctan\left(1/2\,\frac{2\,eg^2x+2\,efg}{g\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^2)^q),x)
```

```
[Out] ln(c*(e*g^2*x^2+2*e*f*g*x+e*f^2+d)^q)*x-2*q*x+q/g*f*ln(e*g^2*x^2+2*e*f*g*x+
e*f^2+d)+2*q/g*d/(d*e)^(1/2)*arctan(1/2*(2*e*g^2*x+2*e*f*g)/g/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6336, size = 458, normalized size = 7.27

$$\left[-\frac{2\,gqx-gx\log\left(c\right)-q\sqrt{-\frac{d}{e}}\log\left(\frac{eg^{2}x^{2}+2\,efgx+ef^{2}+2\,\left(egx+ef\right)\sqrt{-\frac{d}{e}}-d}{eg^{2}x^{2}+2\,efgx+ef^{2}+d}\right)-\left(gqx+fq\right)\log\left(eg^{2}x^{2}+2\,efgx+ef^{2}+d\right)}{g},-\frac{2\,gqx-\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [-(2*g*q*x - g*x*log(c) - q*sqrt(-d/e)*log((e*g^2*x^2 + 2*e*f*g*x + e*f^2 +
 2*(e*g*x + e*f)*sqrt(-d/e) - d)/(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d)) - (g*
q*x + f*q)*log(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g, -(2*g*q*x - g*x*log(c
) - 2*q*sqrt(d/e)*arctan((e*g*x + e*f)*sqrt(d/e)/d) - (g*q*x + f*q)*log(e*g
^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**2)**q),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26182, size = 130, normalized size = 2.06

$$qx\log\left(g^{2}x^{2}e+2\,fgxe+f^{2}e+d\right)+\frac{2\,\sqrt{d}q\arctan\left(\frac{\left(gxe+fe\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{g}-2\,qx+\frac{fq\log\left(g^{2}x^{2}e+2\,fgxe+f^{2}e+d\right)}{g}+x\log\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] q*x*log(g^2*x^2*e + 2*f*g*x*e + f^2*e + d) + 2*sqrt(d)*q*arctan((g*x*e + f*
e)*e^(-1/2)/sqrt(d))*e^(-1/2)/g - 2*q*x + f*q*log(g^2*x^2*e + 2*f*g*x*e + f
^2*e + d)/g + x*log(c)
```

3.631 $\int \log\left(c(d+e(f+gx))^q\right)\,dx$

Optimal. Leaf size=35

$$\frac{(d+ef+egx)\log\left(c(d+e(f+gx))^q\right)}{eg}-qx$$

```
[Out] -(q*x) + ((d + e*f + e*g*x)*Log[c*(d + e*(f + g*x))^q])/(e*g)
```

Rubi [A] time = 0.0160286, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2444, 2389, 2295}

$$\frac{(d+ef+egx)\log\left(c(d+e(f+gx))^q\right)}{eg}-qx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x))^q],x]
```

```
[Out] -(q*x) + ((d + e*f + e*g*x)*Log[c*(d + e*(f + g*x))^q])/(e*g)
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(c(d+e(f+gx))^q\right)\,dx &= \int \log\left(c(d+ef+egx)^q\right)\,dx\\
&= \frac{\operatorname{Subst}\left(\int \log\left(cx^q\right)\,dx, x, d+ef+egx\right)}{eg}\\
&= -qx+\frac{(d+ef+egx)\log\left(c(d+e(f+gx))^q\right)}{eg}
\end{aligned}$$

Mathematica [A] time = 0.031426, size = 47, normalized size = 1.34

$$\frac{(f+gx)\log\left(c(d+e(f+gx))^q\right)}{g}+\frac{dq\log(d+ef+egx)}{eg}-qx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x))^q],x]
```

```
[Out] -(q*x) + (d*q*Log[d + e*f + e*g*x])/(e*g) + ((f + g*x)*Log[c*(d + e*(f + g*
x))^q])/g
```

Maple [A] time = 0.122, size = 57, normalized size = 1.6

$$\ln\left(c\left(egx+fe+d\right)^{q}\right)x-qx+\frac{q\ln\left(egx+fe+d\right)f}{g}+\frac{q\ln\left(egx+fe+d\right)d}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+(g*x+f)*e)^q),x)
```

```
[Out] ln(c*(e*g*x+e*f+d)^q)*x-q*x+q/g*ln(e*g*x+e*f+d)*f+q/e/g*ln(e*g*x+e*f+d)*d
```

Maxima [A] time = 1.01673, size = 73, normalized size = 2.09

$$-egq\left(\frac{x}{eg}-\frac{\left(ef+d\right)\log\left(egx+ef+d\right)}{e^{2}g^{2}}\right)+x\log\left(\left(\left(gx+f\right)e+d\right)^{q}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="maxima")
```

```
[Out] -e*g*q*(x/(e*g) - (e*f + d)*log(e*g*x + e*f + d)/(e^2*g^2)) + x*log(((g*x +
 f)*e + d)^q*c)
```

Fricas [A] time = 1.50005, size = 108, normalized size = 3.09

$$-\frac{egqx-egx\log\left(c\right)-\left(egqx+\left(ef+d\right)q\right)\log\left(egx+ef+d\right)}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="fricas")
```

```
[Out] -(e*g*q*x - e*g*x*log(c) - (e*g*q*x + (e*f + d)*q)*log(e*g*x + e*f + d))/(e
*g)
```

Sympy [A] time = 1.51682, size = 85, normalized size = 2.43

$$\begin{cases} x\log\left(cd^{q}\right) & \text{for}\: e=0\wedge g=0 \\ x\log\left(c\left(d+ef\right)^{q}\right) & \text{for}\: g=0 \\ x\log\left(cd^{q}\right) & \text{for}\: e=0 \\ \frac{dq\log\left(d+ef+egx\right)}{eg}+\frac{fq\log\left(d+ef+egx\right)}{g}+qx\log\left(d+ef+egx\right)-qx+x\log\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f))**q),x)
```

```
[Out] Piecewise((x*log(c*d**q), Eq(e, 0) & Eq(g, 0)), (x*log(c*(d + e*f)**q), Eq(
g, 0)), (x*log(c*d**q), Eq(e, 0)), (d*q*log(d + e*f + e*g*x)/(e*g) + f*q*lo
g(d + e*f + e*g*x)/g + q*x*log(d + e*f + e*g*x) - q*x + x*log(c), True))
```

Giac [A] time = 1.17378, size = 93, normalized size = 2.66

$$\frac{(gxe + fe + d)qe^{(-1)}\log(gxe + fe + d)}{g} - \frac{(gxe + fe + d)qe^{(-1)}}{g} + \frac{(gxe + fe + d)e^{(-1)}\log(c)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="giac")
```

```
[Out] (g*x*e + f*e + d)*q*e^(-1)*log(g*x*e + f*e + d)/g - (g*x*e + f*e + d)*q*e^(
-1)/g + (g*x*e + f*e + d)*e^(-1)*log(c)/g
```

3.632 $\int \log \left(c \left(d+\frac{e}{f+gx}\right)^q\right) dx$

Optimal. Leaf size=45

$$\frac{(f+gx)\log \left(c \left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log (d(f+gx)+e)}{dg}$$

```
[Out] ((f + g*x)*Log[c*(d + e/(f + g*x))^q])/g + (e*q*Log[e + d*(f + g*x)])/(d*g)
```

Rubi [A] time = 0.0267869, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2483, 2448, 263, 31}

$$\frac{(f+gx)\log \left(c \left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log (d(f+gx)+e)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] ((f + g*x)*Log[c*(d + e/(f + g*x))^q])/g + (e*q*Log[e + d*(f + g*x)])/(d*g)
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x}\right)^q\right)dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g} + \frac{(eq)\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x}dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g} + \frac{(eq)\,\text{Subst}\left(\int \frac{1}{e+dx}dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g} + \frac{eq\log(e+d(f+gx))}{dg}$$

Mathematica [A] time = 0.0468237, size = 56, normalized size = 1.24

$$\frac{dgx\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)+q(df+e)\log(df+dgx+e)-dfq\log(f+gx)}{dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] (-(d*f*q*Log[f + g*x]) + (e + d*f)*q*Log[e + d*f + d*g*x] + d*g*x*Log[c*(d
+ e/(f + g*x))^q])/(d*g)
```

Maple [A] time = 0.131, size = 74, normalized size = 1.6

$$\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^q\right)x-\frac{qf\ln(gx+f)}{g}+\frac{q\ln(dgx+df+e)f}{g}+\frac{eq\ln(dgx+df+e)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f))^q),x)
```

```
[Out] ln(c*((d*g*x+d*f+e)/(g*x+f))^q)*x-1/g*q*f*ln(g*x+f)+1/g*q*ln(d*g*x+d*f+e)*f
+1/g*e*q/d*ln(d*g*x+d*f+e)
```

Maxima [A] time = 1.02127, size = 88, normalized size = 1.96

$$-egq\left(\frac{f\log(gx+f)}{eg^2}-\frac{(df+e)\log(dgx+df+e)}{deg^2}\right)+x\log\left(c\left(d+\frac{e}{gx+f}\right)^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="maxima")
```

```
[Out] -e*g*q*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2))
+ x*log(c*(d + e/(g*x + f))^q)
```

Fricas [A] time = 1.602, size = 163, normalized size = 3.62

$$\frac{dgqx\log\left(\frac{dgx+df+e}{gx+f}\right)-dfq\log\left(gx+f\right)+dgx\log\left(c\right)+\left(df+e\right)q\log\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="fricas")
```

```
[Out] (d*g*q*x*log((d*g*x + d*f + e)/(g*x + f)) - d*f*q*log(g*x + f) + d*g*x*log(
c) + (d*f + e)*q*log(d*g*x + d*f + e))/(d*g)
```

Sympy [A] time = 2.37966, size = 109, normalized size = 2.42

$$\begin{cases} x\log\left(c\left(\frac{e}{f}\right)^{q}\right) & \text{for}\: d=0\wedge g=0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^{q}\right) & \text{for}\: g=0 \\ -\frac{fq\log\left(f+gx\right)}{g}+qx\log\left(e\right)-qx\log\left(f+gx\right)+qx+x\log\left(c\right) & \text{for}\: d=0 \\ \frac{fq\log\left(d+\frac{e}{f+gx}\right)}{g}+qx\log\left(d+\frac{e}{f+gx}\right)+x\log\left(c\right)+\frac{eq\log\left(df+dgx+e\right)}{dg} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f))**q),x)
```

```
[Out] Piecewise((x*log(c*(e/f)**q), Eq(d, 0) & Eq(g, 0)), (x*log(c*(d + e/f)**q),
 Eq(g, 0)), (-f*q*log(f + g*x)/g + q*x*log(e) - q*x*log(f + g*x) + q*x + x*
log(c), Eq(d, 0)), (f*q*log(d + e/(f + g*x))/g + q*x*log(d + e/(f + g*x)) +
 x*log(c) + e*q*log(d*f + d*g*x + e)/(d*g), True))
```

Giac [A] time = 1.30012, size = 116, normalized size = 2.58

$$\frac{dgqx\log\left(dgx+df+e\right)-dgqx\log\left(gx+f\right)+dfq\log\left(dgx+df+e\right)-dfq\log\left(-gx-f\right)+dgx\log\left(c\right)+qe\log\left(dgx+\right.}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="giac")
```

```
[Out] (d*g*q*x*log(d*g*x + d*f + e) - d*g*q*x*log(g*x + f) + d*f*q*log(d*g*x + d*
f + e) - d*f*q*log(-g*x - f) + d*g*x*log(c) + q*e*log(d*g*x + d*f + e))/(d*
g)
```

3.633 $\int \log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right) \, dx$

Optimal. Leaf size=59

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{2\sqrt{e}q\tan^{-1}\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}g}$$

```
[Out] (2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/(Sqrt[d]*g) + ((f + g*x)*
Log[c*(d + e/(f + g*x)^2)^q])/g
```

Rubi [A] time = 0.0356361, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2483, 2448, 263, 205}

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{2\sqrt{e}q\tan^{-1}\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}g}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x)^2)^q],x]
```

```
[Out] (2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/(Sqrt[d]*g) + ((f + g*x)*
Log[c*(d + e/(f + g*x)^2)^q])/g
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x^2}\right)^q\right)dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g} + \frac{(2eq)\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^2}\right)x^2}dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g} + \frac{(2eq)\,\text{Subst}\left(\int \frac{1}{e+dx^2}dx, x, f+gx\right)}{g}$$

$$= \frac{2\sqrt{e}q\tan^{-1}\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}g} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

Mathematica [A] time = 0.0468646, size = 61, normalized size = 1.03

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g} - \frac{2\sqrt{e}q\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}(f+gx)}\right)}{\sqrt{d}g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x)^2)^q],x]
```

```
[Out] (-2*Sqrt[e]*q*ArcTan[Sqrt[e]/(Sqrt[d]*(f + g*x))])/(Sqrt[d]*g) + ((f + g*x)
*Log[c*(d + e/(f + g*x)^2)^q])/g
```

Maple [B] time = 0.133, size = 115, normalized size = 2.

$$\ln\left(c\left(\frac{dg^2x^2+2dfgx+df^2+e}{(gx+f)^2}\right)^q\right)x - 2\frac{qf\ln(gx+f)}{g} + \frac{qf\ln\left(dg^2x^2+2dfgx+df^2+e\right)}{g} + 2\frac{eq}{g\sqrt{de}}\arctan\left(1/2\frac{2dg^2\ldots}{g\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f)^2)^q),x)
```

```
[Out] ln(c*((d*g^2*x^2+2*d*f*g*x+d*f^2+e)/(g*x+f)^2)^q)*x-2/g*q*f*ln(g*x+f)+1/g*q
*f*ln(d*g^2*x^2+2*d*f*g*x+d*f^2+e)+2/g*e*q/(d*e)^(1/2)*arctan(1/2*(2*d*g^2*
x+2*d*f*g)/g/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7431, size = 660, normalized size = 11.19

$$\left[\frac{gqx\log\left(\frac{dg^2x^2+2\,dfgx+df^2+e}{g^2x^2+2\,fgx+f^2}\right)+fq\log\left(dg^2x^2+2\,dfgx+df^2+e\right)-2\,fq\log\left(gx+f\right)+gx\log\left(c\right)+q\sqrt{-\frac{e}{d}}\log\left(\frac{dg^2x^2+2}{\cdots}\right.}{g}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [(g*q*x*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2))
+ f*q*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log
(c) + q*sqrt(-e/d)*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + 2*(d*g*x + d*f)*sqr
t(-e/d) - e)/(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)))/g, (g*q*x*log((d*g^2*x^2
 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2)) + f*q*log(d*g^2*x^2 +
2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log(c) + 2*q*sqrt(e/d)*ar
ctan((d*g*x + d*f)*sqrt(e/d)/e))/g]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f)**2)**q),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.57269, size = 185, normalized size = 3.14

$$dg^4q\left(\frac{fe^{(-1)}\log\left(dg^2x^2+2\,dfgx+df^2+e\right)}{dg^5}-\frac{2\,fe^{(-1)}\log\left(|gx+f|\right)}{dg^5}+\frac{2\,\arctan\left(\frac{(dgx+df)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{d^{\frac{3}{2}}g^5}\right)e+qx\log\left(dg^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] d*g^4*q*(f*e^(-1)*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(d*g^5) - 2*f*e^(-
1)*log(abs(g*x + f))/(d*g^5) + 2*arctan((d*g*x + d*f)*e^(-1/2)/sqrt(d))*e^(
-1/2)/(d^(3/2)*g^5))*e + q*x*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - q*x*l
og(g^2*x^2 + 2*f*g*x + f^2) + x*log(c)
```

3.634 $\int \log \left(c \left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx$

Optimal. Leaf size=165

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}-\frac{\sqrt[3]{e}q\log\left(d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}\right)}{2\sqrt[3]{d}g}+\frac{\sqrt[3]{e}q\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{\sqrt[3]{d}g}-\frac{\sqrt{3}\sqrt[3]{e}q}{}$$

```
[Out] -((Sqrt[3]*e^(1/3)*q*ArcTan[(e^(1/3) - 2*d^(1/3)*(f + g*x))/(Sqrt[3]*e^(1/3
))])/(d^(1/3)*g)) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g + (e^(1/3)*q
*Log[e^(1/3) + d^(1/3)*(f + g*x)])/(d^(1/3)*g) - (e^(1/3)*q*Log[e^(2/3) - d
^(1/3)*e^(1/3)*(f + g*x) + d^(2/3)*(f + g*x)^2])/(2*d^(1/3)*g)
```

Rubi [A] time = 0.172958, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2483, 2448, 263, 200, 31, 634, 617, 204, 628}

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}-\frac{\sqrt[3]{e}q\log\left(d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}\right)}{2\sqrt[3]{d}g}+\frac{\sqrt[3]{e}q\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{\sqrt[3]{d}g}-\frac{\sqrt{3}\sqrt[3]{e}q}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x)^3)^q],x]
```

```
[Out] -((Sqrt[3]*e^(1/3)*q*ArcTan[(e^(1/3) - 2*d^(1/3)*(f + g*x))/(Sqrt[3]*e^(1/3
))])/(d^(1/3)*g)) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g + (e^(1/3)*q
*Log[e^(1/3) + d^(1/3)*(f + g*x)])/(d^(1/3)*g) - (e^(1/3)*q*Log[e^(2/3) - d
^(1/3)*e^(1/3)*(f + g*x) + d^(2/3)*(f + g*x)^2])/(2*d^(1/3)*g)
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)dx &= \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x^3}\right)^q\right)dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{(3eq)\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^3}\right)x^3}dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{(3eq)\,\text{Subst}\left(\int \frac{1}{e+dx^3}dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{\left(\sqrt[3]{e}q\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{e}+\sqrt[3]{d}x}dx, x, f+gx\right)}{g}+\frac{\left(\sqrt[3]{e}q\right)\text{Su}}{}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}g}-\frac{\left(\sqrt[3]{e}q\right)\text{Subst}\left(\int \frac{-}{e^{2/3}}\right.}{}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}g}-\frac{\sqrt[3]{e}q\log\left(e^{2/3}-\sqrt[3]{d}\sqrt{}\right.}{}\\
&= -\frac{\sqrt{3}\sqrt[3]{e}q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}}{\sqrt{3}}\right)}{\sqrt[3]{d}g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(\right.}{\sqrt[3]{d}g}
\end{aligned}$$

Mathematica [C] time = 0.333291, size = 66, normalized size = 0.4

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}-\frac{3eq\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{e}{d(f+gx)^3}\right)}{2dg(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x)^3)^q],x]
```

```
[Out] (-3*e*q*Hypergeometric2F1[2/3, 1, 5/3, -(e/(d*(f + g*x)^3))])/(2*d*g*(f + g
*x)^2) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g
```

Maple [C] time = 0.554, size = 157, normalized size = 1.

$$\ln\left(c\left(\frac{dg^3x^3+3dfg^2x^2+3df^2gx+df^3+e}{(gx+f)^3}\right)^q\right)x-3\frac{qf\ln(gx+f)}{g}+\frac{q}{dg}\sum_{\_R=\mathrm{RootOf}(dg^3\_Z^3+3dfg^2\_Z^2+3df^2g\_Z+df^3+e)}\frac{(\_R^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f)^3)^q),x)
```

```
[Out] ln(c*((d*g^3*x^3+3*d*f*g^2*x^2+3*d*f^2*g*x+d*f^3+e)/(g*x+f)^3)^q)*x-3/g*q*f
*ln(g*x+f)+1/g*q/d*sum((_R^2*d*f*g^2+2*_R*d*f^2*g+d*f^3+e)/(_R^2*g^2+2*_R*f
*g+f^2)*ln(x-_R),_R=RootOf(_Z^3*d*g^3+3*_Z^2*d*f*g^2+3*_Z*d*f^2*g+d*f^3+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$3q\int\frac{dfg^2x^2+2df^2gx+df^3+e}{dg^3x^3+3dfg^2x^2+3df^2gx+df^3+e}\,dx-\frac{3fq\log(gx+f)-gx\log\left((dg^3x^3+3dfg^2x^2+3df^2gx+df^3+e)^q\right)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="maxima")
```

```
[Out] 3*q*integrate((d*f*g^2*x^2 + 2*d*f^2*g*x + d*f^3 + e)/(d*g^3*x^3 + 3*d*f*g^
2*x^2 + 3*d*f^2*g*x + d*f^3 + e), x) - (3*f*q*log(g*x + f) - g*x*log((d*g^3
*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)^q) + 3*g*x*log((g*x + f)^q)
 - g*x*log(c))/g
```

Fricas [C] time = 9.93869, size = 3146, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log((d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)/(g^3
*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)) - 4*sqrt(3)*g*sqrt((((-1/2*f^3*q^3/g
```

```
^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3)
+ 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f
^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2
)/g^2)*arctan(-1/24*(2*sqrt(3)*sqrt(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f
^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f
^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) - 2*f*q/g))*(((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f
^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*d*g^2 + 2*d*f*g*q
)*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d
*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*
e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*
f*q/g)*f*g*q + 4*f^2*q^2)/g^2) - sqrt(3)*(8*d*f*g^2*q^2*x + ((-1/2*f^3*q^3/
g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*f*q/g)^2*d*g^3 + 12*d*f^2*g*q^2 + 4*(d*g^3*q*x + 2*d*f*g^2*q)*((-
1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3
)*(I*sqrt(3) + 1) - 2*f*q/g))*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3)
+ 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 +
 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3)
)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2))/(e*q^3)) - 12*f
*q*log(g*x + f) - 2*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3
 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*log(q*x - 1/2*(-1/2*f
^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g
^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g +
 6*f*q)*log(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(
d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^
2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(
d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g))
)/g
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f)**3)**q),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="giac")

[Out] Timed out
```

3.635 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n,x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

Rubi [A] time = 0.0059268, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
```

```
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$$

Mathematica [A] time = 0.406539, size = 0, normalized size = 0.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

Maple [A] time = 0.659, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

```
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^{p}\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

3.636 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx$

Optimal. Leaf size=221

$$\frac{24b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}-\frac{12b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right.\right.}{dg}$$

```
[Out] (-4*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^4)/(d*g) - (12*b
^2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])^2*PolyLog[2, 1 + e/(d*(f + g*x)
)])/(d*g) + (24*b^3*e*p^3*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[3, 1 +
 e/(d*(f + g*x))])/(d*g) - (24*b^4*e*p^4*PolyLog[4, 1 + e/(d*(f + g*x))])/(
d*g)
```

Rubi [A] time = 0.27778, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2483, 2449, 2454, 2396, 2433, 2374, 2383, 6589}

$$\frac{24b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}-\frac{12b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right.\right.}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```


```
[Out] (-4*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^4)/(d*g) - (12*b
^2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])^2*PolyLog[2, 1 + e/(d*(f + g*x)
)])/(d*g) + (24*b^3*e*p^3*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[3, 1 +
 e/(d*(f + g*x))])/(d*g) - (24*b^4*e*p^4*PolyLog[4, 1 + e/(d*(f + g*x))])/(
d*g)
```


Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^4 dx, x, f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} + \frac{(4bep)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^3}{x}\right.}{dg}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{(4bep)\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^p))^3}{x}\right.}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

Mathematica [B] time = 1.36468, size = 739, normalized size = 3.34

$$\frac{4b^3p^3\left(6e\text{PolyLog}\left(3,\frac{e}{df+dgx}+1\right)-6e\log\left(d+\frac{e}{f+gx}\right)\text{PolyLog}\left(2,\frac{e}{df+dgx}+1\right)+\left((df+dgx+e)\log\left(d+\frac{e}{f+gx}\right)-3e\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

```
[Out] (-4*b*p*(d*f*Log[f + g*x] - (e + d*f)*Log[e + d*f + d*g*x] - d*g*x*Log[(e +
 d*f + d*g*x)/(f + g*x)])*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f
 + g*x))^p])^3 + d*g*x*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
g*x))^p])^4 + 6*b^2*p^2*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
 g*x))^p])^2*(2*d*f*Log[-(e/(d*f + d*g*x))]*Log[(e + d*f + d*g*x)/(f + g*x)
] + 2*(e + d*f)*Log[e + d*f + d*g*x]*Log[(e + d*f + d*g*x)/(f + g*x)] + d*g
*x*Log[(e + d*f + d*g*x)/(f + g*x)]^2 - d*f*(Log[-(e/(d*f + d*g*x))]*(Log[-
(e/(d*f + d*g*x))] + 2*Log[(e + d*f + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g
*x))/e)]) + (e + d*f)*((2*Log[-((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*L
og[e + d*f + d*g*x] + 2*PolyLog[2, (e + d*f + d*g*x)/e])) + 4*b^3*p^3*(a -
b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(Log[d + e/(f + g*
x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*f + d*g*x)*Log[d + e/(f + g*x)
]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 + e/(d*f + d*g*x)] + 6*e*PolyLog
[3, 1 + e/(d*f + d*g*x)]) - b^4*p^4*(4*e*Log[-(e/(d*f + d*g*x))]*Log[d + e/
```

```
(f + g*x)]^3 - e*Log[d + e/(f + g*x)]^4 - d*f*Log[d + e/(f + g*x)]^4 - d*g*
x*Log[d + e/(f + g*x)]^4 + 12*e*Log[d + e/(f + g*x)]^2*PolyLog[2, 1 + e/(d*
f + d*g*x)] - 24*e*Log[d + e/(f + g*x)]*PolyLog[3, 1 + e/(d*f + d*g*x)] + 2
4*e*PolyLog[4, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Maple [F] time = 0.599, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)

[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="maxima")

[Out] -4*a^3*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*
e*g^2)) + 4*a^3*b*x*log(c*(d + e/(g*x + f))^p) + a^4*x + (b^4*d*g*x*log((d*
g*x + d*f + e)^p)^4 - 4*(b^4*d*f*p*log(g*x + f) + b^4*d*g*x*log((g*x + f)^p
) - (d*f*p + e*p)*b^4*log(d*g*x + d*f + e) - (b^4*d*g*log(c) + a*b^3*d*g)*x
)*log((d*g*x + d*f + e)^p)^3)/(d*g) + integrate(((d*f + e)*b^4*log(c)^4 + 4
*(d*f + e)*a*b^3*log(c)^3 + 6*(d*f + e)*a^2*b^2*log(c)^2 + (b^4*d*g*x + (d*
f + e)*b^4)*log((g*x + f)^p)^4 - 4*((d*f + e)*b^4*log(c) + (d*f + e)*a*b^3
+ (b^4*d*g*log(c) + a*b^3*d*g)*x)*log((g*x + f)^p)^3 + 6*(2*b^4*d*f*p^2*log
(g*x + f) + (d*f + e)*b^4*log(c)^2 - 2*(d*f*p^2 + e*p^2)*b^4*log(d*g*x + d*
f + e) + 2*(d*f + e)*a*b^3*log(c) + (d*f + e)*a^2*b^2 + (b^4*d*g*x + (d*f +
 e)*b^4)*log((g*x + f)^p)^2 + (a^2*b^2*d*g - 2*(d*g*p - d*g*log(c))*a*b^3 -
 (2*d*g*p*log(c) - d*g*log(c)^2)*b^4)*x - 2*((d*f + e)*b^4*log(c) + (d*f +
e)*a*b^3 + (a*b^3*d*g - (d*g*p - d*g*log(c))*b^4)*x)*log((g*x + f)^p))*log(
(d*g*x + d*f + e)^p)^2 + 6*((d*f + e)*b^4*log(c)^2 + 2*(d*f + e)*a*b^3*log(
c) + (d*f + e)*a^2*b^2 + (b^4*d*g*log(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d
*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^4 + 4*a*b^3*d*g*log(c)^3 + 6*a^
2*b^2*d*g*log(c)^2)*x + 4*((d*f + e)*b^4*log(c)^3 + 3*(d*f + e)*a*b^3*log(c
)^2 + 3*(d*f + e)*a^2*b^2*log(c) - (b^4*d*g*x + (d*f + e)*b^4)*log((g*x + f
)^p)^3 + 3*((d*f + e)*b^4*log(c) + (d*f + e)*a*b^3 + (b^4*d*g*log(c) + a*b^
3*d*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^3 + 3*a*b^3*d*g*log(c)^2 + 3
*a^2*b^2*d*g*log(c))*x - 3*((d*f + e)*b^4*log(c)^2 + 2*(d*f + e)*a*b^3*log(
c) + (d*f + e)*a^2*b^2 + (b^4*d*g*log(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d
*g)*x)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p) - 4*((d*f + e)*b^4*log(c)
^3 + 3*(d*f + e)*a*b^3*log(c)^2 + 3*(d*f + e)*a^2*b^2*log(c) + (b^4*d*g*log
(c)^3 + 3*a*b^3*d*g*log(c)^2 + 3*a^2*b^2*d*g*log(c))*x)*log((g*x + f)^p))/(
d*g*x + d*f + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^4 \log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^4 + 4\,ab^3 \log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^3 + 6\,a^2b^2 \log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2 + 4\,a^3b \log\left(c\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="fricas")
```

```
[Out] integral(b^4*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^4 + 4*a*b^3*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^3 + 6*a^2*b^2*log(c*((d*g*x + d*f + e)/(g*x + f
))^p)^2 + 4*a^3*b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^4, x)
```

3.637 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3\,dx$

Optimal. Leaf size=168

$$-\frac{6b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{6b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)}{}$$

```
[Out] (-3*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*g) - (6*b^
2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[2, 1 + e/(d*(f + g*x))])
/(d*g) + (6*b^3*e*p^3*PolyLog[3, 1 + e/(d*(f + g*x))])/(d*g)
```

Rubi [A] time = 0.184101, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2483, 2449, 2454, 2396, 2433, 2374, 6589}

$$-\frac{6b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{6b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```


```
[Out] (-3*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*g) - (6*b^
2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[2, 1 + e/(d*(f + g*x))])
/(d*g) + (6*b^3*e*p^3*PolyLog[3, 1 + e/(d*(f + g*x))])/(d*g)
```


Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^3 dx, x, f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(3bep)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^2}{x}\right.}{dg}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} - \frac{(3bep)\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^p))^2}{x}\right.}{dg}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\right.\right.}{dg}$$

Mathematica [B] time = 0.686147, size = 415, normalized size = 2.47

$$3b^2p^2\left(2e\text{PolyLog}\left(2,\frac{d(f+gx)}{e}+1\right)+d(f+gx)\log^2\left(d+\frac{e}{f+gx}\right)+e\left(2\log\left(-\frac{d(f+gx)}{e}\right)-\log(df+dgx+e)+2\log\left(d+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

```
[Out] (3*b*d*p*(f + g*x)*Log[d + e/(f + g*x)]*(a - b*p*Log[d + e/(f + g*x)] + b*L
og[c*(d + e/(f + g*x))^p])^2 + d*(f + g*x)*(a - b*p*Log[d + e/(f + g*x)] +
b*Log[c*(d + e/(f + g*x))^p])^3 + 3*b*e*p*(a - b*p*Log[d + e/(f + g*x)] + b
*Log[c*(d + e/(f + g*x))^p])^2*Log[e + d*(f + g*x)] + 3*b^2*p^2*(a - b*p*Lo
g[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(d*(f + g*x)*Log[d + e/(
f + g*x)]^2 + e*(2*Log[-((d*(f + g*x))/e)] - Log[e + d*f + d*g*x] + 2*Log[d
 + e/(f + g*x)])*Log[e + d*(f + g*x)] + 2*e*PolyLog[2, 1 + (d*(f + g*x))/e]
) + b^3*p^3*(Log[d + e/(f + g*x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*
f + d*g*x)*Log[d + e/(f + g*x)]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 +
e/(d*f + d*g*x)] + 6*e*PolyLog[3, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int\left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*
e*g^2)) + 3*a^2*b*x*log(c*(d + e/(g*x + f))^p) + a^3*x + (b^3*d*g*x*log((d*
g*x + d*f + e)^p)^3 - 3*(b^3*d*f*p*log(g*x + f) + b^3*d*g*x*log((g*x + f)^p
) - (d*f*p + e*p)*b^3*log(d*g*x + d*f + e) - (b^3*d*g*log(c) + a*b^2*d*g)*x
)*log((d*g*x + d*f + e)^p)^2)/(d*g) + integrate(((d*f + e)*b^3*log(c)^3 + 3
*(d*f + e)*a*b^2*log(c)^2 - (b^3*d*g*x + (d*f + e)*b^3)*log((g*x + f)^p)^3
+ 3*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^2 + (b^3*d*g*log(c) + a*b^2*d*g)*
x)*log((g*x + f)^p)^2 + (b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(c)^2)*x + 3*(2*
b^3*d*f*p^2*log(g*x + f) + (d*f + e)*b^3*log(c)^2 - 2*(d*f*p^2 + e*p^2)*b^3
*log(d*g*x + d*f + e) + 2*(d*f + e)*a*b^2*log(c) + (b^3*d*g*x + (d*f + e)*b
^3)*log((g*x + f)^p)^2 - (2*(d*g*p - d*g*log(c))*a*b^2 + (2*d*g*p*log(c) -
d*g*log(c)^2)*b^3)*x - 2*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^2 + (a*b^2*d
*g - (d*g*p - d*g*log(c))*b^3)*x)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p
```

```
) - 3*((d*f + e)*b^3*log(c)^2 + 2*(d*f + e)*a*b^2*log(c) + (b^3*d*g*log(c)^
2 + 2*a*b^2*d*g*log(c))*x)*log((g*x + f)^p))/(d*g*x + d*f + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^3+3\,ab^2\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2+3\,a^2b\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^3 + 3*a*b^2*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^2 + 3*a^2*b*log(c*((d*g*x + d*f + e)/(g*x + f))
^p) + a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^3, x)
```

3.638 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx$

Optimal. Leaf size=115

$$-\frac{2b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d\cdots\right.\right.\right.}{dg}$$

```
[Out] (-2*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p]))/(d*g)
 + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*g) - (2*b^2*
e*p^2*PolyLog[2, 1 + e/(d*(f + g*x))])/(d*g)
```

Rubi [A] time = 0.0900471, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2483, 2449, 2454, 2394, 2315}

$$-\frac{2b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d\cdots\right.\right.\right.}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] (-2*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p]))/(d*g)
 + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*g) - (2*b^2*
e*p^2*PolyLog[2, 1 + e/(d*(f + g*x))])/(d*g)
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx=\frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^2 dx,x,f+gx\right)}{g}$$

$$=\frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}+\frac{(2bep)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{x}dx\right.}{dg}$$

$$=\frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}-\frac{(2bep)\,\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^p)}{x}dx,\right.}{dg}$$

$$=-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(e+d(f+gx))\left(a+b\log\left(c\left(\right.\right.\right.}{dg}$$

$$=-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(e+d(f+gx))\left(a+b\log\left(c\left(\right.\right.\right.}{dg}$$

Mathematica [A] time = 0.270018, size = 219, normalized size = 1.9

$$x\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2-\frac{bp\left(bdfp\left(\log(f+gx)\left(\log(f+gx)-2\log\left(\frac{df+dgx+e}{e}\right)\right)-2\text{PolyLog}\left(2,-\frac{d(f+gx)}{e}\right)\right)-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/(f + g*x))^p])^2 - (b*p*(2*d*f*Log[f + g*x]*(a + b*Lo
g[c*(d + e/(f + g*x))^p]) - 2*(e + d*f)*(a + b*Log[c*(d + e/(f + g*x))^p])*
Log[e + d*(f + g*x)] + b*d*f*p*(Log[f + g*x]*(Log[f + g*x] - 2*Log[(e + d*f
 + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g*x))/e)]) - b*(e + d*f)*p*((2*Log[-
((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*Log[e + d*(f + g*x)] + 2*PolyLog
[2, (e + d*f + d*g*x)/e])))/(d*g)
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,abegp\left(\frac{f\log\left(gx+f\right)}{eg^2}-\frac{\left(df+e\right)\log\left(dgx+df+e\right)}{deg^2}\right)+2\,abx\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a^2x+b^2\left(\frac{dgx\log\left(\left(dgx+df\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*
g^2)) + 2*a*b*x*log(c*(d + e/(g*x + f))^p) + a^2*x + b^2*((d*g*x*log((d*g*x
 + d*f + e)^p)^2 + d*g*x*log((g*x + f)^p)^2 - (d*f*p^2 + e*p^2)*log(d*g*x +
 d*f + e)^2 + 2*(d*f*p^2 + e*p^2)*log(d*g*x + d*f + e)*log(g*x + f) - 2*(d*
f*p*log(g*x + f) + d*g*x*log((g*x + f)^p) - d*g*x*log(c) - (d*f*p + e*p)*lo
g(d*g*x + d*f + e))*log((d*g*x + d*f + e)^p) + 2*(d*f*p*log(g*x + f) - d*g*
x*log(c) - (d*f*p + e*p)*log(d*g*x + d*f + e))*log((g*x + f)^p))/(d*g) - in
tegrate(-(d*g^2*x^2*log(c)^2 + (d*f^2 + e*f)*log(c)^2 + (2*e*g*p*log(c) + (
2*d*f*g + e*g)*log(c)^2)*x - 2*(d*f^2*p^2 + 2*e*f*p^2 + (d*f*g*p^2 + e*g*p^
2)*x)*log(g*x + f))/(d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x), x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2+2\,ab\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*g*x
 + d*f + e)/(g*x + f))^p) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^2, x)
```

3.639 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx$

Optimal. Leaf size=50

$$ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(d(f+gx)+e)}{dg}$$

```
[Out] a*x + (b*(f + g*x)*Log[c*(d + e/(f + g*x))^p])/g + (b*e*p*Log[e + d*(f + g*
x)])/(d*g)
```

Rubi [A] time = 0.0376744, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2483, 2448, 263, 31}

$$ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(d(f+gx)+e)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

```
[Out] a*x + (b*(f + g*x)*Log[c*(d + e/(f + g*x))^p])/g + (b*e*p*Log[e + d*(f + g*
x)])/(d*g)
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = ax+b\int \log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)dx$$
$$= ax+\frac{b\,\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x}\right)^p\right)dx, x, f+gx\right)}{g}$$
$$= ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{(bep)\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x}dx, x, f+gx\right)}{g}$$
$$= ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{(bep)\,\text{Subst}\left(\int \frac{1}{e+dx}dx, x, f+gx\right)}{g}$$
$$= ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(e+d(f+gx))}{dg}$$

Mathematica [A] time = 0.0408769, size = 70, normalized size = 1.4

$$ax+bx\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)-begp\left(\frac{f\log(f+gx)}{eg^2}-\frac{(df+e)\log(df+dgx+e)}{deg^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

```
[Out] a*x - b*e*g*p*((f*Log[f + g*x])/(e*g^2) - ((e + d*f)*Log[e + d*f + d*g*x])/
(d*e*g^2)) + b*x*Log[c*(d + e/(f + g*x))^p]
```

Maple [A] time = 0.106, size = 81, normalized size = 1.6

$$ax+b\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)x-\frac{bpf\ln\left(gx+f\right)}{g}+\frac{bp\ln\left(dgx+df+e\right)f}{g}+\frac{ebp\ln\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/(g*x+f))^p),x)
```

```
[Out] a*x+b*ln(c*((d*g*x+d*f+e)/(g*x+f))^p)*x-b/g*p*f*ln(g*x+f)+b/g*p*ln(d*g*x+d*
f+e)*f+b*e/g*p/d*ln(d*g*x+d*f+e)
```

Maxima [A] time = 1.02494, size = 95, normalized size = 1.9

$$-begp\left(\frac{f\log\left(gx+f\right)}{eg^2}-\frac{\left(df+e\right)\log\left(dgx+df+e\right)}{deg^2}\right)+bx\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="maxima")
```

```
[Out] -b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2)
) + b*x*log(c*(d + e/(g*x + f))^p) + a*x
```

Fricas [A] time = 1.94278, size = 190, normalized size = 3.8

$$\frac{bdgpx\log\left(\frac{dgx+df+e}{gx+f}\right) - bdfp\log\left(gx+f\right) + bdgx\log\left(c\right) + adgx + \left(bdf+be\right)p\log\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="fricas")
```

```
[Out] (b*d*g*p*x*log((d*g*x + d*f + e)/(g*x + f)) - b*d*f*p*log(g*x + f) + b*d*g*
x*log(c) + a*d*g*x + (b*d*f + b*e)*p*log(d*g*x + d*f + e))/(d*g)
```

Sympy [A] time = 2.29712, size = 114, normalized size = 2.28

$$ax + b\left(\begin{cases} x\log\left(c\left(\frac{e}{f}\right)^{p}\right) & \text{for}\: d = 0 \wedge g = 0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^{p}\right) & \text{for}\: g = 0 \\ -\frac{fp\log\left(f+gx\right)}{g} + px\log\left(e\right) - px\log\left(f+gx\right) + px + x\log\left(c\right) & \text{for}\: d = 0 \\ \frac{fp\log\left(d+\frac{e}{f+gx}\right)}{g} + px\log\left(d+\frac{e}{f+gx}\right) + x\log\left(c\right) + \frac{ep\log\left(df+dgx+e\right)}{dg} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/(g*x+f))**p),x)
```

```
[Out] a*x + b*Piecewise((x*log(c*(e/f)**p), Eq(d, 0) & Eq(g, 0)), (x*log(c*(d + e
/f)**p), Eq(g, 0)), (-f*p*log(f + g*x)/g + p*x*log(e) - p*x*log(f + g*x) +
p*x + x*log(c), Eq(d, 0)), (f*p*log(d + e/(f + g*x))/g + p*x*log(d + e/(f +
 g*x)) + x*log(c) + e*p*log(d*f + d*g*x + e)/(d*g), True))
```

Giac [A] time = 1.31974, size = 123, normalized size = 2.46

$$ax + \frac{\left(dgpx\log\left(dgx+df+e\right) - dgpx\log\left(gx+f\right) + dfp\log\left(dgx+df+e\right) - dfp\log\left(-gx-f\right) + dgx\log\left(c\right) + pe\,\mathrm{lo}\right.}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="giac")
```

```
[Out] a*x + (d*g*p*x*log(d*g*x + d*f + e) - d*g*p*x*log(g*x + f) + d*f*p*log(d*g*
x + d*f + e) - d*f*p*log(-g*x - f) + d*g*x*log(c) + p*e*log(d*g*x + d*f + e
))*b/(d*g)
```

3.640 $\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

Rubi [A] time = 0.0059505, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]

[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Mathematica [A] time = 0.538266, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]

[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

Maple [A] time = 0.123, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

```
[Out] int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^{p}\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

3.641 $\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Rubi [A] time = 0.0057474, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Mathematica [A] time = 0.858967, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

```
[Out] int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{dg^2x^2 + df^2 + ef + (2\,dfg + eg)x}{b^2egp\log\left((dgx + df + e)^p\right) - b^2egp\log\left((gx + f)^p\right) + b^2egp\log(c) + abegp} - \int \frac{2\,dgx +}{b^2ep\log\left((dgx + df + e)^p\right) - b^2ep\,\mathrm{lo}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

```
[Out] (d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x)/(b^2*e*g*p*log((d*g*x + d*f +
 e)^p) - b^2*e*g*p*log((g*x + f)^p) + b^2*e*g*p*log(c) + a*b*e*g*p) - integ
rate((2*d*g*x + 2*d*f + e)/(b^2*e*p*log((d*g*x + d*f + e)^p) - b^2*e*p*log(
(g*x + f)^p) + b^2*e*p*log(c) + a*b*e*p), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^2\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2 + 2\,ab\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*
g*x + d*f + e)/(g*x + f))^p) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\log\left(c\left(d + \frac{e}{gx+f}\right)^p\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^(-2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```